PHYSICS & TECHNOLOGY

TARGET SKILLS

- Analyzing and interpreting
- Modelling concepts

Sound Technology in the Movies

Today's movies aim to give moviegoers a total sensory experience. This includes not only stunning visual special effects, but spectacular sound as well. If a plane in the distance on the lower left of the screen zooms overhead, sound engineers want viewers to hear the sound pass overhead, too, and fade away behind their right shoulder. How is this done?

The invention of two-track stereo sound was a first step. The action was recorded with two separated microphones and played back on speakers at each side of the screen. This allowed the sound to follow a car moving across the screen. The effect could be enhanced by using four tracks, with two extra speakers at the back of the theatre.

In the 1980s, the producers of the film *Earthquake* wanted moviegoers to feel the ground shake. In theatres specially fitted for the film, many large speakers were placed around the walls and under the seats. When the earthquake began, these speakers pumped out loud, low frequency sound that caused the seats and floor to vibrate.

The newest advance in sound technology, three-dimensional sound, does more than just record and reproduce sound. Instead, audio engineers attempt to generate the sound you would have heard had you been in the scene. Computers analyze and reproduce the tiny delays and echoes that occur when you hear a sound. Imagine that someone near you drops a wineglass. The ear slightly closer to the event hears the sound first. Sound rebounding off the ceiling comes to you from above. Echoes arrive from the back of the room a fraction of a second later, although the time lag is so small that you do not actually distinguish these echoes from the original crash. Higher-pitched sounds bounce off you and rebound from the front. Lower-pitched sounds flow around you, and you hear them from behind once again. All of these audio clues help you to sense that the glass shattered in front of you, slightly to the right, and below.

The physical principles governing reflection, diffraction, and interference are all used to reproduce three-dimensional sound correctly. This advanced audio technology is in use in theatre sound systems and quality headsets.

Analyze

1. What frequency might have been used in the movie *Earthquake* to produce a vibration of the floor but not make an audible note?
2. Explain how you are able to locate the source of a sound.
3. Describe how your life might be affected if you were unable to locate sound sources.

Interference Patterns in Water Waves

If plane waves pass through two openings in a barrier that are close together, diffraction of the waves creates a unique pattern. A similar pattern is created by two point sources, located close together, generating circular waves that are in phase. As the waves from the two sources meet, interference creates the distinctive pattern seen in Figure 7.26. This pattern results from alternating nodes and antinodes radiating outward from the sources. Although the situation is much like standing waves on a string, the pattern spreads over the two-dimensional water surface.

You can analyze, even predict, the pattern by considering several lines radiating out from the centre and then determining how waves will interfere along these lines. Start with the perpendicular bisector of a line connecting the two sources. Then, pick any point on that line and draw lines from each source to the point, as shown in Figure 7.27 (A). Notice that these lines form congruent triangles because the bases are equal, the angles are equal, and they share a side. Therefore, the lines from the sources are equal in length. Since the two sources are emitting waves in phase, when the two waves pass through each other at point *P*, they will be at exactly the same phase in their cycles. Figure 7.27 (B) shows two crests meeting at point *P*. They will add constructively, making the amplitude at that point double that of each individual wave. A moment later, troughs will meet. This makes a trough that is double the size of each individual trough. This process occurs at every point along this central line. Thus, every point on the surface will be oscillating maximally.

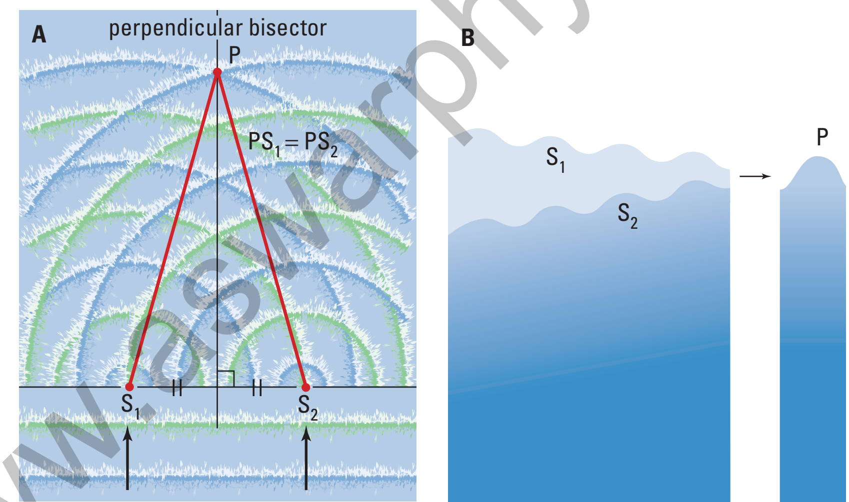
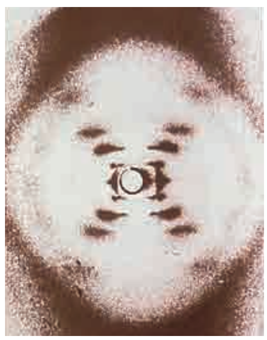


Figure 7.27 **(A)** Constructive interference occurs for a point on the perpendicular bisector because it is an equal distance from both sources. Since the sources are creating waves in phase, they will still be in phase at point *P*. Crests are represented in blue and troughs in green. **(B)** Looking from the side, you can see crests superimposed at point *P*.

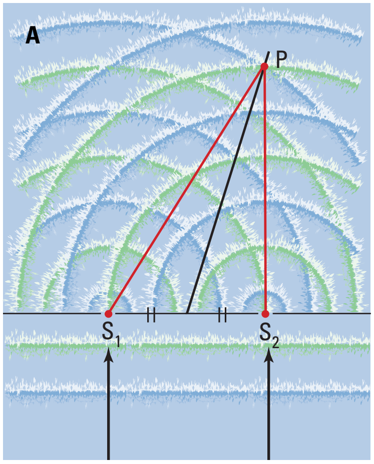
Figure 7.26 A series of antinodes run along the vertical line up the centre. The lines spreading out beside the antinodes are formed by nodes.

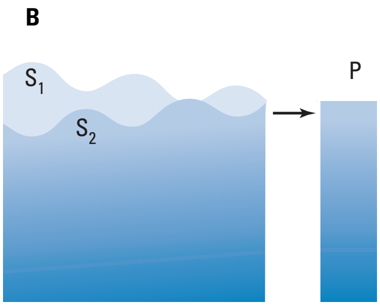
Biology Link

The phenomenon of diffraction provides an important tool in several fields of science. Rosalind Franklin obtained the pattern in the photograph shown here by passing X rays through a crystal of DNA. The pattern produced by the diffraction of X rays is shown on the film. This pattern provided James Watson and Francis Crick with key information that helped them discover the three-dimensional structure of DNA. Find out more about these scientists' amazing discoveries in your biology and chemistry courses, or go to print resources and the Internet for more.



Chemists and biochemists use X-ray diffraction to determine the structure of many crystals and biological molecules.





Next, consider a line radiating outward to the right (or left) of the centre line as shown in Figure 7.28 (A). This line is carefully chosen so that the difference in the distances from the sources to any point on this line is exactly one half of a wavelength. Again, Figure 7.28 (B) shows what you would see if you looked, from the side, at waves travelling along these lines. Notice that the waves that pass through each other at point P_1 on this line are exactly out of phase. No matter what stage of the cycle is passing this point, the waves will destructively interfere. The surface of the water along this line will not move. There are nodes all along the line, so it is called a **nodal line**. Although these lines appear straight in Figure 7.26, they are, in reality, slightly curved.

If you were to continue to draw lines radiating outward at greater and greater angles from the perpendicular bisector, you would reach points where the distance from one source would be one full wavelength longer than the other. Again, the waves would constructively interfere as they passed through each other, forming an **antinodal line**. When the distance to points on a line is 1.5 wavelengths, you would find another nodal line.

Figure 7.28 **(A)** Destructive interference occurs at points that are a distance $\frac{1}{2}\lambda$ farther from one source than the other, because the waves will always be half a wavelength out of phase when they reach this line. **(B)** Looking from the side, you can see the result of a crest superimposed on a trough at point *P*.

7.4 Section Review

1. **C** Sketch the wave produced by dipping a finger into water in a ripple tank. Add rays to your diagram to illustrate the directions of wave movement.

2. **I** Two circular waves are sent out from points about 15 cm apart in a ripple tank.
   (a) Sketch their appearance a short period of time after they have met.
   (b) What does this tell you about how the two waves have moved?

3. **I** Sketch the appearance of a straight wave after it has passed through a small opening in a straight barrier. Add rays to your diagram to illustrate the directions of wavefront movement.

4. **C** Sketch a typical interference pattern produced by two point sources vibrating in phase.

UNIT ISSUE PREP

Diffraction is an important concept to consider in a noise policy document.
- Have you ever heard sound waves diffract around barriers?
- What is the relationship between the amount of diffraction and the sound wave frequency?

CHAPTER 7 Review

REFLECTING ON CHAPTER 7

- Periodic motion occurs when an object moves in a repeated pattern (a cycle) over equal periods of time, T.
- The frequency of the motion, f, is the number of cycles completed in 1 s.
  $f = \frac{1}{T}$ and is measured in hertz (Hz):
  $$1\ \text{Hz} = \frac{1}{\text{s}} = \text{s}^{-1}$$
- The amplitude, A, of the vibration is the distance from the maximum displacement to the rest position.
- When an object is vibrated, even gently, at one of its natural frequencies, the amplitude of its vibration will increase, sometimes very dramatically. This phenomenon is known as resonance.
- A mechanical wave is a disturbance that transfers vibrational energy through a medium. A mechanical wave requires a medium.
- In a transverse wave, the vibration of the medium is at right angles to the direction of the wave. In a longitudinal wave, the vibration of the medium is parallel to the direction of the wave.
- The wavelength of a wave, λ, is the shortest distance between two points in the medium that are vibrating in phase; for example, the distance between two adjacent crests (or troughs).
- The vibrating source that produces the wave determines the frequency, f, of the wave. The frequency is equal to the number of wavelengths produced in 1 s.
- A wave travels with a constant speed in a homogeneous medium predicted by the wave equation $v = f\lambda$.
- When a wave passes from one medium into another, it is partially transmitted and partially reflected.
- If two waves are moving toward each other, they pass through each other without any permanent change in either wave.
- According to the principle of superposition, when two or more component waves are at the same point in a medium at the same time, the resultant displacement of the medium is equal to the sum of the amplitudes of the component waves.
- Interference occurs when two or more waves meet at the same point in a medium.
- Interference may be either constructive or destructive.
- A standing wave with stationary nodes and antinodes is produced when two periodic waves with the same shape, amplitude, and wavelength travel in opposite directions in the same linear medium. Adjacent nodes are spaced half a wavelength ($\frac{1}{2}\lambda$) apart, as are adjacent antinodes.
- Standing waves can be set up in a linear medium by vibrating one end of the medium at the natural frequency for the medium. The lowest natural frequency is referred to as the fundamental.
- Waves that originate from a point source move outward in circular wavefronts because the speed of the wave in the medium is the same in all directions.
- Two-dimensional waves are reflected from straight barriers so that the angle of reflection is equal to the angle of incidence. Straight waves are reflected from a parabolic barrier so that they converge through a single point.
- When straight waves pass through an opening in a barrier, they diffract around the edges of the barrier and spread out in all directions. Diffraction is greater for smaller openings and larger wavelengths.
- The circular waves moving out from two point sources will produce two-dimensional interference patterns consisting of nodal and antinodal lines. If the two sources are vibrating in phase, there will be an antinodal line along the perpendicular bisector of the lines.

Knowledge/Understanding

1. Explain in your own words what periodic motion is. What quantities involved in periodic motion are variables? What concepts do we use to describe these variables?
2. How are frequency and period related? What is a hertz?
3. What is resonance, and how is it related to the natural frequency of an object?
4. In your own words, explain what a wave is. How do transverse and longitudinal waves differ?
5. What determines the frequency of a wave?
6. How could you increase the speed of a wave pulse in a large-diameter spring?
7. Both amplitude and wavelength are linear measurements used to describe waves. Explain the difference between these measurements. If you wanted to increase the amplitude of a wave in a large-diameter spring, what would you do? If you wanted to increase the wavelength of a wave in the spring, what would you do?
8. If the frequency of a wave travelling in a rope is doubled, what will happen to the speed of the wave? What will happen to the wavelength of the wave?
9. A 1 cm-high wave crest is travelling toward a 2 cm-high wave crest in the same spring. What will be produced when they meet? What kind of interference is this?
10. A 1-cm high wave crest is travelling toward a 2-cm deep wave trough in the same medium. What will be produced when they meet? What kind of interference is this?
11. What is a standing wave? What conditions are necessary to produce a standing wave? What are nodes? How far apart are adjacent nodes?
12. What happens to straight waves when they pass through an opening in a barrier? What do we call this effect?

Inquiry

13. Suppose an upright wave pulse travels from a spring where its speed is 20 cm/s into a second spring where its speed is 10 cm/s.
    - **(a)** What will happen to its frequency and wavelength in the second spring?
    - **(b)** Describe what the two springs will look like 2 s after the incident pulse has reached the boundary between the two springs.
    - **(c)** Suppose the wave pulse had gone from the 10 cm/s spring into the 20 cm/s spring. Describe and draw the two springs, 2 s after the incident pulse had reached the boundary between the two springs.
14. A large erect wave pulse is moving to the left on a large spring at the same time that a smaller inverted wave pulse is moving to the right. Draw what the spring would look like
    - **(a)** shortly before they meet
    - **(b)** shortly after they meet
15. Design an experiment to determine the speed of a wave. You are free to select any equipment you need in your summarized design.

Communication

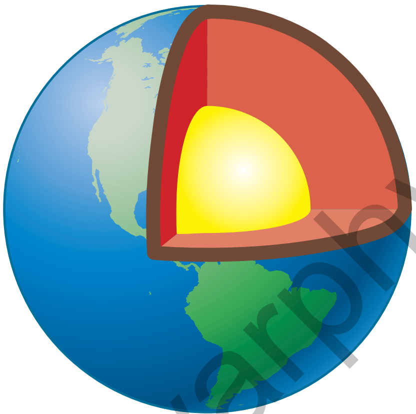
16. Sketch a diagram illustrating how a straight wavefront is reflected from a straight barrier. (Be sure to include the rays that indicate the direction in which the wavefront is moving.)
17. Sketch a diagram illustrating how a straight wavefront is reflected from a parabolic barrier.
18. Sketch the kind of interference pattern produced by two in-phase point sources in a two-dimensional medium.
    - **(a)** What kind of interference will there be on the perpendicular bisector of the line connecting the two sources? Why is this?
    - **(b)** Even though the two sources are in phase and produce crests and troughs at the same time, there are nodal lines in the pattern where destructive interference occurs. Explain why this is the case.

Making Connections

19. The speed of a wave in a string depends on its tension (the greater the tension, the greater the speed) and its mass per unit length (the greater the mass per unit length, the lower the speed). Multi-stringed musical instruments, such as the

guitar or violin, typically play high frequency notes on strings that are under considerable tension and are relatively thin. They play low frequency notes on strings that are under less tension and relatively thick. Explain why this is the case. (You may want to examine a guitar or violin to help you with this question.)

20. Scientists have no way of observing Earth's centre directly. What data, then, gave them evidence that Earth has a solid core surrounded by a thick layer of molten, liquid rock? The answer lies in the study of earthquakes and the types of waves that they generate. Research earthquake waves and how these waves were used to hypothesize the characteristics of our Earth's core.



Problems for Understanding

21. A pendulum takes 1.0 s to swing from the rest line to it's highest point. What is the frequency of the pendulum?

22. By what factor will the wavelength change if the period of a wave is doubled?

23. A wave with an amplitude of 50.0 cm travels down a 8.0 m spring in 4.5 s. The person who creates the wave moves her hand through 4 cycles in 1 second. What is the wavelength?

24. A sound wave has a frequency of 60.0 Hz. What is its period? If the speed of sound in air is 343 m/s, what is the wavelength of the sound wave?

25. Water waves in a ripple tank are 2.6 cm long. The straight wave generator used to produce the waves sends out 60 wave crests in 42 s.
(a) Determine the frequency of the wave.
(b) Determine the speed of the wave.

26. A rope is 1.0 m long and the speed of a wave in the rope is 3.2 m/s. What is the frequency of the fundamental mode of vibration?

27. A tsunami travelled 3700 km in 5.2 h. If its frequency was 2.9×10^{-4} Hz, what was its wavelength?

28. A storm produces waves of length 3.5 m in the centre of a bay. The waves travel a distance of 0.50 km in 2.00 min.
(a) What is the frequency of the waves?
(b) What is the period of the waves?

29. A grandfather clock has a long pendulum with an adjustable mass that is responsible for the clock's ability to keep regular time. This pendulum is supposed to have a period of 1.00 s. You discover that the pendulum executes 117 compete vibrations in 2.00 min.
(a) Calculate the period of the pendulum.
(b) Calculate the percentage error in the time the clock records.
(c) How many hours slow will the clock be after a year?
(d) How might you adjust the pendulum so that its period is exactly 1.00 s?

Numerical Answers to Practice Problems

1. 0.98 Hz; 1.0 s **2.** 7.5 to 11 Hz **3.** 27.9 s **4.** 0.40 Hz; 2.5 s **5.** 7.5 m/s; 0.80 s **6.** 1.4×10^{9} Hz **7.** 3.1×10^{-4} Hz **8.** **(a)** 8.80 Hz **(b)** 853 m **(c)** constant frequency **9.** **(a)** 1.34 m **(b)** 0.670 m

CHAPTER 8 Exploring a Wave Model for Sound

CHAPTER CONTENTS

The howl of a wolf, the cry of a baby, the beat of a drummer — all of these sounds focus your attention and provide information about events in the world around you. More than any other mode of communication, sound evokes strong emotions — fear, concern, happiness, or excitement. Human ingenuity has fashioned sound into complex forms, such as language and music that are characteristic of cultures, nations, and generations. Sound is at the heart of who you are, what groups you bond with, and how you perceive yourself.

Much the same as light, sound is a powerful tool for investigating the world. Bats and dolphins explore their environments and locate prey by emitting high-frequency sound pulses and interpreting the resultant echo. Physicians use ultrasound imaging devices to obtain critical information about a beating heart or a fetus in the womb.

Sound is a complex phenomenon that can enrich your own life and the lives of those around you. However, sound can also be problematic — provoking conflict and even causing physical harm. While the wise use of sound involves much more than science, a study of the physics of sound provides important knowledge for thinking critically and creatively about the multifaceted role that sound plays in society.

MULTI LAB

Sounds and Their Sources

TARGET SKILLS
- Predicting
- Performing and recording
- Communicating results

Musical Rulers

How can you make a ruler produce different notes? Position a ruler so that about two thirds of its length projects over the edge of your desk. Firmly clamp the end of the ruler to the desktop with one hand, and then pluck the free end with your other hand. Change the length of the ruler projecting beyond the desk, and observe the effect on the sound produced.

Analyze and Conclude

1. What vibrates to produce the sound? Describe the vibration.
2. What determines the pitch of the sound produced?
3. How can the sound be made louder or softer?

Sound from a Tuning Fork

Hypothesize about how a tuning fork produces sound. Strike a tuning fork with a rubber mallet. Record what is vibrating and explain how you can tell. Strike the fork harder and record how the sound changes. Compare the sound produced by two different tuning forks.

Analyze and Conclude

1. Sketch the way that the tuning fork moves when you strike it with a rubber mallet.
2. How does striking the tuning fork harder affect the sound?
3. What is responsible for the difference in the sounds of the two tuning forks?

Sound from a Graduated Cylinder

How can you make a 100 mL graduated cylinder produce different notes? Hold the open end of a clean graduated cylinder just below your lower lip and blow strongly across the top. Practise this a few times, until you can produce a sound consistently. Fill the cylinder about one third full of water, and blow again. Produce sounds with different water levels. Record what you hear each time.

Predict how the sound will change as you slowly fill the cylinder with water, while blowing across it. Now test your prediction.

Analyze and Conclude

1. Describe how the sound changes when you change the water level in the cylinder.
2. How can you make the sound louder?
3. What is vibrating to make the sound?
4. How did the sound change when you were adding water and blowing at the same time? Was your prediction correct?
5. Give a possible explanation for the change in the sound.

8.1

A Simple Wave Theory of Sound

SECTION EXPECTATIONS

- Describe and illustrate how sound is produced.
- Analyze and interpret the properties of sound.
- Identify the relationship between velocity, frequency, and wavelength of sound modelled as longitudinal waves.

KEY TERMS

- loudness
- pitch
- quality
- natural frequency
- oscilloscope
- compression
- rarefaction

To the average person, sound is simply part of the everyday world — something that is used for communication and entertainment. To a physicist, however, sound is a more complex entity that, as it turns out, can be explained in terms of mechanical waves. How did physicists make the connection between sound and waves?

Waves and Sound — Some Interesting Similarities

Physicists who first studied sound discovered that it had many properties in common with mechanical waves. As you saw in Chapter 7, waves diffract around corners. Sound also travels around corners. Similarly, the way that waves are reflected by barriers could nicely account for the echo that is heard a short time after shouting toward a rocky cliff.

Table 8.1 provides a summary of the basic properties that physicists first observed to be shared by mechanical waves and sounds.

Table 8.1 Early Comparisons of Mechanical Waves and Sounds

Property	Mechanical waves	Sounds
transmit energy	yes	yes
travel around corners	yes	yes
pass through each other	yes	yes
reflect off barriers	yes	yes
require a medium	yes	?
exhibit constructive and destructive interference	yes	?

The similar characteristics of mechanical waves and sounds that are listed in Table 8.1 illustrate why researchers would have considered using a wave model to explain the properties and behaviour of sound. Like mechanical waves, sounds transmit energy, travel around corners, pass through each other, and are reflected by barriers. The fact that some wave-like properties had still not been confirmed (for example, the requirement of a medium and the ability to produce interference effects) only served to motivate physicists to conduct further research into the possibility of explaining sound by means of a wave model. And, as you will discover throughout this chapter, it is a model that works very well.

PHYSICS FILE

For Sir Karl Popper (1902-1994), a philosopher of science, scientific theories had to be testable, and the role of experiments was to test theories. In fact, he believed experiments should try to *disprove* theories. Scientific theories are reliable because they have survived many attempts to prove them wrong.

Testing a Prediction: Does Sound Require a Medium?

In order to test the prediction that sound requires a medium through which to travel, a method of producing a vacuum was needed. It is not surprising, then, that it was Otto von Guericke, the inventor of the air pump, who carried out the task. In 1654, von Guericke demonstrated that the sound from a bell inside a jar decreased in intensity as air was removed from the jar. A modern demonstration of this effect is shown in Figure 8.1.

If the vacuum pump is a good one, the sound of the bell will almost be eliminated. It would be interesting to perform this experiment in the nearly perfect vacuum of space where no sound would be expected at all (see Figure 8.2).

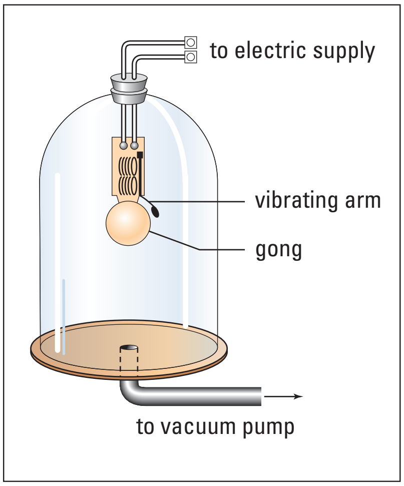


Figure 8.1 An electric bell is sealed inside a bell jar and a vacuum pump removes the air. When the electric bell is turned on, it produces a loud ringing sound. As the vacuum pump removes the air from the bell jar, the loudness of the ringing decreases.

Figure 8.2 What would these astronauts have to do to hear each other speak?

Describing Sound with a Wave Theory

Humans can distinguish between sounds in a variety of ways. Sounds vary in **loudness** (perceived intensity). Jet aircraft engines are so loud that airport workers have to wear ear protection when working near them. On the other hand, the breathing of a sleeping baby is so quiet that new parents can become anxious about their child's welfare.

Sounds also vary in **pitch** (perceived frequency). Flutes and piccolos produce very similar sounds. The main difference between the two is a matter of pitch. In general, the sound of the piccolo is higher and that of the flute is lower. Sounds also vary in another important way called **quality**. The sound of a flute or whistle is described as pure, and that of a cello or organ as rich. It is the quality of a sound that enables you to identify it as being made by a piano rather than a trumpet, even when the two instruments play notes with the same loudness and pitch.

A wave model for sound must relate the loudness, pitch, and quality of the sounds that you hear to specific properties of sound

Computer Link

The difference in sound quality produced by different voices has been the major difficulty in producing word processors that recognize human speech. To use current voice-recognition software, speakers need to "train" the program to recognize the unique quality of their voices. This involves a person correcting the errors that the program makes in typing out words. These corrections are then stored in a particular file for that person that can be accessed when that same person wants to use the software again.

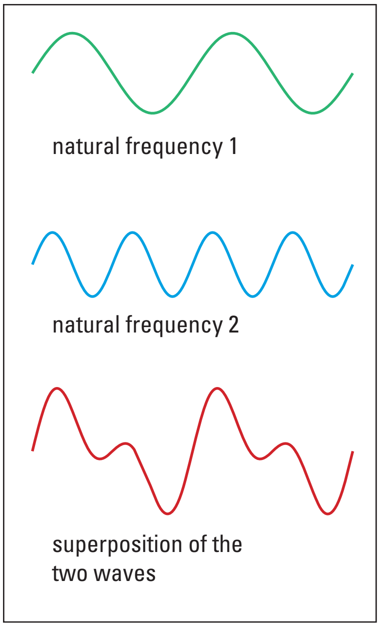


Figure 8.3 When only two frequencies are added together, the resultant wave becomes complex. The quality of this sound is richer than a pure fundamental tone.

waves. Everyday experience makes it clear that loudness is related to energy. To produce a louder sound from a bell, you have to hit it with more force. Yelling requires significantly more effort than whispering. Loudness, then, is in some manner connected to the **amplitude** of the sound wave. Pitch, on the other hand, is related to the frequency of the sound wave. You might recall from the "musical ruler" activity on page 365 that the shorter projections of the ruler that produced higher frequencies of vibrations also produced higher-pitched notes. Likewise, the longer projections produced lower frequencies and lower-pitched notes.

Pure sounds are produced by sources vibrating at only one **natural frequency**. Sound quality arises when the source of the sound vibrates at *several* of its natural frequencies at the same time. As shown in Figure 8.3, the superposition of these component waves — even just two of them — produces a complex wave form with a variety of smaller crests and troughs.

The conceptual links between sound perceptions and their corresponding sound wave characteristics are summarized in Figure 8.4.

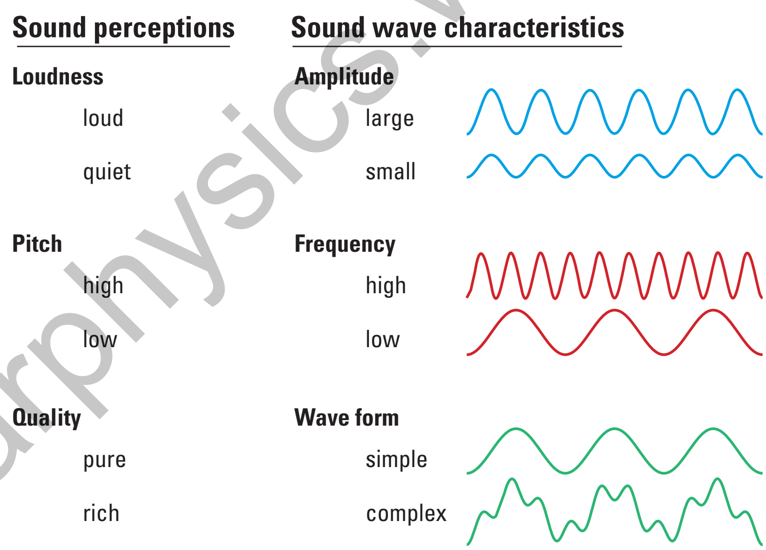


Figure 8.4 Characteristics of sounds and sound waves

"Seeing" Sound Waves

An **oscilloscope** is an electronic instrument that displays the form of electronic signals on a small monitor similar to a television screen. Sounds can be "seen" by using a microphone to convert them into electronic signals, an amplifier to amplify these signals, and an oscilloscope to display their form (see Figure 8.5). With an oscilloscope, you can visualize the difference between the sounds made by a variety of musical instruments, as shown in Figure 8.6.

ELECTRONIC LEARNING PARTNER

Go to your Electronic Learning Partner to enhance your learning about vibrations and sound.

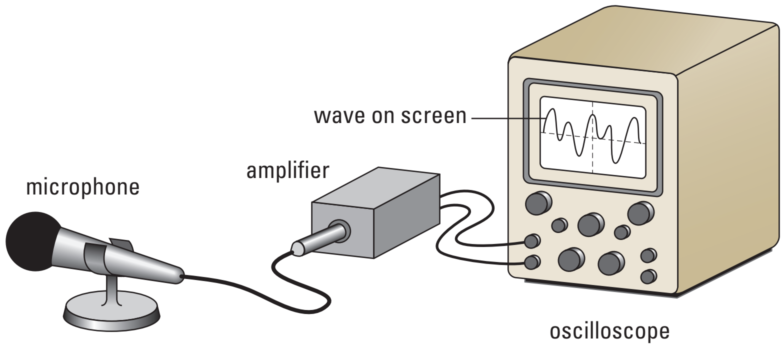


Figure 8.5 By using an oscilloscope, you can "see" sound waves.

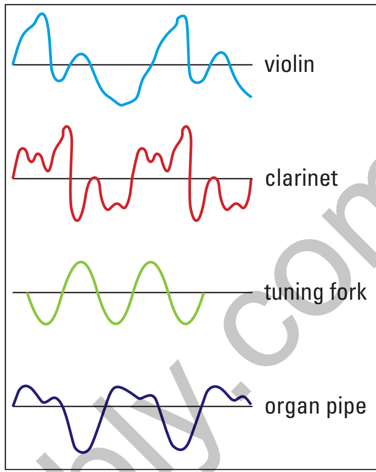


Figure 8.6 These oscilloscope tracings show why you can distinguish among different instruments.

Sound Waves Are Longitudinal Waves

If you use a strobe light to make the vibrations of a large speaker cone appear in slow motion, you will see that the cone is moving in and out, toward and away from the listener. When the speaker cone moves out, the air molecules in front of it are pushed together to produce a small volume of higher pressure air called a **compression**. When the speaker cone moves back, it produces an expanded space for the air molecules to spread out in. The result is a volume of lower pressure air called a **rarefaction**. This alternating pattern of compressions and rarefactions spreads outward through the room.

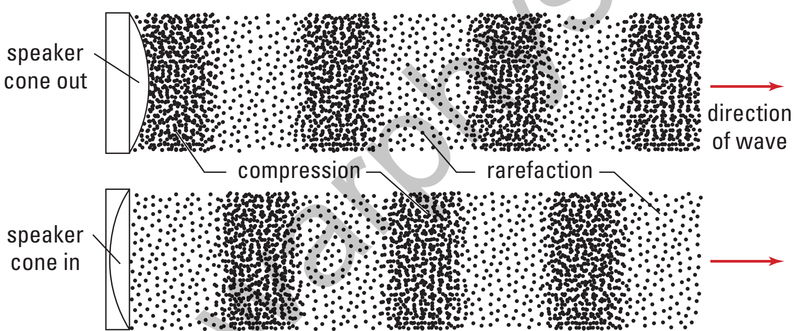


Figure 8.7 When a loudspeaker cone moves out, it exerts a force on the molecules in the air. The molecules move outward until they collide with more molecules. Individual molecules vibrate back and forth, but the collisions carry the sound energy throughout the room.

Tuning forks produce compressions and rarefactions in a somewhat different manner than speaker cones do. When one prong of a tuning fork is struck with a rubber mallet, both prongs move in and out together. As they move away from each other, they produce compressions on their outward sides (and a rarefaction between them). As they move toward each other, they produce rarefactions on their outward sides (and a compression between them).

PHYSICS FILE

In a typical conversation, the amplitude of the pressure wave is about 3×10^2 Pa. Compared to normal atmospheric pressure of 1.01×10^5 Pa, sound wave pressure fluctuations are extremely small. In order to detect such small variation in pressure, the human ear has to be very sensitive. To make hearing easier, the ear also has an amplifying structure to magnify sound.

MISCONCEPTION

But That's Just Theory!

In everyday conversation, people often dismiss ideas with the phrase, "Yes, but that's just theory." This implies a misconception that theory is unreliable. While some untested theories may well be unreliable, the theories that you study in physics have been subjected to very rigorous testing. Scientists generally trust these theories much more than they trust the experiences of particular individuals. If you claimed to have invented a perpetual motion machine (a machine that would keep running forever without energy inputs), it would be very unlikely that physicists would take you seriously, even if they had not tested your machine. According to thermodynamic theory (in particular, the law of conservation of energy) such a machine is impossible. Scientists consider thermodynamic theory, because of the rigorous testing that it has undergone, extremely reliable.

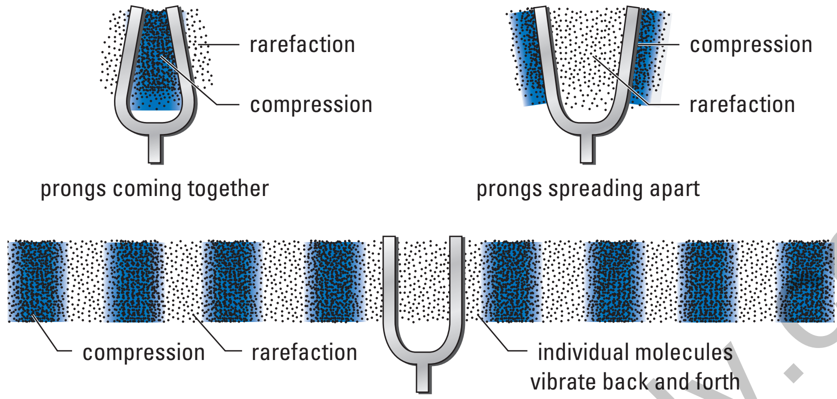


Figure 8.8 A tuning fork directs sound waves outward in two directions.

As you will recall from Chapter 7, there are two distinct types of waves — transverse waves and longitudinal waves. For transverse waves, the vibrations are perpendicular to the direction of the wave motion; for longitudinal waves, the vibrations are parallel to the direction of the wave motion. The above analysis of the sound produced by speakers and tuning forks demonstrates that sound behaves as a longitudinal wave. As shown in Figure 8.9, the vibrations in a sound wave correspond to the changes in air pressure at a point in space — that is, crests that are produced by compressions and troughs that are produced by rarefactions.

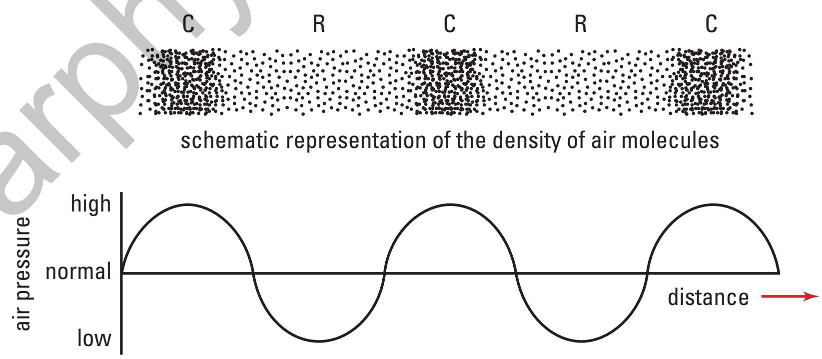


Figure 8.9 Compressions are volumes of maximum pressure, and rarefactions are volumes of minimum pressure.

8.1 Section Review

1. K/U Identify the vibrating source of sound for each of the following:
   (a) guitar
   (b) drum
   (c) clarinet
   (d) singer
   (e) piano

2. C Describe in your own words how the wave theory of sound explains the following phenomena involving sound:
   (a) Sounds can be heard around barriers.
   (b) Sometimes after a sound is produced, an echo is heard.
   (c) Two people can be talking, face to face, at the same time and still hear what the other is saying.
   (d) A large explosion occurs in a fireworks-manufacturing plant, and several seconds later windows shatter in a nearby housing development.

3. C Mechanical waves require a medium to travel through. Thus, the wave theory of sound predicts that sound cannot travel through a vacuum. Describe an experiment designed to test this prediction. Explain how the results of this experiment support the wave theory of sound.

4. I Use your knowledge of mechanical waves to make two more predictions about the behaviour of sound. Describe an experimental set-up that could be used to test each of your predictions. In each case, what results would you expect to observe?

5. C Sounds can be described in terms of their loudness, pitch, and quality.
   (a) Explain how each concept enables you to differentiate sounds.
   (b) How is each of these characteristics of sound represented in a sound wave?

6. K/U The molecules in air are relatively far apart (about 10 molecular diameters). Thus, there are basically no forces of attraction among them. Explain why this fact would lead you to conclude that transverse waves cannot travel through air. How, then, can longitudinal sound waves travel through air?

7. K/U Sketch a graph of pressure versus position for a sound wave of wavelength 15 cm at some instant in time. Mark appropriate scales for pressure on the vertical axis and for position on the horizontal axis.

UNIT ISSUE PREP

Is it possible to predict noise pollution hazards within a community by considering the wave nature of sound?

- Identify specific properties of sound waves that should be considered when creating a noise policy document.
- Identify specific activities that would generate pervasive sounds in a community setting.

8.2 Exploring Wave Characteristics of Sound

SECTION EXPECTATIONS

- Conduct an experiment to investigate factors that affect the speed of sound.
- Analyze the factors that affect sound intensity and its effect in nature.
- Compare sound travelling in different media.

KEY TERMS

- infrasonic
- audible
- ultrasonic
- sound intensity level
- bel
- decibel

Biology Link

Your range of hearing can be affected by age, illness, or injury. Injury can be caused by, among other things, prolonged exposure to loud noise or music. Research what percentage of teenagers have permanently lost some hearing by listening to loud music.

Convinced that sound is a type of mechanical wave, investigators set out to discover more about the properties of sound waves. What is the speed of sound in various media? What is the range of frequencies of sound? What frequencies within the full range can humans hear?

The Range of Hearing in Humans

Scientific investigation always begins with the senses. Because the senses, including hearing, have limited precision, more precise and reliable measuring instruments for measuring phenomena are continually being developed.

The Frequency Range of Human Hearing

There are both upper and lower limits to the sound frequencies that humans can hear. A healthy young person can typically hear frequencies in a range from about 20 to 20 000 Hz (20 kHz). You may well have had experience with a dog whistle that seems to produce no sound at all when blown, but still brings your pet dog bounding back. The frequency of the sound produced by these whistles is higher than 20 kHz. While it is outside the audible range for humans, it is obviously not outside the audible range for dogs.

As you will observe in the Quick Lab opposite, individuals have quite different ranges of hearing. Thus, in order to accurately study sound, physicists have developed instruments that measure the frequency and intensity of sound waves with increased reliability and precision. These instruments provide more objective measures of the frequency and intensity of sound waves than our more subjective perceptions of pitch and loudness. Using these instruments, investigators have discovered that different animals can hear sounds over extremely different frequency ranges (see Figure 8.10). Nevertheless, physicists have established a three-part classification of sound, based on the range of human hearing. Sound frequencies lower than 20 Hz are referred to as **infrasonic**, those in the 20 to 20 000 Hz range are **audible**, and those higher than 20 000 Hz are **ultrasonic**. There is no real qualitative difference in the behaviour of these three kinds of sound.

QUICK LAB

Determining the Upper and Lower Frequency Limits of Hearing

TARGET SKILLS
- Performing and recording
- Communicating results

If you have an audio frequency generator with a loudspeaker available, it is fairly simple to determine the upper and lower frequency limits of your own hearing, and to compare them with those of other students in your class. Start the audio frequency generator at a frequency below 20 Hz and gradually turn it up. Have people in the room raise their hands as soon as they can just hear the sound, so that the lower frequency limit of their hearing can be recorded. Once everyone in the group can hear the sound, the frequency can be turned up more rapidly until it reaches about 12 000 Hz. From 12 000 Hz onward, the frequency should be increased more slowly so that as people lower their hands, when they can no longer hear the sound, the frequency can be noted.

Analyze and Conclude

1. How do the lower frequency limits of individuals in your group compare? How do the upper frequency limits compare?
2. Are there any apparent reasons for the differences you found in question 1?
3. Determine your teacher's lower and upper frequency limits. How do they compare with those of the students in your group? Why might this be?

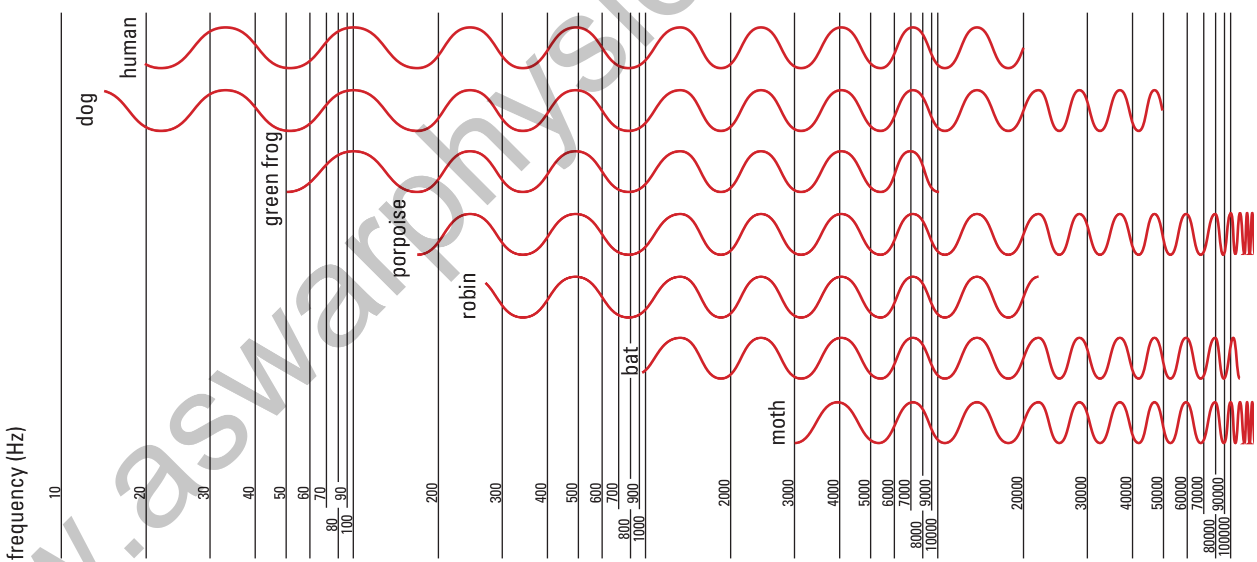


Figure 8.10 The range of hearing in various animals is extremely broad. In this chart showing the audible frequency ranges for some animals, do you see a relationship between the pitch that different animals hear and some characteristic of their bodies?

Math Link

In Figure. 8.10, notice that the distance between the lines corresponding to 10 Hz and 100 Hz is the same as the distance between the lines for 100 Hz and 1000 Hz. In fact, the distance between any two adjacent powers of 10 is always the same. This type of scale is referred to as a *logarithmic* scale. Describe the difficulty you would have drawing this type of graph using a standard, linear scale.

Biology Link

Human ears are very sensitive instruments. They respond to sound intensities as low as a few picowatts per square metre. The ear also deals with a wide range of intensities, from a few picowatts per square metre to ten billion (10^{10}) picowatts per square metre! How does this compare with microphone technologies?

Intensity Range of Human Hearing

Loudness is a measure of the response of the ear to sound waves. Individuals, however, may hear the same sound wave as differing in loudness, depending on how sensitive their ears are. In the previous section, you saw that humans are unable to hear sounds whose frequencies are less than 20 Hz or more than 20 000 Hz. In other words, the loudness of these sounds to humans is zero, although other animals can hear them. In order to accurately compare sound waves, investigators use instruments to measure a property of sounds called intensity. Intensity is a measure of the amount of sound energy reaching a unit of area in 1 s (power per unit area). Sound intensity is measured in units of picowatts per square metre (pW/m^2). (Note that 1.0 pW is 1.0×10^{-12} W.) Intensity is an *objective* property of the sound wave — in fact, it is related to the square of the wave amplitude, and does not depend on the particular characteristics of a person's ears. Loudness, on the other hand, is a *subjective* property of the sound that depends on the human ear, the sensitivity of the ear to the frequency of the sound, and the distance from the source of the sound.

The range from 1.0×10^0 (one) to 1.0×10^{10} (ten billion) picowatts is an extremely wide range to describe and compare. So, scientists decided that it might be easier to describe intensity according to the exponent of 10 instead of the number itself. They defined a new measurement, in terms of the exponent of 10, called **sound intensity level** and named the unit the **bel** (B), after Alexander Graham Bell. In math, you can find the exponent of 10 that corresponds to any specific number by taking the logarithm of the number. For example, the logarithm of 1000 is 3. Thus, the scale in bels is called a logarithmic scale. When the sound intensity increases by a factor of 10 (is 10 times larger), the sound intensity *level* increases by 1 bel. When the sound intensity increases by a factor of 100 (is 100 times larger), the sound intensity *level* increases by 2 bels. Notice the relationship between the number of bels and the number of zeros in the factor by which the sound intensity increases.

1 bel is equal to 10 pW/m^2, the lower threshold of human hearing. Detailed hearing tests have shown that the smallest increase in sound intensity level that humans can distinguish is 0.1 bels, or a **decibel**. Therefore, the decibel (dB) has become the most commonly used unit to describe sound intensity levels.

CANADIANS IN PHYSICS

Telephone Invention an Accident

Alexander Graham Bell (1847–1922)

Few inventors have had as great an impact on our everyday lives as Alexander Graham Bell, creator of the world's first telephone. The invention of the telephone in 1876 changed our social lives and the world of business, and it paved the way for today's information age.

Bell was born in Edinburgh, Scotland, but moved to Brantford, Ontario, with his family as a young adult. Although he eventually became a U.S. citizen and taught the deaf in the United States, he is often considered a "Canadian inventor" because he did much of his scientific work in his summer home on Cape Breton Island in Nova Scotia, and developed his idea for the telephone in Brantford.

From an early age, Bell had a keen interest in speech and sound. It was a passion that came naturally — his mother was deaf and his father was a speech therapist who developed an alphabet to teach deaf people to speak.

Despite his knowledge and his enthusiasm for sound and human communication, Bell did not set out to invent the telephone. He was working on improving the telegraph, the first instrument used to send a message using electricity. The telegraph was already in full use but it could not send multiple messages simultaneously. Bell and his assistant, Thomas Watson, were trying to solve this problem when Bell discovered he could send sound using electrical current.

Bell created the beginnings of the telephone using reeds arranged over a magnet. The reeds vibrated up and down, toward and away from the magnet in response to sound wave variations. The vibrations of the reeds generated a current that could then be carried by wire to a receiver and converted back into sound waves.

On March 10, 1876, Bell accidentally spilled acid while he and Watson were working on the apparatus. He called out, "Mr. Watson. Come here. I want to see you." Watson, working in another room, heard Bell's voice over his receiver. The first transmission of the human voice over a telephone had occurred.

The telephone may be the greatest of Bell's inventions, but it hardly sums up his accomplishments. After inventing the telephone, he continued his experiments in communications and developed the photophone — a device that transmitted a voice signal on a beam of light, and was a forerunner of today's fibre-optic technology.

His interests also extended beyond human communications. The *Silver Dart*, an airplane he played a large part in creating, made aviation history in 1909 as Canada's first successful heavier-than-air flying machine. Yet another of Bell's many inventions was an electric probe that was used in surgery before X rays were discovered.

Web Link

www.school.mcgrawhill.ca/resources/

For more information about Alexander Graham Bell, including diagrams by him and photographs, go to the above Internet site. Follow the links for **Science Resources** and **Physics 11** to find out where to go next.

Math Link

The logarithm of a number is equal to the exponent to which the base 10 must be raised in order to be equal to the number. For example, the logarithm of 10 is 1 ($10 = 10^1$); the logarithm of 100 is 2 ($100 = 10^2$); and the logarithm of 1000 is 3 ($1000 = 10^3$). It follows that the logarithm of 1 is zero ($1.0 = 10^0$) and that the logarithm of 2 is greater than 0 but less than 1. Actually, the logarithm of 2 is about 0.30103. If you have a calculator with a "log" button on it, try entering the number 2 and then pressing the log button to confirm this.

Even though sound intensity levels are almost always given in decibels, it is easier to see the logarithmic nature of this scale by looking at their values in bels. Table 8.2 provides equivalent values for the sound intensity levels in bels and decibels with the sound intensity in picowatts per square metre.

Table 8.2 Logarithmic Intensity Levels and Sound Intensity

Intensity level (dB)	Intensity level (B)	Intensity (pW/m^2)
0	0	1
10	1	10
20	2	100
30	3	1 000
50	5	100 000
100	10	10 000 000 000

Sound intensity levels range from barely audible (0 dB), to the threshold of pain (130 dB), to a space rocket booster (160 dB). The sound of the rocket booster would instantly break workers' eardrums if they did not wear the appropriate ear protection.

Figure 8.11 The sound intensity levels of some common sounds

Because the human ear is not equally sensitive to all frequencies, sounds of different frequencies may be perceived as equally loud even though they have quite different sound intensity levels. Figure 8.12 illustrates this disparity by displaying a set of *constant-loudness* curves. For these graphs, the sound frequencies are plotted along the horizontal axis and the sound intensity levels (in decibels) are plotted on the vertical axis. The curves are called constant-loudness curves because they display the sound intensity level required at each frequency for a sound to be perceived as having the same loudness. The curves are labelled with their sound intensity levels at 1000 Hz. This means that the curve labelled 40 represents all sounds perceived as equally loud as a 1000 Hz sound with an intensity level of 40 dB. The lowest curve can be thought of as the threshold-of-hearing curve. It displays the intensity levels at which sounds of different frequencies are barely audible (0 dB). For example, a 60 Hz sound requires an intensity level just under 50 dB in order to be barely audible, while a 1000 Hz sound requires an intensity level of 0 dB. This means that the 60 Hz sound needs to have an intensity 10^5 or 100 000 times greater to be barely audible.

Note the way that these constant-loudness curves become much "flatter" as the loudness increases. This relative flatness indicates that the ear is almost equally sensitive to all frequencies when the sound is loud. Thus, when the volume on a sound system is turned up, the low, middle, and high frequencies are heard about equally well. However, when the volume is decreased to produce a quieter sound, the low and high frequencies are not heard nearly as well, destroying the balance of the music. Many better sound systems have controls that enable you to individually adjust the amplification of the sound for different frequency ranges.

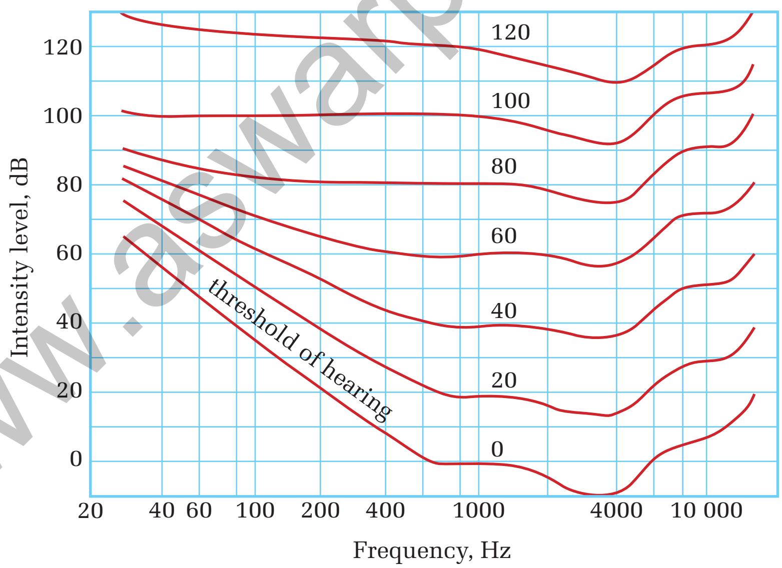


Figure 8.12 Curves of constant loudness for different frequencies

MISCONCEPTION

It's Zero, Not Nothing!

A sound intensity level of 0 dB does not indicate that the sound wave has no intensity or amplitude. An intensity level of 0 dB corresponds to an intensity of $10^0 = 1\ pW/m^2$. The sound intensity level scale is similar to the Celsius temperature scale — it can have negative values. A sound intensity level of –1 dB corresponds to a sound intensity of 10^{-1} or $0.1\ pW/m^2$. Similarly, an intensity level of –2 dB corresponds to an intensity of $0.01\ pW/m^2$.

COURSE CHALLENGE

Intensity and Power Density

Research common sources of microwave radiation, such as cell phones, and their relative power densities.

Learn more from the **Science Resources** section of the following web site: ***www.school.mcgrawhill.ca/resources/*** and find the *Physics 11 Course Challenge*.

The Speed of Sound in Air: A Wave Property

As you learned in Chapter 7, the speed of a wave is determined by the properties of the medium through which it travels. Friction in a medium slowly decreases the amplitude of a wave, but does not affect its speed. If sound is a wave, then it should have a constant speed that is determined by the medium through which it travels. Sound should also travel at different speeds in different media.

Sound waves travel through air by means of moving molecules. Molecular speeds in a gas, however, depend on the temperature of the gas. Therefore, the speed of sound in air should increase with temperature. This is indeed the case. At a temperature of 0˚C and a pressure of 101 kPa, the speed of sound in air is 331 m/s, and for each 1˚C rise in temperature, the speed of sound increases by 0.59 m/s.

PROBEWARE

If your school has probeware equipment, visit the **Science Resources** section of the following web site: ***www.school.mcgrawhill.ca/resources/*** and follow the **Physics 11** links for several laboratory activities on the speed of sound.

THE SPEED OF SOUND IN AIR

The speed of sound in air is 331 plus the product of 0.59 and the Celsius temperature.

$$v = 331 + 0.59T_C$$

Quantity	Symbol	SI unit
speed of sound	v	$\frac{m}{s}$ (metres per second)
temperature of air	T_C	not applicable* (˚C is not an SI unit)

Unit Analysis

$$\frac{m}{s} + \frac{\frac{m}{s}}{\cancel{°C}}(\cancel{°C}) = \frac{m}{s}$$

***Note:** This formula is based on the Celsius temperature scale and cannot be used with the Kelvin scale. How would you modify the equation so that it would apply to the Kelvin scale?

The Speed of Sound in Solids and Liquids

In general, sound travels fastest in solids, slower in liquids, and slowest in gases. The speed of sound in water is almost five times faster than its speed in air. This difference is great enough to be noticed by the human ear. A swimmer who is 1500 m away from a loud noise (perhaps a cannon being fired) would hear the sound that travelled through the water 1 s after it was produced. The same sound travelling through the air, however, would not be heard until 5 s after it was produced.

Knowledge of the speed of sound in different materials is the basis for a variety of techniques used in exploring for oil and minerals, investigating the interior structure of Earth, and locating objects in the ocean depths.

Table 8.3 The Speed of Sound in Some Common Materials

Material	Speed (m/s)
Gases (0°C and 101 kPa)	
carbon dioxide	259
oxygen	316
air	331
helium	965
Liquids (20°C)	
ethanol	1162
fresh water	1482
seawater (depends on depth and salinity)	1440–1500
Solids	
copper	5010
glass (heat-resistant)	5640
steel	5960

PHYSICS FILE

Scientists are continually striving to improve the accuracy of their measurement. In 1986, Dr. George Wong, a senior research officer with the National Research Council of Canada, published a correction to the speed of sound in air. In the course of his investigations into how microphones can be calibrated as accurately as possible, Dr. Wong discovered a 1942 calculation error. This error had gone undetected and had resulted in the scientific community accepting a value for the speed of sound in air that was too large. As a result of Dr. Wong's work, the value of the speed of sound in air at 0°C was revised from 331.45 m/s down to 331.29 m/s.

MODEL PROBLEMS

Applying the Speed of Sound Equation

1. Suppose the room temperature of a classroom is 21°C. Calculate the speed of sound in the classroom.

Frame the Problem

- At a *temperature of 0°C* and a pressure of 101 kPa, the *speed of sound in air* is 331 m/s.
- For each *1°C rise in temperature* the speed of sound increases by 0.59 m/s.

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Identify the Goal

The speed of sound, v

Variables and Constants

Involved in the problem	Known	Unknown
T_C	$T_C = 21°C$	v
v		

Strategy

Use the formula for the velocity (speed) of sound in air.

$v = 331 + 0.59T_C$

Substitute in the known values.

$v = 331\,\frac{m}{s} + 0.59\,\frac{\frac{m}{s}}{\cancel{°C}}(21\cancel{°C})$

Simplify.

$v = 331\,\frac{m}{s} + 12.39\,\frac{m}{s}$

$v = 343.39\,\frac{m}{s}$

The speed of sound in the classroom is 3.4×10^2 m/s.

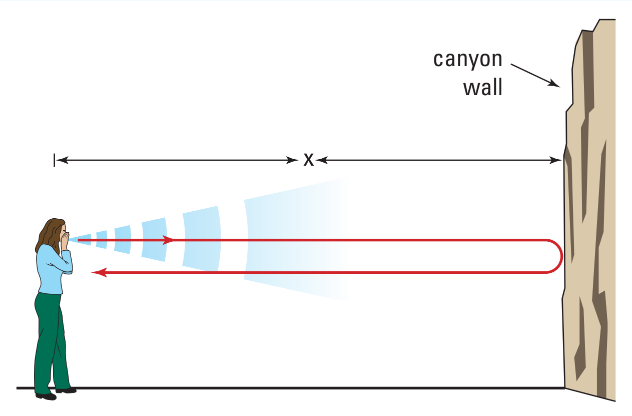
Validate

The speed of sound increases with temperature. At 0°C, the speed is about 330 m/s. The temperature is about 20°C above that, and increases approximately 0.5 m/s for each degree of increase in temperature. Thus, the speed should be about 340 m/s.

2. The temperature was 4.0°C one morning as Marita hiked through a canyon. She shouted at the canyon wall and 2.8 s later heard an echo. How far away was the canyon wall?

Frame the Problem

- The *speed of sound* can be calculated from the *temperature*, which is given.
- The *distance* the sound travels can be calculated from the *speed* of sound and the *time* for the echo to return.
- The sound travels to the wall and back, so the distance to the wall is half the distance that the sound travels.



Sound echoing off a canyon wall

Identify the Goal

The distance, x, to the wall

Variables and Constants

Involved in the problem		Known	Unknown
Δt	Δd	$\Delta t = 2.8\ \text{s}$	v
T_C	x	$T_C = 4.0^\circ\text{C}$	Δd
v			x

Strategy	Calculations
Use the formula for the *velocity* (speed) of sound in air.	$v = 331 + 0.59T_C$
Substitute in the known values.	$v = 331\ \frac{\text{m}}{\text{s}} + 0.59\ \frac{\frac{\text{m}}{\text{s}}}{\cancel{^\circ\text{C}}}(4.0\cancel{^\circ\text{C}})$
Simplify.	$v = 331\ \frac{\text{m}}{\text{s}} + 2.36\ \frac{\text{m}}{\text{s}}$ $v = 333.36\ \frac{\text{m}}{\text{s}}$
Find the *distance* to the wall and back by rearranging the formula for the *velocity* of any entity.	$v = \frac{\Delta d}{\Delta t}$
Substitute in the known values.	$\Delta d = v(\Delta t)$
Simplify.	$\Delta d = 333.36\ \frac{\text{m}}{\cancel{\text{s}}}\ (2.8\ \cancel{\text{s}})$ $\Delta d = 933.41\ \text{m}$
The distance to the wall, x, is half the distance to the wall and back, Δd.	
Substitute in the known values.	$x = \frac{\Delta d}{2}$
Simplify.	$x = \frac{933.41\ \text{m}}{2}$ $x = 466.7\ \text{m}$
The canyon was $4.7 \times 10^2\ \text{m}$ away.	

Validate

Sound at 4°C is not travelling much faster than it is at 0°C. Thus, the sound is travelling at approximately 330 m/s. In 2.8 s, this sound will have travelled about $3 \times 330\ \text{m}$, or 1000 m. However, the sound travelled to the wall and back, so the distance to the wall is only half of 1000 m, or about 500 m.

PROBLEM TIP

In problems such as this one, always remember that the sound travels from the source *to the reflector and back*. Thus, the distance from the source to the reflector is only half of the distance that the sound travelled.

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PRACTICE PROBLEMS

1. What is the speed of sound in air at each of the following temperatures?
   (a) –15˚C **(b)** 15˚C
   (c) 25˚C **(d)** 33˚C
2. For each speed of sound listed below, find the corresponding air temperature.
   (a) 352 m/s **(b)** 338 m/s
   (c) 334 m/s **(d)** 319 m/s
3. A ship's horn blasts through the fog. The sound of the echo from an iceberg is heard on the ship 3.8 s later.
   (a) How far away is the iceberg if the temperature of the air is –12˚C?
   (b) How might weather conditions affect the accuracy of this answer?
4. An electronic fish-finder uses sound pulses to locate schools of fish by echolocation. What would be the time delay between the emission of a sound pulse and the reception of the echo if a school of fish was located at a depth of 35 m in a lake? Assume the temperature of the water is 20˚C.
5. You want to estimate the length of a large sports complex. You generate a loud noise at one end of the stadium and hear an echo 1.2 s later. The air temperature is approximately 12˚C. How far away is the far wall of the stadium from your position?
6. **(a)** How long does it take for sound to travel 2.0 km in air at a temperature of 22˚C?
   (b) The speed of light is 3.0×10^8 m/s. How long does it take for light to travel 2.0 km?
   (c) The rumble of thunder is heard 8.0 s after a flash of lightning hits a church steeple. How far away is the church in 22˚C air?

QUICK LAB

At the Speed of Sound

TARGET SKILLS
- **Performing and recording**
- **Analyzing and interpreting**
- **Communicating results**

A rough determination of the speed of sound in air can be performed using echolocation. Stand at least 50 m from a large outside wall of your school. Measure the distance to the wall. Clap two wooden blocks together, and listen for the echo. Estimate the time interval (a fraction of a second) that it takes for the echo to return. Clap the blocks together repeatedly with a steady rhythm, so that the time interval between a clap and its echo is the same as the time between the echo and the next clap. When you do this, the time between your claps is equal to twice the time that it takes the sound to travel to the wall and back. Thus, the period of your clapping is equal to the length of time it takes sound to travel four times the distance to the wall. Maintaining this rhythm, have your partner measure the length of time that it takes for 20 claps and echoes.

Analyze and Conclude

1. Determine the period of your clapping and the time it takes the sound to travel to the wall. Calculate the speed of sound in air. How accurate is your answer?
2. Assume that your answer to question 1 is correct. Calculate the temperature of the air. Is this a reasonable value? Explain.
3. Use a thermometer to measure the actual temperature of the air. Discuss the discrepancy between your predicted value and your measured value.

8.2 Section Review

1. K/U
   (a) What is the range of sound frequencies audible to the human ear?
   (b) What is the term for sound frequencies that are higher than those humans can hear? Name four animals that can hear these frequencies.
   (c) What is the term for sound frequencies that are lower than those humans can hear? Name two animals that can hear these frequencies.
   (d) How does the human range of hearing compare with that of other animals?

2. C For a physicist, sound includes pressure waves of all frequencies, even though many of them are inaudible to humans. Explain how this definition is more general and less subjective than a definition that includes only those frequencies that are audible to humans.

3. C Explain the difference between *loudness* and *sound intensity*.

4. K/U The sound intensity level measured in bels or decibels is expressed on a logarithmic scale.
   (a) If a sound has an intensity of 10 pW/m^2, what is the sound intensity level in bels? In decibels?
   (b) How much greater than the intensity of a 1 dB sound is the intensity of a
   - 2 dB sound?
   - 3 dB sound?
   - 4 dB sound?

5. I Estimate the decibel reading on a sound meter for
   (a) a quiet room
   (b) a school cafeteria at lunchtime
   (c) a rock concert

6. K/U Each of two students, when talking, produces a 60 dB reading on a sound meter.
   (a) Explain why you would not expect to get a reading of 120 dB when these two students were speaking at the same time.
   (b) What would be a reasonable reading to expect?

7. C
   (a) What do the constant-loudness curves on page 377 tell you about how the human ear hears different frequencies?
   (b) As the loudness increases, what does the increased "flatness" of these curves tell you?

8. C Explain why you would expect that an increase in air temperature would increase the speed of sound in air.

9. I When jelly sets, it changes from a liquid to a solid. What would you expect to happen to the speed of sound in the material as the jelly sets? Explain your reasoning.

10. K/U When a batter hits a home run during a baseball game, it is possible to notice a time delay between when the bat is seen to hit the ball and when the bat hitting the ball is heard. Explain why this is the case.

UNIT ISSUE PREP

Investigate how sound intensity changes as the distance from the source changes.

- Numerical simulations or actual data will help clarify the type of noise pollution policy required for an area.

8.3 Interference of Sound Waves

SECTION EXPECTATIONS

- Explain and illustrate the principle of superposition.
- Identify examples of constructive and destructive interference.
- Investigate interference and compare predicted results with experimental results.

KEY TERMS

- beats
- beat frequency

TRY THIS...

Working with a partner, try to describe the location of the quiet regions around a tuning fork. Strike a tuning fork and then hold it vertically beside the ear of your partner. Slowly rotate the fork, asking your partner to indicate when the sound is soft and then loud. After you have identified the locations of the quiet regions that your partner observed, reverse roles so that your partner can locate the quiet regions that you observed. Repeat the activity until you can agree on the location of the quiet regions.

There Must Be Interference!

The property that seems most characteristic of waves is their ability to interfere. Look back at Table 8.1 on page 366. If sound is a wave, then it must also experience constructive and destructive interference. In Chapter 7, you saw how two waves could combine to produce a resultant wave of either an increased or decreased amplitude. In looking for interference in sound, then, you will look for increases and decreases in loudness and intensity level.

Quiet Regions Near a Tuning Fork

The arms of a tuning fork vibrate back and forth symmetrically so that the air between them is first squeezed (producing a compression) and then pulled apart (producing a rarefaction). At the same time as a compression is produced between the arms, the arms are pulling apart the air outside the arms (producing rarefactions). As a result, we can think of sound waves radiating outward from one source at a point between the arms and from two other sources outside the arms. The source between the arms, however, is in the opposite phase to the sources outside the arms. Where do you expect to find regions of destructive interference around a tuning fork?

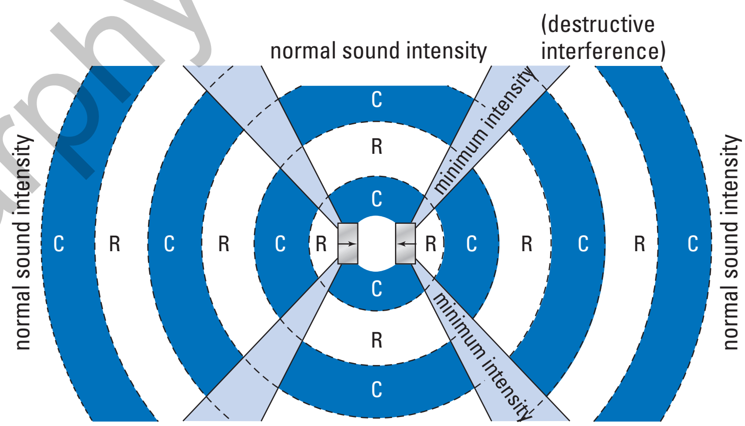


Figure 8.13 Destructive interference occurs in the regions where compressions and rarefactions overlap.

Beats: A More Complex Interference Effect

So far in this section, you have observed interference effects as differences in loudness at *different positions* relative to two or more sound sources. Interference can also be observed as a difference in loudness at different *times*. In this case, what you hear is a kind of wavering of the sound intensity as it becomes alternately louder and quieter. This phenomenon is known as **beats** and is the consequence of two sources of similar (but not identical) frequency producing sound waves at the same time. One beat is a complete cycle from loud to quiet to loud, and the **beat frequency** is the number of cycles of loud-quiet-loud produced per second. The beat frequency depends on the difference in frequency of the two sounds. A larger difference in frequency produces a greater beat frequency, and a smaller difference in frequency produces a smaller beat frequency.

PROBEWARE

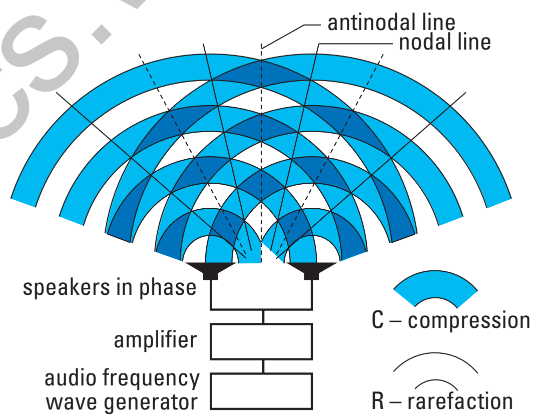
Visit the **Science Resources** section of the following web site if your school has probeware equipment: ***www.school.mcgrawhill.ca/resources/*** and follow the **Physics 11** links for interesting activities on the interference of sound.

QUICK LAB — Locating Interference Between Two Loudspeakers

TARGET SKILLS
- Performing and recording
- Predicting
- Communicating results

Two point wave sources vibrating in phase produce a characteristic interference pattern of nodal and antinodal lines. To create sound interference, you can use two loudspeakers placed side by side, driven by the same audio frequency generator, as two in-phase sound sources. Since destructive interference is produced when one sound wave travels half a wavelength farther than the other (the crest of one wave meets the trough of the second wave), you should be able to locate a point of destructive interference. A frequency of 340 Hz will produce a wavelength of approximately 1 m, so you are looking for a point that is 0.5 m farther from one loudspeaker than the other. By walking across the expected pattern, you should be able to hear a variation of loudness as you walk through nodal and antinodal lines. Set up speakers as shown in the illustration and locate the nodal and antinodal line. Walk back and forth in front of the speakers until you are convinced that you have heard interference in sound.

(**Note:** Unwanted reflections of sound waves from hard surfaces in the room can complicate the sound pattern produced and make it difficult to hear clear points of destructive interference. Try to keep your loudspeakers away from the walls of the room or any other potential sound reflectors.)

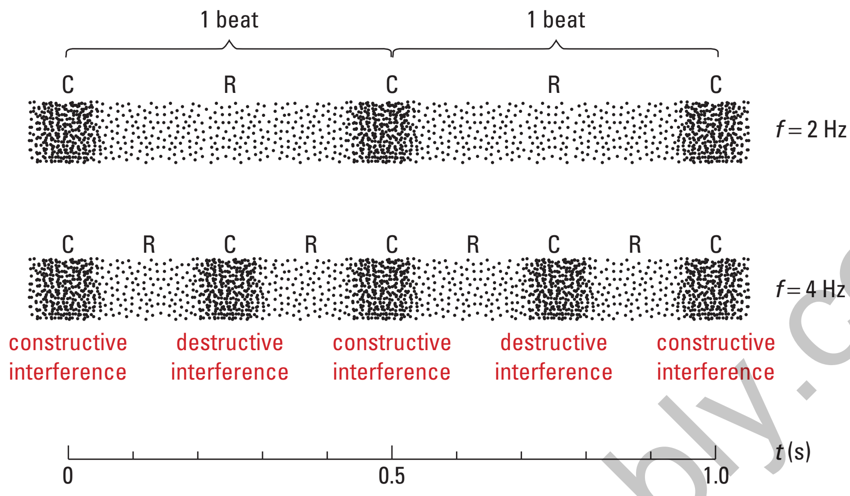


Sound interference produced by two loudspeakers vibrating in phase

Analyze and Conclude

1. What do you expect to hear as you cross the centre line (perpendicular bisector) of a line between the two loudspeakers?
2. How much farther along your path do you expect the first quiet point to be?
3. How far apart do you expect two adjacent quiet points to be?
4. How could you change your walking path to make these quiet points farther apart and, hence, easier to identify?

Figure 8.14 Beats are produced by alternating instances of constructive and destructive interference over time.



Using the principle of superposition, the resultant pressure wave produced by two component sound waves of similar frequency can be constructed, as shown above in Figure 8.14. This resultant wave clearly illustrates the regular variations in loudness characteristic of beats. A second method of visualizing beats is shown in Figure 8.15. Here, air pressure is plotted against time.

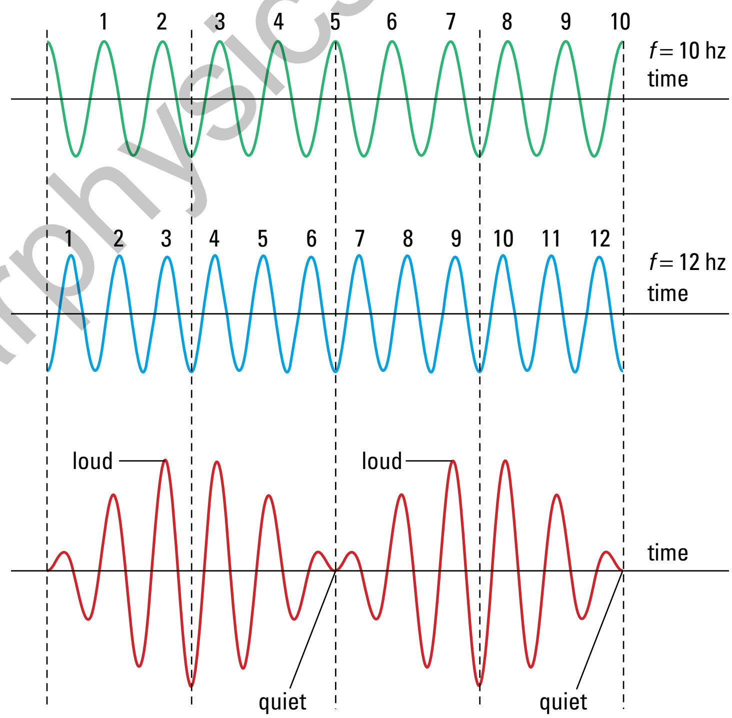


Figure 8.15 The superposition of two similar sound waves produces a resultant sound wave with an intensity that alternates between loud and quiet.

TRY THIS...

Sketch several pairs of component waves and their resultant wave to convince yourself that the beat frequency is equal to the difference in frequency of the two component waves.

BEAT FREQUENCY

The beat frequency is the absolute value of the difference of the frequencies of the two component waves.

$$f_{beat} = |f_2 - f_1|$$

Quantity	Symbol	SI unit
beat frequency	f_{beat}	Hz (hertz)
frequency of one component wave	f_1	Hz (hertz)
frequency of other component wave	f_2	Hz (hertz)

Music Link

When musicians playing in an ensemble are tuning their instruments, they listen for beats produced by the note they are sounding interfering with a reference note sounded by one musician. They then adjust their own instruments to reduce the beat frequency to zero. Which instrument in an orchestra plays the reference note, and why?

MODEL PROBLEM

Finding the Unknown Frequency

A tuning fork of unknown frequency is sounded at the same time as one of frequency 440 Hz, resulting in the production of beats. Over 15 s, 46 beats are produced. What are the possible frequencies of the unknown-frequency tuning fork?

Frame the Problem

- Two tuning forks of different *frequency* are sounding at the same time.
- This results in the production of *beats*.
- The absolute value of the difference between the two *frequencies* is equal to the beat *frequency*.

Identify the Goal

The possible frequencies, f_2, of the unknown-frequency tuning fork

Variables and Constants

Involved in the problem		Known	Unknown
f_1	N	$f_1 = 440$ Hz	f_2
f_2	Δt	$N = 46$	f_{beat}
f_{beat}		$\Delta t = 15$ s	

continued ►

continued from previous page

Strategy

You can find the beat *frequency* because you know the number of beats and the time interval. Substitute these values into the equation.

Calculations

$f_{beat} = N/\Delta t$

$f_{beat} = 46\ \text{beats}/15\ \text{s}$

$f_{beat} = 3.1\ \text{Hz}$

The absolute value of the difference between the two frequencies is equal to the beat frequency. Substitute the known values into the equation.

$f_{beat} = |f_2 - f_1|$

$3.1\ \text{Hz} = |f_2 - 440\ \text{Hz}|$

Simplify. Notice that when you remove the absolute value sign, you cannot know whether the value is positive or negative. Therefore, use both possibilities.

$(f_2 - 440\ \text{Hz}) = \pm 3.1\ \text{Hz}$

Using the positive value

$f_2 - 440\ \text{Hz} = 3.1\ \text{Hz}$
$f_2 = 440\ \text{Hz} + 3.1\ \text{Hz}$
$f_2 = 443.1\ \text{Hz}$

Using the negative value

$f_2 - 440\ \text{Hz} = -3.1\ \text{Hz}$
$f_2 = 440\ \text{Hz} - 3.1\ \text{Hz}$
$f_2 = 436.9\ \text{Hz}$

The frequency of the second tuning fork is either 443 Hz or 437 Hz.

Validate

Having 46 beats in 15 s gives a beat frequency of about 3 Hz, but the beat frequency is equal to the absolute value of the difference between the two tuning forks. Thus, the unknown frequency is either 3 Hz above 440 Hz or 3 Hz below 440 Hz. It is either 443 Hz or 437 Hz.

PROBLEM TIP

The beat frequency provides you with only the *difference* in frequency between two sounds. If the frequency of one of the sounds is known, then the other sound could have one of two possible frequencies — one higher and one lower than the known frequency. You would need some additional information to determine which of these two possible frequencies is the right one.

PRACTICE PROBLEMS

7. Two tuning forks of frequencies 512 Hz and 518 Hz are sounded at the same time.

(a) Describe the resultant sound.

(b) Calculate the beat frequency.

8. A 440 Hz tuning fork is sounded at the same time as a 337 Hz tuning fork. How many beats will be heard in 3.0 s?

9. A trumpet player sounds a note on her trumpet at the same time as middle C is played on a piano. She hears 10 beats over 2.0 s. If the piano's middle C has been tuned to 256 Hz, what are the possible frequencies of the note she is sounding?

10. A string on an out-of-tune piano is struck at the same time as a 440 Hz tuning fork is sounded. The piano tuner hears 12 beats in 4.0 s. He then slightly increases the tension in the string in order to increase the pitch of the note. Now he hears 14 beats in 4.0 s.

(a) What was the original frequency of the string on the out-of-tune piano?

(b) Is the piano more or less in tune after he tightens the string? Explain.

QUICK LAB

Determining the Effect of a Load on the Frequency of a Tuning Fork

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

Strike two identical tuning forks and listen to the sound that they produce. Then, using a small quantity of modelling clay, add equal loads to both arms of one of the forks, equal distances from the ends. Again, strike the two forks and listen to the sound produced. Repeat with an increased load, then a load at a different distance from the ends.

Analyze and Conclude

1. How do you know that the two tuning forks were emitting sounds of the same frequency?
2. How do you know that the loaded tuning fork had a different frequency? Was it higher or lower?
3. Determine the frequency of the loaded tuning fork.
4. How was the frequency of the loaded tuning fork affected by (a) an increase in the load? (b) an increase in the distance of the load from the end of the fork?

Apply and Extend

The A above middle C on a piano is supposed to be tuned to 440 Hz. Use a 440 Hz tuning fork and your knowledge of beats to determine how far a piano is out of tune. Provide a quantitative measure for how far out of tune the piano is.

8.3 Section Review

1. K/U
   - **(a)** What would you listen for when searching for evidence of interference effects in sound?
   - **(b)** What would you expect to hear as a result of destructive interference? As a result of constructive interference?
2. I Two loudspeakers are placed side by side about 0.5 m apart, and driven in phase with the same audio frequency signal.
   - **(a)** Describe the sound that you expect to hear along the perpendicular bisector of the line that connects the two speakers. Why do you expect to hear this along the perpendicular bisector?
   - **(b)** If a frequency of 340 Hz is used, it will produce sound waves with a wavelength of about 1 m. Where would you expect destructive interference to occur? Use a diagram to help you explain.
   - **(c)** If the two speakers were driven in opposite phase to each other, what would you expect to hear along the perpendicular bisector? Why?
3. C
   - **(a)** Explain how a tuning fork vibrates.
   - **(b)** Explain how this mode of vibration can be seen as resulting from more than one sound source.
   - **(c)** Where would you expect to find quiet regions near the tuning fork? Why?
4. C What are "beats"? What conditions are needed to produce them? What is the beat frequency?
5. C Outline the arguments and the evidence that you might use to convince someone that the properties of sound can be explained very effectively by using a wave model.

8.4 Vibrating Columns of Air

SECTION EXPECTATIONS

- Identify the conditions required for resonance.
- Analyze, through experimentation, the components of resonance.
- Describe how knowledge of the properties of sound have been applied to musical instruments.

KEY TERMS

- noise
- music
- harmonics
- fundamental frequency
- sound spectrum
- closed air column
- resonance length
- displacement node
- displacement antinode
- open air column

Music, Noise, and Resonance in Air Columns

If you strike two stones together, you produce a sound that is immediately recognizable, but which has no specific pitch. A **noise**, such as this, is a mixture of many sound frequencies with no recognizable relationship to each other. **Music**, on the other hand, is a mixture dominated by sound frequencies known as **harmonics** that are whole-number multiples of the lowest frequency or **fundamental frequency**. By plotting the intensity of the various sound frequencies that make up a sound, you can see graphically the difference between music and noise. The **sound spectrum** of music consists of a number of discrete frequencies, while the sound spectrum for noise shows a continuous or nearly continuous range of frequencies.

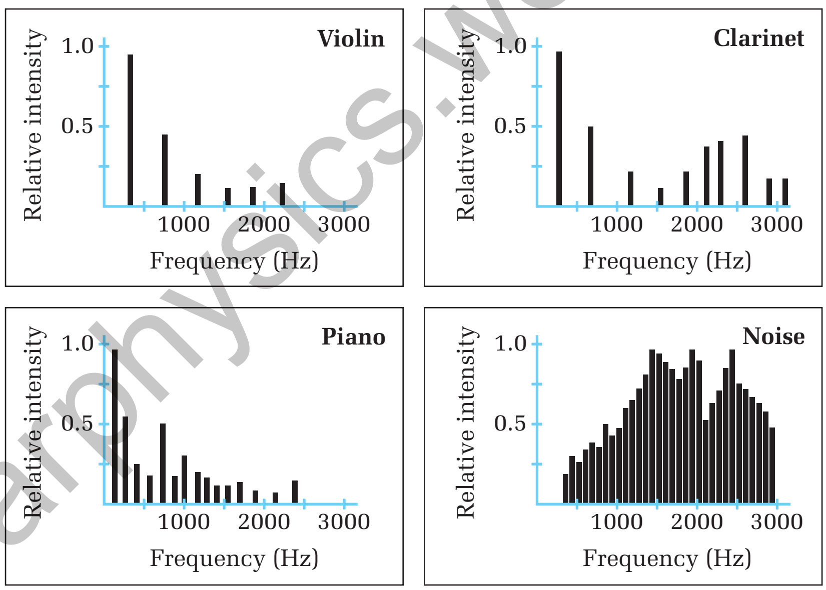


Figure 8.16 Sound spectra of notes played on musical instruments and of a noise

Musical instruments can produce sounds that are harmonious because the vibrations that are set up are essentially in one dimension only. In a guitar or violin, the vibrations are set up in a string. In a wind instrument, like a trombone or clarinet, vibrations are set up in a narrow column of air. Many musical sounds, including the human voice, are produced by resonance in air columns. Resonance in a linear air column occurs when a standing wave is established. The air column (the same as a spring) can sustain a standing wave only for frequencies of vibration that are whole-number multiples of a fundamental. Thus, the sound waves that are emitted from the air column are all whole-number multiples (or integral multiples) of a fundamental, and the total effect is perceived as musical.

Resonance Lengths of a Closed Air Column

An air column that is closed at one end and open at the other is called a **closed air column**. If a tuning fork is held over the open end and the length of the column is increased, the loudness of the sound will increase very sharply for specific lengths of the tube, called **resonance lengths**. If a different tuning fork is used, there will still be distinct resonance lengths, but they will be different than those you found with the first fork.

Resonance in a Closed Air Column

Resonance occurs in an air column when the length of the air column meets the criteria for supporting a standing wave. The

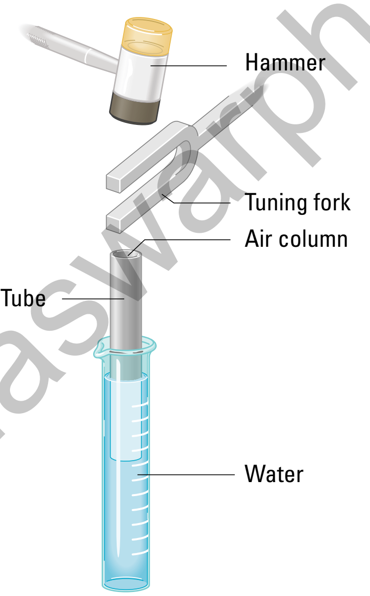
TRY THIS...

Borrow a trumpet, trombone, or other brass instrument. Take the mouthpiece out of the instrument and blow through it. The sound that your vibrating lips make is noise. Re-insert the mouthpiece into the instrument, and then blow again. (You might need a few practice tries to produce a note.) The air column of the instrument will resonate with only one of the frequencies your lips are producing. It is this frequency that you hear.

QUICK LAB Resonance Lengths of a Closed Air Column

TARGET SKILLS

- Performing and recording
- Identifying variables



Place a 50 cm long piece of plastic pipe inside a large graduated cylinder almost completely filled with water that is at room temperature. Sound a 512 Hz tuning fork and hold it over the open end of the plastic pipe. Raise the pipe slowly out of the water while keeping the tuning fork positioned over the open end. Measure the lengths of the air column for which resonance occurs. Repeat the procedure using a 1024 Hz tuning fork.

Analyze and Conclude

1. Use a thermometer to measure the room temperature. Calculate the speed of sound in air, and from that, the wavelength of the sound produced by the 512 Hz tuning fork.
2. By how much is one resonance length longer than the previous one? (If you were able to determine three or more resonance lengths, was this increase in length constant?) What fraction of a wavelength is this increase in resonance length?
3. Repeat questions 1 and 2 for the 1024 Hz tuning fork.

tuning fork produces a sound wave, which travels down the air column and is reflected off the closed end. This reflected wave interferes with the wave from the tuning fork, producing a standing wave.

It is easier to see how resonance occurs if sound waves are represented in the same manner as standing waves on a rope or spring, as presented in Chapter 7. In the case of sound waves, however, the amplitude of the wave does not represent the actual position of the molecules in the air as it does for the rope. The amplitude of the sound wave represents the extent of the longitudinal displacement of the molecules in the air at that specific point along the air column.

In Figure 8.17, the arrows on the right hand side show the distance that the air molecules move (displacement) at different points along the tube. At the closed end of a tube, the air molecules are prevented from moving entirely. Thus, when the standing wave is set up, there has to be a **displacement node** at this closed end. (There will also be a pressure antinode at the same point because the closed end will experience the greatest variation in pressure over time.) At the open end of the air column, however, the air molecules are free to move back and forth relatively easily. Thus, when a standing wave is set up in the air column, there must be a **displacement antinode** at this open end.

ELECTRONIC LEARNING PARTNER

Your Electronic Learning Partner provides more examples of resonance.

PHYSICS FILE

Theories attempt to explain the complexities of real-world phenomena. In deriving the resonance lengths for a closed air column, it was assumed that the behaviour of the air in the column could be treated as being one-dimensional (like the standing waves in a spring). This is a very good assumption for the air inside the air tube, but is not as good at the open end, particularly if the diameter of the tube is large. As a result, the antinode at the open end of the tube actually lies a short distance inside the end. The nodes inside the tube, however, are spaced one half wavelength apart, regardless of the diameter of the tube.

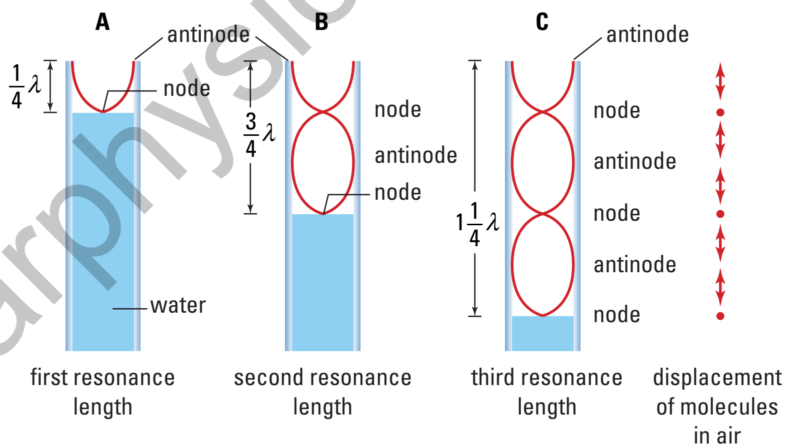


Figure 8.17 First three resonance lengths of a closed air column and their displacement standing wave patterns

In Figure 8.17 (A), you can see that the first resonance length is equal to one quarter of a wavelength, or $\frac{\lambda}{4}$. In (B), the second resonance length is equal to three quarters of a wavelength, or $\frac{3\lambda}{4}$. Finally, in (C), the third resonance length is equal to one and a quarter wavelengths, or $\frac{5\lambda}{4}$. Each of the subsequent resonance lengths is half a wavelength longer than the previous one. Thus, for the first three resonance lengths, the pattern is $\lambda = 4L_1$, $\lambda = \frac{4L_2}{3}$, $\lambda = \frac{4L_3}{5}$, $\lambda = \frac{4L_4}{7}$. To find the resonance lengths, solve for L_n in the equations, where n is a positive integer.

RESONANCE LENGTHS OF A CLOSED AIR COLUMN

The resonance lengths of a closed air column are odd integer multiples of the first resonance length, $\frac{1}{4}\lambda$.

$$L_n = (2n - 1)\frac{\lambda}{4}$$

where n is a positive integer.

Resonance Lengths of an Open Air Column

An air column that is open at both ends is called an **open air column**. At these open ends, the air molecules are free to move easily, so that when a standing wave is set up, there is a displacement antinode at each end, as shown in Figure 8.18.

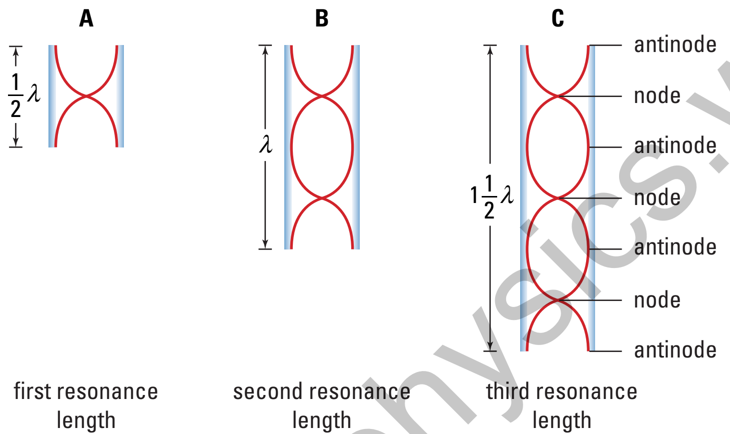


Figure 8.18 First three resonance lengths of an open air column and their displacement standing wave patterns

As you can see in Figure 8.18, the first three resonance lengths are $\frac{\lambda}{2}$, λ, $\frac{3\lambda}{2}$, with each of the subsequent resonance lengths half a wavelength longer. Thus, for the first three resonance lengths, the pattern is $\lambda = 2L_1$, $\lambda = L_2$, $\lambda = \frac{3L_3}{2}$.

RESONANCE LENGTHS OF AN OPEN AIR COLUMN

The resonance lengths of an open air column are integral multiples of the first resonance length, $\frac{1}{2}\lambda$.

$$L_n = \frac{n\lambda}{2}$$

TRY THIS...

Convince yourself that the equation in the box will give you the expressions for the resonance wavelengths by first solving for each L in the expressions at the bottom of page 392, and then by substituting the values one through four into the equation.

Music Link

Wind instrument players play different notes by varying the lengths of the air columns in their instruments. In a trombone, this is most evident as the slide is pushed in and out. In trumpets, tubas, and other brass instruments, different valves are used to open or close auxiliary lengths of tubing. In clarinets, saxophones, and other woodwinds, key pads are lifted off holes in the side of the instrument. In what other ways do these instruments vary?

MODEL PROBLEM

Resonance Lengths of a Closed Air Column

A vibrating tuning fork is held near the mouth of a narrow plastic pipe partially submerged in water. The pipe is raised, and the first loud sound is heard when the air column is 9.0 cm long. The temperature in the room is 20°C.

(a) Calculate the wavelength of the sound produced by the tuning fork.

(b) Calculate the length of the air column for the second and third resonances.

(c) Estimate the frequency of the tuning fork.

Frame the Problem

- Since one end of the pipe is submerged in water, it is a *closed air column*.
- The shortest *resonance length* is one quarter of the wavelength of the sound.
- The resonance lengths of a closed air column are $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$.
- The *wave equation* applies to all kinds of waves.

Identify the Goal

(a) The wavelength, λ, of the sound

(b) The length of the air column for the second and third resonances, L_2 and L_3

(c) The frequency, f, of the tuning fork

Variables and Constants

Involved in the problem		Known	Unknown	
L_1	v	$L_1 = 9.0$ cm	L_2	v
L_2	f	$T = 20°C$	L_3	f
L_3	T		λ	
λ				

Strategy

You can find the wavelength of the sound because you know the first resonance length. Substitute this value into the equation.

Calculations

$$L_1 = \frac{\lambda}{4}$$

$$\lambda = 4L_1$$

$$\lambda = 4\,(9.0 \text{ cm})$$

$$\lambda = 36 \text{ cm or } 0.36 \text{ m}$$

(a) The wavelength of the sound is 0.36 m.

Strategy

You can find the second and third resonance lengths because you know the wavelength and the sequence of formulas for resonance lengths.

Calculations

$L_2 = \frac{3\lambda}{4}$

$L_2 = \frac{3}{4}(36\text{ cm})$

$L_2 = 27\text{ cm or }0.27\text{ m}$

Substitute the wavelength into the equations for second and third resonance lengths.

$L_3 = \frac{5\lambda}{3}$

$L_3 = \frac{5}{4}\ (36\text{ cm})$

$L_3 = 45\text{ cm or }0.45\text{ m}$

(b) The second and third resonance lengths are 0.27 m and 0.45 m, respectively.

You can find the frequency of the tuning fork if you know the speed of sound in the room and the wavelength of the sound.

$v = f\lambda$

You can find the speed of sound if you know the temperature in the room.

$v = 331 + 0.59T_C$

Substitute the value of the temperature in the formula for the speed of sound.

$v = 331\text{ m/s} + 0.59\ \frac{\frac{\text{m}}{\text{s}}}{\cancel{^\circ\text{C}}}\ (20\cancel{^\circ\text{C}})$

$v = 331\text{ m/s} + 11.8\text{ m/s}$

$v = 342.8\text{ m/s}$

Substitute the values of the wavelength (in metres) and the speed of sound in the formula for the frequency.

Substitute first

$v = f\lambda$

$342.8\text{ m/s} = f(0.36\text{ m})$

$f = \frac{342.8\ \frac{\cancel{\text{m}}}{\text{s}}}{0.36\ \cancel{\text{m}}}$

$f = 952.2\text{ s}^{-1}$

Solve for *f* first

$v = f\lambda$

$f = \frac{v}{\lambda}$

$f = \frac{342.8\ \frac{\cancel{\text{m}}}{\text{s}}}{0.36\ \cancel{\text{m}}}$

$f = 952.2\text{ s}^{-1}$

(c) The frequency of the tuning fork is 9.5×10^2 Hz.

Validate

If the first resonance length is 9.0 cm, the second is three times the first (27 cm), and the third is five times the first (45 cm).

At a temperature of 20˚C, the speed of sound is about 340 m/s. The wavelength of the sound is $4 \times 9 = 36$ cm, which is about $\frac{1}{3}$ m. The frequency is then approximately 340 m/s divided by $\frac{1}{3}$ m, or 1000 Hz.

continued ►

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PRACTICE PROBLEMS

11. A narrow plastic pipe is almost completely submerged in a graduated cylinder full of water, and a tuning fork is held over its open end. The pipe is slowly raised from the water. An increase in loudness of the sound is heard when the pipe has been raised 17 cm and again when it has been raised 51 cm.

(a) Determine the wavelength of the sound produced by the tuning fork.

(b) If the pipe continues to be raised, how far from the top of the pipe will the water level be when the next increase in loudness is heard?

12. The first resonance length of an air column, resonating to a fixed frequency, is 32 cm.

(a) Determine the second and third resonance lengths, if the column is closed at one end.

(b) Determine the second and third resonance lengths, if the column is open at both ends.

13. The third resonance length of a closed air column, resonating to a tuning fork, is 95 cm. Determine the first and second resonance lengths.

14. The second resonance length of an air column, open at both ends and resonating to a fixed frequency, is 64 cm. Determine the first and third resonance lengths.

15. A particular organ pipe, open at both ends, needs to resonate in its fundamental mode with a frequency of 128 Hz. The organ has been designed to be played at a temperature of 22°C.

(a) How long does the organ pipe need to be?

(b) If this pipe is closed at one end by a stopper, at what fundamental frequency will it resonate?

Figure 8.19 Bugles have no valves, keys, or slides, so how can the bugler play a tune?

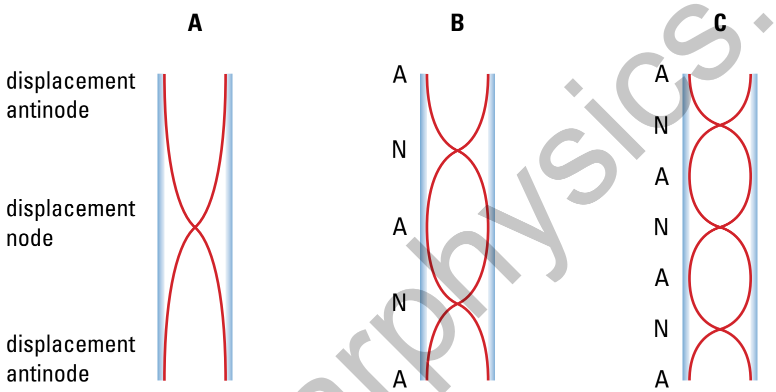
Resonance Frequencies for Fixed-Length Air Columns

How can a bugler play a melody when there are no valves, keys, or slides? The length of the air column is fixed, so the fundamental frequency is fixed. When trumpeters and trombonists blow into the mouthpiece, the vibration of their lips creates a range of frequencies. The length of the air column determines the fundamental frequency and overtones, which it "amplifies" by setting up a standing wave. The resonating air column is, in fact, what makes the sound musical.

The pitch of a wind instrument can be changed not only by increasing or decreasing the length of its air column, but also by increasing or decreasing the tension in the player's lips (the embouchure). This change in the player's embouchure produces a different range of frequencies as the player blows through the mouthpiece. High tension of the lips creates only frequencies that are higher than the fundamental. The result is that the pitch is perceived as the lowest frequency of the overtone frequencies that set up standing waves in the air column. Although buglers have no means of varying the length of the air columns, they can still play a melody by varying their embouchures. They can access a range of different notes when playing “Reveille,” “Last Post,” or “Taps.”

Open Air Columns

Most wind instruments, such as the bugle and the flute, behave the same as air columns that are open at both ends. The major exceptions are reed instruments such as the clarinet, oboe, and bassoon. They behave like closed air columns. For an open air column of given length L, the fundamental and overtone frequencies can be calculated in a straightforward way. The wave equation tells you that the frequency is the quotient of the speed of the wave and the wavelength, $f = \frac{v}{\lambda}$. Thus, you can find the relationship between the frequency and the length of the air column by substituting the expression for wavelength in terms of air column length, L, as shown in Figure 8.20.



$\lambda_1 = 2L$ $\quad$ $\lambda_2 = L$ $\quad$ $\lambda_3 = \frac{2L}{3}$

$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$ $\quad$ $f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1$ $\quad$ $f_3 = \frac{v}{\lambda_3} = \frac{v}{\frac{2L}{3}} = \frac{3v}{2L} = 3f_1$

Figure 8.20 **(A)** Fundamental mode or first harmonic; **(B)** first overtone or second harmonic; **(C)** second overtone or third harmonic

In general, the harmonics of an open air column form the series f_1, $2f_1$, $3f_1$, $4f_1$, $5f_1$, and so on. That is, all integer multiples of the fundamental frequency are produced when an air column is open at both ends. Thus, the effect of these frequencies sounding together is music rather than noise. This is why all of these frequencies are referred to as harmonics — the fundamental frequency as the first harmonic, the first overtone as the second harmonic, and so on.

Math Link

By doubling the frequency of a note, a wind player jumps an octave in pitch. For brass players, this is easily accomplished by tightening up their embouchure to move from the first to the second harmonic of an open air column. For a clarinet player, however, this will not work because of the odd integer harmonic structure of a closed air column. Use the equation for resonance frequencies in closed air columns to show why this is true.

Clarinet players cannot jump an octave by changing their embouchure.

RESONANCE FREQUENCIES OF A FIXED-LENGTH OPEN AIR COLUMN

The resonance frequencies of a fixed-length open air column are integral multiples of the first resonance frequency, f_1.

$$f_n = nf_1$$

where $f_1 = \dfrac{v}{2L}$

Closed Air Columns

For the clarinet, which behaves like a closed air column, the overtones produced are somewhat different.

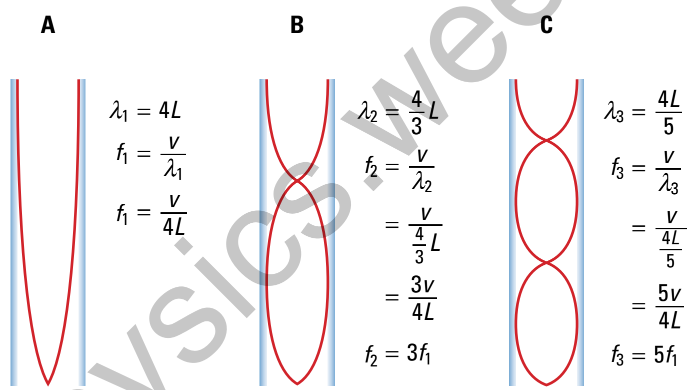


Figure 8.21 **(A)** Fundamental mode or first harmonic; **(B)** first overtone or second harmonic; **(C)** second overtone or third harmonic

In general, the harmonics of a closed air column form the series f_1, $3f_1$, $5f_1$, $7f_1$, $9f_1$, and so on. That is, only odd integral multiples of the fundamental frequencies are produced when the air column is closed at one end.

RESONANCE FREQUENCIES OF A FIXED-LENGTH CLOSED AIR COLUMN

The resonance frequencies of a fixed-length closed air column are odd integer multiples of the first resonance frequency, f_1.

$$f_n = (2n - 1)f_1$$

where $f_1 = \dfrac{v}{4L}$

INVESTIGATION 8-A

Determining the Speed of Sound in Air

TARGET SKILLS
- Predicting
- Performing and recording
- Identifying variables
- Communicating results

Problem

How can resonance in air columns be used to obtain a more accurate value for the speed of sound in air?

Prediction

Make a prediction about the relative precision of measurements of the speed of sound. Will measuring the speed of sound using air columns have greater precision than the method you used in the Quick Lab on page 382? On what do you base your prediction?

Equipment

- 512 Hz and 1024 Hz tuning forks
- rubber mallet
- hollow plastic pipe (do *not* use a glass pipe)
- 1000 mL graduated cylinder
- water at room temperature
- thermometer
- metre stick

CAUTION Use only plastic pipes with tuning forks. If glass pipes are used, there is a danger of flying glass if the vibrating tuning fork touches the pipe.

Procedure

1. After placing the plastic pipe in the graduated cylinder, fill the cylinder with water, as near to the top as possible. Using the thermometer, measure the temperature of the air in the room.
2. Sound the 512 Hz tuning fork, and hold it over the open end of the plastic pipe. Raise the pipe slowly out of the water while keeping the tuning fork positioned over the end of the plastic pipe. Locate the positions for which the sound increases dramatically. (Ignore positions of slightly increased sound of different frequencies from your tuning fork.) Have your partner carefully measure the length of the air column for each of these resonance points.
3. Arrange your data in four columns, with the headings: Resonance length, Change in resonance length, Wavelength, Speed of sound ($v = 512 \times \lambda$).

   Frequency = 512 Hz
   Temperature = ________
4. Repeat the procedure with the 1024 Hz tuning fork. Arrange your data and calculations in a similar table.

Analyze and Conclude

1. Using the data you obtained with the 512 Hz tuning fork, calculate an average value for the speed of sound in air. Estimate the precision of this value.
2. Using the data you obtained with the 1024 Hz tuning fork, calculate an average value for the speed of sound in air. Estimate the precision of this value.
3. Using your measurement of the temperature of the air, calculate a value for the speed of sound in air, using $v = 331 + 0.59T_C$.
4. Earlier in this chapter, you used an echolocation procedure to determine the speed of sound in air. Compare the two values you obtained in this experiment with each other and with your earlier result. Compare the precisions of the three results. Which result do you think is the most accurate? Why?
5. How do your experimental results for the speed of sound in air compare with the calculated value? Which do you trust the most? Why?

MODEL PROBLEMS

Harmonics in a Fixed-Length Air Column

1. An air column, open at both ends, has a first harmonic of 330 Hz.

(a) What are the frequencies of the second and third harmonics?

(b) If the speed of sound in air is 344 m/s, what is the length of the air column?

Frame the Problem

- The air column is *open at both ends*, so the *harmonics* are integral multiples of f_1.
- The frequency of the first harmonic, or f_1, is 330 Hz.
- The frequency of the first harmonic is equal to the speed of sound in air divided by twice the length of the air column.

Identify the Goal

(a) The frequencies of the second and third harmonics, f_2 and f_3

(b) The length, L, of the air column

Variables and Constants

Involved in the problem		Known	Unknown
f_1	v	$f_1 = 330$ Hz	f_2
f_2	L	$v = 344$ m/s	f_3
f_3			L

Strategy

You can find the frequencies of the second and third harmonics because you know the frequency of the first harmonic.

Substitute this value into the equations.

Calculations

$f_n = nf_1$

$f_2 = 2f_1$

$f_2 = 2(330\ \text{Hz})$

$f_2 = 660$ hz

$f_3 = 3f_1$

$f_3 = 3(330\ \text{Hz})$

$f_3 = 990$ Hz

(a) The frequencies of the second and third harmonics are 660 Hz and 990 Hz, respectively.

Strategy

You can find the length of the air column because you know the speed of sound in air and the frequency of the first harmonic.

Substitute these values into the equation.

Calculations

Substitute first

$$f_1 = \frac{v}{2L}$$

$$330\ \text{Hz} = \frac{344\frac{\text{m}}{\text{s}}}{2L}$$

$$330\ s^{-1}(2L) = \frac{344\frac{\text{m}}{\text{s}}}{\cancel{2L}}(\cancel{2L})$$

$$\frac{\cancel{660\ \text{s}^{-1}}\,L}{\cancel{660\ \text{s}^{-1}}} = \frac{344\frac{\text{m}}{\cancel{\text{s}}}}{660\ \cancel{\text{s}^{-1}}}$$

$$L = 0.5212\ \text{m}$$

Solve for *L* first

$$f_1 = \frac{v}{2L}$$

$$f_1 L = \frac{v}{2\cancel{L}}(\cancel{L})$$

$$\frac{\cancel{f_1}L}{\cancel{f_1}} = \frac{v}{2f_1}$$

$$L = \frac{344\frac{\text{m}}{\cancel{\text{s}}}}{2\left(330\ \cancel{\text{s}^{-1}}\right)}$$

$$L = 0.5212\ \text{m}$$

(b) The length of the air column is 0.52 m.

Validate

For an open air column, the harmonics are all integral multiples of the first harmonic. Thus, the frequency of the second harmonic is $2 \times 330 = 660$ Hz, and the frequency of the third harmonic is $3 \times 330 = 990$ Hz.

For the first harmonic, the resonance length is $\frac{1}{2}\lambda$ and the wavelength can also be calculated from $\lambda = \frac{v}{f}$. The length is equal to $\frac{1}{2} \times \frac{344}{330}$ or approximately $\frac{1}{2} \times 1$ or 0.5 m. These values are very close to the calculated values.

2. An air column, closed at one end, has a first harmonic of 330 Hz. If the speed of sound in air is 344 m/s, what is the length of the air column?

Frame the Problem

- The air column is *closed at one end*, so the *harmonics* are $f_1, 3f_1, 5f_1, 7f_1, 9f_1, \ldots$.
- The frequency of the first harmonic, or f_1, is 330 Hz.
- The frequency of the first harmonic is equal to the speed of sound in air divided by four times the length of the air column.

Identify the Goal

The length, L, of the air column

Variables and Constants

Involved in the problem	Known	Unknown
f_1	$f_1 = 330$ Hz	L
v	$v = 344$ m/s	
L		

continued ►

continued from previous page

Strategy

You can find the length of the air column because you know the speed of sound in air and the frequency of the first harmonic.

Substitute these values into the equation.

The length of the air column is 0.26 m.

Calculations

Substitute first

$$f_1 = \frac{v}{4L}$$

$$330\ \text{Hz} = \frac{344\,\frac{\text{m}}{\text{s}}}{4L}$$

$$330\ \text{Hz}\,(L) = \frac{344\,\frac{\text{m}}{\text{s}}}{4\cancel{L}}(\cancel{L})$$

$$\frac{\cancel{330\ \text{s}^{-1}}\,L}{\cancel{330\ \text{s}^{-1}}} = \frac{86\,\frac{\text{m}}{\cancel{\text{s}}}}{330\ \cancel{\text{s}^{-1}}}$$

$$L = 0.2606\ \text{m}$$

Solve for *L* first

$$f_1 = \frac{v}{4L}$$

$$f_1 L = \frac{v}{4\cancel{L}}(\cancel{L})$$

$$\frac{\cancel{f_1}L}{\cancel{f_1}} = \frac{v}{4f_1}$$

$$L = \frac{344\,\frac{\text{m}}{\cancel{\text{s}}}}{4\left(330\ \cancel{\text{s}^{-1}}\right)}$$

$$L = 0.2606\ \text{m}$$

Validate

For the first harmonic of a closed air column, the resonance length is $\frac{1}{4}\lambda$ and the wavelength can be calculated from $\lambda = \frac{v}{f}$. Thus, the length is $\frac{1}{4} \times \frac{344}{330}$, or approximately $\frac{1}{4} \times 1$, or 0.25 m.

PRACTICE PROBLEMS

16. An air column, open at both ends, resonates with a fundamental frequency of 256 Hz. Determine the frequencies of its first and second overtones (second and third harmonics).

17. A bugle is essentially a 2.65 m pipe that is open at both ends.

(a) Determine the lowest frequency note that can be played on a bugle.

(b) Determine the next two higher frequencies that will produce resonance.

18. A trombone is playing F (87.3 Hz) as its first harmonic. A trombone functions as an air column that is open at both ends.

(a) Determine the second harmonic.

(b) If the speed of sound is 344 m/s, what is the length of the tubing being used? Why would this note be difficult to play?

8.4 Section Review

1. C Explain the difference between noise and music.

2. K/U Why are the resonance lengths of open and closed air columns different?

3. I A fixed-length air column has only one fundamental frequency. How is it possible to play a melody on an instrument such as a bugle, which has a fixed-length air column?

4. C Explain the statement: "The first overtone is the second harmonic."

CHAPTER 8 Review

REFLECTING ON CHAPTER 8

- A wave model of sound predicts that sound requires a medium and that sound waves will exhibit interference effects with each other.
- Sound does not travel through a vacuum.
- For a wave theory of sound:
  - loudness corresponds to amplitude
  - pitch corresponds to frequency
  - quality corresponds to harmonic structure
- Sound waves are longitudinal waves.
- Human beings can hear sounds in a frequency range from 20 Hz to 20 000 Hz.
- Sound frequencies are classified as infrasonic (less than 20 Hz), audible (20 to 20 000 Hz), and ultrasonic (greater than 20 000 Hz).
- Loudness is what the ear perceives. Sound intensity is the energy delivered by a sound wave in 1 s over an area of 1m^2. Intensity is measured in picowatts per square metre (pW/m^2).
- The intensity level of a sound wave is a logarithmic scale of sound intensity measured in decibels. A 20 dB sound has a sound intensity 10 times greater than a 10 dB sound. A 30 dB sound has an intensity 100 times greater than a 10 dB sound (10 dB = 1 B = 10pW/m^2).
- The speed of sound in air at standard atmospheric pressure (101 kPa) is constant for a given temperature, and given by the equation $v = 331 + 0.59T_C$.
- In general, the speed of sound is slowest in gases, faster in liquids, and fastest in solids.
- When two sounds of similar frequency are sounded at the same time, alternately loud and quiet sounds are produced. This wavering effect is called beats.

  The beat frequency is given by the equation
  $$f_{beat} = |f_2 - f_1|$$
- Music is a sound made up of whole number multiples of a lowest or fundamental frequency. Noise is a sound made up of a multitude of sound frequencies with no recognizable relationship to each other.
- A closed air column (closed at one end) has resonance lengths
  $$L_n = \frac{(2n-1)\lambda}{4}$$
- An open air column (open at both ends) has resonance lengths
  $$L_n = \frac{n\lambda}{2}$$
- A closed air column of fixed length has resonance frequencies
  $$f_n = (2n-1)f_1$$
  where $f_1 = \frac{v}{4L}$.
- An open air column of fixed length has resonance frequencies
  $$f_n = nf_1$$
  where $f_1 = \frac{v}{2L}$.

Knowledge/Understanding

1. What properties of sound suggest a wave theory for sound? How does the wave theory explain each?
2. What properties of sound does a wave theory predict should exist that otherwise might not have been considered?
3. Explain ways in which the simple wave theory of sound is testable.
4. Describe the bell-jar experiment and its observations. How do these observations help increase our confidence in the wave theory of sound?
5. Sketch pairs of sound waves that illustrate the following contrasts in sound.
   (a) pitch (low versus high)
   (b) loudness (quiet versus loud)
   (c) quality (pure versus rich)
6. Why, in physics, is sound not defined in terms of the pressure waves that humans can hear?

7. (a) What is the standard audible range of frequencies for the human ear?
   (b) Explain the meaning of the terms "infrasonic" and "ultrasonic."
   (c) How does the human audible frequency range compare to the range for dogs? Bats? Porpoises?
8. (a) A sound intensity level of 1 bel is equal to an intensity of 10pW/m^2. What are the intensities for intensity levels of 2 bels? For levels of 3 bels?
   (b) What term is used to describe a scale such as the sound intensity scale?
9. Estimate a reasonable sound intensity level in decibels for
   (a) students whispering
   (b) a passing train
   (c) a jet engine
10. As the temperature of air increases, what happens to a sound's (a) speed? (b) frequency? (c) wavelength?
11. Does sound travel faster in a gas or in a solid? Explain why you think that this is the case.
12. Some animals use short pulses of high-frequency sound to locate objects. If the echo is received a time interval Δt after the pulse of sound is transmitted, how could you use the Δt and the speed of sound to calculate the distance between the animal and the object?
13. Describe the location of the quiet regions near a tuning fork. Explain how these are created.
14. Describe the phenomenon of beats and the conditions necessary to produce them.
15. A trumpet player is tuning the instrument to concert A (440 Hz) being played by the oboe. As the trumpet player moves the tuning slide in, beat frequency increases. Is the trumpet getting closer to, or farther away from, the correct pitch? Explain.
16. How is music different from noise?
17. When a trombone player pushes the slide of the instrument out, the pitch of the sound being produced decreases. Explain why this happens.
18. When a stream of water is used to fill a graduated cylinder, a sound of steadily rising pitch is heard. Explain why this is the case.
19. When a stream of air is directed over the end of a 40 cm long piece of plastic pipe, open at both ends, a sound is produced.
   (a) Explain why this occurs.
   (b) If the bottom end of the pipe is covered, what will happen to the pitch of the sound? Explain.
20. How do the harmonics of a fixed air column closed at one end compare with the harmonics of a fixed air column open at both ends?

Inquiry

21. Design an experiment to test the assumption that sound waves are influenced by the medium through which they travel. Be sure to identify the variables that you would control and those you would test.
22. A student constructs a home-made flute. She collects the data below, which represent different effective lengths and different harmonics, at an air temperature of 22°C. Copy the table and fill in the third column with an appropriate fraction. Should her instrument be considered an open or closed resonator?

Length at which resonance occurs (m)	Frequency of sound	Resonance length as fraction of wavelength
0.39	440 Hz	
0.67	512 Hz	
0.88	584 Hz	

Communication

23. Typically, thunder is heard a short time after lightning is observed. Describe how this time interval can be used to calculate the distance that the lightning is away. Explain why this strategy yields a reasonable answer. Would you be able to tell if a storm is approaching using this method?

24. What are the first three resonance lengths of a closed air column? Sketch displacement standing wave patterns for each of these.
25. What are the first three resonance lengths of an open air column? Sketch displacement standing wave patterns for each of these.

Making Connections

26. Human hearing receptors are more sensitive to frequencies above 400 Hz than those below. Consider how this variation affects (a) hearing speech on a telephone, (b) noise concerns in a community, and (c) the intensity level at which various types of music sound most pleasant.
27. Thoughtfully consider and discuss how you believe science affected the production of musical instruments, and conversely, how music affected scientific inquiry into sound.

Problems for Understanding

28. Calculate the speed of sound in air for each temperature.
    (a) –40.0˚C
    (b) 5.0˚C
    (c) 21.0˚C
    (d) 35.0˚C
29. Calculate the temperature of the air if the speed of sound is
    (a) 355 m/s
    (b) 344 m/s
    (c) 333 m/s
    (d) 318 m/s
30. A hunter wanted to know the air temperature. The echo from a nearby cliff returned 1.5 s after he fired his rifle. If the cliff is 250 m away, calculate the air temperature.
31. On a crisp fall day, a cottager looks across the lake and sees a neighbour chopping wood. As he watches, he notices that there is a time delay of 2.1 s between the time the axe hits the log and when he hears the sound of its impact. If the air temperature is 8.0˚C, how far is he from his neighbour?
32. A guitar player is tuning his guitar to A (440 Hz) on the piano. He hears 14 beats in 4.0 s when he tries to play A on his guitar.
    (a) What two frequencies might he be playing?
    (b) Explain how he could determine which of the two he was playing.
33. Two tuning forks of frequencies 441 Hz and 444 Hz are sounded at the same time.
    (a) Describe what is heard.
    (b) Calculate the beat frequency, and explain what it is.
34. When a violin plays concert A, it produces a sound spectrum with a very intense line at 440 Hz, and less intense lines at 880 Hz, 1320 Hz, 1760 Hz, 2200 Hz, and 2640 Hz.
    (a) Explain why this sound would be described as music.
    (b) Sketch a possible sound spectrum (intensity against frequency) for noise.
35. A narrow plastic pipe is placed inside a large graduated cylinder almost filled with water. An 880 Hz tuning fork is held over the open end as the pipe is slowly raised out of the water.
    (a) Describe what will be heard as the pipe is raised.
    (b) Assuming the air temperature to be 22˚C, calculate the first four resonance lengths.
36. A slightly smaller-diameter plastic pipe is inserted inside a second plastic pipe to produce an air column, open at both ends, whose length can be varied from 35 cm to 65 cm.
    A small loudspeaker, connected to an audio frequency generator, is held over one of the open ends. As the length of the air column is increased, resonance is heard first when the air column is 38 cm long and again when it is 57 cm long.
    (a) Calculate the wavelength of the sound produced by the audio frequency generator.
    (b) Assuming the air temperature to be 18˚C, calculate the frequency setting of the audio frequency generator.

Numerical Answers to Practice Problems

1. (a) 3.2×10^2 m/s **(b)** 3.4×10^2 m/s **(c)** 3.5×10^2 m/s **(d)** 3.2×10^2 m/s **2. (a)** 35.6˚C **(b)** 11.9 ˚C **(c)** 5.1 ˚C **(d)** –20.3 ˚C **3. (a)** 6.2×10^2 m **4.** 0.005 s **5.** 2.0×10^2 m **6. (a)** 5.8 s **(b)** 6.7×10^{-6} m **(c)** 2.8 km **7. (b)** 6.00 Hz **8.** 9.0 beats **9.** 251 Hz or 261 Hz **10. (a)** 443 Hz **11. (a)** 68 cm **(b)** 85 cm **12. (a)** 96 cm, 160 cm **(b)** 64 cm, 96 cm **13.** 19 cm, 57 cm **14.** 32 cm **15. (a)** 1.34 m **(b)** 64 Hz **16.** 512 Hz, 768 Hz **17. (a)** 64.9 Hz **(b)** 130 Hz, 195 Hz **18. (a)** 175 Hz **(b)** 1.97 m

CHAPTER 9

Sound in Our World

CHAPTER CONTENTS

A blue whale's 180 dB rumble is the loudest animal sound ever recorded. Whale sounds also appear to be part of a highly evolved communication system. Some whales are thought to communicate over hundreds and maybe thousands of kilometers. This is possible, in part, because sound waves travel five times faster in water than in air. In addition, the temperature characteristics of ocean water — decrease in temperature with depth — create a unique sound phenomenon. Layers of ocean water at specific temperatures "focus" the sound energy in channels that allow the sound to travel tremendous distances with little dissipation.

Artificial sounds in the oceans may be partially responsible for some recent threats to whale populations. In June of 2000, 16 whales and dolphins were found dead or dying on beaches in the Bahamas. Scientists found that the whales had torn and bleeding eardrums consistent with a "distant explosion or intense acoustic event." Some people attributed the beachings to ongoing U.S. Navy sonar experiments involving sound intensities of over 200 dB. More and more, the oceans are becoming awash in noise pollution, from the constant hum of cargo ships to remote sensing systems like sonar. The background noise in much of the ocean has reached the 85 dB level. The increased noise pollution of the underwater environment is prompting exciting research dealing with the mysteries of sound in the depths of our oceans.

This one example alone shows you the importance of understanding sound energy and its applications in the living world. In this chapter you will learn about many practical applications of sound as well as concerns relating to sound.

MULTI LAB

Analyzing Sound

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

Musical Pipes

You can make musical instruments out of PVC™ pipe by cutting it into specific lengths. Make several pipes and determine the pitch of each by comparing the sound to a piano or other tuned instrument. Label the pipes with their musical notes. Form small groups and work out some tunes.

Try this:
F F C C D D C
B♭ B♭ A A G G F

Analyze and Conclude

1. How is the resonant frequency of each pipe related to its length?
2. Sketch the standing waves in a pipe. Label nodes and antinodes.

Seeing Speech

Cut or melt a hole in the bottom of a plastic cup. Cut out a piece of balloon or cellophane that is larger than the top of the cup. Stretch the piece of balloon over the mouth of the cup and secure it with an elastic band. Observe the stretched balloon while your lab partner speaks into the hole in the bottom of the cup. When you speak into the hole, touch the balloon lightly to see how sound "feels."

Analyze and Conclude

1. Describe and explain your observations of the balloon.
2. Would this device work if a balloon is attached to both ends of the cup? Explain.
3. Would the device in question 2 work if the space between the balloons is a vacuum? Explain.

Motion and Sound

Strike a tuning fork and then rest it gently against a suspended pith ball. Observe the effect the tuning fork has on the pith ball. Hold your palm tightly against your ear and press the base of a struck tuning fork on your elbow.

Analyze and Conclude

1. Describe and explain the motion of the pith ball.
2. Describe and explain the results of holding a sounded tuning fork against your elbow. What general statements can you make about sound waves based on this activity?

9.1 Human Hearing and Speech

SECTION EXPECTATIONS

- Describe the perception mechanisms that allow the ear to distinguish between different frequency sounds.
- Analyze and interpret energy transformations in the human ear.

KEY TERMS

- outer ear
- middle ear
- inner ear
- place theory of hearing
- temporal theory of hearing
- conductive hearing loss
- sensorineural hearing loss
- vocal cords
- articulators
- resonators

The human body is one of the finest examples of physics at work that you will ever discover. In your study of hearing, what you might expect to be a simple process, you will encounter compressional waves in air, resonance in pipes, mechanical vibrations, compressional waves in fluids, resonance in a membrane, and electrical signals.

The Human Ear

You have already learned that the human ear can detect sound waves with frequencies between 20 Hz and 20 000 Hz. As well, you learned that the ear can detect sound intensities as low as a few picowatts per square metre and may withstand sound intensities as high as ten billion picowatts per square metre. The next important question to ask is, "How does the ear accomplish these tremendous feats?" In this section, you will look at the structure and function the ear as an instrument of sound detection and interpretation.

Scientists divide the ear into three parts, the outer, middle, and inner ear. The **outer ear** captures and guides sound waves in air. The **middle ear** converts sound waves into mechanical vibrations of three tiny bones. The **inner ear** converts these vibrations into sound waves in a fluid. The fluid motion stimulates specialized cells in the inner ear, which communicate with neurons (nerve cells) that send coded messages to the brain. Parts of the inner ear also sense changes in motion and the direction of the gravitational force. These functions, however, do not involve waves or sound so you will not examine the structures.

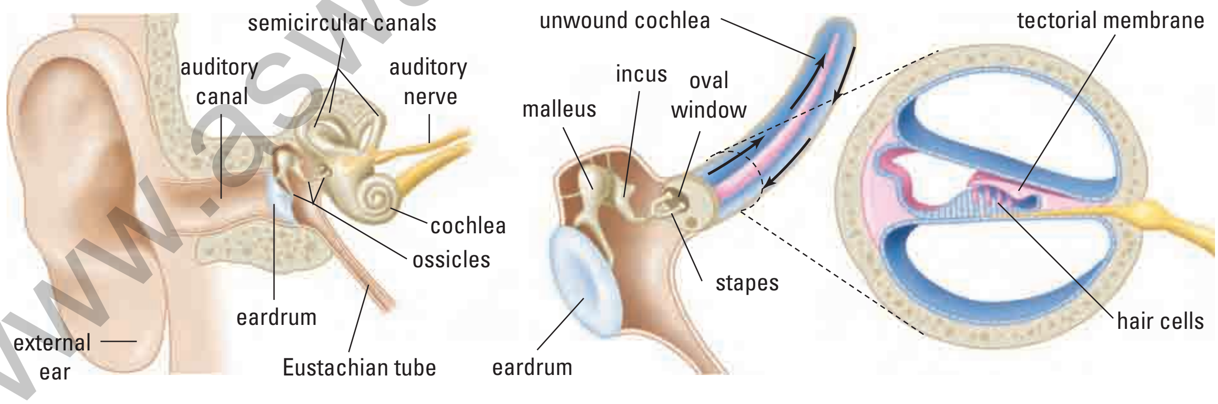


Figure 9.1 The outer, middle, and inner ear.

The Outer Ear

The external ear and the auditory canal, shown in Figure 9.1, constitute the outer ear. The external ear captures sound waves and funnels them down the auditory canal. As you might expect from its shape, the auditory canal functions as a closed-end, resonating air column. Its resonant frequencies vary from approximately 2.5 kHz to 4.5 kHz, corresponding to the range of frequencies to which the human ear is most sensitive.

The Middle Ear

The middle ear begins with the eardrum or tympanic membrane, a cone-shaped membrane that stretches across the end of the auditory canal. Sound waves arriving at the eardrum carry slight changes in pressure, causing the eardrum to vibrate at the frequency of the sound waves (Figure 9.2). Since the eardrum must respond to very small variations in pressure, it is critical that, in the absence of sound, the pressure is the same on the two sides of the delicate membrane. This "resting" condition is maintained by the eustachian tube by connecting the air filled cavity of the middle ear to the outside air through the throat and mouth. If the outside pressure changes significantly and puts pressure on the eardrum, the eustachian tube might open suddenly, causing the familiar "popping" of the ear.

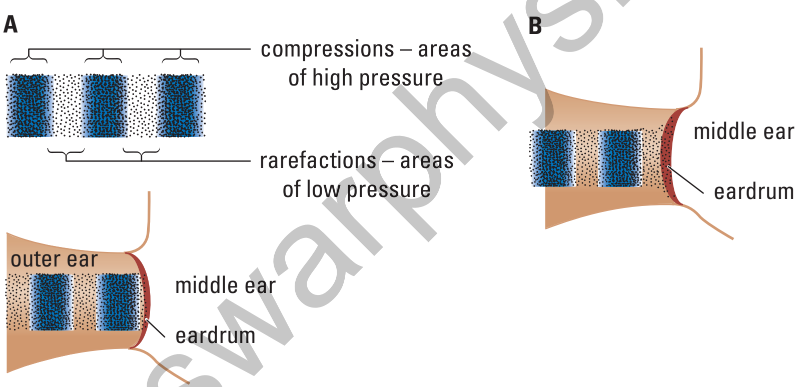


Figure 9.2 **(A)** Compressions cause the eardrum to bend inward. **(B)** Rarefactions cause the eardrum to bend outward because their pressure is lower than that of the middle ear.

Three small bones, called ossicles, form a chain that transmits and amplifies the vibrations of the eardrum across the middle ear to the oval window of the inner ear. The first bone, the malleus, attaches to the eardrum from one edge to the centre, causing the eardrum to assume its cone shape. The malleus vibrates with the eardrum and transmits the vibrations on to the incus, which then pushes and pulls the stapes. The stapes passes the amplified

PHYSICS FILE

Biophysicist John van Opstal, who works at the University of Nijmegen in the Netherlands, conducted a six-week-long study into the function of the external ear. He and his students wore plastic inserts in their ears continuously for six weeks. The inserts effectively changed the shape of their sound-capturing external ears. Our brains compare the relative loudness and arrival time of sounds reaching each ear in order to locate the sound source in the horizontal plane. Dr. Opstal found that vertical source location was related to the shape of our external ear's nooks and crannies and was learned by our brain over time. Students wearing the plastic inserts were unable to determine if a sound was originating above or below them. During the study, each person began to be able to detect vertical location, until finally they had all learned to do so with their "new" ears. Interestingly, when they removed the plastic inserts, they were still able to detect vertical changes in a source's position, demonstrating that our brains are able to store more than one acoustic imprint.

PHYSICS FILE

The stapes is the smallest bone in the human body, averaging less than 0.33 cm in length with a mass of less than 4 mg.

motion to the oval window, part of the inner ear. You may have heard of the ossicles by their common names, hammer, anvil, and stirrup, which describe the shapes of these tiny bones.

Muscles in the middle ear respond to loud sounds by putting tension on the stapes, thus preventing it from causing excessively large vibrations of the oval window. Researchers have theorized that in some young children the stapes actually loses contact with the oval window during periods of excessive noise, thus protecting the child's hearing.

The Inner Ear

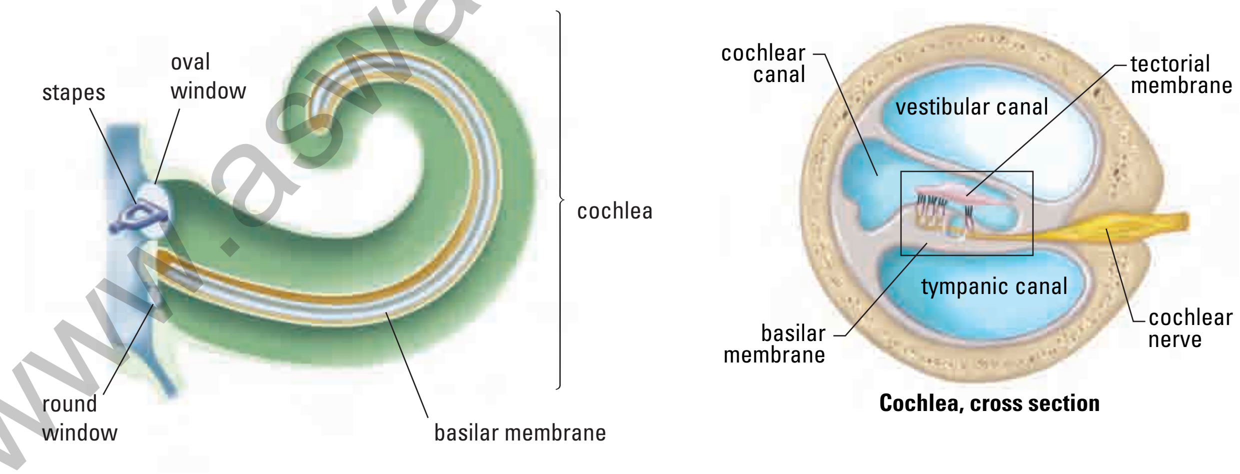
PHYSICS FILE

The minimum distance that the eardrum must move, in order for the motion to be detected as sound, is smaller than the diameter of a hydrogen atom!

The inner ear is a small bony structure containing membranous, fluid-filled tubes and cavities. The cochlea is the snail-shaped segment of the inner ear that functions in hearing. Figure 9.3 shows the cochlea as though it were unwound. In the figure you can see the continuous channel of fluid that vibrates back and forth with the vibrations of the oval window. At the far end of the bony channel is the round window with its flexible membrane that relieves the pressure of the moving fluid.

The cross section of the channels, also shown in Figure 9.3, shows yet another, smaller channel. The sensory cells, called hair cells, sit on the basilar membrane of this channel. Vibrations of the fluid in the outer channel initiate vibrations of the basilar membrane. The tiny protrusions on the hair cells are imbedded in a gelatinous structure called the tectoral membrane. When the basilar membrane vibrates up and down, the tectorial membrane pulls and pushes the "hairs" from side to side. The hair cells are not themselves neurons (nerve cells) but they respond to the motion of the "hairs" by releasing chemicals that stimulate neurons. The neurons then carry electrical impulses to the brain where the impulses are interpreted as sound.

Figure 9.3 If you could unwind the cochlea, you would see two outer channels that connect at the apex (tip) of the cochlea. Vibration of the stapes starts compressional waves in the fluid that initiate vibrational waves in the flexible basilar membrane.



All hair cells are fundamentally the same. How, then, does the ear distinguish so precisely between such a wide range of frequencies of sound, and send information to the brain that can be interpreted as pitch? Scientists have been studying this problem for hundreds of years and have provided several models that can be divided into two general theories, the place theory and the temporal theory.

The term "place" in the **place theory of hearing** refers to the position along the basilar membrane that resonates in response to a given frequency of vibration. The shape of the basilar membrane changes gradually along the cochlea. Near the oval window, it is narrow and stiff but becomes wider and more flexible as it approaches the apex or tip of the winding cochlea. The narrow, less flexible end of the basilar membrane resonates at high frequencies much like a taut string. As the frequency decreases, portions of the basilar membrane further from the oval window resonate. Figure 9.4 shows the positions along the cochlea that resonate at various frequencies of the compressional waves in the fluid. You can see an example of larger vibrations in one region of the membrane than in the rest of the membrane.

The **temporal theory of hearing** proposes that the brain receives information about sound waves in the form of impulses arriving at the brain with the same frequency as the sound wave. A single neuron is not capable of firing at any but the very lowest frequencies of sound waves that the ear detects. Thus the theory proposes that a group of nerve fibres works as a unit to send impulses to the brain with the same frequency as the incoming sound wave.

The place theory is firmly established by experimental observations but the temporal theory cannot be ruled out. It quite possibly plays a role in coding messages for the brain to interpret. As researchers learn more about the details of the coding and decoding of sound messages to the brain, scientists and physicians will be better able to develop technologies to assist the hearing-impaired.

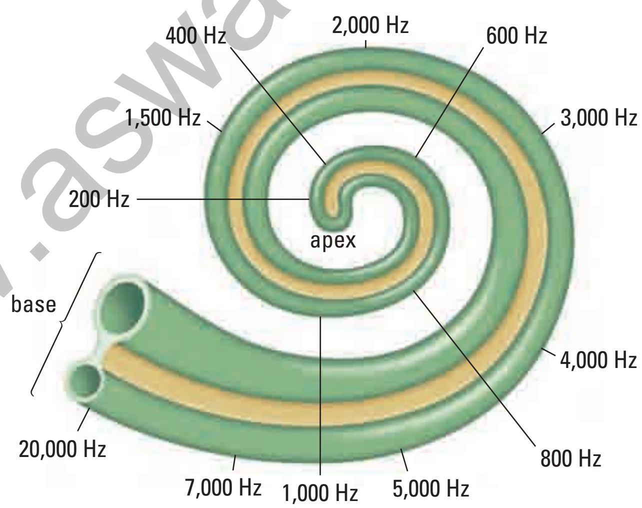


Figure 9.4 When a wave travels down the basilar membrane, the part of the membrane with a natural frequency similar to the travelling wave vibrates at a much greater amplitude than the rest of the membrane.

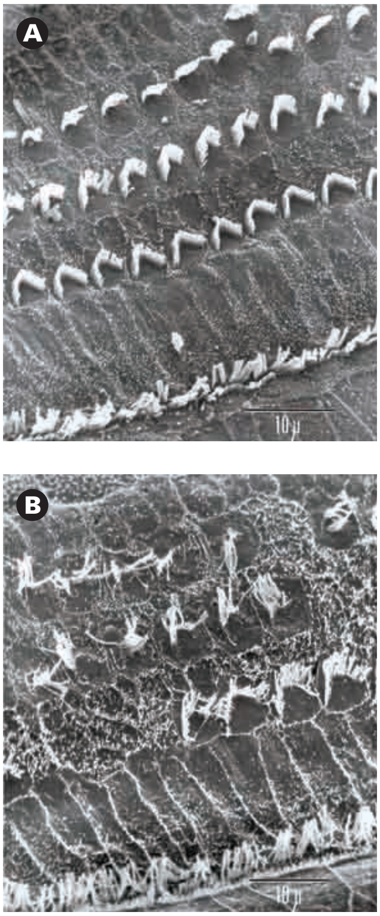


Figure 9.5 The hair cells in **(A)** are from a healthy inner ear of a guinea pig. The hair cells in **(B)** are from a guinea pig that had been exposed to 120 dB sound, very similar to sound at a loud concert.

Hearing Impairment

Hearing loss is divided into two categories based on the structures in the ear that are responsible for the impairment. **Conductive hearing loss**, as the name suggests, results from improper conduction of sound waves through the ear passages. The simplest form of conductive hearing loss is the accumulation of earwax in the auditory canal. Ear wax, a sticky substance secreted in the ear canal, serves several purposes such as catching dirt particles, preventing germs from reaching the eardrum, and, due to its unpleasant odour, repelling insects. If, however, an excess of earwax accumulates, it will reduce the amount of sound energy reaching the eardrum. The problem is easily corrected by removal of the wax by a physician.

Middle ear infections also cause conductive hearing loss. Infections often cause an accumulation of liquid in the normally air-filled cavity around the tiny ossicles. The amount of energy required for the bones to vibrate is much greater in a liquid than in air, because liquid friction is much greater than air friction. Hearing is usually restored when the infection subsides. Persistent infections, however, can cause permanent damage. For example, frequent or long-term infections might stimulate the growth of a type of skin over the eardrum that makes it harder and less flexible, making it difficult to hear anything but loud sounds. Other types of growths may also form as a result of long term infections.

The second category of impairment, **sensorineural hearing loss**, includes damage to the sensory cells (hair cells) or to the nerves that conduct the electrical impulses to the brain. The two most common causes of sensorineural hearing loss are aging and long-term exposure to loud sounds. Roughly one third of the population over age 65 experiences some loss of hearing, especially in the high frequency range, the frequencies detected by the part of the basilar membrane nearest the oval window. Most people in this category retain their sensitivity to mid-range and low frequencies and thus can still hear normal speech. The loss of high frequency sound often makes it difficult to distinguish certain words, as though the speaker was mumbling.

Prolonged exposure to sounds over 85 dB can permanently damage the sensory hair cells. Figure 9.5 shows the damage to guinea pig hair cells after the animal was exposed to sounds equivalent to a loud concert. Table 9.1 is a guide to the length of time the ear might endure a sound level before permanent damage is likely to occur.

Table 9.1 General Exposure Guidelines

Exposure time (h)	Sound level (dB)
8	90
4	95
2	100
1	105
1/2	110
<1/4	115

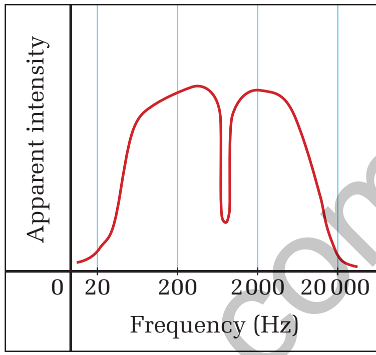


Figure 9.6 In a test, a person was exposed to sounds at the same intensity level through the entire range of frequencies. The apparent intensity of sound that the person heard shows that he or she had lost the ability to hear a very narrow range of frequencies.

Think It Through

- Notice, in Table 9.1, that the exposure time is cut in half when the sound goes up by five decibels. Based on your knowledge of the decibel scale, explain why a small increase on the decibel scale results in a much greater danger of suffering permanent damage to the sensory cells in the ear.

Exposure to sounds of a specific, small range of freqecies over a long period of time, can permanently damage just those sensory cells that respond to that particular frequency. For example, a person might operate a piece of machinery that emits a sound with a frequency around 1000 Hz. The individual would eventually lose the ability to hear those frequencies, as illustrated in Figure 9.6.

Sound Technologies

Hearing aid technology has seen dramatic advances in recent decades. Digital hearing aids, such as the one in Figure 9.7, that the wearer can adjust by remote control are now available. The wearer can turn them up or down, tune out background noise, and even set the amplification for specific frequencies. Such hearing aids are often the best solution to partial hearing loss caused by damage or degeneration of sensory cells.

Total hearing loss can sometimes be treated with cochlear implants. The "implant" consists of an array of tiny wire electrodes that are threaded directly into the cochlea. The electrodes are connected to a receiver-stimulator that is implanted in the bone just behind the ear. An external microphone receives sound waves and a processor converts the sound into electrical signals. A transmitter, attached just behind the ear, sends signals through the skin to the receiver that then stimulates the auditory nerve in a combination of positions along the cochlea. The implant recipient does not hear sounds in the same way that a hearing person does.

Figure 9.7 A hearing aid is composed of three distinct components: a microphone, an amplifier, and a speaker. Improvements in technology allow for higher quality components of smaller and smaller sizes. The amplifier often contains a programmable computer chip.

Nevertheless, with practice, they are able to recognize words and phrases — in some cases, for the first time in their lives. As the technology improves, cochlear implants may dramatically increase the quality of life for individuals with complete hearing loss.

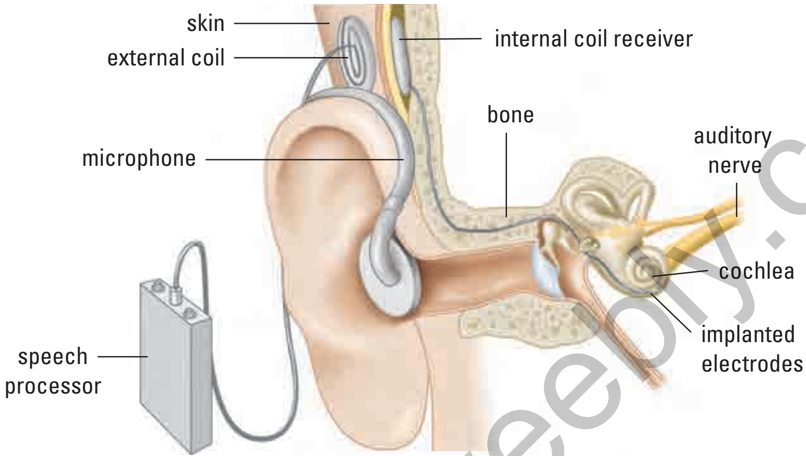


Figure 9.8 A cochlear implant

QUICK LAB: Test Your Hearing

TARGET SKILLS
- Analyzing and interpreting
- Communicating results

Read and answer each of the following questions:

- Do you have trouble hearing over the telephone?
- Do you have trouble following conversation when two or more people are talking at the same time?
- Do you have to strain to understand conversations?
- Do others complain that you turn the TV volume too loud?
- Do you miss hearing common sounds like the phone or doorbell ringing?
- Do you get confused about the direction from which a sound comes?
- Do you misunderstand some words in a sentence and need to ask people to repeat themselves?
- Do you especially have trouble understanding the speech of small children or people with high voices?
- Do you have a history of working in noisy environments?
- Do you experience ringing or buzzing sounds in your ears?

Analyze and Conclude

If you answered yes to any of the questions, you may have hearing loss.

Explain the reason why F, S, P, and TH sounds are commonly the first sounds that people who develop hearing loss find difficult to hear.

The Human Voice

When you blow on PVC pipe, your lips vibrate and create sounds having a wide range of frequencies. The length of the bottle or pipe cause certain frequencies to resonate and those are the sounds that you hear. Your voice works in much the same way. Your **vocal cords** create the initial vibrations and your throat and oral cavity act somewhat like a resonating closed-end pipe, with your vocal cords at the closed end and your mouth at the open end. In fact, your vocal cords are never completely closed, but their small size makes a pipe closed at one end an acceptable model of your vocal tract.

Your vocal cords are two thin folds of muscle and elastic tissue that relax when you are not speaking. To speak, your muscles stretch your vocal cords. Your diaphragm (a wall of muscle underneath your lungs that acts like a bellows) creates air pressure behind your vocal cords. A steady pressure from your diaphragm causes your vocal cords to vibrate. Classical singers know how to concentrate their energy in the diaphragm, so the air passes effortlessly over the vocal cords. Rock singers usually over-tense their vocal cords, which creates a dramatic sound but can cause vocal injuries such as nodules (small growths on the vocal cords that must be surgically removed). Greater tension in the vocal cords and higher air pressure create vibrations at higher frequencies.

Biology Link

Healthy vocal cords are smooth and moist. Laryngitis is an inflammation and swelling of the vocal cords. What causes your vocal cords to become inflamed? How can you prevent this from occurring? How should you treat inflamed vocal cords?

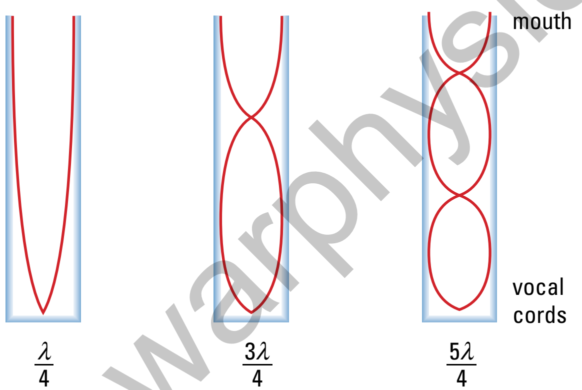


Figure 9.9 Sound is manipulated by changing the resonance of your vocal tract. The vocal tract behaves somewhat like a closed-end resonator.

The typical adult vocal tract functions as a tube approximately 17 cm to 18 cm long. As shown in Figure 9.9, resonant frequencies occur at $\frac{\lambda}{4}$, $\frac{3\lambda}{4}$, and $\frac{5\lambda}{4}$. For a 17 cm tube, these frequencies are 500 Hz, 1500 Hz, and 2500 Hz. Most sounds in normal human speech occur between 300 Hz and 3000 Hz. Thus the first three harmonics of the vocal tract are the most important.

TRY THIS...

Try making the sounds in Table 9.2 and see if you can "feel" how the parts of your oral cavity are functioning in the production of these sounds. Compare the following combinations of sounds and decide how they are alike and how they differ. What is the fundamental difference between the pairs of sounds. (a) *k* and *g*, (b) *sh* and *ch*, (c) *s* and *z*, (d) *t* and *d*.

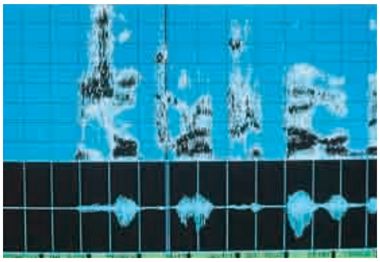


Figure 9.10 This is an actual voice print. Compare it with the simulated voice prints in Figure 9.11. Do you think you could determine the sound a person was making from their voice print alone?

Human speech is far more complex than simply producing one frequency followed by another. Your brain has learned how to direct your vocal tract to create the extremely large variety of sounds required by language. One technique is to modify sounds by changing the shape and position of your **articulators**, which are your lips, tongue, and teeth. Table 9.2 shows which articulators you use to make some very common consonant sounds. To produce different vowel sounds, you also make use of your **resonators**: the elements of your vocal tract, which include your mouth (oral and laryngial pharynx) and your nasal cavities.

Table 9.2 Sounds and Related Articulators and Resonators

Sound	Articulator/Resonator
p, b, m, w	lips
t, d, th	tongue and teeth
s, z, sh, ch	tip of tongue
n	tongue
k, g	middle or back of tongue

Although everyone must create sounds in much the same way, no two voices sound identical. Every person's vocal tract is unique and creates a specific combination of frequencies that can be identified by a "voice print" like the one shown in Figure 9.10. You create particular sounds by combining a specific set of frequencies. A few examples are shown in Figure 9.11.

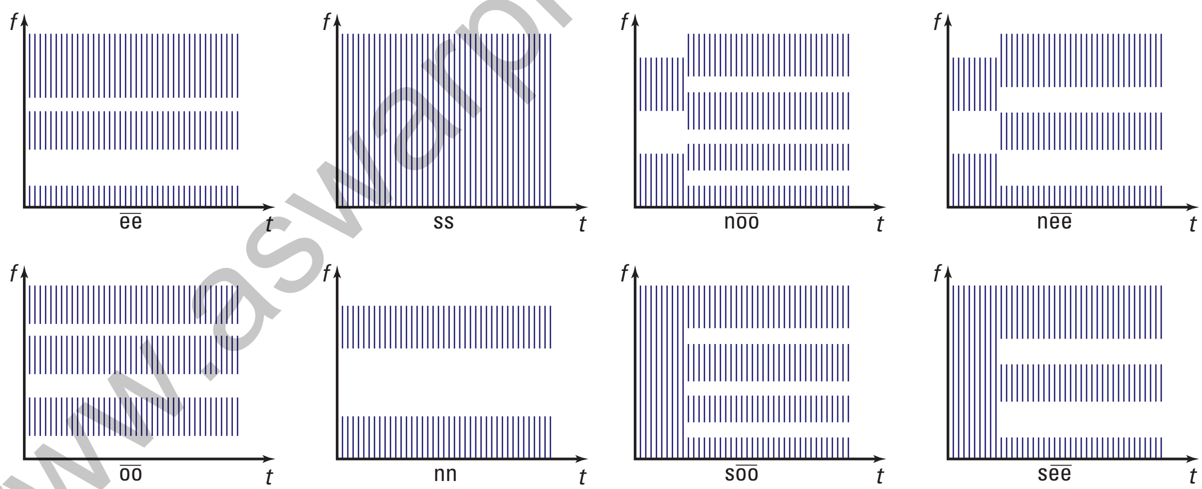


Figure 9.11 Idealized voice prints on frequency versus time axes. Try making each of the sounds represented here. Notice which part of your vocal tract you adjust and how you modify it to make each sound.

9.1 Section Review

1. K/U State the range of hearing of an average human being.
2. K/U Trace the energy path from a sound wave reaching your ear to the signal in your brain.
3. C Describe how the ossicles function to affect the way we hear very soft sounds and very loud sounds.
4. C Describe the process that converts mechanical motion into electrical signals in the cochlea.
5. K/U Summarize (a) the place theory and (b) the temporal theory of hearing.
6. K/U What range of frequencies can an average human generate?
7. C A friend tells you that a dog whistle does not actually make a sound, but rather that the dog is reacting to your breathing change when you blow into the whistle. Describe how you would explain the physics behind a dog whistle to your friend.
8. C Several animal species rely on their hearing as a means of survival. Often, these animals have very large, visible external ears. Is there an advantage for animals to have large external ears? Explain.
9. C Explain why it is difficult to talk while breathing in.
10. K/U List and describe the function of human vocal articulators and resonators.
11. MC Describe each of the two main categories of hearing loss.
12. MC How does the temporary hearing loss associated with a middle ear infection illustrate conservation of energy?
13. MC How will the type of hearing loss associated with long-term exposure to a specific frequency sound over 85 dB most likely affect someone as they age?
14. C As people age, some hearing loss is inevitable. Explain how a digital hearing aid is able to improve hearing by doing more than simply amplifying the sound.
15. C Several friends talking on a front porch hear the screeching of tires, and all turn immediately to face the same direction. Explain how is it possible that each person was able to know in which direction to look.

UNIT ISSUE PREP

Your hearing system is both incredibly dynamic, able to sense varied intensities and frequencies, and very sensitive and susceptible to damage.

- Identify specific mechanisms within your ear that are most susceptible to damage.
- Investigate types of sound energy exposure that can result in damage.

9.2 Motion and Sound

SECTION EXPECTATIONS

- Explain the Doppler effect.
- Investigate applications of ultrasonic sound.
- Analyze environmental impacts of a sonic boom.

KEY TERMS

- Doppler effect
- sonic boom
- Mach number
- echolocation
- sonar
- ultrasound

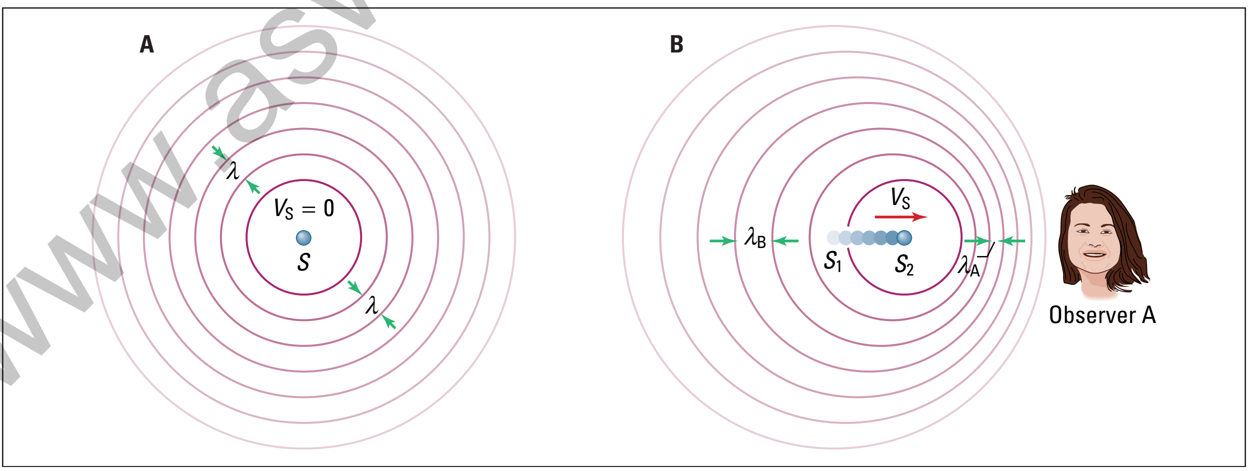
Submarines navigate through the murky depths of the oceans by sending and receiving sound waves. Whales and bats do the same, but with considerably more precision. For people, the ability to develop sound technologies as tools for navigation requires an understanding of the effect of motion on sound waves. If the source of the sound is moving, how does that affect the perception of the observer? Does motion of the observer influence the perception of sound?

The Doppler Effect

Have you ever noticed how the sound of an emergency vehicle's siren seems to increase in pitch as it approaches? Then, just as the vehicle passes, the pitch suddenly appears to drop. The phenomenon responsible for the apparent change in pitch is called the **Doppler effect**. The siren of an emergency vehicle generates exactly the same sound at all times. The apparent changes in pitch as the vehicle approaches, then recedes, results from the motion of the vehicle as the source of the sound.

When the source of a sound is stationary, the sound will radiate outward in the shape of a three-dimensional sphere. This effect is shown in two dimensions in Figure 9.12 A. When the source of a sound is moving relative to the observer, the wave fronts appear as shown in Figure 9.12 B. During the time between successive compressions, which initiate a wave front, the source has moved toward the observer. Therefore, each new compression is nearer to the front of the previously created compression. This motion reduces the effective wavelength of the sound wave and the frequency appears greater. The same line of reasoning explains why the pitch seems to drop as the source of the sound moves away from an observer.

Figure 9.12 The Doppler effect



Think It Through

- Draw a diagram similar to Figure 9.12 B, that demonstrates the Doppler effect as (a) a source moves away from your stationary position and (b) you move toward a stationary source.
- A friend puts a battery and a siren from a toy into a Nerf™ ball. She connects the battery and tosses you the ball with the siren wailing. Describe what you will hear and what she will hear as the ball moves through the air.
- Why does the wavelength of a sound generated from a moving source *decrease* as the speed that the source moves toward you increases? Why (in terms of the wave equation) does the frequency *increase*?
- Draw a diagram that illustrates sound waves surrounding a source that is moving at the speed of sound.

TRY THIS...

Tie a tuning fork securely to a string, strike it, and then let it swing like a pendulum. How does the pitch change as the tuning fork swings toward you and away from you? Explain why. Try it again but stand in front of the swinging tuning fork (so that you are facing it as if it was a grand-father clock). Does the pitch change? Why or why not?

Sonic Booms

An extreme case of the Doppler effect occurs when an object travels at or beyond the speed of sound as shown in Figure 9.13. Figure 9.14 demonstrates compression waves generated by a source moving at various speeds.

If you have ever put your hand out of the window of a moving car, you will know that the pressure of the air on your hand is substantial. Your hand is colliding with air molecules and generating a longitudinal wave that moves out from your hand. Moving your hand through water provides a more visual example of the same phenomenon, except with transverse waves.

Imagine that you are able to keep holding your hand outside of the car as it accelerates toward the speed of sound. The pressure on your hand would become immense, but nothing compared to the pressure that you would feel as you reach and exceed the

PROBEWARE

The **Science Resources** section on the following web site: **www.school.mcgrawhill.ca/resources/** has an excellent laboratory activity on the Doppler effect using probeware equipment. Navigate to the investigation by following the **Physics 11** links.

Figure 9.13 The shock wave as a jet breaks the sound barrier is visible because the increased pressure causes water vapour to condense.

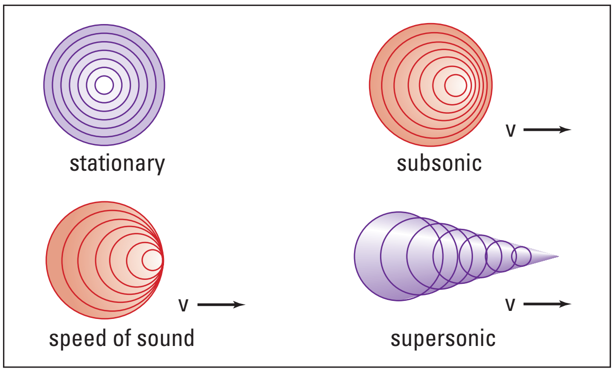


Figure 9.14 Sound waves propagating outward from a source moving at various speeds

Figure 9.15 The large bow wave is the result of the boat moving faster than the speed of the waves in water.

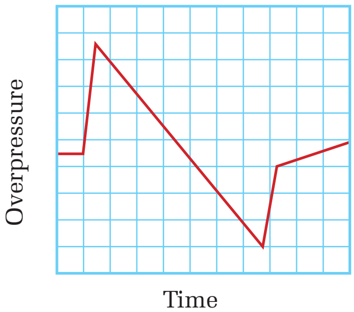


Figure 9.16 Pressure, measured on the ground, versus time as an F-18 fighter jet flies overhead at an altitude of 9000 m and a speed of Mach 1.20. The "N-wave" change in pressure is characteristic of sonic booms.

MISCONCEPTION

The Sound Barrier

Is it true that only fast vehicles and aircraft can break the sound barrier? No! We can "snap" a wet towel, "crack" a whip, and "pop" a balloon, each of which is an example of breaking the sound barrier and producing a mini sonic boom.

speed of sound. If you are travelling slower than the speed of sound, although the pressure is great because of the increased number of air molecule collisions that you are experiencing, the wave fronts are still moving away from your hand. At the speed of sound, you are moving with the same speed as the wave fronts. Now the compressive wave fronts that you generate with your hand cannot move away from your hand. Each successive wave front combines with the ones made before it, creating a massive compression. This area is called the overpressure.

The photo of the large bow wave in front of the boat moving through the water in Figure 9.15 demonstrates how the wave fronts build up when the source is travelling faster than the waves can move in the medium. Likewise, behind your hand a massive rarefaction, or area of extremely low pressure, exists. This rapid change in air pressure, from very high to very low, is called a shock wave and is heard as a **sonic boom**. Figure 9.16 shows what a typical sonic boom pressure profile looks like.

Examine the wavefronts for supersonic motion in Figure 9.14. Notice that many wavefronts converge along a V-shaped path behind the source. Of course, in three dimensions, it forms a cone. This superposition of compressional wavefronts creates extremely large pressure changes as the cone moves. You hear the results as a sonic boom. Figure 9.17 shows you how sonic boom trails behind the source that is moving faster than the speed of sound. Supersonic jets usually fly at very high altitudes because although the sonic boom cone then covers more area on the ground, it is significantly weaker. The sonic boom cone sweeps over a path on the ground that is approximately 1 km wide for every 250 m of altitude. Therefore, a jet flying faster than the speed of sound at an altitude of 10 000 m will create a sonic boom across a 40 km wide path of land beneath the jet. The higher a jet flies, the wider the path of the sonic boom, but the lower the energy level on the ground.

A certain jet flying at an altitude of 9000 m generates a 128 dB sonic boom on the ground. The same jet, flying at an altitude of 30 m would generate a 263 dB sonic boom! The pressure change from typical atmospheric pressure to the maximum pressure of the compressional wave of a sonic boom occurs over a time interval of 100 ms for a fighter jet and in approximately 500 ms for the

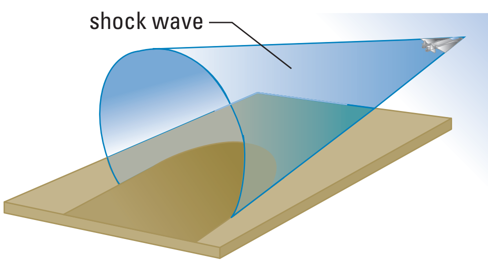


Figure 9.17 The sonic boom trails behind the jet, growing larger but weaker as it expands.

space shuttle or the *Concorde*. Such a sudden, drastic pressure change can force windows to flex beyond their elastic limit, and shatter.

Since the speed of sound in air varies with temperature and pressure, it is not possible to classify specific speeds as subsonic, sonic, and supersonic. Austrian physicist Ernst Mach devised a method to describe these classes of speeds as ratios of the speed of the jet (or other object) to the speed of sound in air that has the temperature and pressure of the air in which the jet is flying. The ratio is now known as the **Mach number**. A mach number of less than one indicates that an object is moving slower than the speed of sound. Mach one means that it is flying at precisely the speed of sound and a Mach number greater than one indicates that the object is moving faster than the speed of sound.

MACH NUMBER

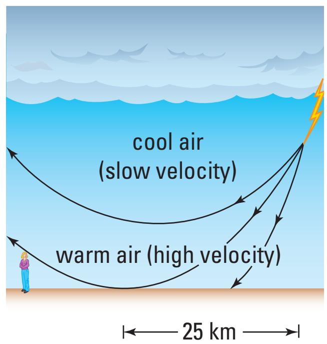
The Mach number of a moving object is the ratio of its speed to the speed of sound in air at conditions identical to those in which the object is moving.

$$\text{Mach number} = \frac{v_{\text{object}}}{v_{\text{sound}}}$$

Quantity	Symbol	SI Unit
speed of object	v_{object}	m/s (metres/second)
speed of sound	v_{sound}	m/s (metres/second)
Mach number		pure number (no units)

PHYSICS FILE

A sudden crack of lightning is often followed by the violent rumble of thunder. The super-heated air actually expands faster than the speed of sound in air, generating a sonic boom. Interestingly, thunder, although extremely intense at its source, is rarely heard more than 25 km away due to the changing air temperature that refracts the sound back up into the sky. During most thunderstorms, the air nearer to the ground is warmer than the air above. The speed of sound is faster in warmer air and therefore causes the rumbling to be refracted up and away from the surface.

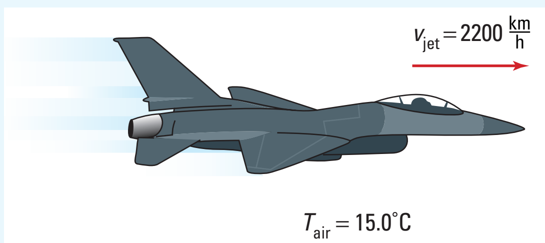


MODEL PROBLEM

Calculating Mach Number

Calculate the Mach number of a Canadian Forces jet flying through 15.0˚C air at standard atmospheric pressure with a velocity of 2.20×10^3 km/h near Cold Lake, Alberta.

Frame the Problem



- *Mach number* is a *ratio* of the *speed of the jet* compared to the *speed of sound*.
- Both speed values must have the same units, so that they will cancel, leaving a Mach number without units.
- The *speed of sound* in air depends on the temperature.

continued ►

continued from previous page

Identify the Goal

The speed of the plane in terms of the Mach number

Variables and Constants

Involved in the problem	Known	Unknown
v_{jet}	$v_{jet} = 2.20 \times 10^3$ km/h	v_{sound}
v_{sound}	$T = 15.0^\circ C$	Mach number
T		
Mach number		

Strategy

Convert v_{jet} to SI units.

Calculations

$$2.20 \times 10^3 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} = 6.11 \times 10^2\ \frac{\text{m}}{\text{s}}$$

Calculate v_{sound} using approximation given by equation (from Chapter 8, Section 2):

$$v_{sound} = 331 + 0.59\ T$$

$$v_{sound} = 331\ \frac{\text{m}}{\text{s}} + 0.59\ \frac{\frac{\text{m}}{\text{s}}}{\cancel{^\circ\text{C}}}(15.0\ \cancel{^\circ\text{C}})$$

$$v_{sound} = 339.85\ \frac{\text{m}}{\text{s}}$$

Both v_{jet} and v_{sound} are in the same units, so you may substitute them into the Mach number equation.

$$\text{Mach number} = \frac{v_{jet}}{v_{sound}}$$

$$\text{Mach number} = \frac{611.11\ \cancel{\text{m/s}}}{339.85\ \cancel{\text{m/s}}}$$

$$\text{Mach number} = 1.80$$

The jet is flying at Mach 1.80.

Validate

A jet flying faster than the speed of sound will have a Mach number greater than 1.0, which is the case here.

PRACTICE PROBLEMS

1. Calculate the Mach number of a bullet travelling at 385 m/s in air at standard conditions.
2. How fast is a jet flying if it is travelling at Mach 0.500 through 25.0° C air at standard atmospheric pressure?
3. What is the minimum speed with which the tip of a whip must travel in air at standard pressure and temperature to make a cracking sound?

Echolocation

Sound can be used to measure distance. The slight time difference between a single sound reaching our two ears allows our brain to pinpoint the source. Some animals have a far more refined auditory system. Toothed whales or *odontocetes* and most species of bats are able to generate and interpret ultrasonic pulses that reflect off obstacles and prey. This process, called **echolocation,** allows animals to pinpoint not only the object's exact location, but also (using Doppler shifts) its speed and direction.

Dolphins, part of the toothed whale family, have very specialized vocal and auditory systems. They make whistle-type vocal sounds using their larynx, which does not contain vocal cords. The echolocation and navigation clicks are produced in their nasal sac region. Dolphins are able to generate sounds ranging in frequency from 250 Hz to 150 kHz. The lower range frequencies, from 250 Hz to 50 kHz, are thought to be used primarily for communication between dolphins, while the higher frequencies are used for echolocation.

Echolocation clicks produced by dolphins, each lasting from 50 ms to 128 ms, are grouped into "trains." A large, flexible, gelatinous outcropping on the front of the dolphin's skull, called a melon, focuses the clicks. The melon is filled mostly with fatty tissue and is easily shaped by the muscles to act as an acoustical lens for the high-pitched clicks. Research has shown that the high-pitched sounds do not travel as far in water as lower frequency noises, resulting in effective echolocation ranges from 5 to 200 m away for targets of 5 cm to 15 cm in length. Dolphins produce a wide range of sounds, varying in frequency, volume, wavelength, and pattern. They are able to identify size, shape, speed, distance, direction, and even some internal structure of the objects in the water. For example, a dolphin can detect flatfish lying beneath a layer of sand on the seabed.

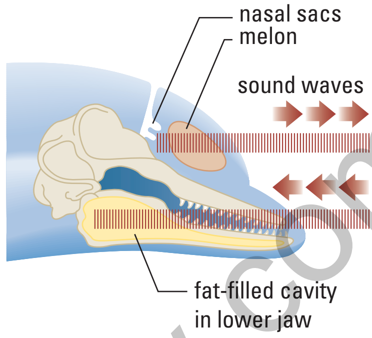


Figure 9.18 The echolocation clicks are generated in the nasal sacs, focussed by the melon, and received by the fat-filled region in the lower jaw.

Figure 9.19 Notice the slightly contorted shape of the melon as this dolphin focusses echolocation clicks. Tests have demonstrated that dolphins are able to distinguish between different-shaped objects contained within a closed box. Essentially, they are able to accomplish naturally what we do with ultrasounds to examine expectant mothers.

Figure 9.20 Most species of bats are able to "see" using ultrasonic sounds, enabling them to catch their prey in the darkness of night.

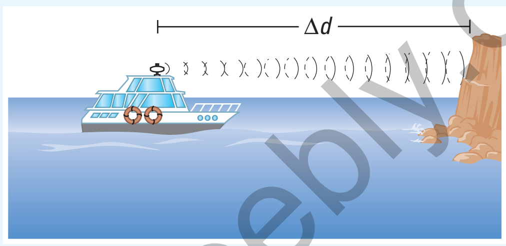
MODEL PROBLEM

Determining Distance

A boat approaches a large cliff in a dense fog. The captain sounds his foghorn and hears an echo 2.4 s later. He assumes the speed of sound to be 343 m/s. Calculate the distance the boat is from the cliff.

Frame the Problem

- Sketch the situation.
- The *sound* must *travel* to the cliff and then *back again*.
- The sound travels *twice the distance* the boat is from the cliff.
- Sound travels at a *constant speed*.



Identify the Goal

The distance Δd, of the boat from the cliff

Variables and Constants

Involved in the problem		Known	Unknown
Δd	Δt_{total}	$v_{sound} = 343$ m/s	Δd
v_{sound}	$\Delta t_{1/2}$	$\Delta t_{total} = 2.4$ s	$\Delta t_{1/2}$

Strategy

Divide the total time in half to find the time required for the sound to reach the cliff.

Calculations

$$\Delta t_{1/2} = \frac{\Delta t_{total}}{2} = \frac{2.4\text{ s}}{2} = 1.2\text{ s}$$

Calculate the distance to the cliff using the speed of sound and the time for half the round trip.

$$\Delta d = v_{sound}\Delta t_{1/2}$$

$$\Delta d = \left(343\ \frac{\text{m}}{\cancel{\text{s}}}\right)(1.2\ \cancel{\text{s}})$$

$$\Delta d = 412\text{ m}$$

The boat is 4.1×10^2 m from the cliff.

Validate

Using the round-trip time and then dividing the final distance in half could also complete this question. The trick to echo questions is remembering that the sound makes a round trip from the source to the reflecting barrier and back.

PRACTICE PROBLEMS

4. While vacationing in northern Ontario, an ingenious physics student decides to calculate the distance from her cottage to the cliff at the far edge of the lake. She finds that the time between her dog barking and the echo returning is 3.5 s. The temperature of the air is 26˚C. How far away is the cliff?

5. The human ear is just able to distinguish between two sounds about 0.10 s apart. How far from a large wall must someone stand in order to just hear an echo? (Assume the speed of sound is 343 m/s.)

CAREERS IN PHYSICS

Sounds from the Seabed

The survey ship glides across the inky-blue ocean. Sensors on its hull sweep back and forth, sending sound waves to the seabed and receiving the sound waves that bounce back. This is multi-beam sonar (SOund Navigation And Ranging). The exact amount of sound energy bouncing back is determined by what is on the seabed. Harder materials send back more energy than softer ones. The sensors on the hull, called transducers, convert the sound waves they receive into electrical signals. These are processed by computer to produce three-dimensional, photographic-quality maps of what is on and beneath the seabed, perhaps an offshore oil pipeline, debris from a downed aircraft, or a sea cage used in fish farming. Viewers can even “fly through” the data to see the seabed in real time.

Scientists in the University of New Brunswick’s Geodesy and Geomatics Engineering Department (GGE) are among those who collect, process, and interpret such data. Dr. Susan E. Nichols and her colleagues in the GGE Department apply their expertise to projects involving ocean governance. For example, countries sometimes disagree over offshore boundaries. Fish farmers sometimes conflict with people in traditional fisheries. Companies interested in developing offshore oil or gas need to know where to explore and where to lay pipelines. Federal, provincial, and municipal governments want to ensure that petroleum development will not conflict with property rights or harm environmentally sensitive areas. In such cases, three-dimensional maps like those from the GGE are a valuable basis for discussing and resolving conflicts.

Besides ocean mapping, other uses for sonar include finding and sizing schools of fish, detecting submarines or icebergs, and determining whether there are valuable gravel, gold, or other mineral deposits in the ocean floor.

Going Further

1. Research sonar in encyclopedias, Internet sites, and/or other sources. Prepare a labelled diagram showing how it works.

2. Find out what is involved in geodesy and geomatics. What are some career possibilities in these fields?

Exploring salmon and lobster farming sites in the Bay of Fundy. Professional geomatics engineer Dr. Susan E. Nichols, is second from the right. With her are (left to right) researchers Michael Sutherland, Rosa Tatasciore, and Sue Hanham.

QUICK LAB

Locating a Low-Pitched Sound

TARGET SKILLS
- Performing and recording
- Communicating results

Test your ability to locate a sound. Two funnels, some rubber tubing, and a T-connector are all you need. Cut one tube to 2.0 m and the other to 3.0 m long. Attach the tubes using the T-connector. Connect a funnel to the end of each tube. Hold the funnels to each ear and have a partner gently tap the open end of the T-connector. From which side of your head does the sound seem to be originating? Switch the funnels to the opposite ears and repeat the experiment.

Analyze and Conclude

1. Did the sound seem to be originating from the funnel attached to the longer or shorter tube?
2. Explain how this demonstration provides insight into how the human hearing system locates the horizontal position of low-pitched sounds.

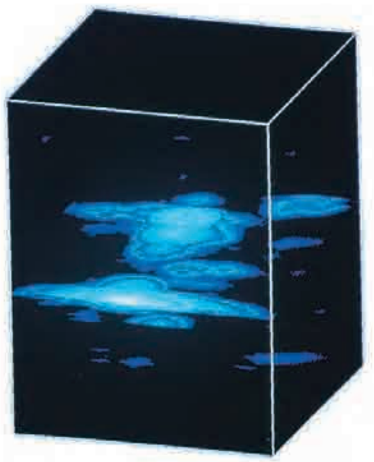


Figure 9.21 Enhanced technology allows very detailed, colour images to be constructed from sonar data.

SONAR

Sonar was developed during World War I (1914–1918) to detect German submarines. The technology has progressed dramatically and so have the applications. If you enjoy fishing, you may have used a fish-finder or depth meter that uses sonar. Figure 9.22 demonstrates the basic principle. In **sonar**, sound pulses are sent out and the reflected signal received. A computer is able to measure the time between the outgoing and incoming signals, which is then used to calculate an object's depth. The computer is also able to determine the relative size of an object based on the intensity of the reflected signal.

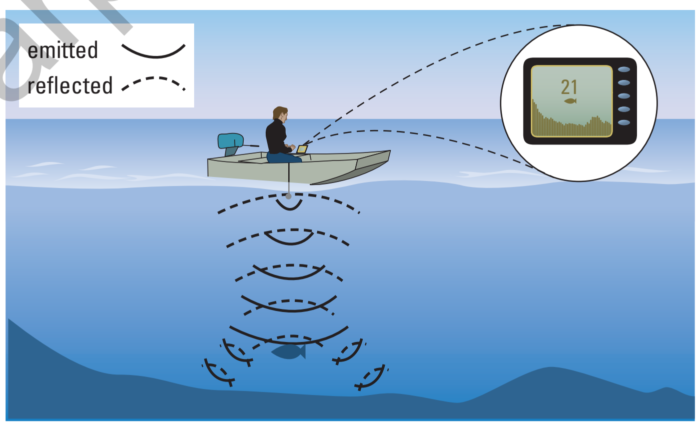


Figure 9.22 Ultrasonic pulses are sent toward the lake bottom. They reflect off fish and off the bottom of the lake. A computer measures the time between the emitted and the received signal and calculates the object's depth. The intensity of the signals provides information about the size of the object.

Uses for sound technologies have grown as computer technology developed speeds required to handle more and more data in real time. Figure 9.23 shows some of the varied applications of **ultrasound** (frequencies above 200 kHz), audible sounds, and infrasound in use today.

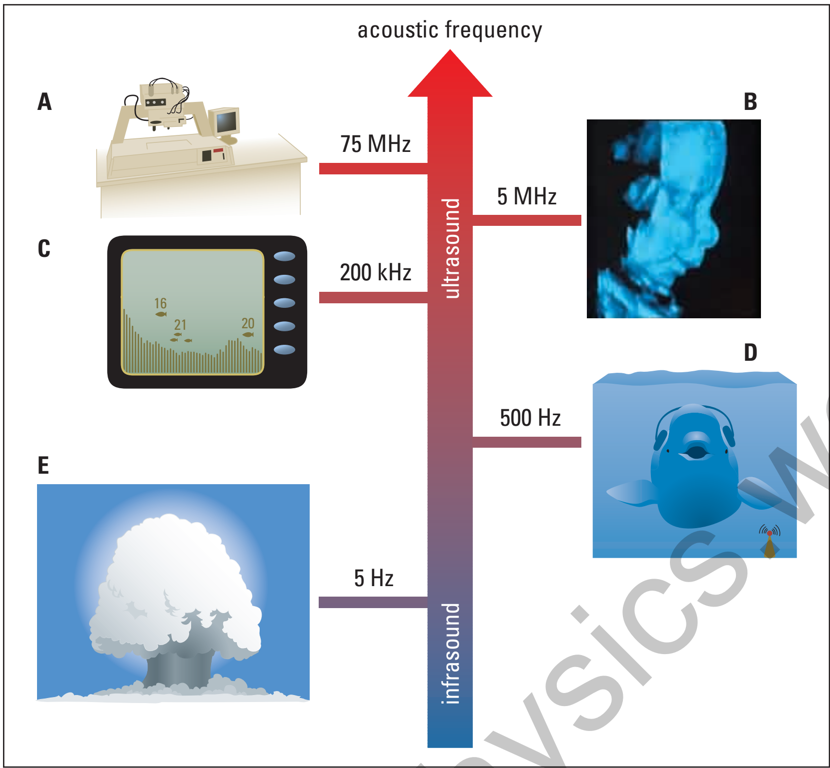


Figure 9.23 **(A)** An acoustic microscope can detect defects in certain materials. **(B)** Doctors use three-dimensional ultrasound images during pregnancy. **(C)** Fish-finders locate fish. **(D)** Long-range military sonar may affect marine life. **(E)** Infrasonic vibration detectors warn of volcanoes, earthquakes, avalanches, or nuclear explosions.

QUICK LAB “Sonar” in the Classroom

TARGET SKILLS
- Identifying variables
- Performing and recording

In a location where your partner cannot see, set up a “rocky ocean floor,” with high ridges and deep valleys. Use boxes, books, basketballs, and anything else that will provide an interesting profile. Once your “sea floor” is complete, your partner will attempt to build a map of the underwater terrain using a motion sensor and computer interface. Work together to determine the best way to conduct the experiment, but remember that your partner needs to construct a terrain map without ever actually looking at the model set-up. Repeat the procedure with your partner building the model and you using the probeware to make the map.

Analyze and Conclude

1. How is the computer able to measure the distance from the “surface” to the “sea bottom”?
2. Describe the factors that you needed to control to get the best image possible.

Apply and Extend

3. How could you improve the mapping procedure to differentiate between large fish and slender, tall peaks rising from the sea floor or perhaps to make a two-dimensional map? If time permits, test your ideas.

9.2 Section Review

1. C Explain the cause of the Doppler effect.
2. K/U Sketch a frequency versus position graph to describe the pitch of a train whistle as it approaches and then recedes from your position.
3. MC You are in a car travelling toward the base of a large cliff in an otherwise completely open area. As you approach the massive wall of rock, you sound the horn.
   (a) Will there be a change in frequency of the reflected sound?
   (b) Is the change due to a moving source or receiver? Explain.
4. C Explain why a sonic boom is capable of breaking windows.
5. K/U Which of the following photographs illustrate a situation or item that could produce a sonic boom?

6. K/U Explain why jets that fly at supersonic speeds should maintain very high altitudes?
7. (a) K/U What frequencies does a dolphin use to echolocate its food?
   (b) MC Give two reasons why the frequency from part (a) is a good choice for tracking fast moving prey.
8. C Explain how the human auditory system determines the direction from which a sound originated. Use diagrams to support your answer.
9. K/U How does sonar detect objects?
10. K/U What is ultrasound and how is it used?
11. MC Ultrasound can be produced by applying an alternating voltage across opposite faces of piezoelectric crystals such as quartz. The applied voltage causes the crystal to expand and contract extremely quickly with the same frequency as the alternating current. Certain frequencies will cause the crystals to resonate, producing very strong ultrasonic waves. Identify as many applications of this effect as you can.

9.3 Musical Instruments

Music has been an important part of human culture throughout history and even in prehistoric times. Some exquisite musical instruments date back thousands of years. Clearly, the technology for making fine instruments existed long before scientists discovered the principles of sound waves and resonance. Understanding the nature of the sounds that blend together to make music has not changed the importance of the music itself to culture and society. The knowledge has, however, made it possible to mimic the sounds of instruments electronically and also to record and reproduce music. Try to imagine the reaction of some great, historic musician such as Mozart if you played a CD for him for the first time.

Modern recording techniques are only the most recent innovations in the performance and enjoyment of music. Every instrument in the classical orchestra has evolved over the centuries, in reaction to new material technologies and the demands of composers such as Bach, Beethoven, and Mahler. More recently, rock instruments have benefited from the invention of multitrack recording, the wow pedal, and the electronic drum machine. Nevertheless, it is still possible to categorize all these instruments according to the way in which they produce their sound.

SECTION EXPECTATIONS

- Explain and illustrate the principle resonance in musical instruments.
- Define standing waves and harmonic structure.
- Investigate how notes are produced.

KEY TERMS

- brass instruments
- woodwind instruments
- stringed instruments
- consonance
- dissonance
- timbre
- harmonic structure
- percussion instruments

Wind Instruments

Today there are two main categories of wind instruments — brass and woodwind. The fundamental difference between the categories is the source of the vibrations that produce initial sound. The musicians' lips provide the vibration for **brass instruments** such as the trumpet and trombone. Trumpet players tighten their lips then force air through them, creating vibrations having a wide range of frequencies. A reed that vibrates when air flows by it supplies the initial sounds for **woodwinds** such as the clarinet and saxophone.

Figure 9.24 An instrument's mouthpiece produces sound, but the instrument's resonant cavity produces music.

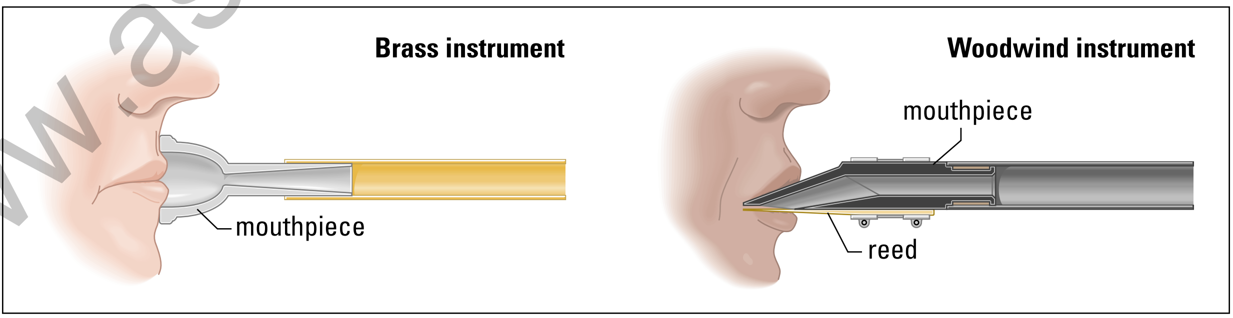


Figure 9.25 Each instrument produces a characteristic sound.

Figure 9.26 A seashell acts as a resonator for selected background noise frequencies, producing a sound like distant ocean waves.

PROBEWARE

Visit the **Science Resources** section of the following web site if your school has probeware equipment: **www.school.mcgrawhill.ca/resources/** and follow the **Physics 11** links for an interesting activity on the quality of sound.

The sounds from the mouthpieces alone are far from musical. Only when they are attached to the instruments are pleasant tones heard. Different notes are obtained in brass instruments by varying both the frequency of the initial sound waves produced by the lips and the length of the resonating column. Woodwind instruments change the effective length of the resonating column by covering and uncovering small finger holes along the instrument.

The mouthpiece also determines the type of resonances available. Brass instruments and flutes act as open-end resonating columns. Other woodwinds, like the oboe, clarinet, and basson, function as closed-end resonators, because they have reeds in the mouthpiece that behave like human vocal cords.

The length and shape of an instrument determine the range of musical notes that it can produce. Early instruments were created through trial and error. Musicians experimented with the length of the resonating column, the position of the finger holes, and the internal shape (cylindrical or conical) of an instrument in an attempt to create pleasant sounds. Table 9.3 provides a list of some instruments and their approximate musical ranges. The sequence of notes in a single octave is C D E F G A B C; subscripts represent higher or lower octaves. Composers must be aware of these ranges because it would be silly to write, say D_2, to be played on the violin since that note is below the instrument's range.

Table 9.3 Selected Instruments and Their Musical Ranges

Instrument		Lowest note	Frequency (Hz)	Highest note	Frequency (Hz)
woodwind	soprano recorder in C	C_5	523.2	D_7	2349.3
	flute	C_4	261.6	D_7	2349.3
	soprano clarinet in A	C_3	130.8	A_6	1760.0
	baritone saxophone in $E^\flat$	D_2	73.4	A_4	440.0
	oboe	B_5	987.8	G_6	1568.0
brass	C trumpet	$F_3^\sharp$	174.6	C_6	1046.5
	French horn in F	B_1	61.7	F_5	698.5
	trombone in $B^\flat$	E_2	82.4	D_5	587.3
	tuba in $B^\flat$	$B_0^\flat$	30.9	$B_3^\flat$	246.9
string	violin	G_3	196.0	E_7	2637.0
	double bass	E_1	41.2	B_3	246.9
	harp	C_1	32.7	G_7	3136.0
	guitar	E_2	82.4	A_5	880.0
keyboard	piano	A_0	27.5	C_8	4186.0

QUICK LAB

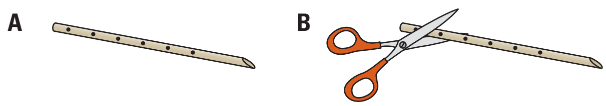
Singing Straws

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting

Press one end of each of two straws flat and then cut them as shown in the diagram. Set one straw aside. Carefully cut small holes into the top of one straw. (Alternatively, a heated nail melts nice clean holes into the straw.) Blow into the straw end that has been cut. Cover and uncover the holes and notice how the pitch changes. Begin with all of the holes covered and then uncover them one at a time starting from the far end and moving toward your mouth.

Using the second straw, carefully cut small pieces off the far end of the straw as you blow through it. Observe the change in pitch.

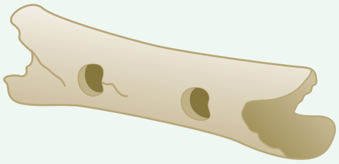


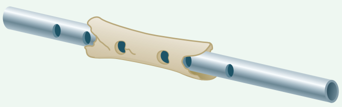
Analyze and Conclude

1. Did covering and uncovering the holes in the straw affect the pitch of the sound produced?
2. Describe a relationship between the pitch and the distance from the flattened end to the first uncovered hole.

TARGET SKILLS

- Identifying variables
- Analyzing and interpreting





PHYSICS & SOCIETY

The Neanderthal Flute

In 1995, in a cave occupied by Neanderthals between 40 000 and 80 000 years ago, paleontologist Dr. Ivan Turk discovered a broken section of the leg bone of a bear. Close examination revealed two holes drilled into the sides and two more at each of the broken ends. Dr. Turk realized that he had found part of a flute, the oldest known musical instrument.

Specialists in radiocarbon dating at Simon Fraser University in British Columbia determined that the flute, and pieces of charcoal found in the soil layers around it, were approximately 45 000 years old. This discovery was exciting because it suggested that Neanderthals, distant cousins of humans thought to have become extinct some 20 000 years ago, had a culture that included musical entertainment.

A short time later, the flute was examined by Dr. Bob Fink, a Saskatchewan musicologist. Dr. Fink carefully measured the spacing of the holes. Comparing the distances to the separation of his fingers, he proposed the length and structure of the original instrument. Then, making use of the physics of open air columns, Dr. Fink determined what notes the flute would have played. His findings created quite a stir: Fink's analysis suggested that the flute would have played a standard major or Do-Re-Mi scale familiar to any piano student or viewer of *The Sound of Music*. The major scale forms the basis of most music in the Western world.

Fink's claim is very interesting and quite controversial. The major scale is only one of many possible musical scales. Much Chinese music is based on a five-note or pentatonic scale. Gregorian chants composed in the Middle Ages are based on six-note scales called hexachords. Since the origin of the major scale is the strongest harmonics of the fundamental note, Fink argued that it is a natural scale. The fact that the Neanderthals used that scale, he said, showed its universality and their sophistication.

The discussion about the validity of Dr. Fink's claim continues. It is amazing how a couple of holes in an old bone can lead to so much good scientific and artistic discussion, and tell us so much about our ancestors and ourselves.

Analysis

1. What does a musicologist do?
2. On what factors did Dr. Fink base his claim that Neanderthals used our familiar musical scale?
3. Find examples of speculation (as opposed to solid evidence) in the article.

Stringed Instruments

Strumming stretched elastic bands will produce sound, but only someone very near to you will hear the music you are creating. All **stringed instruments** include strings that can either be plucked, strummed, or bowed, and some form of a resonator, or sound box. An acoustic guitar has a uniquely shaped body filled with air and a large central hole. A violin has a characteristic shape with two openings into the air-filled centre. A harp and a harpsichord have their strings tightly bound to the large frame or soundboard. You can demonstrate the process of forcing the vibration of another object to amplify sound by pressing the base of a vibrating tuning fork against your desk. The desktop will be made to oscillate with the same frequency as the tuning fork and, because the surface area of the desk is large, the amount of sound generated will be amplified.

Figure 9.27 illustrates the standing wave pattern that is set up in a vibrating guitar string. Pressing the string down onto the frets effectively reduces the length of the string and raises the note. The vibrations of the string alone can be heard only by those very close to the instrument. The vibrating string causes the air in the cavity of an acoustic guitar to resonate, providing the energy necessary to project the sound. An electric guitar uses a device called a pickup to transform the string's vibration energy into electric energy fed to an amplifier.

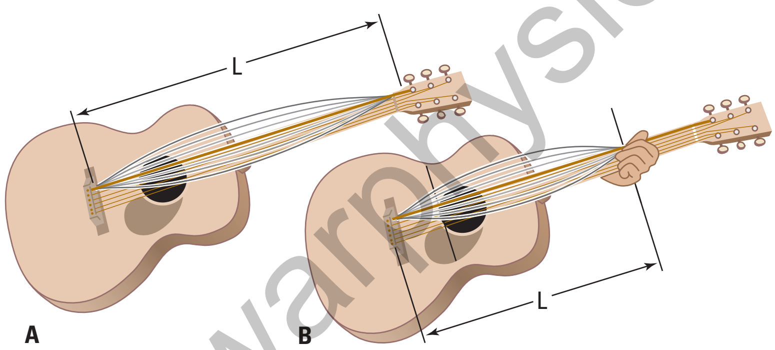


Figure 9.27 The frequency of the longer wavelength is lower than that of the shortened string. Only the fundamental frequency is shown.

A well-designed and well-built violin will have a low and high resonance due to the wood and a third resonance due to the air in the cavity. All three resonant cavities should be of approximately equal loudness. Figure 9.28 illustrates two loudness curves (sound created by bowing equally up the scale), one from a good violin and one from a poor violin. Violins are amazing instruments, and each has its very own characteristic sound due to specific harmonics that result from both the material used and its design. In the late seventeenth century, an Italian craftsman named Stradivarius built beautiful-sounding violins, of which 650 are still played today.

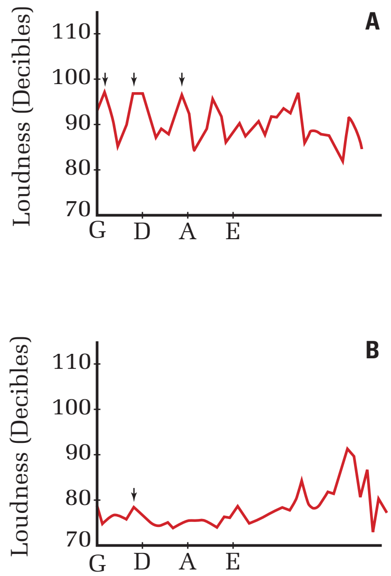


Figure 9.28 These are loudness curves for **(A)** a good violin and **(B)** a poor violin. Notice low and high resonance due to the wood and the middle resonance due to the air inside the cavity.

Interestingly, a violin actually sounds better the more it is played. This is probably because the wood making up the body of the instrument gains flexibility in specific areas due to repeated harmonic oscillations in the two-dimensional surfaces.

The Piano

Figure 9.29 Ray Charles exploits the piano's versatility.

The only musical constellation in the sky is Lyra, The Lyre, easily recognized high overhead in summer and fall by its bright star, Vega. The lyre is a plucked instrument, like a harp, and was popular in Greece more than 2500 years ago. The piano is a distant cousin to the ancient lyre. Instead of using fingers to pluck strings, the piano uses a keyboard-operated mechanism to "hammer" strings. Over the last 200 years, the piano has evolved into one of the most popular musical instruments, thanks to its range in both pitch and loudness and to the beauty of the music it can produce. Its popularity is due to the advent of cast-iron frames. Each of the 230 metal strings is under tremendous tension, resulting in a very strong combined compressive force, up to 2.6×10^5 N, that wood frames could not withstand.

When a pianist presses down on the keys, felt-covered wooden hammers strike metal strings, causing them to vibrate. (Technically, this makes the piano a percussion instrument.) Vibrating strings themselves would be too soft to hear, so they are connected to the soundboard, which generates the large amplitude sound waves. High tension in the strings ensures that the transfer of energy from the strings to the soundboard is very efficient. Notes of higher pitch have triplets of strings, middle notes are strung in pairs, and low notes have only one string. This distribution of strings helps to equalize the loudness of all notes played. Finally, the large amplitude sound waves reflect off the open lid of a grand piano, or off a nearby wall for an upright piano.

Language Link

The name piano is shortened from pianoforte, a word which reflected the instrument's ability to hammer the strings to play both soft and loud, or, in Italian, "piano" and "forte." What is a "fortepiano"?

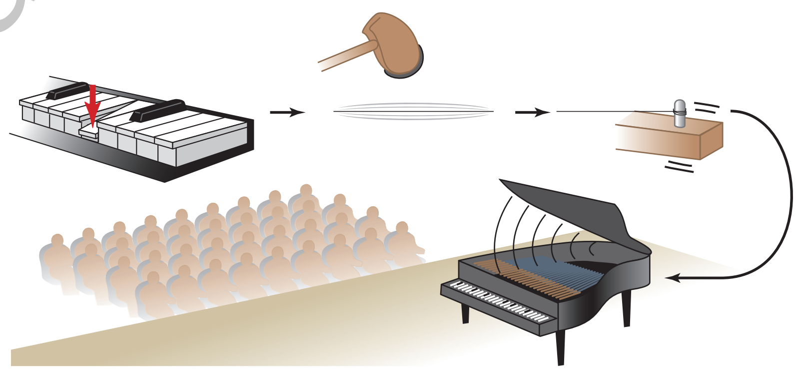
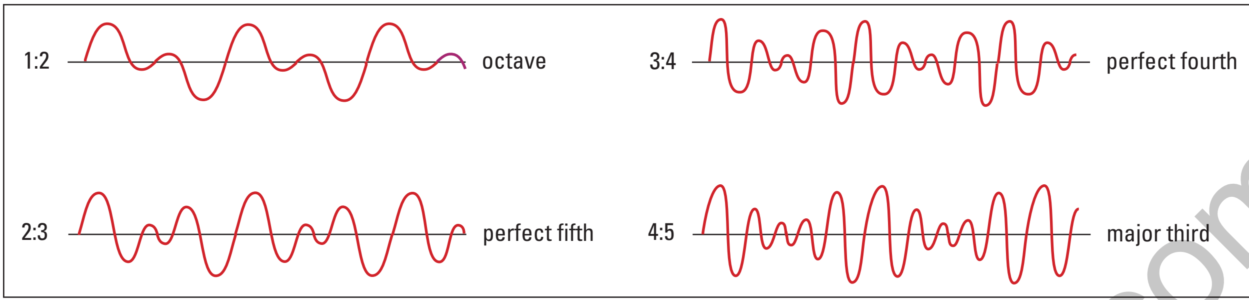


Figure 9.30 Mechanical energy is transformed into sound energy.



Consonance and Dissonance

Figure 9.31 Common whole number ratios of frequencies that produce pleasant sounds

What sounds good to you may or may not sound good to someone else. Surprisingly, though, almost all humans are generally pleased by certain sounds and displeased by others. When more than one pitch is played at a time, the resulting sound is called a chord. Different cultures find different chords to be more pleasing, and therefore often have very distinctive-sounding music.

The ancient Greek mathematician Pythagoras, famous for the Pythagorean theorem, conducted extensive research into the mathematics of pleasing chords. He used identical strings under identical tensions, varying only the length. He found that pleasing sounds were generated when the strings' lengths were whole number fractions of the original length, such as 1/2, 2/3, and 3/4. As you have learned, the length differences translate directly into frequency or pitch differences. Musicians have discovered that several pleasant-sounding musical intervals exist in those ratios as shown in Figure 9.31. Musical notes that sound pleasant together are said to be in **consonance**. Combinations that sound unpleasant are said to be in **dissonance**.

Musical instruments each produce a distinctive sound. Even when different instruments are playing the same note, they produce sounds with different qualities that are created by each instrument's specific harmonic structure. The difference in sound is described as a difference in musical **timbre**. Figure 9.32 shows three sound intensity versus frequency graphs illustrating the relative strengths of different harmonics for each instrument. This pattern of intensities is the **harmonic structure**.

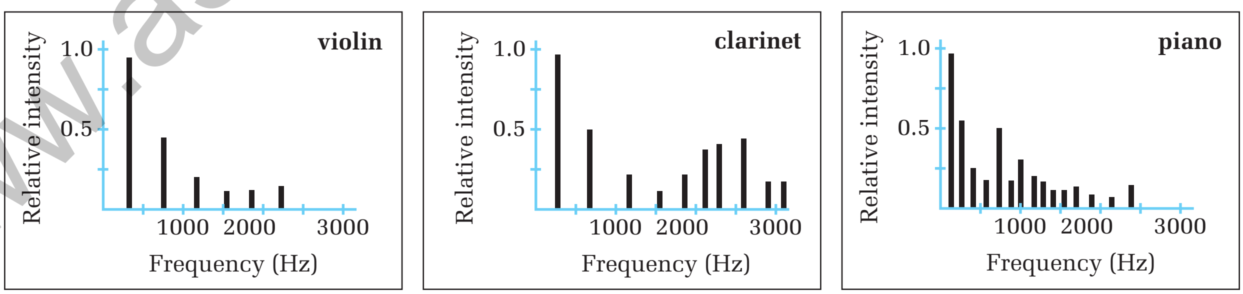


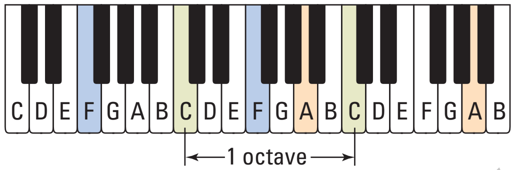
Figure 9.32 A violin, clarinet, and piano produce characteristic sound spectra.

QUICK LAB

Consonance versus Dissonance

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting

The simplest Pythagorean ratio that produces consonance is 1/2. That is, reducing the string length by a factor of 1/2 causes the pitch to go up an octave. Using a piano or an electronic keyboard, play two notes exactly one octave apart simultaneously. Repeat the procedure for several different notes. What you are hearing is said to be in *consonance*. Now return to your original position on the keyboard and begin to vary only one of the previous notes. Move up one note at a time for three or four notes. Repeat the procedure moving down one note at a time.



Analyze and Conclude

1. Describe the note combinations that sounded pleasant to your ear.
2. Determine the exact frequency of the note pairs and then find their ratios.

ELECTRONIC LEARNING PARTNER

View musical vibrations by going to your Electronic Learning Partner.

Percussion Instruments

Pounding on a drum, chiming a bell, or making music on a xylophone creates specific sounds based on each instrument's shape and the way in which it is struck. A large bass drum produces a substantially lower note than a much smaller snare drum. A large church bell rings out much lower tones than a small hand-held bell, and a xylophone generates a range of notes based on the length of the bar you choose to strike. **Percussion instruments** (instruments that sound when struck) have specific fundamental frequencies and harmonics associated with their size and the way they are played, as do wind and stringed instruments.

The membrane of a drum oscillates when struck. Unlike a string that settles into a one-dimensional mode of oscillation, a two-dimensional surface like the surface of a drum oscillates in more complicated two-dimensional modes. Figure 9.33 illustrates three possible modes of vibration for the surface of a drum. Striking a drum near the edge or near its centre will produce different modes of vibration and therefore different sounds.

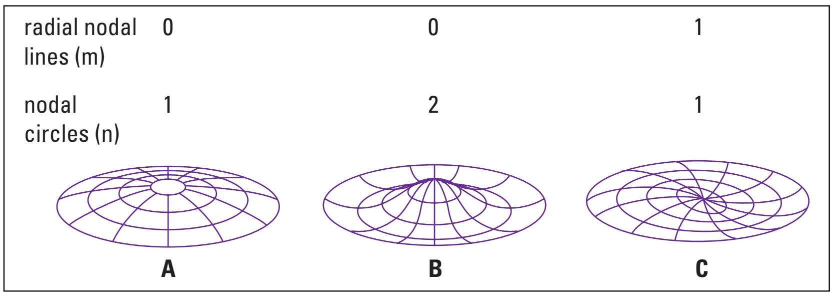


Figure 9.33 Three modes of vibration of a two-dimensional surface. Several much more complicated modes also exist.

The bars of a xylophone behave in the same way as open-ended resonant air columns. The bars are fastened to the frame at nodal points of the fundamental frequency of vibration for each bar (see Figure 9.34). The bars also resonate at various harmonic frequencies. Xylophones vary widely in quality, as you can easily hear by the purity of the tones they produce.

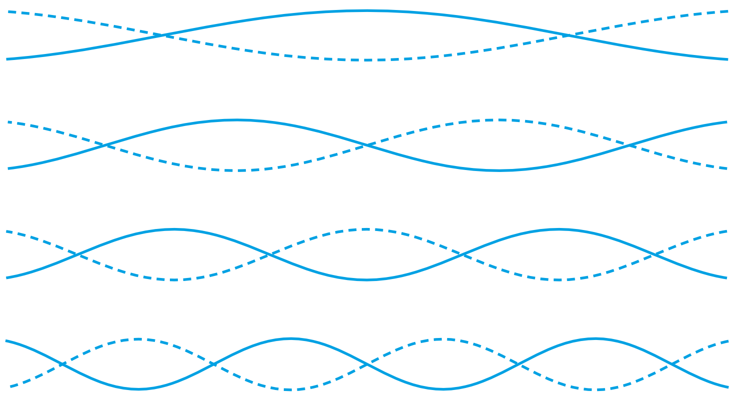
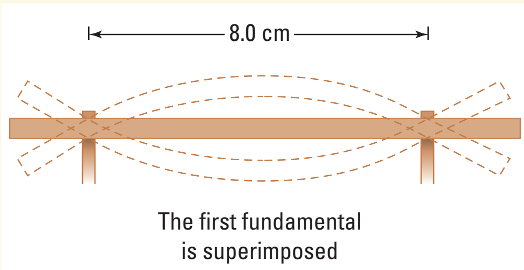


Figure 9.34 Bars on a xylophone vibrate in a similar fashion to open-ended resonant columns.

9.3 Section Review

1. K/U How does the length of a brass instrument affect its sound?
2. K/U What is the purpose of the body of a guitar or a violin?
3. MC You and a friend are learning to play the recorder. Your instrument is less than half the length of your friend's. Whose instrument will resonate with a lower pitch? Why?
4. C While playing your recorder, you cover all of the finger holes, and then uncover just one near the middle. Describe what will happen to the sound as you uncover the single hole and explain why.
5. C Describe how a xylophone is able to produce a range of pitches.
6. K/U Figure 9.32 illustrates the harmonic structure of three instruments. Draw the harmonic structure of an ideal tuning fork with a fundamental frequency of 512 Hz.
7. K/U Calculate the fundamental frequency of a metal bar held in position by supports 8.0 cm apart. The speed of sound in the metal is 5050 m/s.



8. MC For some of the composers named on page 429, or other historic composers or modern musicians of your choosing, describe innovations in instrument design that the musician either brought about or popularized.

9.4 Building Acoustics

SECTION EXPECTATIONS

- Describe how knowledge of the properties of waves is applied to the design of buildings.
- Analyze the acoustical properties of technological equipment and natural materials.

KEY TERMS

- acoustics
- reverberation time
- absorption coefficient
- liveness
- intimacy
- fullness
- clarity
- warmth
- brilliance
- texture
- ensemble
- blend
- anechoic chamber
- acoustical shadow

Why do speeches and oral presentations sound clear and easy to understand in some large rooms while you can hardly hear a speaker in other rooms of the same size? How do theatres designed for orchestras differ from movie theatres? Why is it difficult to understand the words from a television set in certain rooms, even when the volume is turned up? Answers to all of these questions all relate to acoustic qualities of the rooms. How do sound waves interact with the walls, floor, ceiling, and the contents of the room, including people?

Acoustics of a Room

Acoustics is the science or nature of sound quality. Thus, to design a room with appropriate acoustics, you need a scientific understanding of echoes, frequency variations and reflection and absorption of sound waves. Echoes are one of the most important factors affecting the acoustics of a room. Too many echoes cause speech to sound muddled while too few echoes make a room sound eerily dead. The term **reverberation time** describes the relative sound intensity of echoes in a room in terms of the time required for echoes to become inaudible. Quantitatively, reverberation time is the time for a sound to drop by 60 dB from its maximum intensity.

You can engineer a room's reverberation time to a desired length by carefully choosing the texture of the wall coverings, the ceiling texture and even the properties of the furniture. A theatre will have a substantially different reverberation time when empty compared to when it is full of people, because reflected sound is the main factor contributing to reverberation time. Materials that absorb sound will limit the amount of reflected energy in a room and thereby reduce the reverberation time. Likewise, surfaces that reflect a great deal of sound will increase the reverberation time of a room. Table 9.4 lists **absorption coefficients** of several different materials for specific frequencies. The higher the coefficient, the more sound is absorbed and the less sound is reflected. Perfectly reflective material would have an absorption coefficient of 0.00.

Multi-purpose rooms in many schools and public gathering places create problems for an acoustical engineer. The appropriate room acoustics for a presentation such as a speech differ from the acoustics that are desirable for a musical performance. In fact, the appropriate acoustics for a string ensemble are very different than those for a rock band. The engineer must make serious compromises in the design of a multi-purpose room.

Table 9.4 Absorption Coefficients for Different Materials

Material	125 Hz	500 Hz	4000 Hz
concrete, bricks	0.01	0.02	0.03
glass	0.19	0.06	0.02
drywall	0.20	0.10	0.02
plywood	0.45	0.13	0.09
carpet	0.10	0.30	0.60
curtains	0.05	0.25	0.45
acoustical board	0.25	0.80	0.90

QUICK LAB

Reverberation Time

TARGET SKILLS

- Predicting
- Performing and recording
- Analyzing and interpreting

How do the rooms in your school measure up in regard to appropriate acoustics? In this lab, you will select a variety of rooms and measure the reverberation times using a "loudness" or decibel meter and a stopwatch.

Make a list of the rooms you will test such as a classroom, the cafeteria, gymnasium, and music room. Before making any measurements, predict the relative reverberation times. List the rooms in order from longest to shortest reverberation time according to your predictions.

Select a sound source such as a trumpet. Keep the sound intensity level and the frequency of your source the same for all trials. A frequency near 250 Hz (C_4) would be appropriate. Since reverberation times will be quite short, possibly on the order of 1.0 s, they will be difficult to measure accurately. Therefore, carry out several trials for each room and use the average value of the results for your final comparison of the different rooms. Position your sound source in the place where speakers or performers would normally be located. Take your sound measurements in the centre of the room.

Analyze and Conclude

1. Organize your results from longest to shortest reverberation times. How accurate were your predictions?
2. Attempt to find similarities or differences between the rooms as the reverberation times increase down your list (for example, do the rooms get larger as reverberation times increase?).
3. Compare the surface of the walls and the ceiling between the rooms with the shortest and longest reverberation times. Suggest how the surfaces might affect the reverberation times of each room.

This graph illustrates typical reverberation times for performance-specific rooms versus the volume of the rooms. Rooms designed for speaking (Figure 9.36) require much shorter reverberation times than rooms constructed for musical performances (Figure 9.37).

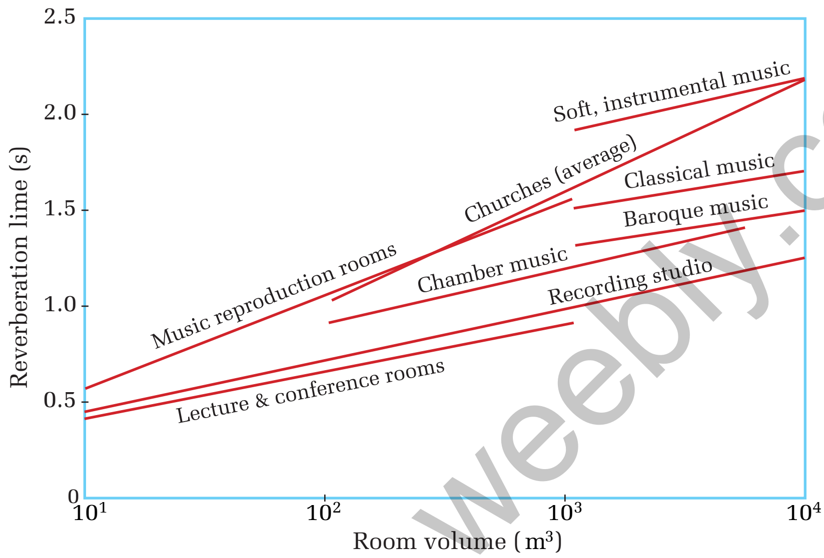


Figure 9.35 Notice that the volume scale is logarithmic, beginning in a small room with dimensions 3 m × 3 m × 3 m and moving to a very large auditorium 30 m × 30 m × 30 m.

Figure 9.36 Speaking requires a room that is not too live, and has good clarity and brilliance.

Figure 9.37 Singing with the quiet accompaniment of an acoustic guitar sounds best in a room that is very live and has good intimacy and fullness.

The entertainment industry has coined several terms to describe the acoustical properties of a room. Each term may sound very qualitative but, in fact, can be attributed to some very specific and measurable properties of sound. Physicists have quantified the terms that describe how well a room reacts to different sound situations, as shown in Table 9.5 on the next page.

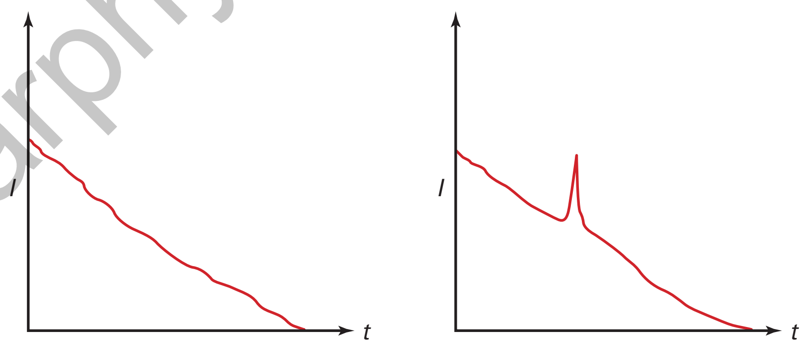


Figure 9.38 The first graph shows a room with good texture; the sound intensity drops off consistently. The second graph illustrates a room that has poor texture; a large late echo peaks the sound intensity, distorting what the audience hears.

Table 9.5 Acoustical Properties of Buildings

Term	Description	Adjusting techniques	Performance type
liveness	Liveness is a direct measure of reverberation time. The longer the reverberation time, the more "live" a room is.	Increase reverberation time by using more reflective materials on walls and ceiling.	soft, slow tempo instrumental music
intimacy	A room is more intimate if the listener feels very close to the performing group. This is accomplished if the first reflected sound reaches the listener 20 ms or less after the direct sound.	Placing a reflective canopy above the performers improves intimacy by reflecting sound to the audience.	all music and all speaking
fullness	The closer the reflected sound intensity is to the direct sound intensity, the more fullness a room has.	Fullness is generally increased by increasing reverberation time.	soft, slow tempo instrumental music, pipe organs
clarity	Clarity is the acoustical opposite of fullness. Reflected sound intensity should be very low for good clarity. Clarity is measured by repeating consonants. The maximum allowable loss of understanding is taken to be 15%.	Reducing reflected sound intensity is usually accomplished by reducing reverberation time.	speaking
warmth	Reverberation time for low frequencies (< 500 Hz) should be up to 1.5 times longer than frequencies above 500 Hz.	Select wall and ceiling materials to enhance low frequency reverberation time.	all music
brilliance	A brilliant room has the reverberation time of all frequencies nearly equal. If high-frequency reverberation time is too long, a constant high-pitched sound may occur.	Wall and ceiling materials can be selected to maintain consistent reverberation time for all frequencies.	speaking
texture	Good texture is achieved when at least 5 separate echoes reach the listener within 60 ms of the direct sound. Also, the sum of the reflected sound intensity should decrease uniformly.	The shape of the room is most related to texture. Sound focussing or the creation of large late groups of reflections produces poor texture.	all music
ensemble	Good ensemble means that each member of the performing group is able to hear what every other member is doing. No strong reflections should exist past the shortest notes being played by the group.	Like texture, the shape of the performing area is crucial to achieving good ensemble. Sounds must be softly reflected to all participants.	all music
blend	Blend is ensemble, but for the listening audience. Good blend is desired in every location of the room, but is much more easily achieved in the middle of any auditorium.	Good blend is achieved by mixing the sound before it is dispersed to the audience, using reflecting surfaces around the performing group.	all music

MODEL PROBLEM

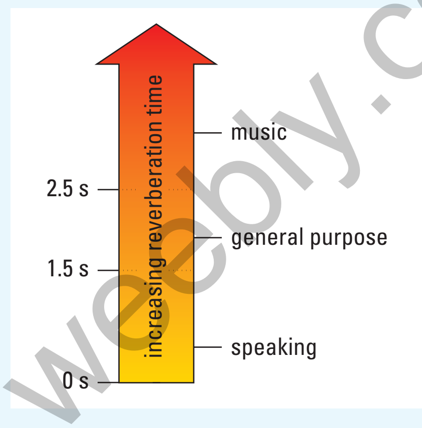
Creating Appropriate Acoustics

A singer is preparing to give a performance in your school's auditorium and complains that the room is too lively and lacks acoustical warmth. Describe the problem and a possible solution.

Frame the Problem

The singer is identifying two acoustical properties of the room.

- Liveness: Longer reverberation times make a room more lively.
- Acoustical warmth is a room's tendency to have longer reverberation times for lower frequencies (< 500 Hz).
- Materials exist that preferentially reflect lower frequencies (see Table 9.5 on the previous page).



Identify the Goal

To decrease the room's liveness (shorten reverberation time)
To increase the room's acoustical warmth (longer reverberation time for lower frequencies)

Variables and Constants

Involved in the problem	Known	Unknown
materials covering walls	room acoustics:	low frequency reflecting materials
room acoustics	(a) too lively	materials covering walls
low frequency reflecting materials	(b) lacks warmth	

Strategy

Identify the definition of acoustical liveness.
Identify the definition of acoustical warmth.

Identify materials to

(a) decrease reverberation time (high absorption coefficient)

(b) decrease reverberation time of higher frequencies more than lower frequencies (high absorption coefficient for higher frequencies)

Refer to Table 9.5 for absorption coefficients.

Calculations

Acoustical board has an absorption coefficient of 0.25 for 125 Hz and 0.90 for 4000 Hz.

The acoustical board will reduce the room's overall reverberation time, making the room less acoustically lively.

The acoustical board will absorb more of the high frequencies and less of the lower ones, making the room increase in acoustical warmth.

Validate

Acoustical board may be available in the music room of the school, making the proper adjustments to the auditorium possible.

PRACTICE PROBLEMS

6. Why does a room designed for speaking require a relatively short reverberation time?
7. A physics student moves about in an auditorium as an orchestra plays. He notices that a region exists near the back where the bass sounds drown out the treble. What is a possible cause of the poor acoustics and how might it be corrected?
8. A classroom has one entire side wall made of windows. During oral presentations, the room lacks clarity. Should the curtains be open or drawn closed during the speeches? Explain.
9. Examine the Physics File below. Why does a person seated in the audience have an absorption coefficient greater than 1?

Sound Focussing

For optimal presentations, domes, curved walls, and other shapes that resemble ellipses or parabolas should be avoided. Curved surfaces focus sound to specific areas of a room, rather than spreading it evenly throughout the audience. Focussing sound using a microphone with a parabolic reflector demonstrates the principle that causes problems with curved surface construction. Concave surfaces focus sound and convex surfaces diffuse sound.

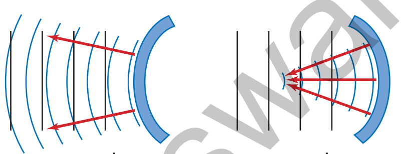


Figure 9.39 Convex surfaces (left) diffuse sound while concave surfaces focus sound.

Figure 9.40 A parabolic reflector around a microphone focusses sounds and allows distant noises to be clearly recorded.

PHYSICS FILE

An empty auditorium has a substantially different reverberation time than when filled with an audience. An adult person absorbs a great deal of the direct and reflected sound during a performance. Consider the absorption coefficients at 1000 Hz:

Wooden seat	0.28
Upholstered seat	3.2
Adult person	4.6
Adult person in an upholstered seat	5.0

Both the person and the seat have absorption coefficients greater than one because sound is not only absorbed but also blocked from progressing through the room. It is analogous to computing a brick wall's absorption coefficient, not from the reflected sound, but rather the transmitted sound that manages to penetrate the wall.

QUICK LAB Acoustical Diffraction

TARGET SKILLS

- Performing and recording
- Communicating results

Obtain a very high-pitched and a very low-pitched tuning fork. Find a very quiet location away from walls or other sound-reflecting surfaces. Gently sound the high-pitched fork and hold it at arm's length out from your partner's right ear. Practise striking the tuning fork to give it just enough energy to be barely audible. Repeat this procedure, but this time, have your partner cover his or her right ear. Can your partner hear the sound? Repeat the entire procedure with the low-pitched tuning fork. Repeat the procedure and have your partner hold the tuning fork about 1 m from your right ear.

Analyze and Conclude

1. Were you able to hear the high-pitched tuning fork when your right ear was covered?
2. Were you able to hear the low-pitched tuning fork when your right ear was covered?
3. Explain how this experiment tests the diffraction of sound. Make a general statement about how the amount of diffraction of sound depends on frequency.

Figure 9.41 An anechoic chamber that is literally "without echo" yields sound power reduction that follows the inverse square law ($1/r^2$).

Echoes

Outdoor band shells are often in the shape of a large parabola with the performers positioned at the focal point. In the open expanse of the outdoors, this design projects the sound out to the audience very effectively. This design causes a drastic echo problem however when used indoors. An audience will hear the direct sound followed by a large echo caused by the sound reflecting off the rear wall. Interior design requires that the sound be mixed at the source by using small reflective boards and then evenly dispersed to the audience. Large flat surfaces of an auditorium should be covered with a sound-absorbing material to reduce the amount of reflected sound, thereby minimizing echoes.

Anechoic chambers are used in the study of acoustics. The term anechoic means "without echo" and is pronounced "ann-e-KO-ic." An **anechoic chamber** has sound absorbing material on the floor, walls, and ceiling. Speaking in a room that has absolutely no echoes generates a feeling that your words are being sucked out of you, only to disappear.

Shadows

In Chapter 8, you discovered that sound waves, like all waves, can interact to cause interference. Constructive interference, when compressions overlap with compressions and rarefactions with rarefactions, produces an increase in the loudness of a sound experienced at that point. Destructive interference occurs when compressions overlap with rarefactions causing a reduction in the loudness. If destructive interference occurs in an auditorium, the region where it is occurring is called an **acoustical shadow**. An audience member in an acoustical shadow may have trouble hearing a performance.

Shadows often occur in places where large objects, such as balconies, protrude into the hall. The sound diffracts around the objects. As you have learned, the amount of diffraction depends on the frequency of the sound. A large amount of diffraction causes the performance to be distorted. Limiting the intrusion of balconies into the main performance hall reduces shadows.

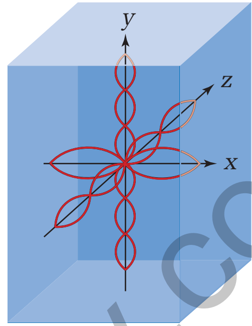


Figure 9.42 Each pair of parallel surfaces provides a resonant chamber for a standing wave to form. Combine the three resonant lengths associated with a standard rectangular shower stall and the very reflective surfaces and it is no wonder your voice in the shower does not sound half bad.

Resonance

Resonance becomes an issue only in small rooms, where the length of the room may be only a few wavelengths of some lower frequencies. A bathroom shower is a good example of a room exhibiting resonance. The material, glass or tile, is highly reflective and the shower's sides are symmetrical. The reflecting surfaces come in pairs, allowing standing waves to set up in every dimension of the room (see Figure 9.42). Auditoriums and theatres avoid the use of parallel walls to help reduce the resonance of certain frequencies.

QUICK LAB

Singing in the Shower

TARGET SKILLS

- Predicting
- Performing and recording
- Analyzing and interpreting

Measure the dimensions of an enclosed shower stall. Measure the height, width, and depth of the shower. Calculate the fundamental resonant frequency for each dimension. Determine what notes should resonate in your shower. Be sure to include one or two harmonics for each fundamental. Test your predictions.

Analyze and Conclude

1. Why does your voice seem to sound "better" when you sing in the shower?
2. Calculate the wavelength range of the musical spectrum, approximately 20 Hz to 15 000 Hz.

PHYSICS & TECHNOLOGY

TARGET SKILLS

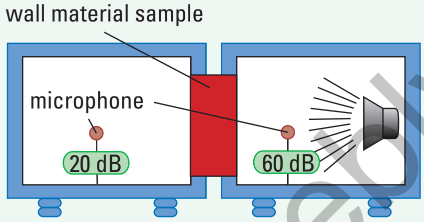
- Identifying variables
- Analyzing and interpreting

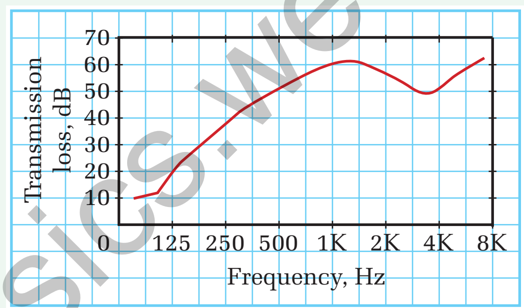
Acoustic Design in Buildings

The performer on stage is speaking, but you cannot quite make out the words. This problem, so annoying for theatregoers, is not always the actor's fault. Some elegant theatres have bad acoustics.

Acoustics is the physics of sound. When designing theatres and concert halls, architects consult acoustic engineers, in the hope that music and speech will sound perfect in their buildings. Sometimes experimentation and changes are required after the hall is constructed. Roy Thomson Hall, home of the Toronto Symphony Orchestra, has large circular panels hanging from the ceiling. These can be raised, lowered, or tilted in an attempt to reflect sounds equally to all regions of the theatre. An extensive renovation in 2002 should further improve the hall's acoustics.

For residential, office, and industrial buildings, the architect's concerns are different. Noise reduction becomes important. Acoustic engineers have to consider the sources, frequencies, and intensities of the sounds produced in various parts of the building. Then, they recommend materials to use to block the sound. For example, various materials transmit sound differently, depending on the pitch. To test a material for its sound-blocking ability, a wall of the material is placed between a speaker and a microphone. The intensity is plotted on a graph against the frequency. From the graph, the acoustic engineer can suggest suitable materials for the specific building being designed.





Sometimes the best-designed concert halls have disappointing sound qualities. Even the most well-planned buildings also can sometimes have regions where noise pollution is high. Fixing such problems involves knowledge of physics and experimentation skills. The application of acoustics to building design is as much an art as a science.

Analysis

1. Why might "dead spaces" occur in concert halls where some notes produced by a singer cannot be heard?

2. Despite the use of appropriate materials in the walls, ceiling, and floor, sometimes even low intensity sounds at a certain pitch can produce annoying buzzes in rooms, such as the rattling of windows. Why might this happen?

INVESTIGATION 9-A

Design and Test Your Own Theatre

TARGET SKILLS
- Initiating and planning
- Performing and recording
- Modelling concepts

Models are often used to perform critical tests before a full-scale, expensive structure is built. In this activity, you are to use the acoustical design knowledge you have gained to design a movie theatre. Produce a scale drawing of your theatre design that conforms to the specifications listed below. Remember to choose a scale that will fit into a wave tank.

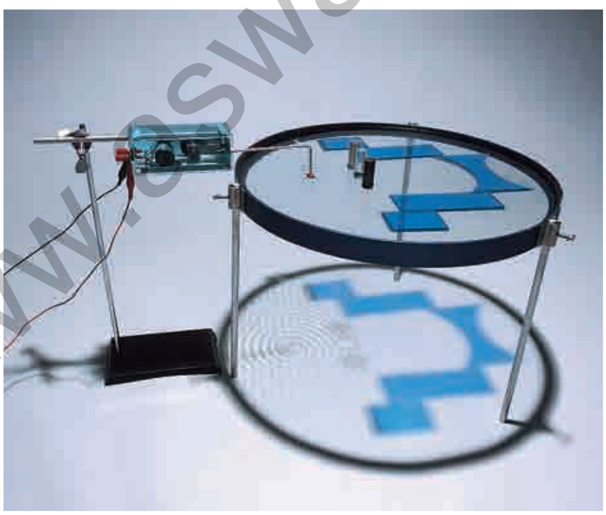
Theatre Specifications

- seating capacity: 75
- seat dimensions (including personal space): 0.60 m × 1.20 m to 0.80 m × 1.60 m
- aisle minimum width: 3.0 m
- minimum distance from screen: 5.0 m
- maximum distance from screen: 20 m
- screen dimensions: 10.0 m × 8.0 m

Sound will be generated at the front of the theatre only.

Building your Model

Construct a scale model of your theatre's *perimeter* in a wave tank. Use waterproof materials arranged so that the interior of the model matches the design of your theatre. The photograph shows a simple example.



Testing your Model

Record the scale used to construct the model.

Test your model using (a) a straight wave source and (b) a point source.

Look for acoustical problems such as

- areas of sound focussing
- large reflections
- large late reflections
- sound shadows
- resonance (interference)
- reverberation time between low and high frequencies (acoustical warmth)

(Refer to Table 9.5, Acoustical Properties of Buildings, for possible properties to test.)

Mark problem areas on your scale model drawing. Attempt to improve the acoustical properties of your theatre by employing strategies discussed in Table 9.5, Acoustical Properties of Buildings. Test your modified design.

Analyze and Conclude

1. Describe acoustical problems you discovered in your initial design.
2. Assuming the model and wave tank test was valid, describe the acoustical properties of your theatre using the terms of the music industry (Table 9.5) before and after design revisions.
3. How valid was the wave tank test? Explain.

Apply and Extend

4. Is it possible to equate the speed of the water waves in the tank to the speed of sound in air? How?
5. Use your results from question 4 to determine the frequencies of sound that were actually tested for during the wave tank testing.

PHYSICS FILE

The Pentagon in the United States is not designed to keep external noise out, but to keep internal noise in. Top-secret discussions remain secret because part of the Pentagon is actually a box sealed within a box. The two boxes are separated by a layer of sound absorbing material.

External Noise

The final consideration architects of auditoriums must take into account is the amount of external noise. Designing the perfect room can be rendered useless if, during each performance, a rumbling train roars by and drowns out the famous singers' voices. If highway or rail traffic noise exists outside of a theatre, engineers must insulate the building to prevent the unwanted noises from entering the room.

9.4 Section Review

1. C Describe the physical characteristics a physicist attributes to:
   (a) acoustical warmth
   (b) acoustical clarity
   (c) acoustical texture
2. C Describe the main acoustical difference between an auditorium designed for speaking and one designed for pipe organ music.
3. K/U You wish to renovate your bedroom so that you can enjoy music the way you like it—lots of bass and very little treble or high frequencies. What material(s) would be best for you to put on your walls?
4. K/U What is an anechoic chamber?
5. MC Could you use some very powerful speakers to mask the noise of a helicopter's rotors? If so, how?
6. C Explain why singing in the shower sounds better than singing in a large room.
7. C Consider the graphs in Figure 9.38 showing how the intensity of sound within a room fades. Describe the acoustical problem that is demonstrated by the graph.
8. C Based on your wavelength calculation, would it be wise for a movie theatre to have a length of 17.0 m? Explain.

UNIT ISSUE PREP

Acoustical design principles are applied to the design and layout of communities.

- Identify materials and design features that would help control sound pollution.
- Determine which acoustical characteristics generate noise pollution complaints by members of a community.

CHAPTER 9 Review

REFLECTING ON CHAPTER 9

- The outer ear collects sound energy and directs it toward the eardrum.
- The eardrum, in the middle ear, vibrates in response to the compressional waves in the air. The eardrum initiates the mechanical vibrations of the three ossicles.
- The stapes, the third ossicle, vibrates the oval window on the surface of the inner ear, starting compressional waves in the liquid in the cochlea. Travelling waves move down the basilar membrane and create resonance in specific locations.
- Hair cells on the basilar membrane stimulate neurons in the auditory nerve, which then carries signals to the brain.
- Programmable hearing aids can benefit people with partial deafness. Cochlear implants, which stimulate auditory neurons directly, can benefit profoundly hearing impaired persons.
- Air forced through the vocal cords cause them to vibrate. The sounds resonate in the vocal tract including the throat, mouth, and nasal cavity. Articulators create specific sounds from resonating air columns.
- The Doppler effect is the apparent change in the frequency of a sound due to relative motion of the source of sound and the observer.
- When an object is moving faster than the speed of sound in the air through which it is travelling, each new compressional wavefront is ahead of the previous one. The overlapping of wavefronts along a cone creates extremely large compressions that are heard as a "sonic boom."
- Mach number $= \dfrac{\text{speed of object}}{\text{speed of sound}}$
- Most bats and whales use echolocation to detect prey or solid objects.
- Technologies such as sonar and ultrasound are based on the principle of echolocation.
- Musical instruments create sound with a wide spectrum of frequencies. The resonant properties of the instruments determine which frequencies are amplified.
- The reverberation time of a room describes the time required for the intensity of echoes to drop by 60 dB. The purpose of the room dictates the appropriate reverberation time.

Knowledge/Understanding

1. Does the human ear have any mechanisms to protect against extremely loud sounds?
2. Draw an energy path diagram detailing the path of a sound from when it is generated by a falling tree right up to when the electrical signal reaches the brain.
3. Name and explain the function of the three smallest bones in the body.
4. What is the frequency range of normal human speech?
5. A friend comes to school and is barely able to speak, claiming to have laryngitis. How does laryngitis limit the ability to generate sound?
6. What physical characteristic(s) makes human voices unique?
7. Define and provide an example of conductive and sensorineural hearing loss.
8. Complete the following chart.

Sound	Articulator/resonator
p, b, m, w	
	tip of tongue
k, g	

9. Sketch a diagram that traces the energy path from the beginning of an incident sound wave all the way to the electrical signal reaching the brain of a patient with a cochlear implant.
10. Sketch a diagram to model the sound waves emanating from an object that is travelling away from you at a speed that is slower than the speed of sound.

11. Dolphins are able to use ultrasonic waves to locate and track prey.
    (a) How do they generate and receive the sounds?
    (b) What frequencies do the dolphins use primarily for echolocation?
    (c) What are the advantages of using the frequencies from part (b)?
12. Draw an energy path diagram that traces the energy imparted to a string on an acoustic guitar by someone strumming it right up to the eventual sound that is heard by the audience.
13. Define reverberation time and explain how it affects a room's acoustical properties.

Communication

14. Describe the features of the human ear that aid in the amplification of sound.
15. The auditory canal of most humans has resonant frequencies that vary between 2.5 kHz to 4.5 kHz. Explain the significance of this resonance in terms of hearing sensitivity and communication.
16. Human speech is an incredibly complex process that enables us to communicate effectively. Explain how are we able to produce so many complex sounds.
17. **(a)** Briefly describe the place theory of hearing.
    (b) Describe the temporal theory of hearing.
18. While waiting at a railway crossing, you hear the approaching train sound its horn. Describe the change in frequency of the sound you hear as it approaches and then departs from your position at high speed.
19. High frequency and low frequency sounds behave differently. High frequency sounds diffract (bend) less around obstacles than lower frequency sounds. Describe why this characteristic makes it beneficial to put high frequency sirens on emergency vehicles and low frequency fog horns on boats.
20. Sketch and label a pressure versus time graph to describe how a sonic boom is able to break a window.
21. Explain the relationship between the absorption coefficient of wall and ceiling material and reverberation time.
22. Describe the acoustical property of a room with good
    (a) liveness
    (b) fullness
    (c) brilliance
    (d) blend
23. Describe the acoustical property of a room that has a reverberation time for low frequencies (< 500 Hz) that is 1.5 times longer than for frequencies above 500 Hz.
24. Describe how each of the following acoustical problems could be reduced.
    (a) poor texture
    (b) sound focussing
    (c) excessive liveness
25. Describe the acoustical properties of a room that is well designed for speaking performances.
26. Describe the acoustical effects that are represented by this photograph of a water droplet.

Making Connections

27. While driving in a car, your favourite music is replaced by a newsperson giving a winter storm report dealing with school closures. You find that it is difficult to hear what is being said, even when the volume is increased. How should you adjust the treble and bass settings to help you hear?
28. Canadian Forces jets flying from Cold Lake, Alberta are restricted from flying at supersonic speeds over certain areas. Discuss the possible effects on wildlife subjected to regular low-level supersonic air traffic. Explain why high-altitude supersonic flights affect a greater area, but have a diminished impact.

29. Current oceanographic research involves deploying high frequency sound beacons that may travel hundreds of kilometres throughout the oceans. They are used to measure the temperature of the oceans or for military surveillance. Discuss whether the benefits of these applications are worth the possible damage caused to marine life.

30. Mice hear sounds that are well above the range of human hearing. Devices that emit continuous, high intensity ultrasonic sounds will drive away mice without being heard by human ears. Do you believe this type of device is safe? Is the use of such a device ethical, not only in terms of the mice, but also in terms of the unknown effects it may have on other nearby wildlife or on neighbours' pets?

Problems for Understanding

31. A neighbour explains that the old well behind his house is 500 m deep. You decide to see for yourself. You drop a stone from rest and measure the time interval until you hear the splash of the stone striking the water. You find it to be 6.0 s. You assume the speed of sound in air to be 343 m/s. How deep is the well?

32. A car horn produces a 4200 Hz sound. You hear the sound as 4700 Hz. Is the vehicle approaching or leaving your position?

33. Calculate the wavelength of the sound from the car horn in question 32 when (a) stationary and when (b) perceived by you as 4700 Hz.

34. The siren of an emergency vehicle produces a 5500 Hz sound. If the speed of sound is 340 m/s in air, calculate the wavelength of the sound.

35. A jet is travelling at Mach 2.4 in air, with a speed of sound of 320 m/s. How fast is the jet flying in km/h?

36. A bullet leaves the barrel of a gun at 458 m/s and goes into air that has an ambient temperature of 26.5°C. Determine the Mach number of the bullet.

37. A sonar depth finder is capable of receiving a reflected signal up to 1.75 s after sending it. Calculate the maximum depth that the device could accurately measure in fresh water.

38. A vacant city lot sits next to a large building. You hope to convince City Council to turn the lot into a soccer field. First you need to determine how far it is from the edge of the lot to the building. You bark out a loud call and the echo returns in 0.75 s. Calculate the distance to the building if the air temperature is 31°C.

39. Intimacy is achieved when the first reflected sound arrives less than 20 ms after the direct sound. Calculate the maximum extra path length that the reflected sound may travel in order to arrive in 20 ms. (Assume air temperature of 20°C.)

40. Calculate three fundamental resonant frequencies for a small room with dimensions of $2.0\ \text{m} \times 3.0\ \text{m} \times 1.5\ \text{m}$. (Assume standard atmospheric pressure at 0°C.)

41. Some friends drop a water balloon out of a window 12.0 m above the ground. As it falls, one of the pranksters cries out to warn the person below, 1.5 s after releasing the balloon. If the air temperature is 28.0°C and if the person below is able to move infinitely fast upon hearing the warning, will the person avoid the balloon?

Numerical Answers to Practice Problems

1. 1.16 **2.** 173 m/s **3.** 331 m/s **4.** 6.1×10^2 m **5.** 17 m **6.** To prevent reflections or echoes that can make it difficult to understand the speaker's words. **7.** The back of the room is too warm, possibly due to materials, like carpet, that preferentially absorb high frequencies. Replace carpet with plywood or drywall, materials which absorb a greater fraction of low frequencies than carpet does, while reflecting the higher frequencies. **8.** Curtains should be closed as they have a higher absorption coefficient at the intermediate frequencies appropriate for human voices. Increasing the absorption results in a decrease in reflection, which leads to a smaller reverberation time and a corresponding increase in clarity. **9.** A person blocks a fraction of the sound getting through the room, and absorbs some of the sound. Comparing the sum of the amount that got through and the amount reflected to the amount emitted gives an absorption coefficient greater than one.

A Policy for Noise Pollution

The environment is alive with sounds.

Background

Sounds that are continuous, or loud, or both, are called noise pollution. Air and road traffic, construction, and loud music, are common forms. The effects of noise pollution on hearing loss are well documented. Studies have found that people living in remote regions, far from the noises of an industrialized environment, have much better hearing in old age than people that have lived in urban settings. These studies suggest that although hearing loss may be a natural result of aging, external factors can also have a dramatic impact. Studies also suggest that noise pollution can have an effect on our emotional well-being. As a result, many towns and cities attempt to protect citizens by passing legislation aimed at regulating noise levels.

Hearing Loss with Age at 3000 Hz

In recent court proceedings brought by a town in British Columbia against a local company, noise pollution bylaws were put to the test. Because of the company's proximity to a residential area, bylaws prohibited the company staff from making noise outside the hours of 6:00 a.m. to 8:00 p.m. on weekdays. Apparently, however, company personnel were routinely receiving shipments at 5:00 a.m., which prompted a group of angry residents to file a complaint. The judge in the case threw out the complaint, claiming the town's bylaws did not sufficiently define unacceptable noise. The judge further suggested that noise deemed annoying by one person could easily go unnoticed by another.

This case raises important questions about the planning and implementation of noise pollution bylaws. What constitutes acceptable versus unacceptable levels of noise? Who should have a say? What steps must municipal planners and lawmakers take to ensure that such bylaws do not unfairly discriminate against businesses, or prevent people from going about everyday tasks, while at the same time ensuring that residents of local communities have periods of quiet time?

Bearing these issues in mind, your task will be to apply your knowledge of waves and sound to draft a noise pollution policy for a specific community.

Many regions have by-laws governing when construction noise is allowed.

ASSESSMENT CHECKLIST

After you complete this issue analysis:

- **assess your ability to conduct research: did you find current and relevant information?**
- **assess your communication skills: how effectively were you able to share your ideas with the other members of your group?**
- **assess your ability to make connections to the world outside the classroom: did you address social and economic considerations in your policy document?**

Plan and Present

1. As a class, establish clear guidelines for the finished product. Discuss specifics such as overall length, required sections, proper sourcing, and timelines.
2. In groups of three or four, brainstorm potential communities for which a noise pollution policy could be useful. Communities to consider include a hospital, an apartment building, a retirement residence, or your school. Each group should share its choice of community with the class.
3. Once your group has selected a community, begin identifying the stakeholders (any person or group that will be affected by the policy).
4. Develop your proposal by:
   - conducting experiments to determine current noise levels
   - interviewing stakeholders
   - researching existing policies
   - researching appropriate noise levels as prescribed by unbiased scientific studies
   - determining reasonable penalties for bylaw infractions

Logging vehicles are often restricted from operating during specific times of the day. In some areas seasonal operating restrictions also exist.

Evaluate

1. What components were common to each group's policy document, regardless of the community?
2. List three items that you found most interesting after analyzing all of the documents. Explain your selections.
3. Describe two challenges and two successes associated with your group's attempt to draft a useful and enforceable document.
4. Describe how you feel about noise pollution. Respond to the comment, "Societal noise regulation has often been left to individual municipalities because of the perception that noise pollution is simply an irritation."

Urban planners often set aside specific geographic regions for industrial, entertainment, and housing developments to help reduce problems one may cause another.

UNIT 3 Review

Knowledge and Understanding

True/False

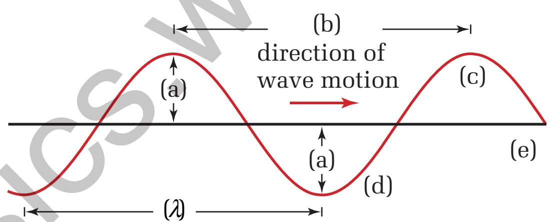
In your notebook, indicate whether each statement is true or false. Correct each false statement.

1. The time required to complete one cycle is called the frequency.
2. When a particle oscillates, its maximum displacement from its rest position is called its amplitude.
3. Pushing a friend on a swing is an example of mechanical resonance.
4. The medium through which a wave is passing experiences a net movement in the direction of travel of the wave.
5. The speed of a wave depends on the amount of energy used to create it.
6. Water waves are an example of transverse waves.
7. When two waves interact to produce destructive interference, there is a net loss of energy.
8. Sound travels around corners in a phenomenon known as refraction.
9. Constructive and destructive interference cannot occur in longitudinal waves.
10. Infrasonic sound waves have frequencies below what humans can hear.
11. A sound intensity of 0 dB indicates that a sound wave has no amplitude.
12. When the source of a sound moves toward a stationary observer, the apparent pitch of the sound seems to be lower than the actual pitch of the source.

Multiple Choice

In your notebook, write the letter of the best answer for each of the following questions.

13. The period of a pendulum oscillating in periodic motion is
    (a) the time to complete one cycle.
    (b) the position the pendulum assumes when allowed to hang freely.
    (c) one complete repeat of the pattern.
    (d) the number of cycles completed in a specific time interval.
    (e) the amplitude of the pendulum divided by its velocity.
14. The main reason the Tacoma Narrows bridge collapsed was
    (a) extremely high winds
    (b) the steel used was faulty, and could not support the weight of the traffic
    (c) certain wind speeds caused the bridge to vibrate at its natural frequency
    (d) the bridge was not made massive enough
    (e) all of the above
15. In the diagram below, which of the features labelled (a) – (e) is the amplitude?



16. In the diagram for question 19, which of the features labelled (a)–(e) is the wavelength?
17. In the diagram for question 19, which of the features labelled (a)–(e) is the rest position?
18. Identify the false statement. When a wave travels from one medium to another,
    (a) the frequency of the wave changes
    (b) the amplitude of the wave changes
    (c) the speed of the wave changes
    (d) some of the energy is transmitted and some is reflected
    (e) the period of the wave remains constant
19. The frequency range of human hearing is approximately
    (a) 2 Hz to 2000 Hz
    (b) 20 Hz to 20 000 Hz
    (c) 200 Hz to 200 000 Hz
    (d) 2 Hz to 2 kHz
    (e) 2 kHz to 2 MHz
20. Two sound sources that vary in frequency of only a few hertz, when sounded together will produce
    (a) resonance
    (b) acoustical liveness

(c) a closed air column
(d) beats
(e) nodal lines

21. Where are incident sounds amplified in the human ear?
(a) ear drum, ossicles, cochlea
(b) external ear, ear canal, oval window
(c) ear canal, ear drum, ossicles
(d) ear drum, Eustachian tube, auditory nerve
(e) external ear, ear canal, ossicles

22. Pleasant-sounding combinations of notes are said to
(a) be in dissonance with each other
(b) be in consonance with each other
(c) interfere destructively
(d) interfere constructively
(e) produce harmonic structure

23. An auditorium is designed so that at least five reflected sounds reach the listener within 60 ms of the direct sound. This auditorium would be said to have good
(a) blend
(b) warmth
(c) texture
(d) ensemble
(e) clarity

Short Answer

In your notebook, write a sentence or a short paragraph to answer each of the following questions.

24. Compare the detected wavelength and frequency of a source of sound as it moves toward a stationary listener with the detected wavelength and frequency as the source moves away from the stationary listener.

25. If the fundamental mode of vibration for a standing wave on a vibrating string is 200 Hz, could a standing wave of 250 Hz exist on the same string? Explain why or why not.

26. How does a tuning fork produce sound? Why is the sound produced at a specific frequency?

27. What evidence exists to support the assertion that sound requires a medium to travel through?

28. How would the intensity of a sound be affected if you moved
(a) twice as far away from the source?
(b) seven times farther away from the source?
(c) one third as far away from the source?

29. How does the distance between the first and second resonant lengths compare for an air column that is closed at one end and an air column that is open at both ends?

30. How is a harmonic mode of vibration related to the fundamental mode of vibration for a string?

31. Why is sound intensity, measured in dB, presented using a logarithmic scale?

32. List and describe the three main functional areas of the human ear.

33. (a) How does the human hearing system amplify sounds?
(b) How does the human hearing system protect against very loud sounds?

34. Summarize the place theory and the temporal theory of hearing. Ensure that you identify their strengths and weaknesses.

35. Describe how a sonic boom is an extreme case of the Doppler effect.

36. Define the term echolocation and provide two examples of animals that use it.

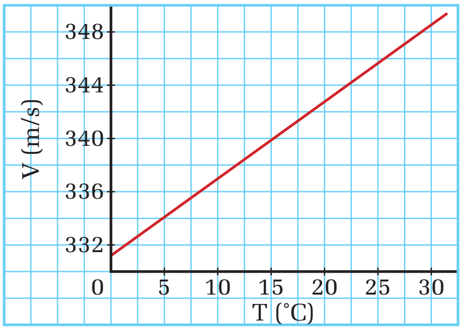
37. How can a single drum be made to produce different sounds?

38. (a) Define reverberation time.
(b) What types of performances are better suited for rooms with long reverberation times? Why?

Inquiry

39. Using your understanding of the diffraction of waves in general, describe how you could carry out an experiment to show that sound waves do or do not diffract.

40. Two physics students conducted an experiment to test how the speed of sound in air varied with changes in temperature. Use the graph of their data (below) to develop a mathematical relationship that could be used to predict speed of sound in air at specific temperatures.



41. Describe how you could use a meter stick, a pair of binoculars, a classmate with a drum, and a large pendulum hanging from the branch of a tree in an open valley to assist you in the approximate calculation of the speed of sound.

42. Your school plans to conduct a "coffee house" to allow students to share their musical talents. Two locations are suggested, the cafeteria and the gymnasium. The cafeteria is a large L-shaped room with sound-absorbing ceiling tiles and wall coverings. The gymnasium is a large rectangular box with concrete walls and a flat steel ceiling. Design a series of tests and plan a report format to provide the "coffee house" committee with a detailed acoustical analysis of each room, with highlighted strengths and weaknesses of each.

Communication

43. Extremely high water waves at sea are sometimes known as rogue waves. They may occur following a storm and seem to come out of nowhere. Explain, using your knowledge of wave interference why these waves occur and have these special characteristics.

44. A police car is equipped with a siren type horn located under its hood that rotates very quickly. Explain the reason for the rotation of this horn as it emits a sound of constant pitch and loudness.

45. Dolphins and whales often make use of submarine "sound channels" to communicate over very large distances. Find out and describe how these sound channels operate and are able to carry sound so far without losing much of its intensity.

Making Connections

46. Explain why an empty auditorium has a substantially different reverberation time when it is filled with an audience. Does the audience increase or decrease the reverberation time?

47. Draw an energy path diagram illustrating how incident sound energy is eventually transformed into electrical impulses by the human hearing system.

48. Many countries use the sky above the wilderness area around Goose Bay, Labrador to train fighter pilots in the skills of low level flying using supersonic military aircraft. Write a research paper on the environmental impact of low level flying of supersonic aircraft over wilderness areas such as in the case of the Labrador wilderness.

Problems for Understanding

Show complete solutions for all problems that involve equations and numbers.

49. Calculate the speed of water waves hitting the shore if adjacent crests are 3.0×10^1 m apart and a wave hits the beach every 1.0×10^1 s.

50. A pendulum takes 1.50 s to swing from the rest line to its highest point. What is the frequency of the pendulum?

51. A wave with an amplitude of 50.0 cm travels down a 12.0 m spring in 3.00 s. The student who creates the wave moves his hand through 5 cycles in 1 s. What is the wavelength?

52. A klystron tube in a microwave oven generates radiation of wavelength 4.20 cm. What is the frequency of the microwave radiation? (Microwaves travel at the same speed as light.)

53. The international tuning note (A above middle C) has a frequency of 440 Hz. If the speed of sound in air is 320 m/s, what is the wavelength of the note in air?

54. A sound wave reflects from the end of an air column with a distance between any two consecutive nodes of 54.0 cm. If the air temperature is 10.0˚C, what is the frequency of the vibration?

55. A violin string vibrates with a frequency of 990 Hz at the second octave. If the speed of sound is 343 m/s, what is the wavelength of the same string vibrating at its fundamental frequency?

56. A tuning fork with a frequency of 324 Hz is held over a tube whose length can be changed by raising and lowering a column of water in the tube. The surface of the water, initially very near to the top of the tube, is gradually lowered. If the speed of sound in air is 336 m/s, how far from the top of the tube is the surface of the water when the first point of constructive interference is detected?

57. When two tuning forks vibrate simultaneously the sound grows louder and softer, with 100 intensity peaks every 80.00 seconds. If one tuning fork is known to have a true frequency of 384.0 Hz, what are the possible frequencies of the other tuning fork?

58. While scuba diving the Pacific Ocean you bump into your partner and your oxygen tanks make a loud clank. How far are you from the nearest underwater reflecting surface if the sound returns to you in 3.00 s? (Assume speed of sound in salt water to be 1440 m/s)

59. **(a)** What is the Mach number of an airplane travelling at 2.0×10^3 km/h through 8°C air?
(b) Will the Mach number increase or decrease as the temperature falls?

60. Calculate the range of wavelengths of sound travelling through air at 20°C that are audible to the human ear.

61. In archery class you shoot an arrow with a constant speed of 22.0 m/s at a target that is 5.0×10^1 m away. How long after you release the arrow will you hear it hit the target?

62. Professional cliff diving experts often leap from heights of 5.0×10^1 m into deep water. How much time would pass between the moment the professional jumped to the moment spectators would hear a splash if they were watching from a boat located 35 m from where the diver strikes the water?

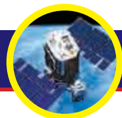
63. A bat uses echolocation to find insects. Waves can only be used to detect objects that are one wavelength long or greater. If the bat's echolocation is to be able to detect an insect that is 0.450 cm long on a night when the temperature is 15.0 °C, what must be the frequency that it uses to locate the insect? (Hint: Some bats use frequencies as high as 150 kHz.)

64. The sonar of a submarine uses a sonic "ping" with a frequency of 698 Hz. The echo returns from a distant submarine 5.60 km away after 8.00 s. What is the wavelength of the sound the first submarine is using to echolocate the other submarine?

65. People love to sing in the shower, since the dimensions of the shower cabinet are usually such that it causes resonance, making their voices sound better than normal. If a shower cabinet measures 1.0 m by 1.0 m square and 2.0 m high, would a male or a female singer most likely appreciate the effect of the resonance? Explain.

66. A student had no thermometer, so in order to measure the temperature, she resourcefully used the following procedure. She accurately measured a distance of 1.00×10^2 m from a wall, and then struck two stones together so that each new strike coincided with the echo from the previous strike. The student found that the time to make 1.50×10^2 strikes was 92 seconds. To the nearest degree, what was the air temperature?

COURSE CHALLENGE



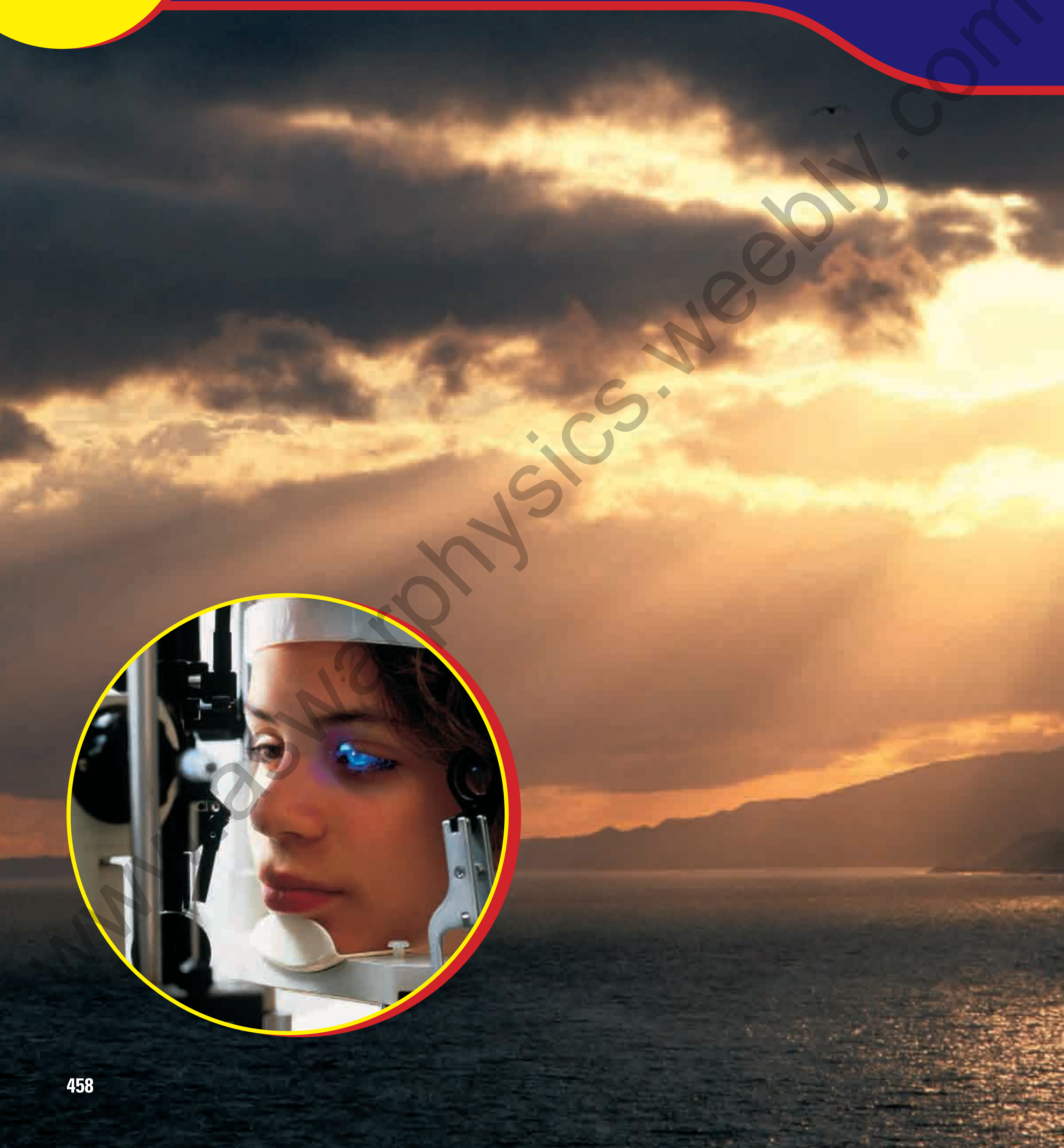
Space-Based Power

Plan for your end-of-course project by considering the following:

- Are you able to incorporate analysis of waves and wave properties into your project?
- Begin to consider time and equipment requirements that may arise as you design project-related investigations.
- Examine the information that you have gathered to this point. Produce a detailed plan, including a timeline, to guide you as you continue working on your project portfolio.

UNIT 4

Light and Geometric Optics



OVERALL EXPECTATIONS

DEMONSTRATE an understanding of the properties of light and its transmission

INVESTIGATE and predict the behavior of light using ray diagrams and algebraic equations

EVALUATE the contributions to entertainment, communications, and health made by optical devices

UNIT CONTENTS

Sometimes, when the conditions are just right, you can see sunbeams piercing the clouds during a sunset. Aside from their beauty, these sunbeams convey information about the nature of light. For example, you can see that light travels in straight lines. What property of light is revealed when tiny waves on the water sparkle brightly? When and why can you see images reflected from the surface of still water? Sometimes, on a hot summer day, you may look at a road surface off in the distance and think you see shimmering water, but there is no water there at all. How is this mirage effect created?

Early observers and scientists asked and found answers to questions similar to those above. In the process, they learned so much about light that they were able to develop a variety of technologies that enhance everyday life. For example, the woman in the small photograph may no longer have to wear corrective lenses. Her doctor is using specialized lenses to examine her eyes. He will decide whether he can use laser light to change the shape of the corneas of her eyes. In this unit you will develop an understanding of the properties of light that make these incredible technologies such as laser eye surgery possible.

UNIT PROJECT PREP

Refer to pages 596–597. In this unit project, you will have the opportunity to build a reflecting telescope or a periscope.

- How will the light ray model help you in the design of your optical instrument?
- What is the nature of the images produced by concave and convex lenses?

CHAPTER 10 Reflection of Light

CHAPTER CONTENTS

Have you ever been in a house of mirrors at an amusement park? Finding your way out of a mirror maze can sometimes be quite challenging. The placement of the mirrors and the lighting effects create so many different images that some people become quite disoriented as they try to find the exit. You can also create some amazing optical illusions by using different combinations of curved mirrors instead of plane mirrors. However, mirrors serve many functions other than having fun.

Interior decorators use mirrors to make rooms appear larger or to create dramatic effects. You use mirrors at home to check your own appearance. Mirrors in cars help you avoid collisions. Also, curved reflectors help focus car headlights to provide maximum visibility without blinding oncoming drivers. How many ways do you use mirrors in a single day?

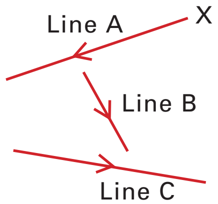
MULTI LAB

Mirrors and Images

TARGET SKILLS

- Identifying variables
- Performing and recording
- Analyzing and interpreting

Reflection from Plane and Curved Mirrors



Draw and label three lines at random on a blank page, like the ones shown here. Mark arrows on each line to indicate a direction. Place a ray box at point X on line A, and shine a single light ray along the line. Position a plane mirror at the other end of line A, such that the reflected light ray touches the beginning of line B. Mark the position of the mirror, and draw in the light ray from A to B. Repeat for lines B and C, so that the final reflected light ray touches point X.

Analyze and Conclude

1. Find a pattern in the angles of the incident and reflected rays.
2. Repeat the procedure, using the concave and convex mirror shapes. Does the same relationship exist for the curved mirrors as for the plane mirror? Explain your answer.
3. What difficulties did you encounter measuring the angles?
4. Suggest a practical solution to the problem.

Your Mirror Image

Hold a plane mirror so close to your face that you cannot see your image. Very slowly move it away from your face. Observe the size and orientation (upright or inverted) of your image as you move the mirror further from your face. Repeat the procedure with concave and convex mirrors.

Analyze and Conclude

1. At what relative distances and with which mirrors was your image (a) upright, (b) inverted, (c) larger than your face, and (d) smaller than your face?
2. For each mirror, summarize the changes that took place as you moved the mirror further from your face.

Reflecting Images

Place a concave mirror behind a light source such as a small bulb or a candle. Move a paper screen around through many positions until you can see an image of the light source on the screen. You may have to move the light source also. Once you have found an image, note the distances of the mirror and screen from the light source. Also note the size and orientation of the image compared to the light source. Attempt to carry out the procedure with plane and convex mirrors.

Analyze and Conclude

1. Under which conditions could you project an image of the light source onto the screen?
2. Describe the conditions under which it was not possible to form an image on the screen.
3. Make a general statement about the conditions needed to project an image on a screen using a mirror.

10.1 Reflection and the Light Ray Model

SECTION EXPECTATIONS

- Describe the wave theory and the wavefront and light ray models of light, and use them to explain how we see objects.
- Illustrate and predict the reflection of light, according to the law of reflection.

KEY TERMS

- wave theory of light
- wavefront model
- linear propagation of light
- ray model
- light ray
- law of reflection
- regular reflection
- diffuse reflection

PHYSICS FILE

Isaac Newton believed that light consisted of extremely rapidly moving *particles* and many other scientists of his time followed his teachings. Later, Christian Huygens demonstrated that light travelled with *wave*-like properties such as constructive and destructive interference. Eventually, Max Plank, Albert Einstein, and others demonstrated that, when light interacts with matter, it behaves like particles or packets of energy now called photons. Today, scientists accept the "wave-particle" duality of the nature of light. Light travels like a wave and interacts with matter like a particle. Nevertheless, physicists still seek a more complete model of light.

More than 80% of the information humans receive about their surroundings is carried by the light that enters their eyes. You can read this book and see your classmates because light reflected from all these objects and people reaches your eyes. The information that you will learn from this book about light and images will come to you in the form of light reflected from the pages of this book and form images on the retinas of your eyes.

The Wave Theory of Light

When possible, scientists try to develop a theory that can explain as many aspects as possible of the phenomena they are investigating. As scientists gathered data and information about the propagation of light, they discovered many wave-like properties. Light can be reflected, refracted, and diffracted in essentially the same way as water waves and other forms of mechanical waves. However, no material objects are in motion when light energy travels from one location to another so light cannot be a mechanical wave. In fact, light can travel through a vacuum. Nevertheless, the similarities outweigh the differences and scientists now accept the **wave theory of light**, in which light energy travels in the form of electromagnetic waves.

You can apply all of the principles of waves that you studied in Chapter 7 to the transmission of light, except that light does not need a medium in which to travel. You cannot see the wavefronts of light but you can envision them as very similar to water wavefronts. However, you need to try to picture them in three dimensions as shown in Figure 10.1. Light wavefronts from a point source (very small source) move out as spherical surfaces. If you use a parabolic reflector like the one you used to experiment on water waves, you can guide the waves so that the wavefronts are flat, planar surfaces. The **wavefront model** of light allows you to explain and predict many of the properties of light propagation.

The observation in the unit opener on page 459 that light travels in straight lines is such a fundamental property that it has been accorded the status of a principle. The principle of **linear propagation of light** leads to the very useful **ray model** of light. A **light ray** is an imaginary arrow that points in the direction of the propagation of light. Like mechanical waves, the direction in which light energy travels is perpendicular to the wavefronts. Thus light rays are always perpendicular to wavefronts. The wavefront model can explain the properties of light, but wavefronts are

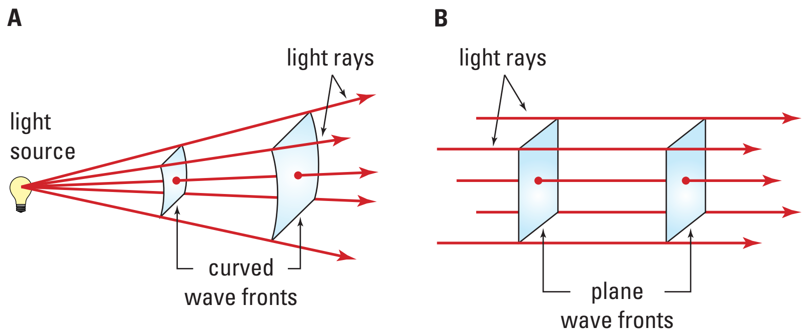


Figure 10.1 **(A)** When the light waves are close to the light source, the wavefronts appear as parts of a sphere, and the light rays gradually diverge (spread out) from one another. **(B)** When the light waves are at a great distance from the light source, the wavefronts can be considered parallel to one another. The light rays are also parallel, indicating the direction in which the light is travelling.

more difficult to sketch and interpret than light rays. Therefore, you will be using light rays to understand, describe, and predict the properties of light through out this unit.

Reflection

In Chapter 7, you briefly considered plane water waves reflecting from a straight barrier. The law of reflection, introduced in connection with water waves, applies to all types of waves but it takes on greater importance as you learn about the formation of images by light rays. Examine Figure 10.2 to review the terms and symbols scientists use in discussing reflection. You always draw a normal (or perpendicular) line from the reflecting surface from the point at which a light ray strikes a reflecting surface. The angle between the incident ray and the normal line is the angle of incidence, θ_i. The angle between the reflected ray and the normal line is the angle of reflection, θ_r. The **law of reflection** states that the angle of reflection is always equal to the angle of incidence. This law applies to all surfaces.

THE LAW OF REFLECTION

The angle of incidence, θ_i, equals the angle of reflection, θ_r, and the incident light ray, the reflected light ray, and the normal to the surface all lie in the same plane.

Language Link

The word "ray" is derived from the Latin word *radius*. Usually, light fans out radially from a point source of light. What are some other examples of radial phenomena?

PHYSICS FILE

In Unit 3, Mechanical Waves, it was established that all waves travelled through a medium. In the seventeenth century, Huygens, a Dutch physicist, developed the first significant theory proposing that light travelled as a wave. Scientists supporting this theory believed that light (electromagnetic) waves travelling through space from the Sun must also travel through a medium. By the mid-nineteenth century the wave theory was well established, and scientists "invented" a medium, called "ether," that was considered to permeate the entire universe. The electromagnetic waves were called "ether waves." In 1887, a series of brilliant experiments by Michelson and Morley proved conclusively that ether did not exist, and it was established that electromagnetic waves like light could, in fact, travel through a vacuum without the need of a medium of any kind.

Language Link

The word "normal" is derived from the Latin word *norma*. What does *norma* mean? Why is this appropriate for the use of "normal" in mathematics and physics?

History Link

There is a legend that the ancient Greek mathematician and inventor, Archimedes, used mirrors to set fire to Roman ships around 212 B.C.E. In 1973, the Greek historian, I. Sakkas, set out to test whether Archimedes could have done so. He lined up 70 soldiers with flat copper shields and directed them to reflect sunlight to a rowboat anchored 50 m from shore. The boat soon caught fire.

This doesn't prove the story was true, but it does show that Archimedes could have used such a method.

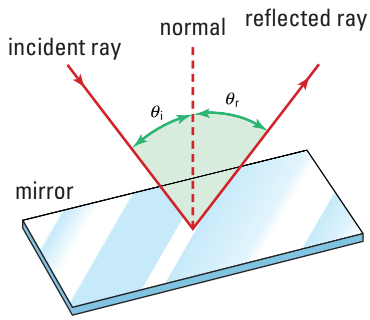


Figure 10.2 The angle of incidence, θ_i, equals the angle of reflection, θ_r. Both of these angles are measured with respect to the normal, a line drawn at right angles to the mirror surface at the point of incidence.

Regular and Diffuse Reflection

Why is it that you can see yourself in a mirror, but you cannot see yourself in a piece of paper, such as this page in your textbook, when you hold it in front of your face? The quick answer is that the mirror surface is much smoother than the surface of the paper. Try looking at other shiny surfaces, such as the cover of your textbook. You might be able to see a faint reflection of your face on the book cover.

How smooth does a surface have to be to behave like a mirror? If you magnify the surface of a mirror several hundred times, the surface still appears flat and smooth. However, the magnified surface of a page in a book is quite irregular, as shown in Figure 10.3.

To understand what happens to the light when it strikes these two kinds of surfaces, use the light ray model to see the path of the light. When a set of parallel light rays hit the mirror surface in Figure 10.4, the reflected light rays are also parallel to one another. This type of reflection is called **regular** (or specular) **reflection**. Each of the parallel light rays that hit the irregular surface in Figure 10.4 is reflected in a different direction. This type of reflection is called **diffuse reflection**. However, the law of reflection still applies for every light ray reflecting from the irregular surface. The surfaces of many common objects, such as most types of cloth, paper, brick and stone surfaces, and pieces of wallpaper, reflect light in this way.

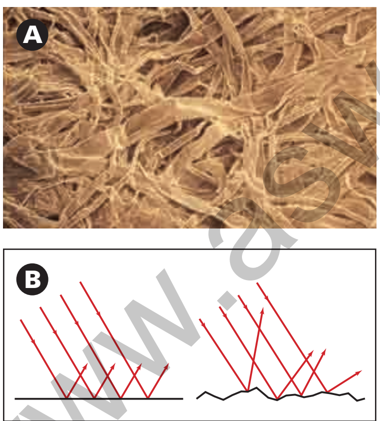


Figure 10.3 **(A)** The page of a book viewed under a microscope. **(B)** Which of the ray diagrams illustrates reflection off paper?

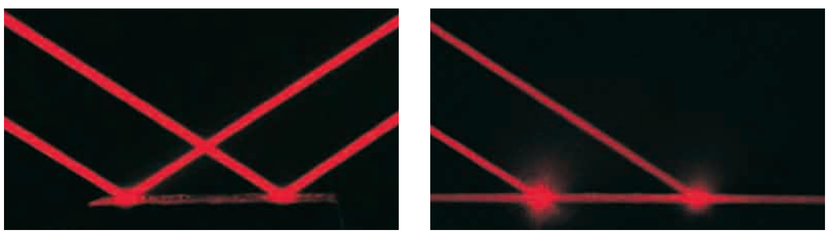


Figure 10.4 A laser beam is a group of parallel light waves that are close enough together to form a fine beam of light. Notice that the laser beams reflected from the mirror are parallel, but those that strike the rough surface appear as fuzzy, round dots because the incoming parallel light rays in the beam are reflected in many different directions.

Concept Organizer

Wave Theory → Wavefront Model
Linear Propagation of Light → Wavefront Model

Wavefront Model → Light Ray Model → Light Ray Diagrams

The wave theory is used to explain and predict effects produced by light waves. Because light travels in a straight line in a uniform medium, straight lines, called light rays, can be drawn to indicate the path of the wavefronts of the light waves. The light ray model uses light rays and the geometry of straight lines to produce a light ray diagram. This is a simplified graphical technique to help explain how we see objects and images. Although each point on the book gives off light in all directions, the eye sees only a diverging cone of rays.

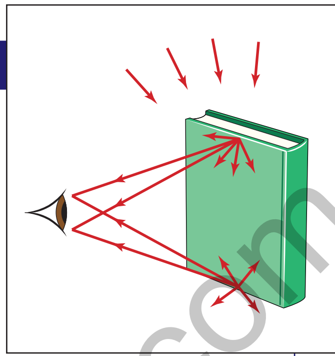


Figure 10.5 Using light ray diagrams to model how you see an object.

QUICK LAB Predicting Reflections

TARGET SKILLS

- Predicting
- Performing and recording
- Analyzing and interpreting

Obtain a white piece of paper, a pencil, a ruler, a plane mirror, and a ray box. Draw a line across the upper left corner at approximately a 60° angle with the side of the page to represent a mirror surface. Near the middle of the line, mark a point and label it A. On the right side of the paper, mark a point and label it B. Imagine a light ray coming from B and hitting the "mirror surface" at A. Predict the direction of the reflected ray and mark it on the paper. Repeat the procedure for two more pairs of points A_2, B_2 and A_3, B_3.

Place a mirror directly on top of the "mirror" line on the paper. With the ray box, find and mark the direction of the reflected rays for each of the "rays" that you marked on the paper. Compare the directions of the actual ray with your predictions.

Analyze and Conclude

1. Explain the reasoning that you used when you were making your predictions about the directions of the reflected rays.
2. Comment on the accuracy of your predictions.
3. Formulate a method that would allow you to make accurate predictions. Then test your method.

10.1 Section Review

1. K/U List four effects that can be observed that demonstrate light travels in straight lines.

2. C
   (a) State the law of reflection for light.
   (b) Draw a fully labelled diagram to show the reflection of light from a plane mirror surface.

3. K/U Why does it become more difficult to read a page in a glossy magazine than a page in this book when reading in bright sunlight?

4. C
   (a) Explain the difference between the wavefront and light ray models of light.
   (b) Which of the two models is most useful for explaining effects produced by optical devices? Why?

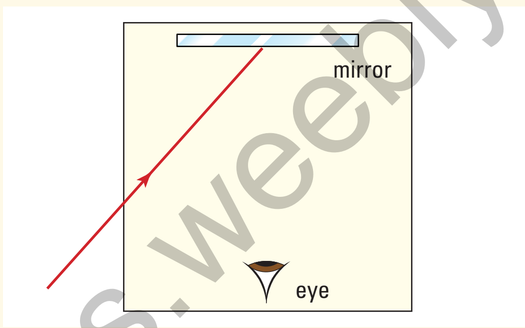
5. K/U A light ray strikes a mirror at an angle of 72° to the normal.
   (a) What is the angle of reflection?
   (b) What is the angle between the incident ray and the reflected ray?

6. K/U A ray of light strikes a plane mirror at an angle of 26° to the mirror surface. What is the angle between the incident ray and the reflected ray?

7. I Design and carry out an experiment, using one or more ray boxes and several reflecting surfaces, to show the law of reflection still applies when light undergoes diffuse reflection.

8. K/U Light is shining onto a plane mirror at an angle of incidence of 48°. If the plane mirror is tilted such that the angle of incidence is increased by 17°, what will be the total change in the angle of reflection from the original reflected light?

9. C In the room illustrated in the figure, the room is dark and has perfectly black walls. The air in the room is clear, without dust or smoke. If you stay at the position indicated by the eye, can you see the mirror on the opposite wall when a collimated beam of light enters the room in the direction indicated by the line segment? Explain your answer.



10. MC The windows of some large buildings are coated with a very thin of layer of gold. Why was this material used? From an economic point of view, is the use of gold justifiable for this purpose?

COURSE CHALLENGE

We're Losing Power

Signal attenuation refers to energy loss. This energy loss can be result of absorption or reflection. Design an experiment to test the attenuation properties of various substances to microwaves radiation. (Hint: Start with an ice cube and a microwave oven.) Have your teacher verify that your procedures are safe before conducting any experiments.

Learn more from the Science Resources section of the following web site: ***www.school.mcgrawhill.ca/resources/*** and find the *Physics 11 Course Challenge*

10.2 Images and Light Rays

The dentist in Figure 10.6 is using a mirror to see inner side of the patient's teeth. Dentists can determine precisely where an object is by using mirrors. When mirrors are built into instruments such as periscopes, cameras, telescopes, and other devices, they must be placed in precisely the correct position before the instrument is ever used. The engineer that designs the instrument must have a method to determine exactly how light rays will be reflected, in order to design the instrument. In the remainder of this chapter, you will learn how to use the law of reflection and the geometry of light rays to predict the location and characteristics of images formed by mirrors.

SECTION EXPECTATIONS

- Classify images formed by reflection according to their characteristics.
- Use the light ray model to predict image characteristics for a plane mirror, and test these predictions experimentally.

KEY TERMS

- object
- image
- real image
- virtual image
- plane mirror
- geometric optics

Figure 10.6 A dentist is using a mirror to see the inner side of the patient's teeth.

Objects and Images

As you have read many times in this text, everyday terms used in physics often have specific meanings. In the study of geometric optics, two of the most important terms are object and image. The **object** is the tangible item that you see in the absence of any optical devices. You can see an object because light rays are spreading out or diverging from every point on the object. In most cases, light from a luminous source is shining on the object and the object reflects the light by diffuse reflection. Your eyes collect the diverging rays and focus them on the retinas of your eyes. When an optical device redirects the diverging light rays and makes the rays appear to be coming from a point that is not really on the object, the device has formed an **image**.

History Link

Polished metal mirrors used in Egypt around 2000 B.C.E. were used until the sixteenth century when the first glass mirrors were made in Venice. Early glass mirrors were sheets of glass attached to very thin sheets of tin coated with mercury. In 1857, Foucault developed a process for coating glass surfaces with silver. How have mirrors developed since then?

When you were observing images in different mirrors in the Multi-Lab on page 461, you saw a variety of sizes, shapes, and orientations of images. You can completely describe any image by defining four characteristics. The magnification is the ratio of the image size to the object size. If the magnification is greater than one, the image is larger than the object. When it is equal to one, the object and image are the same size. If the magnification is less than one, the image is smaller than the object. The attitude of an image indicates whether the image is oriented the same way as the object (upright) or upside down (inverted) with respect to the object. The image position is the distance between the image and the optical device — mirror or lens.

The fourth characteristic, the type of image, indicates whether the image is real or virtual. An image is **real** if light rays are actually converging at a point then continuing on beyond that point and diverging. In other words, if you placed a screen at the image position, the image would appear on the screen. The student in Figure 10.7 B is using his hand as a screen to demonstrate the meaning of a real image.

If an image is not real, it is **virtual**. If you placed a screen at the position of a virtual image, nothing would appear on the screen. There are no light rays actually converging on the image position, as Figure 10.7 A reveals. Light rays only appear as though they are diverging from the image location. This will become more clear as you practice drawing ray diagrams. The four image characteristics are summarized in Table 10.1.

Figure 10.7 **(A)** No image is formed on the screen behind the plane mirror because a virtual image is formed. **(B)** When the student places his hand in front of an overhead projector screen, a real image is formed on his hand.

Table 10.1 Image Classification Properties

Property or characteristic	Possible values	Comments
magnification	larger same size smaller	
attitude	upright inverted	The image has the same vertical orientation as the object. The image is upside down in comparison to the object.
type	virtual real	If a screen is placed at the image position, no image will appear on the screen. If you placed a screen at the image position, an image would appear on the screen.
position	measured from optical device	

Images and the Light Ray Model

You can determine all of the characteristics of an image by using the light ray model. Since you are very familiar with **plane mirrors** (mirrors with a flat surface), they will provide a good learning tool. By simply looking at an image in a plane mirror, you can qualitatively describe the image. Look at the Canadian flag in Figure 10.8. As you can see, the image is upright. It also appears to be the same size as the object and the same distance behind the mirror as the object is in front of the mirror. Although the image appears to be behind the mirror, it is obvious that there are no light rays coming from behind the mirror and passing through it. Therefore, the image must be virtual.

Figure 10.8 The image of the flag is the same size, is upright, and is the same distance behind the mirror as the flag is in front of it. However, if you hold a screen behind the mirror where the image appears to be, no light from the flag is able to reach the screen. The image formed by any plane mirror is a virtual image.

The light ray model provides a method for predicting the four properties of an image without using a mirror. Study Figure 10.9 to see how light rays and the law of reflection allow you to predict the image characteristics quite precisely. Once again, the Canadian flag is the object. The straight line represents the mirror. As you now know, light rays diverge from every point on the flag. You can choose any point or combination of points and draw several rays emanating outward. In the figure, rays are diverging from Point *O*. Those rays that strike the mirror will reflect according to the law of reflection. You can see that the rays continue to diverge after they reflect from a plane mirror. The point *O′* from which those diverging rays *appear* to be diverging, is the location of the image of point *O* on the flag. To find the image location, you simply continue the light rays back behind the mirror. The point where two or more extended rays meet is the location of the image. You should always use dashed lines to extend the rays to indicate that these are not actually light rays but instead, an imaginary path. When used in this way, dashed lines will always indicate that the image is virtual. You can repeat the process from as many points on the image as you choose in order to visualize the image as a whole. For example, if you sketched rays from point *A* on the flag, you would find that they appear to be diverging from point *A′* behind the mirror. For practice using the ray model, complete the following investigation.

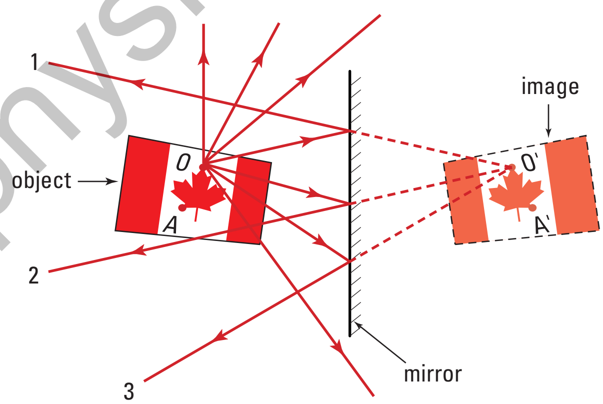


Figure 10.9 The image of point *O* can be seen in the mirror from any location in front of the mirror where the reflected light rays from point *O* are travelling.

Notice that point *A* is on the left side of the flag but *A′* is on the right side of the image of the flag. This apparent right-left reversal is a characteristic of all mirror images.

INVESTIGATION 10-A

Predicting Image Characteristics Using the Light Ray Model

TARGET SKILLS
- Predicting
- Hypothesizing
- Performing and recording
- Analyzing and interpreting

Problem

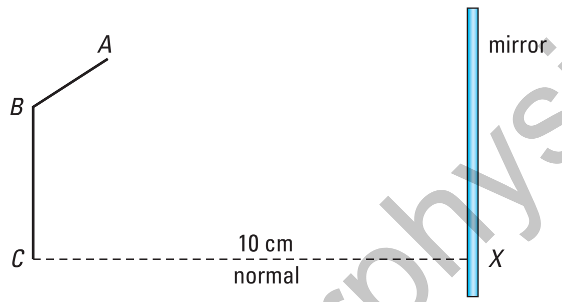
How can you use light ray diagrams to predict the characteristics of an image formed by a plane mirror?

Equipment

- plane mirror
- mirror supports
- ray box with single ray

Procedure

1. Draw a diagram similar to the one shown here. Place the "object" about 10 cm in front of the centre of the mirror.



2. Draw a normal line from point *C* to the mirror. This is your reference line.
3. Support the mirror vertically. Using the ray box, shine a single light ray so that it appears to travel from point *A* toward the mirror surface along a normal line to the mirror.
4. Mark a small dot just in front of the mirror to indicate the path of the light ray on the paper.
5. Move the ray box, and shine a second light ray toward the mirror so that it appears to travel from point *A* to point *X* where the reference line meets the mirror. Mark dots on the paper to indicate the path of both the incident and reflected rays.
6. Move the ray box, and shine a third light ray from point *A* toward the mirror so that it reflects from the mirror surface at any point above or below the other two rays. Mark the path of the incident and reflected rays.
7. Remove the ray box and mirror from the paper. Draw lines to represent the three incident and reflected rays. Draw arrows on the lines to represent the direction in which the light rays are travelling.
8. Draw dotted lines that extend the reflected ray behind the mirror line until they cross one another.
9. Repeat steps 3 to 8 for the point marked "*B*" on the object in the diagram.
10. Modify the procedure in steps 3 to 8 as required, and draw the path of three light rays coming from point *C*.
11. Draw a dotted line to show the position of the image behind the mirror.

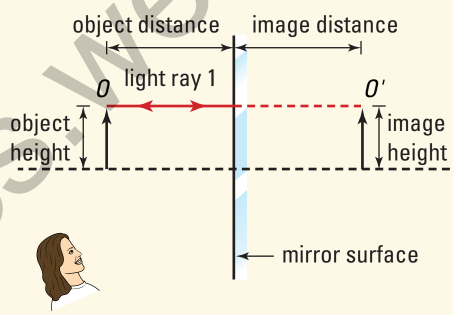
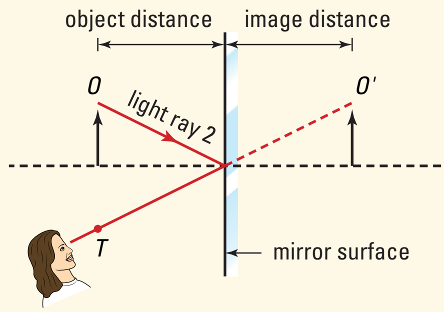
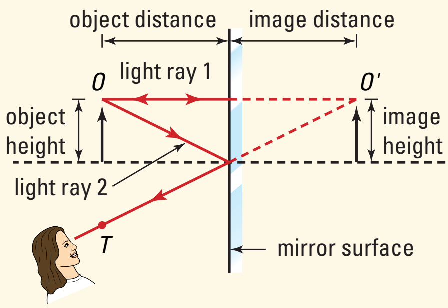
Analyze and Conclude

1. Describe the four characteristics of your image.
2. What is the minimum number of light rays that need to be drawn to establish the location of a point on an image? Why?

Drawing Light Ray Diagrams

You could use any light ray from the object that strikes a mirror to help locate an image. However, certain rays are much easier to use. In fact, two specific rays will allow you to locate an image formed by any type of mirror or lens. Often, you will want to use a third ray, just as a test to ensure that you drew the first two rays correctly. Table 10.2 outlines the procedure for using the first two rays for plane mirrors.

Table 10.2 Light Ray Diagrams for Plane Mirrors

Description	Comments	Illustration
Reference line Draw a normal line from the base of the object to the mirror.	Since the line is perpendicular to the mirror, an incident ray would reflect directly backward. The ray extended behind the mirror would form a continuous, straight line. Thus the base of the image will lie on this line.	
Light ray #1 Draw a ray from the top of the object to the mirror parallel to the reference line.	Since this line is parallel to the reference line, it is normal to the mirror. The reflected ray will go back along the same line.	
Light ray #2 Draw a line from the top of the object to the point where the reference line meets the mirror. Draw the reflected ray according to the law of reflection.	Instead of measuring angles to draw the reflected ray, you can construct congruent triangles. Mark a faint point directly below the object that is as far below the line as the top of the object is above the line (*T*). Draw the reflected ray from the mirror to this point. Since the triangles are congruent, the angle of reflection must be equal to the angle of incidence.	
Extended rays Extend both rays behind the mirror, using dashed lines, until they intersect.	The point at which the extensions of the rays meet is the position of the top of the image.	

Geometry and the Properties of Images

Qualitatively, observation of images in mirrors and light ray diagrams give a strong indication that an image in a plane mirror is the same size as the object. However, physicists always prefer to demonstrate such relationships quantitatively. One reason that the light ray model is so useful is that you can apply the mathematics of geometry and develop quantitative relationships. For this reason, this branch of physics is known as **geometric optics**.

Figure 10.10 shows a ray diagram of an object (an arrow) with the two rays described in Table 10.2 and the image. Notice that the distance from the object to the mirror is labelled d_o and the distance from the mirror to the image is labelled d_i. Consider the following aspects of the figure.

- $\angle\beta = \angle\theta$ because they are opposite angles formed by intersecting lines.
- $\angle\alpha_1 = \angle\alpha_2$ because they are complementary to equal angles.
- The angles at point C are both right angles.
- Triangle ABC is congruent to triangle DBC because they have two equal angles with a common side between the equal angles. (angle-side-angle)
- $d_i = d_o$ because they are equivalent sides of congruent triangles.

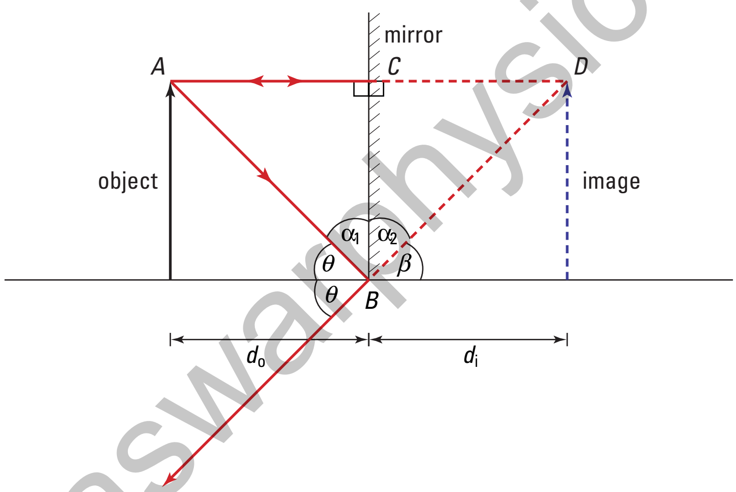


Figure 10.10 The triangles formed by the incident and extended light rays are similar triangles. By using geometry, it is possible to show that the image distance d_i equals the object distance d_o.

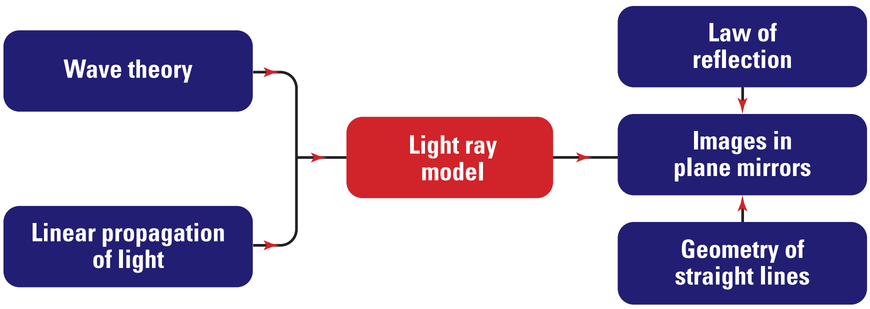
Think It Through

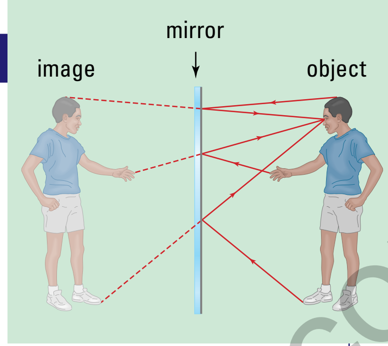
- Make a ray diagram identical to Figure 10.10. Add the labels, h_o for the height of the object and h_i for the height of the image. Use geometry to verify that the heights of the image and object are the same.

Career Link

"Chips" Klein is an inventor and successful businesswoman. She created and produced a special kind of make-up mirror that consists of three mirrors mounted in an arc. The mirrors are being sold throughout the world. Klein is one of the founding members of the Women Inventors Project, an organization that provides information, expertise, and encouragement to women inventors throughout Canada. Find out more about the Women Inventors Project and some of the inventions associated with it.

Concept Organizer





How can images in plane mirrors, like the reflection of this boy, be described? It is possible to model his image by applying the light ray model, which is based on the wave theory and the concept of the linear propagation of light, together with the law of reflection and the geometry of straight lines.

Figure 10.11 Understanding images in plane mirrors

In summary, the characteristics of images in plane mirrors are:

- the image is the same distance from the mirror as is the object
- the image is the same size as the object
- the image is always upright
- the image is always virtual

Applying Plane Mirrors: MEM Chips

Micro-electro-mechanical (MEM) computer chips are silicon chips the size of a thumbnail that have tiny movable plane mirrors mounted on them. When a small electric current is applied to the computer chip, the mirror changes its angle. MEM chips are used in "optical switches." Optical switches are electronic devices that control beams of light travelling inside optical fibres about as thick as a single human hair. The light coming out of one optical fibre can be reflected by the mirror into a nearby optical fibre. A single light beam can carry huge amounts of telecommunications information such as TV channels and telephone messages all at the same time. Bundles of these efficient optical fibres are now connected throughout most parts of North America, and by submarine cable to Europe and the rest of the world.

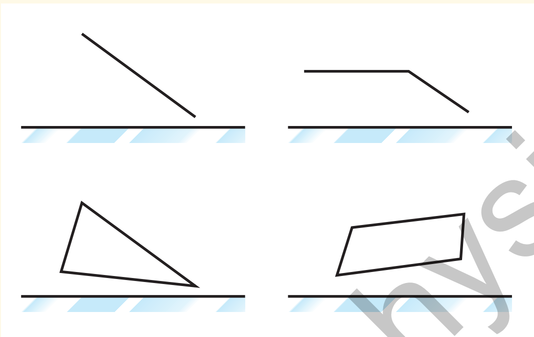
10.2 Section Review

1. **K/U** Distinguish between an object and an image.

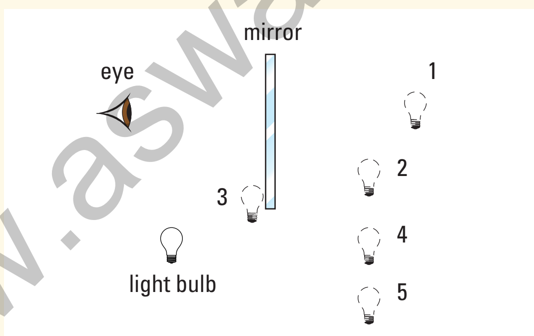
2. **K/U** Distinguish between converging light rays and diverging light rays.

3. **K/U** Why is it necessary to draw only two light rays from any given point on an object to determine the position of the image when light is reflected from a (plane) mirror?

4. **C** Copy the following diagrams into your notebook and draw a light ray diagram for each example to show how the image is formed in the plane mirror. Show all construction lines.



5. **K/U** A person is facing a mirror, observing a light bulb. At what position does the person "see" the image of the light bulb?



6. **C** Explain how it is possible to observe a given point on an object or an image from many different locations.

7. **K/U** State the characteristics of the image formed by a plane mirror. What characteristics of the image remain the same, and what ones change as the object position is changed? Explain your answer.

8. **C** A small, round bead, 1 mm in diameter, is placed 5.0 cm in front of a plane mirror. Draw an eye 7.0 cm to one side of the bead and 6.0 cm in front of the mirror. Draw a light ray diagram to locate the position of the image and to show how the light travels from the object to the eye to form the image.

9. **MC** Predict the shortest length a mirror could be that would enable all members of a family ranging in height from 2.1 m to 0.85 m to see full-length images of themselves. Assume that everyone's eyes are 10 cm below the top of their head, and that they are standing 1.5 m in front of the mirror. Does the distance the person stands in front of the mirror make any difference to the length of mirror required? Explain your answer, and draw a light ray diagram to illustrate the path of the light for each person.

10. **I** Design an experiment to illustrate the difference between real and virtual images.

11. **MC** Why are full-length mirrors in clothing stores sometimes tiltable?

UNIT PROJECT PREP

Plane mirrors are useful in the construction of optical devices.

- How will reflections from plane mirrors affect your final image?

10.3 Images Formed by Curved Mirrors

SECTION EXPECTATIONS

- Describe, with the aid of light ray diagrams, the images formed in concave and convex mirrors by objects at different distances from the mirror.
- Develop and use equations to calculate the heights and distances of images formed by concave and convex mirrors.
- Use the light ray model to predict image characteristics for a concave mirror, and test these predictions experimentally.

KEY TERMS

- spherical mirror
- concave mirror
- convex mirror
- centre of curvature
- radius of curvature
- vertex
- principal axis
- focal point
- focal length
- spherical aberration
- parabolic reflector
- mirror equation
- magnification equation

What do a shiny, metal soupspoon and the reflector in a car headlight have in common? Both are examples of curved mirrors. Some mirrors, such as those used to apply make-up or to shave, are designed to form images. Other mirrors are used for focussing light energy in a particular direction, such as those used in flashlights or in halogen yard lights. Using modern manufacturing techniques, it is possible to produce very complex mirror shapes to suit any required need.

Concave Mirrors

Next time that you are in a parking lot, look at the reflectors in the headlights of the different makes of car. Almost every model of car has a different-shaped headlight reflector. Many car manufacturers design the headlights to be a streamlined part of the car body, and so the curved mirrors in the headlights also have to be a special shape. However, to understand what happens to the light as it is reflected from curved mirrors, you must first analyze how light reflects from a relatively simple shape — a **spherical mirror**.

As Figure 10.12 shows, a spherical mirror has the shape of a section sliced from the surface of a sphere. If the hollowed inside surface is made into the mirror surface, it is called a **concave mirror** (think of the hollow entrance of a cave). If the outside of the sphere is made into a mirror surface, it is called a **convex mirror** (think of how your eyeball bulges outwards).

Consider the concave mirror surface shown in Figure 10.13. The **centre of curvature** (C), is located at the centre of an imaginary sphere with the same curvature as the mirror. The **radius of curvature** (R) is any straight line drawn from the centre of curvature to the curved surface. The geometric centre of the actual curved mirror surface is called the **vertex** (V). The straight line passing through both the vertex, V, and the centre of curvature, C, is called the **principal axis** (PA). One way to visualize the principal

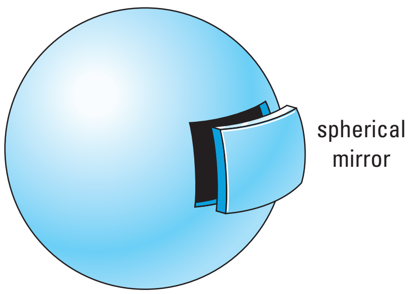


Figure 10.12 A spherical mirror has the shape of a segment of the surface of a sphere.

axis passing through the vertex of the curved mirror is to think of the handle of an open umbrella, and the way it appears to come from the centre of the umbrella covering, as seen in Figure 10.14. Figure 10.14 also illustrates an important point. All normal lines run along radii of curvature, thus they all meet at the centre of curvature. Each incident ray, its normal line, and reflected ray lie in a plane. Therefore, all reflected rays generated by incident rays that are parallel to the principal axis pass through the principal axis.

Biology Link

A number of animals, including cats, have a mirror-like reflecting layer at the back of their eyes. When light is shone into one of their eyes, some of the light is reflected back towards the light source. What is this reflecting layer for?

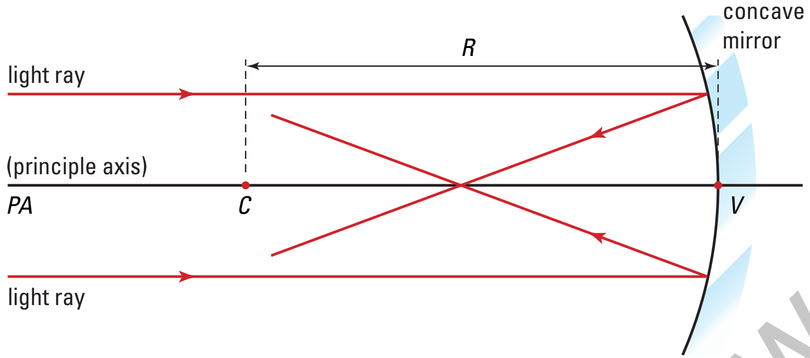


Figure 10.13 *C* on the principal axis of the spherical, concave mirror surface in this diagram is called the "centre of curvature." Light rays travelling parallel to the principal axis reflect through the same point.

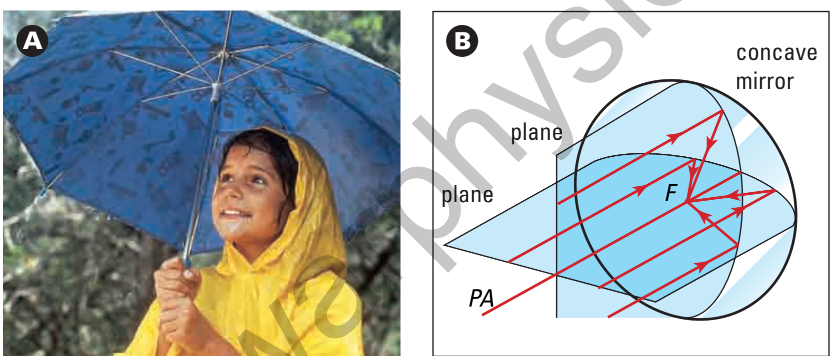


Figure 10.14 **(A)** The handle of an umbrella passes through the fabric in a similar way to the principal axis passing through the vertex of the mirror. **(B)** Incident rays parallel to the principal axis create reflected rays that pass through the principal axis.

Consider what happens when light rays that are parallel to the principal axis, strike a concave mirror. Follow the path of the ray in Figure 10.15 on page 478. The light ray strikes the mirror at point *P*. The line segment *CP* is the radius of the mirror and, therefore, is the normal to the spherical surface of the mirror at *P*. The light ray reflects from the mirror such that the angle of reflection θ_r equals the angle of incidence θ_i. Furthermore, the angle *PCF* is also θ_i, because the radial line segment *CP* crosses two parallel lines. Since two of its angles are equal, the (coloured) triangle *CPF* is an isosceles triangle; thus, sides *CF* and *FP* are equal.

When the incoming parallel light ray lies close to the principal axis, the angle of incidence, θ_i, is small, and the distance *FP* becomes similar in length to distance *FV*. Because θ_i is small,

$CF = PF = FV$. Therefore, $FV = \frac{1}{2}\,CV$ and so point F lies halfway between the centre of curvature and the vertex of the mirror. Point F is called the **focal point**. The distance from the focal point to the vertex is called the **focal length** and is symbolized, f. The focal length, f, is one-half of the radius of curvature, R, for the spherical mirror. Point F is called the focal point because all the incident light rays that are parallel and close to the principal axis of the mirror, reflect from the mirror and pass through that one point.

$$\textbf{Focal length}\text{ of a concave mirror: } f = \frac{1}{2}R$$

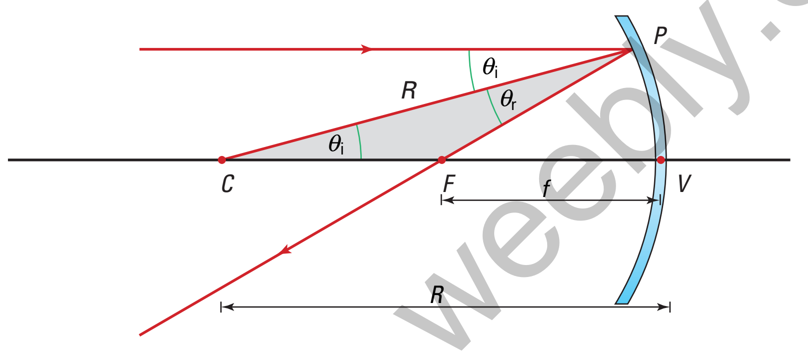


Figure 10.15 When the parallel light ray is close to the principal axis, the length segment *PF* is very close to the length *FV*. This shows that all light rays travelling close to the principal axis reflect through the same point, *F*.

Think It Through

- What are the focal length and radius of curvature of a *plane* mirror? To answer this, imagine a plane mirror as a section of a spherical mirror as in Figure 10.12. For a small sphere, any section is obviously curved. However, considering larger and larger spheres, you can see that the curvature of any subsection appears to "get flatter." A weakly curved, or flatter, surface has a larger radius of curvature than a strongly curved surface. Following this logic, answer the original question.

COURSE CHALLENGE

From the Equator to the Poles

Investigate the efficiency of photovoltaic cells from a local electronics shop. Is it possible to incorporate a parabolic reflector into a photovoltaic system to improve power output?

Learn more from the **Science Resources** section of the following web site: **www.school.mcgrawhill.ca/resources/** and find the *Physics 11 Course Challenge*

Spherical Aberration

The formula above only applies to light rays that are close to the principal axis. As you can see in Figure 10.16 (A), parallel incident light rays that are not close to the principal axis reflect across it between the focal point and the vertex of the mirror. If you use a large spherical concave mirror as a make-up mirror, you may see a partially blurred image of your face, because all the light rays that are not close to the principal axis do not focus as they should. To avoid this **spherical aberration** and reflect all of the parallel incident light to a single focal point, the mirror's surface must have a parabolic shape, as shown in Figure 10.16 (B). A spherical mirror can be used to form reasonable images if the diameter of the mirror is small compared to its focal length.

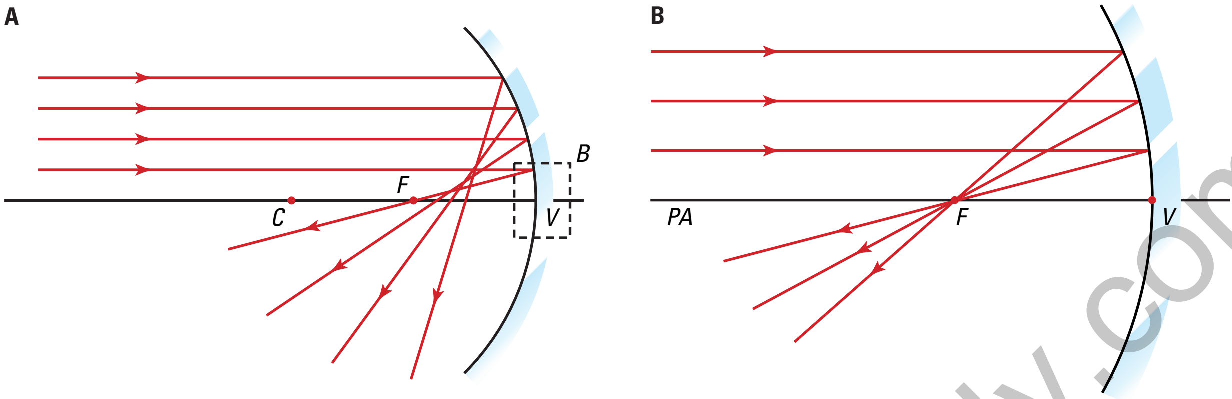


Figure 10.16 **(A)** Incident light rays that are parallel but not close to the principal axis of a spherical concave mirror do not reflect through the focal point. **(B)** All incident light rays that are parallel to the principal axis of a parabolic reflector pass through its focal point.

When concave parabolic mirrors are used to reflect light energy, rather than to produce images, they provide an interesting example of the reversibility of the path of light. When light from a distant source such as the Sun is reflected by a **parabolic reflector** (mirror), all the light energy is focussed at the focal point of the mirror. Concave mirrors are used in parabolic reflectors to focus sunlight for solar-thermal energy facilities, such as those located in the Mojave Desert, near Barstow, California, and in the Pyrénées of southern France. The television satellite dish is another example of a parabolic reflector. In this case, electromagnetic signals are being received from orbiting transmitters. If the path of the light is reversed, and a light source is placed at the focal point of the mirror, light from the bulb is reflected from the mirror in a parallel beam, such as is formed by a searchlight.

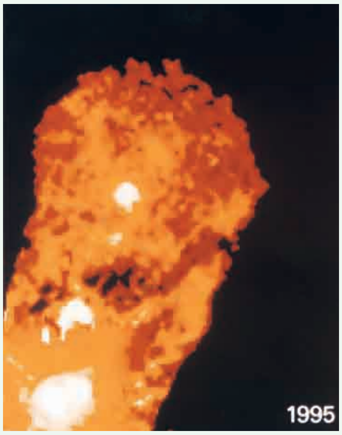
Concave mirrors that are not parabolic also have many practical uses. Most car headlights are not perfectly parabolic because they are designed to reflect a beam of light that spreads out a little to illuminate the complete width of the road (see Figure 10.17). Solar ovens used for cooking are designed to reflect the sunlight onto a small area of the bottom of the pot (cooking utensil). Why would it not be practical to use a parabolic mirror for this purpose?

Figure 10.17 The reflectors used in solar cookers and headlights are deliberately designed so that they do not have a perfectly parabolic shape.

PHYSICS & TECHNOLOGY

TARGET SKILLS

- Analyzing and interpreting
- Conducting research



The Hubble Space Telescope

Astronomers need clear skies for their telescopes. That's why most of the large telescopes are on mountaintops — higher than most of the clouds, pollution, and hot-air currents that could degrade the images. For many years, astronomers realized that a perfect location for telescope would be in space, above the atmosphere. Astronomers' dreams were realized when, on April 24, 1990, the space shuttle *Discovery* carried the Hubble Space Telescope into orbit. Unfortunately, unexpected problems soon arose. As the telescope went from −150˚C temperatures in the dark to +200˚C in the sunlight, Hubble's solar arrays expanded and vibrated. Worse still, the primary mirror produced spherical aberration.

During subsequent space shuttle visits, astronauts corrected the problems, actually improving the telescope beyond the original specifications. Today, the Hubble Space Telescope produces breathtaking new images of the planets, stars, galaxies, and nebulas.

Light from distant objects enters the telescope through a door that can be closed to protect the inside from direct sunlight or space debris. The light reflects off the 2.4 m primary concave mirror to the secondary mirror. This 25 cm convex mirror reflects the light back through a hole in the primary mirror to the focal plane. Here, special mirrors correct the blurring and send the light to various optical instruments.

A Near Infrared Camera and Multi-Object Spectrometer use infrared radiation to produce images that highlight temperature differences. Visible light from nearby planets and distant galaxies is sent to the Wide Field Planetary Camera or to a smaller, high-resolution camera. The Space Telescope Imaging Spectrograph separates ultraviolet light into different frequencies in order to identify the chemical makeup of celestial objects. Special Faint Object Cameras can produce images in the visible and ultraviolet ranges.

The Hubble Space Telescope has recorded explosions of stars, found new planets around nearby stars, produced images of Pluto and its moon, photographed volcanoes on Jupiter's moon Io, and shown amazingly intricate webs of dust and gas in space almost to the edge of the universe.

Analyze and Conclude

1. What is the great advantage of the Hubble Space Telescope?
2. What kinds of lenses or mirrors are used on the telescope?
3. What kinds of radiation does the Hubble Space Telescope detect? What can astronomers learn from each kind?

QUICK LAB

Properties of Concave and Convex Mirrors

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

Place a concave mirror on edge in a piece of modelling clay, and point the mirror toward the Sun, tilting it slightly so that the light is reflected slightly upward. Place an opaque white screen in the path of the reflected light. Move the screen back and forth in the reflected beam, and locate the position where the light is focussed to the smallest possible spot of light. Repeat the procedure with the convex mirror.

CAUTION Take care not to reflect the sunlight into your eyes.

Analyze and Conclude

1. What is special about the position of the bright spot of light?
2. What does the bright spot of light actually represent?
3. Determine the focal length of the concave mirror.
4. Comment on the observations made when using the convex mirror. Suggest reasons for these observations.

The Geometry of Spherical Mirrors

Draw a line representing the principal axis on a blank piece of paper. Position a concave mirror shape at right angles to the principal axis line, with the vertex of the mirror touching the line. With a ray box shine several light rays parallel to the principal axis toward the mirror. Draw the path of the incident and reflected rays on the page.

Predict what will happen if you shine a set of light rays parallel to the principal axis of a convex mirror that is positioned the same way as the concave mirror. Check your prediction. For each light ray, mark the path of the light ray and draw it on the paper.

Analyze and Conclude

1. What is the significance of each of the two points on the principal axis where the light rays cross?
2. What mathematical relationship exists between these two points?
3. Does the concave mirror surface demonstrate spherical aberration?
4. Determine the position of the focal point of the convex mirror, and measure the focal length.

Images in Concave Mirrors

When you move a plane mirror away from your face, the only image characteristic that changes is the distance of the image from the mirror. The other three characteristics remain the same. The images formed by curved mirrors are much more varied, particularly in the case of the concave mirror. If you have a make-up or shaving mirror at home, try propping it up so that you can view your image in the mirror as you back away from it across the room. As you first move away from the mirror, the image of your face is upright and gets larger, then it disappears. As you continue moving away, your image re-appears but it is now upside down. As you continue to move away, the image continues to decrease in size. How can a single mirror shape form so many different

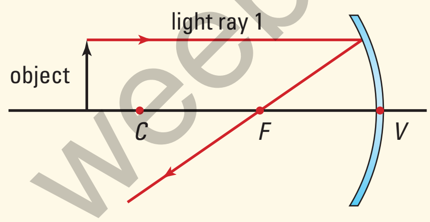
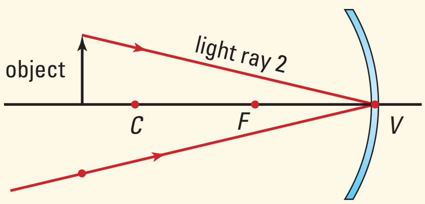
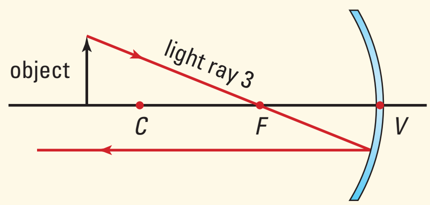
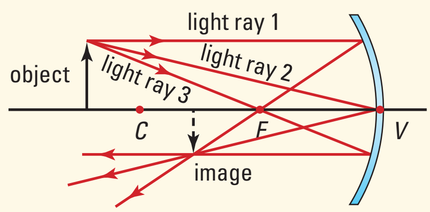
Web Link

www.school.mcgrawhill.ca/resources/

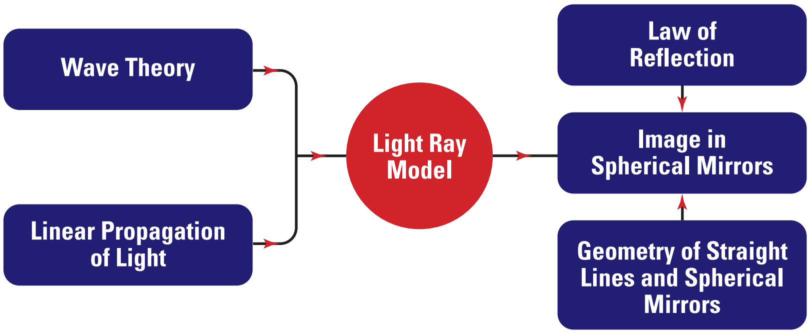
For more pictures and information about the Hubble Space Telescope, go to the above Internet site. Follow the links for **Science Resources** and **Physics 11** to find out where to go next.

kinds of images? You can answer this question by constructing ray diagrams for several different distances from a concave mirror. The directions for drawing ray diagrams for concave mirrors are given in Table 10.3. Note that a third, "test" ray has been added to the list.

Table 10.3 Light Ray Diagrams for Concave Mirrors

Description	Comments	Illustration
Reference line Draw a principal axis from the centre of curvature to the vertex of the mirror. Place the base of the object on this line.	Since the line is perpendicular to the mirror, an incident ray would reflect directly backward. The ray extended behind the mirror would form a continuous, straight line. Thus the base of the image will lie somewhere along this line.	
Light ray #1 Draw a ray from the top of the object to the mirror parallel to the principal axis. Draw the reflected ray through the focal point.	Since this incident ray is parallel to the principal axis, it will reflect through the focal point.	
Light ray #2 Draw a ray from the top of the object to the vertex. Draw the reflected ray according to the law of reflection.	Instead of measuring angles to draw the reflected ray, you can construct congruent triangles. Mark a faint point directly below the object that is as far below the line as the top of the object is above the line. Draw the reflected ray from the mirror to this point. Since the triangles are congruent, the angle of reflection must be equal to the angle of incidence.	
Light ray #3 Draw a ray from the top of the object through the focal point and on to the mirror. Draw the reflected ray parallel to the principal axis.	Any incident light ray passing through the focal point will reflect back parallel to the principal axis.	
Extended rays If the reflected rays are converging, extend them until they all cross. If the reflected rays are diverging, extend them back behind the mirror with dashed lines.	The point at which the rays meet is the position of the top of the image. If the reflected rays meet, the image is real. If the rays must be extended backward, behind the mirror, the image is virtual.	

Concept Organizer



Images in spherical mirrors, like the reflection of this girl, can be modelled in the same way as images in plane mirrors. By applying the light ray model, which is based on the wave theory and the concept of the linear propagation of light, together with the law of reflection and the geometry of straight lines and spherical mirrors, we can understand why this girl's image appears the way it does.

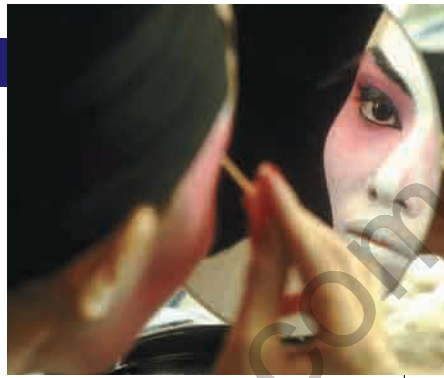


Figure 10.18 Understanding images in spherical mirrors

You can now draw ray diagrams for concave mirrors with the object in three, representative regions. The results will provide examples of all possible classes of images that can be formed by concave mirrors. The regions are (1) beyond the centre of curvature, (2) between the centre of curvature and the focal point, and (3) between the focal point and the vertex.

The ray diagram in Figure 10.19 on page 484 has an object beyond the centre of curvature. As you can see, the reflected rays converge in front of the mirror and below the principal axis, making the image inverted. If you placed a screen at the image position, the reflected rays would focus on the screen and form an image. Therefore, the image is real. In addition, you can see that the image is smaller than the object.

Because your eyes can only focus diverging rays, you must be behind the image in order to see it. After the rays converge at the image position, they pass through each other and begin to diverge. These are the rays that you see.

In Figure 10.20 on page 484, the light ray diagram shows that when the object is placed between the centre of curvature, *C*, and the focal point, *F*, the image is still real and inverted, but is now larger than the object.

Think It Through

- At what position on the principal axis will the size of the image be the same as the size of the object?

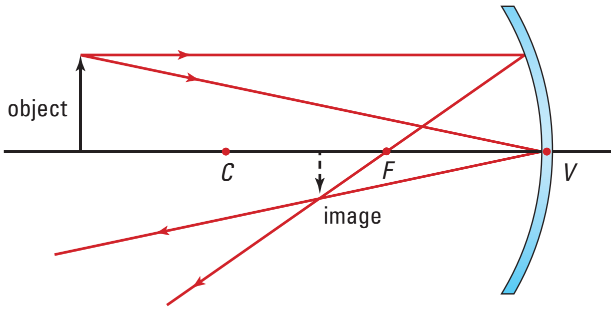


Figure 10.19 When the object is located beyond the centre of curvature, the image is always inverted, real, and smaller than the object, and is located between the focal point and the centre of curvature.

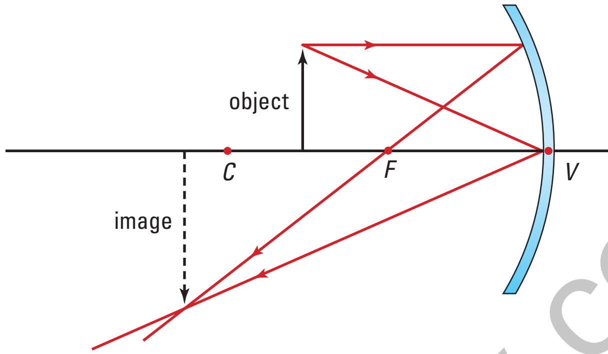


Figure 10.20 When the object is located between the centre of curvature and the focal point, the image is always inverted, real, and larger than the object, and is located beyond the centre of curvature.

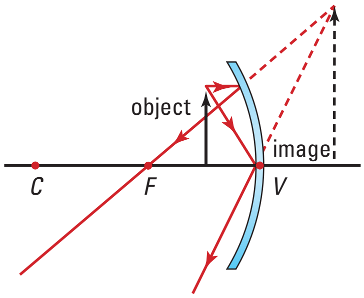


Figure 10.21 When the object is located between the focal point and the vertex of the concave mirror, the image is always upright, virtual, and larger than the object. The image is located behind the mirror.

When the object is placed at the focal point, light rays are reflected parallel to one another and, thus, no image is formed because they never cross in front of the mirror or behind it. When the image of your face in a make-up mirror disappears, it is because your face is positioned at the focal point of the mirror. When the object is positioned between the focal point and the vertex of the mirror, the image properties change completely. As shown in Figure 10.21, the light rays reflecting from the concave mirror are now diverging. When *reflected light rays diverge*, no image will be formed in front of the mirror, and thus the image will be virtual. To locate the position of the image, the reflected light rays must be extended behind the mirror where they converge to a point. The virtual image is upright and larger than the object. A concave mirror can be used as a make-up or shaving mirror when the object, your face, is placed between the focal point and the vertex of the mirror.

Applications of Concave Mirror Images

In movies such as *Top Gun*, some of the most exciting scenes occur when it seems that you are in the pilot's seat looking out through the cockpit window of the fighter plane. Because pilots operating these high performance aircraft do not have time to look down at their instruments, a special information display system called a "head-up display" (HUD) is used. The information given by the critical instruments in the cockpit is displayed directly on the window that the pilot looks through. This method of displaying information is now used in some cars and trucks (see Figure 10.22). The driver sees the speed of the car displayed on the windshield, and does not need to look down at the speedometer.

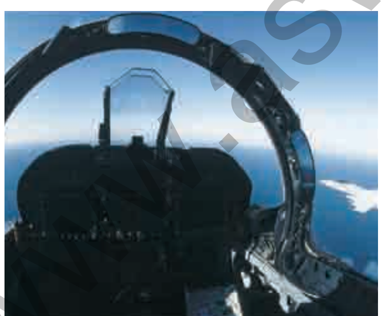


Figure 10.22 HUD display in a jet

A concave mirror is a key part of the HUD technology. As shown in Figure 10.23, the digital readout of the speedometer is placed between the focal point and the vertex of the concave mirror. The mirror produces a virtual, enlarged, and upright image of the digital readout that reflects up onto the windshield of the car. A special layer of material called a "combiner" is imbedded in between the layers of glass in the windshield in front of the driver. The light from the concave mirror is reflected into the driver's eyes by the combiner material. The combiner material has special optical properties. All the colours in the light coming through the windshield to the driver's eyes pass through the combiner, except the colour given off by the digital speedometer display. The combiner also acts as a plane mirror to the colour of the light from the digital display, thus making the digital readout always visible to the driver. The display is usually located at the bottom of the windshield to provide the minimum distraction to the driver.

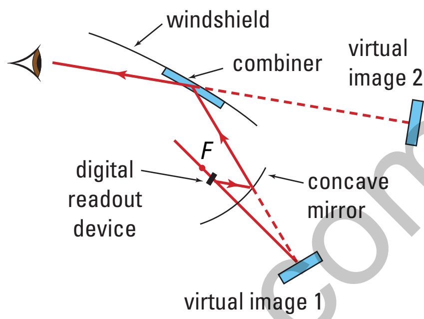


Figure 10.23 The HUD display is formed by combining a concave mirror and a plane mirror (the combiner) that is mounted inside the glass of the windshield.

CAREERS IN PHYSICS

"Good Seeing"

The first time Dr. Neil de Grasse Tyson looked at the Moon through binoculars, he knew he wanted to be a scientist. Today, he is an astrophysicist doing research into dwarf galaxies and the "bulge" at the centre of the Milky Way. At a recent high school reunion, his former classmates voted him the one with the "coolest job."

Astrophysicists such as Dr. Tyson study the physical properties and behaviour of celestial bodies. They make observations using optical telescopes that generate visible images of stars and other celestial bodies by means of concave mirrors. Using computers, they record the images and then examine them in detail.

To find the kinds of images they seek, astrophysicists often need to travel. One of Canada's best optical observation sites is the Dominion Astrophysical Observatory (DAO) near Victoria, B.C. The Victoria area generally has clear nights and stable weather, which make for what astrophysicists call "good seeing." A more recently developed facility is the Canada-France-Hawaii Telescope (CFHT) on the extinct Hawaiian volcano Mauna Kea. Mauna Kea, with its dark skies and super-sharp star images, is the northern hemisphere's best site for optical observations.

Mauna Kea is also one site for a new optical-infrared telescope project. Called the Gemini project, it has another site on Cerro Pachon, a mountain in central Chile. Together, the twin Gemini telescopes give astrophysicists total coverage of both the northern and southern hemispheres' skies. That's "good seeing!"

Going Further

Research one of these: the Dominion Astrophysical Observatory (DAO), the Canada-France-Hawaii Telescope (CFHT), or the Gemini Project.

PHYSICS FILE

Remember: When light rays from a point on an object are reflected from any shape of mirror, the type of image can always be determined by observing the path of the reflected light rays. If the *reflected light rays converge*, the image is *real*. If the *reflected light rays diverge*, the image is *virtual*.

Images in Convex Mirrors

Do you remember reading the warning printed on the convex mirror on the passenger side of a car? This warning gives you a hint about the image formed by the mirror. What does the warning say, and why is it necessary? Next time you sit in the driver's seat of a car, look at an object in the rearview mirror inside the car, and then look at the same object through the passenger-side outside mirror. The image in the passenger-side mirror is smaller than the one in the plane mirror inside the car.

You can determine the properties of an image formed in a convex mirror by using the same light rays as those you used for the concave mirror. The relationship between the focal point of the convex mirror and its centre of curvature is the same as that of the concave mirror, except that for the convex mirror both points are behind the mirrored surface, rather than in front of it. The convex mirror has a virtual focal point. The three light rays used to determine the properties of the image are described in Table 10.4.

The properties of images formed by convex mirrors are very similar to those formed by plane mirrors. Look at the reflected light rays in the last illustration in Table 10.4. The reflected light rays will always diverge for all positions of the object in front of the convex mirror, so the images are always virtual. As well, the images will always be upright. The differences occur in the size and position of the images formed by the mirror. When the object is close to the mirror, the image size is slightly smaller than that of the object. As the object moves away from the mirror, the image becomes smaller and its position moves toward the virtual focal point of the mirror. The image will always be formed within the shaded triangle on the diagram.

Convex mirrors are generally used because they provide a very wide field of view, compared to a plane mirror. In many stores where they are used for security purposes, it is possible to see almost the entire store from one location. Convex mirrors are also used in hospital corridors and in factories for safety purposes to help prevent people and sometimes vehicles from colliding.

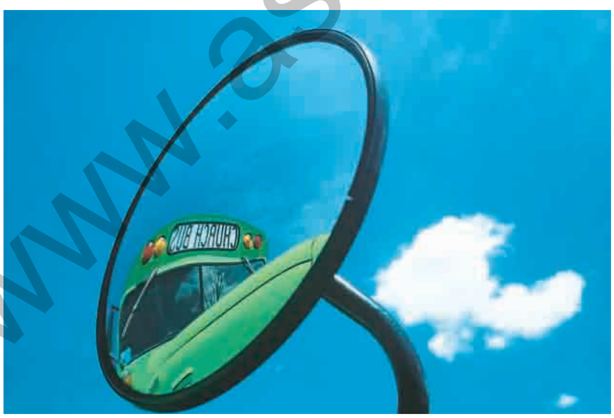
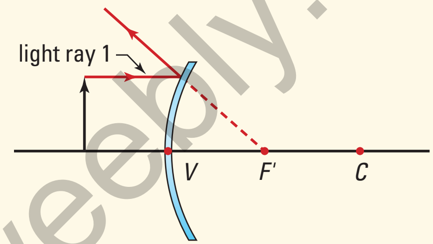
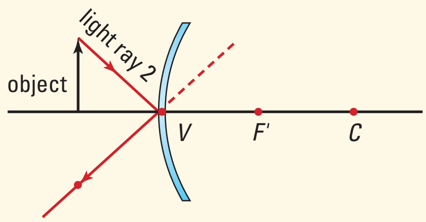
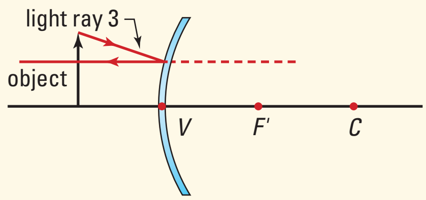
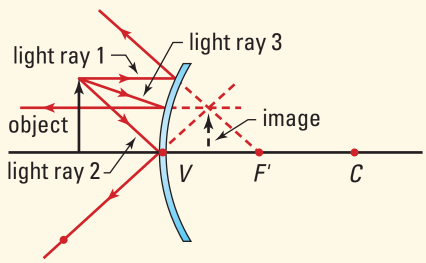


Figure 10.24 Convex mirrors are used when you need to be able to view a large area with just one mirror. The smaller, distorted images formed by convex mirrors can sometimes cause safety problems when used for car rearview mirrors.

Table 10.4 Light Ray Diagrams for Convex Mirrors

Description	Comments	Illustration
Reference line Draw a principal axis from the front of the mirror through the vertex to the centre of curvature. Place the base of the object on this line.	Since the line is perpendicular to the mirror, an incident ray would reflect directly backward. The ray extended behind the mirror would form a continuous, straight line. Thus the base of the image will lie somewhere along this line.	
Light ray #1 Draw a ray from the top of the object to the mirror parallel to the principal axis. Draw the reflected ray as though it were coming from the focal point.	Any incident ray that is parallel to the principal axis, will reflect along a line from the focal point, through the point where the parallel ray meets the mirror.	
Light ray #2 Draw a ray from the top of the object to the vertex. Draw the reflected ray according to the law of reflection.	Instead of measuring angles to draw the reflected ray, you can construct congruent triangles. Mark a faint point directly below the object that is as far below the line as the top of the object is above the line. Draw the reflected ray from the mirror to this point. Since the triangles are congruent, the angle of reflection must be equal to the angle of incidence.	
Light ray #3 Draw a ray from the top of the object toward the focal point behind the mirror. Stop the ray at the mirror. Draw the reflected ray parallel to the principal axis.	Any incident light ray directed toward the virtual focal point will reflect back parallel to the principal axis.	
Extended rays Extend the rays behind the mirror until they cross.	The point at which the rays meet is the position of the top of the image. Since the reflected rays meet behind the mirror, the image is virtual.	

COURSE CHALLENGE

Keeping the Mirror Clean

Design a Moon station for manufacturing photovoltaic cells. Consider social, economic, political, and environmental issues that would need to be addressed to ensure the success of your Moon base.

Learn more from the **Science Resources** section of the following web site: ***www.school.mcgrawhill.ca/resources/*** and find the *Physics 11 Course Challenge*

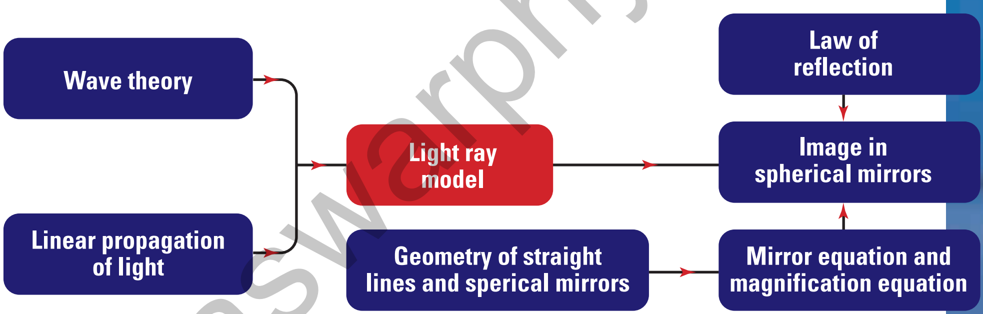
The Mirror Equations

Drawing light ray diagrams provides a strategy for both explaining and predicting the properties of images formed in plane, concave, and convex mirrors. To determine the magnification of the image, and its position with precision, it is necessary to draw very accurate scaled diagrams.

For the best precision, you can calculate the image distance and magnification. When studying the ray diagrams for plane mirrors, you used the geometry of congruent triangles to show that the image distance from the mirror and the height of the image was the same as that of the object. By using a similar strategy, you can use similar triangles to derive two equations, known as the *mirror equation* and the *magnification equation*, to provide a complete quantitative description of the images formed in curved mirrors. The following quantities are included in these two equations:

f = the focal length of the mirror
h_o = the height of the object
h_i = the height of the image
d_o = the distance of the object from the mirror
d_i = the distance of the image from the mirror
m = the magnification of the image

Concept Organizer



Where does the image appear? What are its size and orientation? These questions can be answered by applying both the mirror and the magnification equations. The mirror and magnification equations are used to model and understand images in spherical mirrors. They are based on the application of the light ray model, the law of reflection, and the geometry of straight lines and spherical mirrors.

Figure 10.25 Understanding the properties of images in spherical mirrors

CONVENTION FOR THE MIRROR AND MAGNIFICATION EQUATIONS

To include all of the possible properties of both images and objects, the following sign convention has been established for both concave and convex spherical mirrors.

OBJECT DISTANCE

d_o is positive for objects in front of the mirror (real objects)

IMAGE DISTANCE

d_i is positive for objects in front of the mirror (real images)
d_i is negative for objects behind the mirror (virtual images)

IMAGE ATTITUDE

h_i is positive for images that are upright, compared to the object
h_i is negative for images that are inverted, compared to the object

FOCAL LENGTH

f is positive for concave mirrors
f is negative for convex mirrors

PHYSICS FILE

Could an object be virtual? Yes. If another mirror or lens created an image, it could become an object for a second mirror. If the "image turned object" was located behind the second mirror, it would be virtual and d_o would be negative. The object would be virtual because the light rays would never reach the object position. They would encounter the second mirror and be reflected before they could reach the position behind the mirror.

Examine the ray diagram in Figure 10.26. The shaded triangles are formed by a light ray travelling from the top of the object to the vertex of the mirror and the reflected ray travelling back at an angle that obeys the law of reflection. The angle of incidence is equal to the angle of reflection and the angles at C and at A are right angles. Since the triangles have two angles that are equal to each other, the third angles must be equal and the triangles are similar.

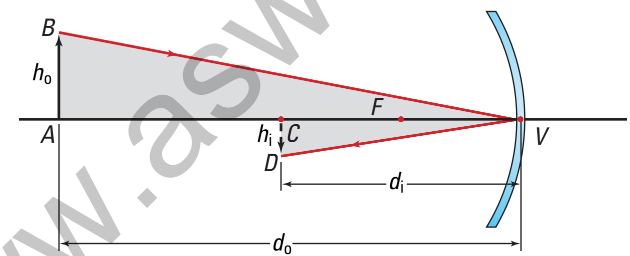


Figure 10.26 The coloured triangles *ABV* and *CDV* are similar triangles. Remember that the mirror equation and the magnification equation are only valid if the parallel light rays are close to the principal axis of the mirror.

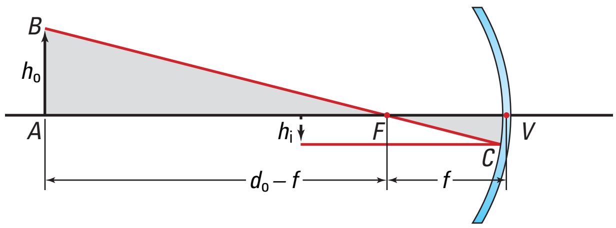


Figure 10.27 The coloured triangles *ABF* and *VCF* are similar triangles.

Figure 10.27 shows different rays for the same object and image. Each of the shaded triangles has a right angle. Also, the small angles in each triangle are equal because they are opposite angles between two intersecting lines. Since the triangles have two equal angles, the third angles must also be equal and the triangles are similar.

The following procedure uses the rules of similar triangles and geometry to develop the mirror equation.

- Ratios of equivalent sides of similar triangles (Figure 10.26) are equal. The image height is negative because it is inverted.

$$\frac{h_o}{-h_i} = \frac{d_o}{d_i}$$

- Using the same rule of similar triangles and the triangles in Figure 10.27, the relationship to the right holds true.

$$\frac{h_o}{-h_i} = \frac{d_o - f}{f}$$

- Since both of the equations have expressions equal to $\frac{h_o}{-h_i}$, you can set those expressions equal to each other.

$$\frac{d_o}{d_i} = \frac{d_o - f}{f}$$

- To isolate f, separate the right side as shown.

$$\frac{d_o}{d_i} = \frac{d_o}{f} - \frac{f}{f}$$

$$\frac{d_o}{d_i} = \frac{d_o}{f} - 1$$

- Add 1 to both sides of the equation.

$$\frac{d_o}{d_i} + 1 = \frac{d_o}{f}$$

- Create a common denominator on the left side.

$$\frac{d_o}{d_i} + \frac{d_i}{d_i} = \frac{d_o}{f}$$

$$\frac{d_o + d_i}{d_i} = \frac{d_o}{f}$$

- Divide both sides by d_o.

$$\frac{d_o + d_i}{d_o d_i} = \frac{\cancel{d_o}}{f\,\cancel{d_o}}$$

$$\frac{1}{f} = \frac{d_o + d_i}{d_o d_i}$$

- Separate the right side into two terms and simplify.

$$\frac{1}{f} = \frac{\cancel{d_o}}{\cancel{d_o}\,d_i} + \frac{\cancel{d_i}}{d_o\,\cancel{d_i}}$$

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

The final equation above is known as the **mirror equation**. It relates the distances of the object and the image to a concave mirror, in terms of the mirror's focal length.

MIRROR EQUATION

The reciprocal of the focal length is the sum of the reciprocals of the object distance and the image distance.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

Quantity	Symbol	SI unit
focal length	f	m (metres)
object distance	d_o	m (metres)
image distance	d_i	m (metres)

Think It Through

- Use the mirror equation to show that when the object distance is very large for concave mirrors, the image will be formed at the focal point. Hint: Let $d_o \rightarrow \infty$ so that $\frac{1}{d_o} \rightarrow 0$ and solve for d_i.

The magnification of the image in a mirror is the ratio of the image height to the object height: $m = \frac{h_i}{h_o}$. If the image is larger than the object, the magnitude of m is greater than one, and if the image is smaller than the object, then m is less than one. Rearranging the first equation in the derivation of the mirror equation, you find:

Magnification Equation $\quad m = \frac{\text{image height}}{\text{object height}} = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$

When determining the magnification, it should be noted that a positive value indicates that the image is upright, and a negative value indicates that the image is inverted, compared to the object.

Think It Through

- Use the mirror and magnification equations to show that the image distance for a plane mirror is negative, and thus a virtual image is formed.
- Show that the magnification for a plane mirror is $m = +1$. Hint: begin by recalling that the focal length for a plane mirror is infinite, so that $\frac{1}{f} = \frac{1}{\infty} = 0$, and solve for d_i.

TRY THIS...

Use a variety of information technology resources to research the use of mirrors in the generation of electric energy. Prepare a written report, with photographs or diagrams to show typical methods of producing renewable forms of electric energy.

Images Formed by Concave and Convex Mirrors

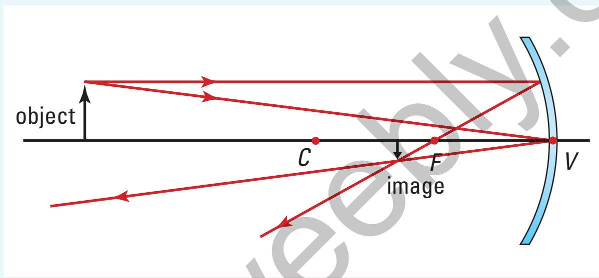
1. A concave mirror has a radius of curvature of 24.0 cm. An object 2.5 cm tall is placed 40.0 cm in front of the mirror.

(a) At what distance from the mirror will the image be formed?

(b) What is the height of the image?

Frame the Problem

- Visualize the problem by sketching the light ray diagram.
- Because the object distance is greater than the radius of curvature, you would expect the image to be inverted, smaller than the object, and real.



Identify the Goal

The distance, d_i, from the mirror that an image is formed
The height, h_i, of the image

Variables and Constants

Involved in the problem		Known	Unknown
d_o	f	$d_o = 40.0$ cm	d_i
d_i	h_o	$R = 24.0$ cm	h_i
R	h_i	$h_o = 2.5$ cm	f

Strategy

Calculate the focal length, f, from the radius of curvature, R.

Use the mirror equation to find the image distance, d_i.

Substitute values for the focal length, f, and the object distance, d_o.

Rearrange.

Calculations

$$f = \frac{R}{2}$$

$$f = \frac{24.0\text{ cm}}{2}$$

$$f = 12.0\text{ cm}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{12.0\text{ cm}} = \frac{1}{40.0\text{ cm}} + \frac{1}{d_i}$$

$$\frac{1}{12.0\text{ cm}} - \frac{1}{40.0\text{ cm}} = \frac{1}{d_i}$$

Find a common denominator.

$$\frac{10}{120\text{ cm}} - \frac{3}{120\text{ cm}} = \frac{1}{d_i}$$

$$\frac{7}{120\text{ cm}} = \frac{1}{d_i}$$

Invert.

$$\frac{120\text{ cm}}{7} = d_i$$

$$d_i = 17.14\text{ cm}$$

(a) The image is formed 17 cm in front of the mirror. Because the image distance is positive, the image is real.

PROBLEM TIP

Remember to take the reciprocal of $1/d_i$ to find d_i.

Use the magnification equation to find h_i.

$$\frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

Substitute in known values.

$$\frac{h_i}{2.5\text{ cm}} = \frac{-17.1\ \cancel{\text{cm}}}{40\ \cancel{\text{cm}}}$$

Simplify.

$$\frac{h_i}{\cancel{2.5\text{ cm}}}\cancel{(2.5\text{ cm})} = \frac{-17.1}{40}(2.5\text{ cm})$$

Round to one decimal place.

$$h_i = -1.07\text{ cm}$$

(b) The height of the image is −1.1 cm.

Validate

The units are in centimetres.

The height of the image is a negative value because the image is inverted (see the previous ray diagram).

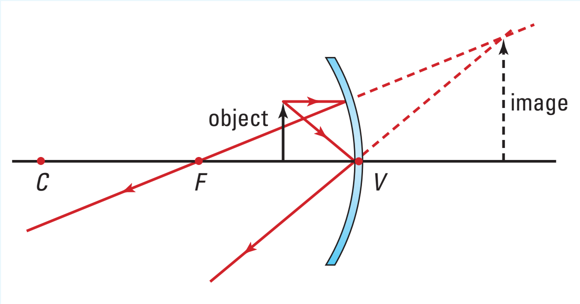
The height of the image is less than the height of the object.

The magnitudes of both the image position and the height seem reasonable when compared to the ray diagram.

2. An object 3.5 cm tall is located 7.0 cm in front of a concave make-up mirror that has a focal length of 10.0 cm. Where is the image located, and what is the height of the image?

Frame the problem

- Visualize the problem by sketching the light ray diagram.
- Because the object distance is less than the focal length, you would expect the image to be upright, larger than the object, and virtual.



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Identify the Goal

The location, d_i, and height, h_i, of the image in a concave mirror

Variables and Constants

Involved in the problem		Known	Unknown
d_o	h_i	$d_o = 7.0$ cm	d_i
d_i	f	$h_o = 3.5$ cm	h_i
h_o		$f = 10.0$ cm	

Strategy	Calculations
Use the mirror equation to find the image location, d_i.	$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$
Substitute in the known values.	$\frac{1}{10.0\text{ cm}} = \frac{1}{7.0\text{ cm}} + \frac{1}{d_i}$
Rearrange.	$\frac{1}{10.0\text{ cm}} - \frac{1}{7.0\text{ cm}} = \frac{1}{d_i}$
Find a common denominator.	$\frac{7}{70\text{ cm}} - \frac{10}{70\text{ cm}} = \frac{1}{d_i}$
	$\frac{-3}{70\text{ cm}} = \frac{1}{d_i}$
Invert.	$\frac{-70\text{ cm}}{3} = d_i$
	$d_i = -23.3$ cm
Use the magnification equation to find h_i.	$\frac{h_i}{h_o} = \frac{-d_i}{d_o}$
Substitute in known values.	$\frac{h_i}{3.5\text{ cm}} = \frac{-(-23.3)\cancel{\text{cm}}}{7.0\cancel{\text{cm}}}$
	$\frac{h_i}{\cancel{3.5\text{ cm}}}\cancel{(3.5\text{ cm})} = \frac{23.3}{7.0}(3.5\text{ cm})$
Simplify.	$h_i = +11.7$ cm

The image is formed 23 cm behind the mirror.

The height of the image is 12 cm.

Validate

The units are in centimetres.

The height of the image is a positive value because the image is upright (see the previous ray diagram).

The image location is a negative value because the image is virtual (behind the concave mirror).

The image height is larger than the object height.

The magnitudes of both the image position and the height seem reasonable when compared to the ray diagram.

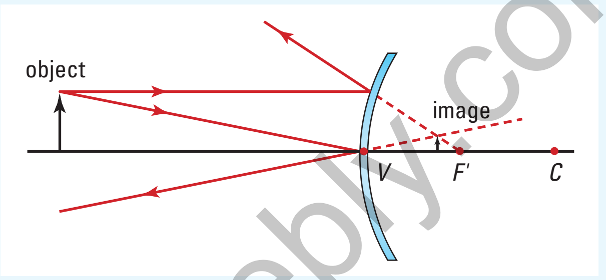
3. **A convex supermarket surveillance mirror has a radius of curvature of 80.0 cm. A 1.7 m tall customer is standing 4.5 m in front of the mirror.**

(a) **What is the location of the customer's image in the mirror?**

(b) **What is the height of the customer's image?**

Frame the problem

- Visualize the problem by sketching the light ray diagram.
- From your knowledge of convex mirrors, you expect the image to be behind the mirror and smaller than the object.
- The focal length is negative for a convex mirror.



Identify the Goal

The location, d_i, and height, h_i, of an image in a convex mirror

Variables and Constants

Involved in the problem		Known	Unknown
d_o	h_i	$d_o = 4.5$ m	d_i
d_i	R	$h_o = 1.7$ m	h_i
h_o	f	$R = 80.0$ cm $= 0.8$ m	f

Strategy

Calculations

Calculate the focal length, f, from the radius of curvature, R.

$$f = \frac{R}{2}$$

$$f = \frac{0.8\text{ m}}{2}$$

$$f = 0.4\text{ m}$$

Use the convention that the focal length is negative for convex mirrors.

$$f = -0.4\text{ m}$$

Use the mirror equation to find the image distance, d_i.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Substitute in known values.

$$\frac{1}{-0.4\text{ m}} = \frac{1}{4.5\text{ m}} + \frac{1}{d_i}$$

Rearrange.

$$\frac{1}{-0.4\text{ m}} - \frac{1}{4.5\text{ m}} = \frac{1}{d_i}$$

Find a common denominator.

$$\frac{-4.5}{1.8\text{ m}} - \frac{0.4}{1.8\text{ m}} = \frac{1}{d_i}$$

$$\frac{-4.9}{1.8\text{ m}} = \frac{1}{d_i}$$

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Invert.

$$d_i = \frac{-1.8 \text{ m}}{4.9}$$

$$d_i = -0.367 \text{ m}$$

(a) The image is formed 0.37 m behind the mirror.

Use the magnification equation to find h_i.

$$\frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

Substitute in known values.

$$\frac{h_i}{1.7 \text{ m}} = \frac{-(-0.367) \cancel{\text{m}}}{4.5 \cancel{\text{m}}}$$

$$\frac{h_i}{\cancel{1.7 \text{ m}}}\cancel{(1.7 \text{ m})} = \frac{0.367}{4.5}(1.7 \text{ m})$$

Simplify.

$$h_i = 0.1386 \text{ m}$$

(b) The height of the image is 0.14 m.

Validate

The units are in metres.

The height of the image is a positive value because the image is upright (see the ray diagram above).

The height of the image is smaller than the height of the object.

The image location is a negative value because the image is virtual (behind the convex mirror).

The magnitude of both the image position and the height seem reasonable when compared to the ray diagram.

PRACTICE PROBLEMS

1. An object 6.0 mm tall is 10.0 cm in front of a concave mirror that has a 6.0 cm focal length. Find the image distance and its height by means of
   (a) a ray diagram drawn to scale.
   (b) the mirror and magnification equations.
2. What is the radius of curvature of a concave mirror that magnifies an object placed 30.0 cm from the mirror by a factor of +3.0?
3. A convex security mirror in a warehouse has a radius of curvature of 1.0 m. A 2.2 m tall forklift is 6.0 m from the mirror. What is the location and size of the image?
4. A dancer is applying make-up in a concave mirror. Her face is 35 cm in front of the mirror. The image is 72 cm behind the mirror. Using the mirror equation, find the focal length of the mirror. What is the magnification of the image?

INVESTIGATION 10-B

Image Formation in Concave Mirrors

TARGET SKILLS
- Predicting
- Hypothesizing
- Performing and recording
- Analyzing and interpreting

One of the fundamental purposes of the scientific theories, models, and equations related to the study of light is that they can be used to make predictions, as well as to provide explanations.

Problem

Can the light ray model and the mirror and magnification equations predict the properties of the images formed by concave mirrors?

Equipment

- concave mirror
- optical bench (with attachments)
- metre stick
- light source (candle)
- screen

Procedure

1. Read the procedure and prepare a suitable table to record the predictions and observations in the investigation.
2. In a darkened room, point the mirror at a distant light source or at an object that can be seen through the window. Move the screen back and forth until the object is clearly focussed on the screen. Measure the distance from the mirror to the screen. Determine the focal length, f, of the mirror.
3. Using the focal length, determine the object distance from the mirror for object positions of $2.5f$, $2.0f$, $1.5f$, and $0.5f$.
4. Draw a scale light ray diagram for each of the four object positions.
5. Interpret and record the four properties of the image in the table.
6. Using the mirror and magnification equations, predict the four characteristics of the image for each object position and record them in the table.
7. Place the mirror at the zero end of the optical bench.
8. Place the object (light source) at the $2.5f$ position from the mirror. Move the screen back and forth until you can see a clearly focussed image on the screen. It might be necessary to tilt the mirror slightly upward to prevent the source light from being blocked by the screen. Record the image properties in the table.
9. Repeat step 8 for the other object positions in step 5. It will not be possible to record the image distance for virtual images.

Analyze and Conclude

1. Compare your predictions with your experimental results for each object position, and account for any discrepancies.
2. Which method for predicting the image properties is most useful? Explain why.
3. What image properties cannot be obtained easily by experiment? Why?
4. In terms of the focal length, f, for what part of the total range of object positions is it most difficult to use each of the three methods for obtaining the image properties?

10.3 Section Review

1. K/U State what happens when a set of parallel light rays is reflected from (a) a plane mirror, (b) a concave mirror, and (c) a convex mirror. Draw a light ray diagram for each of the three mirror shapes to show how the light is reflected.

2. K/U In what position must an object be placed in front of a concave mirror to form (a) enlarged, virtual images, and (b) enlarged, real images? Draw light ray diagrams to show how each of these images is formed.

3. K/U Why are convex mirrors often used instead of plane mirrors in stores?

4. K/U What shape of curved mirror should be used to avoid problems caused by spherical aberration? Draw a light ray diagram to show why spherical aberration occurs.

5. C Explain the basic principle that determines where the image will be formed by light rays when they are reflected from any shape of mirror.

6. C Carry out research to find out how "see-through" mirrors work. Explain their operation, with the aid of light ray diagrams, and list applications for plane and curved mirrors of this type.

7. K/U
   (a) A concave mirror has a focal length of 24 cm. What is the radius of curvature of the mirror?
   (b) A convex mirror has a radius of curvature of 2.4 m. What is the focal length of the mirror?

8. C Draw light ray diagrams to show the characteristics of the image formed by a concave mirror for an object placed at (a) 0.5 f, (b) 1.75 f, and (c) 3.0 f. State the characteristics of the image in each case.

9. I Draw a light ray diagram to show that an object placed at the focal point of a concave mirror will not form an image.

10. I Draw light ray diagrams to determine the image characteristics for an object 1.5 cm high, when it is placed in front of a concave mirror with a focal length of 8.0 cm at a distance of (a) 4.0 cm, (b) 12.0 cm, and (c) 20.0 cm.

11. I Draw scaled light ray diagrams to determine the image characteristics for an object 2.0 cm high, when it is placed in front of a concave mirror with a focal length of 24.0 cm at a distance of (a) 10.0 cm, (b) 40.0 cm, and (c) 60.0 cm.

12. I Determine the image characteristics for the object in question 11 using the mirror equations.

13. I Design and carry out an investigation to measure the focal length of a convex mirror mounted on the door of a car. Check to see if the radius of curvature of the mirror is within the legal requirements for such mirrors. (Refer to the Canadian Motor Vehicle Safety Standards).

14. MC Research how parabolic reflectors are used to focus sunlight and create solar furnaces in solar-thermal electrical energy facilities. How much energy is produced compared to other energy generation methods? Describe some advantages and disadvantages of using solar furnaces to generate energy.

UNIT PROJECT PREP

The mirror and magnification equations can be used to determine the properties of curved mirror images.

- Construct a schematic diagram for your optical device based on the equations.

CHAPTER 10 Review

REFLECTING ON CHAPTER 10

- The wave theory of light is based on the concept that light is a series of electromagnetic waves. Two scientific models, the wavefront model and the light ray model, help explain light phenomena in terms of the wave theory.
- Regular reflection occurs when light reflects from a smooth surface, such as a mirror. Diffuse reflection occurs when light reflects from a rough surface.
- Each point on an object is a source of light rays that diverge in all directions. The rays coming into your eye from any single point on the object form a cone. When all these cones of divergent rays, from every point on the object, come into your eye, you see the object.
- A likeness of an object, such as that seen in a plane mirror, is called an image. A real image is an image that can be projected onto a screen. Actual light rays that converge after reflection from a mirror surface form it.
- The image formed in a plane mirror is the same size as the object, and is virtual and upright. The image is located as far behind the mirror as the object is in front of the mirror.
- The light ray model can be used to determine the properties of the image formed by a plane mirror. The use of the model is based on the geometry related to straight lines.
- A spherical mirror has the shape of a section from the surface of a sphere, and can be used to form concave and convex mirror surfaces. The principal axis of a spherical mirror is the straight line drawn through the centre of curvature and the vertex of the mirror.
- When parallel light rays reflect from a concave mirror, they pass through the focal point. Parallel light rays reflecting from a convex mirror appear to diverge from the virtual focal point behind the mirror surface.
- Concave mirrors form real, inverted images if the object is farther from the mirror than the focal point, and virtual, upright images if the object is between the focal point and the mirror.
- The mirror and magnification equations can be used to determine the location, size, and orientation of an image, and whether the image is real or virtual.

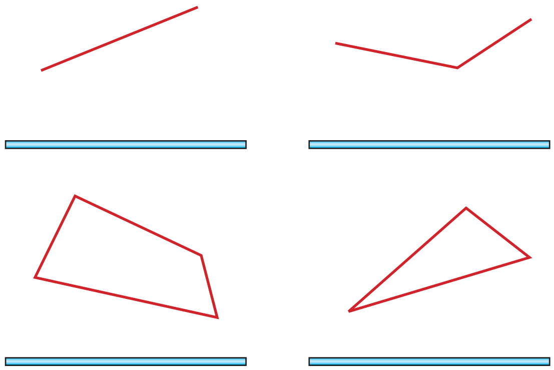
Knowledge/Understanding

1. Why are parallel light rays used to show the difference between regular and diffuse reflection?
2. When driving a car on a rainy night, does a wet part of the road appear lighter or darker than a dry portion of the road in the car's headlights? Explain your answer in terms of the reflection of the light from the surface of the road.
3. List some situations where the letters on signs are reversed. Explain why this is done.
4. How could you set up mirrors to be able to read the image of printing seen in a plane mirror? Explain your answer.
5. As the object is moved along the principal axis toward the vertex of a concave mirror, the type of image changes. Where does the change in type of image occur? Explain why, in terms of the path of the light rays.
6. Explain the difference between a real and a virtual image.
7. Why can you always tell by looking at a light ray diagram whether an image is real or virtual? Why are the lines drawn in this way?
8. Explain why concave mirrors can produce real or virtual images, but convex mirrors can produce only virtual images.

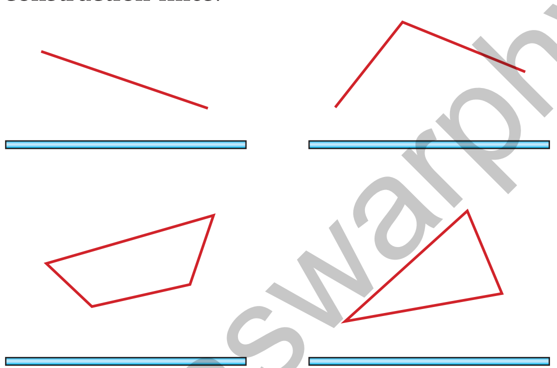
Communication

9. Describe how you can determine the type of image formed in a plane mirror (a) by experiment, and (b) by using the light ray diagram.

10. Copy the diagrams in the following illustration into your notebook, and draw a light ray diagram for each example to show how the image is formed in the plane mirror. Show all construction lines.



11. Copy the diagrams in the following illustration into your notebook and draw a light ray diagram for each example to show how the image is formed in the plane mirror. Show all construction lines.



12. Prepare a series of overhead projector slides to use in a presentation to the class to explain changes in the characteristics of the images formed by (a) a concave mirror, and (b) a convex mirror as the object is moved from the surface of the mirror to infinity.
13. Concave mirrors are used in searchlights and solar ovens. Draw two diagrams to show the path of light in each case, and explain why these two applications demonstrate the principle of the reversibility of light.
14. Using a diagram, explain why a diverging mirror provides a wide field of view.

Inquiry

15. Sometimes campers use a small mirror to light paper to start a campfire. What shape of mirror would they use? What is happening to the sunlight when they do this? Draw a light ray diagram to illustrate your answer.
16. Draw a fully labelled light ray diagram to show the position of the image formed by a concave mirror for each example listed in the table below. The height of the object is 1.5 cm. State the four properties for each image below the ray diagram.

Focal length of mirror (cm)	Distance of object from mirror (cm)
5.0	15.0
12.0	3.0
9.0	16.0

17. Draw a fully labelled light ray diagram to show the position of the image formed by a convex mirror for each example listed in the table below. The height of the object is 1.5 cm. State the image characteristics for each image below the appropriate ray diagram.

Radius of curvature of mirror (cm)	Distance of object from mirror (cm)
16.0	5.0
10.0	15.0

Making Connections

18. Describe the similarities and differences of the properties of the images formed in a plane mirror and a convex mirror. Why are convex mirrors used for security purposes in stores, rather than plane mirrors?
19. Satellite dishes are now commonly used for receiving television and radio signals. What shape are they? What method is used to align them? What is the position of the satellite receiver (the device which detects the signal)?

Problems for Understanding

20. A student wishes to take a photograph of his own image in a plane mirror. At what distance should the camera lens be focussed if it is positioned 2.3 m in front of the mirror?
21. A ray of light strikes a mirror at an angle of 55° to the normal.
    - (a) What is the angle of reflection?
    - (b) What is the angle between the incident ray and the reflected ray?
22. A ray of light strikes a plane mirror at an angle of 57° to the mirror surface. What is the angle between the incident ray and the reflected ray?
23. What is the angle of incidence if the angle between the reflected ray and the mirror surface is 34°?
24. Light is shining on to a plane mirror at an angle of incidence of 27°. If the plane mirror is tilted such that the angle of incidence is reduced by 8°, what will be the total change in the angle of reflection from the original reflected light?
25. A patient is sitting in an optometrist's chair, facing a mirror that is 2.25 m from her eyes. If the eye chart she is looking at is hanging on a wall behind her head, 1.75 m behind her eyes, how far from her eyes does the chart appear to be? Why would charts used for this purpose have to be specially made?
26. If you move directly along a normal line toward a plane mirror at a speed of 3.5 m/s, what is the speed of the image relative to you?
27. What is the speed of the image, relative to you, if you walk away from the mirror surface at 3.5 m/s at an angle of 30° to the mirror surface?
28. Look at a kaleidoscope and determine how many images of the group of brightly coloured objects are formed by the two mirrors inside. Using the equation $N = \frac{360°}{\theta} - 1$ (where N is the number of images and θ is the angle between the mirrors), calculate the angle between the mirrors.
29. A concave mirror has a focal length of 26 cm. What is the radius of curvature of the mirror?
30. The radius of curvature of a convex mirror is 60 cm. What is the focal length of the mirror?
31. The light from a star reflects from a concave mirror with a radius of curvature of 1.70 m. Determine how far the image of the star is from the surface of the mirror.
32. Use the mirror equation to find the image location and the height of an object placed at the centre of curvature of a concave mirror. Also find the magnification. Hint: What is the relation between the focal length and the object distance, d_o, for this situation?
33. Using the mirror equation and the magnification equation, find the four properties of the image formed in a concave mirror with a focal length of 50.0 cm, if the object is 1.5 m from the mirror and is 2.5 cm high.
34. Using the mirror equation and the magnification equation, find the four properties of the image formed in a concave mirror with a focal length of 35 cm, if the object is 50.0 cm from the mirror and is 3.0 cm high.
35. Using the mirror equation and the magnification equation, find the four properties of the image formed in a concave mirror with a focal length of 1.2 m, if the object is 0.80 m from the mirror and is 2.0 cm high.
36. Using the mirror equation and the magnification equation, find the four properties of the image formed in a convex mirror with a focal length of 90 cm, if the object is 2.5 m from the mirror and is 0.4 m high.
37. Using the mirror equation and the magnification equation, find the four properties of the image formed in a convex mirror with a radius of curvature of 1.5 m, if the object is 80.0 cm from the mirror and is 25 cm high.
38. A concave mirror has a radius of curvature of 50.0 cm. At what position should an object be placed to produce an upright virtual image that is 3.0 times as large as the object?

Numerical Answers to Practice Problems

1. (b) $d_i = 15$ cm; $h_i = -0.90$ cm **2.** 9.0×10^1 cm
3. $d_i = -0.46$ m; $h_i = 0.17$ m **4.** $f = 66$ cm

CHAPTER 11

Refraction of Light

CHAPTER CONTENTS

When you pass a jewellery store, look at the displays of diamond necklaces, earrings, and engagement rings in the window. Notice the light sparkling from the jewels, often glistening with different colours. Sometimes, jewellers mount their displays on rotating platforms to enhance the glittering of the colours. How does light interact with diamonds to create colour from an apparently colourless material? What other materials produce a similar effect? Could you get this effect by using glass instead of diamonds?

How could the glittering of diamonds be related to the surgical procedure shown in the small photograph? Some of the same properties of light that make diamonds glitter help the surgeon to see inside a patient's stomach. The device, called an "endoscope," contains optical fibres that carry light into the patient's stomach, allowing the doctor to observe the tissues. If light travels in straight lines, as you learned in Chapter 10, something unusual must be happening to the light inside the endoscope that causes the light to follow the curved path of the tube. In this chapter, you will learn about refraction of light, which is responsible for this and for many other interesting phenomena.

MULTI LAB

Light Travelling Through Matter

TARGET SKILLS
- Hypothesizing
- Analyzing and interpreting
- Communicating results

Beaker Optics

Pour about 300 mL of water into a 500 mL beaker. Hold the beaker with the markings facing away from you. Observe the markings from above the water, then through the water. Finally, look up through the bottom of the beaker.

Analyze and Conclude

Describe the changes in the appearance of the markings as you looked down from the top of the beaker, then directly through the beaker, and, finally, up through the bottom of the beaker.

Apparent Depth

Fill a sink or a plastic bowl or bucket approximately three quarters full of water. Hold a ruler about half-submerged in the water, and slowly change the angle of the ruler relative to the water. Tilt the ruler to the side, and then tilt it away from you. Move your head up and down so that the level of your eyes is at first well above the water and then very close to the water. In each case, note any changes in the appearance of the ruler. Move your head and the ruler until the submerged end of the ruler disappears.

Analyze and Conclude

1. Describe any patterns in the changes of the appearance of the ruler.
2. Formulate a hypothesis that accounts for the fact that the ruler seems to disappear at certain angles. As you study this chapter, look for explanations of the disappearance of the ruler and compare those with your hypothesis.

Shimmering

Adjust a Bunsen or alcohol burner to burn with the hottest flame. From a safe distance, position your eyes level with the flame. Look past the flame as close to its edges as possible.

Analyze and Conclude

Describe how the flame affected the appearance of objects that were beyond and just to the side of the flame. What phenomenon do you think is responsible for these effects?

11.1 Refraction

SECTION EXPECTATIONS

- Define and describe refraction and index of refraction.
- Predict, in quantitative terms, the effect of a medium, on the velocity of light.

KEY TERMS

- index of refraction (refractive index)
- optically dense

When you look through a window, objects usually appear just the same as they do when looking only through air. Why do the windowpanes in Figure 11.1 cause so much distortion? Why does the curved side of a beaker cause distortion? Windowpanes and beakers are transparent, so if light penetrates these materials, why do some of them cause distortions, while others do not? Can you think of any situations in which air alone can cause distortion?

Figure 11.1 Imagine looking at the world through these windowpanes. Why was this kind of glass once used in windows?

Refraction and Light Waves

When you were looking at the ruler that was half-submerged in water, was the ruler really bent? You are probably thinking, "Of course not!" It was actually the light rays that were bent, not the ruler. The change in direction of the light as it passes at an angle from one material into another is a result of refraction.

Refraction is the changing of the speed of a wave when it travels from one medium into another. Light travels as a wave, and therefore experiences refraction. When light travels from one medium into another at an angle, the difference in the speed causes a change in the direction of the light (Figure 11.2). The shimmering effect you see above a barbecue grill, the distorted view of objects viewed through a glass bottle, and even rainbows are all results of the refraction of light.

Light travels through a vacuum at the extremely high speed of 3.00×10^8 m/s (300 000 km/s). The speed of light in a vacuum is such an important fundamental physical constant in the study of physics that it has been assigned its own symbol, *c*.

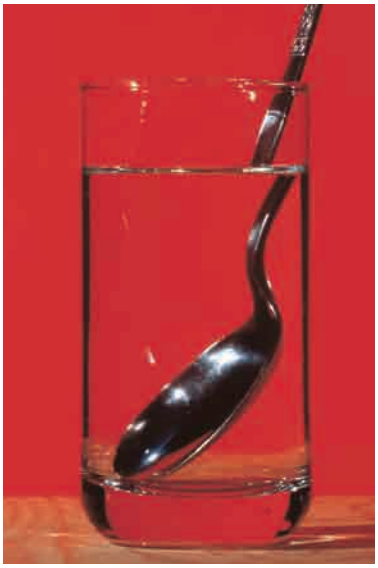


Figure 11.2 Light refracts, or bends, toward the normal when it passes from air into water or glass.

History Link

Until the seventeenth century, people generally accepted that light appeared instantaneously. In fact, the first few scientists who attempted to measure the speed of light were ridiculed by the entire scientific community for the values that they suggested. Finally, in the late 1800s, U.S. physicists A.A. Michelson (1852–1931) and E.W. Morley (1838–1923) performed the critical experiments that established the speed of light. This determination played a major role in the development of both classical and modern physics. What is the connection between the Michelson-Morley experiments and Einstein's formulation of the theory of relativity?

Index of Refraction

The values for the speed of light in different media are extremely large, unwieldy numbers. Physicists developed a more useful constant called the **index of refraction** (or **refractive index**). The index of refraction is a ratio of the speed of light in a vacuum to the speed of light in a specific medium. Since the term represents a ratio of two values having the same units, the units cancel, leaving the constant unitless.

INDEX OF REFRACTION

The index of refraction of a material is the ratio of the speed of light in a vacuum to the speed of light in that material.

$$n = \frac{c}{v}$$

Quantity	**Symbol**	**SI unit**
index of refraction	n	none
speed of light in a vacuum	c	$\frac{\text{m}}{\text{s}}$ (metres per second)
speed of light in a specific medium	v	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$\frac{\text{metres per second}}{\text{metres per second}} = \frac{\cancel{\frac{\text{m}}{\text{s}}}}{\cancel{\frac{\text{m}}{\text{s}}}} = \text{no unit}$$

Language Link

The symbol for the speed of light, *c*, represents the Latin word *celeritas*, which means "speed." What term that you used frequently in your study of forces and motion is also derived from this Latin word for speed?

PHYSICS IN THE NEWS

Light Pulse Breaks Ultimate Speed Limit

Red alert! The nebula your starship is headed toward contains deadly radiation. A faster-than-light signal has arrived from the future, warning you not to enter. You change course, narrowly escaping danger.

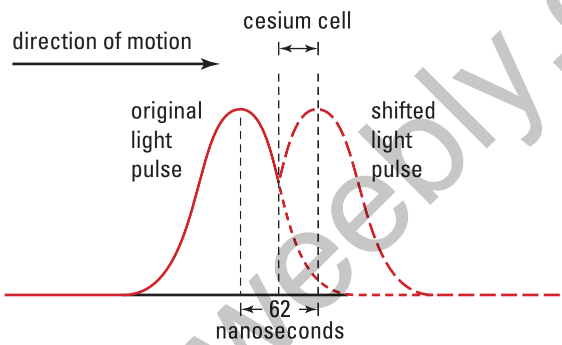
Scenarios like this are common in science fiction. They are based on speculation that if faster-than-light travel were possible, time travel into the past would be possible. Among other things, you could send information back in time and change the course of your life. There's just one problem — if you did this, the version of the future from which you sent the information might no longer exist!

This paradox and Einstein's famous theory of relativity both imply that superluminal (faster-than-light) speeds are impossible. Neither mass nor information should be able to travel faster than the speed of light in a vacuum (3×10^8 m/s). Yet, in laser experiments conducted by Dr. Lijun Wang and his colleagues at the NEC Research Institute in New Jersey, the crest of a light pulse did exactly that. The crest actually exited from the back end of a cesium gas cell before it entered the front end!

Light pulses, which have no mass, can be coaxed into travelling at superluminal speeds. Physicists first predicted that this was possible in 1970, and the work of Wang's team provides the first unambiguous evidence of this fact.

A light pulse is a packet of light waves of various frequencies. Where the crests of these light waves align, their amplitudes add together to produce the pulse.

In Wang's experiment, a light pulse travels through a cell of specially prepared cesium gas that accelerates higher frequency light waves and slows lower frequency light waves. This is the opposite of what a glass prism does to light. The altered light waves within the pulse interfere with each another and rephase, shifting the pulse ahead in time by 62 ns, as shown in the diagram. The shifted crest of the pulse exits from the cesium gas cell well before the original crest enters the cell.



This light pulse leaves the cesium gas cell *before* it enters.

Wang insists that information cannot be transmitted superluminally by this technique. However, not all physicists agree. There is now considerable debate over what counts as "information."

In the meantime, the technique might have practical applications. Today's technologies, such as computers and the Internet, transfer information at velocities well below the speed of light. Wang feels his research might boost transfer velocities to this ultimate speed limit.

Web Link

www.school.mcgrawhill.ca/resources/

For more historical detail on the various methods used to measure the speed of light, go to the above Internet site. Follow the links for **Science Resources** and **Physics 11** to find out where to go next. You will also find links to faster-than-light physics, including the theoretical basis for a "space warp."

Table 11.1 Index of Refraction of Various Substances*

Substance	Index of Refraction (n)
vacuum	1.00000
gases at 0°C, 1.013×10^5 Pa	
hydrogen	1.00014
oxygen	1.00027
air	1.00029
carbon dioxide	1.00045
liquids at 20°C	
water	1.333
ethyl alcohol	1.362
glycerin	1.470
carbon disulfide	1.632

Substance	Index of Refraction (n)
solids at 20°C	
ice (at 0°C)	1.31
quartz (fused)	1.46
optical fibre (cladding)	1.47
optical fibre (core)	1.50
Plexiglas™ or Lucite™	1.51
glass (crown)	1.52
sodium chloride	1.54
glass (crystal)	1.54
ruby	1.54
glass (flint)	1.65
zircon	1.92
diamond	2.42

* Measured using yellow light, with a wavelength of 589 nm in a vacuum.

The index of refraction of a material can be determined by methods that do not involve measuring the speed of light in that material. Once the value is measured, however, you can use it to calculate the speed of light in that material. The index of refraction has been determined for a large number of materials, some of which are listed in Table 11.1.

Notice that the values for the index of refraction are always greater than one, because the speed of light in a vacuum is always greater than the speed of light in any material. Notice, also, that media in which the speed of light is low have large indices of refraction: When you divide a constant by a small number, the ratio is large. For example, the speed of light in water ($n = 1.33$) is 2.26×10^8 m/s, whereas the speed of light in zircon ($n = 1.92$) is 1.56×10^8 m/s.

The term **optically dense** refers to a refractive medium in which the speed of light is low in comparison to its speed in another medium. For example, based on the previous values given for zircon and water, you would say that zircon is more optically dense than water. The speed of light in air and other gases is so close to that of the speed of light in a vacuum that the index of refraction of these materials is considered to have a value of one, or unity, for most practical purposes. You will notice in Table 11.1 that the index of refraction in gases differs from that in a vacuum only in the fifth significant digit.

MODEL PROBLEM

Index of Refraction and Speed of Light

The speed of light in a solid is 1.969×10^8 m/s.

(a) What is the index of refraction of the solid?

(b) Identify the material, using Table 11.1.

Frame the Problem

- The *speed of light in the medium* is known.
- The *index of refraction* is unknown.
- The speed of light in a medium is related to the index of refraction of that medium.

Identify the Goal

(a) The index of refraction, n, of the medium

(b) The identity of the medium

Variables and Constants

Involved in the problem	Known	Implied	Unknown
v	$v = 1.969 \times 10^8 \frac{\text{m}}{\text{s}}$	$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$	n
n			medium
c			
medium			

Strategy

Use the equation that defines the index of refraction.

$$n = \frac{c}{v}$$

All of the values are known, so substitute them into the equation.

$$n = \frac{3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{1.969 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}$$

$$n = 1.5236$$

Calculations

(a) The index of refraction of the material is 1.52.

(b) Based on the information in Table 11.1, the material is probably crown glass. However, more information would be needed to confirm its identity, since many materials do not appear in the table.

Validate

All of the units cancelled, leaving the answer unitless.
This is in agreement with the nature of the index of refraction.

The magnitude is between 1.00 and 2.00, which is quite realistic.

PRACTICE PROBLEMS

1. What is the index of refraction of a solid in which the speed of light is 1.943×10^8 m/s?
2. Determine the speed of light in diamond.
3. What is the speed of light in glycerin?
4. Determine the time taken for light to travel a distance of 3500 km along the core of an optical fibre.
5. Determine the change in the speed of light as it passes from ice into water.

The index of refraction, and therefore the speed of light, for all wavelengths (colours) of light is the same when light is travelling in a vacuum. For any other given material, the index of refraction varies slightly, however, depending on the wavelength of the light passing through it. Therefore, different wavelengths of light travel at different speeds in the same medium. To avoid confusion, physicists have chosen to report the values for the speed of light of a specific colour. They have chosen the yellow light emitted by the sodium atom, which has a wavelength of 589 nm in a vacuum.

Figure 11.3 Streetlights are usually filled with sodium vapour. The yellow light it produces is preferred, because it penetrates fog and mist better than the blue-green light produced by mercury vapour.

PHYSICS FILE

Objects such as a stove burner emit visible light when the temperature is high enough to excite the electrons within an atom. Many substances emit many different wavelengths (colours) of light, which are distributed throughout the visible part of the electromagnetic spectrum. Some elements, such as sodium and hydrogen, release only a few wavelengths of visible light. In fact, sodium releases only two wavelengths of light and they are both in the yellow part of the visible spectrum. The wavelength for yellow light, 589 nm, is the standard wavelength used to determine the index of refraction for all materials.

11.1 Section Review

1. K/U What are the units for the index of refraction?
2. K/U As the speed of light in a medium increases, what happens to the index of refraction of the medium?
3. K/U Explain the term "optically dense."
4. K/U What is the benefit of choosing light of a specific colour to report values for the speed of light?
5. C Describe, in your own words, how the index of refraction is related to the speed of light, and suggest a reason why gases have refractive indices close to 1.

11.2 Snell's Law of Refraction

SECTION EXPECTATIONS

- Apply the wave model of light to explain refraction.
- Predict, in quantitative and qualitative terms, the refraction of light as it passes from one medium to another, using Snell's law.
- Conduct an experiment to verify Snell's law.
- Describe and predict the refraction of light using the ray model for light.

KEY TERMS

- angle of refraction
- Snell's law
- principle of reversibility of light

For thousands of years, people have known that various types of crystals can change the direction of light when it passes from air into the crystals. Scientists also knew long ago that the amount of refraction depended on the both the material that made up crystal and the angle at which the incident light crossed the boundary between the air and the crystal. Numerous scientists tried to identify a mathematical relationship that could describe refraction, but it was not until 1621 that Willebrord Snell (1580–1626), a Dutch mathematician, discovered the relationship experimentally.

Snell's Observations

In Chapter 10, you learned the meaning of the terms "angle of incidence," "angle of reflection," and "normal line." In Figure 11.4, in which a light ray is travelling from air into water, you see a new, but related, angle — the **angle of refraction**, θ_R. The angle of refraction is defined as the angle that the refracted ray makes with the normal line.

Snell discovered that when light travels across a boundary from one medium into another, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant value for all angles of incidence greater than zero. When the angle of incidence is zero, the angle of refraction is also zero and the direction of the light is unchanged. Snell's observations can be described mathematically by the following expression.

$$\frac{\sin \theta_i}{\sin \theta_R} = \text{a constant}$$

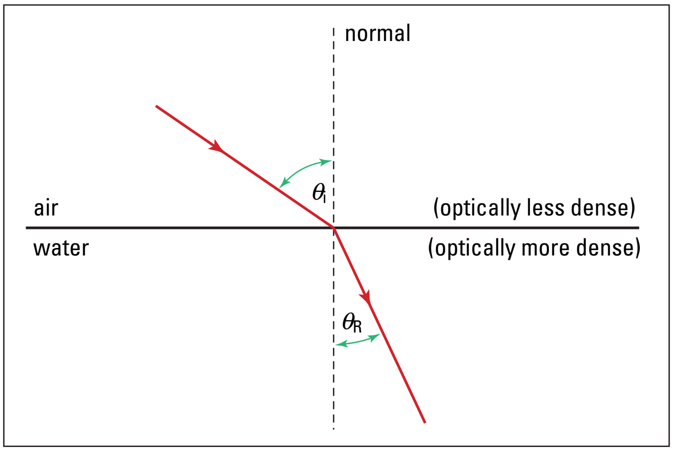


Figure 11.4 Light refracts (bends) toward the normal when passing from air into water.

When the incident medium is a vacuum (or air), the constant in Snell's relationship is actually the same value as the index of refraction, n, of the refracting medium. For example, the constant for light travelling from air into water is 1.33. This relationship provides a means of determining the speed of light in any specific medium. For a given angle of incidence, simply measure the angle of refraction when light travels from air into the medium, and determine n. Use the calculated value of n to determine the velocity, v, for that medium. In addition, since many indices of refraction are known, you can identify an unknown material by measuring the angle of refraction in that medium, calculating n, and then comparing it to values of n of known materials.

History Link

Astronomer and mathematician Ptolemy of Alexandria (Claudius Ptolemaeus, approximately 85–168 B.C.E.) made accurate measurements and kept data tables of angles of incidence and refraction of several materials. At a different time in his career, he also developed a set of trigonometric ratios, including sine ratios. He never identified the link between these ratios and the angles of incidence and refraction, however. Which scientific theory is Ptolemy most famous for?

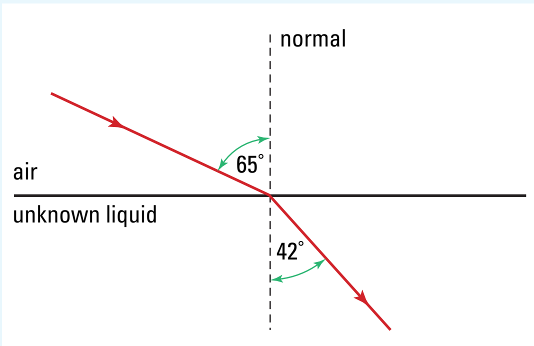
MODEL PROBLEM

Index of Refraction

Light travels from air into an unknown liquid at an angle of incidence of 65.0˚. The angle of refraction is 42.0˚. Determine the index of refraction of the unknown liquid.

Frame the Problem

- Make a sketch of the problem.
- The *angles of incidence and refraction* are known.
- The index of refraction relates the angles of incidence and refraction when the incident medium is *air*.
- *Air* is the incident medium.



Identify the Goal

The index of refraction, n, of the unknown liquid

Variables and Constants

Involved in the problem	Known	Unknown
θ_i	air	n
θ_R	θ_i	
n	θ_R	
incident medium: air		

continued ►

continued from previous page

Strategy

Use Snell's constant

Since the incident medium is air, the constant is the index of refraction, n, of the liquid.

Substitute in the known values.

Calculations

$$\frac{\sin\theta_i}{\sin\theta_R} = \text{a constant}$$

$$n = \frac{\sin\theta_i}{\sin\theta_R}$$

$$n = \frac{\sin 65.0^\circ}{\sin 42.0^\circ}$$

$$n = \frac{0.9063}{0.6691}$$

$$n = 1.354$$

The index of refraction of the unknown liquid is 1.35.

Validate

The absence of units is in agreement with the unitless nature of the index of refraction.

The value is between one and two, which is very reasonable for an index of refraction.

PRACTICE PROBLEMS

6. Light travels from air into a material at an angle of incidence of 59°. If the angle of refraction is 41°, what is the index of refraction of the material? Identify the material by referring to Table 11.1, Index of Refraction of Various Substances.

7. A beam of light travels from air into a zircon crystal at an angle of 72.0°. What is the angle of refraction in the zircon?

8. What is the angle of incidence of light travelling from air into ethyl alcohol when the angle of refraction is 35°?

Snell's constant ratio is observed for any two media, even when neither medium is air. However, the value of the constant is not the same as the index of refraction of either medium. For example, when light is travelling from water into crown glass, the constant is found to be 1.143. This is a different value than that shown in Table 11.1 for either material. An additional observation shows that the constant might be less than one. In the case of light travelling from quartz into water, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is 0.914. Nevertheless, there is still a relationship between Snell's constants and the indices of refraction of any two media. Many observations and comparisons revealed that Snell's constant is, in fact, the ratio of the indices of refraction of the two media:

$$\frac{\sin\theta_i}{\sin\theta_R} = \frac{n_R}{n_i}$$

For example, Snell's constant for light travelling from water into crown glass (1.143) is the ratio of the index of refraction of crown glass (1.523) to that of water (1.333). This relationship, known as **Snell's law**, is usually rearranged and expressed mathematically as shown in the box below.

SNELL'S LAW

The product of the index of refraction of the incident medium and the sine of the angle of incidence is the same as the product of the index of refraction of the refracting medium and the sine of the angle of refraction.

$$n_{\mathrm{i}} \sin \theta_{\mathrm{i}} = n_{\mathrm{R}} \sin \theta_{\mathrm{R}}$$

Quantity	**Symbol**	**SI unit**
index of refraction of the incident medium	n_{i}	unitless
angle of incidence	θ_{i}	none (degree is not a unit)
index of refraction of the refracting medium	n_{R}	unitless
angle of refraction	θ_{R}	none (degree is not a unit)

In addition to the mathematical relationship, a geometric relationship exists between the incident ray and refracted ray. *When light travels across a boundary between two different materials, the refracted ray, the incident ray, and the normal line at the point of incidence all lie in the same plane.*

Inspection of Snell's law reveals an important general result of the relationship. When light travels from an optically less dense medium into an optically more dense medium, the refracted ray bends toward the normal. Conversely, when light travels from an optically more dense medium into an optically less dense medium, the refracted ray bends away from the normal.

Think It Through

- Use Snell's law to verify the statements in the last paragraph about the directions of bending of the refracted ray and the optical density of the incident and refracting media.

INVESTIGATION 11-A

Verifying Snell's Law

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Predicting

When a scientist collects data and finds a mathematical relationship from the data alone, the relationship is said to be based on empirical evidence. The mathematical relationship is not derived from more fundamental principles. Often, the empirical relationship leads a scientist to more experiments that reveal more fundamental scientific principles.

Snell's law was originally based on empirical evidence. It was more than 300 years before scientists determined the speed of light and were then able to explain refraction on the basis of the ratio of the speed of light in a vacuum and in a medium. In this investigation, you will verify Snell's law empirically and then use it to make predictions.

Problem

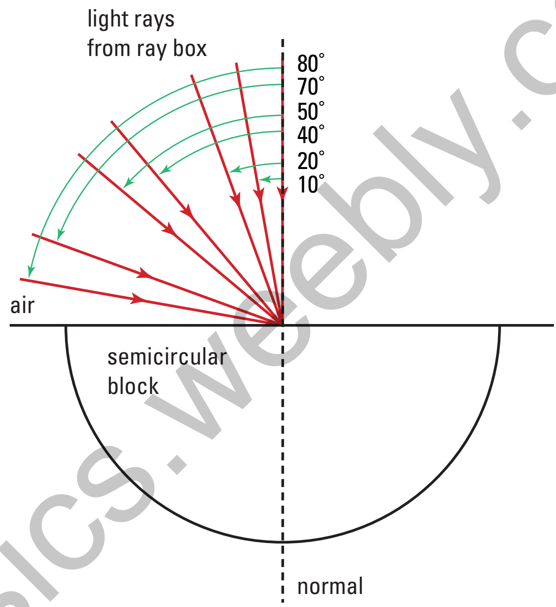
Verify Snell's Law.

Equipment

- ray box
- semicircular block of a material
- protractor
- sheet of paper (or polar coordinate graph paper)

Procedure

1. Make a data table with the following headings: Angle of Incidence (θ_i), Angle of Refraction (θ_R), sin θ_i, and sin θ_R.
2. Draw a horizontal line across the sheet of paper so that the paper is divided into half.
3. Down the centre of the page, from top to bottom, draw a dashed line at right angles to the horizontal line. This line is the normal line for the air-medium interface.
4. Using a protractor, draw lines on the top half of the page to represent the path of the incident light in air for all of the angles shown in the diagram at right.



5. Place the semicircular block of material on the page so that the straight side is positioned on the horizontal line. Position the centre of the flat side of the block where the normal line crosses the horizontal line (refer to the diagram).
6. Shine a single light ray from the ray box along the 10˚ line that you drew on the paper. The ray should strike the flat side of the block of material at the centre and pass through the block. Mark the paper at the point where the light ray exits the semicircular side of the block. Label the incident light ray and the corresponding refracted light ray at the point where it comes out through the semicircular side.
7. Repeat step 6 for each of the angles of incidence shown in the diagram.

8. Remove the semicircular block and draw lines from the centre point where the rays entered the block to the points where they exited. These lines are the paths of the refracted rays.
9. Measure each angle of refraction and enter the data in your table.
10. Determine the values of the sines of the angles of incidence and refraction, and enter the data in your table.

Analyze and Conclude

1. What happens to the light when it passes from air into the refracting medium?
2. As the angle of incidence increases, does the angle of refraction increase more rapidly or less rapidly?
3. Predict what will happen if the angle of incidence continues to increase. Is there a maximum value for the angle of incidence?
4. Plot a graph of θ_i versus θ_R.
5. Plot a graph of sin θ_i versus sin θ_R.
6. Compare the two graphs. How might such a comparison help scientists to develop empirical relationships?
7. Determine the slope of the graph of sin θ_i versus sin θ_R. What does the slope of this line represent?
8. Compare the accepted value of the index of refraction of the material used in the investigation with the experimental value you just obtained.
9. Comment on the experimental results in terms of verification of Snell's law. Suggest reasons for any discrepancies.
10. Using your experimental value of the index of refraction of the material, predict the angles of refraction for angles of incidence of 30° and 60°.
11. Test your predictions.

Apply and Extend

12. You have just learned about the principle of reversibility of light. Imagine doing the experiment in reverse. Your refracted rays become incident rays and your incident rays become refracted rays. Which angles (incidence or refraction) will approach 90° more rapidly?
13. Predict what will happen as the incident angle approaches 90°.
14. If possible, develop a method for testing your prediction and carry out the test.

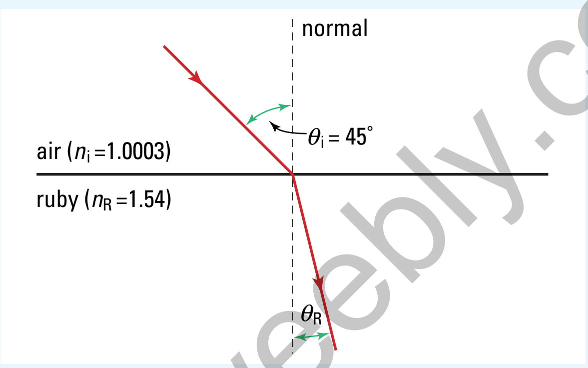
MODEL PROBLEM

Finding the Angle of Refraction

Light travels from air into a ruby crystal at an angle of incidence of 45°. Determine the angle of refraction of the light in the ruby.

Frame the Problem

- Sketch and label a ray diagram.
- Light travels from *air*, an *optically less dense* medium, into ruby, an *optically more dense* medium.
- The refracted ray should *bend toward the normal* line.
- You can use *Snell's law* to determine the extent of the bending of the refracted ray.



Identify the Goal

The angle of refraction, θ_R, in ruby

Variables and Constants

Involved in the problem		Known	Implied	Unknown
θ_i	n_i	$\theta_i = 45°$	$n_i = 1.00$	θ_R
θ_R	n_R		$n_R = 1.54$	

Strategy

Use Snell's law to solve the problem.

Calculations

$$n_i \sin\theta_i = n_R \sin\theta_R$$

Substitute first

$$1.00\ \sin\ 45° = 1.54\ \sin\theta_R$$

$$\frac{1.00(0.7071)}{1.54} = \frac{\cancel{1.54}\sin\theta_R}{\cancel{1.54}}$$

$$\sin\theta_R = 0.4592$$

$$\theta_R = \sin^{-1}(0.4592)$$

$$\theta_R = 27.33°$$

Solve for θ_R first

$$\frac{n_i \sin\theta_i}{n_R} = \frac{\cancel{n_R}\sin\theta_R}{\cancel{n_R}}$$

$$\sin\theta_R = \frac{n_i \sin\theta_i}{n_R}$$

$$\theta_R = \sin^{-1}\left(\frac{n_i \sin\theta_i}{n_R}\right)$$

$$\theta_R = \sin^{-1}\left(\frac{1.00 \sin 45°}{1.54}\right)$$

$$\theta_R = \sin^{-1}\left(\frac{0.7071}{1.54}\right)$$

$$\theta_R = \sin^{-1}(0.4592)$$

$$\theta_R = 27.33°$$

The angle of refraction in ruby is 27°.

Validate

The angle of refraction is less than the angle of incidence, which is to be expected when light travels from an optically less dense medium (air) into an optically more dense medium (ruby). The magnitude of the angle of refraction is realistic.

PRACTICE PROBLEMS

9. A beam of light passes from air into ethyl alcohol at an angle of incidence of 60.0˚. What is the angle of refraction?

10. A beam of light passes from ethyl alcohol into air. The angle of refraction is 44.5˚. Determine the angle of incidence.

Refraction and the Wave Model for Light

Have you been wondering how a change in the speed of a wave can cause it to change its direction? The ray model is excellent for visualizing the angles and helping you to set up the calculations, but it is not as helpful as the wave model for visualizing the *reason* for the change in direction.

Although you cannot see the shape of light waves, recall your study of water waves and envision light waves in the same way. What happens to the shape of the waves when they pass from an optically less dense medium into a more dense medium, where the velocity is slower? Recall from your study of mechanical waves that the frequency of waves does not change. From the wave equation, $v = \lambda f$, you can see that if the velocity decreases and the frequency remains the same, then the wavelength must decrease. Figure 11.5 shows the behaviour of wavefronts of light waves as they pass from air into water at an angle of incidence of 0˚. In this case, there is no change in direction, but the wavefronts are closer together because the wavelength is shorter.

Figure 11.6 (A) on the next page shows light wavefronts approaching the interface between two media (air and water) at an angle. You can take the same approach as shown in Figure 11.5 to see why the change in direction occurs. In Figure 11.6 (A), the left edge of each wavefront reaches the interface and begins to slow down, while the rest of the wavefront continues at the faster speed. The dashed lines show where the wavefronts would have been if the speed had not decreased. In a sense, the wavefronts themselves are bending, because one end is travelling more slowly than the other. The direction of the wavefronts (which is, of course, the direction of a light ray) bends toward the normal.

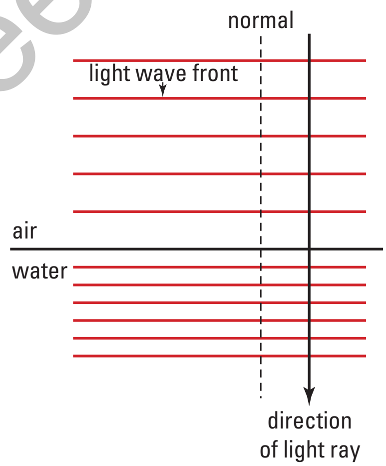


Figure 11.5 Plane light waves, represented by a ray, are travelling from air into water.

COURSE CHALLENGE

Twinkling Stars Point to a Problem

Design an experiment to test whether microwave frequencies are more or less likely than light to be refracted by variations in the atmosphere.

Learn more from the **Science Resources** section of the following web site: ***www.school.mcgrawhill.ca/resources*** and find the *Physics 11 Course Challenge*.

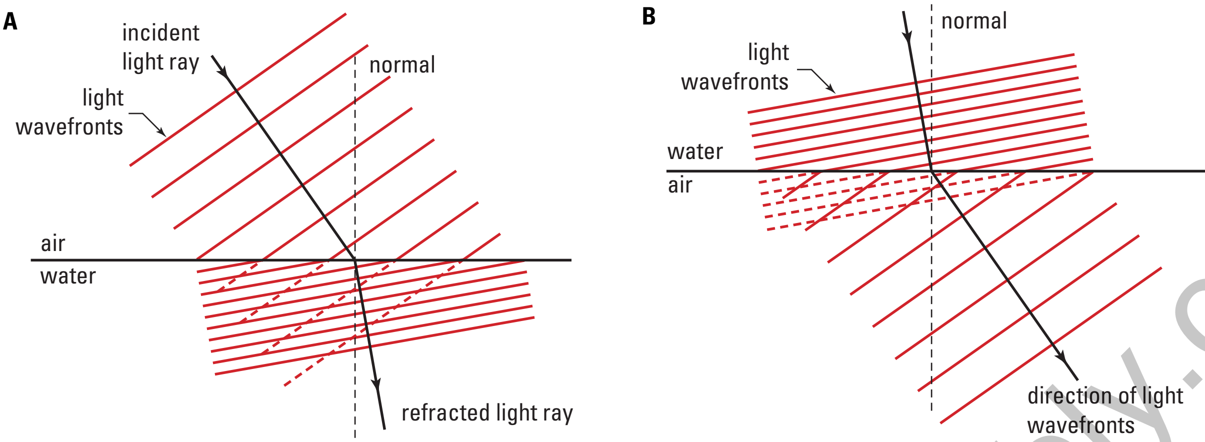


Figure 11.6 **(A)** The left edges of the wavefronts are bent backwards because the waves travel slower in the water than they do in air. **(B)** The left edges of the wavefronts bend forward, because the waves travel faster in air than in water.

The opposite situation is illustrated in Figure 11.6 (B), where light is travelling from water into air. The edge of the wavefront that reaches the air first speeds up, leaving the other side behind. Again, the wavefronts bend, causing the light ray to bend away from the normal.

Reversibility of Light Rays

How do parts (A) and (B) of Figure 11.6 differ? How are they the same? In Figure 11.6 (A), the ray is travelling from air into water, and the refracted ray bends toward the normal. In part (B), the ray is travelling in the opposite direction, from water into air. The refracted ray bends away from the normal. If you rotated Figure 11.6 (B) and then superimposed the two figures, you would see that the rays that are travelling in opposite directions have exactly the same shape. This situation is always true. If a new ray of light is directed backward along the path of a refracted ray, it will follow the same path after passing across the boundary between the two media. This outcome is called the **principle of reversibility of light**.

Think It Through

- Use Snell's law to verify the principle of reversibility. Explain your reasoning in detail.

11.2 Section Review

1. K/U Why is the value for Snell's constant for light travelling from water into glass a different value than that shown in Table 11.1 for either material?

2. K/U Explain, in terms of the wave model, why light changes direction when it crosses the interface between two different media.

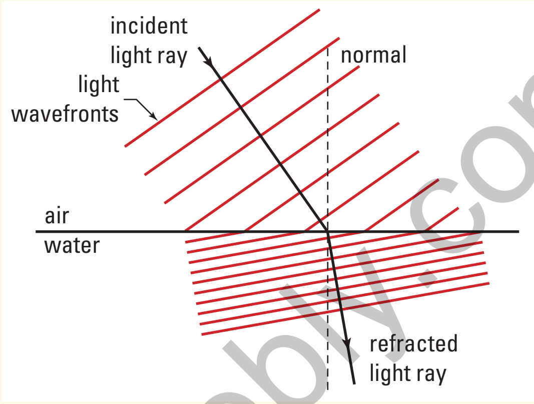
3. C Explain Snell's law without the use of an equation.

4. K/U Is the relationship between the incident angle and the refracted angle linear or non-linear?

5. C Draw ray diagrams to illustrate the principle of the reversibility of light.

6. C Describe why stars twinkle, even when the night sky is cold and clear.

7. C Consider the diagram (above right) showing how wavefronts change direction at the interface between to different optical media.
   - **(a)** Design an analogy to explain this bending behaviour of light as it crosses a boundary between media of different optical densities.
   - **(b)** Recall the reversibility of lights rays as described on the previous page. Analyze your analogy from part (a) to ensure that it also predicts the behaviour of wave-fronts moving in the opposite direction.



8. K/U As the angle of incidence increases, does the angle of refraction increase more or less rapidly?

9. K/U Is there a maximum value for the incident angle?

10. I Investigation 11-A provided opportunity to verify Snell's law. As part of the procedure, you were required to plot a graph of sin θ_i and sin θ_R.
    - **(a)** Describe the type of relationship that was represented by the graph?
    - **(b)** What does the slope of the line represent?

UNIT PROJECT PREP

Snell's law provides a mathematical model to predict the behaviour of light as it passes from one optical medium to another.

- How will an understanding of Snell's law assist you with your project design?
- You may want to individualize your project by selecting materials with extraordinarily high refractive indices. How would these materials affect your design?

11.3 Refraction Effects

SECTION EXPECTATIONS

- Apply the ray model of light to explain optical effects that occur as natural phenomena such as a rainbow.
- Define and describe partial reflection and refraction.

KEY TERMS

- apparent depth
- angle of deviation
- lateral displacement
- partial reflection and refraction of light
- mirage

Without realizing it, you make decisions nearly every waking moment, using information that is transmitted to your eyes by light. When you take a step, you assume that you know where the floor is located. When you reach for an object, your brain tells your hand how far and in what direction to reach. All of these decisions are based on the assumption that light always travels in straight lines. Usually your observations are correct, because the light from the objects you observe is travelling only through air. You start having problems, however, when the light has travelled through more than one medium and refraction changes the path of the light.

Apparent Depth

How can you make people appear to be shorter without physically altering their bodies in any way? At first, this task would seem to be impossible, but by using an optical effect caused by refraction, it is, in fact, quite easy to accomplish. Think about your results from "Apparent Depth" in the Multi-Lab at the beginning of the chapter. Based on these observations, suggest how you could make people appear to be shorter.

When you look at objects that are under water, they appear to be closer to the surface than they actually are. This effect, known as **apparent depth**, is due to the refraction of the light that is coming up from the object through the water and out into the air toward your eyes. The light ray diagram in Figure 11.7 shows what happens to the light travelling from an object on the bottom of a swimming pool up to the eyes of a person standing at the pool's edge.

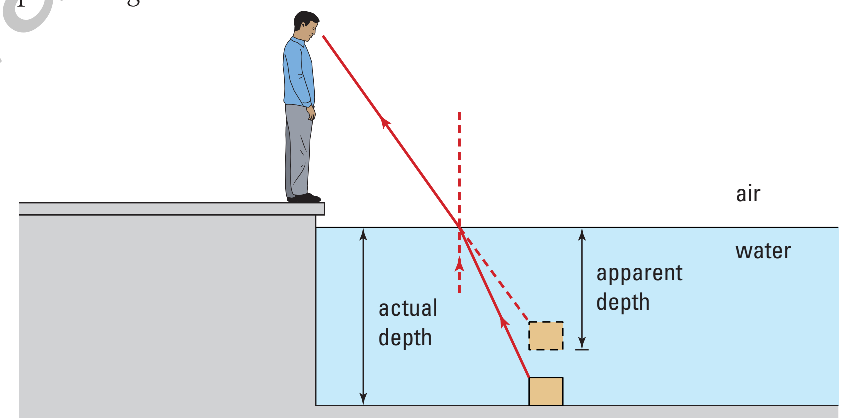


Figure 11.7 The object on the bottom of the pool appears to be closer to the surface than it is, because the light is refracted away from the normal as it enters the air from the water.

Math Link

You can use the formula for Snell's law ($n_i \sin\theta_i = n_R \sin\theta_R$) to develop the relationship shown below for calculating the apparent depth of an object under water, when the object is viewed from directly above. In this example, n_R is the index of refraction of the air, and n_i is the index of refraction of the water.

$$d_{\text{apparent}} = d_{\text{actual}}\left(\frac{n_R}{n_i}\right)$$

So, for $n_R < n_i$, the apparent depth, d_{apparent}, is less than the actual depth, d_{actual}.

Attempt to develop the proof for the relationship and write an explanation, using diagrams. Hint: For small angles, $\sin\theta \approx \tan\theta$.

Water is usually about 30 percent deeper than it appears to be. A person who thinks that the water in a stream is deep enough to come only halfway up his thighs might be unpleasantly surprised to find himself waist-deep in water. This illusion also makes it difficult for people fishing among shallow reefs and shoals to judge when the water is deep enough to operate the outboard motors on their boats.

Biology Link

What problem would the phenomenon of apparent depth pose for diving birds such as ospreys and seagulls?

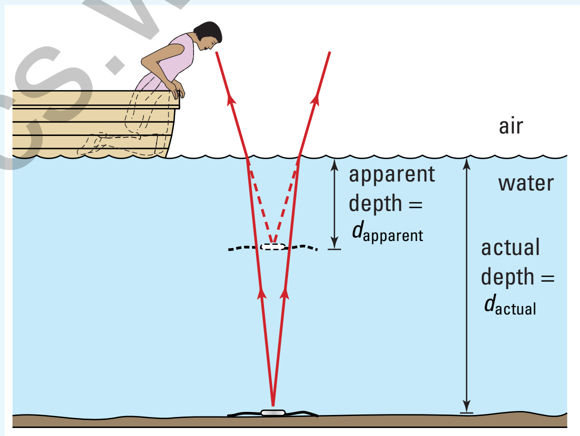
MODEL PROBLEM

Apparent Depth of a Lake

A woman in a motorboat sees a wristwatch on the bottom of the lake, just beside the boat. The boat's depth finder indicates that the water is 6.55 m deep. What is the apparent depth of the water?

Frame the Problem

- Visualize the problem by drawing a light ray diagram.
- Light rays from the watch travel a distance, d_{actual}, through the water and refract on entering the air. Extending the refracted rays straight back gives the image position.
- The indices of refraction of air and water are listed in Table 11.1, Index of Refraction of Various Substances. You can use the equation for the apparent depth to solve the problem.



Identify the Goal

The apparent depth, $d_{apparent}$, of the wristwatch below the water surface

Variables and Constants

Involved in the problem	Known	Implied	Unknown
n_{air}	$d_{actual} = 6.55$ m	$n_{air} = 1.000$	$d_{apparent}$
n_{water}		$n_{water} = 1.333$	
d_{actual}			
$d_{apparent}$			

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Strategy

You can use the formula for the apparent depth, d_{apparent}.

You have the indices of refraction and the actual depth, so all of the required variables are known.

Substitute values into the formula.

Evaluate.

The apparent depth of the wristwatch is 4.91 m.

Calculations

$$d_{\text{apparent}} = d_{\text{actual}}\left(\frac{n_{\text{R}}}{n_{\text{i}}}\right)$$

$$d_{\text{apparent}} = 6.55\text{ m }(1.000/1.333)$$

$$d_{\text{apparent}} = 4.9137\text{ m}$$

Validate

The units are in metres.
The apparent depth is less than the actual depth.

PRACTICE PROBLEMS

11. A girl is holding a clear, thin plastic bag of water that contains a goldfish. If she looks directly at the goldfish when it is 15 cm away from the side through which she is looking at it, how far away from the plastic will the fish appear to be?

12. A worker is looking down at a sample of radioactive waste that is encased in a rectangular glass block. The sample appears to be 0.55 m below the top surface of the glass. What is the actual depth of the sample in the glass block?

Figure 11.8 Why does it appear that the girl in this photograph has two left arms?

The apparent change in position caused by the refraction of light produces some unusual optical illusions. If you dip your leg into water and someone looks at it, the leg will look as though it is broken and bent upward. This is another example of the apparent depth effect. The human eye-brain system assumes that light travels in a straight line and, therefore, that the light is travelling directly from the leg toward the observer's eyes.

Deviation

When light from an incandescent light bulb that has a clear glass envelope shines on a wall or ceiling, the glass has no effect on the light — all of the walls are illuminated evenly. If light from the same bulb shines through crystal chandeliers or crystal glassware, however, beautiful patterns of light appear on the walls and ceiling. You might have seen this effect in displays of chandeliers in stores or when the sun shines through a piece of crystal glassware. The patterns occur because the light is entering, passing through, and then leaving a piece of glass in which the two sides of the glass are not parallel to each other.

In the prism shown in Figure 11.9, the incident light ray travels through the air and enters the left side of the glass. The light bends toward the normal in the glass, because glass has a higher index of refraction (optical density) than the air. When the light leaves the glass and emerges into the air on the other side of the prism, the light is refracted away from the normal. Notice that the direction of the light leaving the glass is different from that of the light entering it. The change in direction of the light as it passes through the glass is known as its deviation. The amount of change is called the **angle of deviation**, θ_{dev}. You can determine the angle of deviation for any shape of prism by applying Snell's law at each air-glass interface.

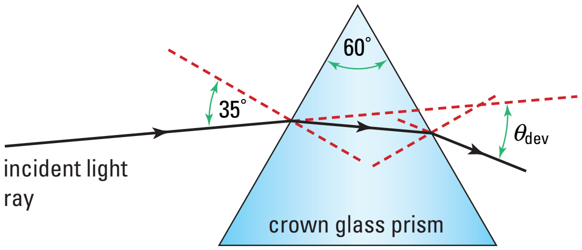


Figure 11.9 The light in a prism is refracted at both air-glass interfaces. The change in direction of the emergent light ray compared to the original incident light ray is called the angle of deviation.

TRY THIS...

Fill a glass beaker with water. Place a 30 cm ruler vertically into the water so that the flat side of the ruler is touching one side of the beaker. The millimetre markings should be facing in toward the water. With your face near the top of the ruler, look straight down into the water. Compare the size of a 1 cm space on the ruler, both above the water and below it. Then, move your head to one side, away from the ruler, to increase the angle of incidence at which you observe the submerged part of the ruler, and make the same comparison. Estimate the minimum and maximum changes in the apparent size of the submerged ruler.

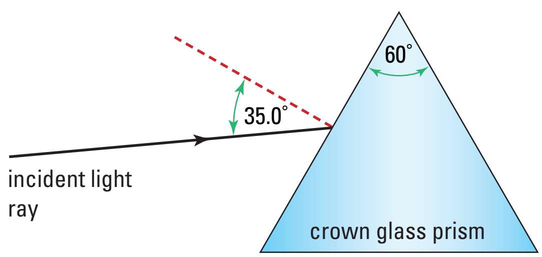
MODEL PROBLEM

Calculating The Angle of Deviation

Light enters the side of an equilateral, crown glass prism at an angle of incidence of 35.0°. Determine the angle of deviation for the light after it has passed through the prism.

Frame the Problem

- Draw a ray diagram of the light passing through the prism.
- Light enters the prism and is *refracted toward the normal*.
- The light *passes through* the prism and exits on the other side.
- As the light re-enters the air, it is *refracted away from the normal*.



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- The *angle of deviation* is the angle formed by the extension of the incident ray and the extension of the emergent ray.
- *Snell's law* applies individually to light travelling across each air-glass interface.
- The *rules of geometry* provide the method for finding the angle of incidence for the second interface.

Identify the Goal

The angle of deviation of the light ray as it passes through the prism

Variables and Constants

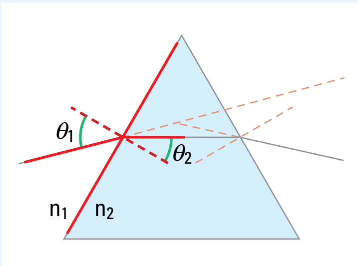
Involved in the problem		Known	Implied	Unknown
θ_1	n_1	$\theta_1 = 35.0^\circ$	$n_1 = 1.000$	θ_2
θ_2	n_2	$\theta_{prism} = 60.0^\circ$	$n_2 = 1.523$	θ_3
θ_3	n_3		$n_3 = 1.523$	θ_4
θ_4	n_4		$n_4 = 1.000$	θ_{dev}
θ_{dev}				
θ_{prism}				

Strategy

Calculations

Determine the angle of refraction in the glass at the first air-glass interface, using Snell's law.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$



All of the values except the angle of refraction are known, so substitute and solve.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\frac{n_1 \sin \theta_1}{n_2} = \frac{\cancel{n_2} \sin \theta_2}{\cancel{n_2}}$$

$$\sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2}$$

$$\sin \theta_2 = \frac{1.000 \sin 35^\circ}{1.523}$$

$$\sin \theta_2 = \frac{0.57357}{1.523}$$

$$\sin \theta_2 = 0.3766$$

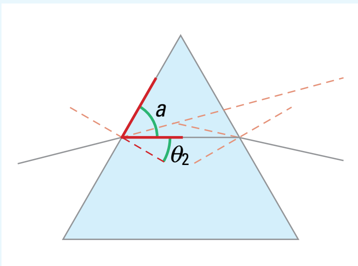
$$\theta_2 = 22.12^\circ$$

Find angle a by using the rule that a normal line forms a 90° angle with the side of the prism.

$$\theta_2 + a = 90^\circ$$

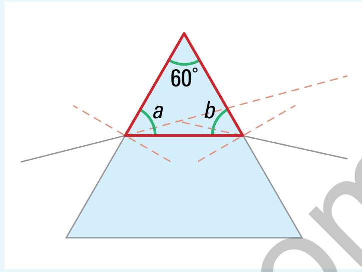
$$a = 90^\circ - 22.12^\circ$$

$$a = 67.88^\circ$$



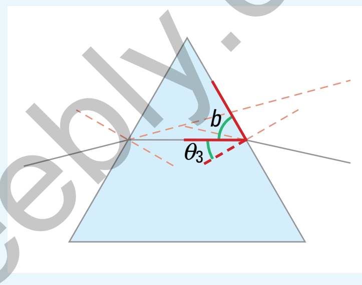
Find angle b from the rule that the sum of the angles in any triangle is 180°.

$$60^\circ + 67.88^\circ + b = 180^\circ$$
$$b = 180^\circ - 127.88^\circ$$
$$b = 52.12^\circ$$



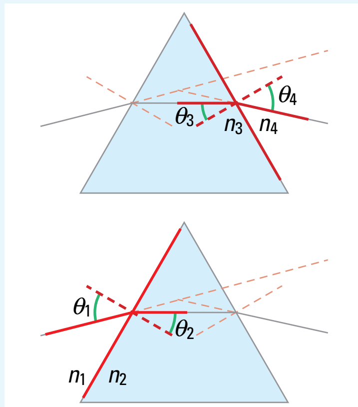
Find θ_3 from the rule that a normal line makes a 90° angle with the side of the prism.

$$b + \theta_3 = 90^\circ$$
$$\theta_3 = 90^\circ - b$$
$$\theta_3 = 90^\circ - 52.12^\circ$$
$$\theta_3 = 37.88^\circ$$

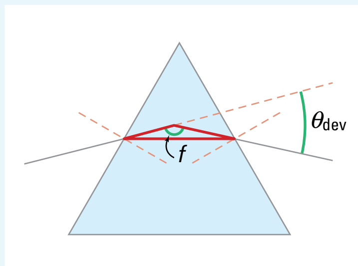


Find the angle of refraction at the second air-glass interface, using Snell's law.

$$n_3 \sin \theta_3 = n_4 \sin \theta_4$$
$$\frac{n_3 \sin \theta_3}{n_4} = \frac{\cancel{n_4} \sin \theta_4}{\cancel{n_4}}$$
$$\sin \theta_4 = \frac{n_3 \sin \theta_3}{n_4}$$
$$\sin \theta_4 = \frac{1.523 \sin 37.88^\circ}{1.000}$$
$$\sin \theta_4 = \frac{0.9351}{1.000}$$
$$\sin \theta_4 = 0.9351$$
$$\theta_4 = 69.24^\circ$$



Now, focus on the small triangle formed by the ray inside the prism and the extensions of the incident and emergent rays. If you can find the value of angle f, you can find the angle of deviation, θ_{dev}, because they are supplementary angles. Supplementary angles add to 180°. If you find angles c and e, you can use them to find f.



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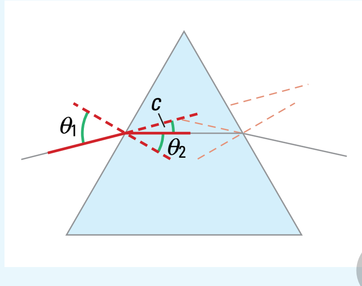
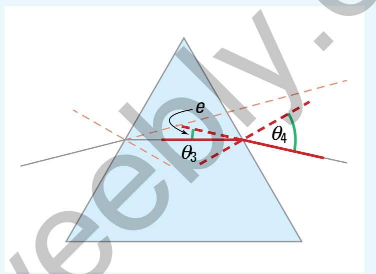
Find angle c by using the rule of geometry that when two straight lines intersect, the opposite angles are equal. The incident ray and its extension intersect with the normal.

$$\theta_1 = c + \theta_2$$
$$35^\circ = c + 22.12^\circ$$
$$35^\circ - 22.12^\circ = c + 22.12^\circ - 22.12^\circ$$
$$c = 12.88^\circ$$



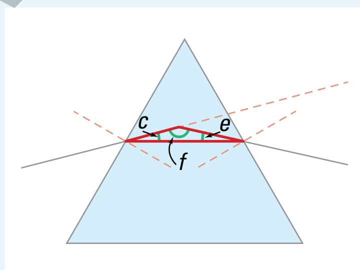
Find angle e, using the same techniques that you used above. The emergent ray and its extension intersect with the normal on the second interface.

$$\theta_4 = e + \theta_3$$
$$69.24^\circ = e + 37.88^\circ$$
$$69.24^\circ - 37.88^\circ = e + 37.88^\circ - 37.88^\circ$$
$$e = 31.36^\circ$$



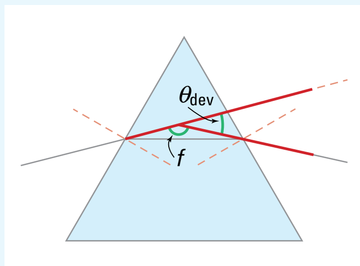
Find angle f from the rule of geometry that the sum of the angles of a triangle is 180°.

$$c + e + f = 180^\circ$$
$$12.88^\circ + 31.36^\circ + f = 180^\circ$$
$$f = 180^\circ - 12.88^\circ - 31.36^\circ$$
$$f = 135.76^\circ$$



Angles f and θ_{dev} are supplementary; therefore, they add to 180°.

$$\theta_{\text{dev}} + f = 180^\circ$$
$$\theta_{\text{dev}} + 135.76^\circ = 180^\circ$$
$$\theta_{\text{dev}} + 135.76^\circ - 135.76^\circ = 180^\circ - 135.76^\circ$$
$$\theta_{\text{dev}} = 44.24^\circ$$



The angle of deviation is 44°.

Validate

From the diagram, the answer appears to be reasonable. You could also draw a scale diagram and determine that the answers were nearly the same.

PRACTICE PROBLEMS

13. Light enters the side of a Plexiglas™ prism, which has an apex angle of 30.0°, at an angle of incidence of 45.0°. Determine the angle of deviation for the light after it has passed through the prism.

14. Light leaves the second interface of a crystal glass prism, which has an apex angle of 60.0°, at an angle of refraction of 45°.

(a) Determine the angle of incidence for the light as it first entered the prism.

(b) Determine the angle of deviation for the light after it has passed through the prism.

Look at the object positioned behind the aquarium in Figure 11.11. Notice how the part of the object that you are viewing through water appears to be shifted to the side relative to the top of the object, which you can see through air only. This apparent shift to the side is a special case of deviation that occurs whenever light penetrates a medium for which the two sides are parallel. In these cases, the angle of deviation is always zero.

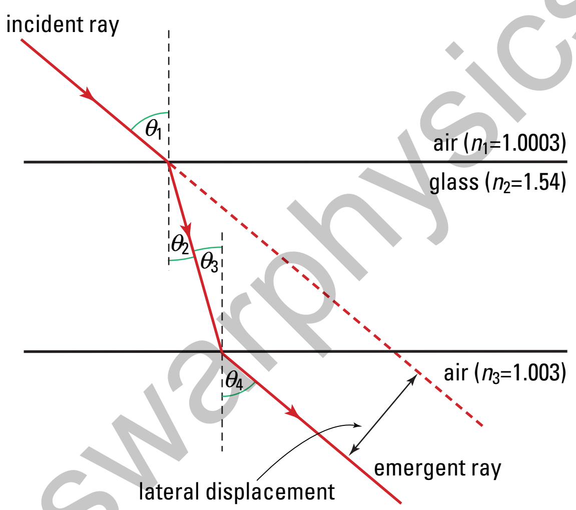


Figure 11.10 The light does not change direction, but is shifted to the side, so that it emerges travelling parallel to the original path of the light.

The path of the light ray is not a straight line, however. Inside the refracting medium, the ray is shifted to the side, but the ray that emerges from the medium is parallel to the incident ray; Figure 11.10 illustrates how this occurs. The incident ray enters the medium and is bent toward the normal. When the ray exits from the other side of the medium, the ray is refracted again and, this time, it is bent away from the normal. This shifting of light to

TRY THIS...

Obtain a triangular-shaped prism, a protractor, and a light source from your teacher. Choose an angle of incidence and use Snell's law to predict the angle of deviation for the prism. Verify your prediction experimentally. Then, make a prediction about how the angle of incidence affects the angle of deviation. Test your prediction.

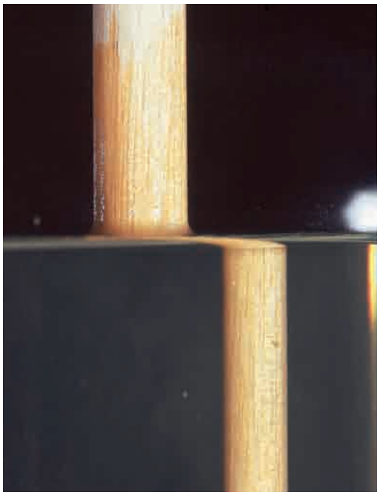


Figure 11.11 The top and bottom parts of the object appear to be separated.

the side is called **lateral displacement**. The amount of lateral displacement depends on the angle of incidence, the thickness of the material, and the index of refraction of the material.

Lateral displacement occurs every time you look at an angle through a window at an object such as a flagpole, so the flagpole is not positioned exactly where it appears to be. The light coming from the flagpole is refracted twice as it passes through the glass, and is therefore shifted to the side.

TRY THIS...

Obtain a rectangular block from your teacher. Choose an angle of incidence and use Snell's law to predict the lateral displacement for the rectangular block. Verify your prediction experimentally.

Think It Through

- Why do you usually not notice lateral displacement? Suggest a way in which you could prove to someone that lateral displacement is occurring when looking at objects such as flagpoles through a window.
- Does the position at which the incident light enters the side of a prism affect the path of the light? Explain your answer.
- Does the position at which the incident light enters the side of a rectangular block affect the path of the light? Explain your answer.

The discussion above describes lateral displacement qualitatively. To prove that the emerging ray is, in fact, exactly parallel to the incident ray, you can use Snell's law, along with the laws of geometry.

- Write Snell's law for the first air-glass interface, using the notations from Figure 11.10.

$$n_{\text{air}} \sin \theta_1 = n_{\text{glass}} \sin \theta_2$$

- Write Snell's law for the second glass-air interface.

$$n_{\text{glass}} \sin \theta_3 = n_{\text{air}} \sin \theta_4$$

- Recall from the rules of geometry that when a straight line (ray) cuts two parallel lines (normal lines), the internal angles are equal.

$$\theta_2 = \theta_3$$

- Substitute θ_2 for θ_3 into Snell's law for the second interface.

$$n_{\text{glass}} \sin \theta_2 = n_{\text{air}} \sin \theta_4$$

- Notice that the first and third statements of Snell's law contain identical terms. Therefore,

$$n_{\text{air}} \sin \theta_1 = n_{\text{air}} \sin \theta_4$$

- Divide both sides of the equation by n_{air}.

$$\frac{\cancel{n_{\text{air}}} \sin \theta_1}{\cancel{n_{\text{air}}}} = \frac{\cancel{n_{\text{air}}} \sin \theta_4}{\cancel{n_{\text{air}}}}$$

$$\sin \theta_1 = \sin \theta_4$$

$$\theta_1 = \theta_4$$

- Based on the rules of geometry, since the lines forming one side of the equal angles are parallel (normal lines), the second sides of those angles must be parallel.

The emergent ray is parallel to the incident ray.

Atmospheric Effects

Since the index of refraction of air is so close to that of a vacuum, you might conclude that you could never detect any refractive effects of air. When you consider that the atmosphere is hundreds of kilometres thick, though, it should not surprise you that the refraction of light from the Sun and stars might produce a noticeable effect.

When the Sun is setting or rising and is very close to the horizon, the bottom half of the Sun appears to be flattened. This illusion occurs because the light from the bottom portion of the Sun is refracted more than the light from the upper portion, as shown in the diagram in Figure 11.12.

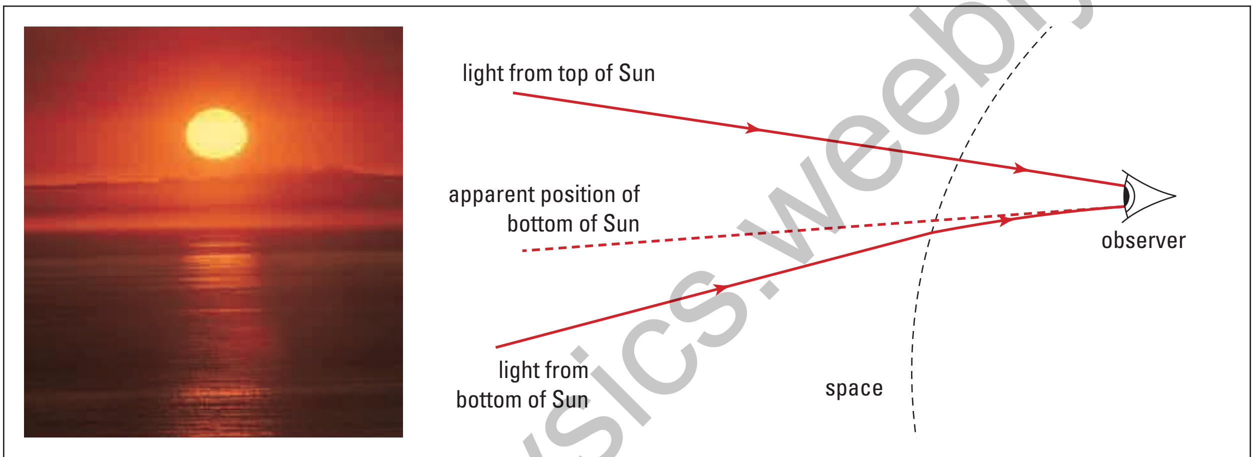


Figure 11.12 Refraction of light from the Sun in the upper atmosphere causes the bottom of the Sun to appear flattened as it rises and sets.

The bending of the light from the Sun just as it is setting produces yet another optical illusion — you can still see the Sun above the horizon after it has actually set. Due to the change in density and temperature, and therefore a gradual change in the refractive index of the atmosphere, the light from the Sun follows a curved path as it passes through the atmosphere, as shown in Figure 11.13.

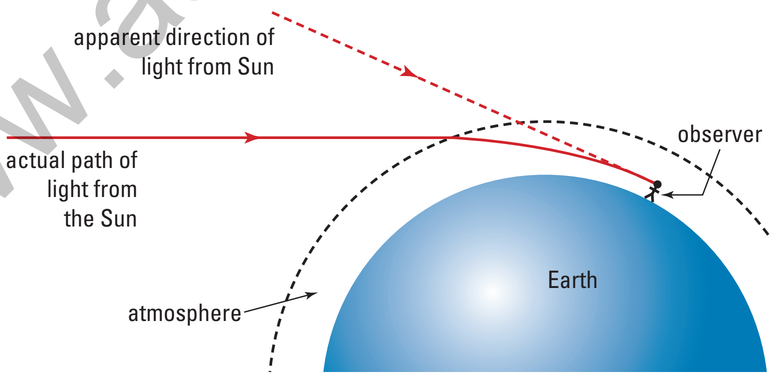


Figure 11.13 The changing optical density of Earth's atmosphere causes sunlight to follow a curved path toward the observer. The Sun still appears to be positioned above the horizon, even though it has already set.

Both effects — the flattened look of the Sun at sunrise and sunset and the fact that you can still see the Sun above the horizon after it has set — are produced in a similar way to the apparent depth effect. The change in the index of refraction of Earth's atmosphere is produced by variations in both the composition and temperature of the air. The index of refraction of a material varies slightly with temperature. In Table 11.1, the refractive index of each of the gases was specified for a temperature of 0°C.

The twinkling of stars at night is another effect of the refraction of light as it travels through Earth's atmosphere. The starlight passes through many different layers of air that are continuously moving and that vary in temperature and density. These variations cause tiny changes in the refractive index of the air and, therefore, in the path of the star's light as it travels to Earth's surface. For a fraction of a second, the direction of the light from the distant source is changed sufficiently that the star's location appears to shift slightly in the sky, and then it quickly shifts back again, close to its original position. The effect occurs so quickly and so often that the star appears to twinkle. The orbiting Hubble Space Telescope is located beyond Earth's atmosphere and provides astronomers with opportunities to view celestial phenomena without the distortions produced by atmospheric refraction.

The variation of the refractive index of air with temperature causes the shimmering effect you see above barbecues and the surface of hot roads. Because the heat that creates these effects only occurs in the daytime, you see a number of objects through the heated air instead of a single one. Therefore, you see a constantly changing (shimmering), distorted image of all of the objects, rather than a twinkling effect.

Partial Refraction

Sometimes, when a car goes by, you are momentarily blinded by a flash of light, due to sunlight reflecting from a car window. At night, when you are inside a lighted room looking out through a window into the darkness, you can see an image of yourself and the room's contents clearly reflected in the glass. In both of these situations, you know that light is passing through the window, as well as being reflected. When light strikes any interface between two media, some of it reflects back, while some light refracts through the second medium.

As illustrated in Figure 11.14, when light travelling through the air (the first medium) strikes the flat surface of a transparent plastic block (the second medium), the incident light separates into two distinct light rays. Most of the light passes (is refracted) into the plastic block. A small amount of the light is reflected back into the air from the surface of the plastic block, according to the law of reflection. This phenomenon is called the **partial reflection and refraction of light**.

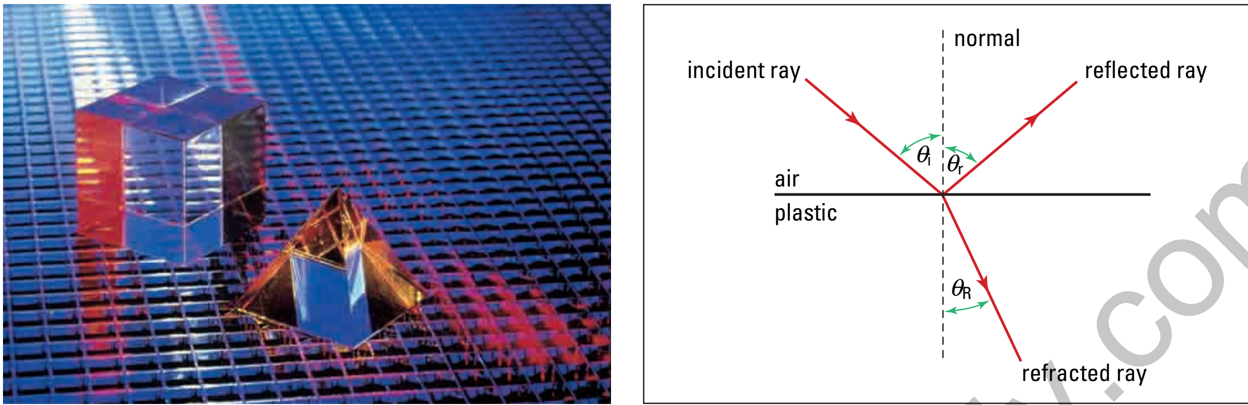


Figure 11.14 Light is simultaneously reflected and refracted when it passes from one medium into another.

The amount of the incident light that is refracted, as opposed to reflected, depends on the angle of incidence and on the indices of refraction of the two transparent materials. When light travelling from air into glass shines directly down onto the glass surface, almost all of the light is refracted and very little is reflected. When the angle of incidence approaches 90˚, however, only a small portion of the light is refracted into the glass and most of it is reflected.

You can observe how the amount of reflected light changes with the angle of incidence by standing at the edge of a shallow pond on a sunny day and looking across the pond's surface. If you look at the surface just in front of you, you can see down into the water quite clearly. Very little sunlight reflects from the water surface toward you. Most of the sunlight refracts down into the water, then reflects off the bottom, and travels back up to your eyes. However, if you look across the surface farther from shore, the amount of sunlight reflecting from the water surface is much greater. Much less of the sunlight is entering the water at this larger angle of incidence. Figure 11.15 illustrates this.

When you are driving at night, the glaring reflection in your rearview mirror of the headlights of vehicles behind you can be a very real safety hazard. Most rearview mirrors in cars have a small lever mechanism that allows the driver to flip the mirror to another preset position, to reduce the glare to safer levels.

As you can see in Figure 11.16 (A), on the following page, the structure of the rearview mirror is quite different from that of normal plane mirrors. The mirror consists of a glass prism (wedge) that has a mirrored back surface. During the day, the mirror is positioned so that the light is refracted as it enters the glass wedge. The light is then reflected from the mirror surface at the back, travels back through the glass, and is refracted out into the air toward the driver's eyes. Glare is usually not a problem in daylight, even if a following vehicle has its headlights switched on.

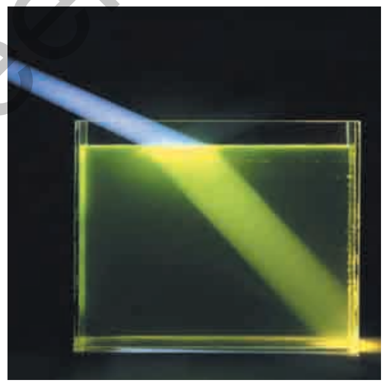


Figure 11.15 As the angle of incidence increases, more light is reflected from the surface than is refracted into the medium.

Figure 11.16 A rearview mirror has a mechanism to reduce the glare of headlights. The mirror can be adjusted so that the light is reflected from the front surface of the glass, rather than being reflected by the mirror surface on the back of the glass.

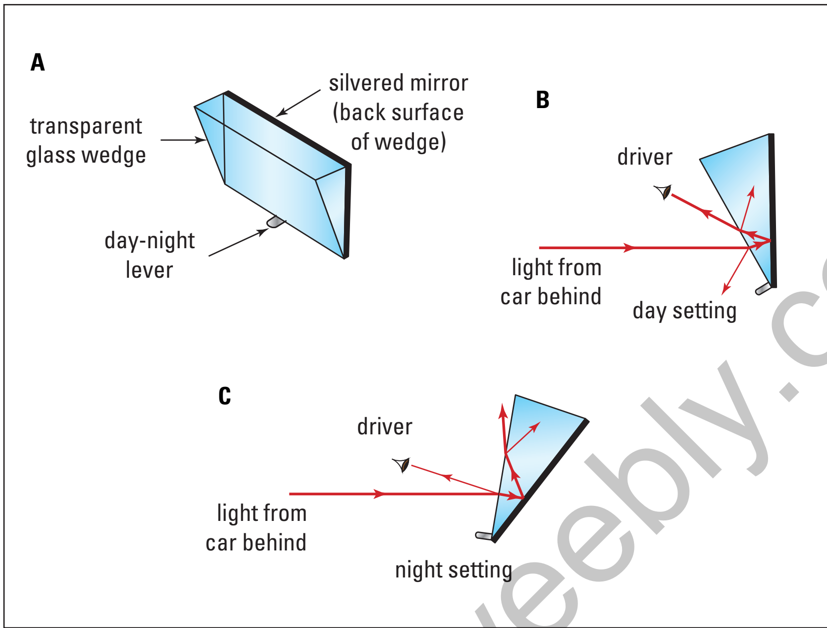


Figure 11.16 (B) shows that a small amount of partial reflection also occurs at the air-glass interface each time the light is refracted. In both cases, however, the reflected light is not directed toward the driver's eyes. At night, when the glare reflected from the mirror is too great, the driver uses the lever to flip the mirror to the preset position shown in Figure 11.16 (C). When the mirror is in the new position, the bright light that is reflected from the mirror surface travels back through the glass wedge, emerges into the air, and shines up toward the roof of the car, thus avoiding the driver's eyes. The shape of the glass wedge is such that its front surface is now in just the right position to act as a mirror surface, reflecting the light toward the driver's eyes. Because the incident light is so intense, the partial reflection that occurs at the front surface is sufficient to provide the driver with enough light to observe the traffic behind.

Language Link

From which French verb is the word "mirage" derived, and what does it mean?

Mirages

A **mirage** is a virtual image that occurs naturally when particular atmospheric conditions cause a much greater amount of refraction of light than usual. A mirage might be observed when some layers of Earth's atmosphere are warmer than others. On a hot, sunny day, the layers of air above a road surface vary in temperature and form a mirage, as shown in Figure 11.17 (A). Light coming from the sky is refracted to such a great extent as it passes through the different layers of air that some of the light follows a curved path and travels up toward the observer's eye. This refraction effect makes objects that are just above the horizon appear as if they are formed as images in a mirror-like road surface, as shown in Figure 11.17 (B).

Sometimes the layers of air are reversed, with the hot air being above a layer of cold air that is next to the ground. When this happens, light from a distant object undergoes refraction in the hot air, and the image appears to be higher in the air than it actually is. These kinds of mirages sometimes make distant mountains seem higher and closer than they actually are.

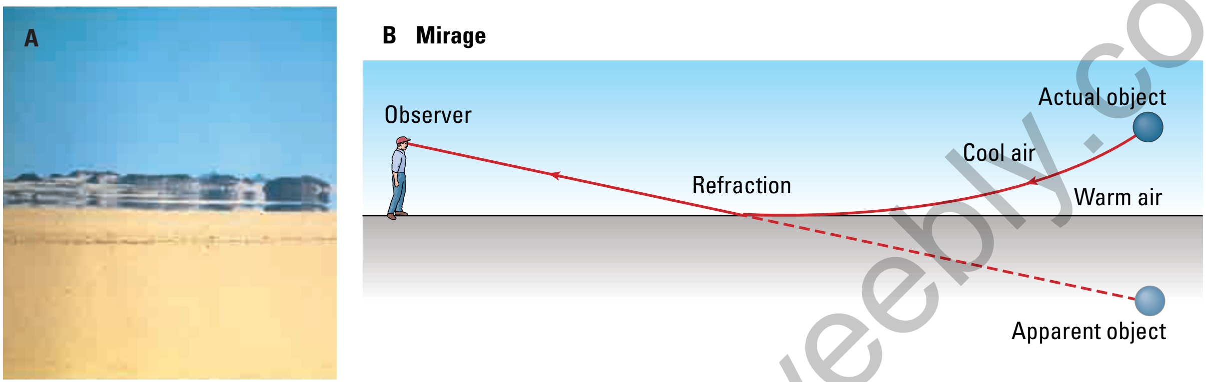


Figure 11.17 Mirages are caused by refraction of light by air when extreme temperature changes occur near the ground. Cool air above very warm air that has been heated by the ground creates ideal conditions for the formation of a mirage.

11.3 Section Review

1. K/U Why is there more glare from car headlights on a rainy night than on a night when the roads are dry?
2. K/U Why is it easier to see objects on the floor of a lake or river when looking directly down, rather than when looking at an angle?
3. K/U List several optical distortions or illusions that you have observed. Explain how they occur.
4. MC Planets do not twinkle, so they can easily be distinguished from stars. Explain why this is the case, based on the effects of refraction.
5. C Explain, using diagrams, why you can still see the Sun even after it has set.

UNIT PROJECT PREP

Refraction effects are a key element of many optical instruments.

- How do refraction effects influence where you place your lenses in your optical instrument?
- Consider lens properties such as size, thickness and index of refraction when designing your optical instrument.

11.4 Total Internal Reflection

SECTION EXPECTATIONS

- Define and describe total internal reflection and critical angle.
- Explain the conditions required for total internal reflection.
- Analyze and describe total internal reflection situations using the light ray model.

KEY TERMS

- critical angle
- total internal reflection
- retroreflector

The next time you are talking on the telephone or watching a television program, look down at the hairs on your arm. A single glass fibre, thinner than one of those hairs, is now able to transmit tens of thousands of telephone calls and several dozen television programs simultaneously. How can such a thin piece of glass fibre carry all of that information? Learning about total internal reflection will help you to understand this remarkable — and extremely useful — technology and the impact it has on your daily life.

Conditions for Total Internal Reflection

So far, you have studied refraction of light as it travels from an optically less dense medium into an optically more dense medium, and as it travels from an optically more dense medium into an optically less dense medium. Have you noticed anything unusual that occurs in the second case, but not in the first? Complete the Quick Lab on the opposite page and focus on any differences that you observe between light travelling from the optically less dense into the more dense medium, compared to light travelling in the opposite direction.

When light travels from an optically more dense material into an optically less dense one, the refracted light bends away from the normal. As you can see in Figure 11.18, for all angles of refraction up to and including 90°, the light is simultaneously reflected and refracted, as expected. The amount of reflected light gradually increases and the amount of refracted light gradually decreases as the angle of incidence increases. Notice that the angle of refraction increases more rapidly than the angle of incidence.

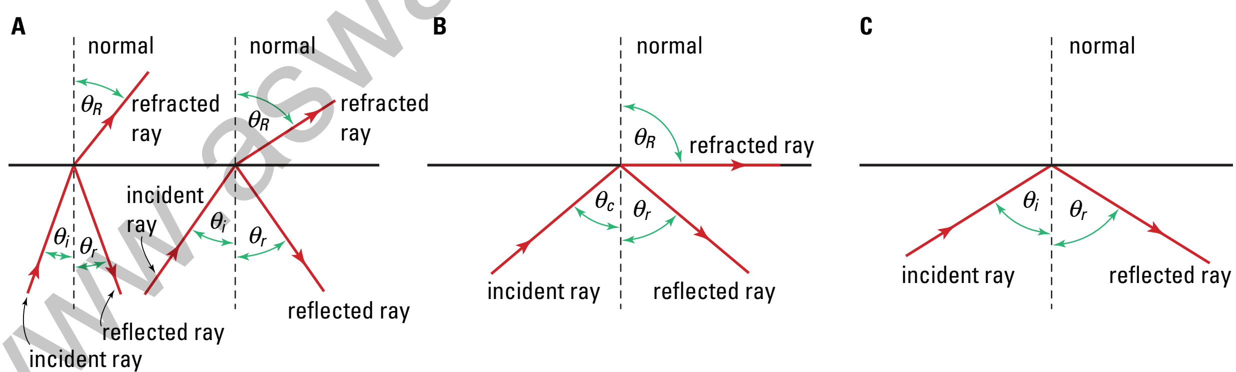


Figure 11.18 The incident light in the Plexiglass™ is both partially refracted and partially reflected for all angles of incidence **(A)** up to the critical angle, θ_c, for the Plexiglas™ and **(B)** at the critical angle. **(C)** As soon as the angle of incidence exceeds the critical angle, total internal reflection occurs.

QUICK LAB

Total Internal Reflection

TARGET SKILLS
- Performing and recording
- Communicating results

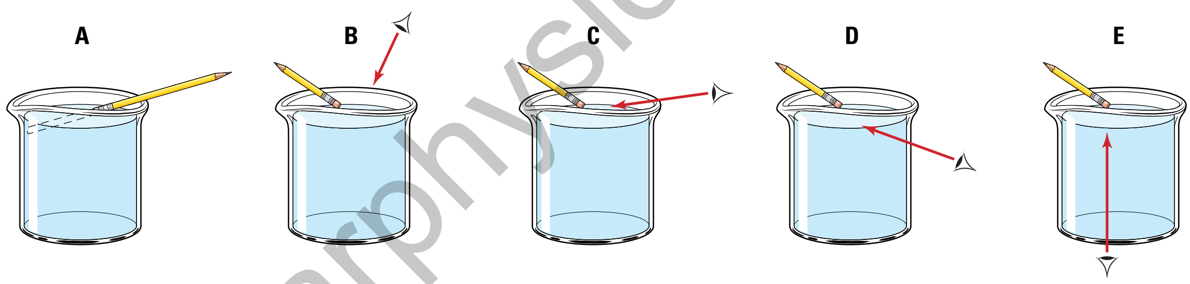
In this lab, you will observe the same object, a pencil, from three different perspectives. Position a 1.0 L beaker so that you can observe it from the top, side, and bottom. Fill the beaker with water. Observe the motion of a pencil as it is slowly submerged into the water at a very small angle with the horizontal, as shown in the first part of the diagram.

In each of the following cases, gradually move the pencil under the surface of the water, after referring to the indicated part of the diagram. Observe the appearance of the pencil:

- from almost directly above the water, as shown in (B)
- from the side, so that you are looking toward the surface of the water as shown in (C)
- from the side, looking at the surface of the water from below, as shown in (D)
- from below, looking up through the bottom of the beaker

Analyze and Conclude

1. Describe in detail the appearance of the pencil as you saw it from each perspective.
2. Describe the most significant difference that you observed between one perspective and the others.
3. Explain how and why the appearance of the pencil differed so greatly from the different perspectives. What properties of light and its interaction with various materials were responsible for these differences?



The angle of refraction can be increased only until it reaches its maximum possible value of 90˚, as seen in Figure 11.18 (A). The maximum angle of refraction in any medium is 90˚, because it is not physically possible to exceed this value without having the light remain in the original medium. In that case, the light would no longer be refracted, but would be reflected.

The angle of incidence, for which the angle of refraction is exactly 90˚, is called the **critical angle**, θ_c as illustrated in Figure 11.18 (B). In fact, something very interesting occurs when the angle of incidence is increased beyond the critical angle — all of the incident light is completely reflected back into the optically more dense Plexiglas™, as shown in Figure 11.18 (C). None of the

TRY THIS...

Pour about 400 mL of water into a 500 mL beaker and place an empty test tube in the water at an angle. Observe the test tube from above and below the water surface, and describe any differences. Now, remove the test tube from the water in the beaker and pour water into the test tube until it is about one third full. Replace the test tube in the water in the beaker and repeat your observations. Explain the effect produced by adding the water to the test tube.

light is refracted into the optically less dense air. The interface between the two media behaves as if it was a perfect mirror surface. This phenomenon is called **total internal reflection**.

Even the most efficient (metallic) mirrored surfaces absorb from 8 percent to 11 percent of the incident light. By contrast, when total internal reflection occurs in a glass prism, *none* of the light is absorbed during reflection and only about 2 percent of the light is absorbed by the glass itself.

TOTAL INTERNAL REFLECTION

The two conditions required for total internal reflection to occur are as follows.

- The light must travel from an optically more dense medium into an optically less dense medium.
- The angle of incidence must exceed the critical angle, θ_c, associated with the material.

You can determine the critical angle, θ_c, for light travelling from diamond into air by using Snell's law, as demonstrated in the following problem.

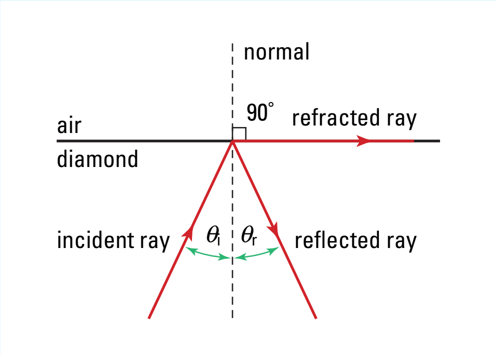
MODEL PROBLEM

Finding θ_c

Determine the critical angle for diamond.

Frame the Problem

- Make a sketch of the problem. Label all of the media, angles, and rays.
- Light is travelling *from* an optically *more dense* material *into* an optically *less dense* material
- The *critical angle* of incidence corresponds to an angle of refraction equal to 90°.
- The needed *indices of refraction* are listed in Table 11.1.



A light ray striking a diamond-air interface, with an angle of incidence equal to θ_c

Identify the Goal

Calculate the critical angle, θ_c, for diamond.

Variables and Constants

Involved in the problem	Implied	Unknown
n_{air}	$n_{air} = 1.0003$	$\theta_{c/diamond}$
$n_{diamond}$	$n_{diamond} = 2.42$	
θ_{air}	$\theta_{air} = 90^\circ$	
$\theta_{c/diamond}$		

Strategy

Calculations

The critical angle of incidence, $\theta_{c/diamond}$, occurs when the angle of refraction is exactly 90°.

Use Snell's law.

$$n_i \sin\theta_i = n_R \sin\theta_R$$

Find the required indices of refraction by using Table 11.1.

Substitute values into the formula, using the fact that $\theta_i = \theta_{c/diamond}$ when $\theta_R = 90^\circ$.

$$2.42 \sin\theta_{c/diamond} = 1.0003 \sin 90^\circ$$

Evaluate.

$$\sin\theta_{c/diamond} = \frac{1.0003 \times 1.000}{2.42}$$

$$\sin\theta_{c/diamond} = 0.4133$$

$$\theta_{c/diamond} = \sin^{-1} 0.4133$$

$$\theta_{c/diamond} = 24.415^\circ$$

The critical angle of incidence for diamond is 24.4°.

Validate

The angle of incidence is less than the angle of refraction.

PRACTICE PROBLEMS

15. Determine the critical angle for ethyl alcohol.

16. The critical angle for a new kind of plastic in air is 40°. What is the critical angle for this plastic if it is immersed in water?

17. Optical fibres, made of a core layer surrounded by cladding, trap transmitted light by ensuring that the light always strikes the core-cladding interface at an angle greater than the critical angle. Calculate the critical angle between the core-cladding interface.

18. While swimming in a friend's pool, you allow yourself to slowly sink to the bottom exactly 3.0 m from the edge of the pool. As you sink you fix your gaze on the edge. Calculate how deep your eyes must be below the surface for total internal reflection to occur.

PROBLEM TIP

When using Snell's law to determine the critical angle for a transparent material, the value for the sine of the angle of refraction will always be unity (one), because the maximum angle of refraction is 90°.

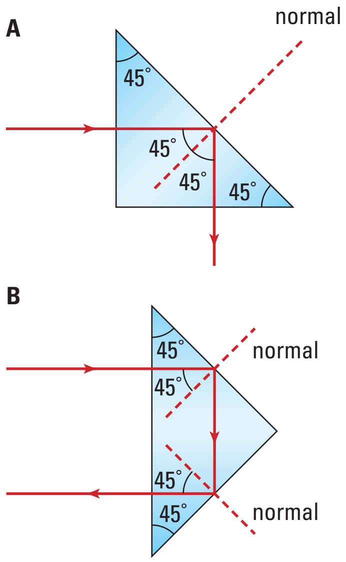


Figure 11.19 The direction of the light can be changed by either 90° or 180° as the light undergoes total internal reflection inside a single 45°–45°–90° prism.

Applications of Total Internal Reflection

What do driveway reflectors, binoculars, and optical fibres all have in common? Each technology is based on total internal reflection of light. You can understand the operation of these devices by studying total internal reflection in a simple glass prism, such as the one illustrated in Figure 11.19. Notice that the cross section of the prism is an isosceles right triangle. The critical angle for most types of glass and plastic is less than 45°. When light enters the prism perpendicular to the glass surface on one of the shorter sides of the prism, it will strike the longer side at an angle of 45°, as shown in Figure 11.19 (A). Because the angle of incidence at the glass-air interface is greater than the critical angle for glass (θ_c is in the range of 40° to 43°), the light undergoes total internal reflection and passes out into the air perpendicular to the third glass surface.

If the light enters the glass prism on the longer side, as shown in Figure 11.19 (B), the light strikes both glass-air interfaces and is totally internally reflected twice inside the prism. The light finally leaves the prism through the same side that it entered, having been reflected back parallel to its original path.

Optical devices such as periscopes, binoculars, and cameras use transparent glass or plastic prisms as reflectors to change the direction of light by either 90° or 180°, as shown in Figure 11.20. In some cameras, a single pentaprism, similar to that shown in Figure 11.20 (C), reflects the light into the viewing eyepiece. As discussed earlier, prisms are particularly efficient at reflecting light — even more efficient than high-quality mirrors. The extra amount of light reflected by prisms provides a brighter image than a mirror, which can be very important when using periscopes, binoculars, cameras, and other optical devices in dim light.

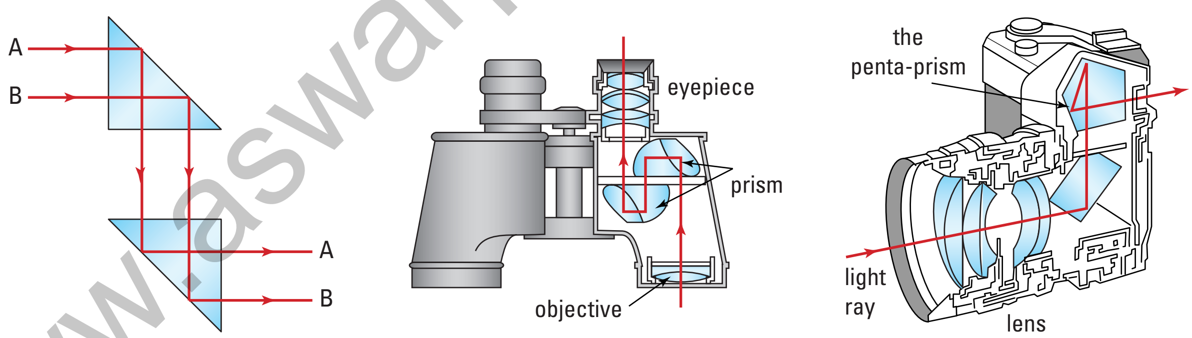


Figure 11.20 **(A)** In a periscope, total internal reflection occurs once in each prism to laterally displace the light the desired amount. Notice that the image remains upright.

(B) In a pair of binoculars, total internal reflection occurs twice in each prism, making it possible both to reduce the length of the binoculars and to achieve lateral displacement of the image.

(C) The principle of total internal reflection is also incorporated into the design of some cameras.

One of the most widely used applications of total internal reflection is found in a familiar device known as a **retroreflector**. Anyone travelling in any type of road vehicle, or even on a bicycle, uses retroreflectors as a safety precaution. A retroreflector is an optical device that reflects light directly backward, parallel to its original path. Car and bicycle reflectors and markers used to identify hazards on highways and roads are all designed to reflect light by means of total internal reflection.

As the light ray diagram in Figure 11.21 shows, light travelling from a car headlight toward a bicycle reflector passes through the reflector's glass and strikes the many sets of flat, angled surfaces at the back of the device. The light undergoes total internal reflection at the 45° surfaces, before passing out through the front of the reflector and back toward the light source. If a single plane mirror was used, the light would be reflected uselessly off to the side of the road.

Yet another application of total internal reflection can be found in gemstones, such as diamonds and emeralds, and various crystal ornaments, such as those seen on elaborate chandeliers — objects that appear to sparkle as you move past them. This sparkling effect is produced by the total internal reflection of light as it enters the many flat surfaces (facets) of a gem or crystal and is then reflected internally and back out through other surfaces toward your eyes. In addition to being valuable, diamond is very popular because its large index of refraction (and, therefore, small critical angle) allows a gem-cutter to cut more facets for total internal reflection than any other gemstone. Each additional facet creates more opportunities for light to reflect and create more "sparkles."

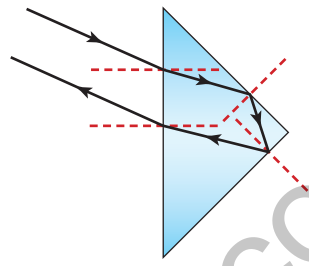


Figure 11.21 To show the basic principles of retroreflectors, light is shown being refracted and then reflected from just two reflecting surfaces. In actual retroreflectors, there are three reflecting surfaces, each positioned at 90° to one another. Why are three reflecting surfaces required?

Optical Fibres

Perhaps one of the most innovative and exciting ways in which total internal reflection is integrated with technology is in the transmission of information carried by light energy in optical fibres. An optical fibre is a very fine strand of a special kind of glass. When light shines into one end of an optical fibre, total internal reflection causes the energy to be confined within the fibre (Figure 11.22). The light travels along the inside of the length of the fibre, carrying information in the form of pulses. Even if the optical fibre is literally tied in knots, the light will still travel through the fibre until it reaches the other end.

A typical optical fibre is about the thickness of a human hair. As illustrated in Figure 11.23, on the next page, the fibre consists of a glass core, roughly 50 μm in diameter, surrounded by a thin layer known as "optical cladding," which is made of another type of glass. The cladding increases the total outside diameter to about 120 μm. The glass core has a slightly higher index of refraction ($n = 1.5$) than the optical cladding layer ($n = 1.47$), as is required for total internal reflection. By surrounding the entire length of the

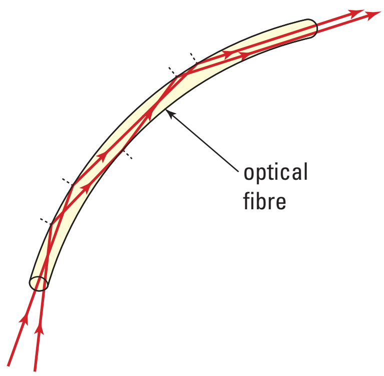


Figure 11.22 The light travelling inside the optical fibre will continue to undergo total internal reflection along the interior wall of the fibre until it reaches the other end.

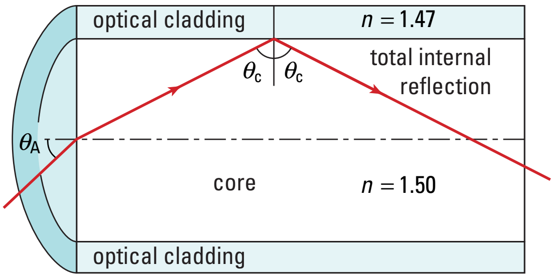


Figure 11.23 Only light that has an angle of incidence of more than 78.5° will be totally internally reflected down the entire length of the optical fibre.

core, the outer layer of glass cladding ensures that the critical angle of incidence of the core remains constant throughout the entire length of the fibre.

Because the difference in refractive indices of the two materials that make up an optical fibre is very small, the critical angle of the glass core can be as high as 78.5°. Consequently, only light entering the optical fibre at angles of incidence greater than 78.5° can be transmitted along it. What happens to any light that enters the optical fibre at angles up to and including the critical angle of incidence for the core? The angle, θ_A, that is shown in the Figure 11.23 is called the "acceptance angle" for the fibre, and in the case of this fibre, the θ_A is 11.5° (90° – 78.5°). Any light entering the fibre at an angle less than the acceptance angle will meet the cladding at an angle of incidence greater than the critical angle, θ_c, and will experience total internal reflection. The light enters the fibre and then travels along it, reflecting from side to side in a zigzag path until it reaches the other end.

Groups of optical fibres are often bundled together to produce what are called "optical fibre cables," which are usually covered with a protective plastic covering (Figure 11.24). Because these cables have a small diameter (some less than a millimetre) and are flexible, they can fit into spaces too small for metallic wires. They can also be used safely in corrosive and explosive environments.

Figure 11.24 Total internal reflection between the optical fibre core and cladding allows the transmission of information in the form of light pulses.

The optical fibres are made from a very special form of ultra-pure fused silica (sand). An optical fibre cannot be made of ordinary glass, because internal impurities reduce by a factor of 100 000 the intensity of any light entering it, after the light travels a distance of only 5 m. Currently, the very pure form of glass used in optical fibres is so transparent that light can travel up to 50 km along the fibre. To appreciate the purity of this glass, note that normal sunlight penetrates to a depth of only about 100 m in ordinary seawater before it is completely absorbed, and at depths below this, there is total darkness.

Clearly, sending information along an optical fibre requires a unique light source. The unique properties of laser light allow audio, video, and text based data to be transmitted simultaneously over long distances. When the signals are received they can be separated and processed individually. The light produced by a laser is called "coherent light," because in laser light, the light waves emitted by the source leave perfectly in phase with each another and are all of the same wavelength. All other types of light sources give off light waves randomly, so the light waves are not in phase.

The development in 1970 of the ultra-pure glass required to transmit light signals many kilometres made it possible to use optical fibres as a practical telecommunications medium.

Developments in laser light sources and signal receivers meant that, by 1980, it was possible to establish a worldwide installation of optical fibre communication systems. Some of the advantages of using optical fibres rather than metal conductors are the ability to significantly increase the amount of transmitted data, reductions in both weight and size, lower operating costs, and increased security in transmitting sensitive data.

CANADIANS IN PHYSICS

Photons, Lasers, and Superatoms

Physicist Milena Imamovic-Tomasovic

Born in Sibenik, Croatia, Milena Imamovic-Tomasovic was fascinated with science in elementary school. In high school, she read Kurt Mendelssohn's book *The Quest for Absolute Zero*, and decided to study low-temperature physics. She was particularly intrigued by the work of Indian physicist, Satyendra Nath Bose.

In 1924, Bose wrote to Albert Einstein, explaining his theory of the statistics of photons (the particles that make up light). Einstein extended Bose's theory to other particles and atoms. He and Bose predicted that, at a low enough temperature, a gas would condense into a "superatom," a completely new state of matter.

This dramatic transformation, called "Bose-Einstein condensation" was not actually observed until 1995, while Imamovic-Tomasovic was studying physics at the University of Belgrade. That year, researchers in Boulder, Colorado, used laser cooling, and other techniques, to cool a gas to less than a millionth of a degree above absolute zero (–273˚C) — and produce the new form of matter.

Bose condensates are now being created every day in more than 30 laboratories around the world. Imamovic-Tomasovic, at the University of Toronto, is one of a number of physicists studying their unique properties.

"Bose-Einstein condensation is still new," Imamovic-Tomasovic explains. "It's hard to tell what practical application one might expect in the future. Bose condensate is very much like laser light. What makes laser light different from ordinary light is that all of its photons are exactly the same (the same energy and the same phase). The same is true for atoms in the Bose condensate; all are exactly the same (the same energy state). Therefore, one could hope to build an atom laser that would be used in very sensitive measurement instruments. In fact, experiments demonstrating the first atom lasers have already been done."

Asked what advice she would give to high school students, Imamovic-Tomasovic recalls how she followed her heart into low-temperature physics: Look closely and deliberately at possible career paths. Then do what your heart tells you. If you work on something that you really like, almost nothing is too hard for you.

Web Link

www.school.mcgrawhill.ca/resources

For more information on Bose condensates and atom lasers, go to the above Internet site. Follow the links for **Science Resources** and **Physics 11** to find out where to go next.

Web Link

www.school.mcgrawhill.ca/resources/

Scientists continue to do research and development related to optical fibres. To find out more about the latest applications of this technology, go to the above Internet site. Follow the links for **Science Resources** and **Physics 11** to find out where to go next.

Biology Link

Naturally occurring optical fibres have existed on Earth for a long time. The white hairs of a polar bear's fur are hollow and filled with air. Sunlight enters the top of each hair and is internally reflected down to the polar bear's skin, where the light energy is absorbed. What colour is the skin of a polar bear? Suggest reasons why.

Optical Fibres and Medicine

In the field of medicine, the use of optical fibres has significantly changed many types of surgical and diagnostic procedures. In arthroscopic surgery, the endoscope is used to carry out many different types of internal surgical operations. The endoscope consists of a flexible tubular sheath only millimetres in diameter, containing two separate optical fibre cables, minute surgical tools, fine tubes to carry cleansing water down to and away from the site of the operation, and a microscope for viewing the interior area of a person's body. One optical cable shines light down to illuminate the interior of the body, and the other cable carries the reflected light back so that the surgeon can see the affected area. The surgeon makes a small incision in the skin and carefully slides the long tube of the endoscope into the area requiring surgery.

The reflected images can also be fed to a video camera and displayed on a television screen. Sometimes, the patient is conscious during the operation. The minimal damage caused to the tissues near the area of the operation allow the patient to recover much more rapidly than with conventional surgery.

One type of endoscope, the bronchoscope, is inserted through the throat or the nose and used to view the bronchial tubes or lungs. A similar instrument can be used to view the upper parts of the digestive system. Tissue samples can be obtained and surgical procedures performed using all of these devices.

QUICK LAB: Determining the Critical Angle

TARGET SKILLS

- Initiating and planning
- Predicting
- Performing and recording

So far in this unit, the light ray model has been a very useful visual scientific model for analyzing and predicting the behaviour of light. A mathematical formula is another very powerful and commonly used type of scientific model that can be used for the same purposes. For example, Snell's law can be used to predict the critical angle of incidence for any material.

- Given the index of refraction for a known material, use Snell's law to predict the critical angle of incidence for the material, and then design an investigation to experimentally verify the prediction.
- Prepare an appropriate list of required equipment and materials. The transparent material to be used in the investigation will be specified by your teacher.
- Design a procedure for testing your prediction and present it to your teacher for approval. After it has been approved, carry out the procedure and record and analyze all required data.

Analyze and Conclude

Compare the theoretical predictions and the experimental (empirical) evidence, and account for any discrepancies.

TARGET SKILLS

- Analyzing and interpreting
- Communicating results

PHYSICS & SOCIETY

Lasers in Medicine

In many action films, lasers have been agents of destruction, used to burn through heavy metal, aim powerful weapons, or act as "trip wires" to detect intruders. Lasers have, however, many peaceful, life-enhancing uses. They read the music engraved on your compact disk, help bricklayers align a wall, and identify the brand and price of items at stores' checkout counters.

The pinpoint accuracy of a laser makes it a useful tool for surgeons. Instead of knotting a thread around a tiny artery that is leaking blood into the abdomen, a surgeon can "zap" the end of the vessel with some laser light. The very local heating cauterizes the artery, effectively sealing the end.

With a laser, an eye surgeon can reshape the eye to correct a problem that would otherwise require glasses or contact lenses. For example, by taking a tiny slice off of the front of the cornea with a high energy CO_2 laser, the eye surgeon can correct a patient's near-sightedness.

Lasers are used in a wide variety of cosmetic surgical procedures, from wrinkle, freckle, and tattoo removal, to repairing drooping eyelids, to reducing snoring.

Lasers also have a place in major operations. Some patients with severe angina, a condition in which portions of the heart are scarred and not working properly, are too ill for open-heart surgery. In a new procedure called "transmyocardial revascularization" (TMR), a CO_2 laser is used to burrow tiny channels in the heart muscle. Small blood vessels grow through these channels, restoring blood flow and oxygen to the muscle. Although experimental and currently being used only for patients with few other options, TMR does not require that the surgeon open the chest wall, so patients can go home in just a few days.

Tattoo removal using a laser can be quite painful.

Tattoo removal before and after images.

Analyze

What property of a laser makes it useful for surgical procedures?

11.4 Section Review

1. K/U List the two conditions required for total internal reflection to occur.
2. K/U Why are prisms, rather than mirrors, often used in optical devices to reflect light?
3. K/U How does the speed of light relate to the critical angle for a given material?
4. MC Why would it be desirable to have such a large critical angle for optical fibres? What problems could be caused if the critical angle was reduced?

11.5 Dispersion

SECTION EXPECTATIONS

- Demonstrate and illustrate, using light ray diagrams, how light behaves at the interface of various media.
- Define and explain dispersion.

KEY TERMS

- dispersion
- visible spectrum
- recombination

The beautiful, sparkling colours produced by ice crystals on a twig in winter, the vibrant colours of a rainbow, and the brilliant flashes of colour you see when light passes through glass chandeliers and diamonds — these are all examples of a phenomenon known as **dispersion**. Dispersion is the separation of visible light into its range of colours. Although this effect has been observed for thousands of years, it was Sir Isaac Newton who, in 1666, initiated the first systematic study of dispersion.

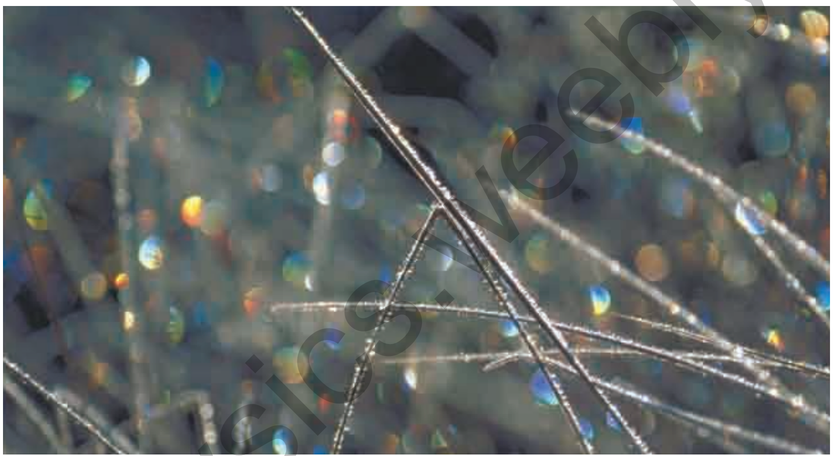


Figure 11.25 Ice crystals formed on blades of grass disperse sunlight separating it into a collage of colour.

Newton's research was prompted by problems that he encountered with the lenses he was using to build a refracting telescope, a device invented by Hans Lippershey (1570–1619), a Dutch lensmaker. The lenses used in early telescopes suffered from a problem now known as "chromatic (colour) aberration." The glass at the outer edges of the lenses refracted the light so much that rainbow-coloured fringes appeared around the perimeter of all objects observed through a telescope.

The Spectrum

Newton found that sunlight was separated into a range of colours (called the **visible spectrum**) when it passed through a glass prism, as shown in Figure 11.26. Some skeptics believed that, instead of being a property of the light, the colours were somehow produced by the prism. So Newton added a lens and a second prism to show that the coloured light could be put back together into white light again through a process referred to as **recombination** — something that should not happen if the prism was the source of the colours. Figure 11.27 shows Newton's experiment.

Sunlight has a continuous visible spectrum that, for convenience, has been grouped into seven colours: red, orange, yellow, green, blue, indigo, and violet. More careful analysis shows that there is an infinite number of different colours that blend together continuously to form the entire visible spectrum.

As Figure 11.26 shows, red light is refracted the least and violet light the most. This difference in the degree of refraction of the colours indicates that the index of refraction of the material varies with the colour of the light. For all commonly used transparent materials, the index of refraction is smaller for red light than it is for violet light. Since the index of refraction is related to the speed of light in a medium, the smaller index of refraction of red light indicates that red light travels through common media faster than violet light.

Each type of light source, such as incandescent, fluorescent, and neon lights, produces a characteristic spectrum of light. When the light from an incandescent lamp passes through a prism, a continuous band of colour is produced, with no one colour being brighter than any other. A normal fluorescent source produces a continuous spectrum of colour, but four discrete wavelengths, or colours, predominate. A neon source produces only a few discrete colours of the spectrum. The laser light is unique because it consists of only one wavelength out of the entire visible spectrum of light.

Technology Link

An optical instrument called a "spectrometer" is used to identify the precise frequencies of light produced by a given light source. The light is passed through either a large prism or a special device called a "diffraction grating" inside the spectrometer. Spectrometers are also used to analyze the chemical composition of unknown substances. The unknown material is heated to incandescence and the light that is produced is passed through the spectrometer. How does this help to identify the material?

Dispersion in Nature

Water droplets and ice crystals in the atmosphere produce some beautiful natural displays of colour. Rainbows form when sunlight passes through raindrops, which disperse the light into the different spectral colours. The various wavelengths in light are refracted by differing amounts when they enter the top of the water droplet.

Figure 11.26 A prism separates white light into its spectrum of colours.

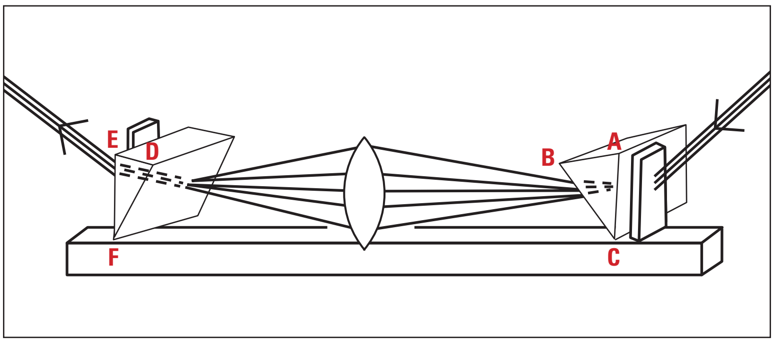


Figure 11.27 This sketch illustrates Newton's demonstration with prisms and lenses. Prism ABC separates white light into the spectral colours. The dispersed light then passes through the lens, and prism DEF recombines the colours back into white light.

The colours then undergo total internal reflection at the back of the droplet and are refracted again as they leave the droplet near the bottom (see Figure 11.28). The different colours exit the droplets at slightly different angles. The red light leaving the droplets makes an angle of 42° in relation to the sunlight entering them. The violet light leaving the droplets makes an angle of 40° in relation to the incident sunlight, and so on.

When you look at a rainbow, you see only the red light from the droplets higher in the sky. The other colours leaving those droplets pass above your eye. You see only the violet light from droplets lower in the sky, because the other colours from these droplets pass below your eye. All of the other different colours of light in the spectrum leave the millions of droplets between the droplets that produce the red and violet light, allowing you to see the complete spectrum in the rainbow.

As shown in Figure 11.29, conditions sometimes to produce a second rainbow that is higher in the sky and fainter than the first (primary) one. This secondary rainbow is formed by light that undergoes total internal reflection *twice* inside each raindrop.

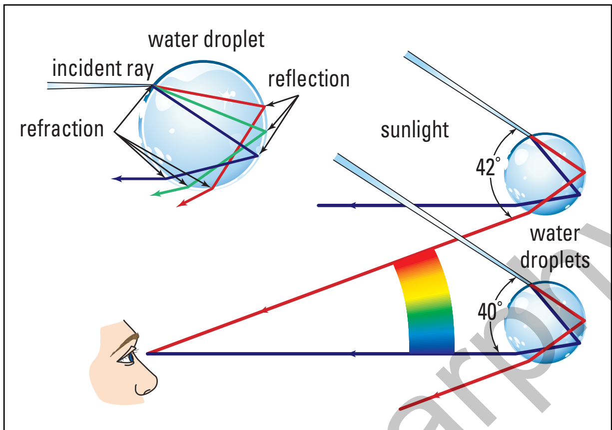


Figure 11.28 Sunlight undergoes both refraction and total internal reflection to produce a rainbow.

Figure 11.29 What do you notice about the colour distribution in this double rainbow?

11.5 Section Review

1. C Describe how Newton proved that sunlight is a combination of all of the spectral colours?
2. K/U Which of the spectral colours travels slowest in a glass prism? Explain.
3. K/U How could the shape of a rainbow be affected by the altitude of your location on the side of a mountain?
4. K/U Which colours in a rainbow do you see at the lowest elevation in the sky? Explain.

CHAPTER 11 Review

REFLECTING ON CHAPTER 11

- Refraction of light, as of other waves, is the change in velocity when light passes from one medium into another. Refracted light may also change direction.
- The index of refraction of a medium is the ratio of the speed of light in a vacuum to the speed of light in the medium: $n = \frac{c}{v}$. A medium with a high index of refraction is optically dense.
- The angle of refraction is the angle of a light ray exiting from a refractive boundary. For any two given media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant: $\frac{\sin\theta_i}{\sin\theta_R}$ = constant.
- Snell's law relates angles of incidence and refraction to indices of refraction: $n_i \sin\theta_i = n_R \sin\theta_R$. The incident ray, the refracted ray, and the normal all lie in the same plane.
- Refraction can be explained in terms of the wave model for light and the wave equation.
- The principle of reversibility of light states that, if a new ray of light is directed backwards along the path of a refracted ray, it will follow the same path after crossing the boundary between the media.
- Refraction causes effects such as the smaller apparent depth of objects submerged in water, deviation in prisms with sides that are not parallel, lateral displacement in prisms with parallel sides, atmospheric refraction in sunsets and mirages, and partial refraction and reflection.
- Total internal reflection occurs when light in an optically more dense medium strikes the boundary with an optically less dense medium at an angle of incidence greater than the critical angle for the medium.
- Applications of total internal reflection include prisms for reflection in periscopes, binoculars and cameras; retroreflectors; and optical fibres, in telecommunication, computing, and medicine.
- Dispersion is the separation of visible light into its component colours resulting from refraction. Water droplets disperse sunlight to create primary and secondary rainbows.

Knowledge/Understanding

1. Define the term index of refraction.
2. An observer looking down at a mug at an angle is not able to see a coin resting on the bottom of the mug. As the mug is filled with water, the coin becomes visible. Explain why.
3. Light travels from medium Y to medium X. The angle of refraction is larger than the angle of incidence. In which medium does the light travel at a lower speed? Explain your logic.
4. When light passes from Plexiglass™ into ice at an angle, which will be smaller, the angle of refraction or the angle of incidence? Explain why.
5. Light travels from medium C to medium D. The angle of incidence is larger than the angle of refraction. Which medium has the lower index of refraction? Explain your logic.
6. How does the size of the critical angle change as the index of refraction decreases?
7. Which pair of media, air and water or air and glass, have the smaller critical angle? Explain why.
8. When white light passes through a prism and is dispersed into the spectral colours, which colour is refracted the least? Explain why in terms of the index of refraction and the speed of light in the medium.

Inquiry

9. **(a)** You are given a rectangular, transparent block of an unknown solid material. Design an experiment to determine the index of refraction of the material and the speed of light in the material.

(b) You are given a sample of an unknown liquid material. Design an experiment to determine the index of refraction of the material.

10. Design an investigation to measure the index of refraction of a sample of a salt solution with a known concentration. Devise and describe a procedure that could be used to determine the concentration of a salt solution (or any other kind of solution) with an unknown concentration.

11. Design an investigation, using scaled light ray diagrams, to measure the change in the apparent depth of an object submerged in water, at a series of angles of incidence, increasing by 10˚, from 0˚ to 70˚. Plot a graph of the ratio of apparent depth to actual depth versus angle of incidence, and interpret the results obtained. Create an activity to demonstrate the effects you have identified.

12. Design an investigation to determine the maximum angle of deviation for a given triangular prism. Identify any practical limitations that restrict the maximum angle of deviation. How does the maximum angle of deviation depend on the angles of the prism?

13. For a given rectangular, transparent glass block, predict the maximum amount of lateral displacement possible, and check your prediction by experiment. Explain your prediction.

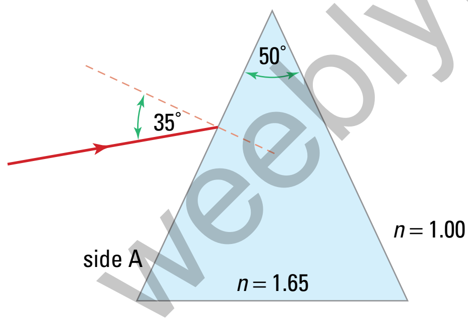
14. Try shining a flashlight at a car or bicycle reflector at different angles. Over what range of angles does the reflector direct the light back to you?

Communicating

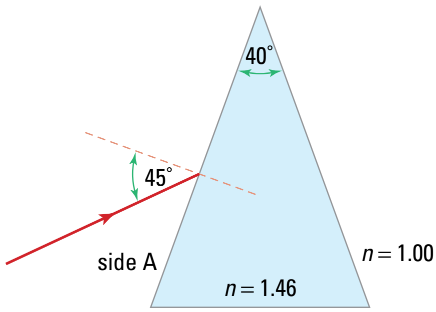
15. Use a variety of information technology resources to research the use of optical fibres in such areas as the transmission of telecommunications signals. Prepare a written report, with photographs or diagrams to show typical uses.

16. Carry out research on optical illusions caused by refraction effects. Summarize and present the general categories of illusions, using diagrams and photographs where appropriate, and explain why each type of illusion is produced.

17. (a) Draw an enlarged copy of the ray diagram below, such that side A is four times the size shown. Using Snell's law and the law of reflection, draw light rays to show the path of the light through the prism.



(b) Draw an enlarged copy of the ray diagram below, such that side A is four times the size shown. Using Snell's law and the law of reflection, draw light rays to show the path of the light through the prism.



Making Connections

18. Evaluate the impact of the development of optical fibres in our modern society. In which area of application will optical fibres have the greatest influence in the near future? Summarize your findings in a report.

19. Imagine you are swimming underwater and wearing goggles. As you look up at the air beyond the water surface, what effect can you expect to see? Draw a diagram to illustrate your answer. What implications does this underwater view of the airspace above the water have for fish? (Hint: Think about the critical angle of water.)

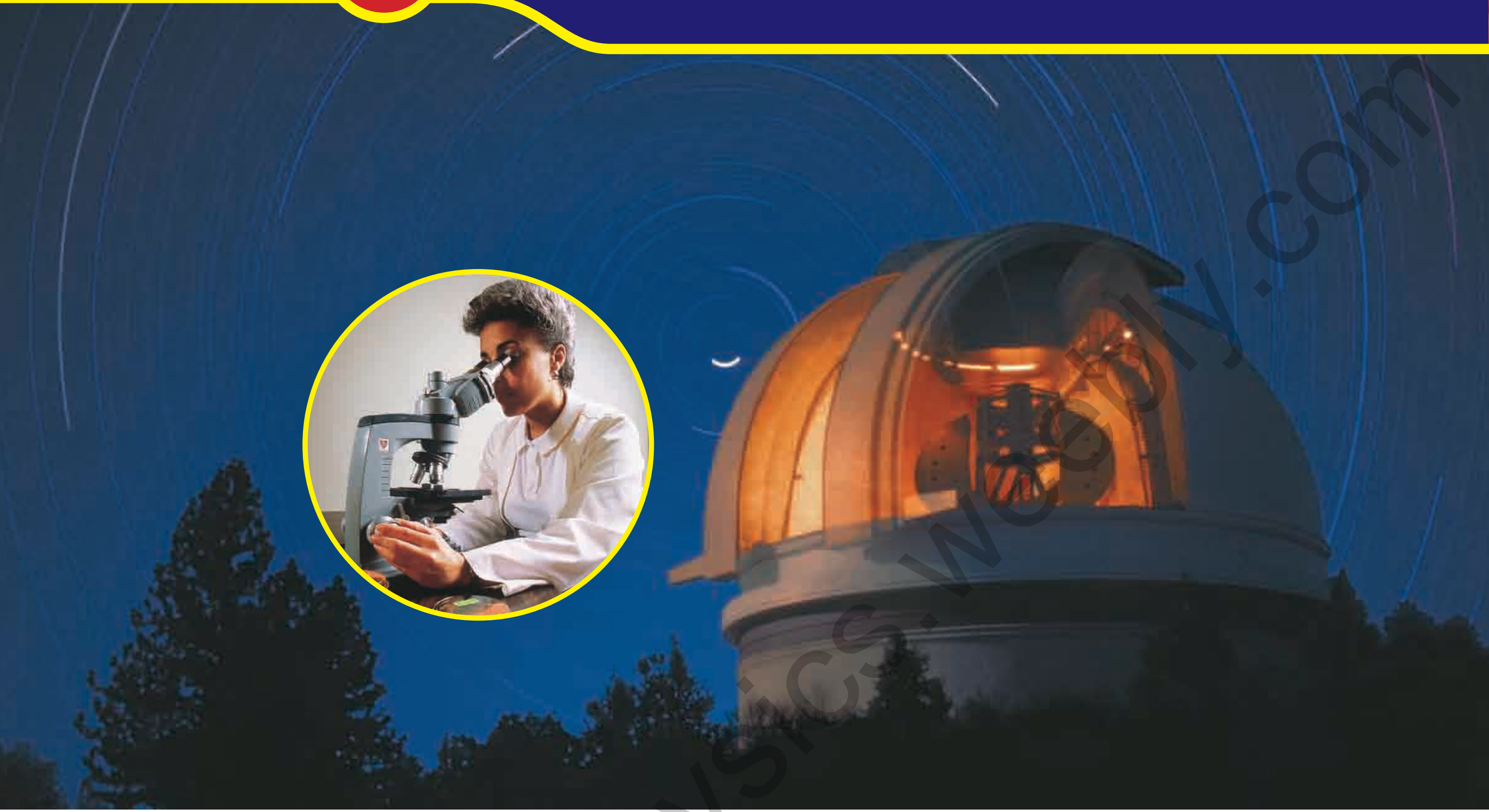
Problems for Understanding

20. Light passes from water into a block of unknown material. If the angle of incidence in the water is 70.0° and the angle of refraction is 40.0°, what is the index of refraction of the unknown material?
21. A ray of light passes from Lucite™ into water at an angle of incidence of 20.0°. What is the angle of refraction in the water?
22. A light ray enters the longest side of a 45°–45° – 90° crown glass retroreflector. Assume that the light ray enters the longest side of the retroreflector at a point one quarter of the length of the side from one corner, at an angle of incidence of 30°. Use Snell's law and the law of reflection to determine the complete path of the light ray until it leaves the retroreflector. Draw an accurate diagram to show the complete path of the light ray. The length of each of the two shorter sides of the retroreflector is 10 cm.
23. Determine the time it takes for light to travel 54 cm through glycerin in an aquarium.
24. **(a)** What is the index of refraction in a medium if the angle of incidence in air is 57°, and the angle of refraction is 44°?
    (b) What is the angle of refraction if the angle of incidence in air is 27°, and the index of refraction of the medium is 2.42?
    (c) What is the angle of incidence in air if the angle of refraction is 28°, and the index of refraction of the medium is 1.33?
25. Light passes from crystal glass into ethyl alcohol. If the angle of refraction is 25°, determine the angle of incidence.
26. Red light travels from air into liquid at an angle of incidence of 39.0° and an angle of refraction of 17.0°. Calculate the wavelength of the red light in the liquid if its wavelength in air is 750 nm.
27. Make a careful copy of Figure 11.6A on page 518, and mark the angles of incidence and refraction. At each end of the boundary, add a line perpendicular to the wavefront to make a right-angled triangle. Using these two triangles, mathematically derive Snell's law.
28. A diver is standing on the end of a diving board, looking down into 2.5 m of water. How far does the bottom of the pool appear to be from the water's surface?
29. The critical angle for a special glass in air is 44°. What is the critical angle if the glass is immersed in water?
30. When astronauts first landed on the Moon, they set up a circular array of retroreflectors on the lunar surface. Scientists on Earth were then able to shoot a laser beam at the array and receive a reflection of the original laser light. By measuring the time interval for the signal's round trip, scientists were able to measure the distance between Earth and the Moon with great accuracy.
    (a) If the time interval could be measured to the nearest 3×10^{-10} s, how accurate would the distance measurement be? (Hint: Remember that the signal makes a round trip.)
    (b) Suggest two reasons why a laser beam was used for the measurement.

Numerical Answers to Practice Problems

1. 1.54 **2.** 1.24×10^8 m/s **3.** 2.04×10^8 m/s **4.** 1.8×10^{-2} s **5.** It slows by 1.5 %. **6.** 1.31, ice **7.** 29.7° **8.** 51° **9.** 39.5° **10.** 31.0° **11.** 11 cm **12.** 0.84 m **13.** 18.1° **14. (a)** 56° **(b)** 41° **15.** 47.2° **16.** 58.9° **17.** 78.5° **18.** 2.6 m

CHAPTER 12 Lenses and Images



CHAPTER CONTENTS

At night, astronomers probe the universe using devices such as the 500 cm telescope at the Hale Observatory on Mount Palomar in California. Built in 1948, until 1976 it was the largest optical telescope ever built. It is able to photograph galaxies as far as five billion light years away. Its monolithic (one-piece) primary mirror is a massive 14 tonnes and is near the upper limit in size for monolithic mirrors. At the other end of the scale, medical and biological researchers use powerful optical microscopes that can magnify images up to 1000 times. Whether they are designed to observe macroscopic or microscopic objects, lenses are a basic element of almost all optical instruments.

In Chapter 10, you learned how mirrors reflect light to create real and virtual images. In this chapter, you will learn how lenses refract light to form images. You will investigate the construction of optical devices such as telescopes, microscopes and cameras. In preparation for an in-depth study of these instruments, you will learn how to apply ray diagrams and the thin lens equations to predict the properties of images formed by lenses.

INVESTIGATION 12-A

Comparing Kepler and Galileo Telescopes

TARGET SKILLS
- Hypothesizing
- Performing and recording
- Analyzing and interpreting

Simple telescopes use two lenses. One lens is called the eyepiece and the second lens is the objective lens. The Galileo telescope (developed by Galileo Galilei in 1608) uses a convex objective lens and a concave eyepiece. An astronomical or Kepler telescope (invented by Johannes Kepler in 1611) uses a convex objective lens and a convex eyepiece.

CAUTION Never look at the Sun through any optical instrument. You could permanently damage your eyes. Handle all lenses with care.

Problem

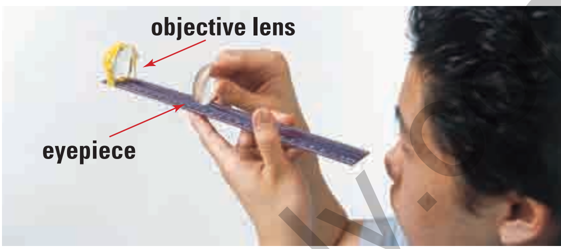
How do the different eyepieces used in Galileo and Kepler telescopes affect the images created?

Equipment and Materials

- 2 convex lenses ($f \approx 20$ cm and $f \approx 5$ cm)
- concave lens ($f \approx 5$ cm)
- 30 cm ruler
- masking tape or modelling clay

Procedure

1. To simulate an astronomical telescope, attach the convex lens with the longer focal length at one end of the ruler with masking tape or a bit of modelling clay. Be sure that the lens is adjusted so that when you look down along the ruler from the other end, you are looking directly through the lens. This will be your objective lens.
2. Hold the ruler with one hand so that you can look through the lens from the other end of the ruler, as shown in the photograph. Hold the second convex lens in your other hand and place it between your eye and the objective lens. This lens acts as your eye-piece. Adjust the eyepiece until you can see a clear image of an object on the other side of the room. Record the distance between the lenses at the point where the image is sharpest. Record the distance between the lenses. Record the appearance of the image.



3. Walk closer to the object and repeat the process. Record the distance between the lenses the point where the image is the sharpest. Record the appearance of the image.
4. To simulate a Galileo telescope, use the same objective lens but use the concave lens as the eyepiece. Carry out the same procedure with the concave eyepiece that you completed with the convex eyepiece. Be sure to stand the same two distances from the same object that you used to take data with the astronomical telescope. Record the same type of information.

Analyze and Conclude

Answer the following questions for each telescope.

1. How does the distance to the object affect the distance between the lenses?
2. Is the image upright or inverted?
3. Is the image larger or smaller than the object? Estimate the difference in size. (Hint: Look at the image through the telescope with one eye and directly at the object with the other eye.)
4. Is the image closer to or farther away from you than the object?

Apply and Extend

5. With objects at the same distance, for which telescope were the lenses closer together?
6. Which telescope gave the largest image for a given object at a given distance? Which was the easiest to adjust to get a clear image?

12.1 Convex Lenses and Images

SECTION EXPECTATIONS

- Explain, using light ray diagrams, the characteristics of images formed by convex lenses.
- Predict, using ray diagrams and the lens equation, the characteristics and positions of images formed by convex lenses.
- Conduct experiments to compare theoretical predictions and empirical evidence.

KEY TERMS

- convex lens
- principal axis
- vertical axis
- secondary axis
- principal focus or focal point
- focal length
- secondary focus or focal point
- biconvex or double convex
- concavo-convex or convex meniscus
- plano-convex
- lens-maker's equation
- thin-lens equations
- magnification equation
- mirror/lens equation

Compare the two images shown in Figure 12.1, which were formed by the same convex lens. Obviously, a convex lens can create an upright image or an inverted image much the same as a concave mirror does. Although it is not obvious from Figure 12.1, the upright image is virtual and the inverted image is real. To find out how the same lens can create such contrasting images, you need to examine convex lenses in detail.

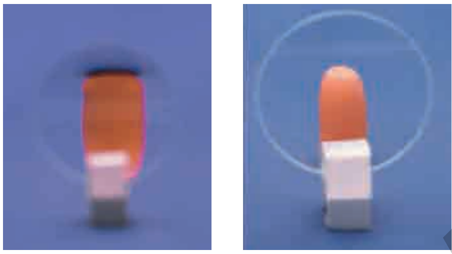


Figure 12.1 These two images were formed by the same convex lens.

Convex lenses are also called converging lenses because any set of parallel rays that strike the lens, will *converge* on a single point on the opposite side of the lens. The concepts you learned about refraction and prisms in Chapter 11 will help you understand how a convex lens functions. In Chapter 11, you traced the path of light through a prism and measured the angle of deviation. If the refracting angle at the apex of a prism increases, the angle of deviation increases as well. Thus, by using proper arrangements of prisms, beams of light can be deviated so that they pass through a common area as shown in Figure 12.2. Such arrangements simulate crude convex lenses. As you can see in Figure 12.2 B, by adding more segments of prisms, you can more closely simulate a convex lens. When the prisms are combined into one solid object with smooth, rounded surfaces, you have a convex lens that causes the light rays to converge to a distinct point, as shown in Figure 12.2 C.

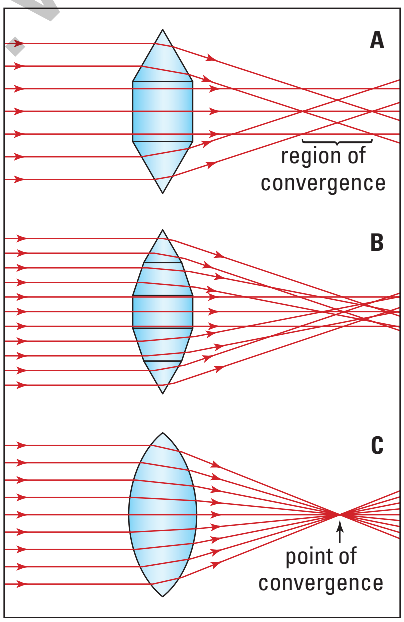


Figure 12.2 Sets of prisms can be used to create a "lens" that crudely focusses a set of parallel incident rays.

Properties of Convex Lenses

When you examine the simple diagram of a convex lens in Figure 12.3, you can see many similarities between mirrors and lenses. The term, convex, means curved outward for both mirrors and lenses. The surfaces of simple mirrors and lenses are spherical and described by radii of curvature. As well, any line passing through the geometric centre of the lens is called an "axis." The axis that passes through the centres of curvature of the lens is the **principal axis**. The upright axis that is perpendicular to the principal axis is the **vertical axis**. All other axes are called **secondary axes**.

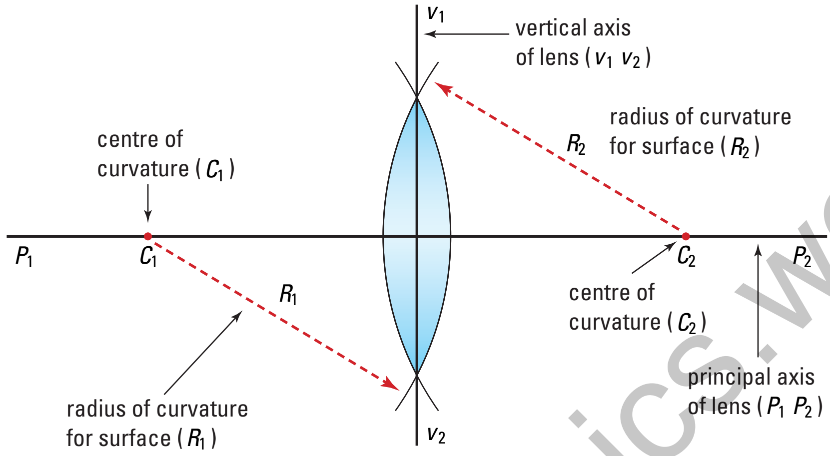


Figure 12.3 The simplest way to draw a representation of a convex lens is to draw two intersecting circular arcs.

The similarities between mirrors and lenses end with the method for determining the focal point and focal length. Mirrors have only one spherical surface and one radius of curvature and the focal length was half the length of the radius of curvature. Lenses, however, have two spherical surfaces and two radii of curvature. As well, light passes through a refracting medium. All of these properties contribute to the focal length. Nevertheless, the focal length is quite easily found experimentally. You have already seen that parallel rays that pass through a convex lens converge to a point on the other side of the lens. As shown in Figure 12.4, the point of convergence of the rays that enter parallel to the principal axis is called the **principal focus** or **focal point**. Since the rays of light can enter the lens from either side, there is a principal focus or focal point on the principal axis on each side of the lens. The distance from the vertical axis to either of the focal points is called the **focal length** of the lens.

Figure 12.4 A convex lens refracts rays that are parallel to its principal axis to converge on the focal point on the opposite side of the lens. Rays can enter the lens from either side; thus, each lens has two principal foci that lie on the principal axis, equidistant from the vertical axis of the lens.

TRY THIS...

Hold up each of the convex lenses from Investigation 12-A, one at a time, close to your eyes, and look through them at an object that is about 1 m away from you. Gradually move the lens farther from your eye until you can see a clear image of the object. Write a brief description of the image that you observed for each lens, including answers to the following questions.

- Is the image upright or inverted?
- Is the image larger or smaller than the object?
- How far from your eye do you have to hold the lens before you can see a clear (in focus) image?

Language Link

When lenses were first discovered, they were thought to resemble the seeds of the lentil plant, known in Latin as *lens culinaris*. Similarly, the word "lenticular" comes from the Latin word *lenticularis*, meaning "like a lentil". What is the exact meaning of lenticular?

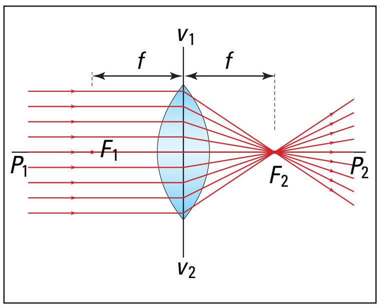
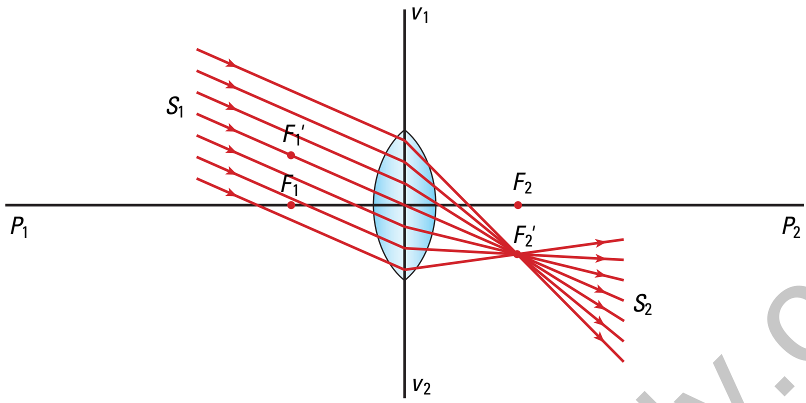


Figure 12.5 The line S_1-S_2 is a secondary axis with secondary focal points F_1' and F_2'. Rays parallel to a secondary axis are refracted to pass through the secondary focal points on that axis.



When rays enter a convex lens parallel to a secondary axis, they converge to a point called a **secondary focal point**, as shown in Figure 12.5. All secondary focal points are the same distance from the lens as the principal focal points.

Practical Lenses and the Lensmakers' Equation

ELECTRONIC LEARNING PARTNER

Go to your Physics 11 Electronic Learning Partner to enhance your learning about convex lenses.

If you examine any pair of eyeglasses, you will discover that both surfaces curve in the same direction. One side is convex but the other side is concave. How can you classify such lenses? Early in this section, you read that convex lenses were also called converging lenses. Any lens that is thicker in the centre than around the edges will cause parallel rays to converge and is therefore classified as a convex lens. Figure 12.6 shows three variations of convex lenses. The first lens, shaped like those you have been examining, is called a **biconvex** or **double-convex** lens. The second lens, similar to eyeglass lenses for people who are farsighted, is called a **concavo-convex** or **convex meniscus** lens. The last lens is similar to one that you might find in a projector. This lens, called a **plano-convex** lens, produces a concentrated light beam that makes a projected image very bright.

It is not difficult to experimentally determine the focal length of a lens, but how does a lensmaker know what curvatures are needed to make a lens of a desired focal length? This knowledge goes back to the time of Galileo and Newton. As the interest in telescopes increased, scientists derived an equation that related the radii of curvature and the index of refraction of the material to the focal length. When using the lensmakers' equation given in the box on the next page, it is necessary to know how to distinguish the difference between convex and concave sides of a lens. By convention, the radius of curvature of a convex surface is positive and of a concave surface is negative. If the lens is plano-convex, the flat side has "infinite radius" and $\frac{1}{R_2} = 0$.

The **lensmakers' equation** allows you to calculate the focal length of a lens from the index of refraction of the lens' material and the radii of curvature of the lens' two surfaces.

LENSMAKERS' EQUATION

The reciprocal of the focal length is the product of one less than the index of refraction and the sum of the reciprocals of the radii of curvature.

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Quantity	Symbol	SI unit
focal length	f	m (metre)
refractive index	n	no unit
first radius of curvature	R_1	m (metre)
second radius of curvature	R_2	m (metre)

Unit Analysis

(no unit)((length)$^{-1}$ + (length)$^{-1}$) = m^{-1} + m^{-1} = m^{-1}

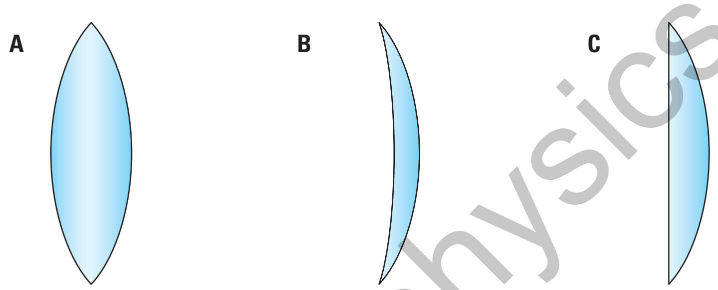


Figure 12.6 Three types of converging lenses: **(A)** double convex; **(B)** convex meniscus or concavo-convex; **(C)** plano-convex.

MODEL PROBLEM

The Focal Length of a Convex Lens

A block of crown glass (n = 1.50) is ground into a biconvex lens. One surface has a radius of curvature of 15.0 cm, while the opposing surface has a radius of curvature of 20.0 cm. What is the focal length of the lens?

Frame the Problem

- The medium is *crown glass* and the *index of refraction* is given.
- The two surfaces of the lens have different *radii* of *curvature*.
- The *lensmakers' equation* applies in this situation.

continued

continued from previous page

Identify the Goal

The focal length, f, of the lens

Variables and Constants

Involved in the problem		Known	Unknown
R_1	n	$n = 1.50$	f
R_2	f	$R_1 = 15.0$ cm	
		$R_2 = 20.0$ cm	

Strategy

Identify the equation that relates the known to the unknown variables.

Calculations

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Since the unknown variable is already isolated in the equation, replace the variables on the right-hand side of the equation with values and solve.

$$\frac{1}{f} = (1.50 - 1)\left(\frac{1}{15.0\text{ cm}} + \frac{1}{20.0\text{ cm}}\right)$$

$$\frac{1}{f} = (0.50)\left(\frac{4}{60\text{ cm}} + \frac{3}{60\text{ cm}}\right)$$

$$\frac{1}{f} = \left(\frac{1}{2}\right)\left(\frac{7}{60\text{ cm}}\right)$$

$$\frac{1}{f} = \frac{7}{120\text{ cm}}$$

The lens has a focal length of 17.1 cm.

$$f = 17.1\text{ cm}$$

Validate

The units of the focal length are centimetres. The focal length is positive, as expected for a convex lens.

PRACTICE PROBLEMS

1. What is the focal length of a biconvex lens made of quartz ($n = 1.46$) if the radii of curvature of the opposing surfaces are 12.0 cm and 15.0 cm?
2. You grind a biconvex lens with opposing surfaces whose radii of curvature are 8.00 cm each. You thought the material was a block of crown glass ($n = 1.50$) but it turns out that it was flint glass ($n = 1.65$). What is the difference in the focal lengths of the lens you thought you were making and the one you did make?
3. A lens is ground using the same radius of curvature ($R = +13.0$ cm) for each of the opposing surfaces. What index of refraction would result in a lens with a focal length of 10.0 cm? Use the table of indices of refraction in Chapter 11 to find out what this material is (see Table 11.1, page 507).
4. Find the focal length of a plano-convex lens made of fused quartz ($n = 1.46$) if the radius of curvature of the spherical surface is 25.0 cm.

Ray Diagrams for Convex Lenses

You can now determine the focal length of a convex lens experimentally and through calculations. The next, and most important, step is using the focal length to predict the nature of an image formed by the lens. Consider any point on an object that is distant from the lens in relation to the focal point. (In optics, this usually means any distance that is greater than two focal lengths from the vertical axis of the lens.) A convex lens causes all of the rays diverging from that point to converge on a single point on the opposite side of the lens (see Figure 12.7). Most importantly, after the rays converge on that point, they continue on, but now the rays are diverging. All of the rays diverging from the object that were incident on the lens are now diverging from a new point on the opposite side of the lens. In order to see the image you must put your eye *in the path* of these diverging rays. The image you see is projected in front of the lens at the point of divergence. If you place your eye in front of the image in the cone where the rays are still converging, you cannot see the image. You can see images only if the rays are diverging. If you always keep this in mind, it will make locating and understanding the properties of images a straightforward task.

Obviously, you do not want to have to draw all of the rays shown in Figure 12.7 when you draw a ray diagram. Since all rays from the object are refracted to one point, any two of the rays are sufficient to locate that point. There are three rays that are very easy to construct when drawing a ray diagram. These rays are described and illustrated in Table 12.1 on the next page. Although two rays are sufficient for locating an image, it is always a good idea to use the third ray as a check to ensure that you drew the first two correctly.

Before starting to draw ray diagrams, there is one important feature to consider. As you know, light rays refract at *both* surfaces of the lens. For ease in drawing, however, physicists approximate the two changes in direction of the ray by one change and place it on the vertical axis. As a result, ray diagrams are *approximations* of the path of the light. Nevertheless, the diagrams are very accurate when the lens is thin relative to the focal length and distances between the lens and the object and image.

TRY THIS...

Find the focal length of a convex lens by holding it in sunlight. Hold the lens near a surface and slowly move it away until the sunlight focuses on a small point. Measure the focal length of the lens.

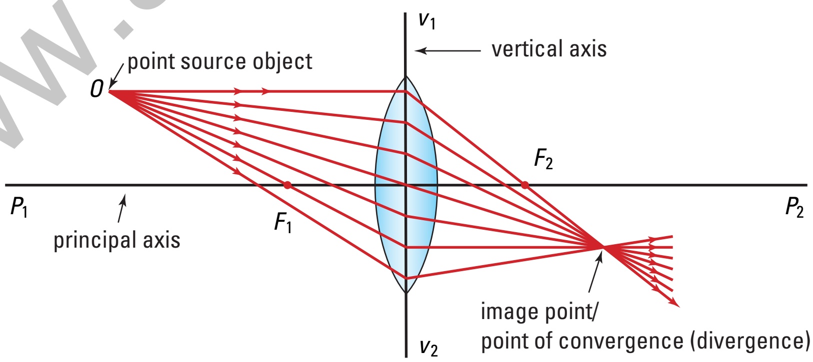
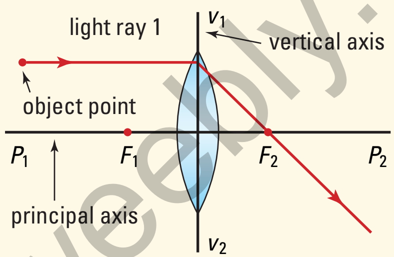
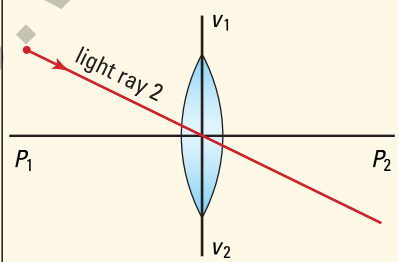
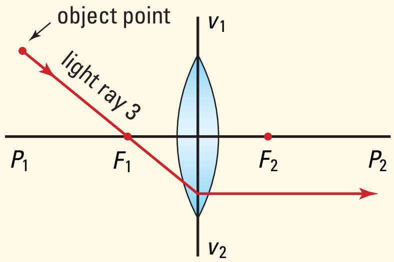
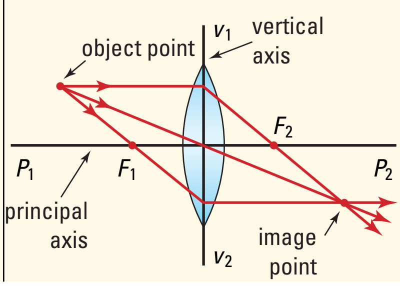


Figure 12.7 All rays diverging from a single point on one side of the lens are refracted so that they converge on, then diverge from, a point on the opposite side of the lens. The image can only be seen by using the diverging rays.

Table 12.1 Light Ray Diagrams for Convex Lenses

Description	Comments	Illustration
Reference line Draw the principal axis through the centre of the lens. Mark and label the focal points on this axis on both sides of the lens. Place the base of the object on this reference line.	Since this line is perpendicular to the lens, any ray travelling along the line will pass through the lens without changing direction. Therefore, the base of the image will lie on this line.	
Light ray #1 Draw a ray from the top of the object to the lens parallel to the principal axis. Draw the refracted ray through the focal point on the far side of the lens.	By the definition of the focal point, any ray entering the lens parallel to the principal axis will converge on the focal point on the far side of the lens.	
Light ray #2 Draw a ray from the top of the object through the centre of the lens and extend it straight through to the far side of the lens.	At the very centre of the lens, the sides are parallel. Recall studying lateral displacement in Chapter 11. When light passes through a refracting medium with parallel sides, the emergent ray is parallel to the incident ray but displaced to the side. In the case of thin lenses, the displacement is so small that it can be ignored.	
Light ray #3 Draw a ray from the top of the object through the focal point and on to the lens. Draw the refracted ray on the far side of the lens parallel to the principal axis.	Two principles determine the path of this ray. As in the case of ray #1, any ray entering the lens parallel to the principal axis will converge on the focal point on the other side of the lens. Then apply the principle of reversibility of light. The result is that any ray entering a lens from a focal point will leave parallel to the principal axis.	
Identify the Image If the refracted rays are converging, extend them until they all cross. If the refracted rays are diverging, extend them backward with dashed lines until they intersect.	The point at which the rays meet is the position of the top of the image. If the refracted rays converge, the image is real. If the rays must be extended backward to converge, the image is virtual.	

Each point on the object is a source of a set of diverging rays that carries the image of that point. In the ray diagrams in this text, the object is always drawn with one end (its bottom) on the principal axis. You only need to locate the top of the image because the base of the image will lie on the principal axis.

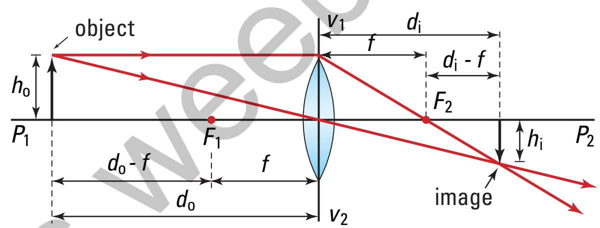
Images Formed by Convex Lenses

The distance between the object and the convex lens determines the nature of the image formed by the lens. The series of ray diagrams below covers all of the general types of images that convex lenses can produce.

Case 1: $d_o > 2f$

When the object distance is greater than twice the focal length,

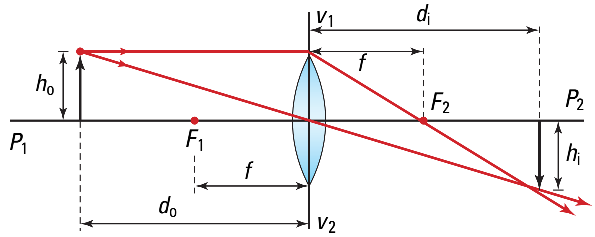
- the image lies on the side of the lens opposite the object,
- the image distance is less than twice the focal length,
- the image is real,
- the image is inverted, and
- the image is smaller than the object.



Case 2: $d_o = 2f$

When the object distance is equal to twice the focal length,

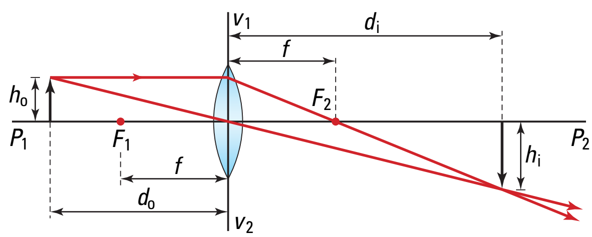
- the image lies on the side of the lens opposite the object,
- the image distance is equal to twice the focal length,
- the image is real,
- the image is inverted, and
- the image is the same size as the object.



Case 3: $2f < d_o < f$

When the object distance is less than twice the focal length but greater than the focal length,

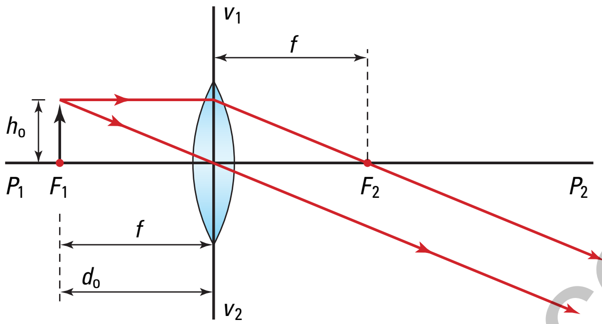
- the image lies on the side of the lens opposite the object,
- the image distance is greater than twice the focal length,
- the image is real,
- the image is inverted, and
- the image is larger than the object.



Case 4: $d_o = f$

When the object distance is equal to the focal length,

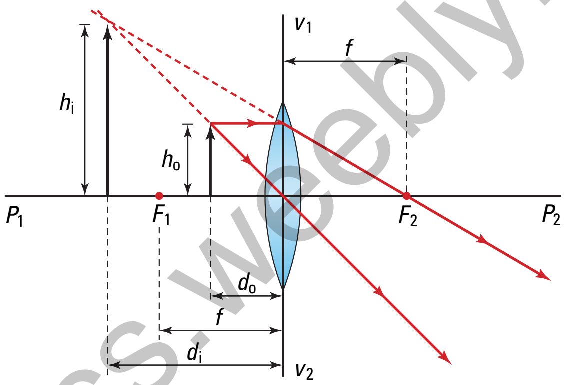
- no image exists because the refracted rays are parallel. You could say that the image lies at infinity.



Case 5: $f > d_o > 0$

When the object distance is less than the focal length but greater than zero,

- the image lies on the same side of the lens as the object,
- the image distance is greater than the object distance,
- the image is virtual,
- the image is upright, and
- the image is larger than the object.



Think It Through

- For cases 1 through 3 above, explain why the image is real.
- Use the concept of the "point of divergence" to explain why no image exists in case 4.
- For case 5, explain why the image is virtual.
- Use the rules of geometry to prove that the size of the object and image in case 2 are the same.

If you follow the sequence presented in the previous cases, it is very easy to see the pattern that exists. Start with the object at a very great distance from the lens. A very tiny, real, inverted image is formed just outside the focal point on the *opposite* side of the lens. As the object moves closer and closer to the focal point of the lens, the real image grows and moves away from the focal point (and the lens).

The point of equilibrium occurs when the object reaches a point that is two focal lengths from the lens. At that instant, the real image is also two focal lengths from the lens and the same size as the object (Case 2). As the object moves toward the focal point, the real image moves farther away from the lens and continues to grow larger (Case 3). Just before the object reaches the focal point, the real image would almost be at an infinite distance and infinitely large. When the object reaches the focal point, there is no image (Case 4). The instant the object moves inside the focal point, a virtual, upright image appears on the *same* side of the lens as the object. The image is larger and farther away than the object (Case 5). As the object moves closer to the lens, the virtual image moves toward the lens as well.

No matter whether a real or virtual image is formed, if the image is farther from the lens than the object, then the image is larger than the object, and vice versa. The relationship between an object's position and the nature of the image is summarized in Table 12.2.

Table 12.2 Relationship between an Object's Position and the Nature of the Image

Position of object (d_o)	Nature of image
$d_o > 2f$	The image is real, inverted, smaller than the object, and closer to the lens than the object.
$d_o = 2f$	The image is real, inverted, and equal in size and distance from the lens as the object.
$f < d_o < 2f$	The image is real, inverted, larger than the object, and farther from the lens than the object.
$d_o = f$	There is no image formed. This is the boundary between the formation of real and virtual images.
$0 < d_o < f$	The image is virtual, upright, larger than the object, and farther away from the lens than the object.

Web Link

www.school.mcgrawhill.ca/resources/

For more animated examples of ray diagrams for mirrors and lenses, go to the above Internet site. Follow the links for **Science Resources** and **Physics 11** to find out where to go next.

Thin-Lens Equations

You can use ray diagrams to develop equations for calculating the relative sizes and positions of the object and images. These are called the **thin-lens equations**, since they are very accurate only if the thickness of the lens is small compared to its diameter.

You probably recall from Chapter 10, that sign conventions for the symbols on the ray diagrams were critical for the development of the mirror equation. The same is true for the lens equations. Table 12.3 defines all of the sign conventions needed for the thin lens equations.

Table 12.3 Sign Conventions for Lenses

Quantity	Sign property
h_o	The height of a real object is always positive .
d_o	The distance from the vertical axis to the object is positive for real objects.
f	The focal length is positive for convex lenses and negative for concave lenses.
h_i	The height of the image is *positive* if the image is *upright* relative to the object; negative if the image is *inverted.*
d_i	The distance from the vertical axis to the image is *positive* if the image is on the *opposite side of the lens* from the object and *negative* if the image is on the *same side* of *the lens* as the object.

Magnification for lenses is defined in the same way as it is for mirrors.

$$M = \frac{h_i}{h_o}$$

To find a second way to calculate magnification, examine Figure 12.8. The shaded triangles to the left and right of the lens are similar, since the corresponding angles for the triangles are equal. Thus, the ratios of corresponding sides must be equal.

Hence, $$\frac{-h_i}{h_o} = \frac{d_i}{d_o}$$

Therefore, $$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

This is the **magnification equation.** The distance from the vertical axis to the object, d_o, the height of the object, h_o, and the distance from the vertical axis to the image, d_i, are all positive. The image is inverted, therefore, its height, h_i, should be represented by a negative number.

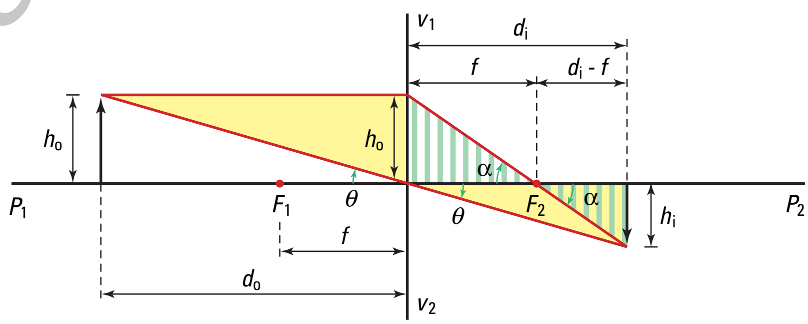


Figure 12.8 The thin-lens equations are approximations based on ray diagrams.

To develop the other thin lens equation, again examine Figure 12.8. The right triangles with striped shading to the right of the lens have equal corresponding angles. By geometry, they are similar and the ratios of corresponding sides for these triangles must be equal. Thus, we can write $\frac{-h_i}{h_o} = \frac{d_i - f}{f}$.

- Compare the two equations that you obtained from similar triangles in Figure 12.8.

$$\frac{-h_i}{h_o} = \frac{d_i}{d_o} \text{ and } \frac{-h_i}{h_o} = \frac{d_i - f}{f}$$

- Since the left sides of the equations are the same, you can set the right sides equal to each other.

$$\frac{d_i}{d_o} = \frac{d_i - f}{f}$$

- Multiply both sides by d_of.

$$\left(\frac{d_i}{\cancel{d_o}}\right)\cancel{d_o}f = \left(\frac{d_i - f}{\cancel{f}}\right)d_o\cancel{f}$$

- Simplify.

$$d_o\,(d_i - f) = d_if$$

- Multiply through by d_o.

$$d_od_i - d_of = d_if$$

- Rearrange to place all terms with f on the right side of the equation.

$$d_od_i = d_of + d_if$$

- Factor out f.

$$d_od_i = (d_o + d_i)f$$

- Divide both sides of the equation by d_id_of.

$$\frac{\cancel{d_o}\cancel{d_i}}{\cancel{d_o}\cancel{d_i}f} = \frac{(d_o + d_i)f}{d_od_i\cancel{f}}$$

- Simplify and write the right side as the sum of two fractions.

$$\frac{1}{f} = \frac{\cancel{d_o}}{\cancel{d_o}d_i} + \frac{\cancel{d_i}}{d_o\cancel{d_i}}$$

- Simplify.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

You will probably recognize this equation as the mirror equation. Because the lens and mirror equations are the same, the equation is often called the **mirror/lens equation**.

THIN-LENS EQUATIONS

Magnification equation: The ratio of the image and object heights is the negative of the ratio of the image and object distances.

$$M = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

Mirror/lens equation: The reciprocal of the focal length is the sum of the reciprocals of the image and object distances.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Quantity	Symbol	SI unit
image height	h_i	m (metre)
object height	h_o	m (metre)
image distance	d_i	m (metre)
object distance	d_o	m (metre)
focal length	f	m (metre)

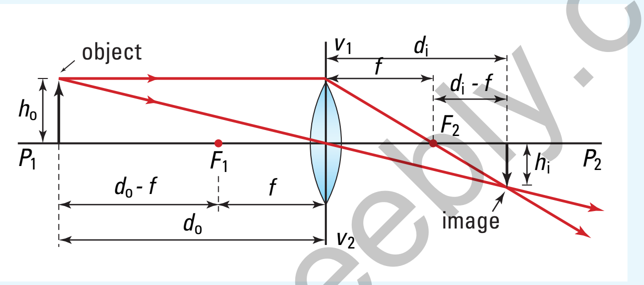
MODEL PROBLEM

Images Formed by a Convex Lens

An object 8.5 cm tall is placed 28 cm from a convex lens that has a focal length of 12 cm. Find the size and location of the image. Describe the image.

Frame the Problem

- According to Table 12.2, we expect the image to be real, inverted, smaller than the object, and closer to the lens than the object.
- Visualize the problem by sketching the light ray diagram.
- The mirror/lens equation and the magnification equation can be applied.



Identify the Goals

(a) The size, h_i, and location, d_i, of the image

(b) A description of the properties of the image: real/virtual, upright/inverted, larger/smaller than the object.

Variables and Constants

Involved in the problem	Known	Unknown
d_o	$d_o = 28$ cm	d_i
d_i	$h_o = 8.5$ cm	h_i
h_o	$f = 12$ cm	
h_i		
f		

Strategy

Use the mirror/lens equation, to find the image distance.

Rearrange.

Calculations

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

Strategy

Substitute the known values into the equation.

Calculations

$$\frac{1}{d_i} = \frac{1}{12\text{ cm}} - \frac{1}{28\text{ cm}}$$

$$\frac{1}{d_i} = \frac{7}{84\text{ cm}} - \frac{4}{84\text{ cm}}$$

Solve.

$$\frac{1}{d_i} = \frac{3}{84\text{ cm}}$$

$$\frac{1}{d_i} = \frac{1}{21\text{ cm}}$$

$$d_i = 21\text{ cm}$$

Now you can use d_i to find h_i.

Begin with the magnification equation.

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Rearrange.

$$h_i = -\frac{h_o d_i}{d_o}$$

Substitute known values.

$$= -\frac{(8.5\text{ cm})(21\text{ cm})}{28\text{ cm}}$$

Solve.

$$= -6.375\text{ cm}$$

(a) The image is 6.4 cm tall and 21 cm from the lens on the opposite side from the object.

(b) The image is real, inverted, and smaller than the object.

Validate

The units are in centimetres.

The height of the image is a negative value, because the image is inverted.

The height of the image is less than the object, and the image is closer to the lens than the object.

PRACTICE PROBLEMS

5. A convex lens has a focal length of 12.0 cm. An object 6.30 cm tall is placed 54.0 cm from the vertical axis of the lens. Find the position and size of the image. Describe the image.

6. An object 7.50 cm tall is placed 1.50 m from a convex lens that has a focal length of 90.0 cm. Find the position and size of the image. Describe the image.

7. Calculate the size and position of the image if an object that is 4.20 cm tall is placed 84.0 cm from a convex lens that has a focal length of 120.0 cm. Describe the image.

8. A real image 96.0 cm tall is formed 144 cm from a convex lens. If the object is 36.0 cm from the lens,

(a) what is the focal length of the lens?

(b) what is the size of the object?

9. A virtual image 35.0 cm tall is formed 49.0 cm from a convex lens. If the object is 25.0 cm tall, find (a) the position of the object and (b) the focal length of the lens.

10. Calculate the position and size of the image if an object is placed at the focal point of a convex lens with a focal length of 20 cm.

INVESTIGATION 12-B

Converging Lens Images

TARGET SKILLS

- Performing and recording
- Modelling concepts
- Analyzing and interpreting
- Communicating results

Problem

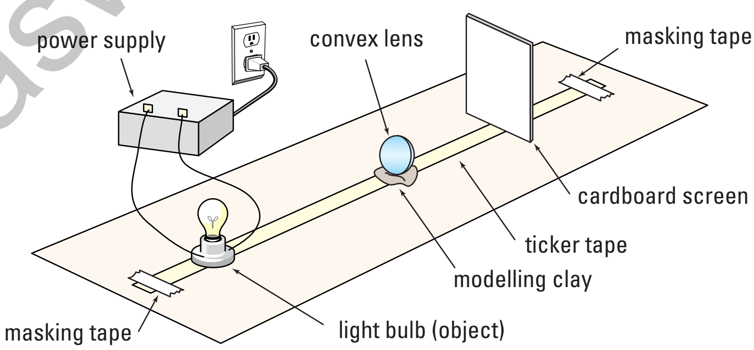
Confirm the thin-lens equations (the mirror/lens and magnifications equations) for convex lenses. Use a screen to project real images (see page 563).

Equipment and Materials

- 2 convex lenses (each with a different focal length)
- ticker tape
- modelling clay
- small light source
- power supply
- square of white cardboard (approximately 10 cm by 10 cm)
- pencil or pen
- vernier callipers

Procedure

1. Attach a long piece of ticker tape to the surface of the lab table with masking tape.
2. Near the middle of the tape, draw a line across the tape, to represent the location of the vertical axis of the lens. Accurately place the vertical axis of the lens, supported by a small piece of modelling clay, over the mark. Be sure to position the lens so that its principal axis is nearly horizontal.
3. Draw a line across the ticker tape near one end of the ticker tape. Place the light source (object) as accurately as possible so that its filament is above the line. (**Note:** If you place the bulb so that the filament is across the tape, rather than parallel, you will get a sharper image to project on the screen. See the diagram of the apparatus set-up.)
4. On the opposite side of the lens, move the screen slowly away from the lens until a clear, sharp image is formed on the screen. (**Note:** Be sure to keep the screen perpendicular to the principal axis of the lens.)
5. Place a line across the ticker tape to mark the position of the image. (**Note:** Place a trial number (1) beside the lines for the object and the image so that later on you can be sure which value for d_o corresponds with which value of d_i.)
6. Draw a line about 10 cm closer to the lens from the position of the object; place the light source (object) on that line and repeat steps 4 and 5.



7. Find images for the position of the object as it is moved closer and closer to the lens until images can no longer be located. Remember to mark lines for the corresponding object and image positions with a trial number.
8. Move the object and the lens off the ticker tape. Measure the distance from the lens to the positions of the object and image for each trial.
9. Record your data in a table as shown below.

Trial	Object distance: d_o (cm)	Image distance: d_i (cm)	Focal length: f(cm)	Percent deviation

10. When the object is at a very large distance from the lens, the rays from the object to the lens are very nearly parallel. Thus, the image is very close to the focal point. Move the object as far from the lens as it is possible for you to do and still locate the image. Mark the image position on the tape. This value of d_o should be very close to the focal length.
11. Change the ticker tape and repeat the experiment for the other convex lens.

Analyze and Conclude

Complete the following for each set of data.

1. For each trial, calculate the focal length from d_o and d_i.
2. Calculate the average focal length for the trials.
3. Compare the measure of the focal length you found in step 10 to the average previously calculated.
4. For each trial, calculate the percent deviation for the focal length from the average focal length.
5. Calculate the average percent deviation for the trials.
6. What are the possible sources of deviation? If the percent deviation for some trials was large, give reasons why this might have occurred.
7. Do the results of your investigation support or reject the mirror/lens equation?
8. Of the lenses used, which one gave the smallest average percent deviation? Suggest reasons why this might occur.

Apply and Extend

9. Do a set of trials as above, but measure the size of the object and image for the trials. The length of the filament for the light bulb can be reasonably accurately measured if you place a set of vernier callipers in front of the bulb and close them until the gap appears to be the same size as the filament.
10. Record your data in a table, as shown.

Trial	h_o (cm)	d_o (cm)	h_i (cm)	d_i (cm)	h_i / h_o	d_i / d_o

11. Make a graph of h_i/h_o versus d_i/d_o.
12. Is the graph a straight line? What is its slope?
13. Which trials were the most prone to errors in measurements?

Projecting Images

It would not be very convenient if the only way you could view images formed at the point of divergence was to put your eye in the path of the diverging rays, as mentioned previously. However, if you place a screen at the exact point at which the rays converge, rather than continuing to travel away from the lens, the rays will be diffusely reflected back into the room (see Figure 12.9). In this way, everyone in the room will be able to see the image "on the screen." Clearly this phenomenon is possible only because real images are the result of "real" points of divergence. Thus, virtual images cannot be projected onto screens since imaginary points of divergence produce them. In the laboratory, real images are almost always studied by projecting them onto a screen. Notice that, if the screen is not exactly at the point of divergence, a small circle of light will exist rather than a point. In this case, the image is "out of focus."

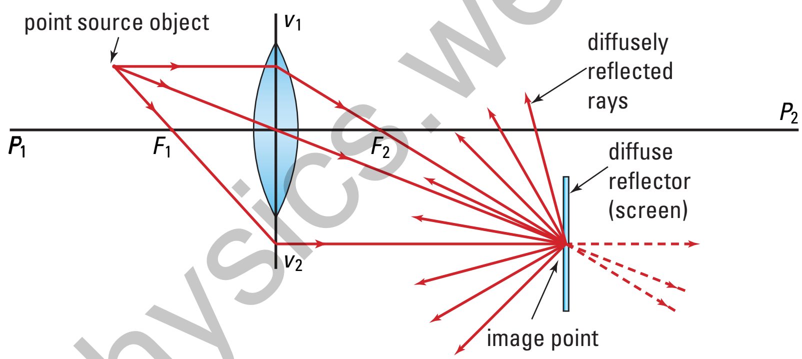


Figure 12.9 Rays that converge on a point on a screen will then, by diffuse reflection, diverge to all points in front of the screen. A sharply focussed image can be seen on the screen.

12.1 Section Review

1. K/U What is a convex lens?
2. K/U Draw an example of the following lenses:
   a) double convex
   b) convex meniscus
   c) plano-convex
3. C Draw and label a convex lens. Include the following in your diagram: principal axis, vertical axis, focal point, focal length, secondary axis, secondary focal point.
4. K/U How can you determine the focal length of a convex lens?
5. C Describe how it is possible for you to see an object.
6. MC Draw ray diagrams and explain similarities between convex lenses and concave mirrors in the creation of (a) real images and (b) virtual images.
7. C Explain how a convex lens forms an image.
8. C Describe what is meant by the term "out of focus."
9. K/U Why does a convex lens invert an image?

12.2 Concave Lenses

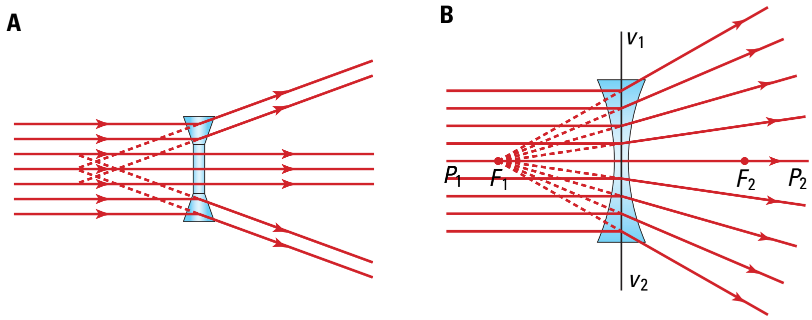


Figure 12.10 **(A)** Prisms can be combined to create a crude concave lens. **(B)** The focal length of concave lenses is negative.

SECTION EXPECTATIONS

- Describe the characteristics of images formed by concave lenses using light ray diagrams.
- Describe the effects of diverging lenses on light.
- Investigate the characteristics of images formed by concave lenses.

KEY TERMS

- concave lens
- convexo-concave or concave meniscus lens
- plano-concave lens

The most common reason for wearing eyeglasses is near-sightedness. This condition is easily corrected with concave lenses. Any lens that causes parallel rays to diverge after passing through the lens, can be classified as a **concave lens**. You can simulate a crude concave lens with prisms in much the same way that you can simulate a convex lens. As you can see in Figure 12.10, rays entering the lens parallel to the principal axis, exit as though they were diverging from a single point on the opposite side of the lens. This point from which the rays *appear* to be coming is the focal point of the concave lens. Because the focal point for the diverging rays is on the side of the lens *from which the light is coming* and because it could be considered as a *virtual* focal point, the focal length must be negative.

Just as convex lenses have three basic forms, so do concave lenses. Figure 12.12 illustrates the shape of the double concave, **convexo-concave or concave meniscus**, and **plano-concave lenses**. Being thinner at the middle, they all act as diverging lenses.

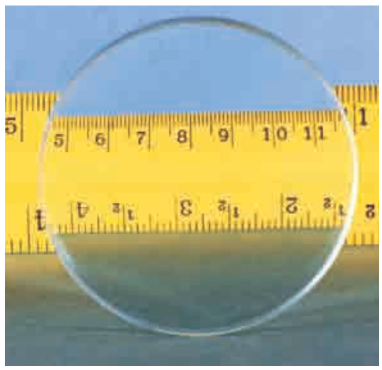


Figure 12.11 A concave lens.

Images from Concave Lenses

The ray diagrams used to locate images are drawn much like those for convex lenses — with one major difference. Rays parallel to the principal axis of a convex lens are refracted to pass through the focal point on the opposite side of the lens. For a concave lens, the rays parallel to the principal axis are refracted so that they appear to be diverging from the focal point on the side of the lens where they began.

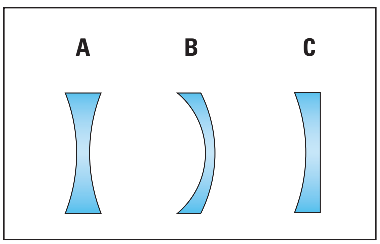
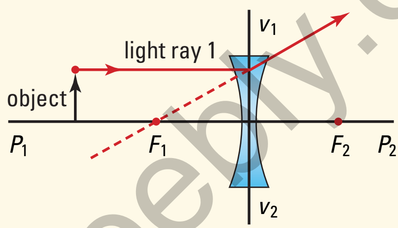
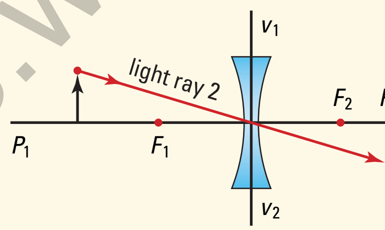
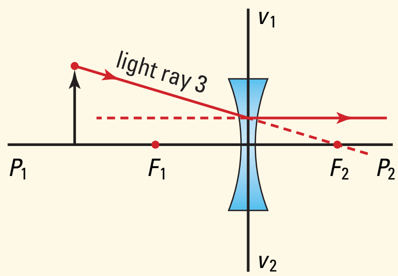
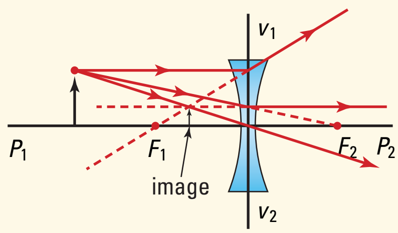


Figure 12.12 Diverging lenses come in three shapes: **(A)** double concave, **(B)** concave meniscus, and **(C)** plano-concave.

Table 12.4 Light Ray Diagrams for Concave Lenses

Description	Comments	Illustration
Reference line Draw the principal axis through the centre of the lens. Mark and label the focal points on this axis on both sides of the lens. Place the base of the object on this reference line.	Since this line is perpendicular to the lens, any ray travelling along the line will pass through the lens without changing direction. Therefore, the base of the image will lie on this line.	
Light ray #1 Draw a ray from the top of the object to the lens parallel to the principal axis. Draw the refracted ray as though it were coming from the focal point on the same side of the lens as the object.	By the definition of the focal point, any ray entering the lens parallel to the principal axis will diverge as though it were coming from the focal point on the same side of the lens as the object.	
Light ray #2 Draw a ray from the top of the object through the centre of the lens and extend it straight through to the far side of the lens.	At the very centre of the lens, the sides are parallel. When light passes through a refracting medium with parallel sides, the emergent ray is parallel to the incident ray but displaced to the side. In the case of thin lenses, the displacement is so small that it can be ignored.	
Light ray #3 Draw a ray from the top of the object directly toward the focal point on the opposite side of the lens. Draw the refracted ray on the far side of the lens parallel to the principal axis.	As in the case of ray #1, any ray entering the lens parallel to the principal axis will diverge as though it were coming from the focal point on the object side of the lens. Then apply the principle of reversibility of light. The result is that any ray entering a lens toward the opposite focal point will leave parallel to the principal axis.	
Identify the Image Refracted rays always diverge. Therefore, extend the rays backward with dashed lines until they intersect.	The point at which the extended rays meet is the position of the top of the image. The image is always virtual.	

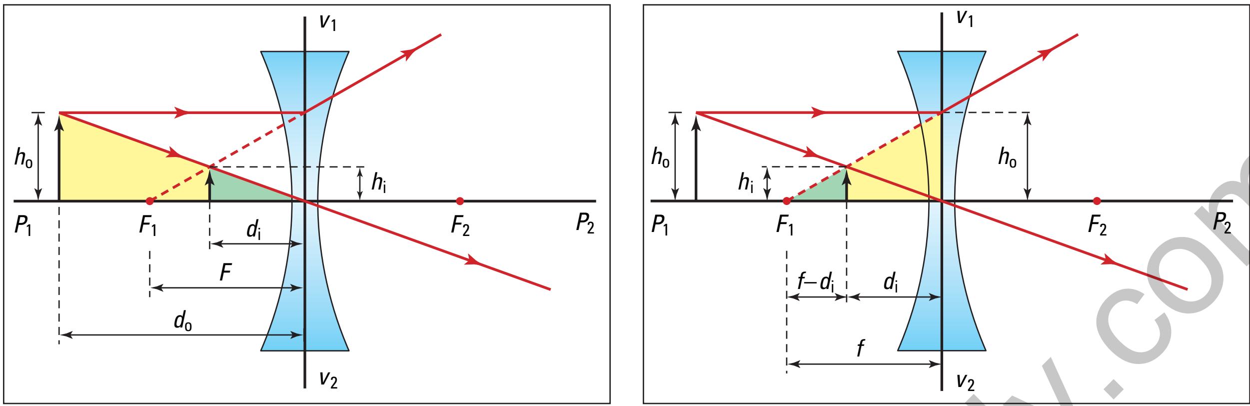


Figure 12.13 The thin-lens equations can be shown to apply to concave lenses by using the property of similar triangles.

You can use Figure 12.13 to prove that the thin-lens equations developed for convex lenses also apply to concave lenses. The triangles in Figure 12.13 (A) are similar. Comparing the ratios of their corresponding sides and applying the sign convensions results in the magnification equation, $\frac{h_i}{h_o} = \frac{-d_i}{d_o}$. Using the shaded triangles in Figure 12.13 (B) you can show that

$$\frac{h_i}{h_o} = \frac{-d_i - (-f)}{f} = \frac{-d_i + (f)}{f} = -\frac{(d_i - f)}{f}$$

Since these are exactly the same relationships you used in your derivation of the lens equation for convex lenses, they will reduce to the same lens equation.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Again, these equations are valid only if the lens thickness is small compared with its diameter.

QUICK LAB

Concave Lens Images

TARGET SKILLS

- **Modelling concepts**
- **Communicating results**

Draw a series of ray diagrams to examine the difference in the image that results when the object moves from a point about 2.5 f from the lens to a point very close to the lens. (You could use the same set of object positions that were used in Cases 1 through 5 in Section 12.1.)

Analyze and Conclude

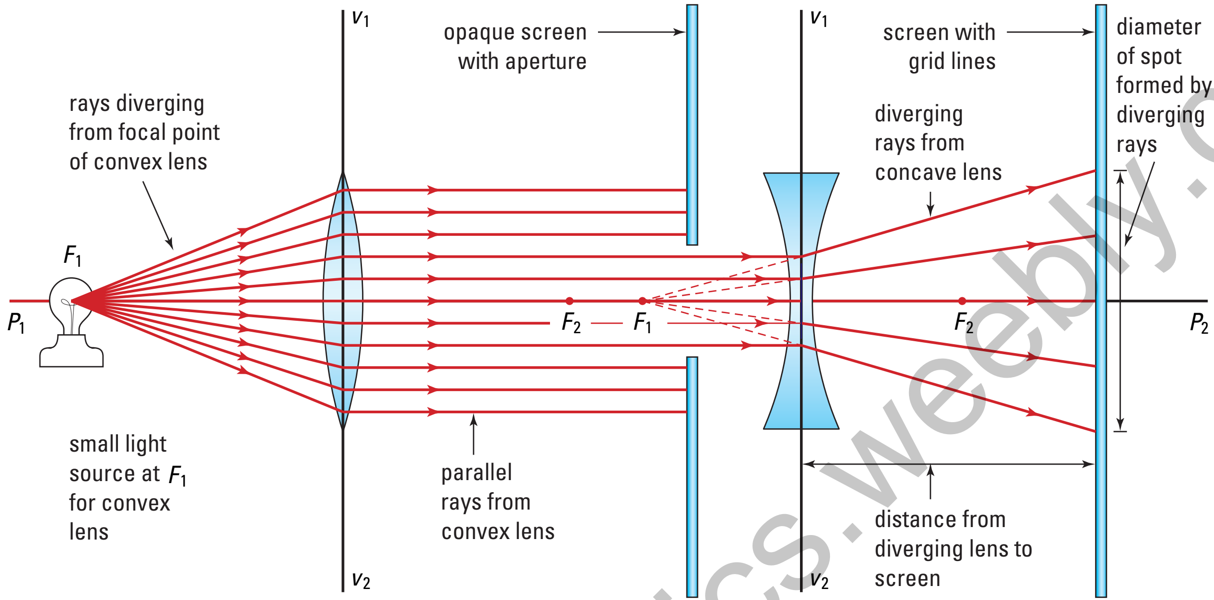
1. Are there any object positions for which a concave lens forms a real image? Explain.
2. What is the greatest distance from the lens that the image can appear?
3. Explain why the ray diagrams predict that the largest image that can be formed is the same size as the object.
4. Write a summary of what happens to the image as the object moves closer and closer to the lens.

INVESTIGATION 12-C

Focal Length of A Concave Lens

TARGET SKILLS

- Predicting
- Performing and recording
- Analyzing and interpreting



Rays parallel to the principal axis of a concave lens are refracted so that they diverge from the focal point of the lens. By measuring the diameter of a set of spots formed by the diverging beam at different distances from the lens, you should be able to predict the point from which they are diverging. The source of the beam of parallel rays is a light source placed at the focal point of a convex lens, as shown in the diagram.

Problem

What is the focal length of a concave lens?

Prediction

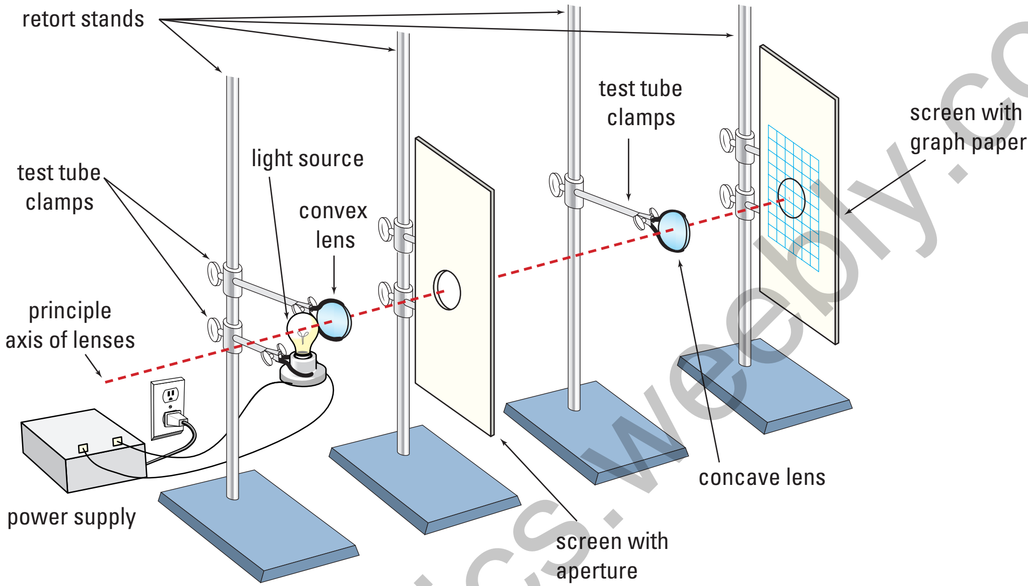
As you are making the measurements, make a prediction of what the focal length of the lens will be.

Equipment

- convex lens (with a known focal length, used to create a parallel light beam)
- concave lens (with unknown focal length)
- 2 retort stands and test tube clamps to support the lenses
- cardboard screen (Hint: Covering the screen with a piece of graph paper makes taking measurements much easier.)
- cardboard screen with a circular hole about 1 cm in diameter
- retort stands to support the screens
- small electric light source
- power supply
- box or stand to support the lamp
- masking tape
- metre stick

Procedure

1. Set up the lamp at the focal point of the convex lens so that the beam is refracted parallel to the principal axis. A spot about the size of the lens should be projected onto the screen, as shown in the diagram below. Place the cardboard screen with the hole in it into the path of the beam to limit the size of the beam. Care must be taken to be sure that the axis of the convex lens and the concave lens are collinear.



2. Place the concave lens in the path of the beam, perpendicular to its path, and project the spot onto the graph paper screen.
3. Move the screen to a distance of 10 cm from the lens and measure the diameter of the spot on the screen.
4. Record your data in a table similar to the one shown below.

Trial	Distance to screen (cm)	Diameter of the light spot (cm)

5. Move the screen 10 cm farther away from the lens and repeat your readings.
6. Repeat steps 4 and 5 until you have completed at least five trials.

Analyze and Conclude

1. Graph the results with the diameter as the dependent variable and the distance from the lens as the independent variable.
2. Extend your graph back to predict the position at which the diameter would be zero. This is the predicted point of divergence and thus the focal point of the lens. (**Note:** If the graph is a straight line, you could find its equation, then calculate the value for distance from the screen that would give a diameter of zero.)
3. What is the focal length of the concave lens?

Apply and Extend

Either a light ray box, or, if your instructor will permit it, a laser can be used as the source of the parallel beam rather than the lens aperture arrangement.

CAUTION You must be very careful when handling a laser. Never let the laser shine into your own or a classmate's eyes or look at reflected laser light; severe retinal damage can occur.

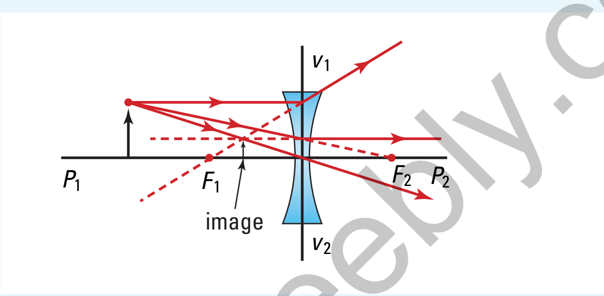
MODEL PROBLEM

Images in Concave Lenses

An object that is 48.0 cm tall is located 60.0 cm from a concave lens with a focal length of 40.0 cm. Calculate the size and position of the image. Describe the image.

Frame the Problem

- Because this is a concave lens, we expect the image to appear on the same side of the lens as the object, and to be upright.
- Visualize the problem by drawing the light ray diagram.
- Use mirror/lens and magnification equations.



Identify the Goals

(a) The size, h_i, and the position, d_i, of the image

(b) A description of the image

Variables and Constants

Involved in the problem		Known	Unknown
d_o	h_o	$d_o = 600$ cm	d_i
d_i	h_i	$h_o = 48.0$ cm	h_i
	f	$f = -40.0$ cm	

Strategy / Calculations

Use the mirror/lens equation to find d_i.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Rearrange.

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

Substitute known values.

$$\frac{1}{d_i} = \frac{1}{-40.0\text{ cm}} - \frac{1}{60.0\text{ cm}}$$

$$\frac{1}{d_i} = -\frac{3}{120\text{ cm}} - \frac{2}{120\text{ cm}}$$

$$\frac{1}{d_i} = -\frac{5}{120\text{ cm}}$$

Solve.

$$\frac{1}{d_i} = -\frac{1}{24.0\text{ cm}}$$

$$d_i = -24.0\text{ cm}$$

Now you can use d_i to find h_i.
Begin with the magnification equation.

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Rearrange. $h_i = -\frac{h_o \cdot d_i}{d_o}$

Substitute known values. $= -\frac{(48.0 \text{ cm})(-24.0 \text{ cm})}{60.0 \text{ cm}}$

Solve. $= 19.2 \text{ cm}$

(a) The image is 19.2 cm tall and 24.0 cm from the lens on the same side as the object.

(b) The image is virtual, upright, and smaller than the object.

Validate

The units are in centimetres.

The negative value for the position of the image is expected since the image from a concave lens is always found on the same side of the lens as the object.

You expect that the image height will be positive since a concave lens produces only virtual, upright images.

PRACTICE PROBLEMS

11. Calculate the size and position of the image formed when an object 84.0 cm tall is placed 120 cm from a concave lens with a focal length of 80.0 cm. Describe the image.

12. A concave lens is used to form the image of an object that is 35.0 cm tall and located 25.0 cm from the lens. If the image is 28.0 cm tall, what is the focal length of the lens?

13. A concave lens with a focal length of 180 cm is used to form an image that is 75.0 cm tall from an object that is 45.0 cm from the lens. What is the height of the object?

12.2 Section Review

1. K/U What is a concave lens?
2. K/U Draw an example of the following lenses:
   - **(a)** double concave,
   - **(b)** convexo-concave, and
   - **(c)** plano-concave.
3. MC Explain, with the aid of ray diagrams, how concave lenses and convex mirrors are similar.
4. C Explain why the image from a concave lens is upright.
5. K/U Where is an image located if it is the same size as the object?

12.3 Optical Instruments

SECTION EXPECTATIONS

- Explain why different lens types are used in optical instruments.
- Analyze, describe, and explain optical effects that are produced by technological devices.
- Evaluate the effectiveness of devices or procedures related to human perception of light.

KEY TERMS

- magnification
- objective lens
- eyepiece lens
- Kepler or astronomical telescope
- spherical aberration
- chromatic aberration
- Galileo telescope
- accommodation
- myopia
- hyperopia
- astigmatism
- presbyopia

Figure 12.14 The Orion nebula

When Galileo turned his telescope on the skies, he changed forever how humans view the universe. Besides offering visual proof that Earth was not the centre of all planetary motion, Galileo mapped the surface of the Moon. Today, using telescopes such as the Hubble Space Telescope, scientists have mapped the universe far beyond what Galileo even imagined might exist. Photographs such as that of the Orion nebula are now available to anyone with an Internet connection.

Magnification and Magnifying Glasses

The simplest and perhaps most commonly used optical apparatus is the simple magnifying glass. As you have learned, **magnification** (M) refers to the ratio of the size of the image to the size of the object. Mathematically, this is

$$M = \frac{h_i}{h_o}$$

and, therefore, can also be found using

$$M = -\frac{d_i}{d_o}$$

For example, if a convex lens creates a real image that is three times as big as the object, then the magnification will equal –3. The negative sign indicates that the real image is inverted.

When a convex lens is used as a magnifier, the lens is moved so that the object is inside the focal point. If you draw ray diagrams for an object at various positions between the convex lens and the focal point, it will become obvious that the closer to the focal point you locate the object, the greater the size of the image.

On average, the closest distance that people can clearly focus on an object is about 25 cm from their eye. You can find the best focussing distance for each of your eyes by bringing this page closer and closer to your eyes (one at a time) until the page begins to become unfocussed. Note the distance at which the print on the page begins to blur. When estimating the magnifying power of a convex lens, assume that the image should be 25 cm from your eye, which, like Sherlock Holmes' eye, is very close to the lens for best vision. Now, to get the greatest magnification the object should be very close to, but inside, the focal point. This means the distance to the object is almost the same as the focal length ($d_o \approx f$). Thus, the magnifying glass can produce effective magnifications of

$$M \approx -\frac{d_i}{d_o}$$

where $d_i = -25$ cm and $d_o \approx f$. Therefore, the best magnification that you can expect from a convex lens being used to produce a virtual image is

$$M \approx -\frac{-25\text{ cm}}{f}$$
$$\approx \frac{25\text{ cm}}{f}$$

Hence, for a convex lens with a focal length of 10 cm, the best you can expect for its magnifying power is

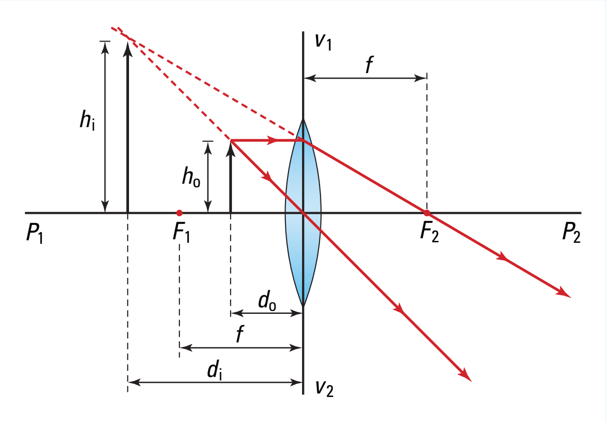
$$M \approx \frac{25\text{ cm}}{10\text{ cm}}$$
$$\approx 2.5$$

The image is a virtual, upright image; thus, the magnification is positive. Obviously, the shorter the focal length of the convex lens, the more powerful it becomes as a magnifying glass.

MODEL PROBLEM

Magnification with a Convex Lens

Find the magnification when an object 45.0 cm tall is placed 48.0 cm from a convex lens with a focal length of 64.0 cm.



Frame the Problem

- The object is located within the focal length of the lens, so we expect the image to be larger than the object and farther from the lens than the object.
- Visualize the problem by sketching the ray diagram.

continued ▶

continued from previous page

Identify the Goal

The magnification, M, of an object by a convex lens

PROBLEM TIP

Be careful not to confuse M (magnification) in optics with m (mass) in dynamics.

Variables and Constants

Involved in the problem		Known	Unknown
d_o	h_i	$d_o = 48.0$ cm	d_i
d_i	f	$h_o = 45.0$ cm	h_i
h_o	M	$f = 64.0$ cm	M

Strategy / Calculations

Use the mirror/lens equation to find the image location, d_i.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Isolate the unknown, d_i.

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

Substitute known values.

$$\frac{1}{d_i} = \frac{1}{64.0\text{ cm}} - \frac{1}{48.0\text{ cm}}$$

$$\frac{1}{d_i} = \frac{3}{192\text{ cm}} - \frac{4}{192\text{ cm}}$$

Solve.

$$\frac{1}{d_i} = \frac{-1}{192\text{ cm}}$$

$$d_i = -192.0\text{ cm}$$

As expected, the image is farther from the lens than the object and on the same side of the lens.

Now, use the magnification equation, to find M.

$$M = -\frac{d_i}{d_o}$$

$$= -\frac{(-192.0\text{ cm})}{48.0\text{ cm}}$$

$$= +4.0$$

The image is a virtual (upright) image that is 4.0 times larger than the object.

Validate

The units for the image distance are in centimetres. There are no units for the magnification.

The value of the magnification seems reasonable.

PRACTICE PROBLEMS

14. Suppose you want to magnify a photograph by a factor of 2.0 when the photograph is 5.0 cm from the lens. What focal length is needed for the lens?

15. What is the magnification of a lens with a focal length of 10.0 cm when the object is 9.5 cm from the lens? Where is the image located?

16. The fine print in a contract that you want to read is only 1.0 mm tall. You have a magnifier with a focal length of 6.0 cm. Where must you hold the magnifier so that the image appears to be 1.0 cm tall? Where is the image located?

Refracting Telescopes

Refracting telescopes generally use two lenses to magnify and focus on a distant object. The **objective lens** converges the almost-parallel rays from the object, and the **eyepiece lens** produces a magnified, virtual image. Modern refracting telescopes are **Kepler** or **astronomical telescopes**, in which the eyepiece and the objective lens are both convex. The object viewed in astronomical observations is obviously very far away; thus, the rays from that object are almost parallel. The refracted rays converge on a point just a bit farther from the objective lens than its second focal point, which lies inside the telescope (See Figure 12.15). The image formed by the objective lens is a very tiny, real (inverted) image. This image is viewed as the object for the eyepiece. The eyepiece is a convex lens used as a magnifying glass. As seen in the previous section, the best magnification is achieved when the object being magnified is just inside the first focal point of the eyepiece. This type of magnifier produces an upright image of the object. Since the object for the eyepiece was an inverted image from the objective, the final image is therefore inverted compared

History Link

Johannes Kepler (1571–1630) was famous as the creator of Kepler's laws of planetary motion. His modifications, in 1611, to Galileo's telescope design enabled him to achieve magnifying powers of almost 1000 times, greatly enhancing its field of view. With his telescope, he observed that the tail of a comet always pointed away from the Sun. He suggested that there must be solar radiation pressure to cause this. It was more than three hundred years before he was proven to be correct.

Figure 12.15 The lens arrangement for an astronomical refracting (Kepler) telescope

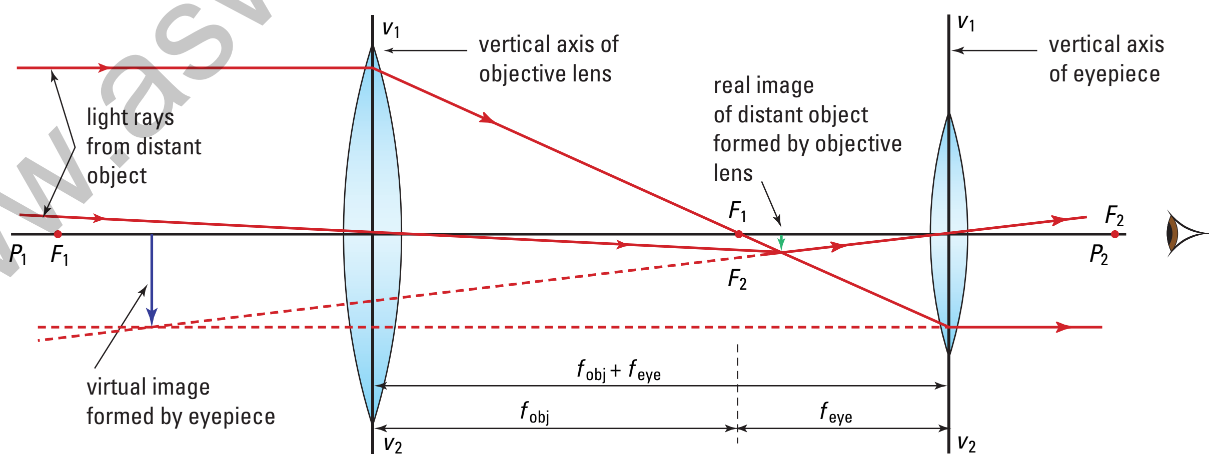


Figure 12.16 High-powered refracting telescopes have very long tubes.

to the original object. For astronomy, it is irrelevant whether the image of the star is upright or inverted.

The telescope is constructed so that the second focal point of the objective lens and the first focal point of the eyepiece are at the same point inside the telescope. Thus, the image from the objective lens that is just outside its focal point will become the object for the eyepiece that is just inside its focal point. Hence, the length of the telescope tube should be $f_{obj} + f_{eye}$, where f_{obj} and f_{eye} are the focal lengths of the objective lens and the eyepiece, respectively. The magnification of the telescope is found by

$$M = -\frac{f_{obj}}{f_{eye}}$$

The negative value tells us that the image is inverted. To produce a high-powered telescope, the focal length of the eyepiece must be small compared with the focal length of the objective. In theory, astronomical (Kepler) telescopes have no limit to their possible magnification. Obviously, that is not the situation in reality. Other factors come into play.

Telescopes with very high powers need to have an objective lens with a long focal length. The magnification is obviously increased by using an eyepiece with a short focal length. But if you make the focal length of the eyepiece too small, there is little room for error in the position of the image from the objective. Even minor changes in the length of the telescope tube make locating the image impossible.

The longer you make the focal length of the objective lens, the longer you must make the length of the telescope. As a result, most astronomical telescopes must be mounted on a tripod or other base to hold them still, as in Figure 12.16. If you want to use an astronomical telescope design to make binoculars, two problems arise. The first is that they become very long and unwieldy. The second problem is that the image is inverted (not a problem in astronomy but a definite drawback for bird watching).

Fortunately, both problems are solved by the same design adaptation. The characteristic “shifted” tube shape of high quality binoculars comes from using prismatic mirrors to make the light path double back on itself to shorten the tube.

By arranging the prismatic mirrors as shown in Figure 12.17, not only is the length of the tube shortened, but also the image is inverted while being reflected. This re-inverts the image so that it is seen as being upright.

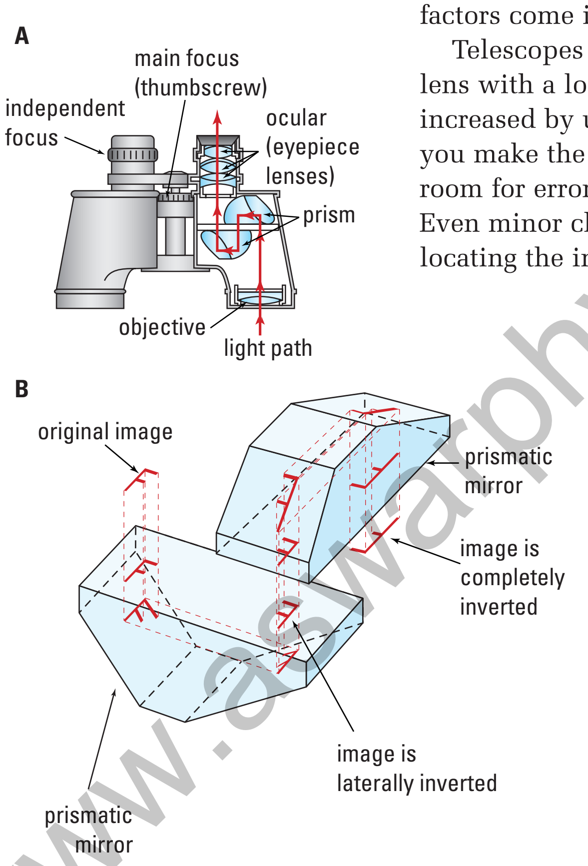


Figure 12.17 **(A)** Binoculars use prismatic mirrors to shorten the tube length and **(B)** re-invert the image to make it appear upright.

Think It Through

- Return to your notes on Investigation 12-A, at the beginning of this chapter, and review your findings on the Kepler telescope. How accurately did you answer the questions at that time?

Design Limitations for Refracting Telescopes

Regardless of magnification, if you want to see very distant and very dim objects, you need to increase the light-gathering ability of the telescope. This means that you need to make the surface area of the objective lens larger, so that more light falls on it. As the diameter of the lens increases, the lens becomes thicker and the problems of lens construction increase. If you are viewing very tiny images of distant stars, even a very slight loss in image quality can render the image useless.

The first problem in the production of large lenses is the process of construction itself. Manufacturing a large piece of flawless glass is very difficult. Even small internal flaws will disturb the image quality. At large magnifications, even small flaws can produce a significant decrease in image quality. It is, therefore, very expensive to make glass of a sufficiently high quality for a large lens. Once the high-quality glass base for the lens is formed, the surfaces must then be ground to an extremely high degree of accuracy. Computers have increased our ability to do this but it is still a very challenging technological process. That is why very large astronomical telescopes use concave mirrors rather than convex lenses for the objective.

Second, the thin-lens equations no longer apply to such thick lenses. The image is still formed but as you move from the centre of the lens toward its edges, the focal length of the lens changes causing the image to become blurry (see Figure 12.18). This is called **spherical aberration**.

Technology Link

Since the mid-1970s, optical engineers and astronomers have been designing huge reflecting optical telescopes with composite primary mirrors that are upwards of 50 m across — bigger than the infield of a baseball diamond. Once built, they will greatly improve the images available in optical astronomy. (For comparison, the largest optical telescope at present is the Keck telescope, located in Hawaii. It is 10 m across and comprised of 36 segments of 2 m each.) Presently, many different telescopes are used to observe the universe using non-visible "light" such as radio waves. Because the mirrors for radio-telescopes do not use specular reflectors, these can generally be made much larger than the mirrors for telescopes designed to reflect visible light. The world's largest radio telescope, at Arecibo in Puerto Rico, is 305 m across. Arrays of radio telescopes are now being used to increase resolution and sensitivity. Find out more about the sensitivity of radio telescopes. What is a jansky?

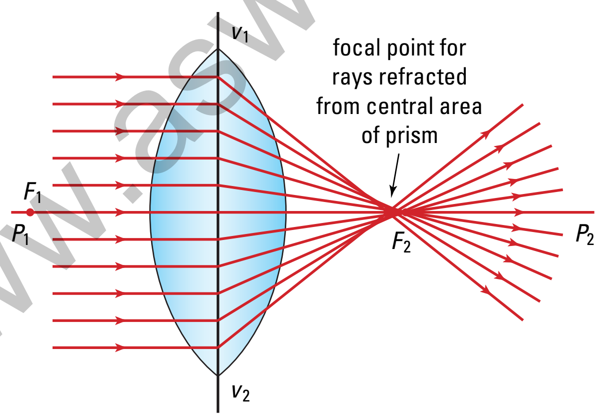


Figure 12.18 Spherical aberration causes deterioration in the quality of an image.

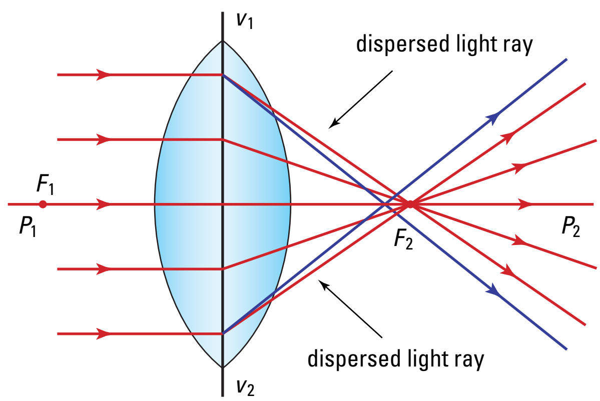


Figure 12.19 Chromatic aberration is the result of dispersion of the light by the edges of the lens.

Third, the edges of the lens become very fat prisms and dispersion of the light from the object into its spectrum disturbs image formation. Because the angle of deviation for violet light is greater than that for red light, the edge of the lens, acting as a prism, refracts the violet light closer to the lens than the red light. Thus, the focal length for violet light is shorter than the focal length for red light. This produces an effect called **chromatic aberration**, as illustrated in Figure 12.19 on the previous page. The result is images that are tinged with a halo of colour.

PHYSICS & TECHNOLOGY

TARGET SKILLS

- Communicating results
- Conducting research

Infrared photographs can be used to increase the contrast and reduce atmospheic distortion.

Camera lenses might have an infrared focussing scale to allow you to take pictures using infrared light.

Focussing on Infrared

If you are a photographer, you can photograph objects using infrared light, otherwise known as radiant heat. Because the index of refraction for infrared light is even smaller than the index of refraction for red light, for any given lens its focal length is even longer than the focal length of red light.

So that you can be sure your infrared pictures are in focus, the lenses on many professional cameras have an infrared focussing scale marked on them. First, the camera is focussed for visible light; then, the setting for accurate focus is turned to the infrared focussing mark. This adjusts the focal distance from visible light to infrared light.

The leaves of plants are good reflectors of the Sun's infrared rays, while the ground and water are very good absorbers of infrared light. The result is that landscapes show an almost eerie increase in contrast when photographed using infrared light.

A further effect results because it is the blue end of the spectrum that is most strongly scattered by the atmosphere, while infrared light at the opposite end of the spectrum undergoes very little scattering. Hence, photographs made with infrared light can often be used to "see" through haze and give a clear picture when a photograph made with visible light would not.

Going Further

1. Find and explain several examples of how infrared photographs are used to gain information that is difficult or impossible to obtain using visible light.
2. Explain clearly why the sky is blue and why the Sun appears red at sunset.

Galileo Telescopes

The Galileo telescope is popular today in the form of opera glasses. Some binoculars (usually inexpensive) that have straight tubes are forms of Galileo telescopes. Galileo telescopes are, however, more limited in their magnification compared to Kepler telescopes.

The advantage offered by **Galileo telescopes** is that the image is upright, compared to the original object, without the use of prismatic mirrors. This is the result of a very clever application of a concave eyepiece. Galileo placed a concave lens in the path of the converging rays from the convex objective lens. He made the length of the telescope tube so that the second focal point of the objective lens coincided with the second focal point of the eyepiece *outside* the telescope tube. Thus, the length of a Galileo telescope tube was $f_{obj} - f_{eye}$. In other words, the rays had not yet reached the point at which they would form an image.

If these rays had not passed through the eyepiece, they would have converged just beyond the second focal point of the eyepiece. They appeared to diverge from a point in front of the concave eyepiece inside the telescope. Because the rays converging from the objective lens had not yet reached the point of convergence they had not crossed and the image was still upright. The image from the objective lens could be considered to be a virtual (or an imaginary) object for the concave lens (see Figure 12.21). For more detail, see Section 12.4, which discusses virtual objects.

Figure 12.20 Opera glasses are usually a pair of Galileo telescopes.

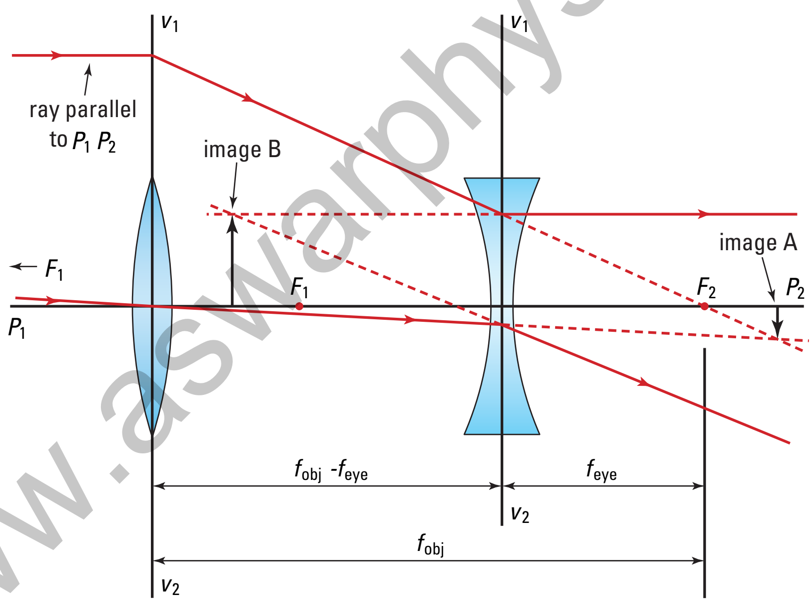


Figure 12.21 In this diagram, the rays from the objective lens are converging to a point beyond the concave eyepiece. The eyepiece causes these rays to diverge from a point inside the telescope, the same point where you see the image.

Web Link

www.school.mcgrawhill.ca/resources/

For more pictures and information about the world's largest optical telescopes, the radio-telescope located at Arecibo, Puerto Rico, and radio astronomy observatories, go to the above Internet site. Follow the links for **Science Resources** and **Physics 11** to find out where to go next.

Think It Through

- Check your notes from Investigation 12-A at the start of this chapter and see how you explained the Galileo telescope at that time. How accurately have you explained how the telescope formed its image? Did you find that the distance between the lenses for the Galileo telescope was less than for the Kepler (astronomical) telescope?

Microscopes

The structures of an astronomical telescope and a microscope are very similar. They both use a convex eyepiece to observe the real image formed by a convex objective. While the telescope has an objective lens with a long focal length to view large objects at great distances, the microscope has an objective lens with a very short focal length to view small objects at very small distances.

Unlike a telescope, the objective lens for a microscope has the shorter focal length and the eyepiece has the longer focal length. But, like the telescope, the real image from the objective lens is projected inside the focal point of the eyepiece. The eyepiece is being used as a standard magnifying glass, which leaves the image inverted. In order to make the image from the objective lens as large as possible, you put the object as close as possible to, but still outside the focal point of, the objective lens, as shown in Figure 12.23 (see also the convex lens ray diagrams on pages 559–560). The length of the microscope tube then is the sum of the length of the image distance for the objective lens plus the focal length of the eyepiece.

Figure 12.22 The lens arrangement of a microscope is similar to that of an astronomical telescope. In a binocular microscope, the light from the objective lens is split in two by prismatic mirrors and directed to the eyepiece lenses.

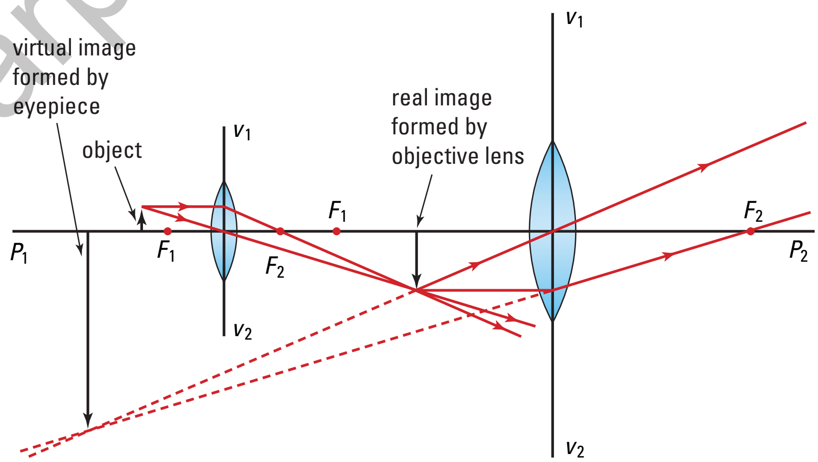


Figure 12.23 A ray diagram for a microscope is illustrated here. For maximum magnification, the real image from the objective lens of a microscope is located just inside the focal point of the eyepiece.

The Eye

The human eye is a marvel of biological design. It is able to focus on objects at different distances, record images, and detect subtle changes in colour and brightness. The focussing occurs at the front of the eye and everything else occurs at the back, or in the brain.

Figure 12.24 shows how the cornea refracts incoming light and projects an image onto the retina. In fact, the cornea contributes approximately seventy percent of the total converging power of the eye. The purpose of the lens, located behind the cornea, is to make focal length adjustments as the distance to the object changes. As the distance from an object to the eye decreases, the distance from the lens to the image increases. The image, originally on the retina, shifts to a position behind the retina. To move the image back onto the retina, the ciliary muscle contracts, allowing the lens, by virtue of its elasticity, to become more convex, shortening its focal length. By shortening its focal length, the lens moves the image so that it falls exactly on the retina. This phenomenon is known as **accommodation**. When an object is far from the eye, the ciliary muscle relaxes allowing ligaments to increase tension on the lens and cause it to flatten.

The retina is spread across the back of the eye. It contains millions of specialized receptor cells of two types: rod and cone cells. Rod cells are highly sensitive and enable you to see in dim light. Cone cells are comparatively less sensitive — they do not work in dim light — but they are able to distinguish fine details and colours. Light from an object you are looking directly at, falls on a small region of your retina called the fovea. The fovea contains only cone cells making the image very clear and distinct. The rest of the retina has a combination of rod and cone cells. Both the rods and cones respond to light from the image and trigger nerve cells to send impulses along the optic nerve to the visual cortex of the brain where "seeing" is actually done.

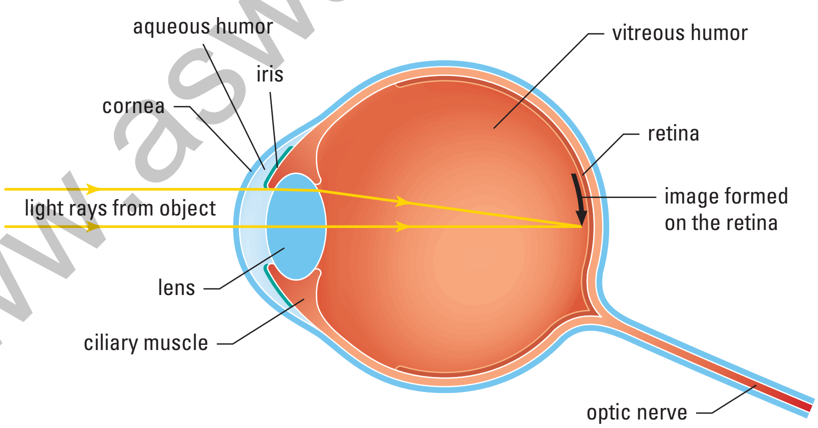


Figure 12.24 The eye focusses an image on its retina.

PHYSICS FILE

Looking straight at an object is not always the best way to see it, especially if the object is faint. When you look at an object directly, you focus it on the centre of your retina, where there are few rods. If you look slightly away, or "avert" your vision, you use more rods and can then see more faint detail of the object. The next time you are looking at the night sky through a telescope or binoculars, remember to use "averted vision"—you'll see more stars!

Language Link

The term "myopia," commonly known as near-sightedness, comes from Greek for "shut-vision." To improve their vision, people suffering from myopia will squint or partially close their eyes. The term "hyperopia," commonly known as far-sightedness, comes from the Greek for "vision beyond." Many words in English are combined to form words from other languages. In Greek, the word *presbus* means "old man." Can you create a word that means "old man's vision"? This term will be explained later in this chapter.

Myopia (Near-sightedness)

As people grow older, the shape of their eye sockets sometimes changes, which affects the shape of their eyeballs. If an eyeball is forced out of a round shape, the lens might not be able to accommodate enough to focus the image exactly on the retina.

In teenagers, their growth spurt often results in the eyeball's shape becoming slightly elongated. The lens cannot project the image far enough back in the eyeball to fall on the retina. This condition is called "**myopia**" or "near-sightedness." Using a diverging lens in front of the eye causes the rays from the object to diverge slightly before they strike the cornea. If the amount of increased divergence is correct, the image will be formed a bit farther behind the lens on the retina. The diverging lens is usually in the form of a concave meniscus (see Figure 12.25). Recently, medical scientists have learned how to use lasers to reshape the cornea to correct for myopia.

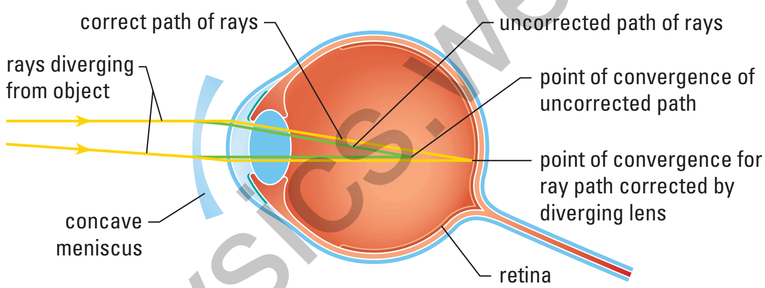


Figure 12.25 A concave meniscus is used to correct myopia.

Hyperopia (Far-sightedness)

When the eyeball becomes too short for the lens to focus the image to the retina, the condition is known as "**hyperopia**" or "far-sightedness." This condition is corrected by the use of a converging lens in the form of a convex meniscus. The effect of the corrective lens is to assist the eye to shorten the distance from the lens to the image, as shown in Figure 12.26. Although laser surgery has been developed to correct hyperopia, it is much more complicated, and thus not as popular, as the laser surgery used to correct myopia.

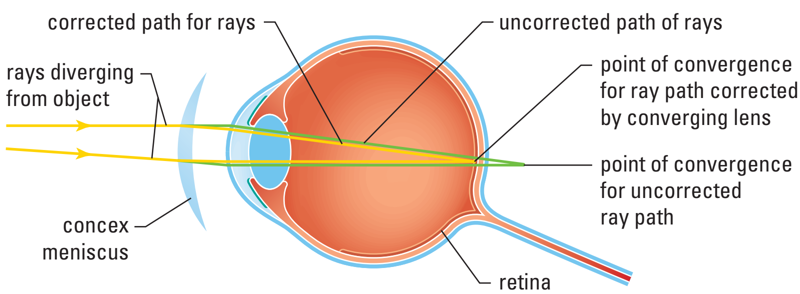


Figure 12.26 A convex meniscus lens is used to correct hyperopia.

CAREERS IN PHYSICS

TARGET SKILLS

- Analyzing and interpreting
- Conducting research

Have your eyes ever been checked?

When you look at a cherry pie, grizzly bear, or anything else, the rays of light from the image ideally focus right on your retinas, the light-sensitive tissues lining the backs and sides of your eyes. However, if you are nearsighted, images focus in front of your retinas because your eyes are longer than normal. If you are farsighted, images focus behind your retinas because your eyes are shorter than normal. Either way, blurry vision can result. An optometrist or ophthalmologist can check your eyes to detect visual problems, if any, and write a prescription for corrective lenses, if needed.

Dispensing opticians interpret such prescriptions. They take facial measurements and measure the distance between the pupils of your eyes. Based on such measurements and your lifestyle, they recommend eyeglass frames and lens types. If you are considering contact lenses, they inform you of the choices involved.

Once you and the dispensing optician decide what to order, the eyeglasses or contact lenses are made, usually in a laboratory supervised by an optician. Based on the prescription, laboratory technicians make concave spherical lenses for near-sightedness. They make convex spherical lenses for far-sightedness. For astigmatism, a condition which is caused by incorrectly shaped cornea, they make cylindrical lenses. For cases where the pupils do not quite line up, they use prisms to help the eyes work together better. For more than one problem, the laboratory combines lens forms. When your eyeglasses or contact lenses are ready, the dispensing optician checks that they are made correctly. He or she adjusts eyeglass frames so they fit comfortably. In the case of contact lenses, the dispensing optician shows you how to insert, remove, and care for them.

Becoming an optometrist (doctor of optometry) requires about four years of university. Becoming an ophthalmologist takes eight to ten years since an ophthalmologist becomes a Doctor of Medicine (M.D.) first and then specializes in eye disorders. Becoming an optician takes one to three years of community college, depending on where you live. For all eye care professionals, useful high school courses include physics, mathematics, biology, chemistry, and English.

Going Further

1. Ask people wearing eyeglasses or contact lenses about their experiences with eye care professionals.
2. Find out what types of eye care are paid for by your province's or territory's public health system. If you have not had your eyes checked in the last year, consider doing so.

ELECTRONIC LEARNING PARTNER

Enhance your learning about optics and the human eye.

Presbyopia

Rather than resulting from a change in the shape of the eyeball, the inability to see objects that are close to you can also occur because the lens loses its flexibility. This condition is known as **presbyopia**. Because the rays from a far object are diverging less than rays from a near object, the lens can still focus rays from distant objects on the retina. Normally, to cause the more sharply diverging rays from a near object to be focussed on the retina, the lens has to become quite round to shorten its focal length. As you age, the lens often loses its flexibility and cannot become round enough to create clear images of near objects.

Presbyopia can occur in conjunction with myopia or hyperopia. If one already wears glasses to correct for myopia, as presbyopia occurs, the lens prescription becomes one for bifocals, or even trifocals, to accommodate both conditions. People who have undergone laser surgery to correct for myopia often still have to wear reading glasses because of presbyopia.

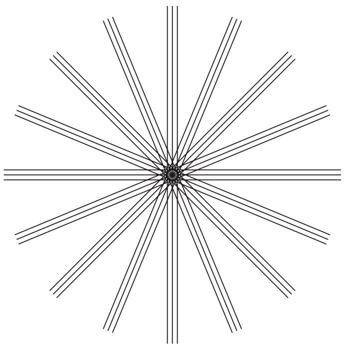


Figure 12.27 If one set of bars seems darker than the rest, it indicates that your cornea is astigmatic.

Astigmatism

Another vision problem is associated with a change in shape of the eyeball. If the cornea becomes "out of round" laterally or vertically, then the focus of either horizontal or vertical lines might be affected. The eye focusses in one plane slightly better than the other planes. This is defined as **astigmatism** and often occurs in conjunction with myopia or hyperopia. In this case, adjustments can be made to the lenses of spectacles to correct this problem. If any lines seem darker than others when you look at Figure 12.27, you might have an astigmatism.

12.3 Section Review

1. K/U Where should the object be placed to obtain the greatest magnification from a convex lens?
2. K/U Define the magnification of a refracting telescope.
3. K/U What are some problems associated with large lens construction?
4. C Discuss the differences and similarities between a telescope and a microscope.
5. K/U List the differences and similarities between a Galileo and a Kepler telescope.
6. K/U Why are Kepler telescopes longer than Galileo telescopes?
7. C Explain accommodation as it applies to vision.
8. K/U What are the roles of the rod and the cone cells in the eye?
9. K/U Define
   - (a) myopia,
   - (b) hyperopia,
   - (c) presbyopia, and
   - (d) astigmatism.

12.4 Virtual Objects

In the Kepler telescope and the microscope, the real image, formed by the objective lens, is used just as a real object by the eyepiece. Even virtual images can be used as objects for other lenses, since the rays from them are diverging. However, in the Galileo telescope, the concave eyepiece intercepts the converging rays from the objective before they reach the point at which the real image would be formed (in other words, the point of divergence). The concave lens (remember this is a diverging lens) then refracts these converging rays so that they appear to diverge from a point inside the telescope. That point of divergence is where the observer sees the image.

Imagine a set of rays that converge on a concave lens, as shown in Figure 12.28. If they had not passed through the lens, the rays would have come to a point of convergence and the real image would have been formed at *P*, just beyond the focal point of the lens (F_2).

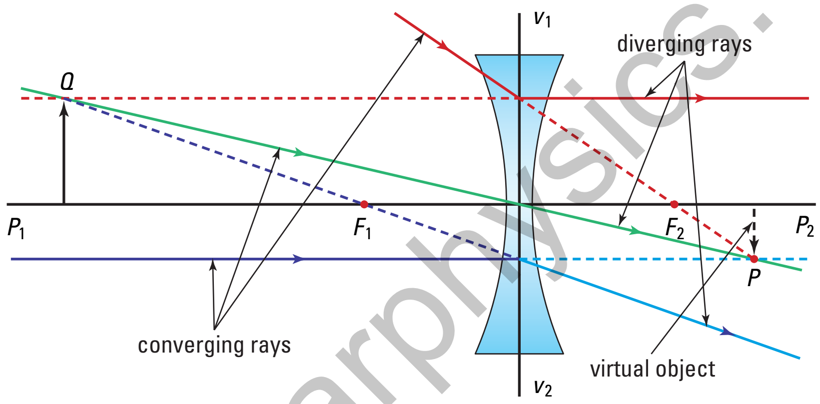


Figure 12.28 When a set of converging rays are incident on a lens, the image they would have formed when they reached the point of convergence acts as a virtual object for the lens. For the object, d_o is negative and the thin-lens equations can be used to calculate the size and position of the image.

The paths of the three rays normally used to locate images are shown. First, the ray through the centre of the lens (in green) goes straight through without bending. Second, the ray that would have passed through F_2 (in red) is refracted parallel to the principal axis. Third, the ray parallel to the principal axis (in blue) refracts so that it appears to have passed through the first focal point (F_1). In each case, the solid line indicates the actual path of the ray, the dotted line indicates the path of the rays had they not been refracted, and the segmented line represents the path of the rays projected back to the imaginary point of divergence at *Q*.

SECTION EXPECTATION

- Analyze, in quantitative terms, the characteristics and positions of images formed by lenses.

KEY TERM

- virtual object

If the rays had not passed through the lens, they would have converged on and then diverged from point P, creating a real image there. Instead, the rays are refracted so that they appear to be diverging from Q; thus, a virtual image will be visible there.

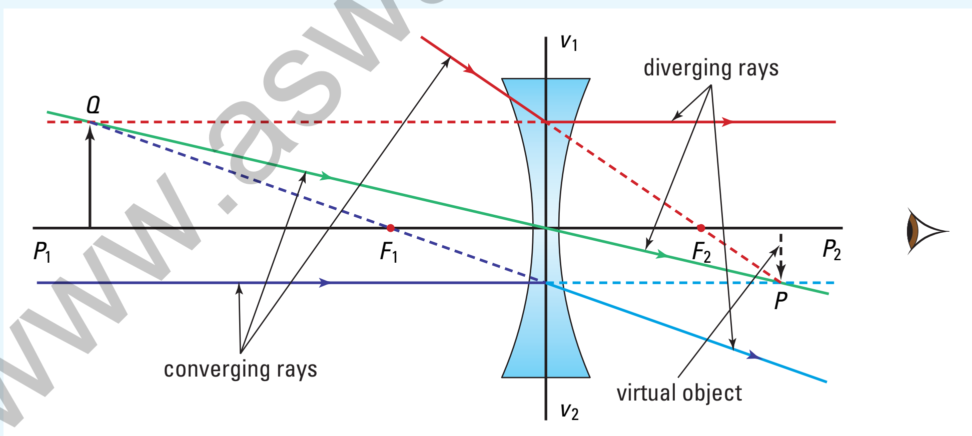
Either concave or convex lenses can create an image when they intercept the converging rays from a convex lens. Depending on the type of lens and where the point of convergence (and thus the location of the unformed real image) would be located in relation to the focal point of the lens, the images might be real or virtual. On the surface, it might seem that the calculations for the position and size of the image might be very complex. But if you think of the not-yet-formed real image as a **virtual object**, it makes calculations using the thin-lens equations quite straightforward. When lenses refract diverging rays to form images, the point of divergence defines the position of the object. The distance from the lens to the object or point of divergence (d_o) is considered positive. For virtual objects, the distance from the lens to the virtual object is considered to be negative. All other values are interpreted as for other conditions, as shown in Table 12.2.

MODEL PROBLEM

Using an Image as an Object

A set of converging rays is incident on a concave lens with a focal length of 25.0 cm. If they had not passed through the concave lens, the rays would have formed an inverted real image 10.0 cm high at a point 50.0 cm beyond the concave lens. Calculate the location and size of the actual image.

Frame the Problem



- Visualize the problem by sketching the light ray diagram.
- The *virtual object* would have been a *real image*, so its *size* is *negative*.
- The lens is *concave*; therefore, *focal length* is *negative*.

Identify the Goal

(a) The location, d_i, of the image

(b) The size, h_i, of the image

Variables and Constants

Involved in the problem		Known	Unknown
f	h_o	$f = -25.0$ cm	d_i
d_o	h_i	$d_o = -50.0$ cm	h_i
d_i		$h_o = -10.0$ cm	

Strategy / Calculations

Use the mirror/lens equation to find d_i.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

Rearrange.

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

Substitute known values.

$$= \frac{1}{-25.0\text{ cm}} - \frac{1}{-50.0\text{ cm}}$$
$$= \frac{-2}{50.0\text{ cm}} + \frac{1}{50.0\text{ cm}}$$

Solve.

$$= \frac{-1}{50.0\text{ cm}}$$
$$d_i = -50.0\text{ cm}$$

Now you can use the magnification equation to find h_i.

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Rearrange.

$$h_i = -h_o\frac{d_i}{d_o}$$

Substitute known values.

$$= -(-10.0\text{ cm})\left(\frac{-50.0\text{ cm}}{-50.0\text{ cm}}\right)$$

Solve.

$$= +10.0\text{ cm}$$

(a) The image is located –50.0 cm from the lens.

(b) The image is +10.0 cm tall.

continued ►

continued from previous page

Validate

The units are in centimetres.

The image is on the same side of the lens as the incident light; therefore, the image distance is negative.

The image is on the opposite side of the lens as the virtual object.

The image is upright and virtual.

PRACTICE PROBLEMS

17. Make an accurately measured scale diagram using the following values. Use a concave lens with a focal length of 3.0 cm. Assuming the light is coming from the right, place point P at a distance of 4.0 cm to the left of the vertical axis and 1.0 cm below the principal axis. Accurately draw the ray diagram as shown in Figure 12.28. Measure the size and location of the image. Use the thin-lens equations to calculate the size and location of the image. Remember that the focal length (f) is –3.0 cm. The distance to the virtual object (d_o) is –4.0 cm since it is on the opposite side of the lens from which the light came. Use the thin-lens equations to calculate the distance to and the size of the image and see if it agrees with the measurements in your diagram.

18. Use the same parameters as in problem 17 but place the virtual object 2.0 cm from the lens.

19. Use the same parameters as in problem 17, but place the virtual object 3.0 cm from the lens.

20. Repeat problems 17 through 19 using a convex lens with at 3.0 cm focal length (f = +3.0 cm). Be careful to make sure you obey the rules of refraction for convex lenses.

12.4 Section Review

1. K/U What is a virtual object?
2. C Explain how a virtual object is produced.
3. K/U How are virtual objects used in optical instruments such as the Galileo telescope?
4. C Use ray diagrams to sketch how Kepler and Galileo telescopes form an image of the Moon.

UNIT PROJECT PREP

Lenses and the principles of refraction have been used to improve eyesight, view microscopic objects, and peer deep into space.

- What advantage, if any, does the application of virtual images provide to the design of your optical instrument?
- Sketch ray diagrams to predict the path of light using the principle of virtual images.

CHAPTER 12 Review

REFLECTING ON CHAPTER 12

- When refracted rays cross at the point of divergence, the image of each point is inverted. If the rays actually pass through the point of divergence, the image is real. If the rays diverge from a point through which they never actually pass, the image is virtual.
- A convex or converging lens is thicker in the middle than at the edges. Parallel rays that strike the lens will converge on the focal point on the opposite side of the lens.
- A concave or diverging lens is thinner in the middle than at the edges. Parallel rays that strike the lens diverge from one another and appear to come from the focal point on the same side of the lens as the incident light.
- The lens-maker's equation is used to predict the focal length of a lens from the index of refraction of the lens material and the radii of curvature of the two surfaces of the lens:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

- The magnification equation relates the image size to the object size, and the image distance to the object distance,

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

  Negative magnifications mean the image is inverted.
- The mirror/lens equation relates the focal length to the object and image distance:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

- A refracting telescope uses an objective lens with a long focal length to view large objects at great distances.
- Keplerian telescopes use convex lenses as an objective and as an eyepiece.
- Galileo telescopes use a convex lens as an objective and a concave lens as an eyepiece.
- Problems with large lenses include spherical aberration and chromatic aberration.
- A microscope has an objective lens with a very short focal length to view small objects at very small distances. Both the objective and eyepiece are convex and a magnified, inverted, virtual image is produced.
- In the eye, the cornea focusses light onto the retina. The focus can be adjusted by changing the tension on the lens, located behind the cornea. Loss of flexibility of the lens or a change in shape of the eyeball affects the focussing of the eye and can be corrected with lenses.
- A virtual object is created when a concave or convex lens is placed *ahead of* the point of convergence.

Knowledge/Understanding

1. What single property involving the shape of a lens indicates that it is a converging lens?
2. Define the term "radius of curvature."
3. Name the three basic shapes for convex lenses.
4. What is the difference between a convex meniscus and a concave meniscus? Illustrate your answer by a diagram.
5. Why can real images be projected onto a screen?
6. How does the shape of the eyeball differ for myopia and hyperopia?
7. Is the image that forms on the retina of the eye a real or a virtual image?
8. What is the difference between the eyepieces of a Galileo and a Kepler telescope?
9. What causes astigmatism?
10. If the magnification for an image is equal to −3.5, what is the nature of the image?
11. What is meant by the term "thin-lens equations"?
12. Why do very large astronomical telescopes use a mirror rather than a lens for the objective optic?

13. Suppose a convex lens produced an image and then was replaced by a larger lens with the same focal length. How would the second image compare to the first image?

Communication

14. Explain why a concave lens cannot create a real image from a real object or real image.
15. Explain how the cornea and the lens of the eye create the image that falls on the retina.
16. The image formed on the retina of the eye, like all real images, is inverted. Discuss why the world does not look inverted to you.
17. Reducing the aperture through which light can enter a lens reduces the amount of spherical aberration. Explain why.
18. Discuss the concept of accommodation.
19. Explain why the tube lengths of telescopes and microscopes are set so that the image formed by the objective lens falls just inside the focal point of the eyepiece.
20. Explain how you would design a microscope that can magnify 1000 times.

Inquiry

21. Eyeglass lens technology continues to improve the quality, durability, and comfort related to wearing glasses. Imagine that you are an entrepreneur, preparing to open an eyeglass manufacturing company. You hope to market your lenses based on new and exciting design advances. Design and conduct several simple experiments to test physical properties of eyeglass lenses (e.g. weight, lateral displacement, etc.). Compare glasses from several years ago with more modern models. Complete your investigation by generating a marketing brochure to promote your new and improved lens design.
22. During an investigation the following data was collected. Complete the table and describe the image characteristics for each scenario.

d_o (cm)	d_i (cm)	f (cm)
60		30
infinity	15	
20		40
30	40	

Making Connections

23. To put a telescope such as the Hubble Space Telescope into space is a very costly venture. Obviously scientists think that it is worth the expense or they would not have done it. Do a research project to find out the advantages and disadvantages of having the telescope in orbit around Earth rather than on the surface.
24. Investigate laser eye surgery to find out what the pros and cons are for using it to correct myopia.
25. In theory, microscopes should be able to produce unlimited magnification. Investigate microscope design to find out why this is not the case.
26. The camera is often compared to an eye. While there are many features that are very similar, there are many significant differences as well. With the aid of diagrams, compare and contrast the camera and the eye.

Problems for Understanding

27. Suppose a convex lens is ground so that one surface has a radius of curvature of 30.0 cm and the opposing surface has a radius of curvature of 40.0 cm. The focal length is 34.2 cm. What is the index of refraction of the lens? What kind of glass is the lens?
28. What is the focal length of a plano-convex lens made of flint glass (n=1.65) if the radius of curvature of the spherical surface is 15.0 cm?
29. An object 17 cm tall is placed 56 cm from a convex lens that has a focal length of 24 cm. Find the size and location of the image. Describe the image.

30. An object 10.0 cm tall is located 20.0 cm from a concave lens with a focal length of 20.0 cm. What is the size and location of the image? Describe the image and draw a ray diagram of the situation.

31. If a single lens forms an image with a magnification of +4.5, describe the lens and the position of the object.

32. Draw an accurate ray diagram to find the size and location of the image formed by a double convex lens with a focal length of 8.0 cm. The object, 3.0 cm tall, is placed 14 cm from the vertical axis of the lens. Check your answers by using the thin-lens equations.

33. Repeat problem 32 using a concave lens rather than a convex lens.

34. Use an accurate ray diagram to establish the size and position of the image formed by a convex lens with a focal length of 10 cm. Place an object that is 2.0 cm tall 5.0 cm from the lens. Check your answers by using the thin-lens equations.

35. A telescope with a magnification of 30.0 has an 18 mm eyepiece. How long is the tube of the telescope?

36. A bug is on a microscope slide 3.4 mm from the objective lens of a microscope. If the focal lengths of the objective lens and eyepiece are 3.2 mm and 1.6 cm, respectively, what is the magnification?

37. A microscope is made in a tube 25.0 cm long. It magnifies 5.00×10^2 times, 50.0 times by the objective lens and 10.0 times by the eyepiece. What are the focal lengths of the two lenses?

38. Determine the focal length of the lens for the situation when an object is placed 12.0 cm in front of it and the magnification is (a) -6.00, and (b) +6.00.

39. What is the ratio of the focal length to the distance from the lens to the image if the image is (a) a real image 0.25 times as big as the object and (b) a virtual image 0.60 times as big as the object?

40. A projector takes an image from a slide that is 35.0 mm tall and projects it onto a screen that is 15.0 m away. If the image on the screen is 2.80 m tall, what is the focal length of the lens and how far from the lens was the slide?

Numerical Answers to Practice Problems

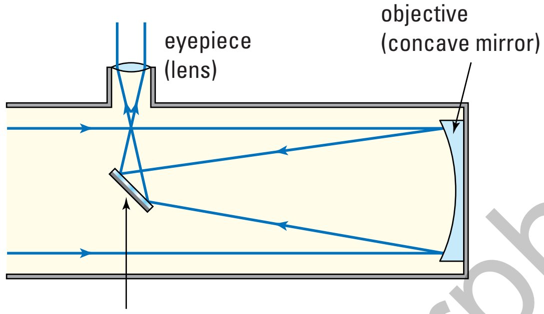
1. 14.5 cm **2.** 1.85 cm **3.** 1.65; it is probably flint glass. **4.** 54.3 cm **5.** $d_i = 15.4$ cm; $h_i = -1.80$ cm. The image is real, inverted, and smaller than the object. It is on the opposite side of the lens from the object. **6.** $d_i = 225$ cm; $h_i = -11.2$ cm. The image is real, inverted, and larger than the object. It is on the opposite side of the lens from the object. **7.** $d_i = -2.80$ m; $h_i = 14.0$ cm. The image is virtual, upright, and 3.33 times larger than the object. It is on the same side of the lens as the object. **8.** $f = 28.8$ cm; $h_o = 24.0$ cm **9.** $d_o = 35.0$ cm; $f = 122$ cm **10.** $\frac{1}{d_i} = 0$; d_i is undefined, as is h_i; no image is formed. **11.** $d_i = -48$ cm; $h_i = 34$ cm; It is a 34 cm tall virtual image that is upright, on the same side as the object, and 48 cm from the lens. **12.** –1.00 m **13.** 94 cm **14.** 0.10 m **15.** 2.0×10^1; $d_i = -1.9 \times 10^2$ cm, the image is farther from the lens than the object and on the same side of the lens **16.** $d_o = 5.4$ cm; $d_i = -54$ cm **17.** $d_i = -0.12$ m; $h_i = 3.0$ cm **18.** $d_i = 6.0$ cm; $h_i = -3.0$ cm **19.** d_i and h_i are undefined **20(17).** $d_i = 1.7$ cm; $h_i = -0.43$ cm **20(18).** $d_i = 1.2$ cm; $h_i = -0.60$ cm **20(19).** $d_i = 1.5$ cm; $h_i = -0.50$ cm

Up Periscope!

Background

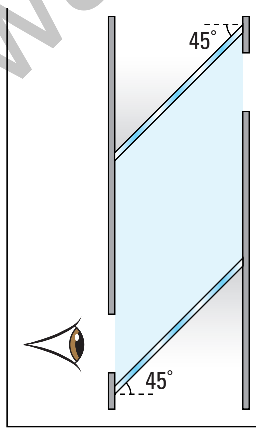
In this unit, several optical instruments that use two lenses were discussed. Other important optical instruments, such as reflecting telescopes, periscopes, and projectors, were omitted. Use your knowledge of lenses and mirrors to create a reflecting telescope or a periscope.

The reflecting telescope is similar to that of the Keplerian telescope except that a concave mirror is used to replace the objective lens, as shown in the following art. This creates viewing problems. Check reference books or the Internet (search: "reflecting+telescopes") for design hints.



A reflecting telescope

The periscope is merely a refracting (Keplerian) telescope that uses two mirrors to look around corners. Check reference books or the Internet (search: "refracting+telescopes") for design hints.



A simple model of a periscope.

Challenge

In this project, you are to research, design, and build a periscope or a reflecting telescope using three to five optical devices capable of focussing light (in other words, convex or concave mirrors and lenses). Both instruments will be based, with refinements, on the design of a refracting (Keplerian) telescope. Since both instruments can be used to view terrestrial objects, the viewed image should be upright and enlarged. Try for a magnification of about seven times. It should be able to focus to create a sharp image of objects from a distance of about 10 m to infinity.

Materials

Aside from lenses and/or curved mirrors, choose materials for building your instrument that are easily available and commonplace. Convex lenses are commercially available in the form of magnifying glasses, and concave mirrors can be found as make-up mirrors. Both convex and concave lenses and mirrors may be obtained from scientific supply companies.

Plane mirrors, prismatic or otherwise, may be required to bend the path of the light. If used, they are not to be considered among the three to five optical components since they bend but do not focus light. Concave and convex mirrors, on the other hand, both bend and focus light.

Be creative in constructing the supporting structure or body of the instrument. For example, plastic (either ABS or PVC) plumbing pipe could be used to build the body of your instrument. Light plywood also makes a good building material. A hot-glue gun can be used to attach together pieces of either material, but the adhesives specially created for ABS or PVC pipe work best with those materials. Both materials may be cut and drilled with normal woodworking tools such as handsaws, power saws, coping saws, and drills.

Safety Precautions

- When using hand or power tools, be sure to wear eye protection.
- Hot-glue guns take several minutes to cool after they are disconnected. If the hot glue gets on your hands, it can cause burns.
- Make sure you clean your hands after using either hot-glue, PVC, or ABS glue.

Design Criteria

A. Work in groups of three or four.

B. Obtain your mirrors and lenses. If the focal lengths are not previously known, do measurements to determine them.

C. Prepare a written presentation, including
- a title page with your names
- a design blueprint (see Action Plan)
- a ray diagram
- a log describing your work
- presentation of the completed instrument

D. Be prepared to have the other students use your telescope or periscope. Students will compare your designs with their own.

Action Plan

1. Brainstorm possible design ideas for your optical instrument. Develop ideas for including more than two optical components. Where would you use them? How would they affect magnification? How do you create the focussing mechanism? Try to create design innovations.
2. Prepare a design brief that explains
   - what you are building and why you have chosen to build it
   - the intended use and users of the instrument

ASSESSMENT

After you complete this project:

- **assess your optical instrument based on the tasks it was designed for. Are the images clear, upright, and enlarged?**
- **assess the instrument by letting others operate it. Is it easy to use?**
- **rate the innovation of your design on a scale of 1 to 5 (1 being very innovative, 5 being not very innovative at all.) In other words, did you get all your design ideas from a manual or did you create some or all of them yourself?**

3. Prepare a full sized blueprint of the instrument you are designing.
4. Create a full sized ray diagram, showing the path of light through the instrument and how the image is formed.
5. Prepare a list of all the materials you used.
6. Keep a log in which you detail the planning and building processes, including any successes and difficulties encountered. Describe how the problems you encountered were solved.
7. Create a construction manual for someone who might want to use your design to build your instrument.

Evaluate

1. Is the magnification as good as you expected? Estimate your actual magnification.
2. Can it be focussed easily for objects at different distances? What are the closest and farthest distances at which an object can be seen clearly?
3. Are the design blueprints and instructions for building the instrument clear? How could you make them clearer and/or simpler?
4. How could you improve your design?

UNIT 4 Review

Knowledge and Understanding

True/False

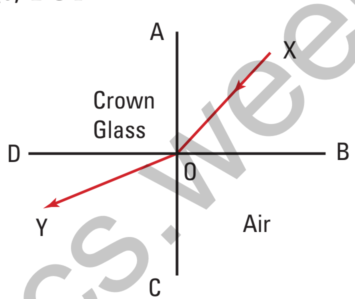
In your notebook, indicate whether each statement is true or false. Correct each false statement.

1. Images in plane mirrors are formed at the surface of the mirror.
2. For a plane mirror, the angle of incidence equals the angle of reflection. This does not apply to curved mirrors.
3. The size of the image formed by a plane mirror depends on the distance the object is from the mirror.
4. The image formed by a concave mirror is always a real image.
5. The focal point of a curved mirror is always in front of the mirror.
6. The lower the index of refraction, the faster the speed of light in the substance.
7. For any given angle of incidence greater than zero, the lateral displacement produced by a rectangular prism decreases as the index of refraction increases.
8. The shimmering effect noticed above a hot fire is due to total internal reflection between the layers of air at different temperatures above the flames.
9. The critical angle for a substance decreases as the index of refraction increases.
10. A convex lens that is curved on one side and flat on the other only has one focal point, and that is on the curved side of the lens.
11. A convex lens can form both upright and inverted images.
12. A concave lens can form both real and virtual images.
13. The focal point of a lens is the point at which the image is formed.
14. Chromatic aberration results because the focal length for red light is less than the focal length for violet light.
15. The cornea, rather than the lens, does most of the refraction in the eye.

Multiple Choice

In your notebook, write the letter of the best answer for each of the following questions.

16. The path of a light ray travelling from crown glass into air is shown in the diagram below. The angle of refraction is
    - **(a)** AOX
    - **(b)** XOB
    - **(c)** DOY
    - **(d)** BOC
    - **(e)** YOC



17. The index of refraction of light in water is 1.33. The speed of light in a vacuum is 3.00×10^8 m/s. The speed of light in water is
    - **(a)** 2.25×10^8 m/s
    - **(b)** 0.443×10^8 m/s
    - **(c)** 3.99×10^8 m/s
    - **(d)** 4.33×10^8 m/s
    - **(e)** 1.67×10^8 m/s
18. The focal length of a double convex lens is 6 cm. If an object is placed 10 cm from the optical centre of the lens, the characteristics of the image formed are
    - **(a)** real, inverted, and larger
    - **(b)** real, inverted, and smaller
    - **(c)** real, upright, and larger
    - **(d)** virtual, upright, and larger
    - **(e)** virtual, inverted, and smaller
19. To rectify the defect of farsightedness in the human eye, the eyeglasses must use
    - **(a)** plano-convex lenses
    - **(b)** diverging lenses
    - **(c)** converging lenses
    - **(d)** double convex lenses
    - **(e)** bifocal lenses
20. A very small light source is placed at the focal point of a convex lens. Which of the following best describes the pattern of light after it passes through the lens?

(a) The rays of light are diverging.
(b) The rays of light are perpendicular.
(c) The rays of light are parallel.
(d) The rays of light are converging.
(e) The rays of light are scattered.

Short Answer

In your notebook, write a sentence or a short paragraph to answer each of the following questions.

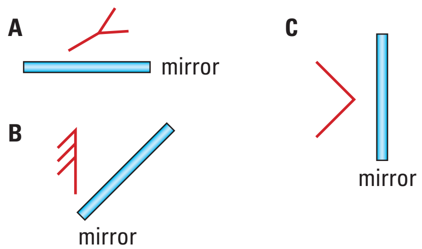
21. Define the terms: light ray, wavefront, normal, regular reflection, and diffuse reflection.
22. Describe an experiment you could do to demonstrate the difference between a real and virtual image.
23. A convex mirror can produce an image that is larger than the object. Comment on this statement, and justify your answer. A diagram may be used.
24. What is the speed of light in fused quartz?
25. When light passes from glycerine into crown glass at an angle, which will be smaller, the angle of incidence or the angle of refraction? Explain why.
26. How does the apparent depth of an object change as the index of refraction of the substance in which the object is immersed increases?
27. List three examples where a shimmering effect can be observed above an object or surface. Select one example, and explain why this effect occurs.
28. Why is it impossible for you ever to stand at the end of a rainbow, even if you are producing the rainbow by using a hose to spray a stream of fine water droplets into the air?
29. A concave lens is used to project a real image onto a screen. Explain how the image would be affected if an opaque piece of cardboard was inserted between the object and the lens so that it gradually covered more and more of the lens.
30. Explain the role of the following parts of the eye: the cornea, the lens, the ciliary muscles, and the retina.

Inquiry

31. Design an experiment, using one or more ray boxes and several reflecting surfaces, to show the law of reflection still applies when light undergoes diffuse reflection.
32. Analyze and describe the structure of a kaleidoscope. Complete a drawing and a set of instructions to show how to make one using cardboard and masking tape.
33. Design an investigation to measure the focal length of a convex mirror mounted on the door of a car. Check to see if the radius of curvature of the mirror is within the legal requirements for such mirrors. (Refer to the Canadian Motor Vehicle Safety Standards).
34. You are given a rectangular, transparent block of an unknown solid material. Design an experiment to determine the critical angle of incidence for the material.
35. Using one or more light ray boxes, design and construct three combinations of mirror, prisms, and lenses to demonstrate the principle of the reversibility of light.
36. Research and analyze data related to the structure of different types of optical fibres. Over what range of angles of incidence is it possible for the light beams to enter each type of optical fibre? Explain why these restrictions are imposed on the angles of incidence.
37. Given a concave lens of unknown focal length,
    (a) design an experiment to determine its focal length a) using natural sunlight, and b) using a light ray box.
    (b) Compare the focal lengths obtained by both methods, and account for any differences.

Communication

38. Copy the diagrams below into your notebook and draw a light ray diagram for each example to show how the image is formed in the plane mirror. Show all construction lines.



39. At the left-hand side of the page, draw a semicircle of a spherical concave mirror shape that has a radius of curvature of 6 cm, with the principal axis drawn across the width of the page. Directly below it construct and draw a parabolic concave mirror shape, also 12 cm in diameter, with the principal axis drawn across the width of the page. By drawing appropriate light ray diagrams, show why spherical aberration occurs, and explain how it is possible to use spherical mirrors to minimize this defect.
40. Design and draw a concept map to show the relationships between the properties and principles of light, the two scientific models, the laws of reflection, plane and curved mirrors, and any associated technological applications.
41. Draw a labelled diagram to show the reflection of three rays of light that are travelling parallel to the principal axis of
    (a) a concave mirror surface
    (b) a convex mirror surface
42. Develop a report on the use of optical fibres in the medical field.
43. Compare and contrast disposable cameras and conventional cameras. Summarize your findings.
44. Interview an optometrist and develop a report that summarizes the relative merits of eye glasses versus contact lenses.

Making Connections

45. The windows of some large buildings are coated with a very thin of layer of gold. Why was this material used? From an economic point of view, is the use of gold justifiable for this purpose?
46. "People should not be viewed by means of a "see-through" mirror, unless they have been informed of its use." Debate this issue in terms of a person's right to privacy.
47. Research the various factors that are considered by jewelers as they determine the best positions and angles to cut the facets on diamonds and other precious stones.
48. Identify the different methods of correcting defects in the human eye related to the cornea and the crystalline lens inside the eye. Propose a set of criteria that could be used to determine which of the different methods should be used for each of the defects, and justify your choice of criteria. Present your findings in an appropriate format.

Problems for Understanding

Show complete solutions for all problems that involve equations and numbers.

49. Draw light ray diagrams to show the characteristics of the image formed by a concave mirror for an object placed at (a) 0.5 f, (b) 1.75 f, and (c) 3.0 f. State the characteristics of the image in each case.
50. A ray of light striking a plane mirror makes an angle of 42° with the mirror surface. What is the angle of reflection to the normal?
51. Light is shining on to a plane mirror at an angle of incidence of 12° to the mirror surface. If the plane mirror is tilted such that the angle of incidence is halved, what will be the total change in the angle of reflection from the original reflected light?
52. A student walks towards a plane mirror at a speed of 1.2 m/s.
    (a) Determine the speed of the image relative to the student when the direction is directly towards the mirror.
    (b) Determine the speed of the image relative to the student when the direction is at an angle of 60.0° with respect to the normal to the mirror.
53. A concave mirror has a focal length of 35 cm. What is the radius of curvature of the mirror?

54. A convex mirror has a radius of curvature of 1.8 m. What is its focal length?

55. Draw scaled light ray diagrams to determine the image characteristics for an object 2.00 cm high when it is placed in front of a concave mirror with a focal length of 18.0 cm at a distance of
(a) 12.0 cm
(b) 24.0 cm
(c) 40.0 cm
In each of parts (a)–(c), determine the image characteristics using the mirror equations.

56. Draw scaled light ray diagrams to determine the image characteristics for an object 2.00 cm high when it is placed in front of a convex mirror with a focal length of 20.0 cm at a distance of
(a) 12.0 cm
(b) 30.0 cm
(c) 50.0 cm
In each of parts (a)–(c), determine the image characteristics using the mirror equations.

57. A small vase is placed 40.0 cm in front of a concave mirror of focal length 15.0 cm. Determine the characteristics of the image, and its magnification.

58. A can on a shelf in a convenience store is 4.50 m away from the surface of a convex mirror on a nearby wall. The mirror has a focal length of 50.0 cm. Use the mirror equations to determine the characteristics of the image, and its magnification.

59. Determine the speed of light in a solid that has an index of refraction of 1.87.

60. Determine the time it takes for light to travel 35 cm through the water in an aquarium.

61. What is the index of refraction a medium in air if the angle of incidence is 68˚, and the angle of refraction is 42˚?

62. What is the angle of refraction if the angle of incidence is 55˚, and the index of refraction of the medium is 1.92?

63. Light passes from hydrogen into sodium chol-ride at an angle of incidence of 57˚. What is the angle of refraction?

64. Yellow light travels from air into a liquid at an angle of incidence of 35.0˚ and an angle of refraction of 20.0˚. Calculate the wavelength of the yellow light in the liquid if its wavelength in air is 580 nm.

65. The speed of light in a clear plastic is 1.9×10^8 m/s. A beam of light strikes the plastic at an angle of 24˚. At what angle is the beam refracted?

66. A diver is standing on the end of a diving board, looking down into 5.5 m of water. How far does the bottom of the pool appear to be from the water's surface?

67. A block of glass has a critical angle of 46.0˚. What is its index of refraction?

68. The critical angle for a special glass in air is 40˚. What is the critical angle if the glass is immersed in water?

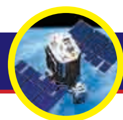
69. Light is travelling from a diamond into the block of plexiglass in which it is embedded. Determine the critical angle of the light in the diamond.

70. The index of refraction of crown glass is 1.51 for red light, and 1.53 for violet light.
(a) What is the speed of red light in crown glass?
(b) What is the speed of violet light in crown glass?

71. Calculate the magnification if an object 12.0 cm tall is placed 25.0 cm from a convex lens with a focal length of 15.0 cm. Describe the image that is formed.

72. The image formed by a concave lens is 24.0 cm tall and 42.0 cm from the lens. Find the size and location of the object if focal points are 60.0 cm from the lens.

COURSE CHALLENGE



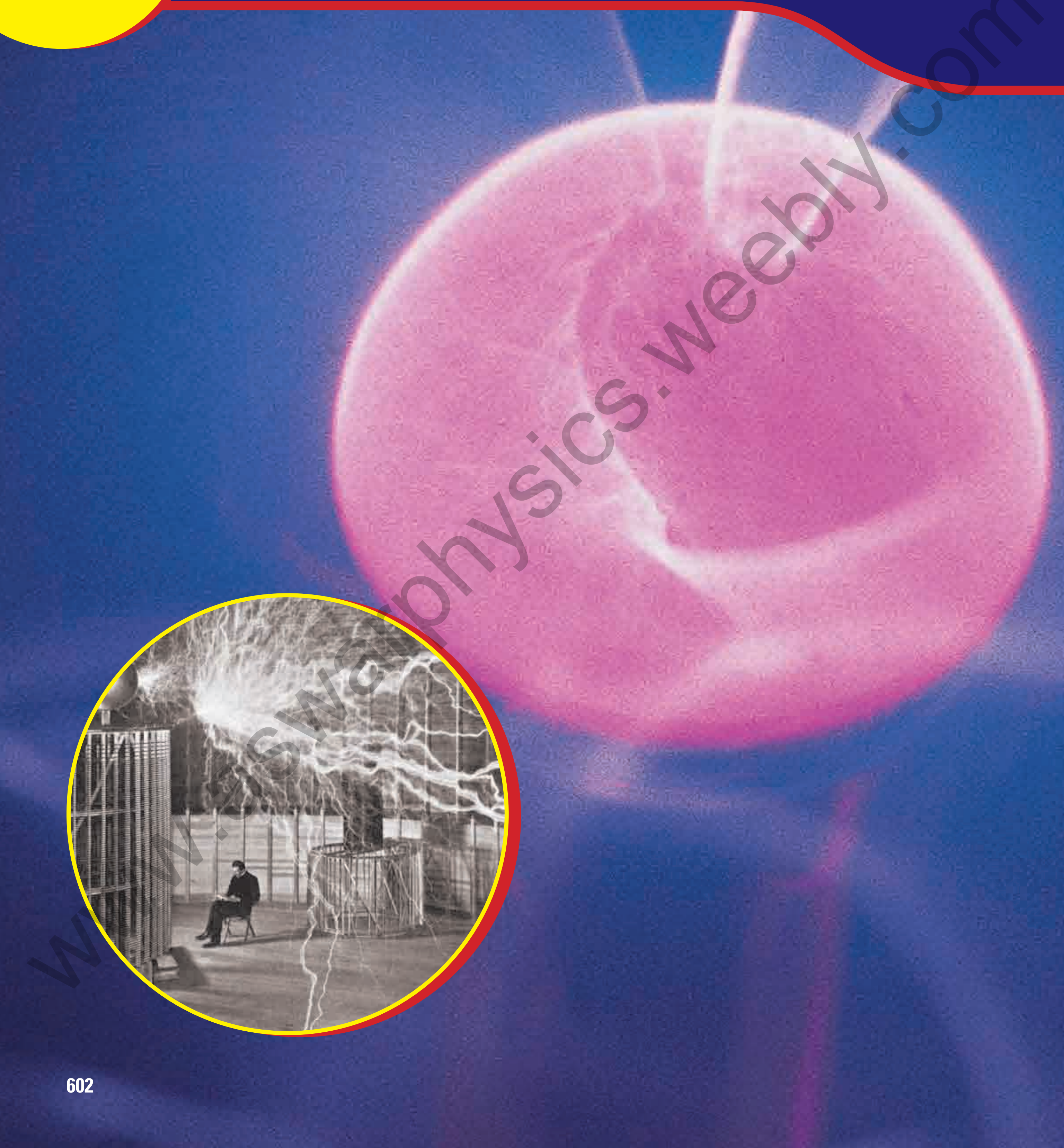
Space-Based Power

Continue to plan for your end-of-course project by con-sidering the following:

- How are the properties and uses of mirrors and lenses related to your project?
- How can you incorporate newly learned skills such as drawing schematic or ray diagrams into your project?
- Analyze the information contained within your research portfolio to identify knowledge or skills gaps that should be filled during the last unit of the course.

UNIT 5

Electricity and Magnetism



OVERALL EXPECTATIONS

DEMONSTRATE an understanding of the principles and laws related to electricity and magnetism.

INVESTIGATE properties of magnetic fields.

DESCRIBE technologies developed on the basis of the scientific understanding of magnetic fields.

UNIT CONTENTS

The lightning bolts in the photograph are radiating from a Tesla coil, invented in 1891 by Nikola Tesla (1856–1943), a Croatian-born U.S. inventor. In 1899, Tesla successfully generated lightning bolts more than 40 m in length — the largest ever artificially created. Unfortunately, Tesla's work set the town's electric generators on fire, and his experiments were cut short.

As a university student, Tesla earned the scorn of his teachers and classmates by arguing that the direct current (DC) electrical systems proposed by Thomas Edison's electric company were too inefficient. Tesla proposed that alternating current (AC) systems were the only practical way to transmit electric energy. As you proceed through this unit, you will learn why Tesla was correct. At the time, no one had ever built an AC dynamo (generator) that ran efficiently. With the backing of George Westinghouse, Tesla invented and built an efficient and inexpensive AC dynamo. As a result, Westinghouse was able to undercut Edison's bid to harness Niagara Falls for power, by about 50%. Tesla built the first power station at Niagara Falls and then transmitted electric energy 35 km to Buffalo. The statue of Tesla at Niagara is a testament to his genius as an inventor and scientist. In this unit, you will explore this and more discoveries that have led to our understanding of magnetism, electricity, and electromagnetism.

UNIT PROJECT PREP

Look ahead to pages 746–747. You will design and build a simple electric motor. Begin to think about questions like:

- What is the electromotive force?
- What design modifications will make your motor more efficient?

CHAPTER 13

Electric Energy and Circuits

CHAPTER CONTENTS

The massive conductors of a power transmission line carry huge currents at high potentials from power stations to your community. The hair-like lines on the printed circuit board in the photograph are the conductors that carry the microcurrents inside your computer and portable music system. Regardless of size, the conductors serve one purpose—to move electric energy, with minimal loss, to a device that will transform it into a desired form of energy.

To ensure that all communities have the electric energy they need, physicists and electrical engineers must know exactly how the properties of the conductor affect the current inside it. To design the circuit board for a computer, an electronic engineer must know how resistance can be used to control the amount of current to the branches of the circuit, so that each computer component gets the correct amount of current at the correct voltage.

In this chapter, you will develop an understanding of how and why electric circuits behave as they do. You will revisit the relationships among potential difference, current, and resistance. You will learn how to diagram, connect, and analyze complex electric circuits to learn how much energy is available to each of the circuits' loads.

INVESTIGATION 13-A

Potential Differences along Current-Carrying Conductors

TARGET SKILLS
- Manipulating and recording
- Analyzing and interpreting
- Communicating results

Have you ever wondered why birds can sit on bare, high voltage power lines without being electrocuted? After you have completed this lab, you should be able to explain why birds seem unaffected by the current in the wire.

Potential difference (or voltage) is a measure of the electrical effort that is being exerted on a system. If a voltage exists between two points on a conductor then a current will flow between those points.

Problem

On a current-carrying conductor, how does the potential difference between two points on the conductor vary with length between the points?

Equipment

- power supply
- metre stick
- Nichrome™ wire (22 gauge)
- insulated conductors with alligator clips
- voltmeter
- thumbtacks

Procedure

1. Connect the apparatus as shown in the diagram.
2. Attach the leads from the voltmeter to the wire near the opposite ends of the metre stick. Have your teacher confirm that the voltmeter is connected correctly.
3. Increase the voltage from the power supply until the voltmeter reads about 2.5 V. Record the length of wire between the clips and the voltmeter reading in a data table.

CAUTION As long as you touch the alligator clips with only one hand you can move them along the wire with no danger of shock.

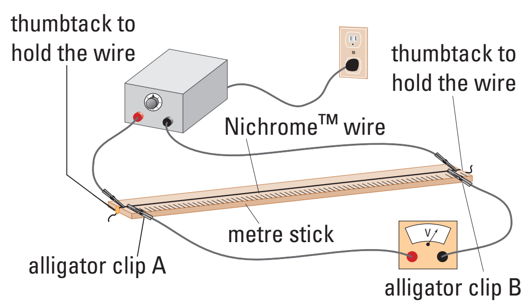
4. Without adjusting the power supply, move one alligator clip along the wire about 15 cm closer to the other and record the length between the clips and the voltmeter reading. Repeat until the length between the clips is zero.
5. Plot a graph of the line of best fit for voltage versus length between clips.

Analyze and Conclude

1. What does the graph tell you about the relationship between voltage and length between clips?
2. In light of your findings, explain why birds are not electrocuted when they sit on bare power lines.

Apply and Extend

3. How is the voltage affected if you move both clips along the wire keeping them a constant distance (say, 15 cm) apart?
4. Does increasing the voltage affect the nature of the result? Try the experiment with the voltage set at 4.0 V.



13.1 Electric Potential Difference

SECTION EXPECTATIONS

- Define and describe electric potential difference.
- Analyze in quantitative terms, problems involving electric potential difference and electric charge.

KEY TERMS

- conductor
- insulator
- electrostatics
- voltaic cell
- battery
- electrode
- electrolyte
- anode
- cathode
- gravitational potential difference
- potential difference

PHYSICS FILE

In addition to insulators and conductors, there is a group of materials called "semiconductors." This group includes substances such as silicon and germanium. Semiconductors, used in the construction of computer chips, make it possible to build the miniature electronic devices now on the market. "Silicon Valley" is the nickname for the area in California where its computer industry is centred. Ottawa, a centre for much of Canada's high-tech industry, has acquired the nickname "Silicon Valley North."

When you comb your hair with a plastic comb, the comb becomes electrically charged and will attract bits of paper. If your comb is made of metal, however, the bits of paper are unaffected by the comb when it is held close to them. Why do metal combs *not* become charged?

Figure 13.1 Combing your hair with a plastic comb results in an electrostatic charge on the comb

Conductors and Insulators

Stephen Gray (1696–1736), an English scientist, made the first recorded explanation of electric conduction in 1729. He classified materials as **conductors** and **insulators**, depending on their ability to allow charges to flow. Although it was a novel idea for him, you probably take for granted that, in general, conductors are metals and insulators are non-metals. Gray also identified Earth as a conductor and gave us the term "ground" to mean "provide a path for charge to escape." Even though Earth is not generally thought of as a metal, it is still a very good conductor, due to its size and the ions dissolved in the moisture in the soil.

The Voltaic Pile

Gray's discovery marked the first step of the journey from **electrostatics** (the study of charges at rest) to the control of electric current. The second, more crucial, step occurred in 1800, when Italian physicist Alessandro Volta (1745–1827) invented the electrochemical cell. Volta discovered that if he placed a layer of salt-water-soaked paper between disks of two different metals, such as silver and zinc, an electric charge appeared on each of the metal disks. When he made a pile of these cells (for example,

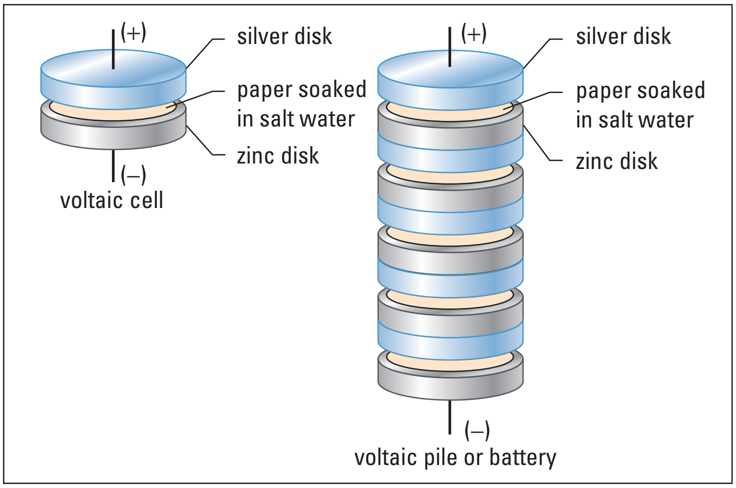


Figure 13.2 The voltaic pile, or battery, supplied scientists with a source of continuous charge flow. For the first time, scientists were able to experiment with steady currents, rather than with the brief bursts of intense charge flow provided by electrostatic generators.

Figure 13.3 Alessandro Volta was a professor at the University of Pavia when he invented the electrochemical (voltaic) cell. For his invention, Napoleon made him a Count of the French Empire. The unit of potential difference, the volt, was named in his honour.

silver/paper/zinc/silver/paper/zinc), the electric strength increased. One pair of such disks became known as a **voltaic cell**; the stack of disks became known as a voltaic pile or **battery** (see Figure 13.2). The metal plates in the cell are the **electrodes**, while the solution between them is the **electrolyte**.

Using an electroscope, Volta determined that the charges on the silver disk of his cell were positive, and the charges on the zinc disk were negative. Since the electron had not yet been discovered, physicists had no way of knowing what positive and negative charges actually were, or which type of charge was moving when they connected conductors to the poles of a voltaic pile. They agreed on the convention that positive charges were moving in electric conductors. Consequently, the positive pole of the battery must be considered to be at a higher electric potential energy than the negative pole. The positive pole would be repelling the positive charges and pushing them "downhill" toward the negative pole. Hence, the positive pole of the battery became known as the **anode** (Greek for "upper path") and the negative pole became the **cathode** (Greek for "lower path").

Language Link

The term "electrode" is derived from two Greek words, *elektron* for "amber" and *hodos* for "way" or "path." Electrode literally means "electric path." How did the Greek word for "amber," which is fossilized resin, become associated with electricity?

Potential Difference

If you imagine a model in which a positive charge moving through a circuit is going downhill, then the battery is analogous to a ski lift taking the charge back to the top of the hill. When you ride a ski lift from the bottom to the top of a hill, the lift uses energy from its motor's fuel and transforms that energy into gravitational potential energy of your body. You probably have gained a

ELECTRONIC LEARNING PARTNER

Go to your Electronic Learning Partner to enhance your learning about electric charge.

PHYSICS FILE

An early theory of electricity suggested that substances had an "electric fluid" within them. The belief was that when two objects were rubbed together, friction would cause some of that fluid to be rubbed off, leaving one of the objects with an excess of the electric fluid and the other object with a deficiency of the fluid. Benjamin Franklin (1706–1790) was doing experiments in 1746 to verify the "fluid theory" of static electricity. Franklin arbitrarily decided that a glass rod rubbed with silk would gain electric fluid from the silk and become "charged with a positive amount of electric fluid." Simultaneously, the silk was "charged with a negative amount of electric fluid." Later, this came to be known simply as "positively charged" and "negatively charged." More than a century later, when the electron was finally discovered, it was found that the electron was repelled by objects that previously had been defined as negatively charged. Hence, today we think of the electron as having a negative charge and the proton as having a positive charge.

different amount of gravitational potential energy from other skiiers during your ride. However, you all gained exactly the same amount of gravitational potential energy per kilogram of your body mass.

By defining the **gravitational potential difference** as the difference in gravitational potential energy per unit mass, $\Delta E_p/m$, you can develop a term that no longer depends on an object's (skier's) mass. Gravitational potential difference depends only on the height of the hill (h) and the acceleration due to gravity (g).

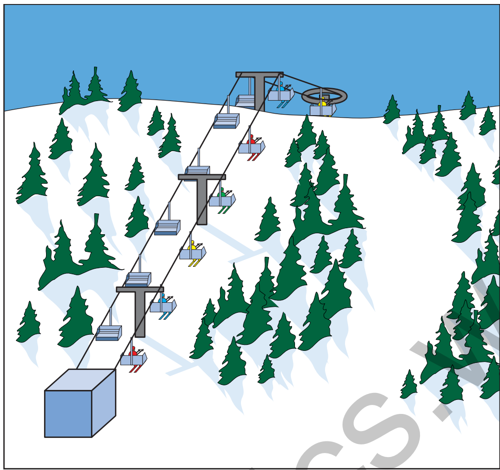


Figure 13.4 A skier of mass *m* riding up a ski lift to the top of a hill gains a potential energy of ΔE_g.

Think It Through

- Think about the skier mentioned in the caption of Figure 13.4. Write the equation for gravitational potential energy. Use the equation to show that the gravitational potential difference, ΔE_g/m, depends on the height (h) of the hill and the gravitational acceleration (g), not the mass of the skier.
- How would the gravitational potential difference change under the following circumstances?
  - **(a)** The skier went three times as far up the hill.
  - **(b)** The skier's mass doubled.
  - **(c)** The skier skied only halfway down the hill.

The skiers on our electric hill are similar to positive charges. The chemical action inside a voltaic cell takes positive charges from the cathode (bottom of the electric hill) to the anode (top of the electric hill), giving them electric potential energy. There is no special term or symbol for gravitational potential difference, but there is a special term for electric potential difference. The difference in electrical potential energy (ΔE_Q) per unit charge (Q) is defined as the **potential difference** (V), sometimes called the "voltage" of the cell, battery, or power supply.

DEFINITION OF ELECTRIC POTENTIAL DIFFERENCE

The electric potential difference between any two points in a circuit is the quotient of the change in the electric potential energy of charges passing between those points and the quantity of the charge.

$$V = \frac{\Delta E_Q}{Q}$$

Quantity	Symbol	SI unit
electric potential difference	V	V (volt)
change in electrical potential energy	ΔE_Q	J (joule)
quantity of charge	Q	C (coulomb)

Unit Analysis

$$\frac{\text{(electric potential energy)}}{\text{(quantity of charge)}} = \frac{\text{J}}{\text{C}} = \text{V}$$

Note: One joule per coulomb is equivalent to one volt.

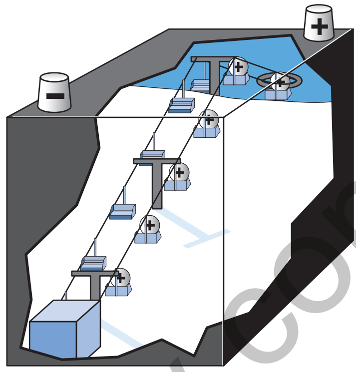


Figure 13.5 The chemical action of the cell gives electric potential energy to the charges deposited on the anode and cathode. This creates a potential difference between the anode and cathode.

One volt (1 V) of potential difference across the poles of a cell is created when the chemical action inside the cell does one joule (1 J) of work on each coulomb of charge (1 C) that it lifts internally from the cathode to the anode.

MODEL PROBLEM

Energy and Potential Difference

A battery has a potential difference of 18.0 V. How much work is done when a charge of 64.0 C moves from the anode to the cathode?

Frame the Problem

- Since the battery has a *potential difference* between the poles, chemical reactions in the battery did *work* to *separate* positive and negative *charges*.
- The *work done* transforms chemical potential energy into *electric potential energy*. Thus, the electric potential energy is equal to the amount of *work* done.
- The expression that defines *potential difference* applies to this problem.

Identify the Goal

The amount of work, W, done to separate charges

continued ▶

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Variables and Constants

Involved in the problem	Known	Unknown
W	$V = 18.0\ \text{V}$	W
ΔE_Q	$Q = 64.0\ \text{C}$	ΔE_Q
V		
Q		

Strategy / Calculations

Use the expression for potential difference.

$$V = \frac{\Delta E_Q}{Q}$$

Substitute first

$$18.0\ \text{V} = \frac{\Delta E_Q}{64.0\ \text{C}}$$

Solve for ΔE_Q.

$$(120\ \text{V})(1440\ \text{C}) = \frac{\Delta E_Q}{\cancel{1440\text{C}}}\cancel{1440\ \text{C}}$$

$$\Delta E_Q = 1150\ \text{VC}$$

$$\Delta E_Q = 1150\ \text{J}$$

Solve for ΔE_Q first

$$VQ = \frac{\Delta E_Q}{\cancel{Q}}\cancel{Q}$$

$$\Delta E_Q = (18.0\ \text{V})(64.0\ \text{C})$$

$$\Delta E_Q = 1150\ \text{VC}$$

$$\Delta E_Q = 1150\ \text{J}$$

Since $1\frac{\text{J}}{\text{C}}$ is equivalent to 1 V, then

$$\text{V} = \frac{\text{J}}{\text{C}}$$

$$\text{VC} = \frac{\text{J}}{\cancel{\text{C}}}\cancel{\text{C}}$$

$$\text{VC} = \text{J}$$

The work done is the same as the potential energy.

$$W = \Delta E_Q$$
$$\Delta E_Q = 1150\ \text{J}$$
$$W = 1150\ \text{J}$$

If a charge of 64.0 C is transferred by a potential difference of 18.0 V, then 1.15×10^3 J of work are done.

Validate

The units cancel to give joules, which is the correct unit for work.

PRACTICE PROBLEMS

1. What is the potential difference of a battery if it does 7.50×10^{-2} J of work when it moves 3.75×10^{-3} C of charge onto the anode?
2. A 9.00 V battery causes a charge of 4.20×10^{-2} C to move through a circuit. Calculate the work done on the charge.
3. A 12 V battery does 0.75 J of work on a quantity of charge it moved through a circuit. Calculate the amount of charge that was moved.

CAREERS IN PHYSICS

TARGET SKILLS
- Communicating results
- Conducting research

Sara Goodchild is a science editor, and also a published author of science articles. She graduated from university with a degree in chemistry, and has combined her interests in both writing and science in her present career.

Robotics, global warming, the space station, genetically modified organisms, and cloning — these are just a few of the hot topics being reported in the media. The demand for science writers is increasing as radio, television, magazines, newspapers, encyclopedias, the Internet and even texts such as this one publish more and more reports on science. Science reporters must be able to distinguish between good and bad science and then present their findings in a clear well-written manner so that their point of view can be understood by the public.

Right now is a great time to be a science writer. Many science topics are becoming increasingly controversial. The ability to research stories and present an accurate balanced report will be extremely difficult and ever more important.

If you have an interest in science and a talent for writing, you may have a career in science journalism.

Going Further

1. Volunteer with a scientific organization like the Royal Astronomical Society to gain experience in writing and editing.
2. Attend meetings of and/or join professional organizations such as the Science Writers' Association of Canada, Periodical Writers' Association of Canada or the Editors' Association of Canada.
3. Submit reports for your local paper on science events such as the Science Fair.

13.1 Section Review

1. MC Why was very little known about current electricity and potential difference before the time of Alessandro Volta?
2. C Explain the difference between electric potential energy and electric potential difference.
3. K/U Which of the following changes would increase the gravitational potential energy of every skier at the top of a chair lift compared to the bottom of the lift?
   (a) Increase the number of runs to accommodate more skiers.
   (b) Extend the top of the lift to a location 20 m higher up the mountain.
   (c) Install a new high-speed quad lift to carry more skiers to the top of the lift at a higher rate.

   Explain your reasoning.
4. C Develop another analogy, different from the ski lift, that would help a classmate understand the concept of electric potential difference.

13.2 Electric Current

SECTION EXPECTATIONS

- Define and describe electric current.
- Describe two conventions used to denote the direction of movement of electric charge.
- Use a circuit diagram to model and quantitatively predict the movement of elementary charge.

KEY TERMS

- current
- electron flow
- elementary charge
- open circuit
- closed circuit
- loads
- power supply
- circuit elements
- ammeter
- voltmeter
- series
- parallel

Volta's invention of the battery provided other scientists with a source of constant electric current for the first time. As a result, many other discoveries relating to current electricity followed quickly. Less than 25 years after Volta published his findings, scientists such as Ohm, Oersted, and Ampère published the results of their experiments, opening the door to the age of electricity.

Electric Current

To develop an understanding of the flow of electric charge, you can compare it to the flow of water. If you were asked to describe the flow of water over Niagara Falls, you might give your answer in litres per second or cubic metres per second. In an electric conductor, **current** (I) is described as a quantity of charge (Q) passing a given point during an interval of time (Δt).

ELECTRIC CURRENT

Electric current is the quotient of the quantity of charge that moves past a point and the time interval during which the charge is moving.

$$I = \frac{Q}{\Delta t}$$

Quantity	Symbol	SI unit
current	I	A (ampere)
amount of charge	Q	C (coulomb)
time interval	Δt	s (second)

Unit Analysis

$$\frac{\text{coulomb}}{\text{second}} = \frac{\text{C}}{\text{s}} = \text{A}$$

Note: One coulomb per second is equivalent to one ampere.

Figure 13.6 In Niagara Falls, Ontario, the rate of water flow over the Canadian (Horseshoe) Falls is approximately 2.25×10^6 L/s.

MODEL PROBLEM

Electric Current and Charge

The electrical system in your home operates at a potential difference of 120.0 volts. A toaster draws 9.60 A for a period of 2.50 min to toast two slices of bread.

(a) Find the amount of charge that passed through the toaster.

(b) Find the amount of energy the toaster converted into heat (and light) while it toasted the bread.

Frame the Problem

- Power lines transport *electric energy* to your home and provide a constant *potential difference*.
- When the toaster is connected to the power source and turned on, the *potential difference* drives a *current* through the toaster elements.
- As *charges* pass through the element, *electric energy* is converted into *heat*.
- The amount of energy that was converted into heat is the same as the *change* in the *potential energy* of the *charges* as they pass through the toaster.

Identify the Goal

The amount of charge, Q, that passes through the toaster elements in a given time

The amount of energy, ΔE_Q, converted into heat (and light)

Variables and Constants

Involved in the problem		Known	Unknown
V	Q	$V = 120.0$ V	Q
I	ΔE_Q	$I = 9.60$ A	ΔE_Q
Δt		$\Delta t = 2.50$ min	

Strategy

Use the definition for current to find the amount of charge.

$$I = \frac{Q}{\Delta t}$$

Convert time to SI units.

$$2.5\ \cancel{\text{min}} \frac{60\ \text{s}}{\cancel{\text{min}}} = 150\ \text{s}$$

Calculations

Substitute first

$$9.60\ \text{A} = \frac{Q}{150\ \text{s}}$$

$$(9.60\ \text{A})(150\ \text{s}) = \frac{Q}{\cancel{150\ \text{s}}}\cancel{150\ \text{s}}$$

Solve for *Q* first

$$(I)(\Delta t) = \frac{Q}{\cancel{\Delta t}}\cancel{\Delta t}$$

$$Q = (9.60\ \text{A})(150\ \text{s})$$

1 A · s is equivalent to 1 C.

Substitute first: $Q = 1440\ \text{A} \cdot \text{s}$, $Q = 1440\ \text{C}$

Solve for *Q* first: $Q = 1440\ \text{A} \cdot \text{s}$, $Q = 1440\ \text{C}$

continued ►

continued from previous page

(a) In 2.5 min, 1.44×10^3 C of charge pass through the toaster.

Strategy

Find the change in potential energy of the charges by using the definition of potential difference.

Calculations

$$V = \frac{\Delta E_Q}{Q}$$

Substitute first

$$120\text{ V} = \frac{\Delta E_Q}{1440\text{ C}}$$

$$(120\text{ V})(1440\text{ C}) = \frac{\Delta E_Q}{\cancel{1440\text{ C}}}\cancel{1440\text{ C}}$$

A V · C is equivalent to a J.

$$\Delta E_Q = 1.73 \times 10^5\text{ V} \cdot \text{C}$$

$$\Delta E_Q = 1.73 \times 10^5\text{ J}$$

Solve for ΔE_q first

$$VQ = \frac{\Delta E_Q}{\cancel{Q}}\cancel{Q}$$

$$\Delta E_Q = (120\text{ V})(1440\text{ C})$$

$$\Delta E_Q = 1.73 \times 10^5\text{ V} \cdot \text{C}$$

$$\Delta E_Q = 1.73 \times 10^5\text{ J}$$

The toaster converted 1.73×10^5 J of electric energy into heat and light while it toasted the bread.

Validate

The units combined to give joules, which is correct for energy. Also, appliances that generate heat, such as a toaster, typically draw a larger current and consume more energy than devices that generate light, such as a light bulb.

PRACTICE PROBLEMS

4. A battery sends a 2.25 A current through a circuit for 1.50 min. If a total of 8.10×10^2 J of work was done by the current, what was the potential difference of the battery?
5. How long would it take a 17 V battery, sending a 5.0 A current through a circuit, to do 680 J of work?
6. How much work is done by a 25.0 V battery when it drives a 4.70 A current through a circuit for 36.0 s?
7. If a 160 V battery did 9.6×10^5 J of work in 2 min, what was the current?
8. A light draws a current of 0.48 A. How long must it be left on for charge of 36 C to pass through it?
9. An electric circuit draws 20 A. If the electric potential drop over the entire circuit is 120 V, calculate the total charge passing through the circuit in 1 h.
10. A celluar phone battery is recharged in 0.25 h after receiving 2.5×10^3 C of charge. Calculate the amount of electric current that the battery draws during recharging?
11. A physics student wishes to determine the amount of electric energy consumed in one day at his school as a result of classroom and hallway lighting. A quick survey revealed that there were approximately 200 40W fluorescent lights operating under a potential difference of 240 V for 16 hours each day.

Current versus Electron Flow

Although physicists began to study and use electric current around 1800, it was not until 1876 that an experiment at Harvard University showed that negative charges were moving in current-carrying conductors. It was another 25 years before J.J. Thomson (1856–1940) discovered the electron, and experiments demonstrated that the moving negative charges were electrons. By this time, the concept of a positive current was entrenched in scientific theory and literature. Fortunately, as long as you use a constant frame of reference, circuit analysis does not depend on knowing whether it is actually positive or negative charges that are moving. All measurable effects, such as the amount of energy transformed, are the same whether positive charges move one way or negative charges move the other way. Today, the term current (I) means the flow of positive charge (from anode to cathode) in a circuit. The flow of negative charge (from cathode to anode) is called **electron flow**. Since a wealth of theory was developed using positive current, the convention for analyzing circuits is still to use positive or conventional current.

Not all charges that move do so inside metals. In other media, either negative or positive (or both) charges can move. The aurora borealis lights up the sky when high-energy electrons from the sun collide with gas molecules in the air and are captured by Earth's magnetic field (see Figure 13.7).

In the process of electroplating with an aqueous salt solution such as silver cyanide (Figure 13.8), the positive silver ions (Ag^+) are attracted to the negative electrode, and the negative cyanide ions (CN^-) are attracted to the positive electrode.

Figure 13.7 The aurora borealis

Figure 13.8 A less expensive metal can be silver-plated to produce an attractive and corrosion-resistant surface. The object to be plated is connected to a circuit as the cathode. It is suspended in a solution containing silver cyanide. The silver ions are attracted to the cathode, where they combine with electrons and become solid silver atoms that remain permanently attached to the surface of the cathode.

COURSE CHALLENGE

Free Energy?

Investigate the efficiency of photovoltaic cells using a small electric toy and photovoltaic cells from a local electronics shop. Learn more from the **Science Resources** section of the following web site: ***www.school.mcgrawhill.ca/resources/***

PHYSICS FILE

During the last 20 years of the 1800s, physicists discovered that light had the ability to cause certain metals to emit negative charges. By 1905, Albert Einstein had created an hypothesis for the cause of, and formulated a law for, the photoelectric effect. In 1916, Robert Andrews Millikan carried out very careful and precise experiments in which he confirmed Einstein's predictions. In 1921, Einstein was awarded the Nobel Prize for "services to Theoretical Physics and the discovery of the law of the photoelectric effect."

Current and the Elementary Charge

Robert Andrews Millikan (1868–1953), a U.S. physicist, won the Nobel Prize in Physics in 1923 for his discovery of the elementary charge and for his research on the photoelectric effect. In 1917, his "oil-drop experiment" revealed that the static charge on a microscopic oil drop was always a whole-number (integral) multiple of a minute electric charge that was fixed in size. He concluded that the minute charge was the smallest size in which electric charge could be found. He designated this minute amount of charge the **elementary charge** (e). His measurements revealed that the size of one elementary charge is $e = 1.60 \times 10^{-19}$ C. (The most precise measurement to date is $e = 1.602\ 177\ 33 \times 10^{-19}$ C.) Today, one elementary charge is known to be the magnitude of the charge on a proton (+1 e) or an electron (–1 e).

When J.J. Thomson (see Figure 13.9) discovered the electron in 1897, he was able to measure only the ratio of the charge to the mass. Many scientists were sceptical about Thomson's proposed charge-carrying particle. They still thought that electric charge might be a fluid that could be divided into infinitely small pieces. However, when Millikan performed his oil-drop experiment in 1917, he established that when charge moved, it moved only as integral (whole-number) multiples of the elementary charge (e), just as water must be moved by at least one molecule at a time. He confirmed Thomson's hypothesis. Scientists now know that every quantity of charge can be expressed as an integral number of elementary charges.

Figure 13.9 J.J. Thomson devised ingenious experiments showing that the mysterious "rays" that caused phosphorus to glow, were in fact, tiny identical particles — he had discovered the electron. Most televisions still use this technology.

ELEMENTARY CHARGE

The amount of charge is the product of the number of elementary charges (electrons or protons) and the magnitude of the elementary charge.

$$Q = Ne$$

Quantity	Symbol	SI unit
amount of charge	Q	C (coulomb)
number of elementary charges	N	integer (pure number, no unit)
elementary charge	e	C (coulomb)

MODEL PROBLEM

Charge and Electrons

A light bulb draws a current of 0.60 A. If the bulb is left on for 8.0 min, how many electrons (elementary charges) pass through the bulb?

Frame the Problem

- When a *current* exists in a light bulb, *electrons* are passing through it.
- If you know the amount of charge that passes through the light bulb, you can use the magnitude of the *elementary charge* to find the number of electrons.

Identify the Goal

The number (N) of electrons passing through the bulb

Variables and Constants

Involved in the problem	Known	Implied	Unknown
I	$I = 0.60$ A	$e = 1.60 \times 10^{-19}$ C	Q
Δt	$\Delta t = 8.0$ min		N
N			
e			
Q			

Strategy

Use the definition of current to find the amount of charge passing through the light bulb in 8.0 min.

$$I = \frac{Q}{\Delta t}$$

First, convert time to SI units.

$$8.0\ \cancel{\text{min}}\ \frac{60\ \text{s}}{\cancel{\text{min}}} = 480\ \text{s}$$

Calculations

Substitute first

$$0.60\ \text{A} = \frac{Q}{480\ \text{s}}$$

$$(0.60\ \text{A})(480\ \text{s}) = \frac{Q}{\cancel{480\ \text{s}}}\cancel{480\ \text{s}}$$

1 A · s is equivalent to 1 C.

$$Q = 288\ \text{A} \cdot \text{s}$$

$$Q = 288\ \text{C}$$

Solve for Q first

$$(I)(\Delta t) = \frac{Q}{\cancel{\Delta t}}\cancel{\Delta t}$$

$$Q = (0.60\ \text{A})(480\ \text{s})$$

$$Q = 288\ \text{A} \cdot \text{s}$$

$$Q = 288\ \text{C}$$

continued

continued from previous page

Strategy

Use the relationship between amount of charge and the elementary charge to find the number of electrons.

Calculations

$Q = Ne$

Substitute first

$288\ \text{C} = N\ 1.60 \times 10^{-19}\ \text{C}$

$$\frac{288\ \cancel{\text{C}}}{1.60 \times 10^{-19}\ \cancel{\text{C}}} = \frac{N\ \cancel{1.60 \times 10^{-19}\ \text{C}}}{\cancel{1.60 \times 10^{-19}\ \text{C}}}$$

$N = 1.80 \times 10^{21}$

Solve for *N* first

$$\frac{Q}{e} = \frac{N\ \cancel{e}}{\cancel{e}}$$

$$N = \frac{288\ \cancel{\text{C}}}{1.60 \times 10^{-19}\ \cancel{\text{C}}}$$

$N = 1.80 \times 10^{21}$

In the 8.0 min that the light bulb was on, 1.8×10^{21} electrons (elementary charges) passed through it.

Validate

In the first part, the units combine to give coulombs, which is correct for charge. In the second part, the units cancel to give a pure number. This is correct, because there are no units for number of electrons. The answer is extremely large, which you would expect because the number of electrons in one coulomb is exceedingly large: $N = \frac{1\ \cancel{\text{C}}}{1.60 \times 10^{-19}\ \cancel{\text{C}}}$ or 6.26×10^{18} electrons.

PRACTICE PROBLEMS

12. Calculate the current if 2.85×10^{20} elementary charges pass a point in a circuit in 5.70 min.

13. A 16.0 V battery does 5.40×10^{4} J of work in 360.0 s.

(a) Calculate the current through the battery.

(b) Calculate the number of elementary charges that pass through the battery.

14. Calculate the number of elementary charges that pass a point in a circuit when a current of 3.50 A flows for 24.0 s.

15. In transferring 2.5×10^{20} elementary charges in 12 s, a battery does 68 J of work.

(a) Calculate the current through the battery.

(b) Calculate the potential difference of the battery.

Electric Circuits

Suppose a power supply (battery) is connected to a load such as a light bulb. A switch allows you to open and close the circuit. An **open circuit** means there is a break (perhaps an open switch) somewhere in the circuit that prevents current from flowing. A **closed circuit** means that all connections are complete. A closed, or continuous, path exists, allowing current to move around the circuit. You could represent the above circuit by using realistic drawings of the apparatus involved, as shown in Figure 13.10. That technique would be very cumbersome, however. It is much more efficient to represent and analyze electric circuits by using the electric-circuit symbols shown in Figure 13.11. The circuit shown in Figure 13.10 is redrawn in Figure 13.12, using these symbols.

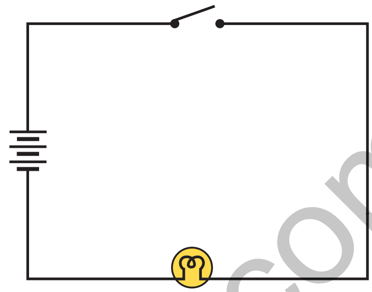


Figure 13.12 This diagram of the same circuit is easier to draw and to analyze.

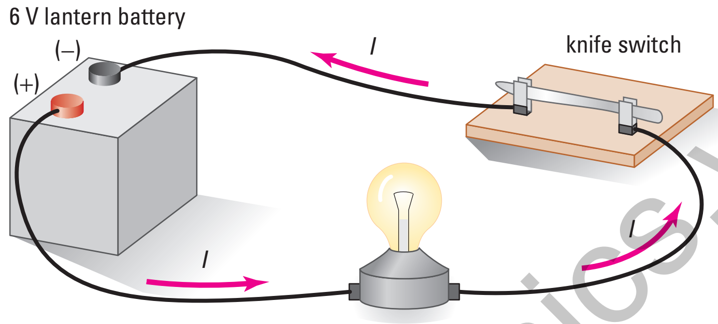


Figure 13.10 A realistic sketch of even a simple circuit is cumbersome.

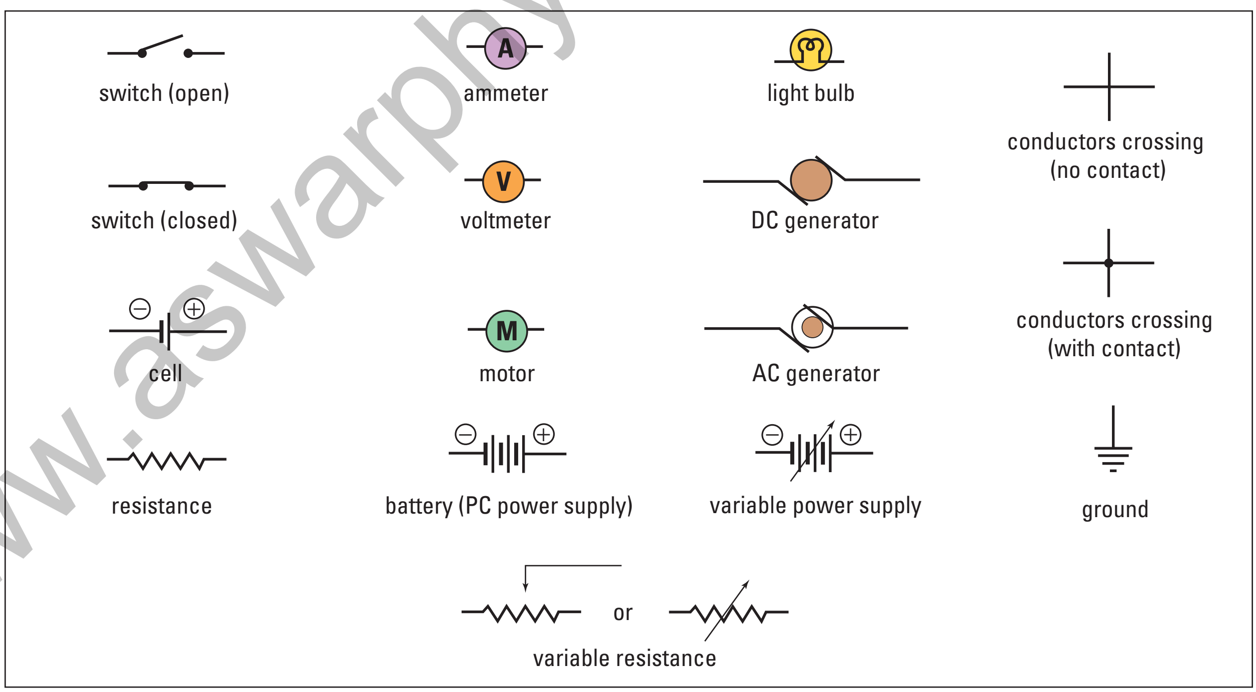
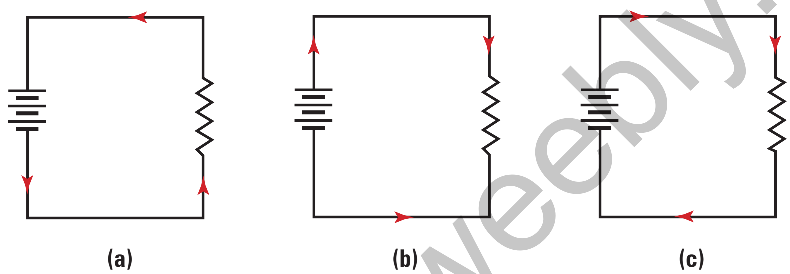


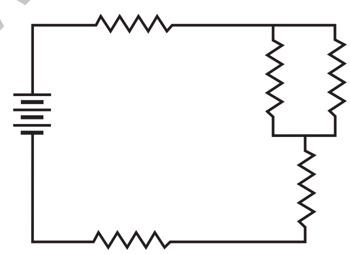
Figure 13.11 Symbols for elements of an electric circuit

Think It Through

- In the circuit symbol for a battery, the longer line represents the positive pole of the battery and the shorter line is the negative pole. In one of the circuits shown here, the arrows represent conventional current. In another, the arrows represent electron flow. One circuit is drawn incorrectly. Neither conventional current nor electron flow could take the directions indicated by the arrows. Analyze the circuits and determine which illustrates conventional current, electron flow, and neither. Explain your reasoning.



- Copy the circuit at the right in your notebook. Add arrows to every branch of the circuit, showing the direction of conventional current.



Ammeters and Voltmeters

To find out what is happening inside the parts of a circuit, scientists use an assortment of devices, such as ammeters, voltmeters, galvanometers, and ohmmeters. A simple circuit is composed of **loads** (for example, light bulbs, resistances, motors) and a **power supply** (cell, battery, or an AC or DC generator). These **circuit elements** (loads and power source) may be connected in series or in parallel to each other. A switch is often included but serves only to open or close the circuit. When meters are used to measure current of potential difference, they are connected in a way that will not interfere with the circuit operation. An **ammeter** measures the electric current to or from a circuit element. A **voltmeter** measures the electric potential difference across a circuit element.

Since ammeters measure the current through a circuit element, they must be inserted into the line before or after the circuit element so that all of the current passing through the circuit element also goes through the ammeter. This is called a **series** connection since the current moves through the circuit element and the ammeter one after the other. On the other hand, a voltmeter measures the potential difference from one side of a circuit element to the other. To function properly, voltmeters must be connected to the opposite sides of the circuit element across which you want to know the potential difference. This is a called a **parallel** connection, since the voltmeter presents a path that runs beside the circuit element. Notice that the ammeter is actually part of the circuit. If you disconnect either pole of the ammeter, the circuit is opened. The voltmeter, on the other hand, makes contact with the circuit at two points to measure the potential difference between those points.If you disconnect either pole of the voltmeter, the circuit is still perfectly functional. Figure 13.13 shows the same circuit as in Figure 13.10, with the addition of an ammeter and a voltmeter showing the proper connection.

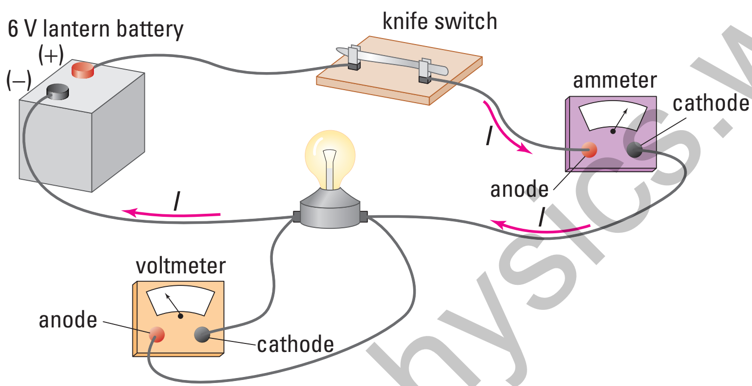


Figure 13.13 Notice the labels indicating the anodes and cathodes of the meters relative to the anode and cathode of the power supply and the direction of the current. How would you connect a voltmeter to measure the potential difference of the battery?

13.2 Section Review

1. **MC** Why do physicists define current, (I), in a circuit to be opposite to the direction in which electrons flow?

2. **K/U** Give an example of a current in which positive charges move.

3. **C** When Millikan measured the amount of charge on oil drops, he obtained data similar to the following. Explain how he used such data to determine (a) that elementary charges existed, and (b) the size of the elementary charge.

   Data: 6.4×10^{-19} C, 1.28×10^{-18} C, 1.92×10^{-18} C, 8.0×10^{-19} C, 1.6×10^{-18} C, 6.4×10^{-19} C, 1.12×10^{-18} C.

4. **C** Explain how a voltmeter must be connected in a circuit in order to measure the potential difference across a light bulb.

13.3 Resistance to Flow of Charge

SECTION EXPECTATION

- Describe the concepts and units related to electricity and magnetism in terms of electron flow.
- Design and conduct an experiment to investigate major variables relating electric potential, current, and resistance.

KEY TERMS

- resistivity
- unit of resistance (ohm)
- non-linear or non-ohmic resistance

Everything that moves experiences frictional forces that resist that motion. The energy used to overcome frictional resistance within mechanical and electrical systems, such as automobile engines and electric motors, costs billions of dollars each year. Like every other system, the frictional effects in conductors offer resistance to the current passing through them. Similar to mechanical systems, friction in electric conductors produces thermal energy that is radiated away from the conductor. This energy is no longer available to do work. Electrical engineers' knowledge of the factors that affect resistance enable them to design systems that reduce energy loss to a minimum.

Figure 13.14 Overcoming resistance is an important aspect of electric-circuit design.

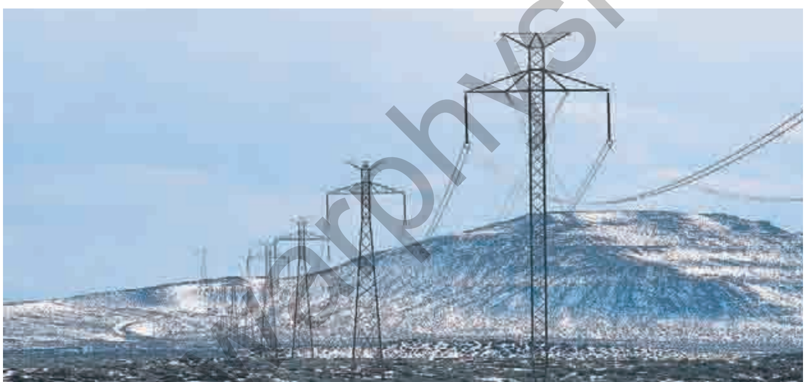
Factors Affecting the Resistance of a Conductor

You can compare a metal conductor carrying electric current to a water pipe carrying water current. For a water pipe of a fixed diameter, the longer the pipe, the greater the drag it exerts on the water passing through it. Similarly, for an electric conductor with a fixed diameter, resistance increases proportionately with the *length* ($R \propto L$). Thus, a 2 m length of a particular conductor has twice as much resistance as a 1 m length of the same conductor.

For two conductors of equal length, changing the cross-sectional area changes the resistance. Again, for a water pipe, the bigger the cross-sectional area of the pipe, the lower the drag on the water moving inside it. It is the cross-sectional area that provides the space in which the current travels. Therefore, doubling the cross-sectional area doubles the space for the current to move and halves the resistance. For electric conductors, resistance varies inversely as the *cross-sectional area* ($R \propto 1/A$). For very long extension cords, the resistance due to the increased length of the cord can cause a significant energy loss. To lower the resistance, long conductors are made of thicker wire, which increases cross-sectional area and thus reduces resistance. Conversely, light bulb filaments must have a large resistance so that the energy will be transformed into light and thermal energy. To increase resistance, filaments are made very short and very thin.

Think It Through

- Consider the different electric cords typically found around the home, such as the cords on an iron, lamp, television set, small space heater, and toaster; a standard extension cord; and the cords for plugging in a vacuum cleaner or a car's block heater. Think about the length and thickness of each cord. Explain why each cord has its own specific size. For example, why is a toaster cord shorter and thicker than a lamp cord?
- Often, electric power generating stations are many kilometres from the communities that they serve. The conductors that carry electric energy over many kilometres have a certain amount of electric resistance for every metre of line. You can calculate the amount of power lost to the resistance and the consequent heating of the lines by using the equation $P = I^2R$. Explain how power companies keep their losses of power to a minimum.



If you combine the relationship of the resistance of a conductor to its length and cross-sectional area, the result is $R \propto \frac{L}{A}$. Any proportionality can be written as an equality if a proportionality constant is included. In the case of resistance, the symbol used for this proportionality constant is the Greek letter *rho* (ρ). The equation for the resistance of a conductor can now be written: $R = \rho\frac{L}{A}$. The value of the proportionality constant (ρ) is called the **resistivity** and is a property of the material from which the conductor is made.

PHYSICS FILE

The thickness of wire is called its "gauge." As the gauge of a wire increases, the wire becomes thinner. When electricians wire the circuits of a house, they usually use 14 gauge wire, while the lighter wire in the cord of a small appliance might be 18 gauge wire.

Diameters/Resistances of Some Gauges of Copper Wire

Gauge	Diameter (mm)	Resistance ($\times 10^{-3}\Omega$/m)
0	9.35	0.31
10	2.59	2.20
14	1.63	8.54
18	1.02	21.90
22	0.64	51.70

RESISTANCE OF A CONDUCTOR

The resistance of a conductor is the product of the resistivity and the length divided by the cross-sectional area.

$$R = \rho\frac{L}{A}$$

Quantity	Symbol	SI unit
resistance	R	Ω (ohm)
resistivity	ρ	$\Omega \cdot$ m (ohm metres)
length of conductor	L	m (metres)
cross-sectional area	A	m^2 (square metres)

Unit Analysis

$$(\text{ohm metres})\frac{\text{metres}}{\text{square metres}} = \Omega \cdot \cancel{\text{m}}\frac{\cancel{\text{m}}}{\cancel{\text{m}^2}} = \Omega$$

The resistance of a conductor with a particular length and cross-sectional area depends on the *material* from which it is made. At room temperature, copper is one of the best conducting (lowest resistance) metals. Table 13.1 includes resistivity values for carbon and germanium, which are semiconductors, and for glass, which is an insulator. Insulators are sometimes thought of as conductors with extremely high resistances. By examining Table 13.1, you can see that glass has about 10^{18} to 10^{22} times the resistance of copper.

Table 13.1 Resistivity of Some Conductor Materials

Material	*Resistivity, ρ ($\Omega \cdot$ m)
silver	1.6×10^{-8}
copper	1.7×10^{-8}
aluminum	2.7×10^{-8}
tungsten	5.6×10^{-8}
Nichrome™	100×10^{-8}
carbon	3500×10^{-8}
germanium	0.46
glass	10^{10} to 10^{14}

*Values given for a temperature of 20°C

Finally, the *temperature* of the conductor affects the resistance. The electrons that move inside a metallic conductor are the electrons from the outermost orbit of the atoms of the metal. Thus, they are the electrons that are most loosely held by the atoms of the metal. These outermost electrons of good conductors can move quite freely within the metal, behaving much like the molecules of a gas. As you heat the metal, these electrons begin to move more randomly at higher speeds inside the metal. As a result, it is more difficult to organize them into a current. Near 20°C, copper increases its resistance by about 0.39% for each degree of temperature increase. Conversely, lowering the temperature reduces the resistance.

MODEL PROBLEM

Using Resistivity

Calculate the resistance of a 15 m length of copper wire, at 20°C, that has a diameter of 0.050 cm.

Frame the Problem

- The *electric resistance* of a conductor depends on its *length*, *cross-sectional area*, the *resistivity* of the conducting material, and the *temperature*.
- These variables are related by the equation for resistance of a conductor.
- The *resistivity* of copper at 20°C is listed in Table 13.1.

Identify the Goal

Resistance, R, of the copper conductor

Variables and Constants

Involved in the problem		Known	Implied	Unknown
R	A	$d = 0.050$ cm	$\rho = 1.7 \times 10^{-8}\ \Omega \cdot \text{m}$	R
L	ρ	$L = 15$ m		A
d (diameter)				

Strategy

Use the equation relating resistance to resistivity and dimensions of the conductor.

$$R = \rho\frac{L}{A}$$

Convert diameter to SI units. (All others are in SI units.)

$$0.050\ \cancel{\text{cm}}\ \frac{\text{m}}{100\ \cancel{\text{cm}}} = 5.0 \times 10^{-4}\ \text{m}$$

Find the cross-sectional area from the diameter.

$$A = \pi r^2$$

$$r = \frac{d}{2}$$

$$r = \frac{5.0 \times 10^{-4}\ \text{m}}{2} = 2.5 \times 10^{-4}\ \text{m}$$

$$A = \pi(2.5 \times 10^{-4}\ \text{m})^2$$

$$A = 1.96 \times 10^{-7}\ \text{m}^2$$

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Strategy

The values are all known, so substitute into the equation for resistance.

The conductor has a resistance of 1.3 Ω.

Calculations

$$R = (1.7 \times 10^{-8}\ \Omega \cdot \text{m})\frac{15\ \text{m}}{1.96 \times 10^{-7}\ \text{m}^2}$$

$$R = 1.3\frac{\Omega \cdot \cancel{\text{m}} \cdot \cancel{\text{m}}}{\cancel{\text{m}^2}}$$

$$R = 1.3\ \Omega$$

Validate

The units cancel to give ohms, which is correct for resistance. At first glance, a resistance of 1.3 Ω seems large for a 15 m length of copper given that copper is a very good conductor. However, the wire is very fine, only 0.5 mm in diameter, giving a cross-sectional area of only 1.96×10^{-7} m^2. This small area accounts for the resistance.

PRACTICE PROBLEMS

Use the data provided in Table 13.1 and the table in the Physics File on page 624 to solve the following problems.

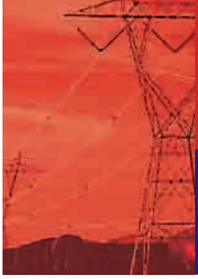
16. What is the resistance of 250 m of aluminum wire that has a diameter of 2.0 mm?

17. What is the length of 18 gauge Nichrome™ wire that has a resistance of 5.00 Ω?

18. The resistance of a 100 W (1.00×10^2 W) light bulb is 144 Ω. If its tungsten filament is 2.0 cm long, what is its radius? The bulb is not turned on.

19. An extension cord is made of 14 gauge aluminum wire. Calculate the resistance of this cord if its length is 35 m.

20. A square carbon rod is 24 m long. If its resistance is 140 Ω, what is its width?



INVESTIGATION 13-B

Current, Resistance, and Potential Difference

TARGET SKILLS
- Hypothesizing
- Performing and recording
- Analyzing and interpreting
- Communicating results

If you change one property in a circuit, such as the potential difference or the resistance, you would expect that another characteristic would change in response. Does it? If so, how does it change?

Problems

- How is current affected by a change in the potential difference if the resistance remains constant?
- How is current affected by a change in the resistance if the potential difference remains constant?

Hypotheses

Formulate hypotheses for the relationships between potential difference and current in a circuit, and between resistance and current in a circuit.

Equipment

- variable DC power supply
- multi-range ammeter
- multi-range voltmeter
- Nichrome™ wire (1 m, approximately 22 gauge)
- metre stick
- insulated connecting leads with alligator clips
- thumbtacks

CAUTION If the current in the Nichrome™ wire is large, the wire will become very hot. When your circuit has more than 1 A of current, you could be burned if you touch the wire.

CAUTION When wiring your circuit, be sure to connect the conductor to the anode last. Making this connection last will ensure that you do not accidentally create a live circuit while you are making other connections.

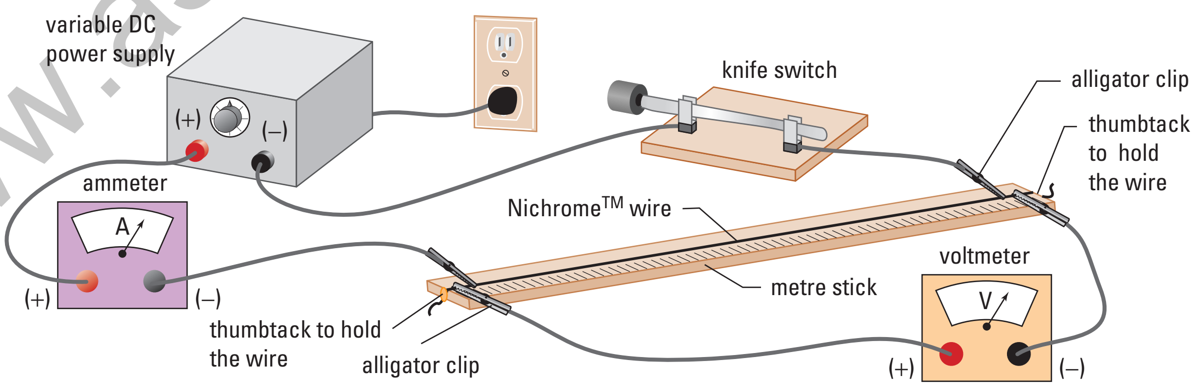
CAUTION DC ammeters and DC voltmeters must be connected properly to avoid damaging them and to ensure that they make proper measurements for the desired parts of the circuit. To avoid damaging the meters, make sure that they are connected in the correct direction. Like DC power supplies, DC meters have an anode (red or positive post) and a cathode (black or negative post). The meter must always be connected so that the current is moving from the red post to the black post (downhill) as it passes through the meter.

Procedure

Part 1

Connecting the Circuit

1. Study the figure to see how all connections will be made. Then, follow the order of making connections in the following steps.



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2. Connect the cathode (black post) of the power supply to one end of the load. (In the first part of the investigation, the load will be about 50 cm of the Nichrome™ wire.)

3. Connect the other end of the load to the black post of the ammeter.

4. Connect the red post of the ammeter to the anode of the power supply. Do *not* connect the voltmeter until these connections are complete and have been checked.

5. Before turning on the power supply, be sure that the knob is turned completely down (counterclockwise). Turn on the power supply and increase the potential difference very slightly. Check the ammeter to see if the current is in the correct direction. If everything is correct, turn off the power supply or disconnect the wire at the anode.

6. Connect the voltmeter across the load (power supply). Be sure that the red post of the voltmeter is connected to the end of the load that is closest to the anode (red post) of the power supply. Reconnect the power supply and increase the potential difference of the power supply slightly. Check that the needle of the voltmeter is moving in the correct direction.

Part 2

Current versus Potential Difference

7. Choose about 50 cm of the Nichrome™ wire as the load. Complete the circuit as described above, checking to see that the ammeter and the voltmeter are connected properly. If the voltmeter and the ammeter are multi-range meters, be sure to start on the least-sensitive range (the highest possible voltage or current readings) and move to more-sensitive ranges as conditions permit. If you are unsure of how to read the meter, consult your teacher.

8. Make a data table with the column headings: Trial, Potential Difference (V), Current (I).

9. With all connections in place, set the power supply at a low value. Read and record the current through, and the potential difference across, the load.

10. Keep the resistance constant; that is, do not move the alligator clips on the Nichrome™ wire. Increase the potential difference of the power supply slightly and, again, read the current and potential difference.

11. Repeat step 10 several times, until you have five or six readings.

Part 3

Current versus Resistance

CAUTION Remember that a large current will cause excessive heating.

12. Use the same circuit connections as in Part 1.

13. Choose a length of Nichrome™ wire (about 12 to 15 cm) as your standard resistance. The resistance of this length of wire will be considered one unit of resistance. (**Note:** Since you have created this standard resistance, you get to name the unit.)

14. Make a data table with the column headings: Trial, Resistance (in the units you gave it), Current. Leave two more columns for data to be used when you make your analysis.

15. Start with one unit of your standard resistance as a load. Increase the potential difference of the power supply until the ammeter registers a current of about 0.5 A. Record this potential difference. In your table, record the current and resistance as Trial 1.

16. Set the resistance of the load to be one unit larger than the previous trial ($R \propto L$). Check the potential difference across the load and reset it to the original value. Read and record the new values of current and resistance.

17. Repeat step 16 until you have four or five separate trials.

Analyze and Conclude

1. Graph your data for current (I) versus potential difference (V). Plot I as a function of V.

2. What does the shape of the line on your graph indicate about the relationship between potential difference and current when resistance is constant? How do these results compare with your hypothesis?

3. Write a summary statement describing your conclusion about the relationship of current to potential difference.

4. Graph your data for current (I) versus resistance (R). Plot I as a function of R.

5. What does the shape of the graph suggest is the probable relationship between the variables, R and I?

6. Based on your interpretation of the graph, decide how to mathematically modify the variable R so you could plot the adjusted value as a function of I and obtain a straight line. (Hint: Mathematical modifications could be such things as squaring, taking the square root, inverting, or multiplying by a constant or by the other variable.)

7. Calculate values for the adjusted independent variable. In one of the empty columns, write your calculated values.

8. Graph the current as a function of the newly created variable.

9. Was your decision in question 6 correct? If not, continue this process until you have found a modified form of R that gives a straight line.

10. Was your original hypothesis about the relationship between current and resistance correct? If not, what is the correct relationship?

11. Write a summary statement about the relationship between current and resistance when the potential difference is held constant.

Apply and Extend

12. Combine these two relationships into one. Write your results in the form of a proportionality. (**Note:** This is the basis of what is known as Ohm's law.)

13. Convert your relationship into an equation by including a constant (k) of proportionality.

14. Calculate the value of your constant by using the data from one of the trials in the second part of the investigation. Rewrite your relationship with your value for k included. This is Ohm's law for your standard resistance.

15. The value you find for k for your apparatus depends on the length of Nichrome϶ wire that you arbitrarily chose as your standard resistance. Calculate the length of wire segment that you would have to use as your standard resistance if you wanted the value for k to equal one. By definition, the length of Nichrome϶ wire that would produce a proportionality constant equal to one ($k = 1$) has a resistance of one ohm.

PROBEWARE

If your school has probeware equipment, visit the Science Resources section of the following web site: ***www.school.mcgrawhill.ca/resources/*** and follow the **Physics 11** links for several laboratory activities.

Ohm's Law

In 1826, German physicist Georg Simon Ohm conducted the original experiments in resistance in electric circuits, using many lengths and thicknesses of wire. He studied the current passing through the wire when a known potential difference was applied across it. From his data, Ohm developed the mathematical relationship that now bears his name, Ohm's law.

OHM'S LAW

The potential difference across a load equals the product of the current through the load and the resistance of the load.

$$V = IR$$

Quantity	Symbol	SI unit
potential difference	V	V (volt)
current	I	A (ampere)
resistance	R	Ω (ohm)

Unit Analysis

(potential difference) = (current)(resistance) = $\mathrm{A \cdot \Omega = V}$

Note: One ampere times one ohm is equivalent to one volt.

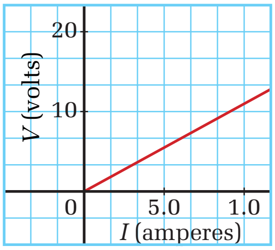
The **unit of resistance**, the **ohm**, is defined in accordance with Ohm's law. One ohm is defined as the amount of electric resistance that will allow one ampere of current to move through the resistor when a potential difference of one volt is applied across the resistor.

$$\left(1\ \Omega = \frac{1\ \mathrm{V}}{1\ \mathrm{A}}\right)$$

Initially, Ohm's law seemed to be the answer to the problem of defining how a load would affect current. Unfortunately, its usefulness is limited to metal conductors at stable temperatures. For the majority of loads, such as motors, electronic capacitors, and semiconductors, the resistance changes with a change in the potential difference. Even a light bulb does not obey Ohm's law, because the heating of the filament causes its resistance to increase. When a load does not obey Ohm's law, the graph of I versus V is not a straight line. Devices and materials that do not obey Ohm's law are said to be **non-linear** or **non-ohmic**. Metallic conductors are, however, a sufficiently important and large class of materials that demonstrate the law is still extremely useful.

Think It Through

- Consider the graph of potential difference versus current such as shown here. What is the significance of the slope of the line? (Hint: The equation for any straight line is $y = mx + b$, where m is the slope of the line and b is the y-intercept.)



MODEL PROBLEM

Applying Ohm's Law

What is the resistance of a load if a battery with a 9.0 V potential difference causes a current of 0.45 A to pass through the load?

Frame the Problem

- A battery creates a *potential difference* that provides energy to cause a *current* to flow in a circuit.
- A *load* resists the flow of current.
- You can find the *resistance* of the load by using Ohm's law.

Identify the Goal

The resistance, R, of the load

Variables and Constants

Involved in the problem	Known	Unknown
V	$V = 9.0$ V	R
I	$I = 0.45$ A	
R		

Strategy

Apply Ohm's law.

Calculations

$V = IR$

Substitute first

$9.0 \text{ V} = (0.45 \text{ A})\, R$

$$\frac{9.0 \text{ V}}{0.45 \text{ A}} = \frac{\cancel{(0.45 \text{ A})}\, R}{\cancel{0.45 \text{ A}}}$$

$R = 20\ \Omega$

Solve for *R* first

$$\frac{V}{I} = \frac{\cancel{I}R}{\cancel{I}}$$

$$\frac{9.0 \text{ V}}{0.45 \text{ A}} = R$$

$R = 20\ \Omega$

If a 9.0 V potential difference across a resistance results in a current of 0.45 A, the resistance is $2.0 \times 10^1\ \Omega$.

continued ►

continued from previous page

Validate

The data fit Ohm's law. The number 9 divided by approximately $\frac{1}{2}$ is the same as $9 \times 2 = 18$. The final answer, 20, is close to 18.

PRACTICE PROBLEMS

21. The heating element of an electric kettle draws 7.5 A when connected to a 120 V power supply. What is the resistance of the element?

22. A toaster is designed to operate on a 120 V (1.20×10^2 V) system. If the resistance of the toaster element is 9.60 Ω, what current does it draw?

23. A small, decorative light bulb has a resistance of 36 Ω. If the bulb draws 140 mA, what is its operating potential difference? (**Note:** The prefix "m" before a unit always means "milli-" or one one-thousandth. 1 mA is 1×10^{-3} A.)

24. The light bulb in the tail-light of an automobile with a 12 V electrical system has a resistance of 5.8 Ω. The bulb is left on for 8.0 min.

(a) What quantity of charge passes through the bulb?

(b) What was the current in the tail-light?

25. An iron transforms 3.35×10^5 J of electric energy to thermal energy in the 4.50 min it takes to press a pair of slacks. If the iron operates at 120 V (1.20×10^2 V), what is its resistance?

26. In Europe, some countries use 240 V (2.40×10^2 V) power supplies. How long will it take an electric kettle that has a resistance of 60.0 Ω to produce 4.32×10^5 J of thermal energy?

13.3 Section Review

1. I By what factor would the resistance of two copper wires differ if the second wire:

(a) was double the length of the first?

(b) was triple the cross-sectional area of the first?

(c) had a radius that was half the radius of the first?

(d) was half as long and twice as thick (twice the diameter) as the first?

(e) was three times as long and a third the cross-sectional area of the first?

2. I What happens to the resistance of a conductor when the temperature of the conductor increases?

3. I Design an experiment that you would carry out to determine whether a particular circuit element was ohmic or non-ohmic. Explain how you would interpret the results.

UNIT PROJECT PREP

Your electric motor will require current to operate.

- What factors affect the current flow within a conductor?
- How can you increase or decrease the current flow within a motor?

13.4 Series and Parallel Circuits

Figure 13.15 Skiers arriving at the top of a ski lift have several different runs down the hill available to them. The route down the hill is a complex circuit of series and parallel runs, taking skiers back to the bottom of the lift.

SECTION EXPECTATIONS

- Demonstrate understanding of the physical quantities of electricity.
- Synthesize information to solve electric energy problems.

KEY TERMS

- series
- parallel
- equivalent resistance
- internal resistance
- electromotive force
- terminal voltage

An extremely simple electric device, such as a flashlight, might have a circuit with one power source (a battery), one load (a light bulb), and one switch. In nearly every practical circuit, however, the power source supplies energy to many different loads. In these practical circuits, the loads may be connected in **series** (Figure 13.16) or in **parallel** (Figure 13.17). The techniques used to analyze these complex circuits are very similar to those you used to analyze simple circuits.

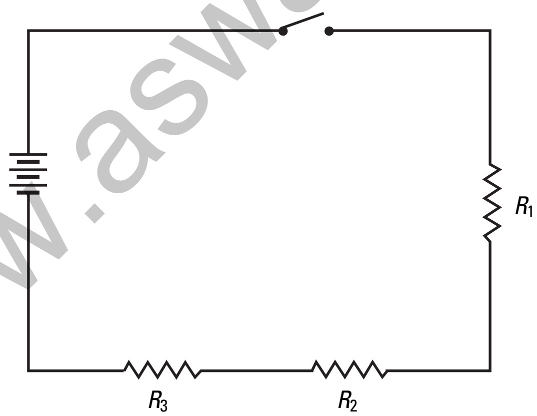


Figure 13.16 A circuit with resistances in series has only one closed path.

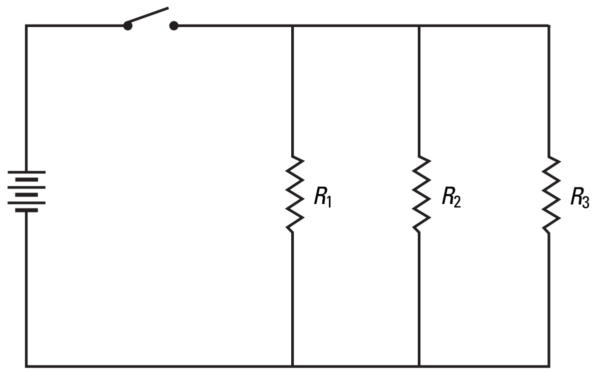


Figure 13.17 A circuit with resistances in parallel has several closed paths.

Series Circuits

A ski hill consisting of three downhill runs, one after the other, with level paths connecting the runs, is an excellent analogy for a series circuit. Since there is only one route down, the number of skiers going down each run would have to be the same. The total height of the three runs would have to equal the total height of the hill, as illustrated in Figure 13.18.

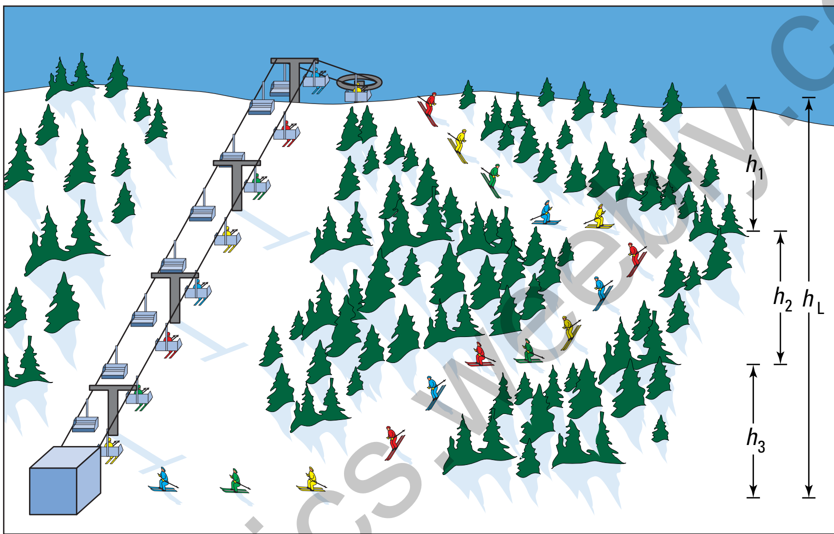


Figure 13.18 As skiers go around a ski circuit, the lift raises them to the top of the hill. Since the ski runs are in series, all of the skiers must ski down each of the runs, so that the number of skiers completing each run is the same. Each of the runs takes the skiers down a portion of the total height given to them by the ski lift. The combined height of the three runs must equal the height of the hill ($h_L = h_1 + h_2 + h_3$).

PHYSICS FILE

Just as skiers' gravitational potential energy drops as they ski down a hill, the electric potential energy of charges moving in a circuit drops when they pass through a load. Physicists often call the potential difference across a load a "potential drop." When the charges are given more potential energy in a battery or power source, they experience a "potential gain."

A series circuit consists of loads (resistances) connected in series, as was shown in Figure 13.16. The current that leaves the battery has only one path to follow. Just as the skiers in the previous analogy must all ski down each run in sequence, all of the current that leaves the battery must pass through each of the loads. An ammeter could be connected at any point in the circuit and each reading would be the same.

Also, just as the total height of the hill must be shared over the three runs, the potential difference of the battery must be shared over all three loads. Thus, a portion of the electric potential of the battery must be used to push the current through each load. If each load had a voltmeter connected across it, the total of the potential differences across the individual loads must equal the potential difference across the battery.

To find the **equivalent resistance** of a series circuit with *N* resistors, as illustrated in Figure 13.19 on the following page, analyze the properties of the circuit and apply Ohm's law.

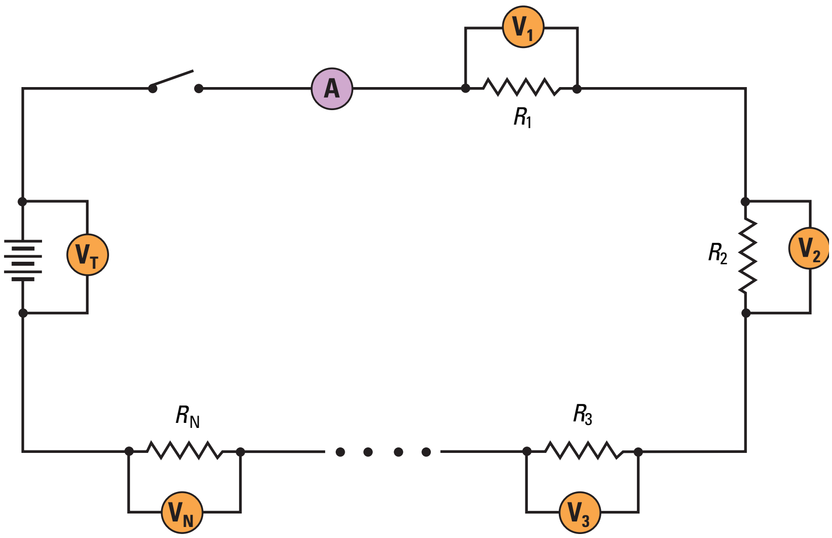


Figure 13.19 A circuit might consist of any number of loads. If this circuit had eight loads, R_N would represent R_8, and eight loads would be connected in series.

- Write, in mathematical form, the statement, "The total of the potential differences across the individual loads must equal the potential difference across the battery." $V_S = V_1 + V_2 + V_3 + \cdots + V_N$
- By Ohm's law, the total potential difference across the battery (source) must be equal to the product of the current through the battery and the equivalent resistance of the circuit. $V_S = I_S R_{eq}$
- For each load, the potential difference across that load must equal the product of the current through the load and its resistance. $V_1 = I_1 R_1,\ V_2 = I_2 R_2,\ V_3 = I_3 R_3, \cdots, V_N = I_N R_N$
- Substitute these expressions into the first expression for potential difference. $I_S R_{eq} = I_1 R_1 + I_2 R_2 + I_3 R_3 + \cdots + I_N R_N$
- Write, in mathematical form, the statement, "All of the current that leaves the battery must pass through each of the loads." $I_1 = I_2 = I_3 = \cdots = I_N = I_S$
- In the expression above, replace the symbol for each separate current with I_S. $I_S R_{eq} = I_S R_1 + I_S R_2 + I_S R_3 + \cdots + I_S R_N$
- Factor I_S out of the right side. $I_S R_{eq} = I_S(R_1 + R_2 + R_3 + \cdots + R_N)$
- Divide both sides by I_S. $R_{eq} = R_1 + R_2 + R_3 + \cdots + R_N$

EQUIVALENT RESISTANCE OF LOADS IN SERIES

The equivalent resistance of loads in series is the sum of the resistances of the individual loads.

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_N$$

Quantity	Symbol	SI unit
equivalent resistance	R_{eq}	Ω (ohm)
resistance of individual loads	$R_{1,2,3,\cdots N}$	Ω (ohm)

MODEL PROBLEM

Resistances in Series

Four loads (3.0 Ω, 5.0 Ω, 7.0 Ω, and 9.0 Ω) are connected in series to a 12 V battery. Find

(a) the equivalent resistance of the circuit

(b) the total current in the circuit

(c) the potential difference across the 7.0 Ω load

Frame the Problem

- Since all of the resistors are in series, the formula for the equivalent resistance for a series circuit applies to the problem.
- All of the loads are in series; thus, the current is the same at all points in the circuit. The current can be found by using the potential difference across the battery, the equivalent resistance, and Ohm's law.
- Ohm's law applies to each individual circuit element.

Identify the Goal

The equivalent resistance, R_S, for the series circuit, the current, I, and the potential difference, V_3, across the 7.0 Ω resistor

Variables and Constants

Involved in the problem		Known	Unknown
R_1	R_{eq}	$R_1 = 3.0\ \Omega$	R_{eq}
R_2	V_S	$R_2 = 5.0\ \Omega$	I
R_3	I_S	$R_3 = 7.0\ \Omega$	V_3
R_4	V_3	$R_4 = 9.0\ \Omega$	
		$V_S = 12$ V	

PROBLEM TIP

Ohm's law may be used several times in one problem, and often needs to be algebraically rearranged. It is convenient to learn all three forms of the law and use the form that gives the desired variable. To solve for potential difference, use the standard form $V = IR$.
To solve for current, use $I = \frac{V}{R}$.
To solve for resistance, use $R = \frac{V}{I}$.

Strategy

Use the equation for the equivalent resistance of a series circuit.

Calculations

$$R_{eq} = R_1 + R_2 + R_3 + R_4$$
$$= 3.0\ \Omega + 5.0\ \Omega + 7.0\ \Omega + 9.0\ \Omega$$
$$= 24\ \Omega$$

(a) The equivalent resistance for the four resistors in series is 24 Ω.

Use Ohm's law, in terms of current, and the equivalent resistance to find the current in the circuit.

$$I_S = \frac{V_S}{R_{eq}}$$

$$I_S = \frac{12\ \text{V}}{24\ \Omega}$$

$1\ \frac{\text{V}}{\Omega}$ is equivalent to 1A.

$$I_S = 0.50 \frac{\text{V}}{\Omega}$$

$$I_S = 0.50\ \text{A}$$

(b) The current in the circuit is 0.50 1A.

Use Ohm's law, the current, and the resistance of a single resistor to find the potential drop across that resistor.

$$V_3 = I_3 R_3$$

$$I_3 = I_S$$

Since the circuit has only one closed loop, the current is the same everywhere in the circuit, so $I_3 = I_S$.

$$V_3 = (0.50\ \text{A})(7.0\ \Omega)$$

$$V_3 = 3.5\ \text{A} \cdot \Omega$$

1 A · Ω is equivalent to 1 V.

$$V_3 = 3.5\ \text{V}$$

(c) The potential drop across the 7.0 Ω resistor is 3.5 V.

Validate

If you now find the potential difference across the other loads, the sum of the potential differences for the four loads should equal 12 V.

$V_1 = (0.50\ \text{A})(3.0\ \Omega)$ $\quad V_2 = (0.50\ \text{A})(5.0\ \Omega)$ $\quad V_4 = (0.50\ \text{A})(9.0\ \Omega)$

$V_1 = 1.5\ \text{V}$ $\quad V_2 = 2.5\ \text{V}$ $\quad V_4 = 4.5\ \text{V}$

$$1.5\ \text{V} + 2.5\ \text{V} + 3.5\ \text{V} + 4.5\ \text{V} = 12\ \text{V}$$

PRACTICE PROBLEMS

27. Three loads, connected in series to a battery, have resistances of 15.0 Ω, 24.0 Ω, and 36.0 Ω. If the current through the first load is 2.2A, calculate

(a) the potential difference across each of the loads

(b) the equivalent resistance for the three loads

(c) the potential difference of the battery

28. Two loads, 25.0 Ω and 35.0 Ω, are connected in series. If the potential difference across the 25.0 Ω load is 65.0 V, calculate

(a) the potential difference across the 35.0 Ω load

(b) the potential difference of the battery

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29. Two loads in series are connected to a 75.0 V battery. One of the loads is known to have a resistance of 48.0 Ω. You measure the potential difference across the 48.0 Ω load and find it is 40.0 V. Calculate the resistance of the second load.

30. Two loads, R_1 and R_2, are connected in series to a battery. The potential difference across R_1 is 56.0 V. The current measured at R_2 is 7.00 A. If R_2 is known to be 24.0 Ω, find

(a) the resistance of R_1

(b) the potential difference of the battery

(c) the equivalent resistance of the circuit

31. A 240 V (2.40×10^2 V) power supply is connected to three loads in series. The current in the circuit is measured to be 1.50 A. The resistance of the first load is 42.0 Ω and the potential difference across the second load is 111 V. Calculate the resistance of the third load.

Resistors in Parallel

The ski hill in Figure 13.20 provides a model for a circuit consisting of a battery and three resistors connected in parallel. The ski lift is, of course, analogous to the battery and the runs represent the resistors. Notice that the hill has three runs beside each other (in parallel) that go all of the way from the top to the bottom of the hill. The height of each of the runs must be equal to the height that the skiers gain by riding the lift up the hill ($h_L = h_1 = h_2 = h_3$).

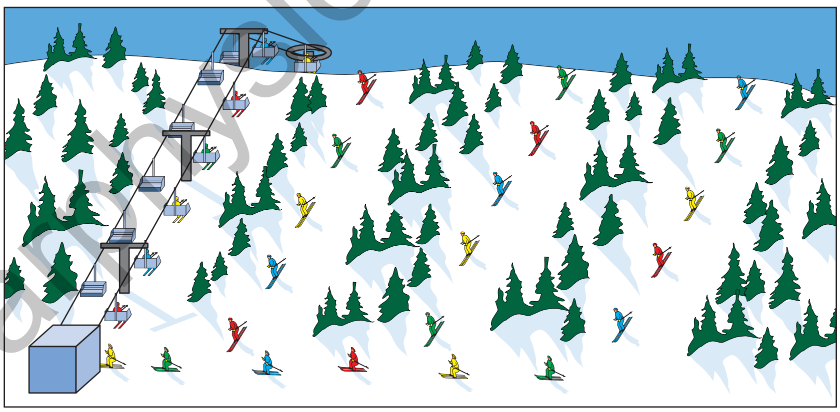


Figure 13.20 When skiers are on a hill where there are several ski runs that all are the same height as the hill, the runs are said to be *parallel* to each other.

The gravitational potential difference of the hill is analogous to the electric potential difference across a battery and resistors connected in parallel in a circuit. Similar to the height of the hill, the potential difference across each of the individual loads in a parallel circuit must be the same as the total potential difference across the battery (V_S). For example, in Figure 13.21, with N resistors in parallel, the mathematical relationship can be written as follows.

$$V_S = V_1 = V_2 = V_3 = \cdots = V_N$$

In the ski-hill analogy, the skiers themselves represent the current. A skier might select any one of the three runs, but can ski down only one of them. Since the skiers riding up the lift can go down only one of the hills, the sum of the skiers going down all three hills must be equal to the number of skiers leaving the lift.

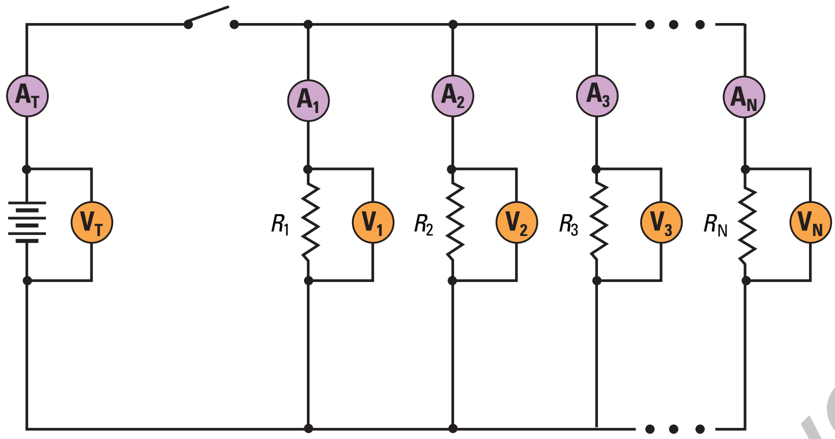


Figure 13.21 The *N* loads in this circuit are all connected in parallel with each other. The dots indicate where any number of additional loads could be connected in parallel with those present.

When the current leaving the battery (I_S) comes to a point in the circuit where the path splits into two or more paths, the current must split so that a portion of it follows each path. After passing through the loads, the currents combine before returning to the battery. The sum of the currents in parallel paths must equal the current entering and leaving the battery.

Knowing the current and potential difference relationships in a parallel circuit, you can use Ohm's law to develop an equation for the equivalent resistance of resistors in a parallel connection.

- Write, in mathematical form, the statement, "The sum of the currents in parallel paths must equal the current through the source."

$$I_S = I_1 + I_2 + I_3 + \cdots + I_N$$

- Write Ohm's law in terms of current.

$$I = \frac{V}{R}$$

- Apply this form of Ohm's law to the current through each individual resistor and for the battery.

$$I_S = \frac{V_S}{R_{eq}} \quad I_1 = \frac{V_1}{R_1} \quad I_2 = \frac{V_2}{R_2}$$
$$I_3 = \frac{V_3}{R_3} \quad I_N = \frac{V_N}{R_N}$$

- Replace the currents in the first equation with the expressions above.

$$\frac{V_S}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \cdots + \frac{V_N}{R_N}$$

- Write the relationship for the potential differences across resistors and the battery, when all are connected in parallel.

$$V_S = V_1 = V_2 = V_3 = \cdots = V_N$$

- Replace the potential differences in the equation above with V_S.

$$\frac{V_S}{R_S} = \frac{V_S}{R_1} + \frac{V_S}{R_2} + \frac{V_S}{R_3} + \cdots + \frac{V_S}{R_N}$$

- Divide both sides of the equation by V_S.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

RESISTORS IN PARALLEL

The inverse of the equivalent resistance for resistors connected in parallel is the sum of the inverses of the individual resistances.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

Quantity	Symbol	SI unit
equivalent resistance	R_{eq}	Ω (ohm)
resistance of the individual loads	$R_1, R_2, R_3, \ldots, R_N$	Ω (ohm)

MODEL PROBLEM

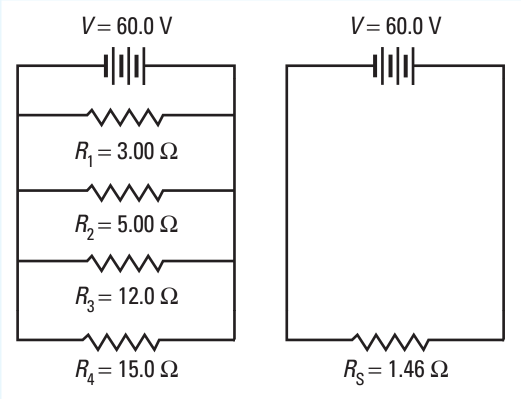
Resistors in Parallel

A 60 V battery is connected to four loads of 3.0 Ω, 5.0 Ω, 12.0 Ω, and 15.0 Ω in parallel.

(a) Find the equivalent resistance of the four combined loads.

(b) Find the total current leaving the battery.

(c) Find the current through the 12.0 Ω load.



Frame the Problem

- The four loads are connected *in parallel*; therefore, the *potential difference* across each load is the same as the *potential difference* provided by the *battery*.
- The *potential difference* across the battery and the *current* entering and leaving the battery would be *unchanged* if the four loads were replaced with one load having the *equivalent resistance*.

- After the current leaves the battery, it reaches branch points where it separates, and part of the current runs through each load.
- *Ohm's law* applies to each *individual load* or to the *combined load*. However, you must ensure that the current and potential difference are correct for the specific resistance that you use in the calculation.

Identify the Goal

The equivalent resistance, R_S, of the four loads
The current, I_S, leaving the battery
The current, I_3, through the 12.0 Ω load

Variables and Constants

Involved in the problem		Known	Unknown
R_1	R_{eq}	$R_1 = 3.00\ \Omega$	R_{eq}
R_2	V_S	$R_2 = 5.00\ \Omega$	I_S
R_3	I_S	$R_3 = 12.0\ \Omega$	I_3
R_4	I_3	$R_4 = 15.0\ \Omega$	
		$V_S = 60.0\ \text{V}$	

Strategy

Use the equation for resistors in parallel and apply it to the four loads.

Substitute values for resistance and add.

Find a common denominator.
Add.

Calculations

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{R_{eq}} = \frac{1}{3.00\ \Omega} + \frac{1}{5.00\ \Omega} + \frac{1}{12.0\ \Omega} + \frac{1}{15.0\ \Omega}$$

$$\frac{1}{R_{eq}} = \frac{20}{60.0\ \Omega} + \frac{12}{60.0\ \Omega} + \frac{5}{60.0\ \Omega} + \frac{4}{60.0\ \Omega}$$

$$\frac{1}{R_{eq}} = \frac{41}{60.0\ \Omega}$$

Strategy

Invert both sides of the equation. (If you invert an equality, it remains an equality.)

Divide.

(a) The equivalent resistance of the four loads in parallel is 1.46 Ω.

Use Ohm's law, in terms of current, and the equivalent resistance to calculate the current entering and leaving the battery.

Calculations

$$R_{eq} = \frac{60.0\ \Omega}{41}$$

$$R_{eq} = 1.46\ \Omega$$

$$I_S = \frac{60.0\ \text{V}}{1.46\ \Omega}$$

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$1\ \dfrac{V}{\Omega}$ is equivalent to an A.

$$I_S = 41.0 \frac{V}{\Omega}$$

$$I_S = 41.0\ A$$

(b) The current entering and leaving the battery is 41.0 A.

Use Ohm's law, in terms of current, to find the current through the 12.0 Ω load.

$$I_3 = \frac{60.0\ V}{12.0\ \Omega}$$

$$I_3 = 5.00 \frac{V}{\Omega}$$

$$I_3 = 5.00\ A$$

(c) Of the 41.0 A leaving the battery, 5.00 A are diverted to the 12.0 Ω load.

PROBLEM TIP

Using the *inverse key* [1/x] on your calculator makes these calculations easy. For example, to solve the above problem, enter the following sequence [3] [1/x] [+] [5] [1/x] [+] [1] [2] [1/x] [+] [1] [5] [1/x] [=]

The total on your calculator should now read 0.683… This is the sum of the inverses of the resistances and is the inverse of the expected answer, so now press [1/x]. The reading, 1.46…, becomes the value for the equivalent resistance. CAUTION: When using your calculator, it is very easy to forget the last step. Be sure to always press the inverse key after obtaining a sum.

Validate

If you do a similar calculation for the current in each of the loads, the total current through all of the loads should equal 41 A.

$$I_1 = \frac{V_1}{R_1} \qquad I_2 = \frac{V_2}{R_2} \qquad I_4 = \frac{V_4}{R_4}$$

$$I_1 = \frac{60.0\ V}{3.00\ \Omega} \qquad I_2 = \frac{60.0\ V}{5.00\ \Omega} \qquad I_4 = \frac{60.0\ V}{15.0\ \Omega}$$

$$I_1 = 20.0\ A \qquad I_2 = 12.0\ A \qquad I_4 = 4.00\ A$$

$$I_{total} = 20.0\ A + 12.0\ A + 5.00\ A + 4.00\ A$$
$$I_{total} = 41.0\ A$$

The answer is validated.

PRACTICE PROBLEMS

Draw a circuit diagram for each problem below. As an aid, write the known values on the diagram.

32. A 9.00 V battery is supplying power to three light bulbs connected in parallel to each other. The resistances, R_1, R_2, and R_3, of the bulbs are 13.5 Ω, 9.00 Ω, and 6.75 Ω, respectively. Find the current through each load and the equivalent resistance of the circuit.

33. A light bulb and a heating coil are connected in parallel to a 45.0 V battery. The current from the battery is 9.75 A, of which 7.50 A passes through the heating coil. Find the resistances of the light bulb and the heating coil, and the equivalent resistance for the circuit.

34. A circuit contains a 12.0 Ω load in parallel with an unknown load. The current in the 12.0 Ω load is 3.20 A, while the current in the unknown load is 4.80 A. Find the resistance of the unknown load and the equivalent resistance for the two parallel loads.

35. A current of 4.80 A leaves a battery and separates into three currents running through three parallel loads. The current to the first load is 2.50 A, the current through the second load is 1.80 A, and the resistance of the third load is 108 Ω. Calculate (a) the equivalent resistance for the circuit, and (b) the resistance of the first and second loads.

Complex Circuits

Many practical circuits consist of loads in a combination of parallel and series connections. To determine the characteristics of the circuit, you must analyze the circuit and recognize the way that different loads are connected in relation to each other. When a circuit branches, the loads in each branch must be grouped and treated as a single, or equivalent, load before they can be used in a calculation with other loads.

Figure 13.22 The circuitry shown here, typical of electronic equipment, illustrates the high level of complexity in circuitry of the household devices that we use every day.

Internal Resistance

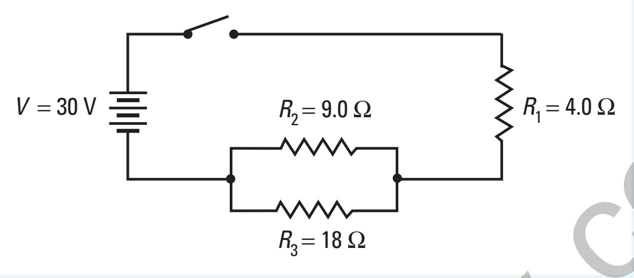
The objective of Part 2 of Investigation 13-B (Current, Resistance, and Potential Difference) was to find out how the current varied with resistance. Each time you changed the resistance, you had to reset the potential difference across the load to the desired value. The circuit behaved as if some phantom resistance was affecting the potential difference. To an extent, that is exactly what was happening — the phantom resistance was the internal resistance of the battery or power supply.

Figure 13.23 A chemical reaction separates electric charge creating a potential difference between the terminals. Internal resistance within the battery reduces the amount of voltage available to an external circuit.

MODEL PROBLEM

Resistors in Parallel

Find the equivalent resistance of the entire circuit shown in the diagram, as well as the current through, and the potential difference across, each load.



Frame the Problem

- The circuit consists of a *battery* and *three loads*. The battery generates a specific *potential difference* across the poles.
- The *current* driven by the potential difference of the battery depends on the *effective resistance* of the entire circuit.
- The circuit has two groups of resistors. Resistors R_2 and R_3 are in parallel with each other. Load R_1 is in series with the R_2–R_3 group.
- Define the R_2–R_3 group as Group A, and sketch the circuit with the equivalent load, R_A.
- Now, define the series group consisting of R_A and R_1 as Group B. Sketch the circuit with the equivalent load, R_B.
- Load R_B is the equivalent resistance of the entire circuit.

Identify the Goal

The equivalent resistance, R_{eq}, of the circuit
The currents, I_1, I_2, and I_3, through the loads
The potential differences, V_1, V_2, and V_3, across the loads

Variables and Constants

Involved in the problem			Known	Unknown	
R_1	R_{eq}	V_1	$R_1 = 4.0\ \Omega$	R_{eq}	V_1
R_2	I_1	V_2	$R_2 = 9.0\ \Omega$	I_1	V_2
R_3	I_2	V_3	$R_3 = 18\ \Omega$	I_2	V_3
V_S	I_3		$V_S = 30\ \text{V}$	I_3	

Strategy

Find the equivalent resistance for the parallel Group A resistors.

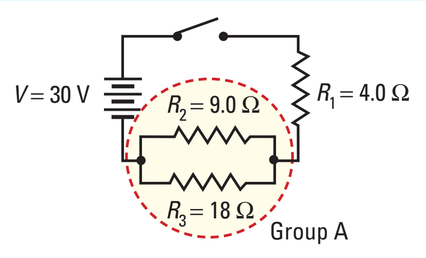
$$\frac{1}{R_A} = \frac{1}{R_2} + \frac{1}{R_3}$$

Find a common denominator.

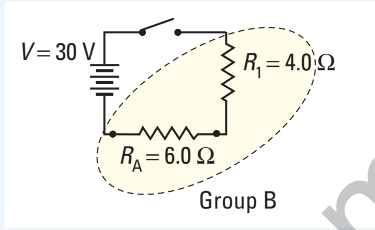
$$\frac{1}{R_A} = \frac{1}{9.0\ \Omega} + \frac{1}{18\ \Omega}$$

$$\frac{1}{R_A} = \frac{2}{18\ \Omega} + \frac{1}{18\ \Omega}$$

Calculations



Strategy | **Calculations**



Simplify.

$$\frac{1}{R_A} = \frac{3}{18\ \Omega}$$

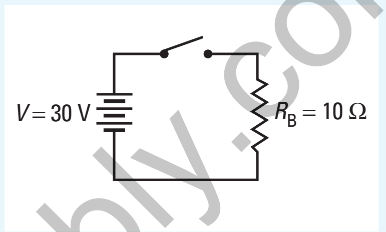
Invert both sides of the equation.

$$\frac{1}{R_A} = \frac{1}{6.0\ \Omega}$$

$$R_A = 6.0\ \Omega$$

Find the equivalent resistance of the series Group B.

$$R_B = R_A + R_1$$
$$R_B = 4.0\ \Omega + 6.0\ \Omega$$
$$R_B = 10\ \Omega$$



Since there is now one equivalent resistor in the circuit, the effective resistance of the entire circuit is $R_{eq} = 10\ \Omega$.

Find the current entering and leaving the battery, using the potential difference of the battery and the total effective resistance of the circuit.

$$I_S = \frac{V_S}{R_S}$$

$$I_S = \frac{30\ \text{V}}{10\ \Omega}$$

$$I_S = 3.0\ \text{A}$$

Since there are no branches in the circuit between the battery and load R_1, all of the current leaving the battery passes through R_1. Therefore, $I_1 = 3.0$ A.

Knowing the current through R_1, and its resistance, you can use Ohm's law to find V_1.

$$V_1 = I_1 R_1$$

$$V_1 = (3.0\ \text{A})(4.0\ \Omega)$$

$$V_1 = 12\ \text{V}$$

The potential difference across load 1 is 12 V.

The loads R_1 and R_A form a complete path from the anode to the cathode of the battery; therefore, the sum of the potential drops across these loads must equal that of the battery.

$$V_S = V_1 + V_A$$
$$30\ \text{V} = 12\ \text{V} + V_A$$
$$30\ \text{V} - 12\ \text{V} = V_A$$
$$V_A = 18\ \text{V}$$

Since the potential difference across a parallel connection is the same for both pathways, $V_2 = 18$ V and $V_3 = 18$ V.

Knowing the potential difference across R_2 and R_3, you can find the current through each load by using Ohm's law.

$$I_2 = \frac{V_2}{R_2} \qquad I_3 = \frac{V_2}{R_3}$$

$$I_2 = \frac{18\ \text{V}}{9.0\ \Omega} \qquad I_3 = \frac{18\ \text{V}}{18\ \Omega}$$

$$I_2 = 2.0\ \text{A} \qquad I_3 = 1.0\ \text{A}$$

The current through load 2 is 2.0 A, and the current through load 3 is 1.0 A.

continued ►

continued from previous page

To summarize, the 30 V battery causes a current of 3.0 A to move through the circuit. All of the current passes through the 4.0 Ω load, but then splits into two parts, with 2.0 A going through the 9.0 Ω load and 1.0 A going through the 18 Ω load. The potential drops across the 4.0 Ω, 9.0 Ω, and 18 Ω loads are 12 V, 18 V, and 18 V, respectively.

Validate

The current to Group A, known to be 3.0 A, was split into two parts. The portion of the current in each of the branches of Group A was inversely proportional to the resistance in the branch. The larger 18 Ω load allowed half the current that the smaller 9.0 Ω load allowed. The sum of the currents through R_2 and R_3 is equal to the current through R_1, which is expected to be true.

The potential difference across Group A should be the same as that of one load with a resistance, R_A, with the total current passing through it. Check this potential difference and compare it with the 18 V found by subtracting 12 V across load 1 from the total of 30 V.

$$V_A = I_S R_A = (3.0\ \text{A})(6.0\ \Omega) = 18\ \text{V}$$

The values are in agreement. The answers are consistent.

PROBLEM TIP

When working with complex circuits, look for the smallest groups of loads that are connected *only* in parallel or *only* in series. Find the equivalent resistance of the group, then redraw the circuit with one load representing the equivalent resistance of the group. Begin again.

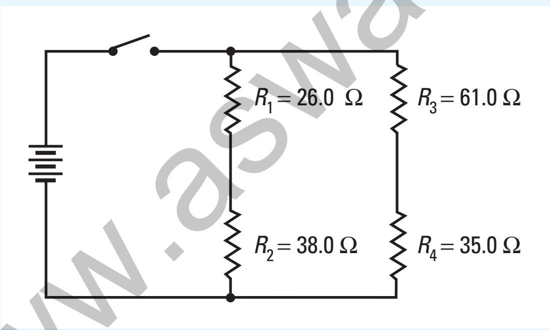
PRACTICE PROBLEMS

36. For the circuit in the diagram shown below, the potential difference of the power supply is 144 V. Calculate

(a) the equivalent resistance for the circuit

(b) the current through R_1

(c) the potential difference across R_3

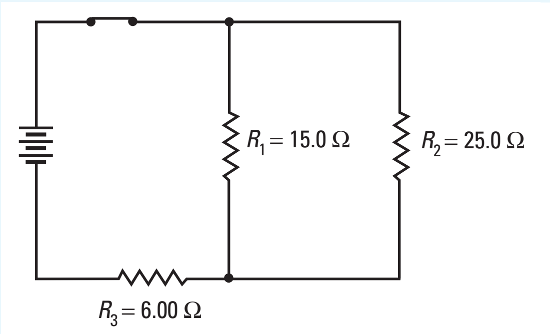


37. For the circuit shown in the diagram below, the potential difference of the power supply is 25.0 V. Calculate

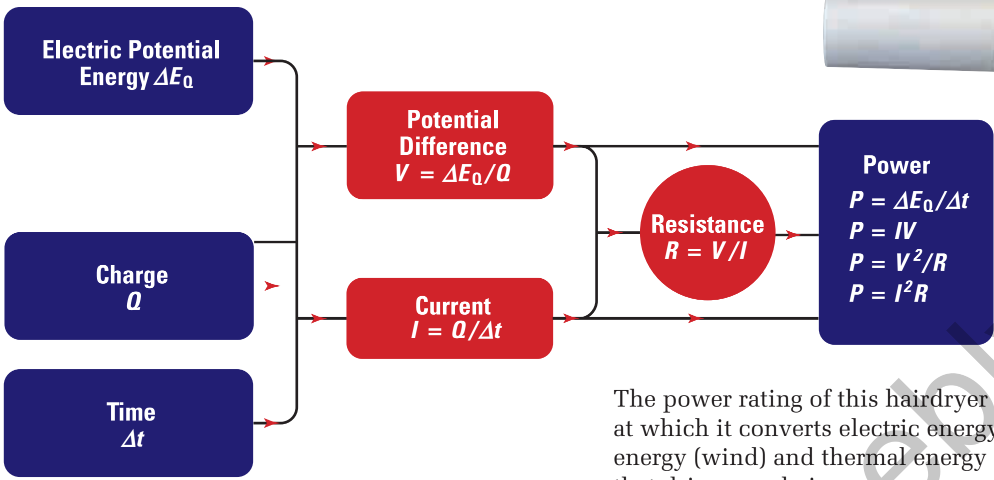
(a) the equivalent resistance of the circuit

(b) the potential difference across R_3

(c) the current through R_1



Concept Organizer



The power rating of this hairdryer describes the rate at which it converts electric energy into the kinetic energy (wind) and thermal energy (heat) of the air that dries your hair.

To develop an understanding of **internal resistance**, consider a gasoline engine being used to power a ski lift. Gasoline engines convert energy from the fuel into mechanical energy, which is then used, in part, to pull the skiers up the hill. No matter how efficient the motor is, however, it must always use some of the energy to overcome the friction inside the motor itself. In fact, as the number of skiers on the lift increases, the amount of energy the motor uses to run itself also increases.

The process involved in an electric circuit is very similar to the motor driving the ski lift. Inside the battery, chemical reactions create a potential difference, called the **electromotive force** (*emf,* represented in equations by $\mathscr{E}$). If there was no internal resistance inside the battery, the potential difference across its anode and cathode (sometimes called the **terminal voltage**) would be exactly equal to the *emf*. However, when a battery is connected to a circuit and current is flowing, some of the *emf* must be used to cause the current to flow through the internal resistance (r) of the battery itself. Therefore, the terminal voltage (V_S) of the battery is always less than the *emf* by an amount equal to the potential difference across the internal resistance ($V_{int} = I \cdot r$). You can find the terminal voltage by using the equation in the following box.

PHYSICS FILE

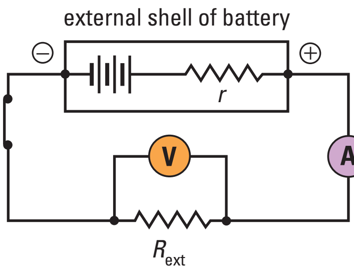
The electromotive force of a battery is not a force at all. It is a potential difference. However, the term "electromotive force" came into common usage when electric phenomena were not as well understood as they are today. Nevertheless, the term "electromotive force" is still used by physicists.

TERMINAL VOLTAGE AND *emf*

The terminal voltage (or potential difference across the poles) of a battery is the difference of the *emf* ($\mathscr{E}$) of the battery and the potential drop across the internal resistance of the battery.

$$V_S = \mathscr{E} - V_{int}$$

Quantity	Symbol	SI unit
terminal voltage	V_S	V (volt)
electromotive force	$\mathscr{E}$	V (volt)
internal potential drop of a battery	V_{int}	V (volt)



If no current is passing through a battery, then the potential difference across the internal resistance will be zero ($V_{int} = 0$). As a result, the potential difference across its terminals (V_S) will be equal to the *emf* ($\mathscr{E}$).

Figure 13.24 A battery has an internal resistance (r) that is in series with the *emf* ($\mathscr{E}$) of the cell.

MODEL PROBLEM

Terminal Voltage versus *emf* of a Battery

A battery with an *emf* of 9.00 V has an internal resistance of 0.0500 Ω. Calculate the potential difference lost to the internal resistance, and the terminal voltage of the battery, if it is connected to an external resistance of 4.00 Ω.

Frame the Problem

- A battery is connected to a closed circuit; thus, a *current* is flowing.
- Due to the *internal resistance* of the battery and the current, a *potential drop* occurs inside the battery.
- The *potential drop* across the internal resistance depends on the amount of *current* flowing in the circuit.
- The *terminal voltage* is lower than the *emf* due to the loss of energy to the internal resistance.

Identify the Goal

The potential difference (V_{int}) lost by current passing through the internal resistance of the battery

The terminal voltage (V_S), or potential difference across the poles, of the battery

Variables and Constants

Involved in the problem		Known	Unknown
$\mathscr{E}$	V_S	$\mathscr{E} = 9.00 \text{ V}$	V_S
r	V_{int}	$r = 0.0500\ \Omega$	V_{int}
R_{emf}	I_S	$R_{ext} = 4.00\ \Omega$	I_S
R_s			$\mathscr{E}_s$

Strategy

To find the current flowing in the circuit, you need to know the equivalent resistance of the circuit. Since the internal resistance is in series with the external resistance, use the equation for series circuits.

Use the *emf* ($\mathscr{E}$), and resistance to the *emf* (R_{emf}) in Ohm's law to find the current in the circuit.

Calculations

$$R_{emf} = r + R_s$$
$$R_{emf} = 0.0500\ \Omega + 4.00\ \Omega$$
$$R_{emf} = 4.05\ \Omega$$

$$I_S = \frac{\mathscr{E}}{R_{emf}}$$
$$I_S = \frac{9.00 \text{ V}}{4.05 \text{ V}}$$
$$I_S = 2.22 \text{ A}$$

Find the internal potential drop of the battery by using Ohm's law, the current, and the internal resistance.

$$V_{int} = I_S r$$
$$V_{int} = (2.22 \text{ A})(0.0500\ \Omega)$$
$$V_{int} = 0.111 \text{ V}$$

Find the terminal voltage from the *emf* and the potential drop due to the internal resistance.

$$V_S = \mathscr{E} - V_{int}$$
$$= 9.00 \text{ V} - 0.111 \text{ V}$$
$$= 8.89 \text{ V}$$

The potential difference across the internal resistance is 0.111 V, causing the *emf* of the battery to be reduced to the terminal voltage of 8.89 V. This is the portion of the *emf* available to the external circuit.

Validate

In order for a battery to be useful, you would expect that the loss of potential difference due to its internal resistance would be very small, compared to the terminal voltage. In this case, the loss (0.111 V) is just over 1 percent of the terminal voltage (8.898 V). The answer is reasonable.

PRACTICE PROBLEMS

38. A battery has an *emf* of 15.0 V and an internal resistance of 0.0800 Ω.

(a) What is the terminal voltage if the current to the circuit is 2.50 A?

(b) What is the terminal voltage when the current increases to 5.00 A?

39. A battery has an internal resistance of 0.120 Ω. The terminal voltage of the battery is 10.6 V when a current of 7.00 A flows from it.

(a) What is its *emf*?

(b) What would be the potential difference of its terminals if the current was 2.20 A?

INVESTIGATION 13-C

Internal Resistance of a Dry Cell

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting
- Communicating results

You can usually measure the resistance of a load by connecting a voltmeter across the load and connecting an ammeter in series with the load. However, you cannot attach a voltmeter across the internal resistance of a battery; you can attach a voltmeter only across the poles of a battery. How, then, can you measure the internal resistance of a dry cell?

Problem

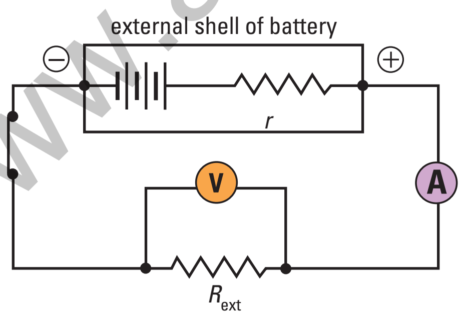
Determine the internal resistance of a dry cell.

Equipment

- $1\frac{1}{2}$ volt D-cell (or 6 V battery)
- variable resistor
- voltmeter
- ammeter
- conductors with alligator clips

Procedure

1. Measure and record the electromotive force (*emf,* represented by E) of the battery. This is the potential difference of the battery before it is connected to the circuit. Record all data to at least three significant digits.
2. Make a data table with the column headings: Trial, Terminal Voltage (V_S), Current (I_S), Equivalent Resistance (R_{eq}), Resistance to the *emf* (R_{emf}), Internal Resistance (r).
3. Connect the apparatus as shown in the diagram.



CAUTION Inspect your connections carefully. Refer to the wiring instructions given in Investigation 13-B on page 627 if you need to refresh your memory.

CAUTION Open the switch or disconnect the circuit when you are not taking readings.

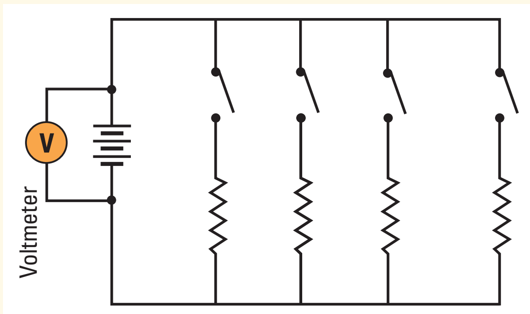
4. Adjust the variable resistor so that the current is about 1.0 A. Then, record the readings for the voltmeter (V_S) and ammeter (I_S).
5. Reduce the resistance of the load and record the voltmeter and ammeter readings.
6. Obtain at least four sets of data readings by reducing the resistance after each trial.
7. After taking the last reading from the circuit, remove the cell from the circuit and measure the *emf* of the cell again to confirm that it has not been diminished significantly.

Analyze and Conclude

1. For each trial, calculate and enter into your data table the value of the equivalent resistance, R_S.
$$\left(R_{eq} = \frac{V_S}{I_S}\right)$$
2. For each trial, calculate and enter into your data table the value of the total resistance, R_T.
$$\left(R_{emf} = \frac{\mathcal{E}}{I_S}\right)$$
3. For each trial, calculate and record in your data table the value of the resistance offered to the *emf* ($r = R_{emf} - R_{eq}$).
4. Explain why the terminal voltage decreases when the current increases.
5. Is the calculated value for the internal resistance constant for all trials? Find the average and the percent error for your results.

13.4 Section Review

1. K/U Classify each of the following statements as characteristics of either a series or a parallel circuit.
   (a) The potential difference across the power supply is the same as the potential difference across each of the circuit elements.
   (b) The current through the power supply is the same as the current through each of the circuit elements.
   (c) The current through the power supply is the same as the sum of the currents through each of the circuit elements.
   (d) The equivalent resistance is the sum of the resistance of all of the resistors.
   (e) The potential difference across the power supply is the same as the sum of the potential differences across all of the circuit elements.
   (f) The current is the same at every point in the circuit.
   (g) The reciprocal of the equivalent resistance is the sum of the reciprocals of each resistance in the circuit.
   (h) The current leaving the branches of the power supply goes through different circuit elements and then combines before returning to the power supply.
2. K/U A series circuit has four resistors, A, B, C, and D. Describe, in detail, the steps you would take in order to find the potential difference across resistor B.
3. K/U A parallel circuit has three resistors, A, B, and C. Describe, in detail, the steps you would take to find the current through resistor C. Describe a completely different method for finding the same value.
4. C A complex circuit has three resistors, A, B, and C. The equivalent resistance of the circuit is greater than the resistance of A, but less than the resistance of either B or C. Sketch a circuit in which this would be possible. Explain why.
5. K/U Under what conditions would the equivalent resistance of a circuit be smaller than the resistance of any one of the loads in the circuit? Explain.
6. I In Investigation 13-D (Internal Resistance of a Dry Cell), why were you cautioned to open the switch when you were not taking readings?
7. I Study the circuit below, in which a battery is connected to four resistors in parallel. Initially, none of the switches is closed. Assume that you close the switches one at a time and record the reading on the voltmeter after closing each switch. Describe what would happen to the voltmeter reading as you close successive switches. Explain why this would happen.



13.5 Electric Power and Energy

SECTION EXPECTATION

- Explain the concept and related units of electric energy.
- Compare and describe the concepts of electric energy and electric power.

KEY TERMS

- kilowatt-hour

When a solar storm sent an unexpectedly high flow of electrons into Earth's magnetic field, the energy surge caused an overload of the power grid serving the northeastern United States. An area of more than 200 000 square kilometres was blacked out, including the cities of New York, Buffalo, and Boston. People were trapped in elevators and on subways; hospitals, including operating rooms, were plunged into darkness.

Figure 13.25 Moonlight reflects from the darkened windows of New York City buildings during a massive power failure in 1965, which affected a large part of both Canada and the United States. As well as major inconveniences, riots and looting marked the event, which highlighted society's dependence on electric energy.

A major power failure is not required to remind us of how dependent we are on electric energy. Even a brief interruption of the electric energy supply demonstrates how reliant we have become on electricity. A major reason why electricity has become such a dominant energy form is the ease with which it can be transmitted and with which it can be converted into other forms of energy.

Power Output, Potential Difference, Current, and Resistance

Every appliance is rated for its power output (P)—the rate at which it can transform electric energy to a desired form (light, sound, or heat, for example). At the high end of the scale, an electric range might have a power rating greater than 12 000 W. An electric clothes dryer might be rated at 5000 W. At the other end of the scale, an electric shaver might be rated at 15 W and a clock at only 5 W.

In Chapter 5, you learned that power is defined as work done per unit time or energy transformed or transferred per unit time.

$$P = \frac{W}{t} \qquad \text{Power} = \frac{\text{work done}}{\text{time}}$$

$$P = \frac{\Delta E}{t} \qquad \text{Power} = \frac{\text{energy transformed}}{\text{time}}$$

The definition is the same whether you are referring to mechanical energy or electric energy. In a circuit, the loads transform electric potential energy into heat, light, motion, or other forms of energy. In power lines, electric energy is transferred from one location to another.

When working with electric circuits and systems, you typically work with potential difference, current, and resistance. It would be convenient to have relationships among power and these commonly used variables. To develop such relationships, start with the definition of potential difference.

- Definition of electric potential difference $$V = \frac{\Delta E_Q}{Q}$$
- Solve for electric potential energy. $$VQ = \frac{\Delta E_Q}{\cancel{Q}}\cancel{Q}$$ $$\Delta E_Q = QV$$
- Recall the definition of power. $$P = \frac{\Delta E}{t}$$
- Substitute the expression for electric potential energy into the equation for power. $$P = \frac{QV}{t}$$
- Recall the definition of current. $$I = \frac{Q}{t}$$
- Substitute I for $\frac{Q}{t}$ in the equation for power. $$P = IV$$

Figure 13.26 The power rating (sometimes called the "wattage") of a light bulb tells you how fast it will convert electric energy into heat and light. For an incandescent bulb, only about 2 percent of the transformed energy is actually emitted as light; the rest is emitted as heat. A fluorescent bulb, on the other hand, converts about 9.5 percent of its energy into light, making it more than four times as efficient as an incandescent bulb.

COURSE CHALLENGE

How Much Electric Power?

Apply the electric power formulas, combined with power density information, to create a numerical simulation that tests the feasibility of space-based power to supply energy to a small community. Use data from Chapter 6, page 313, *Solar Energy Transmission from Space*, as well as other sources. Learn more from the **Science Resources** section of the following web site: ***www.school.mcgrawhill.ca/resources/*** *Physics 11 Course Challenge*

ELECTRIC POWER

Power is the product of current and potential difference.

$$P = IV$$

Quantity	Symbol	SI unit
power	P	W (watt)
current	I	A (ampere)
potential difference	V	V (volt)

Unit Analysis

(power) = (current)(potential difference) $\quad \mathrm{W} = \mathrm{A} \cdot \mathrm{V}$

To verify that a watt is equivalent to an ampere times a volt, recall that an ampere is defined as a coulomb per second. $\quad \mathrm{A} = \frac{\mathrm{C}}{\mathrm{s}}$

Recall that a volt is defined as a joule per coulomb. $\quad \mathrm{V} = \frac{\mathrm{J}}{\mathrm{C}}$

Substitute these units into W = A · V.

$$\mathrm{W} = \mathrm{A} \cdot \mathrm{C}$$

$$\mathrm{W} = \left(\frac{\cancel{\mathrm{C}}}{\mathrm{s}}\right)\left(\frac{\mathrm{J}}{\cancel{\mathrm{C}}}\right)$$

When you cancel units (coulomb), you have shown that a watt is a joule per second, as defined on page 276.

$$\mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}}$$

MODEL PROBLEM

Calculating Electric Power

What is the power rating of a segment of Nichrome™ wire that draws a current of 2.5 A when connected to a 12 V battery?

Frame the Problem

- The *power rating* of an object is the *rate* at which it *converts electric energy* into another form of energy.
- When *current* passes through Nichrome϶ wire, the resistance of the wire causes some energy to be converted into heat.
- You can calculate *power* from the *current* and *potential difference* across the ends of the wire.

Identify the Goal

The power rating, P, of the wire

Variables and Constants

Involved in the problem	Known	Unknown
P	$I = 2.5$ A	P
I	$V = 12$ V	
V		

Strategy / Calculations

Use the equation that relates current and potential difference to power.

$P = IV$

Substitute the known values.

$P = (2.5\text{ A})(12\text{ V})$

$P = 30\text{ A} \cdot \text{V}$

An A · V is equivalent to a W.

$P = 30\text{ W}$

The power rating of the segment of wire is 30 W.

Validate

The units combine to give watts, which is correct for power.

Refer to the derivation on page 653 to review why current times potential difference is a correct expression for power.

PRACTICE PROBLEMS

40. An electric toaster is rated at 875 W at 120 V.

(a) Calculate the current the toaster draws when it is on.

(b) Calculate the electric resistance of the toaster.

41. A light bulb designed for use with a 120 V power supply has a filament with a resistance of 240 Ω.

(a) What is the power output of the bulb when the potential difference is 120 V?

(b) If the bulb is inadvertently connected to an 80.0 V power supply, what would be the power output of the bulb?

(c) If you wanted to construct a bulb to use with an 80.0 V power supply so that it would have the same power output as a 240 Ω bulb connected to a 120 V power supply, what should be the resistance of the bulb's filament?

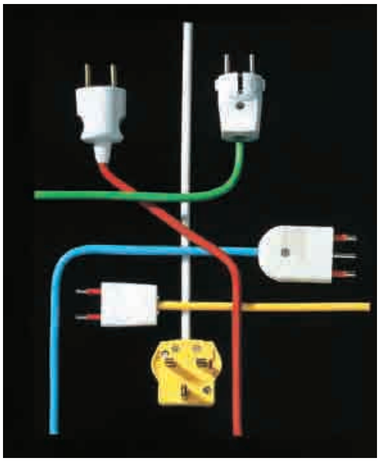
42. A heater has a resistance of 15 Ω.

(a) If the heater is drawing a current of 7.5 A, what is its power output?

(b) If the current to the heater was cut in half, what would happen to the power output?

Technology Link

Different appliances, especially those that have large power outputs, often have a specially designed plug shape that is legal only for that type of appliance. Why is this necessary?



If you read the power rating on any appliance, you will notice that it is always accompanied by the potential difference that produces the specified power output. In Canada and the United States, electric energy is provided at a potential difference of 120 V, while in many other countries (including most of Europe) electric energy is provided at a potential difference of 240 V. Travellers taking appliances such as hair dryers to Europe find that these appliances are often damaged by the higher potential difference of the power supply.

Figure 13.27 Some small hair dryers designed for travel have switches that allow you to select the voltage rating. The dryer will provide the specified power output and not be damaged, whether you plug it into a 120 V line or a 240 V line.

MODEL PROBLEM

Using the Wrong Power Source

In North America, the standard electric outlet has a potential difference of 120 V. In Europe, it is 240 V. How does the dissimilarity in potential difference affect power output? What would be the power output of a 100 W–120 V light bulb if it was connected to a 240 V system?

Frame the Problem

- A light bulb contains a fine filament that has a certain electric *resistance*.
- The specifications (100 W–120 V) on the bulb mean that when it is connected to a 120 V potential difference, it will convert electric energy into light (and thermal energy) at 100 W.
- The resistance will remain approximately the same, even if it is connected to a different potential difference. Therefore, according to *Ohm's law*, the *current* will be different.
- If both the *current* and *potential difference* have *changed*, the *power* will likely be different.
- The key quantity is the *resistance*.

Identify the Goal

The power output, P, when the light bulb is connected to a 240 V line

Variables and Constants

Involved in the problem		Known	Unknown
R	V_{240}	$V_{120} = 120$ V	P_{240}
V_{120}	P_{240}	$P_{120} = 100$ W	I_{240}
P_{120}	I_{240}	$V_{240} = 240$ V	I_{120}
I_{120}			R

(**Note:** Subscripts refer to the potential difference of the line to which the light bulb is connected.)

Strategy

Since there is no direct method to calculate the power, several steps will be involved. A tree diagram will help you determine what steps to take.

You can find the power output at 240 V if you know the current and potential difference at 240 V.

The potential difference is known.

You can find the current at 240 V if you know the potential difference at 240 V and the resistance.

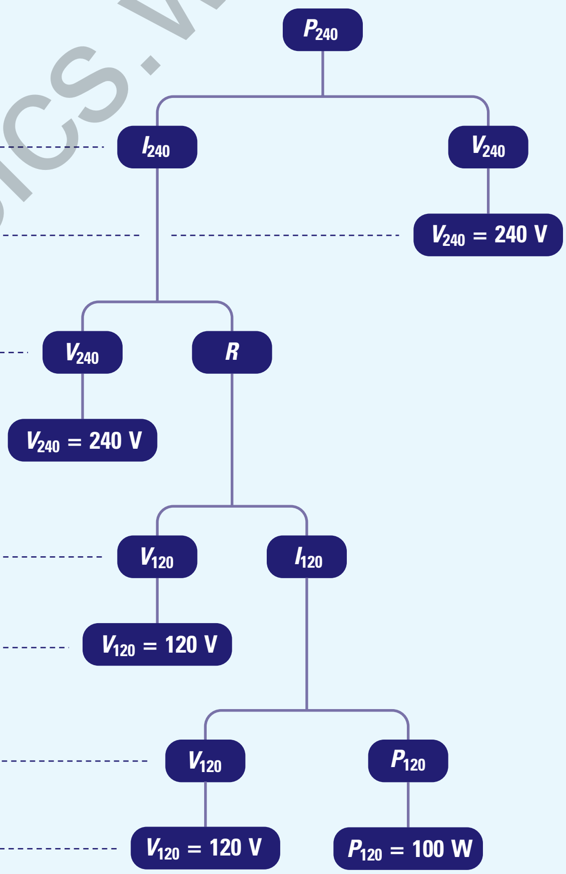
The potential difference is known.

You can find the resistance if you know the current and potential difference at 120 V.

The potential difference is known.

You can find the current at 120 V if you know the potential difference and power at 120 V.

The potential difference and the power at 120 V are known.



The problem is solved. Now do the calculations.

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Strategy	Calculations
Find the current at 120 V.	$P_{120} = I_{120}V_{120}$ $I_{120} = \frac{P_{120}}{V_{120}}$ $I_{120} = \frac{100\text{ W}}{120\text{ V}}$ $I_{120} = 0.867\text{ A}$
Find the resistance.	$R = \frac{V_{120}}{I_{120}}$ $R = \frac{120\text{ V}}{0.867\text{ A}}$ $R = 144\ \Omega$
Find the current at 240 V.	$I_{240} = \frac{V_{240}}{R}$ $I_{240} = \frac{240\text{ V}}{144\ \Omega}$ $I_{240} = 1.73\text{ V}$
Find the power output at 240 V.	$P_{240} = I_{240}V_{240}$ $P_{240} = (1.73\text{ A})(240\text{ V})$ $P_{240} = 400\text{ W}$

The power output at 240 V is 400 W. Notice that the power output is four times as great when you double the potential difference across a given resistance. Such an increase in power output would overheat the filament and melt it.

Validate

Although it might seem strange that the power output quadruples when the potential difference doubles, it is logical. Doubling the potential difference for a given resistance doubles the current. When both potential difference and current are doubled, the power output is quadrupled.

PRACTICE PROBLEMS

43. **(a)** What is the power output of a 45 Ω resistance when connected to
- a 180 V power supply?
- a 270 V power supply?

(b) Find the ratio of the potential differences and the ratio of the power outputs. What is the relationship between the two ratios?

44. A load has a power rating of 160 W when the current in it is 6.0 A. What will be the power output if the current increases to 15 A?

45. **(a)** What is the power output of a circuit that consists of a 25 Ω resistance when connected to a 100 V supply?

(b) If a second 25 Ω resistance is connected in series with the first, what will be the power output of the circuit? Why has the power output of the circuit decreased?

You probably noticed in the model problem and practice problems on the preceding pages that the resistance of a device is often the key to finding the power output. It would be convenient if relationships were available that would relate power output, resistance, and potential difference (P, R, and V) or power output, resistance, and current (P, R, and I). Since Ohm's law relates resistance, current, and potential difference, you can use it to develop more relationships for power.

- Start with the relationship between potential difference, current, and power. $P = IV$
- To eliminate current and to introduce resistance, write Ohm's law in terms of current. $I = \frac{V}{R}$
- Substitute $\frac{V}{R}$ in place of I in the equation for power. $P = \frac{V}{R}V$
- Simplify. $P = \frac{V^2}{R}$

To find a relationship between P, R, and I, use a similar procedure, but eliminate V from the equation for power.

- Start with the first equation for power. $P = IV$
- Express Ohm's law in terms of potential difference. $V = IR$
- Substitute IR in place of V in the equation for power. $P = I(IR)$
- Simplify. $P = I^2R$

ALTERNATIVE EQUATIONS FOR POWER

Power is the quotient of the square of the potential difference and the resistance.

$$P = \frac{V^2}{R}$$

Power is the product of the square of the current and the resistance.

$$P = I^2R$$

Quantity	Symbol	SI unit
power	P	W (watt)
potential difference	V	V (volt)
resistance	R	Ω (ohm)
current	I	A (ampere)

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Unit Analysis

Recall from the definitions of current and potential difference

$$A = \frac{C}{s} \qquad V = \frac{J}{C}$$

From Ohm's law, $V = A \cdot \Omega$, therefore $\frac{V}{\Omega} = A$.

Use these relationships to analyze the units for the two power formulas.

$$\begin{aligned}
(\text{watt}) &= \frac{(\text{volt})^2}{(\text{ohm})} \\
&= \frac{V^2}{\Omega} \\
&= \left(\frac{V}{\Omega}\right) V \\
&= A \cdot V \\
&= \left(\frac{\cancel{C}}{s}\right)\left(\frac{J}{\cancel{C}}\right) \\
&= W
\end{aligned}
\qquad
\begin{aligned}
(\text{watt}) &= (\text{ampere})^2(\text{ohm}) \\
&= (A)^2(\Omega) \\
&= (A)(A \cdot \Omega) \\
&= (A)(V) \\
&= \left(\frac{\cancel{C}}{s}\right)\left(\frac{J}{\cancel{C}}\right) \\
&= \frac{J}{s} \\
&= W
\end{aligned}$$

MODEL PROBLEM

Resistance and Power

An electric kettle is rated at 1500 W for a 120 V potential difference.

(a) What is the resistance of the heating element of the kettle?

(b) What will be the power output if the potential difference falls to 108 V?

Frame the Problem

- An electric kettle will use *energy* at a *rate* of 1500 W if connected to a 120 V line.
- *Power*, *potential difference*, *current*, and *resistance* are all related.

Identify the Goal

The resistance, R, of the heating element of the kettle
The power, P, if the potential difference decreases

Variables and Constants

Involved in the problem		Known	Unknown
P_1	P_2	$P_1 = 1500$ W	R
V_1	V_2	$V_1 = 120$ V	P_2
R		$V_2 = 108$ V	

Strategy

Use the relationship among power, potential difference, and resistance.

Calculations

$$P_1 = \frac{V_1{}^2}{R}$$

Substitute first

$$1.50 \times 10^3 \text{ W} = \frac{(1.20 \times 10^2 \text{ V})^2}{R}$$

$$(1.50 \times 10^3 \text{ W})(R) = \frac{(1.20 \times 10^2 \text{ V})^2}{\cancel{R}}(\cancel{R})$$

$$\frac{\cancel{(1.50 \times 10^3 \text{ W})}(R)}{\cancel{1.50 \times 10^3 \text{ W}}} = \frac{(1.20 \times 10^2 \text{ V})^2}{1.50 \times 10^3 \text{ W}}$$

$$R = 9.60 \frac{\text{V}^2}{\text{W}}$$

$$R = 9.60\ \Omega$$

Solve for R first

$$P_1 R = \frac{V_1{}^2}{\cancel{R}}\cancel{R}$$

$$P_1 R = V_1{}^2$$

$$\frac{\cancel{P_1}R}{\cancel{P_1}} = \frac{V_1{}^2}{P_1}$$

$$R = \frac{V_1{}^2}{P_1}$$

$$R = \frac{(1.2 \times 10^2 \text{ V})^2}{1.50 \times 10^3 \text{ W}}$$

$$R = 9.60 \frac{\text{V}^2}{\text{W}}$$

$$R = 9.60\ \Omega$$

Unit Check: $\dfrac{\text{V}^2}{\text{W}} = \dfrac{\frac{\text{J}^2}{\text{C}^2}}{\frac{\text{J}}{\text{s}}} = \left(\dfrac{\text{J} \cdot \cancel{\text{J}}}{\text{C}^2}\right)\left(\dfrac{\text{s}}{\cancel{\text{J}}}\right) = \left(\dfrac{\text{J}}{\text{C}}\right)\left(\dfrac{\text{s}}{\text{C}}\right) = V\dfrac{1}{\text{A}} = \Omega$

(a) The resistance of the coil in the kettle is 9.60 Ω.

Find the power output when the potential difference is 108 V.

$$P_2 = \frac{V_2^2}{R}$$

$$P_2 = \frac{(108 \text{ V})^2}{9.60\ \Omega}$$

$$P_2 = 1215\frac{\text{V}^2}{\Omega}$$

$$P_2 = 1.22 \times 10^3 \text{ W}$$

See the box on page 660 for a unit analysis.

(b) The power output drops to 1.22×10^3 W when the potential difference drops to 108 V.

(**Note:** This is a 10% loss of potential difference and a 19% loss in power. The percentage decrease in power is nearly double that of the decrease in potential difference because power is proportional to the square of the potential difference.)

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Validate

The units combine properly to give ohms in part (a), and watts in part (b). From past experience, you would expect a proportionately larger drop in the power output than in potential difference.

PRACTICE PROBLEMS

46. A filament in a light bulb rated at 192 W, has a resistance of 12.0 Ω. Calculate the potential difference at which the bulb is designed to operate.

47. An electric kettle is rated at 960 W when operating at 120 V. What must be the resistance of the heating element in the kettle?

48. If a current of 3.50 A is flowing through a resistance of 24.0 Ω, what is the power output?

49. A toaster that has a power rating of 900 W (9.00×10^2 W) draws a current of 7.50 A. If 2.40×10^5 J of electric energy are consumed while toasting some bread, calculate how much charge passed through the toaster.

50. A floodlight filament has an operating resistance of 22.0 Ω. The lamp is designed to operate at 110 V.

(a) What is its power rating?

(b) How much energy is consumed if you use the lamp for 2.50 hours?

MISCONCEPTION

Power or Energy?

Often you hear people speaking of buying power from the "power" company. If you check your monthly electric bill, you will see that you are charged for the energy (measured in kilowatt-hours) that you consumed. Power measures how fast you do the work, not how much work you do. It is electric energy that keeps your lights on and runs your television. People should say that they buy energy from the "energy" company.

$$\begin{aligned} 1\ \text{kW}\cdot\text{h} &= (1000\ \text{W})(1\ \text{h}) \\ &= (1 \times 10^3\ \text{W})(3.6 \times 10^3\ \text{s}) \\ &= 3.6 \times 10^6\ \text{W}\cdot\text{s} \\ &= 3.6 \times 10^6\ \text{J} \end{aligned}$$

(or 3.6 MJ of energy)

Energy Consumption

The electric meter at your home gives readings in **kilowatt-hours** (kW · h). One kilowatt-hour represents the energy transformed by a power output of 1000 W for one hour. This is equivalent to 3.6×10^6 J (3.6 MJ). A typical charge by an energy company for consumed energy might be \$0.07 per kW · h. That means that for only seven cents, you can buy enough energy to lift 360 kg a vertical distance of more than 1 km.

Figure 13.28 The electric meter in your home measures how much energy (in kW · h) that you use.

MODEL PROBLEM

The Cost of Watching Television

A family has its television set on for an average of 4.0 h per day. If the television set is rated at 80 W and energy costs $0.070 per kW · h, how much would it cost to operate the set for 30 days?

Frame the Problem

- The total amount of *electric energy* a television set uses in a specific amount of *time* depends on its *power output*.
- Power companies charge a specific amount of *dollars* per unit of *energy* used.

Identify the Goal

The cost, in dollars, for operating a television set for 30 days

Variables and Constants

Involved in the problem		Known	Unknown
Δt	*Cost*	$\Delta t = (4.0\ \text{h/day})(30\ \text{days})$	*Cost*
P	ΔE_e	rate = \$0.070 kW · h	ΔE_e
rate		$P = 80\ \text{W}$	

Strategy

Find the total time, in hours, that the television set typically is on during one 30-day period.

Find the total amount of energy consumed by using the definition for power.

Find the cost.

The cost of operating a television set for 4.0 h per day is $0.67 for 30 days.

Calculations

$$\Delta t = \left(4.0\frac{\text{h}}{\cancel{\text{day}}}\right)(30\ \cancel{\text{days}})$$

$$\Delta t = 120\ \text{h}$$

$$P = \frac{\Delta E_e}{\Delta t}$$

Substitute first

$$\text{Cost} = \text{rate} \cdot \Delta E_e$$

$$(80\ \text{W})(120\ \text{h}) = \frac{\Delta E_e}{\cancel{120}\ \text{h}}\ \cancel{120\ \text{h}}$$

$$\Delta E_e = 9600\ \text{W} \cdot \text{h}$$

$$\text{Cost} = \text{rate} \cdot \Delta E_e$$

Solve for ΔE_e first

$$(P)(\Delta t) = \frac{\Delta E_e}{\cancel{\Delta t}} = \cancel{\Delta t}$$

$$(P)(\Delta t) = \Delta E_e$$

$$\Delta E_e = (80\ \text{W})(120\ \text{h})$$

$$\Delta E_e = 9600\ \text{W} \cdot \text{h}$$

$$Cost = \frac{\$0.070}{\cancel{\text{kW}} \cdot \cancel{\text{h}}}\ 9600\ \cancel{\text{W}} \cdot \cancel{\text{h}}\ \frac{1\ \cancel{\text{kW}}}{1000\ \cancel{\text{W}}}$$

$$Cost = \$0.67$$

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Validate

The units combine and cancel to give dollars. At first, the cost seems to be low. However, a television set does not use as much energy as many other appliances. As well, electric energy is relatively inexpensive.

PRACTICE PROBLEMS

51. It takes 25.0 min for your clothes dryer to dry a load of clothes. If the energy company charges 7.20 cents per $kW \cdot h$, how much does it cost to dry a load of clothes in a 1250 W dryer?

52. It takes 12.0 min to dry your hair using a blow dryer that has a resistance of 21.0 Ω and draws a current of 5.50 A. Calculate the cost to dry your hair if electricity costs 8.50 cents per $kW \cdot h$.

53. An electric kettle, which operates on a 120 V supply, has a heating element with a resistance of 10.0 Ω. If it takes 3.2 min to boil a litre of water and the energy charge is 6.5 cents per $kW \cdot h$, calculate

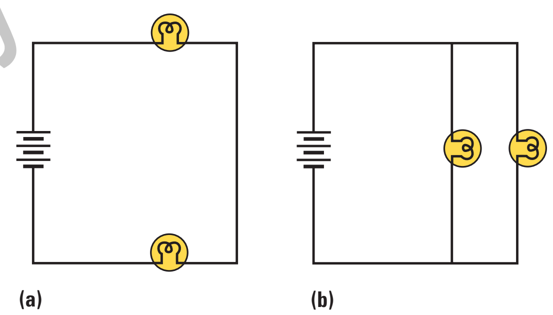
(a) the power rating of the kettle

(b) the cost to boil the water

13.5 Section Review

1. K/U Write four different equations that could be used to find the power consumed by a circuit element. Explain why it is useful to have so many different equations for power.

2. C The brightness of a light bulb is directly related to the power it consumes. In the circuits in the diagram on the right, the light bulbs have the same ratings. In which circuit will the light bulbs glow most brightly? Explain your reasoning.



3. I Using unit analysis, show that a kilowatt-hour is a unit of energy.

UNIT PROJECT PREP

Think about these questions in the analysis of your motor.

- How powerful will your electric motor be?
- What factors will affect the power output of your motor?
- How can you determine the power of your motor?

CHAPTER 13 Review

REFLECTING ON CHAPTER 13

- Materials are classified as conductors, insulators or semi-conductors depending on their ability to pass electric currents.
- The voltaic pile was the first battery. Its invention allowed physicists to carry out the first investigations with current electricity. Potential difference is a measure of the amount of energy that is available per unit charge. One volt is the potential difference that will provide one joule of energy to every coulomb of charge transferred.
- The rate at which charge moves through a circuit is called electric current. The unit of current, the ampere, is the current that transfers charge at the rate of one coulomb per second.
- Current is, by definition, the movement of positive charges. The movement of negative charge is called electron flow.
- Electric charge exists only in whole-number mulitples of the elementary charge, ($1\ e = 1.60 \times 10^{-19}$ C)
- When a current flows through a load, the resistance of the load converts electric energy to heat, light or other forms of mechanical energy. This results in a potential drop across the load.
- The resistance of a conductor of a particular material varies directly as its length and inversely as its cross-sectional area.
- Ammeters are connected in series with loads; voltmeters are connected in parallel with loads.
- Ohm's law describes the relationship between the potential difference across, the current through and the resistance of a load.
- When loads are connected in series, the equivalent resistance is the sum of their resistances. When loads are connected in parallel, the equivalent resistance is the inverse of the sum of the inverses of their resistances.
- The electromotive force (*emf*) of a battery is the maximum potential difference that the battery can create. When a current flows, some of the *emf* is used to move the current through the internal resistance of the battery. The terminal voltage of the battery is the *emf* less the potential difference across the internal resistance. The terminal voltage is the potential difference available to the circuit to which the battery is attached.
- In a circuit composed of several loads, the equivalent resistance of the circuit is the resistance offered to the terminal voltage of the power supply by all the loads in the circuit.
- The power output (rate of energy transfer) of a circuit is measured in watts. Electric energy is sold in units called kilowatt hours (kW · h).

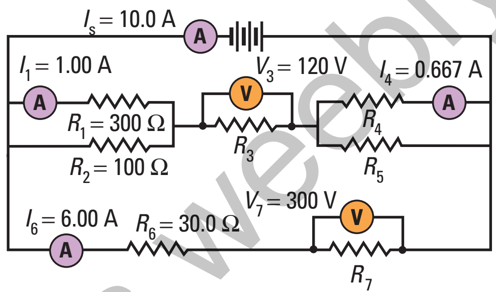
Knowledge/Understanding

1. Describe the difference between potential energy and potential difference.
2. What is the difference between electric current and electron flow?
3. Explain why a light bulb should actually be considered to be a non-ohmic resistor.
4. What is the difference between *emf* and terminal voltage?
5. List the four factors that affect the resistance of a conductor.
6. What is the difference between a battery and a cell?
7. What is meant by the resistivity of a conductor?
8. Define what is meant by the term "elementary charge."
9. Materials are classified by their ability to conduct electricity. Describe the three classifications of materials and give an example of a material in each group.
10. Explain the difference between power and energy.

11. Why does connecting loads in parallel reduce the equivalent resistance of a circuit?
12. A simple circuit consists of a resistance connected to a battery. Explain why connecting two additional resistances in series with the first reduces the power output of the circuit, yet if you connect the two additional resistances in parallel with the first load, the power output increases.
13. What is the significance of Millikan's oil-drop experiment?

Inquiry

14. A 100 W light bulb designed to operate at 120 V has a filament with a resistance of 144 Ω. What constraints are on the design of the filament? Why cannot just any 144 Ω resistance be used as a filament for the bulb?
15. Make voltaic piles by using five pennies and five nickels separated by paper towels soaked in (a) salt water, (b) vinegar, and (c) dilute sulfuric acid.
    - In each case, measure the potential difference of the pile with a voltmeter. Use the voltmeter to measure the potential difference of a single cell. What is the relationship between the values?
    - How does the liquid (the electrolyte) on the paper towel affect the potential difference of the pile?
16. Research the design and make sketches of the (a) dry cell, (b) lead storage battery, and (c) nickel-cadmium battery. (This information is available on the Internet or in chemistry or physics textbooks.) Label and specify the material or chemical used for the anode, cathode, and electrolyte. What are the advantages and disadvantages of each type of cell or battery?
17. The electric circuit to which a television set is connected is protected by a 15 A circuit breaker. If the power rating of the set is 450 W, how many 100 W light bulbs can be operated as well on this circuit without overloading the circuit breaker?
18. In the circuit diagram below, values for some of the quantities for each part of the circuit are given. Calculate the missing currents, resis-tances, and the potential differences for each of the loads in the circuit. Find the equivalent resistance for the circuit and the power output for the circuit.



Communication

19. Describe how series and parallel circuits differ in terms of current flow and potential difference.
20. On a holiday to Canada, a resident of England decides to bring his electric razor. He has heard that the plugs are different so he buys an adapter that will fit into an electric socket in Canada. When he plugs in and turns on his razor in Canada, it runs very slowly and is very weak. Explain why.

Making Connections

21. When you turn up the heating element of an electric stove from low to high, are you increasing or decreasing the resistance of the circuit? Explain.
22. Find the power rating for several of the appliances you have in your home (TV, clothes dryer, electric kettle, iron, hair dryer, and vacuum cleaner, for example). Estimate, for each appliance, how long it is used per month. Make a data table with the column headings: Name of Appliance, Power Rating (kW), Time Used per Month (h), Energy Use per Month (MJ), Cost of Operation (find the charge per kW · h on your electric bill). If you know how, you could use an electronic spreadsheet to organize your calculations.

23. When electricity is transmitted over long distances, very high voltages are used. Consider a hydo-electric plant that has a power output of 25.0 MW. This power output can be achieved by transmitting a small current at a high voltage or a large current at a low voltage.

(a) What would be the current if it was transmitted at a potential difference of 25.0 MV?

(b) What would be the current if it was transmitted at a potential difference of 25 kV?

(c) The transmission lines have a resistance of 0.0100 Ω/km. In each case, how much of the potential difference would be used to push the current through 1000 km of line?

Problems for Understanding

24. A light bulb is rated at 200 W for a 32.0 V power supply. What is its power output if it is inadvertently connected to a 120 V supply?

25. A 1400 W–120 V toaster requires 3.60 minutes to toast a slice of bread.

(a) What current does it draw?

(b) How much charge passes through the toaster in that time?

(c) How much heat and light would be produced in that time?

26. The heating element of a stove operates at 240 V. How much electric energy is converted to heat if it takes 5.50 minutes to bring a pot of water to a boil? The element draws a current of 6.25 A.

27. How much does it cost to run a 15.0 Ω load for 12.0 minutes on a 125 V supply if the rate for electric energy is $0.0850/kWh?

28. The equivalent resistance of two loads connected in parallel is 25.0 Ω. If the resistance of one of the loads is 75.0 Ω, what is the resistance of the other load?

29. A load, R_1, is connected in series with two loads, R_2 and R_3, which are connected in parallel with each other. If the potential difference of the power supply is 180 V, find the current through and the potential difference across each of the loads. The loads have resistances of 25.0 Ω, 30.0 Ω and 6.00 Ω, respectively.

30. A motor draws a current of 4.80 A from a 36.0 V battery. How long would it take the motor to lift a 5.00 kg mass to a height of 35.0 m? Assume 100% efficiency.

31. A 45.0 m extension cord is made using 18 gauge copper wire. It is connected to a 120 V power supply to operate a 1.0×10^2 W-120 V light bulb.

(a) What is the resistance of the extension cord? (Remember that there are two wires to carry the current in the cord.)

(b) What is the resistance of the filament in the light bulb?

(c) What is the current through the cord to the light bulb?

(d) What is the actual power output of the light bulb?

32. When a battery is connected to a load with a resistance of 40.0 Ω, the terminal voltage is 24.0 V. When the resistance of the load is reduced to 15.0 Ω, the terminal voltage is 23.5 V. Find the *emf* and the internal resistance of the battery.

Numerical Answers to Practice Problems

1. 20.0 V **2.** 0.378 J **3.** 6.5×10^{-2} C **4.** 40.0 V **5.** 8.0 s **6.** 4.23×10^3 J **7.** 50 A **8.** 57 s **9.** 7×10^4 C **10.** 2.8 A **11.** 4.6×10^7 J **12.** 0.133 A **13. (a)** 9.38 A **(b)** 2.11×10^{22} elementary charges **14.** 5.25×10^{20} elementary charges **15. (a)** 3.3 A **(b)** 1.7 V **16.** 2.2 Ω **17.** 4.08 m **18.** 1.6×10^{-6} m **19.** 0.45 Ω **20.** 2.4 mm **21.** 16 Ω **22.** 12.5 A **23.** 5.0 V **24. (a)** 9.9×10^2 C **(b)** 2.1 A **25.** 11.6 Ω **26.** 7.50 min **27.** 33 V, 53 V and 79 V respectively **(b)** 75 Ω **(c)** 1.6×10^2 V **28. (a)** 91.0 V **(b)** 156 V **29.** 42.0 Ω **30. (a)** 8.00 Ω **(b)** 224 V **(c)** 32.0 Ω **31.** 44.0 Ω **32.** 0.667 A, 1.00 A and 1.33 A respectively; 3.00 Ω **33.** $R_{coil} = 6.00$ Ω, $R_{bulb} = 20.0$ Ω, $R_S = 4.62$ Ω **34.** $R_{unknown} = 8.00$ Ω, $R_S = 4.80$ Ω **35. (a)** 11.2 Ω **(b)** 21.6 Ω, 30.0 Ω **36. (a)** 38.4 Ω **(b)** 2.25 A **(c)** 91.5 V **37. (a)** 15.4 Ω **(b)** 9.76 V **(c)** 1.02 A **38. (a)** 14.8 V **(b)** 14.6 V **39. (a)** 11.4 V **(b)** 11.2 V **40. (a)** 7.3 A **(b)** 16 Ω **41. (a)** 6.0×10^1 W **(b)** 27 W **(c)** 1.1×10^2 Ω **42. (a)** 840 W **(b)** The power output drops to $\frac{1}{4}$ its original value, or 210 W **43. (a)** $P_a = 720$ W, $P_b = 1.6 \times 10^3$ W **(b)** $P_a/P_b = 4/9$; $V_a/V_b = 2/3$; $P_a/P_b = (V_a/V_b)^2$ **44.** 1.0×10^3 W **45. (a)** 400 W **(b)** 200 W. Increasing resistance decreased the current for the given potential difference. **46.** 48.0 V **47.** 15 Ω **48.** 294 W **49.** 2.00×10^3 C **50. (a)** 550 W **(b)** 5.0×10^6 J **51.** 3.75 cents **52.** 1.08 cents **53. (a)** 1.4×10^2 W **(b)** 0.50 cents