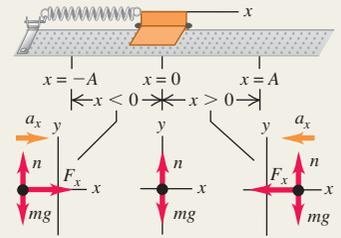


CHAPTER 14 SUMMARY

Periodic motion: Periodic motion is motion that repeats itself in a definite cycle. It occurs whenever a body has a stable equilibrium position and a restoring force that acts when it is displaced from equilibrium. Period T is the time for one cycle. Frequency f is the number of cycles per unit time. Angular frequency ω is 2π times the frequency. (See Example 14.1.)

$$f = \frac{1}{T} \quad T = \frac{1}{f} \quad (14.1)$$

$$\omega = 2\pi f = \frac{2\pi}{T} \quad (14.2)$$



Simple harmonic motion: If the restoring force F_x in periodic motion is directly proportional to the displacement x , the motion is called simple harmonic motion (SHM). In many cases this condition is satisfied if the displacement from equilibrium is small. The angular frequency, frequency, and period in SHM do not depend on the amplitude, but only on the mass m and force constant k . The displacement, velocity, and acceleration in SHM are sinusoidal functions of time; the amplitude A and phase angle ϕ of the oscillation are determined by the initial position and velocity of the body. (See Examples 14.2, 14.3, 14.6, and 14.7.)

$$F_x = -kx \quad (14.3)$$

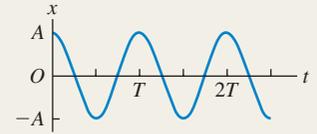
$$a_x = \frac{F_x}{m} = -\frac{k}{m}x \quad (14.4)$$

$$\omega = \sqrt{\frac{k}{m}} \quad (14.10)$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (14.11)$$

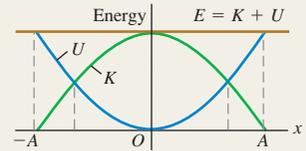
$$T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}} \quad (14.12)$$

$$x = A \cos(\omega t + \phi) \quad (14.13)$$



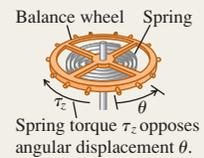
Energy in simple harmonic motion: Energy is conserved in SHM. The total energy can be expressed in terms of the force constant k and amplitude A . (See Examples 14.4 and 14.5.)

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \text{constant} \quad (14.21)$$



Angular simple harmonic motion: In angular SHM, the frequency and angular frequency are related to the moment of inertia I and the torsion constant κ .

$$\omega = \sqrt{\frac{\kappa}{I}} \quad \text{and} \quad f = \frac{1}{2\pi} \sqrt{\frac{\kappa}{I}} \quad (14.24)$$

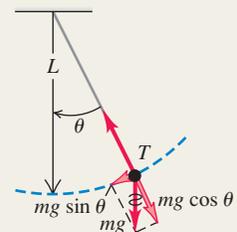


Simple pendulum: A simple pendulum consists of a point mass m at the end of a massless string of length L . Its motion is approximately simple harmonic for sufficiently small amplitude; the angular frequency, frequency, and period then depend only on g and L , not on the mass or amplitude. (See Example 14.8.)

$$\omega = \sqrt{\frac{g}{L}} \quad (14.32)$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \quad (14.33)$$

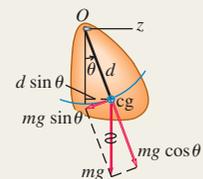
$$T = \frac{2\pi}{\omega} = \frac{1}{f} = 2\pi \sqrt{\frac{L}{g}} \quad (14.34)$$



Physical pendulum: A physical pendulum is any body suspended from an axis of rotation. The angular frequency and period for small-amplitude oscillations are independent of amplitude, but depend on the mass m , distance d from the axis of rotation to the center of gravity, and moment of inertia I about the axis. (See Examples 14.9 and 14.10.)

$$\omega = \sqrt{\frac{mgd}{I}} \quad (14.38)$$

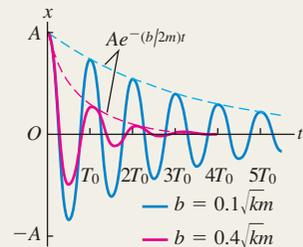
$$T = 2\pi \sqrt{\frac{I}{mgd}} \quad (14.39)$$



Damped oscillations: When a force $F_x = -bv_x$ proportional to velocity is added to a simple harmonic oscillator, the motion is called a damped oscillation. If $b < 2\sqrt{km}$ (called underdamping), the system oscillates with a decaying amplitude and an angular frequency ω' that is lower than it would be without damping. If $b = 2\sqrt{km}$ (called critical damping) or $b > 2\sqrt{km}$ (called overdamping), when the system is displaced it returns to equilibrium without oscillating.

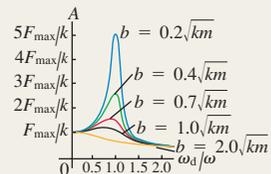
$$x = Ae^{-(b/2m)t} \cos(\omega't + \phi) \quad (14.42)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \quad (14.43)$$



Driven oscillations and resonance: When a sinusoidally varying driving force is added to a damped harmonic oscillator, the resulting motion is called a forced oscillation. The amplitude is a function of the driving frequency ω_d and reaches a peak at a driving frequency close to the natural frequency of the system. This behavior is called resonance.

$$A = \frac{F_{\max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}} \quad (14.46)$$

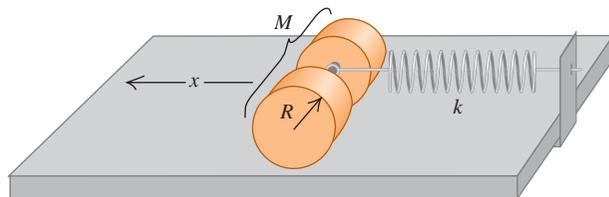


BRIDGING PROBLEM

Oscillating and Rolling

Two uniform, solid cylinders of radius R and total mass M are connected along their common axis by a short, light rod and rest on a horizontal tabletop (Fig. 14.29). A frictionless ring at the center of the rod is attached to a spring with force constant k ; the other end of the spring is fixed. The cylinders are pulled to the left a distance x , stretching the spring, and then released from rest. Due to friction between the tabletop and the cylinders, the cylinders roll without slipping as they oscillate. Show that the motion of the center of mass of the cylinders is simple harmonic, and find its period.

14.29



SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.



IDENTIFY and SET UP

1. What condition must be satisfied for the motion of the center of mass of the cylinders to be simple harmonic? (*Hint:* See Section 14.2.)
2. Which equations should you use to describe the translational and rotational motions of the cylinders? Which equation should you use to describe the condition that the cylinders roll without slipping? (*Hint:* See Section 10.3.)
3. Sketch the situation and choose a coordinate system. Make a list of the unknown quantities and decide which is the target variable.

EXECUTE

4. Draw a free-body diagram for the cylinders when they are displaced a distance x from equilibrium.
5. Solve the equations to find an expression for the acceleration of the center of mass of the cylinders. What does this expression tell you?
6. Use your result from step 5 to find the period of oscillation of the center of mass of the cylinders.

EVALUATE

7. What would be the period of oscillation if there were no friction and the cylinders didn't roll? Is this period larger or smaller than your result from step 6? Is this reasonable?

Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q14.1 An object is moving with SHM of amplitude A on the end of a spring. If the amplitude is doubled, what happens to the total distance the object travels in one period? What happens to the period? What happens to the maximum speed of the object? Discuss how these answers are related.

Q14.2 Think of several examples in everyday life of motions that are, at least approximately, simple harmonic. In what respects does each differ from SHM?

Q14.3 Does a tuning fork or similar tuning instrument undergo SHM? Why is this a crucial question for musicians?

Q14.4 A box containing a pebble is attached to an ideal horizontal spring and is oscillating on a friction-free air table. When the box has reached its maximum distance from the equilibrium point, the pebble is suddenly lifted out vertically without disturbing the box. Will the following characteristics of the motion increase, decrease, or remain the same in the subsequent motion of the box? Justify each answer. (a) frequency; (b) period; (c) amplitude; (d) the maximum kinetic energy of the box; (e) the maximum speed of the box.

Q14.5 If a uniform spring is cut in half, what is the force constant of each half? Justify your answer. How would the frequency of SHM using a half-spring differ from the frequency using the same mass and the entire spring?

Q14.6 The analysis of SHM in this chapter ignored the mass of the spring. How does the spring's mass change the characteristics of the motion?

Q14.7 Two identical gliders on an air track are connected by an ideal spring. Could such a system undergo SHM? Explain. How would the period compare with that of a single glider attached to a spring whose other end is rigidly attached to a stationary object? Explain.

Q14.8 You are captured by Martians, taken into their ship, and put to sleep. You awake some time later and find yourself locked in a small room with no windows. All the Martians have left you with is your digital watch, your school ring, and your long silver-chain necklace. Explain how you can determine whether you are still on earth or have been transported to Mars.

Q14.9 The system shown in Fig. 14.17 is mounted in an elevator. What happens to the period of the motion (does it increase, decrease, or remain the same) if the elevator (a) accelerates upward at 5.0 m/s^2 ; (b) moves upward at a steady 5.0 m/s ; (c) accelerates downward at 5.0 m/s^2 ? Justify your answers.

Q14.10 If a pendulum has a period of 2.5 s on earth, what would be its period in a space station orbiting the earth? If a mass hung from a vertical spring has a period of 5.0 s on earth, what would its period be in the space station? Justify each of your answers.

Q14.11 A simple pendulum is mounted in an elevator. What happens to the period of the pendulum (does it increase, decrease, or remain the same) if the elevator (a) accelerates upward at 5.0 m/s^2 ; (b) moves upward at a steady 5.0 m/s ; (c) accelerates downward at 5.0 m/s^2 ; (d) accelerates downward at 9.8 m/s^2 ? Justify your answers.

Q14.12 What should you do to the length of the string of a simple pendulum to (a) double its frequency; (b) double its period; (c) double its angular frequency?

Q14.13 If a pendulum clock is taken to a mountaintop, does it gain or lose time, assuming it is correct at a lower elevation? Explain your answer.

Q14.14 When the amplitude of a simple pendulum increases, should its period increase or decrease? Give a qualitative argument; do not rely on Eq. (14.35). Is your argument also valid for a physical pendulum?

Q14.15 Why do short dogs (like Chihuahuas) walk with quicker strides than do tall dogs (like Great Danes)?

Q14.16 At what point in the motion of a simple pendulum is the string tension greatest? Least? In each case give the reasoning behind your answer.

Q14.17 Could a standard of time be based on the period of a certain standard pendulum? What advantages and disadvantages would such a standard have compared to the actual present-day standard discussed in Section 1.3?

Q14.18 For a simple pendulum, clearly distinguish between ω (the angular velocity) and ω (the angular frequency). Which is constant and which is variable?

Q14.19 A glider is attached to a fixed ideal spring and oscillates on a horizontal, friction-free air track. A coin is atop the glider and oscillating with it. At what points in the motion is the friction force on the coin greatest? At what points is it least? Justify your answers.

Q14.20 In designing structures in an earthquake-prone region, how should the natural frequencies of oscillation of a structure relate to typical earthquake frequencies? Why? Should the structure have a large or small amount of damping?

EXERCISES

Section 14.1 Describing Oscillation

14.1 • BIO (a) **Music.** When a person sings, his or her vocal cords vibrate in a repetitive pattern that has the same frequency as the note that is sung. If someone sings the note B flat, which has a frequency of 466 Hz , how much time does it take the person's vocal cords to vibrate through one complete cycle, and what is the angular frequency of the cords? (b) **Hearing.** When sound waves strike the eardrum, this membrane vibrates with the same frequency as the sound. The highest pitch that typical humans can hear has a period of $50.0 \mu\text{s}$. What are the frequency and angular frequency of the vibrating eardrum for this sound? (c) **Vision.** When light having vibrations with angular frequency ranging from $2.7 \times 10^{15} \text{ rad/s}$ to $4.7 \times 10^{15} \text{ rad/s}$ strikes the retina of the eye, it stimulates the receptor cells there and is perceived as visible light. What are the limits of the period and frequency of this light? (d) **Ultrasound.** High-frequency sound waves (ultrasound) are used to probe the interior of the body, much as x rays do. To detect small objects such as tumors, a frequency of around 5.0 MHz is used. What are the period and angular frequency of the molecular vibrations caused by this pulse of sound?

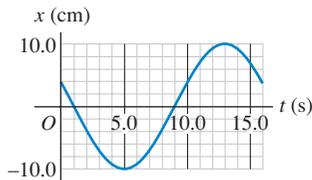
14.2 • If an object on a horizontal, frictionless surface is attached to a spring, displaced, and then released, it will oscillate. If it is displaced 0.120 m from its equilibrium position and released with zero initial speed, then after 0.800 s its displacement is found to be

0.120 m on the opposite side, and it has passed the equilibrium position once during this interval. Find (a) the amplitude; (b) the period; (c) the frequency.

14.3 • The tip of a tuning fork goes through 440 complete vibrations in 0.500 s. Find the angular frequency and the period of the motion.

14.4 • The displacement of an oscillating object as a function of time is shown in Fig. E14.4. What are (a) the frequency; (b) the amplitude; (c) the period; (d) the angular frequency of this motion?

Figure E14.4



14.5 •• A machine part is undergoing SHM with a frequency of 5.00 Hz and amplitude 1.80 cm. How long does it take the part to go from $x = 0$ to $x = -1.80$ cm?

Section 14.2 Simple Harmonic Motion

14.6 •• In a physics lab, you attach a 0.200-kg air-track glider to the end of an ideal spring of negligible mass and start it oscillating. The elapsed time from when the glider first moves through the equilibrium point to the second time it moves through that point is 2.60 s. Find the spring's force constant.

14.7 • When a body of unknown mass is attached to an ideal spring with force constant 120 N/m, it is found to vibrate with a frequency of 6.00 Hz. Find (a) the period of the motion; (b) the angular frequency; (c) the mass of the body.

14.8 • When a 0.750-kg mass oscillates on an ideal spring, the frequency is 1.33 Hz. What will the frequency be if 0.220 kg are (a) added to the original mass and (b) subtracted from the original mass? Try to solve this problem *without* finding the force constant of the spring.

14.9 •• An object is undergoing SHM with period 0.900 s and amplitude 0.320 m. At $t = 0$ the object is at $x = 0.320$ m and is instantaneously at rest. Calculate the time it takes the object to go (a) from $x = 0.320$ m to $x = 0.160$ m and (b) from $x = 0.160$ m to $x = 0$.

14.10 • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the block is at $x = 0.280$ m, the acceleration of the block is -5.30 m/s². What is the frequency of the motion?

14.11 • A 2.00-kg, frictionless block is attached to an ideal spring with force constant 300 N/m. At $t = 0$ the spring is neither stretched nor compressed and the block is moving in the negative direction at 12.0 m/s. Find (a) the amplitude and (b) the phase angle. (c) Write an equation for the position as a function of time.

14.12 •• Repeat Exercise 14.11, but assume that at $t = 0$ the block has velocity -4.00 m/s and displacement $+0.200$ m.

14.13 • The point of the needle of a sewing machine moves in SHM along the x -axis with a frequency of 2.5 Hz. At $t = 0$ its position and velocity components are $+1.1$ cm and -15 cm/s, respectively. (a) Find the acceleration component of the needle at $t = 0$. (b) Write equations giving the position, velocity, and acceleration components of the point as a function of time.

14.14 •• A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. When the ampli-

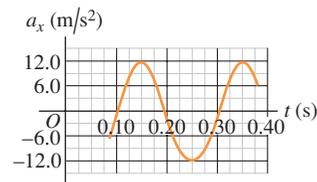
tude of the motion is 0.090 m, it takes the block 2.70 s to travel from $x = 0.090$ m to $x = -0.090$ m. If the amplitude is doubled, to 0.180 m, how long does it take the block to travel (a) from $x = 0.180$ m to $x = -0.180$ m and (b) from $x = 0.090$ m to $x = -0.090$ m?

14.15 • **BIO Weighing Astronauts.** This procedure has actually been used to “weigh” astronauts in space. A 42.5-kg chair is attached to a spring and allowed to oscillate. When it is empty, the chair takes 1.30 s to make one complete vibration. But with an astronaut sitting in it, with her feet off the floor, the chair takes 2.54 s for one cycle. What is the mass of the astronaut?

14.16 • A 0.400-kg object undergoing SHM has $a_x = -2.70$ m/s² when $x = 0.300$ m. What is the time for one oscillation?

14.17 • On a frictionless, horizontal air track, a glider oscillates at the end of an ideal spring of force constant 2.50 N/cm. The graph in Fig. E14.17 shows the acceleration of the glider as a function of time. Find (a) the mass of the glider; (b) the maximum displacement of the glider from the equilibrium point; (c) the maximum force the spring exerts on the glider.

Figure E14.17



14.18 • A 0.500-kg mass on a spring has velocity as a function of time given by $v_x(t) = -(3.60 \text{ cm/s}) \sin[(4.71 \text{ s}^{-1})t - \pi/2]$. What are (a) the period; (b) the amplitude; (c) the maximum acceleration of the mass; (d) the force constant of the spring?

14.19 • A 1.50-kg mass on a spring has displacement as a function of time given by the equation

$$x(t) = (7.40 \text{ cm}) \cos[(4.16 \text{ s}^{-1})t - 2.42]$$

Find (a) the time for one complete vibration; (b) the force constant of the spring; (c) the maximum speed of the mass; (d) the maximum force on the mass; (e) the position, speed, and acceleration of the mass at $t = 1.00$ s; (f) the force on the mass at that time.

14.20 • **BIO Weighing a Virus.** In February 2004, scientists at Purdue University used a highly sensitive technique to measure the mass of a vaccinia virus (the kind used in smallpox vaccine). The procedure involved measuring the frequency of oscillation of a tiny sliver of silicon (just 30 nm long) with a laser, first without the virus and then after the virus had attached itself to the silicon. The difference in mass caused a change in the frequency. We can model such a process as a mass on a spring. (a) Show that the ratio of the frequency with the virus attached (f_{S+V}) to the frequency without the virus (f_S) is given by the formula $\frac{f_{S+V}}{f_S} = \frac{1}{\sqrt{1 + (m_V/m_S)}}$,

where m_V is the mass of the virus and m_S is the mass of the silicon sliver. Notice that it is *not* necessary to know or measure the force constant of the spring. (b) In some data, the silicon sliver has a mass of 2.10×10^{-16} g and a frequency of 2.00×10^{15} Hz without the virus and 2.87×10^{14} Hz with the virus. What is the mass of the virus, in grams and in femtograms?

14.21 •• **CALC Jerk.** A guitar string vibrates at a frequency of 440 Hz. A point at its center moves in SHM with an amplitude of

3.0 mm and a phase angle of zero. (a) Write an equation for the position of the center of the string as a function of time. (b) What are the maximum values of the magnitudes of the velocity and acceleration of the center of the string? (c) The derivative of the acceleration with respect to time is a quantity called the *jerk*. Write an equation for the jerk of the center of the string as a function of time, and find the maximum value of the magnitude of the jerk.

Section 14.3 Energy in Simple Harmonic Motion

14.22 •• For the oscillating object in Fig. E14.4, what are (a) its maximum speed and (b) its maximum acceleration?

14.23 • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. The amplitude of the motion is 0.120 m. The maximum speed of the block is 3.90 m/s. What is the maximum magnitude of the acceleration of the block?

14.24 • A small block is attached to an ideal spring and is moving in SHM on a horizontal, frictionless surface. The amplitude of the motion is 0.250 m and the period is 3.20 s. What are the speed and acceleration of the block when $x = 0.160$ m?

14.25 •• A tuning fork labeled 392 Hz has the tip of each of its two prongs vibrating with an amplitude of 0.600 mm. (a) What is the maximum speed of the tip of a prong? (b) A housefly (*Musca domestica*) with mass 0.0270 g is holding onto the tip of one of the prongs. As the prong vibrates, what is the fly's maximum kinetic energy? Assume that the fly's mass has a negligible effect on the frequency of oscillation.

14.26 •• A harmonic oscillator has angular frequency ω and amplitude A . (a) What are the magnitudes of the displacement and velocity when the elastic potential energy is equal to the kinetic energy? (Assume that $U = 0$ at equilibrium.) (b) How often does this occur in each cycle? What is the time between occurrences? (c) At an instant when the displacement is equal to $A/2$, what fraction of the total energy of the system is kinetic and what fraction is potential?

14.27 • A 0.500-kg glider, attached to the end of an ideal spring with force constant $k = 450$ N/m, undergoes SHM with an amplitude of 0.040 m. Compute (a) the maximum speed of the glider; (b) the speed of the glider when it is at $x = -0.015$ m; (c) the magnitude of the maximum acceleration of the glider; (d) the acceleration of the glider at $x = -0.015$ m; (e) the total mechanical energy of the glider at any point in its motion.

14.28 •• A cheerleader waves her pom-pom in SHM with an amplitude of 18.0 cm and a frequency of 0.850 Hz. Find (a) the maximum magnitude of the acceleration and of the velocity; (b) the acceleration and speed when the pom-pom's coordinate is $x = +9.0$ cm; (c) the time required to move from the equilibrium position directly to a point 12.0 cm away. (d) Which of the quantities asked for in parts (a), (b), and (c) can be found using the energy approach used in Section 14.3, and which cannot? Explain.

14.29 • CP For the situation described in part (a) of Example 14.5, what should be the value of the putty mass m so that the amplitude after the collision is one-half the original amplitude? For this value of m , what fraction of the original mechanical energy is converted into heat?

14.30 • A 0.150-kg toy is undergoing SHM on the end of a horizontal spring with force constant $k = 300$ N/m. When the object is 0.0120 m from its equilibrium position, it is observed to have a speed of 0.300 m/s. What are (a) the total energy of the object at any point of its motion; (b) the amplitude of the motion; (c) the maximum speed attained by the object during its motion?

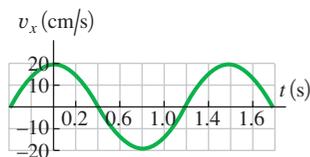
14.31 •• You are watching an object that is moving in SHM. When the object is displaced 0.600 m to the right of its equilibrium position, it has a velocity of 2.20 m/s to the right and an acceleration of 8.40 m/s² to the left. How much farther from this point will the object move before it stops momentarily and then starts to move back to the left?

14.32 •• On a horizontal, frictionless table, an open-topped 5.20-kg box is attached to an ideal horizontal spring having force constant 375 N/m. Inside the box is a 3.44-kg stone. The system is oscillating with an amplitude of 7.50 cm. When the box has reached its maximum speed, the stone is suddenly plucked vertically out of the box without touching the box. Find (a) the period and (b) the amplitude of the resulting motion of the box. (c) Without doing any calculations, is the new period greater or smaller than the original period? How do you know?

14.33 •• A mass is oscillating with amplitude A at the end of a spring. How far (in terms of A) is this mass from the equilibrium position of the spring when the elastic potential energy equals the kinetic energy?

14.34 •• A mass m is attached to a spring of force constant 75 N/m and allowed to oscillate. Figure E14.34 shows a graph of its velocity v_x as a function of time t . Find (a) the period, (b) the frequency, and (c) the angular frequency of this motion. (d) What is the amplitude (in cm), and at what times does the mass reach this position? (e) Find the maximum acceleration of the mass and the times at which it occurs. (f) What is the mass m ?

Figure E14.34



14.35 • Inside a NASA test vehicle, a 3.50-kg ball is pulled along by a horizontal ideal spring fixed to a friction-free table. The force constant of the spring is 225 N/m. The vehicle has a steady acceleration of 5.00 m/s², and the ball is not oscillating. Suddenly, when the vehicle's speed has reached 45.0 m/s, its engines turn off, thus eliminating its acceleration but not its velocity. Find (a) the amplitude and (b) the frequency of the resulting oscillations of the ball. (c) What will be the ball's maximum speed relative to the vehicle?

Section 14.4 Applications of Simple Harmonic Motion

14.36 • A proud deep-sea fisherman hangs a 65.0-kg fish from an ideal spring having negligible mass. The fish stretches the spring 0.120 m. (a) Find the force constant of the spring. The fish is now pulled down 5.00 cm and released. (b) What is the period of oscillation of the fish? (c) What is the maximum speed it will reach?

14.37 • A 175-g glider on a horizontal, frictionless air track is attached to a fixed ideal spring with force constant 155 N/m. At the instant you make measurements on the glider, it is moving at 0.815 m/s and is 3.00 cm from its equilibrium point. Use *energy conservation* to find (a) the amplitude of the motion and (b) the maximum speed of the glider. (c) What is the angular frequency of the oscillations?

14.38 • A thrill-seeking cat with mass 4.00 kg is attached by a harness to an ideal spring of negligible mass and oscillates vertically in SHM. The amplitude is 0.050 m, and at the highest point

of the motion the spring has its natural unstretched length. Calculate the elastic potential energy of the spring (take it to be zero for the unstretched spring), the kinetic energy of the cat, the gravitational potential energy of the system relative to the lowest point of the motion, and the sum of these three energies when the cat is (a) at its highest point; (b) at its lowest point; (c) at its equilibrium position.

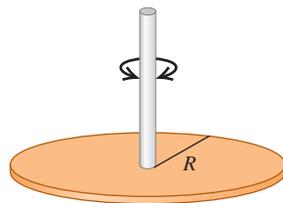
14.39 •• A 1.50-kg ball and a 2.00-kg ball are glued together with the lighter one below the heavier one. The upper ball is attached to a vertical ideal spring of force constant 165 N/m, and the system is vibrating vertically with amplitude 15.0 cm. The glue connecting the balls is old and weak, and it suddenly comes loose when the balls are at the lowest position in their motion. (a) Why is the glue more likely to fail at the *lowest* point than at any other point in the motion? (b) Find the amplitude and frequency of the vibrations after the lower ball has come loose.

14.40 •• A uniform, solid metal disk of mass 6.50 kg and diameter 24.0 cm hangs in a horizontal plane, supported at its center by a vertical metal wire. You find that it requires a horizontal force of 4.23 N tangent to the rim of the disk to turn it by 3.34° , thus twisting the wire. You now remove this force and release the disk from rest. (a) What is the torsion constant for the metal wire? (b) What are the frequency and period of the torsional oscillations of the disk? (c) Write the equation of motion for $\theta(t)$ for the disk.

14.41 •• A certain alarm clock ticks four times each second, with each tick representing half a period. The balance wheel consists of a thin rim with radius 0.55 cm, connected to the balance staff by thin spokes of negligible mass. The total mass of the balance wheel is 0.90 g. (a) What is the moment of inertia of the balance wheel about its shaft? (b) What is the torsion constant of the coil spring (Fig. 14.19)?

14.42 • A thin metal disk with mass 2.00×10^{-3} kg and radius 2.20 cm is attached at its center to a long fiber (Fig. E14.42). The disk, when twisted and released, oscillates with a period of 1.00 s. Find the torsion constant of the fiber.

Figure E14.42



14.43 •• You want to find the moment of inertia of a complicated machine part about an axis through its center of mass. You suspend it from a wire along this axis. The wire has a torsion constant of $0.450 \text{ N} \cdot \text{m}/\text{rad}$. You twist the part a small amount about this axis and let it go, timing 125 oscillations in 265 s. What is the moment of inertia you want to find?

14.44 •• **CALC** The balance wheel of a watch vibrates with an angular amplitude Θ , angular frequency ω , and phase angle $\phi = 0$. (a) Find expressions for the angular velocity $d\theta/dt$ and angular acceleration $d^2\theta/dt^2$ as functions of time. (b) Find the balance wheel's angular velocity and angular acceleration when its angular displacement is Θ , and when its angular displacement is $\Theta/2$ and θ is decreasing. (*Hint*: Sketch a graph of θ versus t .)

Section 14.5 The Simple Pendulum

14.45 •• You pull a simple pendulum 0.240 m long to the side through an angle of 3.50° and release it. (a) How much time does it take the pendulum bob to reach its highest speed? (b) How much time does it take if the pendulum is released at an angle of 1.75° instead of 3.50° ?

14.46 • An 85.0-kg mountain climber plans to swing down, starting from rest, from a ledge using a light rope 6.50 m long. He holds

one end of the rope, and the other end is tied higher up on a rock face. Since the ledge is not very far from the rock face, the rope makes a small angle with the vertical. At the lowest point of his swing, he plans to let go and drop a short distance to the ground. (a) How long after he begins his swing will the climber first reach his lowest point? (b) If he missed the first chance to drop off, how long after first beginning his swing will the climber reach his lowest point for the second time?

14.47 • A building in San Francisco has light fixtures consisting of small 2.35-kg bulbs with shades hanging from the ceiling at the end of light, thin cords 1.50 m long. If a minor earthquake occurs, how many swings per second will these fixtures make?

14.48 • **A Pendulum on Mars.** A certain simple pendulum has a period on the earth of 1.60 s. What is its period on the surface of Mars, where $g = 3.71 \text{ m/s}^2$?

14.49 • After landing on an unfamiliar planet, a space explorer constructs a simple pendulum of length 50.0 cm. She finds that the pendulum makes 100 complete swings in 136 s. What is the value of g on this planet?

14.50 •• A small sphere with mass m is attached to a massless rod of length L that is pivoted at the top, forming a simple pendulum. The pendulum is pulled to one side so that the rod is at an angle Θ from the vertical, and released from rest. (a) In a diagram, show the pendulum just after it is released. Draw vectors representing the *forces* acting on the small sphere and the *acceleration* of the sphere. Accuracy counts! At this point, what is the linear acceleration of the sphere? (b) Repeat part (a) for the instant when the pendulum rod is at an angle $\Theta/2$ from the vertical. (c) Repeat part (a) for the instant when the pendulum rod is vertical. At this point, what is the linear speed of the sphere?

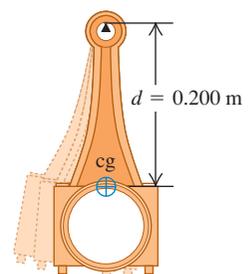
14.51 • A simple pendulum 2.00 m long swings through a maximum angle of 30.0° with the vertical. Calculate its period (a) assuming a small amplitude, and (b) using the first three terms of Eq. (14.35). (c) Which of the answers in parts (a) and (b) is more accurate? For the one that is less accurate, by what percent is it in error from the more accurate answer?

Section 14.6 The Physical Pendulum

14.52 •• We want to hang a thin hoop on a horizontal nail and have the hoop make one complete small-angle oscillation each 2.0 s. What must the hoop's radius be?

14.53 • A 1.80-kg connecting rod from a car engine is pivoted about a horizontal knife edge as shown in Fig. E14.53. The center of gravity of the rod was located by balancing and is 0.200 m from the pivot. When the rod is set into small-amplitude oscillation, it makes 100 complete swings in 120 s. Calculate the moment of inertia of the rod about the rotation axis through the pivot.

Figure E14.53



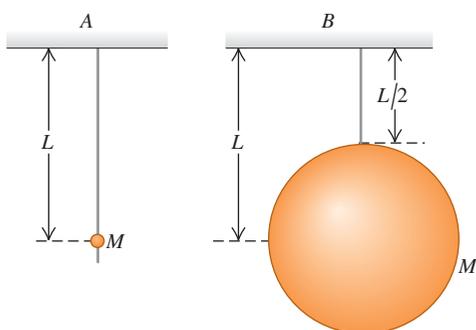
14.54 •• A 1.80-kg monkey wrench is pivoted 0.250 m from its center of mass and allowed to swing as a physical pendulum. The period for small-angle oscillations is 0.940 s. (a) What is the moment of inertia of the wrench about an axis through the pivot? (b) If the wrench is initially displaced 0.400 rad from its equilibrium position, what is the angular speed of the wrench as it passes through the equilibrium position?

14.55 • Two pendulums have the same dimensions (length L) and total mass (m). Pendulum A is a very small ball swinging at the end of a uniform massless bar. In pendulum B , half the mass is in the ball and half is in the uniform bar. Find the period of each pendulum for small oscillations. Which one takes longer for a swing?

14.56 • **CP** A holiday ornament in the shape of a hollow sphere with mass $M = 0.015$ kg and radius $R = 0.050$ m is hung from a tree limb by a small loop of wire attached to the surface of the sphere. If the ornament is displaced a small distance and released, it swings back and forth as a physical pendulum with negligible friction. Calculate its period. (*Hint*: Use the parallel-axis theorem to find the moment of inertia of the sphere about the pivot at the tree limb.)

14.57 • The two pendulums shown in Fig. E14.57 each consist of a uniform solid ball of mass M supported by a rigid massless rod, but the ball for pendulum A is very tiny while the ball for pendulum B is much larger. Find the period of each pendulum for small displacements. Which ball takes longer to complete a swing?

Figure E14.57



Section 14.7 Damped Oscillations

14.58 • A 2.50-kg rock is attached at the end of a thin, very light rope 1.45 m long. You start it swinging by releasing it when the rope makes an 11° angle with the vertical. You record the observation that it rises only to an angle of 4.5° with the vertical after $10\frac{1}{2}$ swings. (a) How much energy has this system lost during that time? (b) What happened to the “lost” energy? Explain *how* it could have been “lost.”

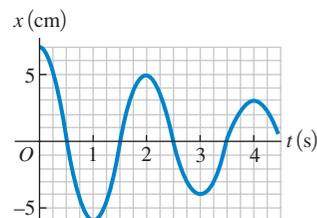
14.59 • An unhappy 0.300-kg rodent, moving on the end of a spring with force constant $k = 2.50$ N/m, is acted on by a damping force $F_x = -bv_x$. (a) If the constant b has the value 0.900 kg/s, what is the frequency of oscillation of the rodent? (b) For what value of the constant b will the motion be critically damped?

14.60 • A 50.0-g hard-boiled egg moves on the end of a spring with force constant $k = 25.0$ N/m. Its initial displacement is 0.300 m. A damping force $F_x = -bv_x$ acts on the egg, and the amplitude of the motion decreases to 0.100 m in 5.00 s. Calculate the magnitude of the damping constant b .

14.61 • **CALC** The motion of an underdamped oscillator is described by Eq. (14.42). Let the phase angle ϕ be zero. (a) According to this equation, what is the value of x at $t = 0$? (b) What are the magnitude and direction of the velocity at $t = 0$? What does the result tell you about the slope of the graph of x versus t near $t = 0$? (c) Obtain an expression for the acceleration a_x at $t = 0$. For what value or range of values of the damping constant b (in terms of k and m) is the acceleration at $t = 0$ negative, zero, and positive? Discuss each case in terms of the shape of the graph of x versus t near $t = 0$.

14.62 • A mass is vibrating at the end of a spring of force constant 225 N/m. Figure E14.62 shows a graph of its position x as a function of time t . (a) At what times is the mass not moving? (b) How much energy did this system originally contain? (c) How much energy did the system lose between $t = 1.0$ s and $t = 4.0$ s? Where did this energy go?

Figure E14.62



Section 14.8 Forced Oscillations and Resonance

14.63 • A sinusoidally varying driving force is applied to a damped harmonic oscillator. (a) What are the units of the damping constant b ? (b) Show that the quantity \sqrt{km} has the same units as b . (c) In terms of F_{\max} and k , what is the amplitude for $\omega_d = \sqrt{k/m}$ when (i) $b = 0.2\sqrt{km}$ and (ii) $b = 0.4\sqrt{km}$? Compare your results to Fig. 14.28.

14.64 • A sinusoidally varying driving force is applied to a damped harmonic oscillator of force constant k and mass m . If the damping constant has a value b_1 , the amplitude is A_1 when the driving angular frequency equals $\sqrt{k/m}$. In terms of A_1 , what is the amplitude for the same driving frequency and the same driving force amplitude F_{\max} , if the damping constant is (a) $3b_1$ and (b) $b_1/2$?

PROBLEMS

14.65 • An object is undergoing SHM with period 1.200 s and amplitude 0.600 m. At $t = 0$ the object is at $x = 0$ and is moving in the negative x -direction. How far is the object from the equilibrium position when $t = 0.480$ s?

14.66 • An object is undergoing SHM with period 0.300 s and amplitude 6.00 cm. At $t = 0$ the object is instantaneously at rest at $x = 6.00$ cm. Calculate the time it takes the object to go from $x = 6.00$ cm to $x = -1.50$ cm.

14.67 • **CP SHM in a Car Engine.** The motion of the piston of an automobile engine is approximately simple harmonic. (a) If the stroke of an engine (twice the amplitude) is 0.100 m and the engine runs at 4500 rev/min, compute the acceleration of the piston at the endpoint of its stroke. (b) If the piston has mass 0.450 kg, what net force must be exerted on it at this point? (c) What are the speed and kinetic energy of the piston at the midpoint of its stroke? (d) What average power is required to accelerate the piston from rest to the speed found in part (c)? (e) If the engine runs at 7000 rev/min, what are the answers to parts (b), (c), and (d)?

14.68 • Four passengers with combined mass 250 kg compress the springs of a car with worn-out shock absorbers by 4.00 cm when they get in. Model the car and passengers as a single body on a single ideal spring. If the loaded car has a period of vibration of 1.92 s, what is the period of vibration of the empty car?

14.69 • A glider is oscillating in SHM on an air track with an amplitude A_1 . You slow it so that its amplitude is halved. What happens to its (a) period, frequency, and angular frequency;

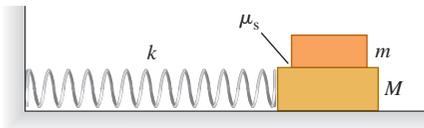
(b) total mechanical energy; (c) maximum speed; (d) speed at $x = \pm A_1/4$; (e) potential and kinetic energies at $x = \pm A_1/4$?

14.70 •• CP A child with poor table manners is sliding his 250-g dinner plate back and forth in SHM with an amplitude of 0.100 m on a horizontal surface. At a point 0.060 m away from equilibrium, the speed of the plate is 0.400 m/s. (a) What is the period? (b) What is the displacement when the speed is 0.160 m/s? (c) In the center of the dinner plate is a 10.0-g carrot slice. If the carrot slice is just on the verge of slipping at the endpoint of the path, what is the coefficient of static friction between the carrot slice and the plate?

14.71 ••• A 1.50-kg, horizontal, uniform tray is attached to a vertical ideal spring of force constant 185 N/m and a 275-g metal ball is in the tray. The spring is below the tray, so it can oscillate up and down. The tray is then pushed down to point A, which is 15.0 cm below the equilibrium point, and released from rest. (a) How high above point A will the tray be when the metal ball leaves the tray? (*Hint:* This does *not* occur when the ball and tray reach their maximum speeds.) (b) How much time elapses between releasing the system at point A and the ball leaving the tray? (c) How fast is the ball moving just as it leaves the tray?

14.72 •• CP A block with mass M rests on a frictionless surface and is connected to a horizontal spring of force constant k . The other end of the spring is attached to a wall (Fig. P14.72). A second block with mass m rests on top of the first block. The coefficient of static friction between the blocks is μ_s . Find the *maximum* amplitude of oscillation such that the top block will not slip on the bottom block.

Figure P14.72



14.73 • CP A 10.0-kg mass is traveling to the right with a speed of 2.00 m/s on a smooth horizontal surface when it collides with and sticks to a second 10.0-kg mass that is initially at rest but is attached to a light spring with force constant 110.0 N/m. (a) Find the frequency, amplitude, and period of the subsequent oscillations. (b) How long does it take the system to return the first time to the position it had immediately after the collision?

14.74 • CP A rocket is accelerating upward at 4.00 m/s^2 from the launchpad on the earth. Inside a small, 1.50-kg ball hangs from the ceiling by a light, 1.10-m wire. If the ball is displaced 8.50° from the vertical and released, find the amplitude and period of the resulting swings of this pendulum.

14.75 ••• An apple weighs 1.00 N. When you hang it from the end of a long spring of force constant 1.50 N/m and negligible mass, it bounces up and down in SHM. If you stop the bouncing and let the apple swing from side to side through a small angle, the frequency of this simple pendulum is half the bounce frequency. (Because the angle is small, the back-and-forth swings do not cause any appreciable change in the length of the spring.) What is the unstretched length of the spring (with the apple removed)?

14.76 ••• CP SHM of a Floating Object. An object with height h , mass M , and a uniform cross-sectional area A floats

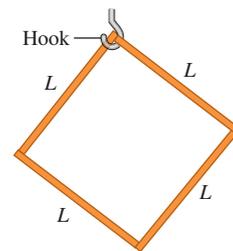
upright in a liquid with density ρ . (a) Calculate the vertical distance from the surface of the liquid to the bottom of the floating object at equilibrium. (b) A downward force with magnitude F is applied to the top of the object. At the new equilibrium position, how much farther below the surface of the liquid is the bottom of the object than it was in part (a)? (Assume that some of the object remains above the surface of the liquid.) (c) Your result in part (b) shows that if the force is suddenly removed, the object will oscillate up and down in SHM. Calculate the period of this motion in terms of the density ρ of the liquid, the mass M , and the cross-sectional area A of the object. You can ignore the damping due to fluid friction (see Section 14.7).

14.77 •• CP A 950-kg, cylindrical can buoy floats vertically in salt water. The diameter of the buoy is 0.900 m. (a) Calculate the additional distance the buoy will sink when a 70.0-kg man stands on top of it. (Use the expression derived in part (b) of Problem 14.76.) (b) Calculate the period of the resulting vertical SHM when the man dives off. (Use the expression derived in part (c) of Problem 14.76, and as in that problem, you can ignore the damping due to fluid friction.)

14.78 ••• CP Tarzan to the Rescue! Tarzan spies a 35-kg chimpanzee in severe danger, so he swings to the rescue. He adjusts his strong, but very light, vine so that he will first come to rest 4.0 s after beginning his swing, at which time his vine makes a 12° angle with the vertical. (a) How long is Tarzan's vine, assuming that he swings at the bottom end of it? (b) What are the frequency and amplitude (in degrees) of Tarzan's swing? (c) Just as he passes through the lowest point in his swing, Tarzan nabs the chimp from the ground and sweeps him out of the jaws of danger. If Tarzan's mass is 65 kg, find the frequency and amplitude (in degrees) of the swing with Tarzan holding onto the grateful chimp.

14.79 •• CP A square object of mass m is constructed of four identical uniform thin sticks, each of length L , attached together. This object is hung on a hook at its upper corner (Fig. P14.79). If it is rotated slightly to the left and then released, at what frequency will it swing back and forth?

Figure P14.79



14.80 ••• An object with mass 0.200 kg is acted on by an elastic restoring force with force constant 10.0 N/m. (a) Graph elastic potential energy U as a function of displacement x over a range of x from -0.300 m to $+0.300 \text{ m}$. On your graph, let 1 cm = 0.05 J vertically and 1 cm = 0.05 m horizontally. The object is set into oscillation with an initial potential energy of 0.140 J and an initial kinetic energy of 0.060 J. Answer the following questions by referring to the graph. (b) What is the amplitude of oscillation? (c) What is the potential energy when the displacement is one-half the amplitude? (d) At what displacement are the kinetic and potential energies equal? (e) What is the value of the phase angle ϕ if the initial velocity is positive and the initial displacement is negative?

14.81 • CALC A 2.00-kg bucket containing 10.0 kg of water is hanging from a vertical ideal spring of force constant 125 N/m and oscillating up and down with an amplitude of 3.00 cm. Suddenly the bucket springs a leak in the bottom such that water drops out at a steady rate of 2.00 g/s. When the bucket is half full, find

(a) the period of oscillation and (b) the rate at which the period is changing with respect to time. Is the period getting longer or shorter? (c) What is the shortest period this system can have?

14.82 •• CP A hanging wire is 1.80 m long. When a 60.0-kg steel ball is suspended from the wire, the wire stretches by 2.00 mm. If the ball is pulled down a small additional distance and released, at what frequency will it vibrate? Assume that the stress on the wire is less than the proportional limit (see Section 11.5).

14.83 •• A 5.00-kg partridge is suspended from a pear tree by an ideal spring of negligible mass. When the partridge is pulled down 0.100 m below its equilibrium position and released, it vibrates with a period of 4.20 s. (a) What is its speed as it passes through the equilibrium position? (b) What is its acceleration when it is 0.050 m above the equilibrium position? (c) When it is moving upward, how much time is required for it to move from a point 0.050 m below its equilibrium position to a point 0.050 m above it? (d) The motion of the partridge is stopped, and then it is removed from the spring. How much does the spring shorten?

14.84 •• A 0.0200-kg bolt moves with SHM that has an amplitude of 0.240 m and a period of 1.500 s. The displacement of the bolt is +0.240 m when $t = 0$. Compute (a) the displacement of the bolt when $t = 0.500$ s; (b) the magnitude and direction of the force acting on the bolt when $t = 0.500$ s; (c) the minimum time required for the bolt to move from its initial position to the point where $x = -0.180$ m; (d) the speed of the bolt when $x = -0.180$ m.

14.85 •• CP SHM of a Butcher's Scale. A spring of negligible mass and force constant $k = 400$ N/m is hung vertically, and a 0.200-kg pan is suspended from its lower end. A butcher drops a 2.2-kg steak onto the pan from a height of 0.40 m. The steak makes a totally inelastic collision with the pan and sets the system into vertical SHM. What are (a) the speed of the pan and steak immediately after the collision; (b) the amplitude of the subsequent motion; (c) the period of that motion?

14.86 •• A uniform beam is suspended horizontally by two identical vertical springs that are attached between the ceiling and each end of the beam. The beam has mass 225 kg, and a 175-kg sack of gravel sits on the middle of it. The beam is oscillating in SHM, with an amplitude of 40.0 cm and a frequency of 0.600 cycle/s. (a) The sack of gravel falls off the beam when the beam has its maximum upward displacement. What are the frequency and amplitude of the subsequent SHM of the beam? (b) If the gravel instead falls off when the beam has its maximum speed, what are the frequency and amplitude of the subsequent SHM of the beam?

14.87 ••• CP On the planet Newtonia, a simple pendulum having a bob with mass 1.25 kg and a length of 185.0 cm takes 1.42 s, when released from rest, to swing through an angle of 12.5° , where it again has zero speed. The circumference of Newtonia is measured to be 51,400 km. What is the mass of the planet Newtonia?

14.88 •• A 40.0-N force stretches a vertical spring 0.250 m. (a) What mass must be suspended from the spring so that the system will oscillate with a period of 1.00 s? (b) If the amplitude of the motion is 0.050 m and the period is that specified in part (a), where is the object and in what direction is it moving 0.35 s after it has passed the equilibrium position, moving downward? (c) What force (magnitude and direction) does the spring exert on the object when it is 0.030 m below the equilibrium position, moving upward?

14.89 •• Don't Miss the Boat. While on a visit to Minnesota ("Land of 10,000 Lakes"), you sign up to take an excursion around one of the larger lakes. When you go to the dock where the 1500-kg boat is tied, you find that the boat is bobbing up and down in the waves, executing simple harmonic motion with amplitude 20 cm. The boat takes 3.5 s to make one complete up-and-down cycle.

When the boat is at its highest point, its deck is at the same height as the stationary dock. As you watch the boat bob up and down, you (mass 60 kg) begin to feel a bit woozy, due in part to the previous night's dinner of lutefisk. As a result, you refuse to board the boat unless the level of the boat's deck is within 10 cm of the dock level. How much time do you have to board the boat comfortably during each cycle of up-and-down motion?

14.90 • CP An interesting, though highly impractical example of oscillation is the motion of an object dropped down a hole that extends from one side of the earth, through its center, to the other side. With the assumption (not realistic) that the earth is a sphere of uniform density, prove that the motion is simple harmonic and find the period. [Note: The gravitational force on the object as a function of the object's distance r from the center of the earth was derived in Example 13.10 (Section 13.6). The motion is simple harmonic if the acceleration a_x and the displacement from equilibrium x are related by Eq. (14.8), and the period is then $T = 2\pi/\omega$.]

14.91 ••• CP A rifle bullet with mass 8.00 g and initial horizontal velocity 280 m/s strikes and embeds itself in a block with mass 0.992 kg that rests on a frictionless surface and is attached to one end of an ideal spring. The other end of the spring is attached to the wall. The impact compresses the spring a maximum distance of 18.0 cm. After the impact, the block moves in SHM. Calculate the period of this motion.

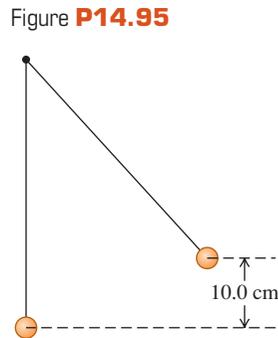
14.92 •• CP CALC For a certain oscillator the net force on the body with mass m is given by $F_x = -cx^3$. (a) What is the potential energy function for this oscillator if we take $U = 0$ at $x = 0$? (b) One-quarter of a period is the time for the body to move from $x = 0$ to $x = A$. Calculate this time and hence the period. [Hint: Begin with Eq. (14.20), modified to include the potential-energy function you found in part (a), and solve for the velocity v_x as a function of x . Then replace v_x with dx/dt . Separate the variable by writing all factors containing x on one side and all factors containing t on the other side so that each side can be integrated. In the x -integral make the change of variable $u = x/A$. The resulting integral can be evaluated by numerical methods on a computer and has the value $\int_0^1 du/\sqrt{1-u^4} = 1.31$.] (c) According to the result you obtained in part (b), does the period depend on the amplitude A of the motion? Are the oscillations simple harmonic?

14.93 • CP CALC An approximation for the potential energy of a KCl molecule is $U = A[(R_0^7/8r^8) - 1/r]$, where $R_0 = 2.67 \times 10^{-10}$ m, $A = 2.31 \times 10^{-28}$ J·m, and r is the distance between the two atoms. Using this approximation: (a) Show that the radial component of the force on each atom is $F_r = A[(R_0^7/r^9) - 1/r^2]$. (b) Show that R_0 is the equilibrium separation. (c) Find the minimum potential energy. (d) Use $r = R_0 + x$ and the first two terms of the binomial theorem (Eq. 14.28) to show that $F_r \approx -(7A/R_0^3)x$, so that the molecule's force constant is $k = 7A/R_0^3$. (e) With both the K and Cl atoms vibrating in opposite directions on opposite sides of the molecule's center of mass, $m_1 m_2 / (m_1 + m_2) = 3.06 \times 10^{-26}$ kg is the mass to use in calculating the frequency. Calculate the frequency of small-amplitude vibrations.

14.94 ••• CP Two uniform solid spheres, each with mass $M = 0.800$ kg and radius $R = 0.0800$ m, are connected by a short, light rod that is along a diameter of each sphere and are at rest on a horizontal tabletop. A spring with force constant $k = 160$ N/m has one end attached to the wall and the other end attached to a frictionless ring that passes over the rod at the center of mass of the spheres, which is midway between the centers of the two spheres. The spheres are each pulled the same distance from the wall, stretching the spring, and released. There is sufficient friction

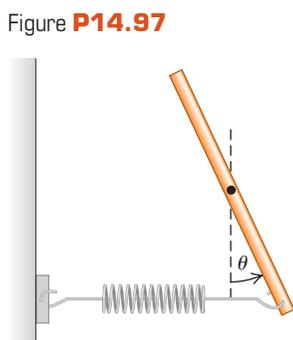
between the tabletop and the spheres for the spheres to roll without slipping as they move back and forth on the end of the spring. Show that the motion of the center of mass of the spheres is simple harmonic and calculate the period.

14.95 • CP In Fig. P14.95 the upper ball is released from rest, collides with the stationary lower ball, and sticks to it. The strings are both 50.0 cm long. The upper ball has mass 2.00 kg, and it is initially 10.0 cm higher than the lower ball, which has mass 3.00 kg. Find the frequency and maximum angular displacement of the motion after the collision.



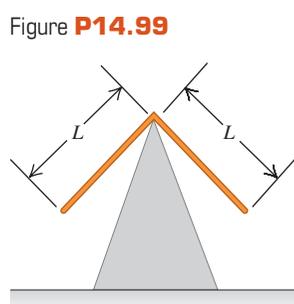
14.96 • CP BIO T. rex. Model the leg of the *T. rex* in Example 14.10 (Section 14.6) as two uniform rods, each 1.55 m long, joined rigidly end to end. Let the lower rod have mass M and the upper rod mass $2M$. The composite object is pivoted about the top of the upper rod. Compute the oscillation period of this object for small-amplitude oscillations. Compare your result to that of Example 14.10.

14.97 • CALC A slender, uniform, metal rod with mass M is pivoted without friction about an axis through its midpoint and perpendicular to the rod. A horizontal spring with force constant k is attached to the lower end of the rod, with the other end of the spring attached to a rigid support. If the rod is displaced by a small angle θ from the vertical (Fig. P14.97) and released, show that it moves in angular SHM and calculate the period. (*Hint:* Assume that the angle θ is small enough for the approximations $\sin \theta \approx \theta$ and $\cos \theta \approx 1$ to be valid. The motion is simple harmonic if $d^2\theta/dt^2 = -\omega^2\theta$, and the period is then $T = 2\pi/\omega$.)



14.98 • The Silently Ringing Bell Problem. A large bell is hung from a wooden beam so it can swing back and forth with negligible friction. The center of mass of the bell is 0.60 m below the pivot, the bell has mass 34.0 kg, and the moment of inertia of the bell about an axis at the pivot is $18.0 \text{ kg} \cdot \text{m}^2$. The clapper is a small, 1.8-kg mass attached to one end of a slender rod that has length L and negligible mass. The other end of the rod is attached to the inside of the bell so it can swing freely about the same axis as the bell. What should be the length L of the clapper rod for the bell to ring silently—that is, for the period of oscillation for the bell to equal that for the clapper?

14.99 ••• Two identical thin rods, each with mass m and length L , are joined at right angles to form an L-shaped object. This object is balanced on top of a sharp edge (Fig. P14.99). If the L-shaped object is deflected slightly, it oscillates. Find the frequency of oscillation.



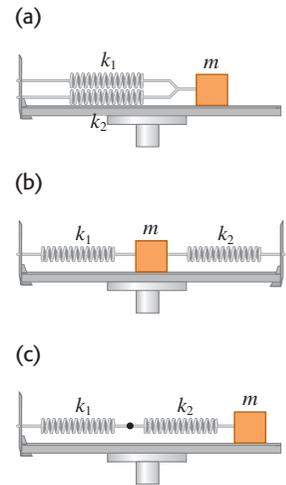
14.100 • CP CALC A uniform rod of length L oscillates through small angles about a point a distance x from its center. (a) Prove that its angular frequency is $\sqrt{gx/[(L^2/12) + x^2]}$. (b) Show that its maximum angular frequency occurs when $x = L/\sqrt{12}$. (c) What is the length of the rod if the maximum angular frequency is $2\pi \text{ rad/s}$?

CHALLENGE PROBLEMS

14.101 ••• The Effective Force Constant of Two Springs.

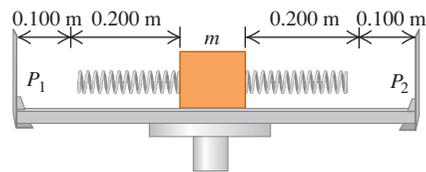
Two springs with the same unstretched length but different force constants k_1 and k_2 are attached to a block with mass m on a level, frictionless surface. Calculate the effective force constant k_{eff} in each of the three cases (a), (b), and (c) depicted in Fig. P14.101. (The effective force constant is defined by $\sum F_x = -k_{\text{eff}}x$.) (d) An object with mass m , suspended from a uniform spring with a force constant k , vibrates with a frequency f_1 . When the spring is cut in half and the same object is suspended from one of the halves, the frequency is f_2 . What is the ratio f_2/f_1 ?

Figure P14.101



14.102 ••• Two springs, each with unstretched length 0.200 m but with different force constants k_1 and k_2 , are attached to opposite ends of a block with mass m on a level, frictionless surface. The outer ends of the springs are now attached to two pins P_1 and P_2 , 0.100 m from the original positions of the ends of the springs (Fig. P14.102). Let $k_1 = 2.00 \text{ N/m}$, $k_2 = 6.00 \text{ N/m}$, and $m = 0.100 \text{ kg}$. (a) Find the length of each spring when the block is in its new equilibrium position after the springs have been attached to the pins. (b) Find the period of vibration of the block if it is slightly displaced from its new equilibrium position and released.

Figure P14.102



14.103 ••• CALC A Spring with Mass. The preceding problems in this chapter have assumed that the springs had negligible mass. But of course no spring is completely massless. To find the effect of the spring's mass, consider a spring with mass M , equilibrium length L_0 , and spring constant k . When stretched or compressed to a length L , the potential energy is $\frac{1}{2}kx^2$, where $x = L - L_0$. (a) Consider a spring, as described above, that has one end fixed and the other end moving with speed v . Assume that the speed of points along the length of the spring varies linearly with distance l from the fixed end. Assume also that the mass M of the spring is distributed uniformly along the length of the spring. Calculate the kinetic energy of the spring in terms of M and v . (*Hint:* Divide the spring into pieces of length dl ; find the speed of each piece in

terms of l , v , and L ; find the mass of each piece in terms of dl , M , and L ; and integrate from 0 to L . The result is *not* $\frac{1}{2}Mv^2$, since not all of the spring moves with the same speed.) (b) Take the time derivative of the conservation of energy equation, Eq. (14.21), for a mass m moving on the end of a *massless* spring. By comparing

your results to Eq. (14.8), which defines ω , show that the angular frequency of oscillation is $\omega = \sqrt{k/m}$. (c) Apply the procedure of part (b) to obtain the angular frequency of oscillation ω of the spring considered in part (a). If the *effective mass* M' of the spring is defined by $\omega = \sqrt{k/M'}$, what is M' in terms of M ?

Answers

Chapter Opening Question ?

The length of the leg is more important. The back-and-forth motion of a leg during walking is like a physical pendulum, for which the oscillation period is $T = 2\pi\sqrt{I/mgd}$ [see Eq. (14.39)]. In this expression I is the moment of inertia of the pendulum, m is its mass, and d is the distance from the rotation axis to the pendulum center of mass. The moment of inertia I is proportional to the mass m , so the mass cancels out of this expression for the period T . Hence only the dimensions of the leg matter. (See Examples 14.9 and 14.10.)

Test Your Understanding Questions

14.1 Answers: (a) $x < 0$, (b) $x > 0$, (c) $x < 0$, (d) $x > 0$, (e) $x > 0$, (f) $x = 0$ Figure 14.2 shows that the net x -component of force F_x and the x -acceleration a_x are both positive when $x < 0$ (so the body is displaced to the left and the spring is compressed), while F_x and a_x are both negative when $x > 0$ (so the body is displaced to the right and the spring is stretched). Hence x and a_x always have *opposite* signs. This is true whether the object is moving to the right ($v_x > 0$), to the left ($v_x < 0$), or not at all ($v_x = 0$), since the force exerted by the spring depends only on whether it is compressed or stretched and by what distance. This explains the answers to (a) through (e). If the acceleration is zero as in (f), the net force must also be zero and so the spring must be relaxed; hence $x = 0$.

14.2 Answers: (a) $A > 0.10$ m, $\phi < 0$; (b) $A > 0.10$ m, $\phi > 0$ In both situations the initial ($t = 0$) x -velocity v_{0x} is nonzero, so from Eq. (14.19) the amplitude $A = \sqrt{x_0^2 + (v_{0x}/\omega)^2}$ is greater than the initial x -coordinate $x_0 = 0.10$ m. From Eq. (14.18) the phase angle is $\phi = \arctan(-v_{0x}/\omega x_0)$, which is positive if the quantity $-v_{0x}/\omega x_0$ (the argument of the arctangent function) is positive and negative if $-v_{0x}/\omega x_0$ is negative. In part (a) x_0 and v_{0x} are both positive, so $-v_{0x}/\omega x_0 < 0$ and $\phi < 0$. In part (b) x_0 is positive and v_{0x} is negative, so $-v_{0x}/\omega x_0 > 0$ and $\phi > 0$.

14.3 Answers: (a) (iii), (b) (v) To increase the total energy $E = \frac{1}{2}kA^2$ by a factor of 2, the amplitude A must increase by a factor of $\sqrt{2}$. Because the motion is SHM, changing the amplitude has no effect on the frequency.

14.4 Answer: (i) The oscillation period of a body of mass m attached to a hanging spring of force constant k is given by

$T = 2\pi\sqrt{m/k}$, the same expression as for a body attached to a horizontal spring. Neither m nor k changes when the apparatus is taken to Mars, so the period is unchanged. The only difference is that in equilibrium, the spring will stretch a shorter distance on Mars than on earth due to the weaker gravity.

14.5 Answer: no Just as for an object oscillating on a spring, at the equilibrium position the *speed* of the pendulum bob is instantaneously not changing (this is where the speed is maximum, so its derivative at this time is zero). But the *direction* of motion is changing because the pendulum bob follows a circular path. Hence the bob must have a component of acceleration perpendicular to the path and toward the center of the circle (see Section 3.4). To cause this acceleration at the equilibrium position when the string is vertical, the upward tension force at this position must be greater than the weight of the bob. This causes a net upward force on the bob and an upward acceleration toward the center of the circular path.

14.6 Answer: (i) The period of a physical pendulum is given by Eq. (14.39), $T = 2\pi\sqrt{I/mgd}$. The distance $d = L$ from the pivot to the center of gravity is the same for both the rod and the simple pendulum, as is the mass m . This means that for any displacement angle θ the same restoring torque acts on both the rod and the simple pendulum. However, the rod has a greater moment of inertia: $I_{\text{rod}} = \frac{1}{3}m(2L)^2 = \frac{4}{3}mL^2$ and $I_{\text{simple}} = mL^2$ (all the mass of the pendulum is a distance L from the pivot). Hence the rod has a longer period.

14.7 Answer: (ii) The oscillations are underdamped with a decreasing amplitude on each cycle of oscillation, like those graphed in Fig. 14.26. If the oscillations were undamped, they would continue indefinitely with the same amplitude. If they were critically damped or overdamped, the nose would not bob up and down but would return smoothly to the original equilibrium attitude without overshooting.

14.8 Answer: (i) Figure 14.28 shows that the curve of amplitude versus driving frequency moves upward at *all* frequencies as the value of the damping constant b is decreased. Hence for fixed values of k and m , the oscillator with the least damping (smallest value of b) will have the greatest response at any driving frequency.

Bridging Problem

Answer: $T = 2\pi\sqrt{3M/2k}$

15

MECHANICAL WAVES

LEARNING GOALS

By studying this chapter, you will learn:

- What is meant by a mechanical wave, and the different varieties of mechanical waves.
- How to use the relationship among speed, frequency, and wavelength for a periodic wave.
- How to interpret and use the mathematical expression for a sinusoidal periodic wave.
- How to calculate the speed of waves on a rope or string.
- How to calculate the rate at which a mechanical wave transports energy.
- What happens when mechanical waves overlap and interfere.
- The properties of standing waves on a string, and how to analyze these waves.
- How stringed instruments produce sounds of specific frequencies.



? When an earthquake strikes, the news of the event travels through the body of the earth in the form of seismic waves. Which aspects of a seismic wave determine how much power is carried by the wave?

Ripples on a pond, musical sounds, seismic tremors triggered by an earthquake—all these are *wave* phenomena. Waves can occur whenever a system is disturbed from equilibrium and when the disturbance can travel, or *propagate*, from one region of the system to another. As a wave propagates, it carries energy. The energy in light waves from the sun warms the surface of our planet; the energy in seismic waves can crack our planet's crust.

This chapter and the next are about mechanical waves—waves that travel within some material called a *medium*. (Chapter 16 is concerned with sound, an important type of mechanical wave.) We'll begin this chapter by deriving the basic equations for describing waves, including the important special case of *sinusoidal* waves in which the wave pattern is a repeating sine or cosine function. To help us understand waves in general, we'll look at the simple case of waves that travel on a stretched string or rope.

Waves on a string play an important role in music. When a musician strums a guitar or bows a violin, she makes waves that travel in opposite directions along the instrument's strings. What happens when these oppositely directed waves overlap is called *interference*. We'll discover that sinusoidal waves can occur on a guitar or violin string only for certain special frequencies, called *normal-mode frequencies*, determined by the properties of the string. The normal-mode frequencies of a stringed instrument determine the pitch of the musical sounds that the instrument produces. (In the next chapter we'll find that interference also helps explain the pitches of *wind* instruments such as flutes and pipe organs.)

Not all waves are mechanical in nature. *Electromagnetic* waves—including light, radio waves, infrared and ultraviolet radiation, and x rays—can propagate even in empty space, where there is *no* medium. We'll explore these and other nonmechanical waves in later chapters.

15.1 Types of Mechanical Waves

A **mechanical wave** is a disturbance that travels through some material or substance called the **medium** for the wave. As the wave travels through the medium, the particles that make up the medium undergo displacements of various kinds, depending on the nature of the wave.

Figure 15.1 shows three varieties of mechanical waves. In Fig. 15.1a the medium is a string or rope under tension. If we give the left end a small upward shake or wiggle, the wiggle travels along the length of the string. Successive sections of string go through the same motion that we gave to the end, but at successively later times. Because the displacements of the medium are perpendicular or *transverse* to the direction of travel of the wave along the medium, this is called a **transverse wave**.

In Fig. 15.1b the medium is a liquid or gas in a tube with a rigid wall at the right end and a movable piston at the left end. If we give the piston a single back-and-forth motion, displacement and pressure fluctuations travel down the length of the medium. This time the motions of the particles of the medium are back and forth along the *same* direction that the wave travels. We call this a **longitudinal wave**.

In Fig. 15.1c the medium is a liquid in a channel, such as water in an irrigation ditch or canal. When we move the flat board at the left end forward and back once, a wave disturbance travels down the length of the channel. In this case the displacements of the water have *both* longitudinal and transverse components.

Each of these systems has an equilibrium state. For the stretched string it is the state in which the system is at rest, stretched out along a straight line. For the fluid in a tube it is a state in which the fluid is at rest with uniform pressure. And for the liquid in a trough it is a smooth, level water surface. In each case the wave motion is a disturbance from the equilibrium state that travels from one region of the medium to another. And in each case there are forces that tend to restore the system to its equilibrium position when it is displaced, just as the force of gravity tends to pull a pendulum toward its straight-down equilibrium position when it is displaced.

Application Waves on a Snake's Body

A snake moves itself along the ground by producing waves that travel backward along its body from its head to its tail. The waves remain stationary with respect to the ground as they push against the ground, so the snake moves forward.

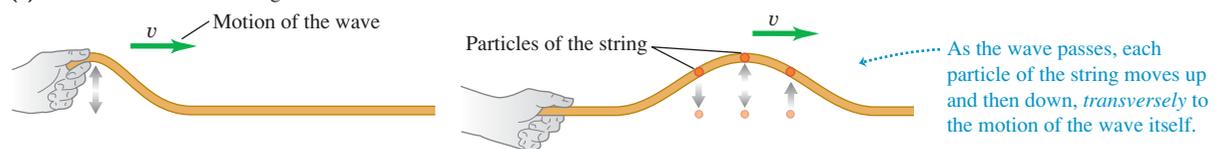


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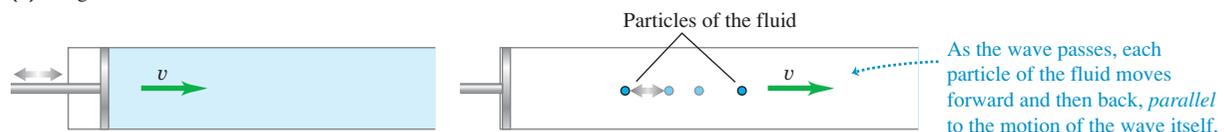
ActivPhysics 10.1: Properties of Mechanical Waves

15.1 Three ways to make a wave that moves to the right. (a) The hand moves the string up and then returns, producing a transverse wave. (b) The piston moves to the right, compressing the gas or liquid, and then returns, producing a longitudinal wave. (c) The board moves to the right and then returns, producing a combination of longitudinal and transverse waves.

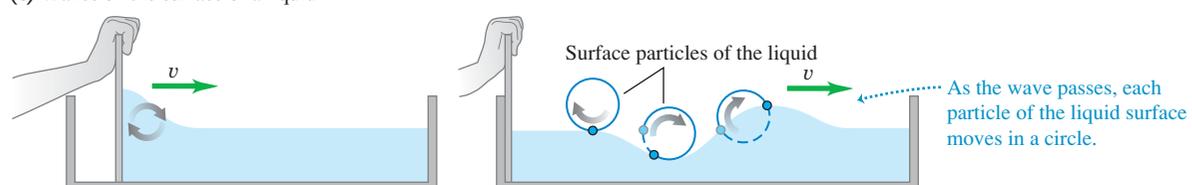
(a) Transverse wave on a string



(b) Longitudinal wave in a fluid



(c) Waves on the surface of a liquid



15.2 “Doing the wave” at a sports stadium is an example of a mechanical wave: The disturbance propagates through the crowd, but there is no transport of matter (none of the spectators moves from one seat to another).



These examples have three things in common. First, in each case the disturbance travels or *propagates* with a definite speed through the medium. This speed is called the speed of propagation, or simply the **wave speed**. Its value is determined in each case by the mechanical properties of the medium. We will use the symbol v for wave speed. (The wave speed is *not* the same as the speed with which particles move when they are disturbed by the wave. We’ll return to this point in Section 15.3.) Second, the medium itself does not travel through space; its individual particles undergo back-and-forth or up-and-down motions around their equilibrium positions. The overall pattern of the wave disturbance is what travels. Third, to set any of these systems into motion, we have to put in energy by doing mechanical work on the system. The wave motion transports this energy from one region of the medium to another. *Waves transport energy, but not matter, from one region to another* (Fig. 15.2).

Test Your Understanding of Section 15.1 What type of wave is “the wave” shown in Fig. 15.2? (i) transverse; (ii) longitudinal; (iii) a combination of transverse and longitudinal.

15.2 Periodic Waves

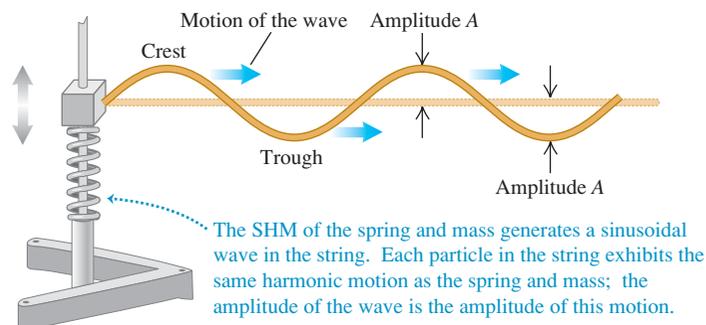
The transverse wave on a stretched string in Fig. 15.1a is an example of a *wave pulse*. The hand shakes the string up and down just once, exerting a transverse force on it as it does so. The result is a single “wiggle,” or pulse, that travels along the length of the string. The tension in the string restores its straight-line shape once the pulse has passed.

A more interesting situation develops when we give the free end of the string a repetitive, or *periodic*, motion. (You may want to review the discussion of periodic motion in Chapter 14 before going ahead.) Then each particle in the string also undergoes periodic motion as the wave propagates, and we have a **periodic wave**.

Periodic Transverse Waves

In particular, suppose we move the string up and down with *simple harmonic motion* (SHM) with amplitude A , frequency f , angular frequency $\omega = 2\pi f$, and period $T = 1/f = 2\pi/\omega$. Figure 15.3 shows one way to do this. The wave that results is a symmetrical sequence of *crests* and *troughs*. As we will see, periodic

15.3 A block of mass m attached to a spring undergoes simple harmonic motion, producing a sinusoidal wave that travels to the right on the string. (In a real-life system a driving force would have to be applied to the block to replace the energy carried away by the wave.)



waves with simple harmonic motion are particularly easy to analyze; we call them **sinusoidal waves**. It also turns out that *any* periodic wave can be represented as a combination of sinusoidal waves. So this particular kind of wave motion is worth special attention.

In Fig. 15.3 the wave that advances along the string is a *continuous succession* of transverse sinusoidal disturbances. Figure 15.4 shows the shape of a part of the string near the left end at time intervals of $\frac{1}{8}$ of a period, for a total time of one period. The wave shape advances steadily toward the right, as indicated by the highlighted area. As the wave moves, any point on the string (any of the red dots, for example) oscillates up and down about its equilibrium position with simple harmonic motion. *When a sinusoidal wave passes through a medium, every particle in the medium undergoes simple harmonic motion with the same frequency.*

CAUTION **Wave motion vs. particle motion** Be very careful to distinguish between the motion of the *transverse wave* along the string and the motion of a *particle* of the string. The wave moves with constant speed v *along* the length of the string, while the motion of the particle is simple harmonic and *transverse* (perpendicular) to the length of the string. **I**

For a periodic wave, the shape of the string at any instant is a repeating pattern. The length of one complete wave pattern is the distance from one crest to the next, or from one trough to the next, or from any point to the corresponding point on the next repetition of the wave shape. We call this distance the **wavelength** of the wave, denoted by λ (the Greek letter lambda). The wave pattern travels with constant speed v and advances a distance of one wavelength λ in a time interval of one period T . So the wave speed v is given by $v = \lambda/T$ or, because $f = 1/T$,

$$v = \lambda f \quad (\text{periodic wave}) \quad (15.1)$$

The speed of propagation equals the product of wavelength and frequency. The frequency is a property of the *entire* periodic wave because all points on the string oscillate with the same frequency f .

Waves on a string propagate in just one dimension (in Fig. 15.4, along the x -axis). But the ideas of frequency, wavelength, and amplitude apply equally well to waves that propagate in two or three dimensions. Figure 15.5 shows a wave propagating in two dimensions on the surface of a tank of water. As with waves on a string, the wavelength is the distance from one crest to the next, and the amplitude is the height of a crest above the equilibrium level.

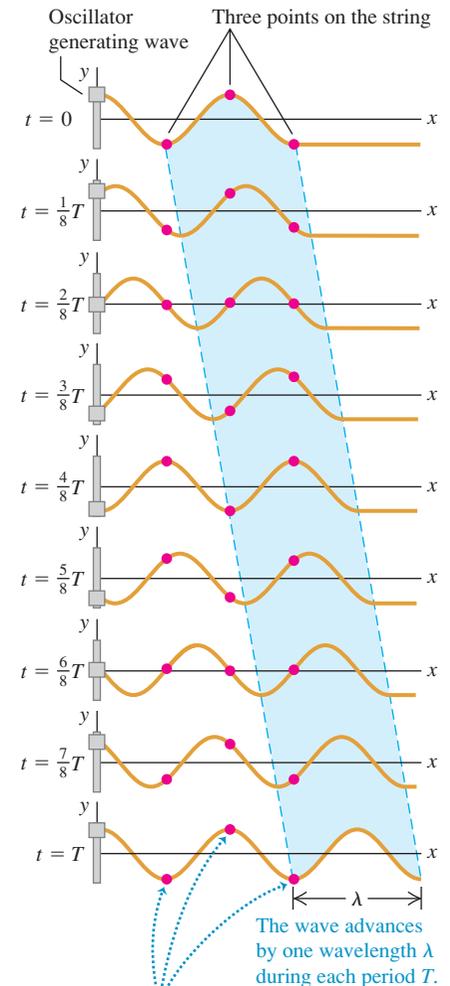
In many important situations including waves on a string, the wave speed v is determined entirely by the mechanical properties of the medium. In this case, increasing f causes λ to decrease so that the product $v = \lambda f$ remains the same, and waves of *all* frequencies propagate with the same wave speed. In this chapter we will consider *only* waves of this kind. (In later chapters we will study the propagation of light waves in matter for which the wave speed depends on frequency; this turns out to be the reason prisms break white light into a spectrum and raindrops create a rainbow.)

Periodic Longitudinal Waves

To understand the mechanics of a periodic *longitudinal* wave, we consider a long tube filled with a fluid, with a piston at the left end as in Fig. 15.1b. If we push the piston in, we compress the fluid near the piston, increasing the pressure in this

15.4 A sinusoidal transverse wave traveling to the right along a string. The vertical scale is exaggerated.

The string is shown at time intervals of $\frac{1}{8}$ period for a total of one period T . The highlighting shows the motion of one wavelength of the wave.



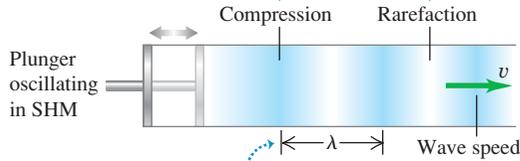
Each point moves up and down in place. Particles one wavelength apart move in phase with each other.

15.5 A series of drops falling into water produces a periodic wave that spreads radially outward. The wave crests and troughs are concentric circles. The wavelength λ is the radial distance between adjacent crests or adjacent troughs.



15.6 Using an oscillating piston to make a sinusoidal longitudinal wave in a fluid.

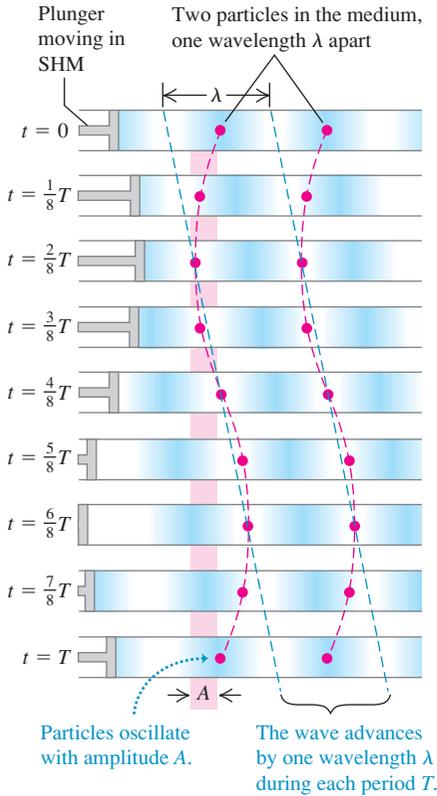
Forward motion of the plunger creates a compression (a zone of high density); backward motion creates a rarefaction (a zone of low density).



Wavelength λ is the distance between corresponding points on successive cycles.

15.7 A sinusoidal longitudinal wave traveling to the right in a fluid. The wave has the same amplitude A and period T as the oscillation of the piston.

Longitudinal waves are shown at intervals of $\frac{1}{8}T$ for one period T .



region. This region then pushes against the neighboring region of fluid, and so on, and a wave pulse moves along the tube.

Now suppose we move the piston back and forth with simple harmonic motion, along a line parallel to the axis of the tube (Fig. 15.6). This motion forms regions in the fluid where the pressure and density are greater or less than the equilibrium values. We call a region of increased density a *compression*; a region of reduced density is a *rarefaction*. Figure 15.6 shows compressions as darkly shaded areas and rarefactions as lightly shaded areas. The wavelength is the distance from one compression to the next or from one rarefaction to the next.

Figure 15.7 shows the wave propagating in the fluid-filled tube at time intervals of $\frac{1}{8}$ of a period, for a total time of one period. The pattern of compressions and rarefactions moves steadily to the right, just like the pattern of crests and troughs in a sinusoidal transverse wave (compare Fig. 15.4). Each particle in the fluid oscillates in SHM parallel to the direction of wave propagation (that is, left and right) with the same amplitude A and period T as the piston. The particles shown by the two red dots in Fig. 15.7 are one wavelength apart, and so oscillate in phase with each other.

Just like the sinusoidal transverse wave shown in Fig. 15.4, in one period T the longitudinal wave in Fig. 15.7 travels one wavelength λ to the right. Hence the fundamental equation $v = \lambda f$ holds for longitudinal waves as well as for transverse waves, and indeed for *all* types of periodic waves. Just as for transverse waves, in this chapter and the next we will consider only situations in which the speed of longitudinal waves does not depend on the frequency.

Example 15.1 Wavelength of a musical sound

Sound waves are longitudinal waves in air. The speed of sound depends on temperature; at 20°C it is 344 m/s (1130 ft/s). What is the wavelength of a sound wave in air at 20°C if the frequency is 262 Hz (the approximate frequency of middle C on a piano)?

SOLUTION

IDENTIFY and SET UP: This problem involves Eq. (15.1), $v = \lambda f$, which relates wave speed v , wavelength λ , and frequency f for a periodic wave. The target variable is the wavelength λ . We are given $v = 344$ m/s and $f = 262$ Hz = 262 s⁻¹.

EXECUTE: We solve Eq. (15.1) for λ :

$$\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{262 \text{ Hz}} = \frac{344 \text{ m/s}}{262 \text{ s}^{-1}} = 1.31 \text{ m}$$

EVALUATE: The speed v of sound waves does *not* depend on the frequency. Hence $\lambda = v/f$ says that wavelength changes in inverse proportion to frequency. As an example, high (soprano) C is two octaves above middle C. Each octave corresponds to a factor of 2 in frequency, so the frequency of high C is four times that of middle C: $f = 4(262 \text{ Hz}) = 1048$ Hz. Hence the *wavelength* of high C is *one-fourth* as large: $\lambda = (1.31 \text{ m})/4 = 0.328 \text{ m}$.

Test Your Understanding of Section 15.2 If you double the wavelength of a wave on a particular string, what happens to the wave speed v and the frequency f ? (i) v doubles and f is unchanged; (ii) v is unchanged and f doubles; (iii) v becomes one-half as great and f is unchanged; (iv) v is unchanged and f becomes one-half as great; (v) none of these.



15.3 Mathematical Description of a Wave

Many characteristics of periodic waves can be described by using the concepts of wave speed, amplitude, period, frequency, and wavelength. Often, though, we need a more detailed description of the positions and motions of individual particles of the medium at particular times during wave propagation.

As a specific example, let's look at waves on a stretched string. If we ignore the sag of the string due to gravity, the equilibrium position of the string is along a straight line. We take this to be the x -axis of a coordinate system. Waves on a string are *transverse*; during wave motion a particle with equilibrium position x is displaced some distance y in the direction perpendicular to the x -axis. The value of y depends on which particle we are talking about (that is, y depends on x) and also on the time t when we look at it. Thus y is a *function* of both x and t ; $y = y(x, t)$. We call $y(x, t)$ the **wave function** that describes the wave. If we know this function for a particular wave motion, we can use it to find the displacement (from equilibrium) of any particle at any time. From this we can find the velocity and acceleration of any particle, the shape of the string, and anything else we want to know about the behavior of the string at any time.

Wave Function for a Sinusoidal Wave

Let's see how to determine the form of the wave function for a sinusoidal wave. Suppose a sinusoidal wave travels from left to right (the direction of increasing x) along the string, as in Fig. 15.8. Every particle of the string oscillates with simple harmonic motion with the same amplitude and frequency. But the oscillations of particles at different points on the string are *not* all in step with each other. The particle at point B in Fig. 15.8 is at its maximum positive value of y at $t = 0$ and returns to $y = 0$ at $t = \frac{2}{8}T$; these same events occur for a particle at point A or point C at $t = \frac{4}{8}T$ and $t = \frac{6}{8}T$, exactly one half-period later. For any two particles of the string, the motion of the particle on the right (in terms of the wave, the "downstream" particle) lags behind the motion of the particle on the left by an amount proportional to the distance between the particles.

Hence the cyclic motions of various points on the string are out of step with each other by various fractions of a cycle. We call these differences *phase differences*, and we say that the *phase* of the motion is different for different points. For example, if one point has its maximum positive displacement at the same time that another has its maximum negative displacement, the two are a half-cycle out of phase. (This is the case for points A and B , or points B and C .)

Suppose that the displacement of a particle at the left end of the string ($x = 0$), where the wave originates, is given by

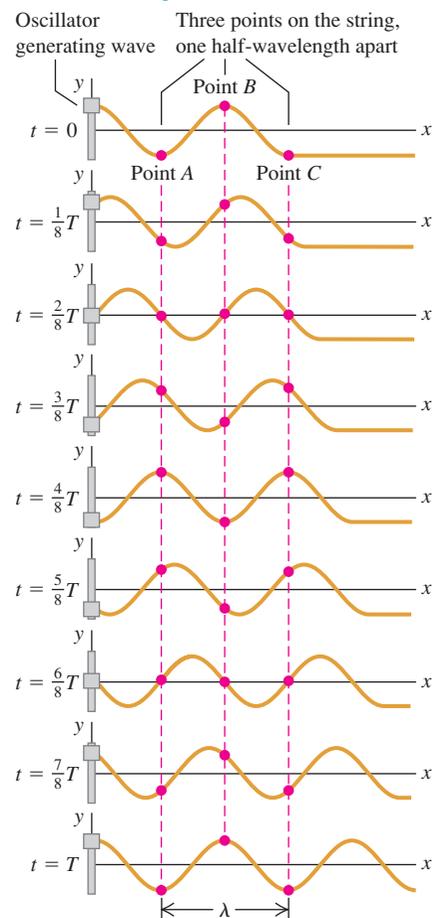
$$y(x = 0, t) = A \cos \omega t = A \cos 2\pi f t \tag{15.2}$$

That is, the particle oscillates in simple harmonic motion with amplitude A , frequency f , and angular frequency $\omega = 2\pi f$. The notation $y(x = 0, t)$ reminds us that the motion of this particle is a special case of the wave function $y(x, t)$ that describes the entire wave. At $t = 0$ the particle at $x = 0$ is at its maximum positive displacement ($y = A$) and is instantaneously at rest (because the value of y is a maximum).

The wave disturbance travels from $x = 0$ to some point x to the right of the origin in an amount of time given by x/v , where v is the wave speed. So the motion of point x at time t is the same as the motion of point $x = 0$ at the earlier time $t - x/v$. Hence we can find the displacement of point x at time t by simply

15.8 Tracking the oscillations of three points on a string as a sinusoidal wave propagates along it.

The string is shown at time intervals of $\frac{1}{8}$ period for a total of one period T .



replacing t in Eq. (15.2) by $(t - x/v)$. When we do that, we find the following expression for the wave function:

$$y(x, t) = A \cos \left[\omega \left(t - \frac{x}{v} \right) \right]$$

Because $\cos(-\theta) = \cos \theta$, we can rewrite the wave function as

$$y(x, t) = A \cos \left[\omega \left(\frac{x}{v} - t \right) \right] = A \cos \left[2\pi f \left(\frac{x}{v} - t \right) \right] \quad \begin{array}{l} \text{(sinusoidal wave} \\ \text{moving in} \\ \text{+}x\text{-direction)} \end{array} \quad (15.3)$$

The displacement $y(x, t)$ is a function of both the location x of the point and the time t . We could make Eq. (15.3) more general by allowing for different values of the phase angle, as we did for simple harmonic motion in Section 14.2, but for now we omit this.

We can rewrite the wave function given by Eq. (15.3) in several different but useful forms. We can express it in terms of the period $T = 1/f$ and the wavelength $\lambda = v/f$:

$$y(x, t) = A \cos \left[2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \right] \quad \begin{array}{l} \text{(sinusoidal wave moving} \\ \text{in +}x\text{-direction)} \end{array} \quad (15.4)$$

It's convenient to define a quantity k , called the **wave number**:

$$k = \frac{2\pi}{\lambda} \quad \text{(wave number)} \quad (15.5)$$

Substituting $\lambda = 2\pi/k$ and $f = \omega/2\pi$ into the wavelength–frequency relationship $v = \lambda f$ gives

$$\omega = vk \quad \text{(periodic wave)} \quad (15.6)$$

We can then rewrite Eq. (15.4) as

$$y(x, t) = A \cos(kx - \omega t) \quad \begin{array}{l} \text{(sinusoidal wave moving} \\ \text{in +}x\text{-direction)} \end{array} \quad (15.7)$$

Which of these various forms for the wave function $y(x, t)$ we use in any specific problem is a matter of convenience. Note that ω has units rad/s, so for unit consistency in Eqs. (15.6) and (15.7) the wave number k must have the units rad/m. (Some physicists define the wave number as $1/\lambda$ rather than $2\pi/\lambda$. When reading other texts, be sure to determine how this term is defined.)

Graphing the Wave Function

Figure 15.9a graphs the wave function $y(x, t)$ as a function of x for a specific time t . This graph gives the displacement y of a particle from its equilibrium position as a function of the coordinate x of the particle. If the wave is a transverse wave on a string, the graph in Fig. 15.9a represents the shape of the string at that instant, like a flash photograph of the string. In particular, at time $t = 0$,

$$y(x, t = 0) = A \cos kx = A \cos 2\pi \frac{x}{\lambda}$$

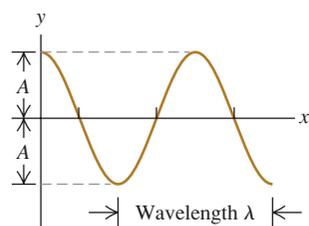
Figure 15.9b is a graph of the wave function versus time t for a specific coordinate x . This graph gives the displacement y of the particle at that coordinate as a function of time; that is, it describes the motion of that particle. In particular, at the position $x = 0$,

$$y(x = 0, t) = A \cos(-\omega t) = A \cos \omega t = A \cos 2\pi \frac{t}{T}$$

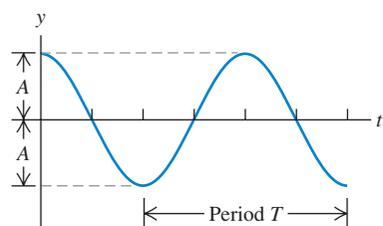
This is consistent with our original statement about the motion at $x = 0$, Eq. (15.2).

15.9 Two graphs of the wave function $y(x, t)$ in Eq. (15.7). (a) Graph of displacement y versus coordinate x at time $t = 0$. (b) Graph of displacement y versus time t at coordinate $x = 0$. The vertical scale is exaggerated in both (a) and (b).

(a) If we use Eq. (15.7) to plot y as a function of x for time $t = 0$, the curve shows the *shape* of the string at $t = 0$.



(b) If we use Eq. (15.7) to plot y as a function of t for position $x = 0$, the curve shows the *displacement* y of the particle at $x = 0$ as a function of time.



CAUTION **Wave graphs** Although they may look the same at first glance, Figs. 15.9a and 15.9b are *not* identical. Figure 15.9a is a picture of the shape of the string at $t = 0$, while Fig. 15.9b is a graph of the displacement y of a particle at $x = 0$ as a function of time. 

More on the Wave Function

We can modify Eqs. (15.3) through (15.7) to represent a wave traveling in the *negative* x -direction. In this case the displacement of point x at time t is the same as the motion of point $x = 0$ at the *later* time $(t + x/v)$, so in Eq. (15.2) we replace t by $(t + x/v)$. For a wave traveling in the negative x -direction,

$$y(x, t) = A \cos \left[2\pi f \left(\frac{x}{v} + t \right) \right] = A \cos \left[2\pi \left(\frac{x}{\lambda} + \frac{t}{T} \right) \right] = A \cos(kx + \omega t) \quad (15.8)$$

(sinusoidal wave moving in $-x$ -direction)

In the expression $y(x, t) = A \cos(kx \pm \omega t)$ for a wave traveling in the $-x$ - or $+x$ -direction, the quantity $(kx \pm \omega t)$ is called the **phase**. It plays the role of an angular quantity (always measured in radians) in Eq. (15.7) or (15.8), and its value for any values of x and t determines what part of the sinusoidal cycle is occurring at a particular point and time. For a crest (where $y = A$ and the cosine function has the value 1), the phase could be $0, 2\pi, 4\pi$, and so on; for a trough (where $y = -A$ and the cosine has the value -1), it could be $\pi, 3\pi, 5\pi$, and so on.

The wave speed is the speed with which we have to move along with the wave to keep alongside a point of a given phase, such as a particular crest of a wave on a string. For a wave traveling in the $+x$ -direction, that means $kx - \omega t = \text{constant}$. Taking the derivative with respect to t , we find $k dx/dt = \omega$, or

$$\frac{dx}{dt} = \frac{\omega}{k}$$

Comparing this with Eq. (15.6), we see that dx/dt is equal to the speed v of the wave. Because of this relationship, v is sometimes called the *phase velocity* of the wave. (*Phase speed* would be a better term.)

Problem-Solving Strategy 15.1 Mechanical Waves



IDENTIFY *the relevant concepts:* As always, identify the target variables; these may include mathematical *expressions* (for example, the wave function for a given situation). Note that wave problems fall into two categories. *Kinematics* problems, concerned with describing wave motion, involve wave speed v , wavelength λ (or wave number k), frequency f (or angular frequency ω), and amplitude A . They may also involve the position, velocity, and acceleration of individual particles in the medium. *Dynamics* problems also use concepts from Newton's laws. Later in this chapter we'll encounter problems that involve the relationship of wave speed to the mechanical properties of the medium.

SET UP *the problem* using the following steps:

1. List the given quantities. Sketch graphs of y versus x (like Fig. 15.9a) and of y versus t (like Fig. 15.9b), and label them with known values.
2. Identify useful equations. These may include Eq. (15.1) ($v = \lambda f$), Eq. (15.6) ($\omega = vk$), and Eqs. (15.3), (15.4), and

(15.7), which express the wave function in various forms. From the wave function, you can find the value of y at any point (value of x) and at any time t .

3. If you need to determine the wave speed v and don't know both λ and f , you may be able to use a relationship between v and the mechanical properties of the system. (In the next section we'll develop this relationship for waves on a string.)

EXECUTE *the solution:* Solve for the unknown quantities using the equations you've identified. To determine the wave function from Eq. (15.3), (15.4), or (15.7), you must know A and any two of v , λ , and f (or v , k , and ω).

EVALUATE *your answer:* Confirm that the values of v , f , and λ (or v , ω , and k) agree with the relationships given in Eq. (15.1) or (15.6). If you've calculated the wave function, check one or more special cases for which you can predict the results.

Example 15.2 Wave on a clothesline

Cousin Throckmorton holds one end of the clothesline taut and wiggles it up and down sinusoidally with frequency 2.00 Hz and amplitude 0.075 m. The wave speed on the clothesline is $v = 12.0$ m/s. At $t = 0$ Throcky's end has maximum positive displacement and is instantaneously at rest. Assume that no wave bounces back from the far end. (a) Find the wave amplitude A , angular frequency ω , period T , wavelength λ , and wave number k . (b) Write a wave function describing the wave. (c) Write equations for the displacement, as a function of time, of Throcky's end of the clothesline and of a point 3.00 m from that end.

SOLUTION

IDENTIFY and SET UP: This is a kinematics problem about the clothesline's wave motion. Throcky produces a sinusoidal wave that propagates along the clothesline, so we can use all of the expressions of this section. In part (a) our target variables are A , ω , T , λ , and k . We use the relationships $\omega = 2\pi f$, $f = 1/T$, $v = \lambda f$, and $k = 2\pi/\lambda$. In parts (b) and (c) our target "variables" are expressions for displacement, which we'll obtain from an appropriate equation for the wave function. We take the positive x -direction to be the direction in which the wave propagates, so either Eq. (15.4) or (15.7) will yield the desired expression. A photograph of the clothesline at time $t = 0$ would look like Fig. 15.9a, with the maximum displacement at $x = 0$ (the end that Throcky holds).

EXECUTE: (a) The wave amplitude and frequency are the same as for the oscillations of Throcky's end of the clothesline, $A = 0.075$ m and $f = 2.00$ Hz. Hence

$$\begin{aligned}\omega &= 2\pi f = \left(2\pi \frac{\text{rad}}{\text{cycle}}\right) \left(2.00 \frac{\text{cycles}}{\text{s}}\right) \\ &= 4.00\pi \text{ rad/s} = 12.6 \text{ rad/s}\end{aligned}$$

The period is $T = 1/f = 0.500$ s, and from Eq. (15.1),

$$\lambda = \frac{v}{f} = \frac{12.0 \text{ m/s}}{2.00 \text{ s}^{-1}} = 6.00 \text{ m}$$

We find the wave number from Eq. (15.5) or (15.6):

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{6.00 \text{ m}} = 1.05 \text{ rad/m}$$

or

$$k = \frac{\omega}{v} = \frac{4.00\pi \text{ rad/s}}{12.0 \text{ m/s}} = 1.05 \text{ rad/m}$$

(b) We write the wave function using Eq. (15.4) and the values of A , T , and λ from part (a):

$$\begin{aligned}y(x, t) &= A \cos 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \\ &= (0.075 \text{ m}) \cos 2\pi \left(\frac{x}{6.00 \text{ m}} - \frac{t}{0.500 \text{ s}} \right) \\ &= (0.075 \text{ m}) \cos [(1.05 \text{ rad/m})x - (12.6 \text{ rad/s})t]\end{aligned}$$

We can also get this same expression from Eq. (15.7) by using the values of ω and k from part (a).

(c) We can find the displacement as a function of time at $x = 0$ and $x = +3.00$ m by substituting these values into the wave function from part (b):

$$\begin{aligned}y(x = 0, t) &= (0.075 \text{ m}) \cos 2\pi \left(\frac{0}{6.00 \text{ m}} - \frac{t}{0.500 \text{ s}} \right) \\ &= (0.075 \text{ m}) \cos (12.6 \text{ rad/s})t \\ y(x = +3.00 \text{ m}, t) &= (0.075 \text{ m}) \cos 2\pi \left(\frac{3.00 \text{ m}}{6.00 \text{ m}} - \frac{t}{0.500 \text{ s}} \right) \\ &= (0.075 \text{ m}) \cos [\pi - (12.6 \text{ rad/s})t] \\ &= -(0.075 \text{ m}) \cos (12.6 \text{ rad/s})t\end{aligned}$$

EVALUATE: In part (b), the quantity $(1.05 \text{ rad/m})x - (12.6 \text{ rad/s})t$ is the *phase* of a point x on the string at time t . The two points in part (c) oscillate in SHM with the same frequency and amplitude, but their oscillations differ in phase by $(1.05 \text{ rad/m})(3.00 \text{ m}) = 3.15 \text{ rad} = \pi$ radians—that is, one-half cycle—because the points are separated by one half-wavelength: $\lambda/2 = (6.00 \text{ m})/2 = 3.00 \text{ m}$. Thus, while a graph of y versus t for the point at $x = 0$ is a cosine curve (like Fig. 15.9b), a graph of y versus t for the point $x = 3.00$ m is a *negative* cosine curve (the same as a cosine curve shifted by one half-cycle).

Using the expression for $y(x = 0, t)$ in part (c), can you show that the end of the string at $x = 0$ is instantaneously at rest at $t = 0$, as stated at the beginning of this example? (*Hint:* Calculate the y -velocity at this point by taking the derivative of y with respect to t .)

Particle Velocity and Acceleration in a Sinusoidal Wave

From the wave function we can get an expression for the transverse velocity of any *particle* in a transverse wave. We call this v_y to distinguish it from the wave propagation speed v . To find the transverse velocity v_y at a particular point x , we take the derivative of the wave function $y(x, t)$ with respect to t , keeping x constant. If the wave function is

$$y(x, t) = A \cos(kx - \omega t)$$

then

$$v_y(x, t) = \frac{\partial y(x, t)}{\partial t} = \omega A \sin(kx - \omega t) \quad (15.9)$$

The ∂ in this expression is a modified d , used to remind us that $y(x, t)$ is a function of *two* variables and that we are allowing only one (t) to vary. The other (x) is constant because we are looking at a particular point on the string. This derivative is called a *partial derivative*. If you haven't reached this point yet in your study of calculus, don't fret; it's a simple idea.

Equation (15.9) shows that the transverse velocity of a particle varies with time, as we expect for simple harmonic motion. The maximum particle speed is ωA ; this can be greater than, less than, or equal to the wave speed v , depending on the amplitude and frequency of the wave.

The *acceleration* of any particle is the *second* partial derivative of $y(x, t)$ with respect to t :

$$a_y(x, t) = \frac{\partial^2 y(x, t)}{\partial t^2} = -\omega^2 A \cos(kx - \omega t) = -\omega^2 y(x, t) \quad (15.10)$$

The acceleration of a particle equals $-\omega^2$ times its displacement, which is the result we obtained in Section 14.2 for simple harmonic motion.

We can also compute partial derivatives of $y(x, t)$ with respect to x , holding t constant. The first derivative $\partial y(x, t)/\partial x$ is the *slope* of the string at point x and at time t . The second partial derivative with respect to x is the *curvature* of the string:

$$\frac{\partial^2 y(x, t)}{\partial x^2} = -k^2 A \cos(kx - \omega t) = -k^2 y(x, t) \quad (15.11)$$

From Eqs. (15.10) and (15.11) and the relationship $\omega = vk$ we see that

$$\frac{\partial^2 y(x, t)/\partial t^2}{\partial^2 y(x, t)/\partial x^2} = \frac{\omega^2}{k^2} = v^2 \quad \text{and}$$

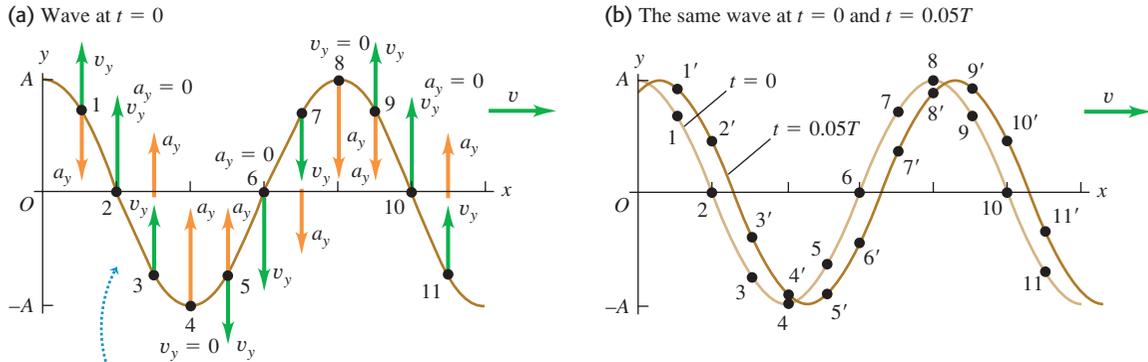
$$\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x, t)}{\partial t^2} \quad (\text{wave equation}) \quad (15.12)$$

We've derived Eq. (15.12) for a wave traveling in the positive x -direction. You can use the same steps to show that the wave function for a sinusoidal wave propagating in the *negative* x -direction, $y(x, t) = A \cos(kx + \omega t)$, also satisfies this equation.

Equation (15.12), called the **wave equation**, is one of the most important equations in all of physics. Whenever it occurs, we know that a disturbance can propagate as a wave along the x -axis with wave speed v . The disturbance need not be a sinusoidal wave; we'll see in the next section that *any* wave on a string obeys Eq. (15.12), whether the wave is periodic or not (see also Problem 15.65). In Chapter 32 we will find that electric and magnetic fields satisfy the wave equation; the wave speed will turn out to be the speed of light, which will lead us to the conclusion that light is an electromagnetic wave.

Figure 15.10a shows the transverse velocity v_y and transverse acceleration a_y , given by Eqs. (15.9) and (15.10), for several points on a string as a sinusoidal wave passes along it. Note that at points where the string has an upward curvature ($\partial^2 y/\partial x^2 > 0$), the acceleration of that point is positive ($a_y = \partial^2 y/\partial t^2 > 0$); this follows from the wave equation, Eq. (15.12). For the same reason the acceleration is negative ($a_y = \partial^2 y/\partial t^2 < 0$) at points where the string has a downward curvature ($\partial^2 y/\partial x^2 < 0$), and the acceleration is zero ($a_y = \partial^2 y/\partial t^2 = 0$) at points of inflection where the curvature is zero ($\partial^2 y/\partial x^2 = 0$). We emphasize again that v_y and a_y are the *transverse* velocity and acceleration of points on the string; these points move along the y -direction, not along the propagation direction of the

15.10 (a) Another view of the wave at $t = 0$ in Fig. 15.9a. The vectors show the transverse velocity v_y and transverse acceleration a_y at several points on the string. (b) From $t = 0$ to $t = 0.05T$, a particle at point 1 is displaced to point 1', a particle at point 2 is displaced to point 2', and so on.



- Acceleration a_y at each point on the string is proportional to displacement y at that point.
- Acceleration is upward where string curves upward, downward where string curves downward.

wave. Figure 15.10b shows the transverse motions of several points on the string.

The concept of wave function is equally useful with *longitudinal* waves. The quantity y still measures the displacement of a particle of the medium from its equilibrium position; the difference is that for a longitudinal wave, this displacement is *parallel* to the x -axis instead of perpendicular to it. We'll discuss longitudinal waves in detail in Chapter 16.

Test Your Understanding of Section 15.3 Figure 15.8 shows a sinusoidal wave of period T on a string at times $0, \frac{1}{8}T, \frac{2}{8}T, \frac{3}{8}T, \frac{4}{8}T, \frac{5}{8}T, \frac{6}{8}T, \frac{7}{8}T,$ and T .

- At which time is point A on the string moving upward with maximum speed?
- At which time does point B on the string have the greatest upward acceleration?
- At which time does point C on the string have a downward acceleration but an upward velocity?



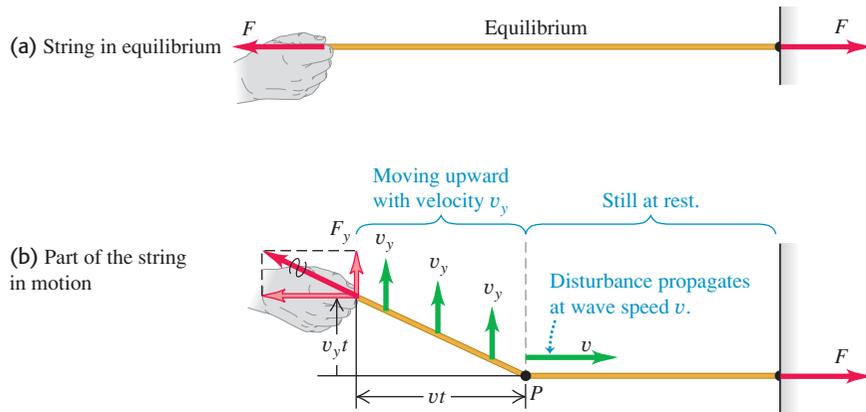
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ActivPhysics 10.2: Speed of Waves on a String

15.4 Speed of a Transverse Wave

One of the key properties of any wave is the wave *speed*. Light waves in air have a much greater speed of propagation than do sound waves in air (3.00×10^8 m/s versus 344 m/s); that's why you see the flash from a bolt of lightning before you hear the clap of thunder. In this section we'll see what determines the speed of propagation of one particular kind of wave: transverse waves on a string. The speed of these waves is important to understand because it is an essential part of analyzing stringed musical instruments, as we'll discuss later in this chapter. Furthermore, the speeds of many kinds of mechanical waves turn out to have the same basic mathematical expression as does the speed of waves on a string.

The physical quantities that determine the speed of transverse waves on a string are the *tension* in the string and its *mass per unit length* (also called *linear mass density*). We might guess that increasing the tension should increase the restoring forces that tend to straighten the string when it is disturbed, thus increasing the wave speed. We might also guess that increasing the mass should make the motion more sluggish and decrease the speed. Both these guesses turn out to be right. We'll develop the exact relationship among wave speed, tension, and mass per unit length by two different methods. The first is simple in concept and considers a specific wave shape; the second is more general but also more formal. Choose whichever you like better.

15.11 Propagation of a transverse wave on a string.**Wave Speed on a String: First Method**

We consider a perfectly flexible string (Fig. 15.11). In the equilibrium position the tension is F and the linear mass density (mass per unit length) is μ . (When portions of the string are displaced from equilibrium, the mass per unit length decreases a little, and the tension increases a little.) We ignore the weight of the string so that when the string is at rest in the equilibrium position, the string forms a perfectly straight line as in Fig. 15.11a.

Starting at time $t = 0$, we apply a constant upward force F_y at the left end of the string. We might expect that the end would move with constant acceleration; that would happen if the force were applied to a *point* mass. But here the effect of the force F_y is to set successively more and more mass in motion. The wave travels with constant speed v , so the division point P between moving and nonmoving portions moves with the same constant speed v (Fig. 15.11b).

Figure 15.11b shows that all particles in the moving portion of the string move upward with constant *velocity* v_y , not constant acceleration. To see why this is so, we note that the *impulse* of the force F_y up to time t is $F_y t$. According to the impulse–momentum theorem (see Section 8.1), the impulse is equal to the change in the total transverse component of momentum ($mv_y - 0$) of the moving part of the string. Because the system started with *no* transverse momentum, this is equal to the total momentum at time t :

$$F_y t = mv_y$$

The total momentum thus must increase proportionately with time. But since the division point P moves with constant speed, the length of string that is in motion and hence the total mass m in motion are also proportional to the time t that the force has been acting. So the *change* of momentum must be associated entirely with the increasing amount of mass in motion, not with an increasing velocity of an individual mass element. That is, mv_y changes because m , not v_y , changes.

At time t , the left end of the string has moved up a distance $v_y t$, and the boundary point P has advanced a distance vt . The total force at the left end of the string has components F and F_y . Why F ? There is no motion in the direction along the length of the string, so there is no unbalanced horizontal force. Therefore F , the magnitude of the horizontal component, does not change when the string is displaced. In the displaced position the tension is $(F^2 + F_y^2)^{1/2}$ (greater than F), and the string stretches somewhat.

To derive an expression for the wave speed v , we again apply the impulse–momentum theorem to the portion of the string in motion at time t —that is, the portion to the left of P in Fig. 15.11b. The transverse *impulse* (transverse

force times time) is equal to the change of transverse *momentum* of the moving portion (mass times transverse component of velocity). The impulse of the transverse force F_y in time t is $F_y t$. In Fig. 15.11b the right triangle whose vertex is at P , with sides $v_y t$ and vt , is similar to the right triangle whose vertex is at the position of the hand, with sides F_y and F . Hence

$$\frac{F_y}{F} = \frac{v_y t}{vt} \quad F_y = F \frac{v_y}{v}$$

and

$$\text{Transverse impulse} = F_y t = F \frac{v_y}{v} t$$

The mass of the moving portion of the string is the product of the mass per unit length μ and the length vt , or μvt . The transverse momentum is the product of this mass and the transverse velocity v_y :

$$\text{Transverse momentum} = (\mu vt)v_y$$

We note again that the momentum increases with time *not* because mass is moving faster, as was usually the case in Chapter 8, but because *more mass* is brought into motion. But the impulse of the force F_y is still equal to the total change in momentum of the system. Applying this relationship, we obtain

$$F \frac{v_y}{v} t = \mu vt v_y$$

Solving this for v , we find

$$v = \sqrt{\frac{F}{\mu}} \quad (\text{speed of a transverse wave on a string}) \quad (15.13)$$

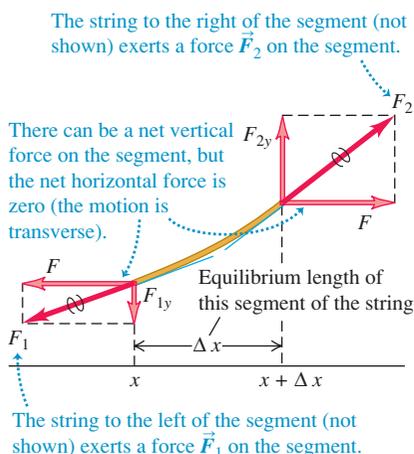
Equation (15.13) confirms our prediction that the wave speed v should increase when the tension F increases but decrease when the mass per unit length μ increases (Fig. 15.12).

Note that v_y does not appear in Eq. (15.13); thus the wave speed doesn't depend on v_y . Our calculation considered only a very special kind of pulse, but we can consider *any* shape of wave disturbance as a series of pulses with different values of v_y . So even though we derived Eq. (15.13) for a special case, it is valid for *any* transverse wave motion on a string, including the sinusoidal and other periodic waves we discussed in Section 15.3. Note also that the wave speed doesn't depend on the amplitude or frequency of the wave, in accordance with our assumptions in Section 15.3.

15.12 These cables have a relatively large amount of mass per unit length (μ) and a low tension (F). If the cables are disturbed—say, by a bird landing on them—transverse waves will travel along them at a slow speed $v = \sqrt{F/\mu}$.



15.13 Free-body diagram for a segment of string. The force at each end of the string is tangent to the string at the point of application.



Wave Speed on a String: Second Method

Here is an alternative derivation of Eq. (15.13). If you aren't comfortable with partial derivatives, it can be omitted. We apply Newton's second law, $\sum \vec{F} = m\vec{a}$, to a small segment of string whose length in the equilibrium position is Δx (Fig. 15.13). The mass of the segment is $m = \mu \Delta x$; the forces at the ends are represented in terms of their x - and y -components. The x -components have equal magnitude F and add to zero because the motion is transverse and there is no component of acceleration in the x -direction. To obtain F_{1y} and F_{2y} , we note that the ratio F_{1y}/F is equal in magnitude to the *slope* of the string at point x and that F_{2y}/F is equal to the slope at point $x + \Delta x$. Taking proper account of signs, we find

$$\frac{F_{1y}}{F} = -\left(\frac{\partial y}{\partial x}\right)_x \quad \frac{F_{2y}}{F} = \left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} \quad (15.14)$$

The notation reminds us that the derivatives are evaluated at points x and $x + \Delta x$, respectively. From Eq. (15.14) we find that the net y -component of force is

$$F_y = F_{1y} + F_{2y} = F \left[\left(\frac{\partial y}{\partial x} \right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x} \right)_x \right] \quad (15.15)$$

We now equate F_y from Eq. (15.15) to the mass $\mu \Delta x$ times the y -component of acceleration $\partial^2 y / \partial t^2$. We obtain

$$F \left[\left(\frac{\partial y}{\partial x} \right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x} \right)_x \right] = \mu \Delta x \frac{\partial^2 y}{\partial t^2} \quad (15.16)$$

or, dividing by $F \Delta x$,

$$\frac{\left(\frac{\partial y}{\partial x} \right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x} \right)_x}{\Delta x} = \frac{\mu}{F} \frac{\partial^2 y}{\partial t^2} \quad (15.17)$$

We now take the limit as $\Delta x \rightarrow 0$. In this limit, the left side of Eq. (15.17) becomes the derivative of $\partial y / \partial x$ with respect to x (at constant t)—that is, the *second* (partial) derivative of y with respect to x :

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F} \frac{\partial^2 y}{\partial t^2} \quad (15.18)$$

Now, finally, comes the punch line of our story. Equation (15.18) has exactly the same form as the *wave equation*, Eq. (15.12), that we derived at the end of Section 15.3. That equation and Eq. (15.18) describe the very same wave motion, so they must be identical. Comparing the two equations, we see that for this to be so, we must have

$$v = \sqrt{\frac{F}{\mu}} \quad (15.19)$$

which is the same expression as Eq. (15.13).

In going through this derivation, we didn't make any special assumptions about the shape of the wave. Since our derivation led us to rediscover Eq. (15.12), the wave equation, we conclude that the wave equation is valid for waves on a string that have *any* shape.

The Speed of Mechanical Waves

Equation (15.13) or (15.19) gives the wave speed for only the special case of mechanical waves on a stretched string or rope. Remarkably, it turns out that for many types of mechanical waves, including waves on a string, the expression for wave speed has the same general form:

$$v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$$

To interpret this expression, let's look at the now-familiar case of waves on a string. The tension F in the string plays the role of the restoring force; it tends to bring the string back to its undisturbed, equilibrium configuration. The mass of the string—or, more properly, the linear mass density μ —provides the inertia that prevents the string from returning instantaneously to equilibrium. Hence we have $v = \sqrt{F/\mu}$ for the speed of waves on a string.

In Chapter 16 we'll see a similar expression for the speed of sound waves in a gas. Roughly speaking, the gas pressure provides the force that tends to return the gas to its undisturbed state when a sound wave passes through. The inertia is provided by the density, or mass per unit volume, of the gas.

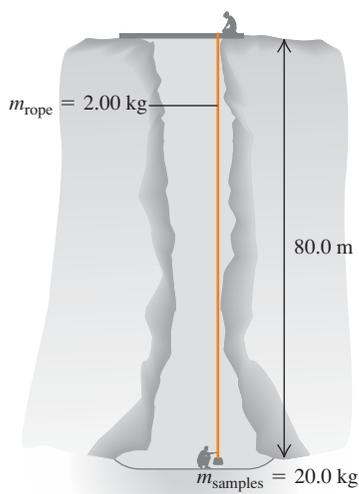
Example 15.3 Calculating wave speed

One end of a 2.00-kg rope is tied to a support at the top of a mine shaft 80.0 m deep (Fig. 15.14). The rope is stretched taut by a 20.0-kg box of rocks attached at the bottom. (a) The geologist at the bottom of the shaft signals to a colleague at the top by jerking the rope sideways. What is the speed of a transverse wave on the rope? (b) If a point on the rope is in transverse SHM with $f = 2.00$ Hz, how many cycles of the wave are there in the rope's length?

SOLUTION

IDENTIFY and SET UP: In part (a) we can find the wave speed (our target variable) using the *dynamic* relationship $v = \sqrt{F/\mu}$

15.14 Sending signals along a vertical rope using transverse waves.



[Eq. (15.13)]. In part (b) we find the wavelength from the *kinematic* relationship $v = f\lambda$; from that we can find the target variable, the number of wavelengths that fit into the rope's 80.0-m length. We'll assume that the rope is massless (even though its weight is 10% that of the box), so that the box alone provides the tension in the rope.

EXECUTE: (a) The tension in the rope due to the box is

$$F = m_{\text{box}}g = (20.0 \text{ kg})(9.80 \text{ m/s}^2) = 196 \text{ N}$$

and the rope's linear mass density is

$$\mu = \frac{m_{\text{rope}}}{L} = \frac{2.00 \text{ kg}}{80.0 \text{ m}} = 0.0250 \text{ kg/m}$$

Hence, from Eq. (15.13), the wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{196 \text{ N}}{0.0250 \text{ kg/m}}} = 88.5 \text{ m/s}$$

(b) From Eq. (15.1), the wavelength is

$$\lambda = \frac{v}{f} = \frac{88.5 \text{ m/s}}{2.00 \text{ s}^{-1}} = 44.3 \text{ m}$$

There are $(80.0 \text{ m})/(44.3 \text{ m}) = 1.81$ wavelengths (that is, cycles of the wave) in the rope.

EVALUATE: Because of the rope's weight, its tension is greater at the top than at the bottom. Hence both the wave speed and the wavelength increase as a wave travels up the rope. If you take account of this, can you verify that the wave speed at the top of the rope is 92.9 m/s?

Application Surface Waves and the Swimming Speed of Ducks

When a duck swims, it necessarily produces waves on the surface of the water. The faster the duck swims, the larger the wave amplitude and the more power the duck must supply to produce these waves. The maximum power available from their leg muscles limits the maximum swimming speed of ducks to only about 0.7 m/s (2.5 km/h = 1.6 mi/h).



Test Your Understanding of Section 15.4 The six strings of a guitar are the same length and under nearly the same tension, but they have different thicknesses. On which string do waves travel the fastest? (i) the thickest string; (ii) the thinnest string; (iii) the wave speed is the same on all strings.

**15.5 Energy in Wave Motion**

Every wave motion has *energy* associated with it. The energy we receive from sunlight and the destructive effects of ocean surf and earthquakes bear this out. To produce any of the wave motions we have discussed in this chapter, we have to apply a force to a portion of the wave medium; the point where the force is applied moves, so we do *work* on the system. As the wave propagates, each portion of the medium exerts a force and does work on the adjoining portion. In this way a wave can transport energy from one region of space to another.

As an example of energy considerations in wave motion, let's look again at transverse waves on a string. How is energy transferred from one portion of string to another? Picture a wave traveling from left to right (the positive x -direction) on the string, and consider a particular point a on the string (Fig. 15.15a). The string to the left of point a exerts a force on the string to the right of it, and vice versa. In Fig. 15.15b the string to the left of a has been removed, and the force it exerts at a is represented by the components F and F_y , as we did in Figs. 15.11 and

15.13. We note again that F_y/F is equal to the negative of the *slope* of the string at a , which is also given by $\partial y/\partial x$. Putting these together, we have

$$F_y(x, t) = -F \frac{\partial y(x, t)}{\partial x} \quad (15.20)$$

We need the negative sign because F_y is negative when the slope is positive. We write the vertical force as $F_y(x, t)$ as a reminder that its value may be different at different points along the string and at different times.

When point a moves in the y -direction, the force F_y does *work* on this point and therefore transfers energy into the part of the string to the right of a . The corresponding power P (rate of doing work) at the point a is the transverse force $F_y(x, t)$ at a times the transverse velocity $v_y(x, t) = \partial y(x, t)/\partial t$ of that point:

$$P(x, t) = F_y(x, t)v_y(x, t) = -F \frac{\partial y(x, t)}{\partial x} \frac{\partial y(x, t)}{\partial t} \quad (15.21)$$

This power is the *instantaneous* rate at which energy is transferred along the string. Its value depends on the position x on the string and on the time t . Note that energy is being transferred only at points where the string has a nonzero slope ($\partial y/\partial x$ is nonzero), so that there is a transverse component of the tension force, and where the string has a nonzero transverse velocity ($\partial y/\partial t$ is nonzero) so that the transverse force can do work.

Equation (15.21) is valid for *any* wave on a string, sinusoidal or not. For a sinusoidal wave with wave function given by Eq. (15.7), we have

$$\begin{aligned} y(x, t) &= A \cos(kx - \omega t) \\ \frac{\partial y(x, t)}{\partial x} &= -kA \sin(kx - \omega t) \\ \frac{\partial y(x, t)}{\partial t} &= \omega A \sin(kx - \omega t) \\ P(x, t) &= Fk\omega A^2 \sin^2(kx - \omega t) \end{aligned} \quad (15.22)$$

By using the relationships $\omega = vk$ and $v^2 = F/\mu$, we can also express Eq. (15.22) in the alternative form

$$P(x, t) = \sqrt{\mu F} \omega^2 A^2 \sin^2(kx - \omega t) \quad (15.23)$$

The \sin^2 function is never negative, so the instantaneous power in a sinusoidal wave is either positive (so that energy flows in the positive x -direction) or zero (at points where there is no energy transfer). Energy is never transferred in the direction opposite to the direction of wave propagation (Fig. 15.16).

The maximum value of the instantaneous power $P(x, t)$ occurs when the \sin^2 function has the value unity:

$$P_{\max} = \sqrt{\mu F} \omega^2 A^2 \quad (15.24)$$

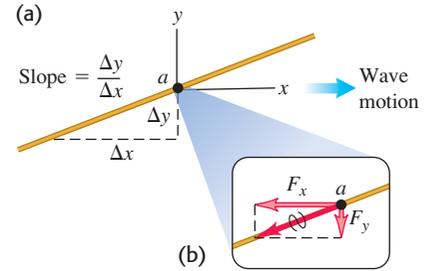
To obtain the *average* power from Eq. (15.23), we note that the *average* value of the \sin^2 function, averaged over any whole number of cycles, is $\frac{1}{2}$. Hence the average power is

$$P_{\text{av}} = \frac{1}{2} \sqrt{\mu F} \omega^2 A^2 \quad (\text{average power, sinusoidal wave on a string}) \quad (15.25)$$

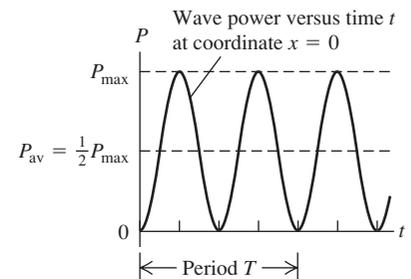
The average power is just one-half of the maximum instantaneous power (see Fig. 15.16).

The average rate of energy transfer is proportional to the square of the amplitude and to the square of the frequency. This proportionality is a general result for mechanical waves of all types, including seismic waves (see the photo that opens this chapter). For a mechanical wave, the rate of energy transfer

15.15 (a) Point a on a string carrying a wave from left to right. (b) The components of the force exerted on the part of the string to the right of point a by the part of the string to the left of point a .



15.16 The instantaneous power $P(x, t)$ in a sinusoidal wave as given by Eq. (15.23), shown as a function of time at coordinate $x = 0$. The power is never negative, which means that energy never flows opposite to the direction of wave propagation.



quadruples if the frequency is doubled (for the same amplitude) or if the amplitude is doubled (for the same frequency).

Electromagnetic waves turn out to be a bit different. While the average rate of energy transfer in an electromagnetic wave is proportional to the square of the amplitude, just as for mechanical waves, it is independent of the value of ω .

Example 15.4 Power in a wave

(a) In Example 15.2 (Section 15.3), at what maximum rate does Throcky put energy into the clothesline? That is, what is his maximum instantaneous power? The linear mass density of the clothesline is $\mu = 0.250$ kg/m, and Throcky applies tension $F = 36.0$ N. (b) What is his average power? (c) As Throcky tires, the amplitude decreases. What is the average power when the amplitude is 7.50 mm?

SOLUTION

IDENTIFY and SET UP: In part (a) our target variable is the *maximum instantaneous* power P_{\max} , while in parts (b) and (c) it is the *average* power. For part (a) we'll use Eq. (15.24), and for parts (b) and (c) we'll use Eq. (15.25); Example 15.2 gives us all the needed quantities.

EXECUTE: (a) From Eq. (15.24),

$$\begin{aligned} P_{\max} &= \sqrt{\mu F \omega^2 A^2} \\ &= \sqrt{(0.250 \text{ kg/m})(36.0 \text{ N})(4.00\pi \text{ rad/s})^2(0.075 \text{ m})^2} \\ &= 2.66 \text{ W} \end{aligned}$$

(b) From Eqs. (15.24) and (15.25), the average power is one-half of the maximum instantaneous power, so

$$P_{\text{av}} = \frac{1}{2}P_{\max} = \frac{1}{2}(2.66 \text{ W}) = 1.33 \text{ W}$$

(c) The new amplitude is $\frac{1}{10}$ of the value we used in parts (a) and (b). From Eq. (15.25), the average power is proportional to A^2 , so the new average power is

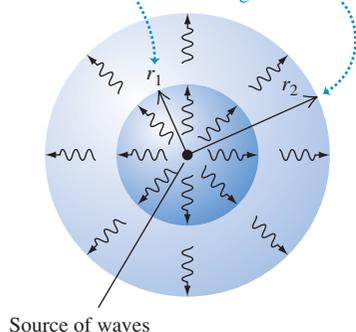
$$P_{\text{av}} = \left(\frac{1}{10}\right)^2(1.33 \text{ W}) = 0.0133 \text{ W} = 13.3 \text{ mW}$$

EVALUATE: Equation (15.23) shows that P_{\max} occurs when $\sin^2(kx - \omega t) = 1$. At any given position x , this happens twice per period of the wave—once when the sine function is equal to $+1$, and once when it's equal to -1 . The *minimum* instantaneous power is zero; this occurs when $\sin^2(kx - \omega t) = 0$, which also happens twice per period.

Can you confirm that the given values of μ and F give the wave speed mentioned in Example 15.2?

15.17 The greater the distance from a wave source, the greater the area over which the wave power is distributed and the smaller the wave intensity.

At distance r_1 from the source, the intensity is I_1 .
At a greater distance $r_2 > r_1$, the intensity I_2 is less than I_1 : the same power is spread over a greater area.



Wave Intensity

Waves on a string carry energy in just one dimension of space (along the direction of the string). But other types of waves, including sound waves in air and seismic waves in the body of the earth, carry energy across all three dimensions of space. For waves that travel in three dimensions, we define the **intensity** (denoted by I) to be *the time average rate at which energy is transported by the wave, per unit area, across a surface perpendicular to the direction of propagation*. That is, intensity I is average power per unit area. It is usually measured in watts per square meter (W/m^2).

If waves spread out equally in all directions from a source, the intensity at a distance r from the source is inversely proportional to r^2 (Fig. 15.17). This follows directly from energy conservation. If the power output of the source is P , then the average intensity I_1 through a sphere with radius r_1 and surface area $4\pi r_1^2$ is

$$I_1 = \frac{P}{4\pi r_1^2}$$

The average intensity I_2 through a sphere with a different radius r_2 is given by a similar expression. If no energy is absorbed between the two spheres, the power P must be the same for both, and

$$4\pi r_1^2 I_1 = 4\pi r_2^2 I_2$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad (\text{inverse-square law for intensity}) \quad (15.26)$$

The intensity I at any distance r is therefore inversely proportional to r^2 . This relationship is called the *inverse-square law* for intensity.

Example 15.5 The inverse-square law

A siren on a tall pole radiates sound waves uniformly in all directions. At a distance of 15.0 m from the siren, the sound intensity is 0.250 W/m^2 . At what distance is the intensity 0.010 W/m^2 ?

SOLUTION

IDENTIFY and SET UP: Because sound is radiated uniformly in all directions, we can use the inverse-square law, Eq. (15.26). At $r_1 = 15.0 \text{ m}$ the intensity is $I_1 = 0.250 \text{ W/m}^2$, and the target variable is the distance r_2 at which the intensity is $I_2 = 0.010 \text{ W/m}^2$.

EXECUTE: We solve Eq. (15.26) for r_2 :

$$r_2 = r_1 \sqrt{\frac{I_1}{I_2}} = (15.0 \text{ m}) \sqrt{\frac{0.250 \text{ W/m}^2}{0.010 \text{ W/m}^2}} = 75.0 \text{ m}$$

EVALUATE: As a check on our answer, note that r_2 is five times greater than r_1 . By the inverse-square law, the intensity I_2 should be $1/5^2 = 1/25$ as great as I_1 , and indeed it is.

By using the inverse-square law, we've assumed that the sound waves travel in straight lines away from the siren. A more realistic solution, which is beyond our scope, would account for the reflection of sound waves from the ground.

Test Your Understanding of Section 15.5 Four identical strings each carry a sinusoidal wave of frequency 10 Hz. The string tension and wave amplitude are different for different strings. Rank the following strings in order from highest to lowest value of the average wave power: (i) tension 10 N, amplitude 1.0 mm; (ii) tension 40 N, amplitude 1.0 mm; (iii) tension 10 N, amplitude 4.0 mm; (iv) tension 20 N, amplitude 2.0 mm.



15.6 Wave Interference, Boundary Conditions, and Superposition

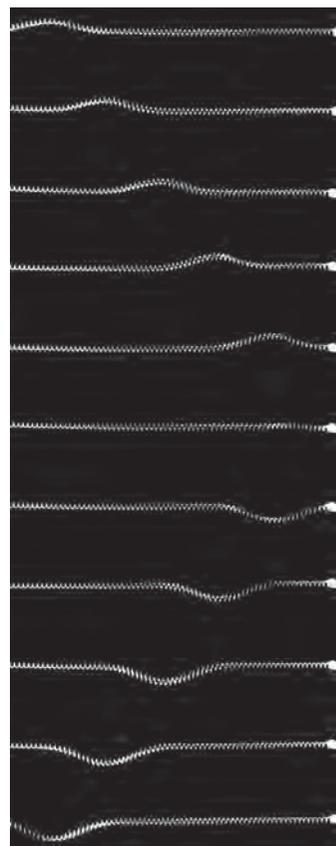
Up to this point we've been discussing waves that propagate continuously in the same direction. But when a wave strikes the boundaries of its medium, all or part of the wave is *reflected*. When you yell at a building wall or a cliff face some distance away, the sound wave is reflected from the rigid surface and you hear an echo. When you flip the end of a rope whose far end is tied to a rigid support, a pulse travels the length of the rope and is reflected back to you. In both cases, the initial and reflected waves overlap in the same region of the medium. This overlapping of waves is called **interference**. (In general, the term "interference" refers to what happens when two or more waves pass through the same region at the same time.)

As a simple example of wave reflections and the role of the boundary of a wave medium, let's look again at transverse waves on a stretched string. What happens when a wave pulse or a sinusoidal wave arrives at the *end* of the string?

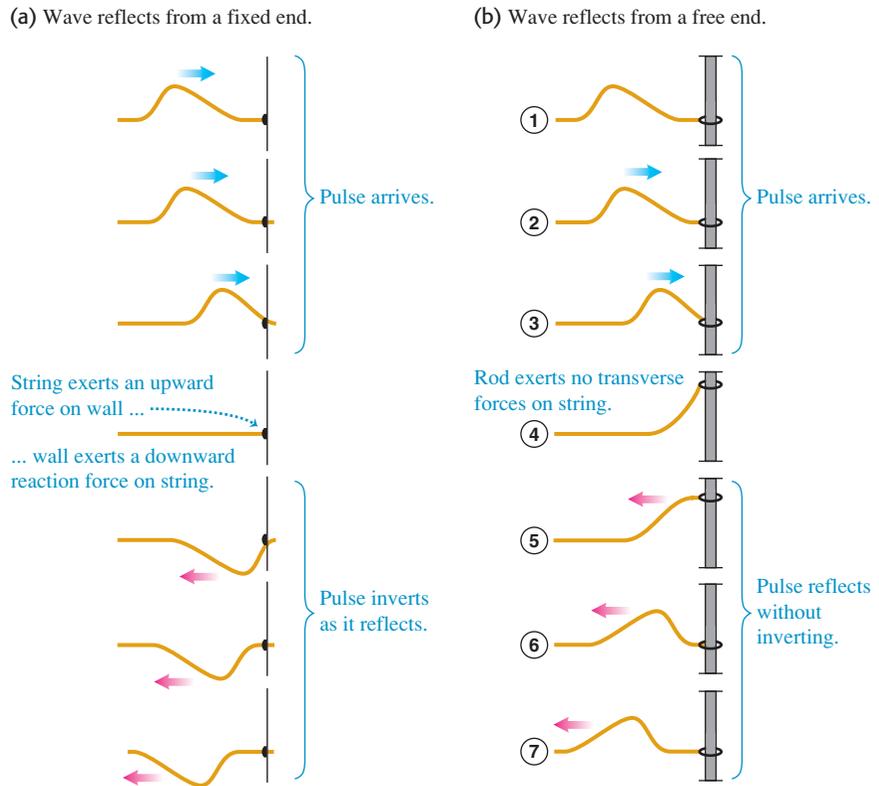
If the end is fastened to a rigid support, it is a *fixed* end that cannot move. The arriving wave exerts a force on the support; the reaction to this force, exerted by the support *on* the string, "kicks back" on the string and sets up a *reflected* pulse or wave traveling in the reverse direction. Figure 15.18 is a series of photographs showing the reflection of a pulse at the fixed end of a long coiled spring. The reflected pulse moves in the opposite direction from the initial, or *incident*, pulse, and its displacement is also opposite. Figure 15.19a illustrates this situation for a wave pulse on a string.

The opposite situation from an end that is held stationary is a *free* end, one that is perfectly free to move in the direction perpendicular to the length of the string. For example, the string might be tied to a light ring that slides on a frictionless rod perpendicular to the string, as in Fig. 15.19b. The ring and rod maintain the tension but exert no transverse force. When a wave arrives at this free end, the ring slides along the rod. The ring reaches a maximum displacement, and both it and the string come momentarily to rest, as in drawing 4 in Fig. 15.19b. But the string is now stretched, giving increased tension, so the free end of the string is

15.18 A series of images of a wave pulse, equally spaced in time from top to bottom. The pulse starts at the left in the top image, travels to the right, and is reflected from the fixed end at the right.

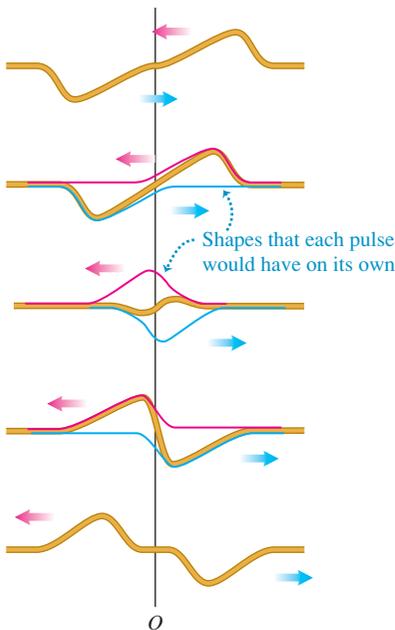


15.19 Reflection of a wave pulse (a) at a fixed end of a string and (b) at a free end. Time increases from top to bottom in each figure.



15.20 Overlap of two wave pulses—one right side up, one inverted—traveling in opposite directions. Time increases from top to bottom.

As the pulses overlap, the displacement of the string at any point is the algebraic sum of the displacements due to the individual pulses.



pulled back down, and again a reflected pulse is produced (drawing 7). As for a fixed end, the reflected pulse moves in the opposite direction from the initial pulse, but now the direction of the displacement is the same as for the initial pulse. The conditions at the end of the string, such as a rigid support or the complete absence of transverse force, are called **boundary conditions**.

The formation of the reflected pulse is similar to the overlap of two pulses traveling in opposite directions. Figure 15.20 shows two pulses with the same shape, one inverted with respect to the other, traveling in opposite directions. As the pulses overlap and pass each other, the total displacement of the string is the *algebraic sum* of the displacements at that point in the individual pulses. Because these two pulses have the same shape, the total displacement at point *O* in the middle of the figure is zero at all times. Thus the motion of the left half of the string would be the same if we cut the string at point *O*, threw away the right side, and held the end at *O* fixed. The two pulses on the left side then correspond to the incident and reflected pulses, combining so that the total displacement at *O* is *always* zero. For this to occur, the reflected pulse must be inverted relative to the incident pulse.

Figure 15.21 shows two pulses with the same shape, traveling in opposite directions but *not* inverted relative to each other. The displacement at point *O* in the middle of the figure is not zero, but the slope of the string at this point is always zero. According to Eq. (15.20), this corresponds to the absence of any transverse force at this point. In this case the motion of the left half of the string would be the same as if we cut the string at point *O* and attached the end to a frictionless sliding ring (Fig. 15.19b) that maintains tension without exerting any transverse force. In other words, this situation corresponds to reflection of a pulse at a free end of a string at point *O*. In this case the reflected pulse is *not* inverted.

The Principle of Superposition

Combining the displacements of the separate pulses at each point to obtain the actual displacement is an example of the **principle of superposition**: When two

waves overlap, the actual displacement of any point on the string at any time is obtained by adding the displacement the point would have if only the first wave were present and the displacement it would have if only the second wave were present. In other words, the wave function $y(x, t)$ that describes the resulting motion in this situation is obtained by *adding* the two wave functions for the two separate waves:

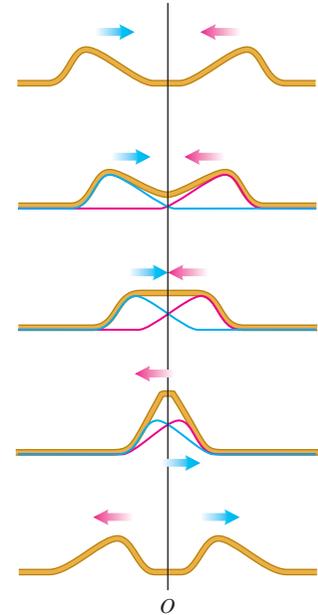
$$y(x, t) = y_1(x, t) + y_2(x, t) \quad (\text{principle of superposition}) \quad (15.27)$$

Mathematically, this additive property of wave functions follows from the form of the wave equation, Eq. (15.12) or (15.18), which every physically possible wave function must satisfy. Specifically, the wave equation is *linear*; that is, it contains the function $y(x, t)$ only to the first power (there are no terms involving $y(x, t)^2$, $y(x, t)^{1/2}$, etc.). As a result, if any two functions $y_1(x, t)$ and $y_2(x, t)$ satisfy the wave equation separately, their sum $y_1(x, t) + y_2(x, t)$ also satisfies it and is therefore a physically possible motion. Because this principle depends on the linearity of the wave equation and the corresponding linear-combination property of its solutions, it is also called the *principle of linear superposition*. For some physical systems, such as a medium that does not obey Hooke's law, the wave equation is *not* linear; this principle does not hold for such systems.

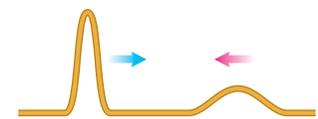
The principle of superposition is of central importance in all types of waves. When a friend talks to you while you are listening to music, you can distinguish the sound of speech and the sound of music from each other. This is precisely because the total sound wave reaching your ears is the algebraic sum of the wave produced by your friend's voice and the wave produced by the speakers of your stereo. If two sound waves did *not* combine in this simple linear way, the sound you would hear in this situation would be a hopeless jumble. Superposition also applies to electromagnetic waves (such as light) and many other types of waves.

Test Your Understanding of Section 15.6 Figure 15.22 shows two wave pulses with different shapes traveling in different directions along a string. Make a series of sketches like Fig. 15.21 showing the shape of the string as the two pulses approach, overlap, and then pass each other.

15.21 Overlap of two wave pulses—both right side up—traveling in opposite directions. Time increases from top to bottom. Compare to Fig. 15.20.



15.22 Two wave pulses with different shapes.



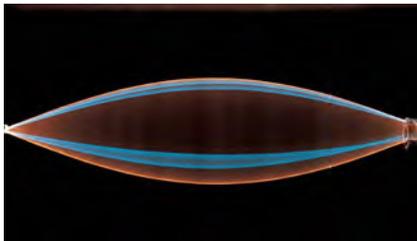
15.7 Standing Waves on a String

We have talked about the reflection of a wave *pulse* on a string when it arrives at a boundary point (either a fixed end or a free end). Now let's look at what happens when a *sinusoidal* wave is reflected by a fixed end of a string. We'll again approach the problem by considering the superposition of two waves propagating through the string, one representing the original or incident wave and the other representing the wave reflected at the fixed end.

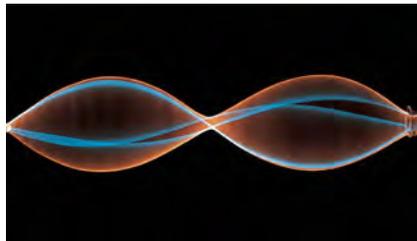
Figure 15.23 shows a string that is fixed at its left end. Its right end is moved up and down in simple harmonic motion to produce a wave that travels to the left; the wave reflected from the fixed end travels to the right. The resulting motion when the two waves combine no longer looks like two waves traveling in opposite directions. The string appears to be subdivided into a number of segments, as in the time-exposure photographs of Figs. 15.23a, 15.23b, 15.23c, and 15.23d. Figure 15.23e shows two instantaneous shapes of the string in Fig. 15.23b. Let's compare this behavior with the waves we studied in Sections 15.1 through 15.5. In a wave that travels along the string, the amplitude is constant and the wave pattern moves with a speed equal to the wave speed. Here, instead, the wave pattern remains in the same position along the string and its amplitude

15.23 (a)–(d) Time exposures of standing waves in a stretched string. From (a) to (d), the frequency of oscillation of the right-hand end increases and the wavelength of the standing wave decreases. (e) The extremes of the motion of the standing wave in part (b), with nodes at the center and at the ends. The right-hand end of the string moves very little compared to the antinodes and so is essentially a node.

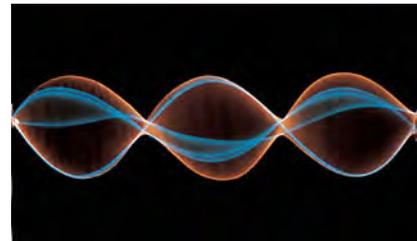
(a) String is one-half wavelength long.



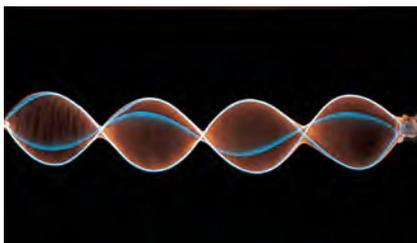
(b) String is one wavelength long.



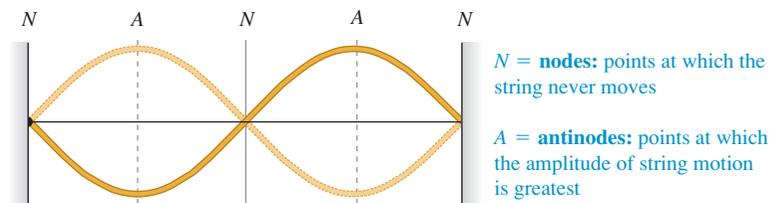
(c) String is one and a half wavelengths long.



(d) String is two wavelengths long.



(e) The shape of the string in (b) at two different instants



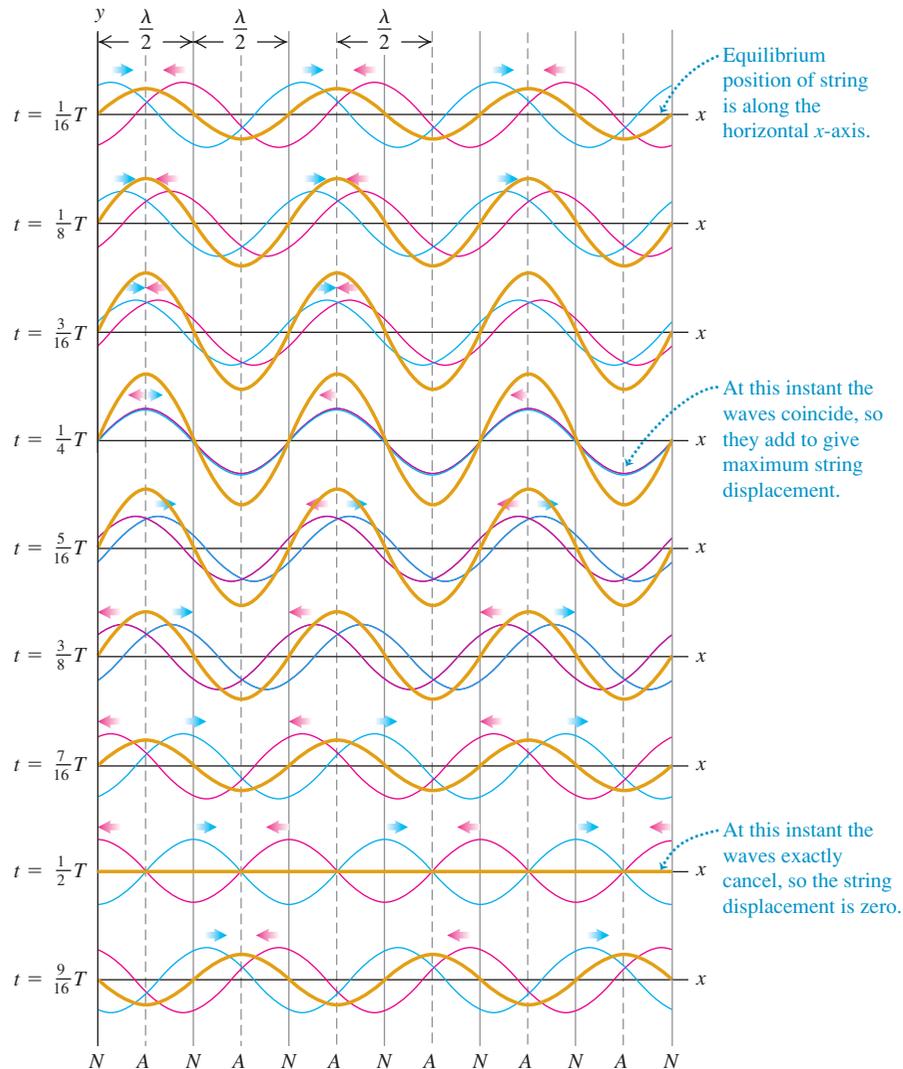
fluctuates. There are particular points called **nodes** (labeled N in Fig. 15.23e) that never move at all. Midway between the nodes are points called **antinodes** (labeled A in Fig. 15.23e) where the amplitude of motion is greatest. Because the wave pattern doesn't appear to be moving in either direction along the string, it is called a **standing wave**. (To emphasize the difference, a wave that *does* move along the string is called a **traveling wave**.)

The principle of superposition explains how the incident and reflected waves combine to form a standing wave. In Fig. 15.24 the red curves show a wave traveling to the left. The blue curves show a wave traveling to the right with the same propagation speed, wavelength, and amplitude. The waves are shown at nine instants, $\frac{1}{16}$ of a period apart. At each point along the string, we add the displacements (the values of y) for the two separate waves; the result is the total wave on the string, shown in brown.

At certain instants, such as $t = \frac{1}{4}T$, the two wave patterns are exactly in phase with each other, and the shape of the string is a sine curve with twice the amplitude of either individual wave. At other instants, such as $t = \frac{1}{2}T$, the two waves are exactly out of phase with each other, and the total wave at that instant is zero. The resultant displacement is *always* zero at those places marked N at the bottom of Fig. 15.24. These are the **nodes**. At a node the displacements of the two waves in red and blue are always equal and opposite and cancel each other out. This cancellation is called **destructive interference**. Midway between the nodes are the points of *greatest* amplitude, or the **antinodes**, marked A . At the antinodes the displacements of the two waves in red and blue are always identical, giving a large resultant displacement; this phenomenon is called **constructive interference**. We can see from the figure that the distance between successive nodes or between successive antinodes is one half-wavelength, or $\lambda/2$.

We can derive a wave function for the standing wave of Fig. 15.24 by adding the wave functions $y_1(x, t)$ and $y_2(x, t)$ for two waves with equal amplitude, period, and wavelength traveling in opposite directions. Here $y_1(x, t)$ (the red curves in Fig. 15.24) represents an incoming, or **incident**, wave traveling to the

15.24 Formation of a standing wave. A wave traveling to the left (red curves) combines with a wave traveling to the right (blue curves) to form a standing wave (brown curves).



left along the $+x$ -axis, arriving at the point $x = 0$ and being reflected; $y_2(x, t)$ (the blue curves in Fig. 15.24) represents the *reflected* wave traveling to the right from $x = 0$. We noted in Section 15.6 that the wave reflected from a fixed end of a string is inverted, so we give a negative sign to one of the waves:

$$y_1(x, t) = -A \cos(kx + \omega t) \quad (\text{incident wave traveling to the left})$$

$$y_2(x, t) = A \cos(kx - \omega t) \quad (\text{reflected wave traveling to the right})$$

Note also that the change in sign corresponds to a shift in *phase* of 180° or π radians. At $x = 0$ the motion from the reflected wave is $A \cos \omega t$ and the motion from the incident wave is $-A \cos \omega t$, which we can also write as $A \cos(\omega t + \pi)$. From Eq. (15.27), the wave function for the standing wave is the sum of the individual wave functions:

$$y(x, t) = y_1(x, t) + y_2(x, t) = A[-\cos(kx + \omega t) + \cos(kx - \omega t)]$$

We can rewrite each of the cosine terms by using the identities for the cosine of the sum and difference of two angles: $\cos(a \mp b) = \cos a \cos b \mp \sin a \sin b$.

Applying these and combining terms, we obtain the wave function for the standing wave:

$$y(x, t) = y_1(x, t) + y_2(x, t) = (2A \sin kx) \sin \omega t \quad \text{or}$$

$$y(x, t) = (A_{\text{SW}} \sin kx) \sin \omega t \quad \text{(standing wave on a string, fixed end at } x = 0) \quad (15.28)$$

The standing-wave amplitude A_{SW} is twice the amplitude A of either of the original traveling waves:

$$A_{\text{SW}} = 2A$$

Equation (15.28) has two factors: a function of x and a function of t . The factor $A_{\text{SW}} \sin kx$ shows that at each instant the shape of the string is a sine curve. But unlike a wave traveling along a string, the wave shape stays in the same position, oscillating up and down as described by the $\sin \omega t$ factor. This behavior is shown graphically by the brown curves in Fig. 15.24. Each point in the string still undergoes simple harmonic motion, but all the points between any successive pair of nodes oscillate *in phase*. This is in contrast to the phase differences between oscillations of adjacent points that we see with a wave traveling in one direction.

We can use Eq. (15.28) to find the positions of the nodes; these are the points for which $\sin kx = 0$, so the displacement is *always* zero. This occurs when $kx = 0, \pi, 2\pi, 3\pi, \dots$, or, using $k = 2\pi/\lambda$,

$$\begin{aligned} x &= 0, \frac{\pi}{k}, \frac{2\pi}{k}, \frac{3\pi}{k}, \dots \\ &= 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \dots \end{aligned} \quad \begin{array}{l} \text{(nodes of a standing wave on} \\ \text{a string, fixed end at } x = 0) \end{array} \quad (15.29)$$

In particular, there is a node at $x = 0$, as there should be, since this point is a fixed end of the string.

A standing wave, unlike a traveling wave, *does not* transfer energy from one end to the other. The two waves that form it would individually carry equal amounts of power in opposite directions. There is a local flow of energy from each node to the adjacent antinodes and back, but the *average* rate of energy transfer is zero at every point. If you evaluate the wave power given by Eq. (15.21) using the wave function of Eq. (15.28), you will find that the average power is zero.

Problem-Solving Strategy 15.2 Standing Waves



IDENTIFY *the relevant concepts:* Identify the target variables. Then determine whether the problem is purely *kinematic* (involving only such quantities as wave speed v , wavelength λ , and frequency f) or whether *dynamic* properties of the medium (such as F and μ for transverse waves on a string) are also involved.

SET UP *the problem* using the following steps:

1. Sketch the shape of the standing wave at a particular instant. This will help you visualize the nodes (label them N) and antinodes (A). The distance between adjacent nodes (or antinodes) is $\lambda/2$; the distance between a node and the adjacent antinode is $\lambda/4$.
2. Choose the equations you'll use. The wave function for the standing wave, like Eq. (15.28), is often useful.

3. You can determine the wave speed if you know λ and f (or, equivalently, $k = 2\pi/\lambda$ and $\omega = 2\pi f$) or if you know the relevant properties of the medium (for a string, F and μ).

EXECUTE *the solution:* Solve for the target variables. Once you've found the wave function, you can find the displacement y at any point x and at any time t . You can find the velocity and acceleration of a particle in the medium by taking the first and second partial derivatives of y with respect to time.

EVALUATE *your answer:* Compare your numerical answers with your sketch. Check that the wave function satisfies the boundary conditions (for example, the displacement should be zero at a fixed end).

Example 15.6 Standing waves on a guitar string

A guitar string lies along the x -axis when in equilibrium. The end of the string at $x = 0$ (the bridge of the guitar) is fixed. A sinusoidal wave with amplitude $A = 0.750 \text{ mm} = 7.50 \times 10^{-4} \text{ m}$ and frequency $f = 440 \text{ Hz}$, corresponding to the red curves in Fig. 15.24, travels along the string in the $-x$ -direction at 143 m/s . It is reflected from the fixed end, and the superposition of the incident and reflected waves forms a standing wave. (a) Find the equation giving the displacement of a point on the string as a function of position and time. (b) Locate the nodes. (c) Find the amplitude of the standing wave and the maximum transverse velocity and acceleration.

SOLUTION

IDENTIFY and SET UP: This is a *kinematics* problem (see Problem-Solving Strategy 15.1 in Section 15.3). The target variables are: in part (a), the wave function of the standing wave; in part (b), the locations of the nodes; and in part (c), the maximum displacement y , transverse velocity v_y , and transverse acceleration a_y . Since there is a fixed end at $x = 0$, we can use Eqs. (15.28) and (15.29) to describe this standing wave. We will need the relationships $\omega = 2\pi f$, $v = \omega/k$, and $v = \lambda f$.

EXECUTE: (a) The standing-wave amplitude is $A_{\text{SW}} = 2A = 1.50 \times 10^{-3} \text{ m}$ (twice the amplitude of either the incident or reflected wave). The angular frequency and wave number are

$$\omega = 2\pi f = (2\pi \text{ rad})(440 \text{ s}^{-1}) = 2760 \text{ rad/s}$$

$$k = \frac{\omega}{v} = \frac{2760 \text{ rad/s}}{143 \text{ m/s}} = 19.3 \text{ rad/m}$$

Equation (15.28) then gives

$$y(x, t) = (A_{\text{SW}} \sin kx) \sin \omega t \\ = [(1.50 \times 10^{-3} \text{ m}) \sin(19.3 \text{ rad/m})x] \sin(2760 \text{ rad/s})t$$

(b) From Eq. (15.29), the positions of the nodes are $x = 0, \lambda/2, \lambda, 3\lambda/2, \dots$. The wavelength is $\lambda = v/f = (143 \text{ m/s})/(440 \text{ Hz})$

$= 0.325 \text{ m}$, so the nodes are at $x = 0, 0.163 \text{ m}, 0.325 \text{ m}, 0.488 \text{ m}, \dots$

(c) From the expression for $y(x, t)$ in part (a), the maximum displacement from equilibrium is $A_{\text{SW}} = 1.50 \times 10^{-3} \text{ m} = 1.50 \text{ mm}$. This occurs at the *antinodes*, which are midway between adjacent nodes (that is, at $x = 0.081 \text{ m}, 0.244 \text{ m}, 0.406 \text{ m}, \dots$).

For a particle on the string at any point x , the transverse (y -) velocity is

$$v_y(x, t) = \frac{\partial y(x, t)}{\partial t} \\ = [(1.50 \times 10^{-3} \text{ m}) \sin(19.3 \text{ rad/m})x] \\ \times [(2760 \text{ rad/s}) \cos(2760 \text{ rad/s})t] \\ = [(4.15 \text{ m/s}) \sin(19.3 \text{ rad/m})x] \cos(2760 \text{ rad/s})t$$

At an antinode, $\sin(19.3 \text{ rad/m})x = \pm 1$ and the transverse velocity varies between $+4.15 \text{ m/s}$ and -4.15 m/s . As is always the case in SHM, the maximum velocity occurs when the particle is passing through the equilibrium position ($y = 0$).

The transverse acceleration $a_y(x, t)$ is the *second* partial derivative of $y(x, t)$ with respect to time. You can show that

$$a_y(x, t) = \frac{\partial v_y(x, t)}{\partial t} = \frac{\partial^2 y(x, t)}{\partial t^2} \\ = [(-1.15 \times 10^4 \text{ m/s}^2) \sin(19.3 \text{ rad/m})x] \\ \times \sin(2760 \text{ rad/s})t$$

At the antinodes, the transverse acceleration varies between $+1.15 \times 10^4 \text{ m/s}^2$ and $-1.15 \times 10^4 \text{ m/s}^2$.

EVALUATE: The maximum transverse velocity at an antinode is quite respectable (about 15 km/h , or 9.3 mi/h). But the maximum transverse acceleration is tremendous, 1170 times the acceleration due to gravity! Guitar strings are actually fixed at *both* ends; we'll see the consequences of this in the next section.

Test Your Understanding of Section 15.7 Suppose the frequency of the standing wave in Example 15.6 were doubled from 440 Hz to 880 Hz . Would all of the nodes for $f = 440 \text{ Hz}$ also be nodes for $f = 880 \text{ Hz}$? If so, would there be additional nodes for $f = 880 \text{ Hz}$? If not, which nodes are absent for $f = 880 \text{ Hz}$? **I**

15.8 Normal Modes of a String

When we described standing waves on a string rigidly held at one end, as in Fig. 15.23, we made no assumptions about the length of the string or about what was happening at the other end. Let's now consider a string of a definite length L , rigidly held at *both* ends. Such strings are found in many musical instruments, including pianos, violins, and guitars. When a guitar string is plucked, a wave is produced in the string; this wave is reflected and re-reflected from the ends of the string, making a standing wave. This standing wave on the string in turn produces a sound wave in the air, with a frequency determined by the properties of the string. This is what makes stringed instruments so useful in making music.

To understand these properties of standing waves on a string fixed at both ends, let's first examine what happens when we set up a sinusoidal wave on such a string. The standing wave that results must have a node at *both* ends of the string. We saw in the preceding section that adjacent nodes are one half-wavelength

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PhET: Fourier: Making Waves

PhET: Waves on a String

ActivPhysics 10.4: Standing Waves on Strings

ActivPhysics 10.5: Tuning a Stringed Instrument: Standing Waves

ActivPhysics 10.6: String Mass and Standing Waves

($\lambda/2$) apart, so the length of the string must be $\lambda/2$, or $2(\lambda/2)$, or $3(\lambda/2)$, or in general some integer number of half-wavelengths:

$$L = n \frac{\lambda}{2} \quad (n = 1, 2, 3, \dots) \quad (\text{string fixed at both ends}) \quad (15.30)$$

That is, if a string with length L is fixed at both ends, a standing wave can exist only if its wavelength satisfies Eq. (15.30).

Solving this equation for λ and labeling the possible values of λ as λ_n , we find

$$\lambda_n = \frac{2L}{n} \quad (n = 1, 2, 3, \dots) \quad (\text{string fixed at both ends}) \quad (15.31)$$

Waves can exist on the string if the wavelength is *not* equal to one of these values, but there cannot be a steady wave pattern with nodes and antinodes, and the total wave cannot be a standing wave. Equation (15.31) is illustrated by the standing waves shown in Figs. 15.23a, 15.23b, 15.23c, and 15.23d; these represent $n = 1, 2, 3$, and 4, respectively.

Corresponding to the series of possible standing-wave wavelengths λ_n is a series of possible standing-wave frequencies f_n , each related to its corresponding wavelength by $f_n = v/\lambda_n$. The smallest frequency f_1 corresponds to the largest wavelength (the $n = 1$ case), $\lambda_1 = 2L$:

$$f_1 = \frac{v}{2L} \quad (\text{string fixed at both ends}) \quad (15.32)$$

This is called the **fundamental frequency**. The other standing-wave frequencies are $f_2 = 2v/2L$, $f_3 = 3v/2L$, and so on. These are all integer multiples of the fundamental frequency f_1 , such as $2f_1$, $3f_1$, $4f_1$, and so on, and we can express *all* the frequencies as

$$f_n = n \frac{v}{2L} = n f_1 \quad (n = 1, 2, 3, \dots) \quad (\text{string fixed at both ends}) \quad (15.33)$$

These frequencies are called **harmonics**, and the series is called a **harmonic series**. Musicians sometimes call f_2 , f_3 , and so on **overtones**; f_2 is the second harmonic or the first overtone, f_3 is the third harmonic or the second overtone, and so on. The first harmonic is the same as the fundamental frequency (Fig. 15.25).

For a string with fixed ends at $x = 0$ and $x = L$, the wave function $y(x, t)$ of the n th standing wave is given by Eq. (15.28) (which satisfies the condition that there is a node at $x = 0$), with $\omega = \omega_n = 2\pi f_n$ and $k = k_n = 2\pi/\lambda_n$:

$$y_n(x, t) = A_{SW} \sin k_n x \sin \omega_n t \quad (15.34)$$

You can easily show that this wave function has nodes at both $x = 0$ and $x = L$, as it must.

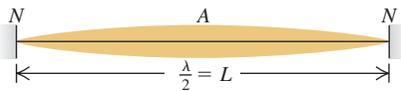
A **normal mode** of an oscillating system is a motion in which all particles of the system move sinusoidally with the same frequency. For a system made up of a string of length L fixed at both ends, each of the wavelengths given by Eq. (15.31) corresponds to a possible normal-mode pattern and frequency. There are infinitely many normal modes, each with its characteristic frequency and vibration pattern. Figure 15.26 shows the first four normal-mode patterns and their associated frequencies and wavelengths; these correspond to Eq. (15.34) with $n = 1, 2, 3$, and 4. By contrast, a harmonic oscillator, which has only one oscillating particle, has only one normal mode and one characteristic frequency. The string fixed at both ends has infinitely many normal modes because it is made up of a very large (effectively infinite) number of particles. More complicated oscillating systems also have infinite numbers of normal modes, though with more complex normal-mode patterns than a string (Fig. 15.27).

15.25 Each string of a violin naturally oscillates at one or more of its harmonic frequencies, producing sound waves in the air with the same frequencies.

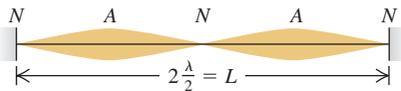


15.26 The first four normal modes of a string fixed at both ends. (Compare these to the photographs in Fig. 15.23.)

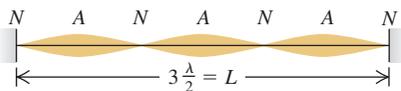
(a) $n = 1$: fundamental frequency, f_1



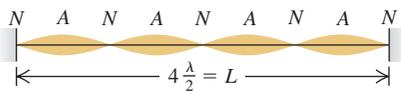
(b) $n = 2$: second harmonic, f_2 (first overtone)



(c) $n = 3$: third harmonic, f_3 (second overtone)



(d) $n = 4$: fourth harmonic, f_4 (third overtone)



Complex Standing Waves

If we could displace a string so that its shape is the same as one of the normal-mode patterns and then release it, it would vibrate with the frequency of that mode. Such a vibrating string would displace the surrounding air with the same frequency, producing a traveling sinusoidal sound wave that your ears would perceive as a pure tone. But when a string is struck (as in a piano) or plucked (as is done to guitar strings), the shape of the displaced string is *not* as simple as one of the patterns in Fig. 15.26. The fundamental as well as many overtones are present in the resulting vibration. This motion is therefore a combination or *superposition* of many normal modes. Several simple harmonic motions of different frequencies are present simultaneously, and the displacement of any point on the string is the sum (or superposition) of the displacements associated with the individual modes. The sound produced by the vibrating string is likewise a superposition of traveling sinusoidal sound waves, which you perceive as a rich, complex tone with the fundamental frequency f_1 . The standing wave on the string and the traveling sound wave in the air have similar **harmonic content** (the extent to which frequencies higher than the fundamental are present). The harmonic content depends on how the string is initially set into motion. If you pluck the strings of an acoustic guitar in the normal location over the sound hole, the sound that you hear has a different harmonic content than if you pluck the strings next to the fixed end on the guitar body.

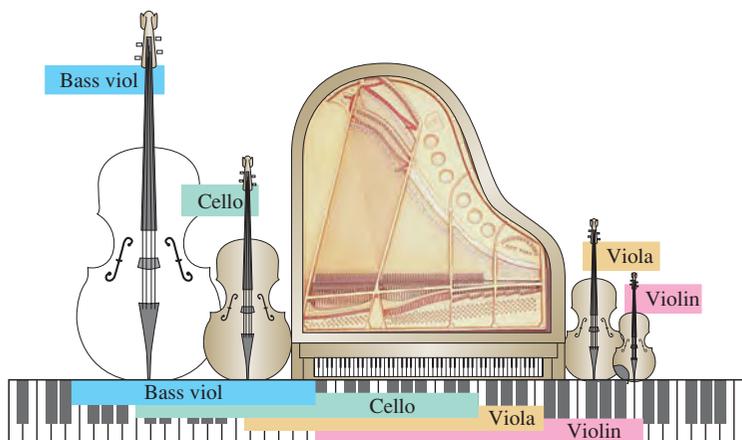
It is possible to represent every possible motion of the string as some superposition of normal-mode motions. Finding this representation for a given vibration pattern is called *harmonic analysis*. The sum of sinusoidal functions that represents a complex wave is called a *Fourier series*. Figure 15.28 shows how a standing wave that is produced by plucking a guitar string of length L at a point $L/4$ from one end can be represented as a combination of sinusoidal functions.

Standing Waves and String Instruments

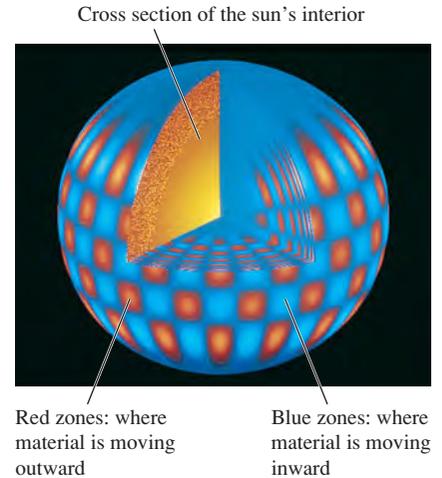
As we have seen, the fundamental frequency of a vibrating string is $f_1 = v/2L$. The speed v of waves on the string is determined by Eq. (15.13), $v = \sqrt{F/\mu}$. Combining these equations, we find

$$f_1 = \frac{1}{2L} \sqrt{\frac{F}{\mu}} \quad (\text{string fixed at both ends}) \quad (15.35)$$

This is also the fundamental frequency of the sound wave created in the surrounding air by the vibrating string. Familiar musical instruments show how f_1 depends on the properties of the string. The inverse dependence of frequency on length L is illustrated by the long strings of the bass (low-frequency) section of the piano or the bass viol compared with the shorter strings of the treble section of the piano or the violin (Fig. 15.29). The pitch of a violin or guitar is usually



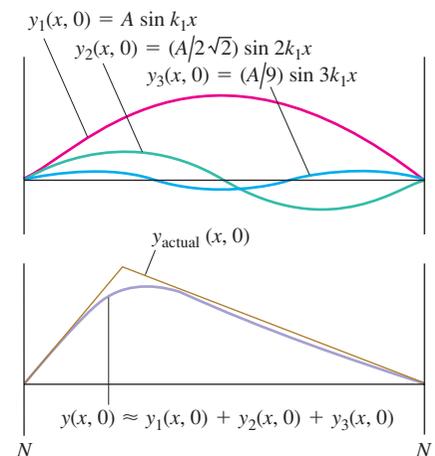
15.27 Astronomers have discovered that the sun oscillates in several different normal modes. This computer simulation shows one such mode.



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ActivPhysics 10.10: Complex Waves: Fourier Analysis

15.28 When a guitar string is plucked (pulled into a triangular shape) and released, a standing wave results. The standing wave is well represented (except at the sharp maximum point) by the sum of just three sinusoidal functions. Including additional sinusoidal functions further improves the representation.



15.29 Comparing the range of a concert grand piano to the ranges of a bass viol, a cello, a viola, and a violin. In all cases, longer strings produce bass notes and shorter strings produce treble notes.

varied by pressing a string against the fingerboard with the fingers to change the length L of the vibrating portion of the string. Increasing the tension F increases the wave speed v and thus increases the frequency (and the pitch). All string instruments are “tuned” to the correct frequencies by varying the tension; you tighten the string to raise the pitch. Finally, increasing the mass per unit length μ decreases the wave speed and thus the frequency. The lower notes on a steel guitar are produced by thicker strings, and one reason for winding the bass strings of a piano with wire is to obtain the desired low frequency from a relatively short string.

Wind instruments such as saxophones and trombones also have normal modes. As for stringed instruments, the frequencies of these normal modes determine the pitch of the musical tones that these instruments produce. We’ll discuss these instruments and many other aspects of sound in Chapter 16.

Example 15.7 A giant bass viol

In an attempt to get your name in *Guinness World Records*, you build a bass viol with strings of length 5.00 m between fixed points. One string, with linear mass density 40.0 g/m, is tuned to a 20.0-Hz fundamental frequency (the lowest frequency that the human ear can hear). Calculate (a) the tension of this string, (b) the frequency and wavelength on the string of the second harmonic, and (c) the frequency and wavelength on the string of the second overtone.

SOLUTION

IDENTIFY and SET UP: In part (a) the target variable is the string tension F ; we’ll use Eq. (15.35), which relates F to the known values $f_1 = 20.0$ Hz, $L = 5.00$ m, and $\mu = 40.0$ g/m. In parts (b) and (c) the target variables are the frequency and wavelength of a given harmonic and a given overtone. We determine these from the given length of the string and the fundamental frequency, using Eqs. (15.31) and (15.33).

EXECUTE: (a) We solve Eq. (15.35) for F :

$$\begin{aligned} F &= 4\mu L^2 f_1^2 = 4(40.0 \times 10^{-3} \text{ kg/m})(5.00 \text{ m})^2(20.0 \text{ s}^{-1})^2 \\ &= 1600 \text{ N} = 360 \text{ lb} \end{aligned}$$

(b) From Eqs. (15.33) and (15.31), the frequency and wavelength of the second harmonic ($n = 2$) are

$$\begin{aligned} f_2 &= 2f_1 = 2(20.0 \text{ Hz}) = 40.0 \text{ Hz} \\ \lambda_2 &= \frac{2L}{2} = \frac{2(5.00 \text{ m})}{2} = 5.00 \text{ m} \end{aligned}$$

(c) The second overtone is the “second tone over” (above) the fundamental—that is, $n = 3$. Its frequency and wavelength are

$$\begin{aligned} f_3 &= 3f_1 = 3(20.0 \text{ Hz}) = 60.0 \text{ Hz} \\ \lambda_3 &= \frac{2L}{3} = \frac{2(5.00 \text{ m})}{3} = 3.33 \text{ m} \end{aligned}$$

EVALUATE: The string tension in a real bass viol is typically a few hundred newtons; the tension in part (a) is a bit higher than that. The wavelengths in parts (b) and (c) are equal to the length of the string and two-thirds the length of the string, respectively, which agrees with the drawings of standing waves in Fig. 15.26.

Example 15.8 From waves on a string to sound waves in air

What are the frequency and wavelength of the sound waves produced in the air when the string in Example 15.7 is vibrating at its fundamental frequency? The speed of sound in air at 20°C is 344 m/s.

SOLUTION

IDENTIFY and SET UP: Our target variables are the frequency and wavelength for the *sound wave* produced by the bass viol string. The frequency of the sound wave is the same as the fundamental frequency f_1 of the standing wave, because the string forces the surrounding air to vibrate at the same frequency. The wavelength of the sound wave is $\lambda_{1(\text{sound})} = v_{\text{sound}}/f_1$.

EXECUTE: We have $f = f_1 = 20.0$ Hz, so

$$\lambda_{1(\text{sound})} = \frac{v_{\text{sound}}}{f_1} = \frac{344 \text{ m/s}}{20.0 \text{ Hz}} = 17.2 \text{ m}$$

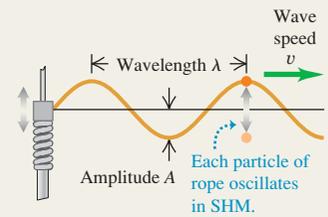
EVALUATE: In Example 15.7, the wavelength of the fundamental on the string was $\lambda_{1(\text{string})} = 2L = 2(5.00 \text{ m}) = 10.0$ m. Here $\lambda_{1(\text{sound})} = 17.2$ m is greater than that by the factor of $17.2/10.0 = 1.72$. This is as it should be: Because the frequencies of the sound wave and the standing wave are equal, $\lambda = v/f$ says that the wavelengths in air and on the string are in the same ratio as the corresponding wave speeds; here $v_{\text{sound}} = 344$ m/s is greater than $v_{\text{string}} = (10.0 \text{ m})(20.0 \text{ Hz}) = 200$ m/s by just the factor 1.72.

Test Your Understanding of Section 15.8 While a guitar string is vibrating, you gently touch the midpoint of the string to ensure that the string does not vibrate at that point. Which normal modes *cannot* be present on the string while you are touching it in this way?

Waves and their properties: A wave is any disturbance that propagates from one region to another. A mechanical wave travels within some material called the medium. The wave speed v depends on the type of wave and the properties of the medium.

In a periodic wave, the motion of each point of the medium is periodic with frequency f and period T . The wavelength λ is the distance over which the wave pattern repeats, and the amplitude A is the maximum displacement of a particle in the medium. The product of λ and f equals the wave speed. A sinusoidal wave is a special periodic wave in which each point moves in simple harmonic motion. (See Example 15.1.)

$$v = \lambda f \quad (15.1)$$



Wave functions and wave dynamics: The wave function $y(x, t)$ describes the displacements of individual particles in the medium. Equations (15.3), (15.4), and (15.7) give the wave equation for a sinusoidal wave traveling in the $+x$ -direction. If the wave is moving in the $-x$ -direction, the minus signs in the cosine functions are replaced by plus signs. (See Example 15.2.)

The wave function obeys a partial differential equation called the wave equation, Eq. (15.12).

The speed of transverse waves on a string depends on the tension F and mass per unit length μ . (See Example 15.3.)

$$y(x, t) = A \cos \left[\omega \left(\frac{x}{v} - t \right) \right] = A \cos 2\pi f \left(\frac{x}{v} - t \right) \quad (15.3)$$

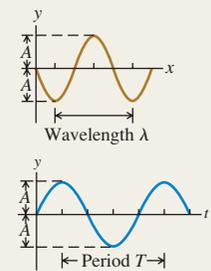
$$y(x, t) = A \cos 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \quad (15.4)$$

$$y(x, t) = A \cos(kx - \omega t) \quad (15.7)$$

where $k = 2\pi/\lambda$ and $\omega = 2\pi f = vk$

$$\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x, t)}{\partial t^2} \quad (15.12)$$

$$v = \sqrt{\frac{F}{\mu}} \quad (\text{waves on a string}) \quad (15.13)$$



Wave power: Wave motion conveys energy from one region to another. For a sinusoidal mechanical wave, the average power P_{av} is proportional to the square of the wave amplitude and the square of the frequency. For waves that spread out in three dimensions, the wave intensity I is inversely proportional to the square of the distance from the source. (See Examples 15.4 and 15.5.)

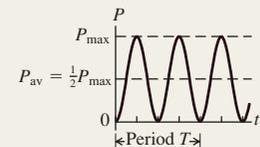
$$P_{av} = \frac{1}{2} \sqrt{\mu F} \omega^2 A^2 \quad (15.25)$$

(average power, sinusoidal wave)

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad (15.26)$$

(inverse-square law for intensity)

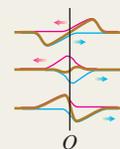
Wave power versus time t at coordinate $x = 0$



Wave superposition: A wave reflects when it reaches a boundary of its medium. At any point where two or more waves overlap, the total displacement is the sum of the displacements of the individual waves (principle of superposition).

$$y(x, t) = y_1(x, t) + y_2(x, t) \quad (15.27)$$

(principle of superposition)



Standing waves on a string: When a sinusoidal wave is reflected from a fixed or free end of a stretched string, the incident and reflected waves combine to form a standing sinusoidal wave with nodes and antinodes. Adjacent nodes are spaced a distance $\lambda/2$ apart, as are adjacent antinodes. (See Example 15.6.)

When both ends of a string with length L are held fixed, standing waves can occur only when L is an integer multiple of $\lambda/2$. Each frequency with its associated vibration pattern is called a normal mode. (See Examples 15.7 and 15.8.)

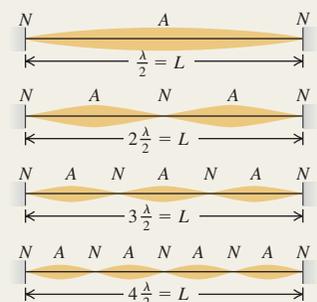
$$y(x, t) = (A_{SW} \sin kx) \sin \omega t \quad (15.28)$$

(standing wave on a string, fixed end at $x = 0$)

$$f_n = n \frac{v}{2L} = n f_1 \quad (n = 1, 2, 3, \dots) \quad (15.33)$$

$$f_1 = \frac{1}{2L} \sqrt{\frac{F}{\mu}} \quad (15.35)$$

(string fixed at both ends)



BRIDGING PROBLEM

Waves on a Rotating Rope

A uniform rope with length L and mass m is held at one end and whirled in a horizontal circle with angular velocity ω . You can ignore the force of gravity on the rope. (a) At a point on the rope a distance r from the end that is held, what is the tension F ? (b) What is the speed of transverse waves at this point? (c) Find the time required for a transverse wave to travel from one end of the rope to the other.

SOLUTION GUIDE

See MasteringPhysics® Study Area for a Video Tutor solution.



IDENTIFY and SET UP

1. Draw a sketch of the situation and label the distances r and L . The tension in the rope will be different at different values of r . Do you see why? Where on the rope do you expect the tension to be greatest? Where do you expect it will be least?
2. Where on the rope do you expect the wave speed to be greatest? Where do you expect it will be least?
3. Think about the portion of the rope that is farther out than r from the end that is held. What forces act on this portion? (Remember that you can ignore gravity.) What is the mass of this portion? How far is its center of mass from the rotation axis?

4. Make a list of the unknown quantities and decide which are your target variables.

EXECUTE

5. Draw a free-body diagram for the portion of the rope that is farther out than r from the end that is held.
6. Use your free-body diagram to help you determine the tension in the rope at distance r .
7. Use your result from step 6 to find the wave speed at distance r .
8. Use your result from step 7 to find the time for a wave to travel from one end to the other. (*Hint:* The wave speed is $v = dr/dt$, so the time for the wave to travel a distance dr along the rope is $dt = dr/v$. Integrate this to find the total time. See Appendix B.)

EVALUATE

9. Do your results for parts (a) and (b) agree with your expectations from steps 1 and 2? Are the units correct?
10. Check your result for part (a) by considering the net force on a small segment of the rope at distance r with length dr and mass $dm = (m/L)dr$. [*Hint:* The tension forces on this segment are $F(r)$ on one side and $F(r + dr)$ on the other side. You will get an equation for dF/dr that you can integrate to find F as a function of r .]

Problems

For instructor-assigned homework, go to www.masteringphysics.com



•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q15.1 Two waves travel on the same string. Is it possible for them to have (a) different frequencies; (b) different wavelengths; (c) different speeds; (d) different amplitudes; (e) the same frequency but different wavelengths? Explain your reasoning.

Q15.2 Under a tension F , it takes 2.00 s for a pulse to travel the length of a taut wire. What tension is required (in terms of F) for the pulse to take 6.00 s instead?

Q15.3 What kinds of energy are associated with waves on a stretched string? How could you detect such energy experimentally?

Q15.4 The amplitude of a wave decreases gradually as the wave travels down a long, stretched string. What happens to the energy of the wave when this happens?

Q15.5 For the wave motions discussed in this chapter, does the speed of propagation depend on the amplitude? What makes you say this?

Q15.6 The speed of ocean waves depends on the depth of the water; the deeper the water, the faster the wave travels. Use this to explain why ocean waves crest and “break” as they near the shore.

Q15.7 Is it possible to have a longitudinal wave on a stretched string? Why or why not? Is it possible to have a transverse wave on a steel rod? Again, why or why not? If your answer is yes in either case, explain how you would create such a wave.

Q15.8 An echo is sound reflected from a distant object, such as a wall or a cliff. Explain how you can determine how far away the object is by timing the echo.

Q15.9 Why do you see lightning before you hear the thunder? A familiar rule of thumb is to start counting slowly, once per second, when you see the lightning; when you hear the thunder, divide the number you have reached by 3 to obtain your distance from the lightning in kilometers (or divide by 5 to obtain your distance in miles). Why does this work, or does it?

Q15.10 For transverse waves on a string, is the wave speed the same as the speed of any part of the string? Explain the difference between these two speeds. Which one is constant?

Q15.11 Children make toy telephones by sticking each end of a long string through a hole in the bottom of a paper cup and knotting it so it will not pull out. When the string is pulled taut, sound can be transmitted from one cup to the other. How does this work? Why is the transmitted sound louder than the sound traveling through air for the same distance?

Q15.12 The four strings on a violin have different thicknesses, but are all under approximately the same tension. Do waves travel faster on the thick strings or the thin strings? Why? How does the fundamental vibration frequency compare for the thick versus the thin strings?

Q15.13 A sinusoidal wave can be described by a cosine function, which is negative just as often as positive. So why isn't the average power delivered by this wave zero?

Q15.14 Two strings of different mass per unit length μ_1 and μ_2 are tied together and stretched with a tension F . A wave travels

along the string and passes the discontinuity in μ . Which of the following wave properties will be the same on both sides of the discontinuity, and which ones will change? speed of the wave; frequency; wavelength. Explain the physical reasoning behind each of your answers.

Q15.15 A long rope with mass m is suspended from the ceiling and hangs vertically. A wave pulse is produced at the lower end of the rope, and the pulse travels up the rope. Does the speed of the wave pulse change as it moves up the rope, and if so, does it increase or decrease?

Q15.16 In a transverse wave on a string, the motion of the string is perpendicular to the length of the string. How, then, is it possible for energy to move along the length of the string?

Q15.17 Both wave intensity and gravitation obey inverse-square laws. Do they do so for the same reason? Discuss the reason for each of these inverse-square laws as well as you can.

Q15.18 Energy can be transferred along a string by wave motion. However, in a standing wave on a string, no energy can ever be transferred past a node. Why not?

Q15.19 Can a standing wave be produced on a string by superposing two waves traveling in opposite directions with the same frequency but different amplitudes? Why or why not? Can a standing wave be produced by superposing two waves traveling in opposite directions with different frequencies but the same amplitude? Why or why not?

Q15.20 If you stretch a rubber band and pluck it, you hear a (somewhat) musical tone. How does the frequency of this tone change as you stretch the rubber band further? (Try it!) Does this agree with Eq. (15.35) for a string fixed at both ends? Explain.

Q15.21 A musical interval of an *octave* corresponds to a factor of 2 in frequency. By what factor must the tension in a guitar or violin string be increased to raise its pitch one octave? To raise it two octaves? Explain your reasoning. Is there any danger in attempting these changes in pitch?

Q15.22 By touching a string lightly at its center while bowing, a violinist can produce a note exactly one octave above the note to which the string is tuned—that is, a note with exactly twice the frequency. Why is this possible?

Q15.23 As we discussed in Section 15.1, water waves are a combination of longitudinal and transverse waves. Defend the following statement: “When water waves hit a vertical wall, the wall is a node of the longitudinal displacement but an antinode of the transverse displacement.”

Q15.24 Violins are short instruments, while cellos and basses are long. In terms of the frequency of the waves they produce, explain why this is so.

Q15.25 What is the purpose of the frets on a guitar? In terms of the frequency of the vibration of the strings, explain their use.

EXERCISES

Section 15.2 Periodic Waves

15.1 • The speed of sound in air at 20°C is 344 m/s. (a) What is the wavelength of a sound wave with a frequency of 784 Hz, corresponding to the note G₅ on a piano, and how many milliseconds does each vibration take? (b) What is the wavelength of a sound wave one octave higher than the note in part (a)?

15.2 • **BIO Audible Sound.** Provided the amplitude is sufficiently great, the human ear can respond to longitudinal waves over a range of frequencies from about 20.0 Hz to about 20.0 kHz. (a) If you were to mark the beginning of each complete wave pattern with a red dot for the long-wavelength sound and a blue dot

for the short-wavelength sound, how far apart would the red dots be, and how far apart would the blue dots be? (b) In reality would adjacent dots in each set be far enough apart for you to easily measure their separation with a meter stick? (c) Suppose you repeated part (a) in water, where sound travels at 1480 m/s. How far apart would the dots be in each set? Could you readily measure their separation with a meter stick?

15.3 • **Tsunami!** On December 26, 2004, a great earthquake occurred off the coast of Sumatra and triggered immense waves (tsunami) that killed some 200,000 people. Satellites observing these waves from space measured 800 km from one wave crest to the next and a period between waves of 1.0 hour. What was the speed of these waves in m/s and in km/h? Does your answer help you understand why the waves caused such devastation?

15.4 • **BIO Ultrasound Imaging.** Sound having frequencies above the range of human hearing (about 20,000 Hz) is called *ultrasound*. Waves above this frequency can be used to penetrate the body and to produce images by reflecting from surfaces. In a typical ultrasound scan, the waves travel through body tissue with a speed of 1500 m/s. For a good, detailed image, the wavelength should be no more than 1.0 mm. What frequency sound is required for a good scan?

15.5 • **BIO** (a) **Audible wavelengths.** The range of audible frequencies is from about 20 Hz to 20,000 Hz. What is the range of the wavelengths of audible sound in air? (b) **Visible light.** The range of visible light extends from 400 nm to 700 nm. What is the range of visible frequencies of light? (c) **Brain surgery.** Surgeons can remove brain tumors by using a cavitron ultrasonic surgical aspirator, which produces sound waves of frequency 23 kHz. What is the wavelength of these waves in air? (d) **Sound in the body.** What would be the wavelength of the sound in part (c) in bodily fluids in which the speed of sound is 1480 m/s but the frequency is unchanged?

15.6 • A fisherman notices that his boat is moving up and down periodically, owing to waves on the surface of the water. It takes 2.5 s for the boat to travel from its highest point to its lowest, a total distance of 0.62 m. The fisherman sees that the wave crests are spaced 6.0 m apart. (a) How fast are the waves traveling? (b) What is the amplitude of each wave? (c) If the total vertical distance traveled by the boat were 0.30 m but the other data remained the same, how would the answers to parts (a) and (b) be affected?

Section 15.3 Mathematical Description of a Wave

15.7 • Transverse waves on a string have wave speed 8.00 m/s, amplitude 0.0700 m, and wavelength 0.320 m. The waves travel in the $-x$ -direction, and at $t = 0$ the $x = 0$ end of the string has its maximum upward displacement. (a) Find the frequency, period, and wave number of these waves. (b) Write a wave function describing the wave. (c) Find the transverse displacement of a particle at $x = 0.360$ m at time $t = 0.150$ s. (d) How much time must elapse from the instant in part (c) until the particle at $x = 0.360$ m next has maximum upward displacement?

15.8 • A certain transverse wave is described by

$$y(x, t) = (6.50 \text{ mm}) \cos 2\pi \left(\frac{x}{28.0 \text{ cm}} - \frac{t}{0.0360 \text{ s}} \right)$$

Determine the wave's (a) amplitude; (b) wavelength; (c) frequency; (d) speed of propagation; (e) direction of propagation.

15.9 • **CALC** Which of the following wave functions satisfies the wave equation, Eq. (15.12)? (a) $y(x, t) = A \cos(kx + \omega t)$; (b) $y(x, t) = A \sin(kx + \omega t)$; (c) $y(x, t) = A(\cos kx + \cos \omega t)$. (d) For the wave of part (b), write the equations for the transverse velocity and transverse acceleration of a particle at point x .

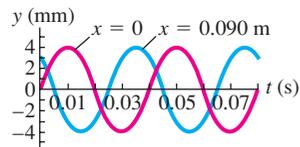
15.10 • A water wave traveling in a straight line on a lake is described by the equation

$$y(x, t) = (3.75 \text{ cm}) \cos(0.450 \text{ cm}^{-1} x + 5.40 \text{ s}^{-1} t)$$

where y is the displacement perpendicular to the undisturbed surface of the lake. (a) How much time does it take for one complete wave pattern to go past a fisherman in a boat at anchor, and what horizontal distance does the wave crest travel in that time? (b) What are the wave number and the number of waves per second that pass the fisherman? (c) How fast does a wave crest travel past the fisherman, and what is the maximum speed of his cork floater as the wave causes it to bob up and down?

15.11 • A sinusoidal wave is propagating along a stretched string that lies along the x -axis. The displacement of the string as a function of time is graphed in Fig. E15.11 for particles at $x = 0$ and at $x = 0.0900 \text{ m}$. (a) What is the amplitude of the wave? (b) What is the period of the wave? (c) You are told that the two points $x = 0$ and $x = 0.0900 \text{ m}$ are within one wavelength of each other. If the wave is moving in the $+x$ -direction, determine the wavelength and the wave speed. (d) If instead the wave is moving in the $-x$ -direction, determine the wavelength and the wave speed. (e) Would it be possible to determine definitively the wavelength in parts (c) and (d) if you were not told that the two points were within one wavelength of each other? Why or why not?

Figure E15.11



15.12 • **CALC** Speed of Propagation vs. Particle Speed. (a) Show that Eq. (15.3) may be written as

$$y(x, t) = A \cos\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

(b) Use $y(x, t)$ to find an expression for the transverse velocity v_y of a particle in the string on which the wave travels. (c) Find the maximum speed of a particle of the string. Under what circumstances is this equal to the propagation speed v ? Less than v ? Greater than v ?

15.13 •• A transverse wave on a string has amplitude 0.300 cm, wavelength 12.0 cm, and speed 6.00 cm/s. It is represented by $y(x, t)$ as given in Exercise 15.12. (a) At time $t = 0$, compute y at 1.5-cm intervals of x (that is, at $x = 0, x = 1.5 \text{ cm}, x = 3.0 \text{ cm}$, and so on) from $x = 0$ to $x = 12.0 \text{ cm}$. Graph the results. This is the shape of the string at time $t = 0$. (b) Repeat the calculations for the same values of x at times $t = 0.400 \text{ s}$ and $t = 0.800 \text{ s}$. Graph the shape of the string at these instants. In what direction is the wave traveling?

15.14 • A wave on a string is described by $y(x, t) = A \cos(kx - \omega t)$. (a) Graph y , v_y , and a_y as functions of x for time $t = 0$. (b) Consider the following points on the string: (i) $x = 0$; (ii) $x = \pi/4k$; (iii) $x = \pi/2k$; (iv) $x = 3\pi/4k$; (v) $x = \pi/k$; (vi) $x = 5\pi/4k$; (vii) $x = 3\pi/2k$; (viii) $x = 7\pi/4k$. For a particle at each of these points at $t = 0$, describe in words whether the particle is moving and in what direction, and whether the particle is speeding up, slowing down, or instantaneously not accelerating.

Section 15.4 Speed of a Transverse Wave

15.15 • One end of a horizontal rope is attached to a prong of an electrically driven tuning fork that vibrates the rope transversely at 120 Hz. The other end passes over a pulley and supports a 1.50-kg mass. The linear mass density of the rope is 0.0550 kg/m.

(a) What is the speed of a transverse wave on the rope? (b) What is the wavelength? (c) How would your answers to parts (a) and (b) change if the mass were increased to 3.00 kg?

15.16 • With what tension must a rope with length 2.50 m and mass 0.120 kg be stretched for transverse waves of frequency 40.0 Hz to have a wavelength of 0.750 m?

15.17 •• The upper end of a 3.80-m-long steel wire is fastened to the ceiling, and a 54.0-kg object is suspended from the lower end of the wire. You observe that it takes a transverse pulse 0.0492 s to travel from the bottom to the top of the wire. What is the mass of the wire?

15.18 •• A 1.50-m string of weight 0.0125 N is tied to the ceiling at its upper end, and the lower end supports a weight W . Neglect the very small variation in tension along the length of the string that is produced by the weight of the string. When you pluck the string slightly, the waves traveling up the string obey the equation

$$y(x, t) = (8.50 \text{ mm}) \cos(172 \text{ m}^{-1} x - 4830 \text{ s}^{-1} t)$$

Assume that the tension of the string is constant and equal to W . (a) How much time does it take a pulse to travel the full length of the string? (b) What is the weight W ? (c) How many wavelengths are on the string at any instant of time? (d) What is the equation for waves traveling *down* the string?

15.19 • A thin, 75.0-cm wire has a mass of 16.5 g. One end is tied to a nail, and the other end is attached to a screw that can be adjusted to vary the tension in the wire. (a) To what tension (in newtons) must you adjust the screw so that a transverse wave of wavelength 3.33 cm makes 875 vibrations per second? (b) How fast would this wave travel?

15.20 • **Weighty Rope.** If in Example 15.3 (Section 15.4) we do *not* neglect the weight of the rope, what is the wave speed (a) at the bottom of the rope; (b) at the middle of the rope; (c) at the top of the rope?

15.21 • A simple harmonic oscillator at the point $x = 0$ generates a wave on a rope. The oscillator operates at a frequency of 40.0 Hz and with an amplitude of 3.00 cm. The rope has a linear mass density of 50.0 g/m and is stretched with a tension of 5.00 N. (a) Determine the speed of the wave. (b) Find the wavelength. (c) Write the wave function $y(x, t)$ for the wave. Assume that the oscillator has its maximum upward displacement at time $t = 0$. (d) Find the maximum transverse acceleration of points on the rope. (e) In the discussion of transverse waves in this chapter, the force of gravity was ignored. Is that a reasonable approximation for this wave? Explain.

Section 15.5 Energy in Wave Motion

15.22 •• A piano wire with mass 3.00 g and length 80.0 cm is stretched with a tension of 25.0 N. A wave with frequency 120.0 Hz and amplitude 1.6 mm travels along the wire. (a) Calculate the average power carried by the wave. (b) What happens to the average power if the wave amplitude is halved?

15.23 • A horizontal wire is stretched with a tension of 94.0 N, and the speed of transverse waves for the wire is 492 m/s. What must the amplitude of a traveling wave of frequency 69.0 Hz be in order for the average power carried by the wave to be 0.365 W?

15.24 •• A light wire is tightly stretched with tension F . Transverse traveling waves of amplitude A and wavelength λ_1 carry average power $P_{\text{av},1} = 0.400 \text{ W}$. If the wavelength of the waves is doubled, so $\lambda_2 = 2\lambda_1$, while the tension F and amplitude A are not altered, what then is the average power $P_{\text{av},2}$ carried by the waves?

15.25 •• A jet plane at takeoff can produce sound of intensity 10.0 W/m² at 30.0 m away. But you prefer the tranquil sound of

normal conversation, which is $1.0 \mu\text{W}/\text{m}^2$. Assume that the plane behaves like a point source of sound. (a) What is the closest distance you should live from the airport runway to preserve your peace of mind? (b) What intensity from the jet does your friend experience if she lives twice as far from the runway as you do? (c) What power of sound does the jet produce at takeoff?

15.26 • Threshold of Pain. You are investigating the report of a UFO landing in an isolated portion of New Mexico, and you encounter a strange object that is radiating sound waves uniformly in all directions. Assume that the sound comes from a point source and that you can ignore reflections. You are slowly walking toward the source. When you are 7.5 m from it, you measure its intensity to be $0.11 \text{ W}/\text{m}^2$. An intensity of $1.0 \text{ W}/\text{m}^2$ is often used as the “threshold of pain.” How much closer to the source can you move before the sound intensity reaches this threshold?

15.27 • Energy Output. By measurement you determine that sound waves are spreading out equally in all directions from a point source and that the intensity is $0.026 \text{ W}/\text{m}^2$ at a distance of 4.3 m from the source. (a) What is the intensity at a distance of 3.1 m from the source? (b) How much sound energy does the source emit in one hour if its power output remains constant?

15.28 • A fellow student with a mathematical bent tells you that the wave function of a traveling wave on a thin rope is $y(x, t) = 2.30 \text{ mm} \cos[(6.98 \text{ rad}/\text{m})x + (742 \text{ rad}/\text{s})t]$. Being more practical, you measure the rope to have a length of 1.35 m and a mass of 0.00338 kg. You are then asked to determine the following: (a) amplitude; (b) frequency; (c) wavelength; (d) wave speed; (e) direction the wave is traveling; (f) tension in the rope; (g) average power transmitted by the wave.

15.29 • At a distance of $7.00 \times 10^{12} \text{ m}$ from a star, the intensity of the radiation from the star is $15.4 \text{ W}/\text{m}^2$. Assuming that the star radiates uniformly in all directions, what is the total power output of the star?

Section 15.6 Wave Interference, Boundary Conditions, and Superposition

15.30 • Reflection. A wave pulse on a string has the dimensions shown in Fig. E15.30 at $t = 0$. The wave speed is 40 cm/s. (a) If point O is a fixed end, draw the total wave on the string at $t = 15 \text{ ms}$, 20 ms, 25 ms, 30 ms, 35 ms, 40 ms, and 45 ms. (b) Repeat part (a) for the case in which point O is a free end.

Figure E15.30

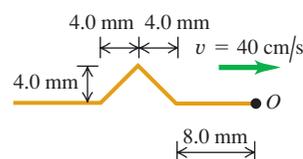
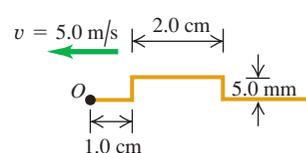


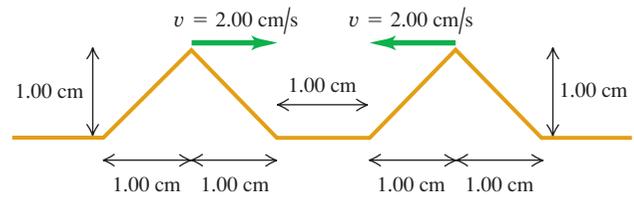
Figure E15.31



15.31 • Reflection. A wave pulse on a string has the dimensions shown in Fig. E15.31 at $t = 0$. The wave speed is 5.0 m/s. (a) If point O is a fixed end, draw the total wave on the string at $t = 1.0 \text{ ms}$, 2.0 ms, 3.0 ms, 4.0 ms, 5.0 ms, 6.0 ms, and 7.0 ms. (b) Repeat part (a) for the case in which point O is a free end.

15.32 • Interference of Triangular Pulses. Two triangular wave pulses are traveling toward each other on a stretched string as shown in Fig. E15.32. Each pulse is identical to the other and travels at 2.00 cm/s. The leading edges of the pulses are 1.00 cm apart at $t = 0$. Sketch the shape of the string at $t = 0.250 \text{ s}$, $t = 0.500 \text{ s}$, $t = 0.750 \text{ s}$, $t = 1.000 \text{ s}$, and $t = 1.250 \text{ s}$.

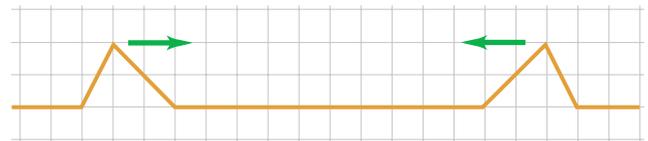
Figure E15.32



15.33 • Suppose that the left-traveling pulse in Exercise 15.32 is *below* the level of the unstretched string instead of above it. Make the same sketches that you did in that exercise.

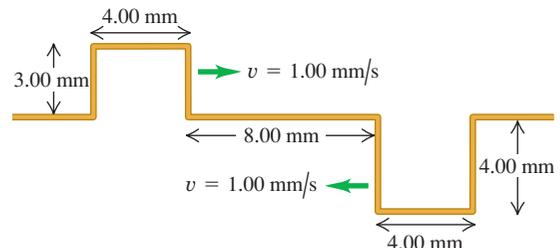
15.34 • Two pulses are moving in opposite directions at 1.0 cm/s on a taut string, as shown in Fig. E15.34. Each square is 1.0 cm. Sketch the shape of the string at the end of (a) 6.0 s; (b) 7.0 s; (c) 8.0 s.

Figure E15.34



15.35 • Interference of Rectangular Pulses. Figure E15.35 shows two rectangular wave pulses on a stretched string traveling toward each other. Each pulse is traveling with a speed of 1.00 mm/s and has the height and width shown in the figure. If the leading edges of the pulses are 8.00 mm apart at $t = 0$, sketch the shape of the string at $t = 4.00 \text{ s}$, $t = 6.00 \text{ s}$, and $t = 10.0 \text{ s}$.

Figure E15.35



Section 15.7 Standing Waves on a String

Section 15.8 Normal Modes of a String

15.36 • CALC Adjacent antinodes of a standing wave on a string are 15.0 cm apart. A particle at an antinode oscillates in simple harmonic motion with amplitude 0.850 cm and period 0.0750 s. The string lies along the $+x$ -axis and is fixed at $x = 0$. (a) How far apart are the adjacent nodes? (b) What are the wavelength, amplitude, and speed of the two traveling waves that form this pattern? (c) Find the maximum and minimum transverse speeds of a point at an antinode. (d) What is the shortest distance along the string between a node and an antinode?

15.37 • Standing waves on a wire are described by Eq. (15.28), with $A_{\text{SW}} = 2.50 \text{ mm}$, $\omega = 942 \text{ rad}/\text{s}$, and $k = 0.750\pi \text{ rad}/\text{m}$. The left end of the wire is at $x = 0$. At what distances from the left end are (a) the nodes of the standing wave and (b) the antinodes of the standing wave?

15.38 • CALC Wave Equation and Standing Waves. (a) Prove by direct substitution that $y(x, t) = (A_{\text{SW}} \sin kx) \sin \omega t$ is a solution of

the wave equation, Eq. (15.12), for $v = \omega/k$. (b) Explain why the relationship $v = \omega/k$ for *traveling* waves also applies to *standing* waves.

15.39 • CALC Let $y_1(x, t) = A \cos(k_1x - \omega_1t)$ and $y_2(x, t) = A \cos(k_2x - \omega_2t)$ be two solutions to the wave equation, Eq. (15.12), for the same v . Show that $y(x, t) = y_1(x, t) + y_2(x, t)$ is also a solution to the wave equation.

15.40 • A 1.50-m-long rope is stretched between two supports with a tension that makes the speed of transverse waves 48.0 m/s. What are the wavelength and frequency of (a) the fundamental; (b) the second overtone; (c) the fourth harmonic?

15.41 • A wire with mass 40.0 g is stretched so that its ends are tied down at points 80.0 cm apart. The wire vibrates in its fundamental mode with frequency 60.0 Hz and with an amplitude at the antinodes of 0.300 cm. (a) What is the speed of propagation of transverse waves in the wire? (b) Compute the tension in the wire. (c) Find the maximum transverse velocity and acceleration of particles in the wire.

15.42 • A piano tuner stretches a steel piano wire with a tension of 800 N. The steel wire is 0.400 m long and has a mass of 3.00 g. (a) What is the frequency of its fundamental mode of vibration? (b) What is the number of the highest harmonic that could be heard by a person who is capable of hearing frequencies up to 10,000 Hz?

15.43 • CALC A thin, taut string tied at both ends and oscillating in its third harmonic has its shape described by the equation $y(x, t) = (5.60 \text{ cm}) \sin[(0.0340 \text{ rad/cm})x] \sin[(50.0 \text{ rad/s})t]$, where the origin is at the left end of the string, the x -axis is along the string, and the y -axis is perpendicular to the string. (a) Draw a sketch that shows the standing-wave pattern. (b) Find the amplitude of the two traveling waves that make up this standing wave. (c) What is the length of the string? (d) Find the wavelength, frequency, period, and speed of the traveling waves. (e) Find the maximum transverse speed of a point on the string. (f) What would be the equation $y(x, t)$ for this string if it were vibrating in its eighth harmonic?

15.44 • The wave function of a standing wave is $y(x, t) = 4.44 \text{ mm} \sin[(32.5 \text{ rad/m})x] \sin[(754 \text{ rad/s})t]$. For the two traveling waves that make up this standing wave, find the (a) amplitude; (b) wavelength; (c) frequency; (d) wave speed; (e) wave functions. (f) From the information given, can you determine which harmonic this is? Explain.

15.45 •• Consider again the rope and traveling wave of Exercise 15.28. Assume that the ends of the rope are held fixed and that this traveling wave and the reflected wave are traveling in the opposite direction. (a) What is the wave function $y(x, t)$ for the standing wave that is produced? (b) In which harmonic is the standing wave oscillating? (c) What is the frequency of the fundamental oscillation?

15.46 •• One string of a certain musical instrument is 75.0 cm long and has a mass of 8.75 g. It is being played in a room where the speed of sound is 344 m/s. (a) To what tension must you adjust the string so that, when vibrating in its second overtone, it produces sound of wavelength 0.765 m? (Assume that the breaking stress of the wire is very large and isn't exceeded.) (b) What frequency sound does this string produce in its fundamental mode of vibration?

15.47 • The portion of the string of a certain musical instrument between the bridge and upper end of the finger board (that part of the string that is free to vibrate) is 60.0 cm long, and this length of the string has mass 2.00 g. The string sounds an A_4 note (440 Hz) when played. (a) Where must the player put a finger (what distance x from the bridge) to play a D_5 note (587 Hz)? (See Fig. E15.47.)

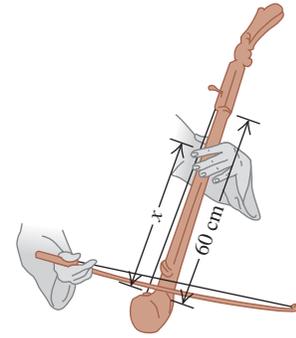
For both the A_4 and D_5 notes, the string vibrates in its fundamental mode. (b) Without retuning, is it possible to play a G_4 note (392 Hz) on this string? Why or why not?

15.48 •• (a) A horizontal string tied at both ends is vibrating in its fundamental mode. The traveling waves have speed v , frequency f , amplitude A , and wavelength λ . Calculate the maximum transverse velocity and maximum transverse acceleration of points located at (i) $x = \lambda/2$, (ii) $x = \lambda/4$, and (iii) $x = \lambda/8$ from the left-hand end of the string. (b) At each of the points in part (a), what is the amplitude of the motion? (c) At each of the points in part (a), how much time does it take the string to go from its largest upward displacement to its largest downward displacement?

15.49 • Guitar String. One of the 63.5-cm-long strings of an ordinary guitar is tuned to produce the note B_3 (frequency 245 Hz) when vibrating in its fundamental mode. (a) Find the speed of transverse waves on this string. (b) If the tension in this string is increased by 1.0%, what will be the new fundamental frequency of the string? (c) If the speed of sound in the surrounding air is 344 m/s, find the frequency and wavelength of the sound wave produced in the air by the vibration of the B_3 string. How do these compare to the frequency and wavelength of the standing wave on the string?

15.50 • Waves on a Stick. A flexible stick 2.0 m long is not fixed in any way and is free to vibrate. Make clear drawings of this stick vibrating in its first three harmonics, and then use your drawings to find the wavelengths of each of these harmonics. (*Hint:* Should the ends be nodes or antinodes?)

Figure E15.47



PROBLEMS

15.51 • CALC A transverse sine wave with an amplitude of 2.50 mm and a wavelength of 1.80 m travels from left to right along a long, horizontal, stretched string with a speed of 36.0 m/s. Take the origin at the left end of the undisturbed string. At time $t = 0$ the left end of the string has its maximum upward displacement. (a) What are the frequency, angular frequency, and wave number of the wave? (b) What is the function $y(x, t)$ that describes the wave? (c) What is $y(t)$ for a particle at the left end of the string? (d) What is $y(t)$ for a particle 1.35 m to the right of the origin? (e) What is the maximum magnitude of transverse velocity of any particle of the string? (f) Find the transverse displacement and the transverse velocity of a particle 1.35 m to the right of the origin at time $t = 0.0625$ s.

15.52 • A transverse wave on a rope is given by

$$y(x, t) = (0.750 \text{ cm}) \cos \pi[(0.400 \text{ cm}^{-1})x + (250 \text{ s}^{-1})t]$$

(a) Find the amplitude, period, frequency, wavelength, and speed of propagation. (b) Sketch the shape of the rope at these values of t : 0, 0.0005 s, 0.0010 s. (c) Is the wave traveling in the $+x$ - or $-x$ -direction? (d) The mass per unit length of the rope is 0.0500 kg/m. Find the tension. (e) Find the average power of this wave.

15.53 •• Three pieces of string, each of length L , are joined together end to end, to make a combined string of length $3L$. The first piece of string has mass per unit length μ_1 , the second piece

has mass per unit length $\mu_2 = 4\mu_1$, and the third piece has mass per unit length $\mu_3 = \mu_1/4$. (a) If the combined string is under tension F , how much time does it take a transverse wave to travel the entire length $3L$? Give your answer in terms of L , F , and μ_1 . (b) Does your answer to part (a) depend on the order in which the three pieces are joined together? Explain.

15.54 •• CP A 1750-N irregular beam is hanging horizontally by its ends from the ceiling by two vertical wires (A and B), each 1.25 m long and weighing 0.360 N. The center of gravity of this beam is one-third of the way along the beam from the end where wire A is attached. If you pluck both strings at the same time at the beam, what is the time delay between the arrival of the two pulses at the ceiling? Which pulse arrives first? (Neglect the effect of the weight of the wires on the tension in the wires.)

15.55 • CALC Ant Joy Ride. You place your pet ant Klyde (mass m) on top of a horizontal, stretched rope, where he holds on tightly. The rope has mass M and length L and is under tension F . You start a sinusoidal transverse wave of wavelength λ and amplitude A propagating along the rope. The motion of the rope is in a vertical plane. Klyde's mass is so small that his presence has no effect on the propagation of the wave. (a) What is Klyde's top speed as he oscillates up and down? (b) Klyde enjoys the ride and begs for more. You decide to double his top speed by changing the tension while keeping the wavelength and amplitude the same. Should the tension be increased or decreased, and by what factor?

15.56 •• Weightless Ant. An ant with mass m is standing peacefully on top of a horizontal, stretched rope. The rope has mass per unit length μ and is under tension F . Without warning, Cousin Throckmorton starts a sinusoidal transverse wave of wavelength λ propagating along the rope. The motion of the rope is in a vertical plane. What minimum wave amplitude will make the ant become momentarily weightless? Assume that m is so small that the presence of the ant has no effect on the propagation of the wave.

15.57 • CP When a transverse sinusoidal wave is present on a string, the particles of the string undergo SHM. This is the same motion as that of a mass m attached to an ideal spring of force constant k' , for which the angular frequency of oscillation was found in Chapter 14 to be $\omega = \sqrt{k'/m}$. Consider a string with tension F and mass per unit length μ , along which is propagating a sinusoidal wave with amplitude A and wavelength λ . (a) Find the "force constant" k' of the restoring force that acts on a short segment of the string of length Δx (where $\Delta x \ll \lambda$). (b) How does the "force constant" calculated in part (b) depend on F , μ , A , and λ ? Explain the physical reasons this should be so.

15.58 •• Music. You are designing a two-string instrument with metal strings 35.0 cm long, as shown in Fig. P15.58. Both strings are under the same tension. String S_1 has a mass of 8.00 g and produces the note middle C (frequency 262 Hz) in its fundamental mode. (a) What should be the tension in the string? (b) What should be the mass of string S_2 so that it will produce A-sharp (frequency 466 Hz) as its fundamental? (c) To extend the range of your instrument, you include a fret located just under the strings but not normally touching them.

How far from the upper end should you put this fret so that when you press S_1 tightly against it, this string will produce C-sharp (frequency 277 Hz) in its fundamental? That is, what is x in the figure?

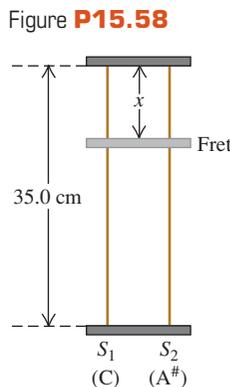


Figure P15.58

(d) If you press S_2 against the fret, what frequency of sound will it produce in its fundamental?

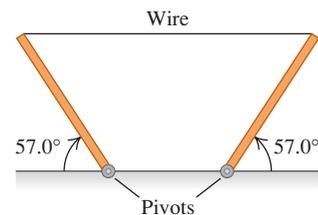
15.59 ••• CP The lower end of a uniform bar of mass 45.0 kg is attached to a wall by a frictionless hinge. The bar is held by a horizontal wire attached at its upper end so that the bar makes an angle of 30.0° with the wall. The wire has length 0.330 m and mass 0.0920 kg. What is the frequency of the fundamental standing wave for transverse waves on the wire?

15.60 ••• CP You are exploring a newly discovered planet. The radius of the planet is 7.20×10^7 m. You suspend a lead weight from the lower end of a light string that is 4.00 m long and has mass 0.0280 kg. You measure that it takes 0.0600 s for a transverse pulse to travel from the lower end to the upper end of the string. On earth, for the same string and lead weight, it takes 0.0390 s for a transverse pulse to travel the length of the string. The weight of the string is small enough that its effect on the tension in the string can be neglected. Assuming that the mass of the planet is distributed with spherical symmetry, what is its mass?

15.61 •• For a string stretched between two supports, two successive standing-wave frequencies are 525 Hz and 630 Hz. There are other standing-wave frequencies lower than 525 Hz and higher than 630 Hz. If the speed of transverse waves on the string is 384 m/s, what is the length of the string? Assume that the mass of the wire is small enough for its effect on the tension in the wire to be neglected.

15.62 ••• CP A 5.00-m, 0.732-kg wire is used to support two uniform 235-N posts of equal length (Fig. P15.62). Assume that the wire is essentially horizontal and that the speed of sound is 344 m/s. A strong wind is blowing, causing the wire to vibrate in its 5th overtone. What are the frequency and wavelength of the sound this wire produces?

Figure P15.62



15.63 ••• CP A 1.80-m-long uniform bar that weighs 536 N is suspended in a horizontal position by two vertical wires that are attached to the ceiling. One wire is aluminum and the other is copper. The aluminum wire is attached to the left-hand end of the bar, and the copper wire is attached 0.40 m to the left of the right-hand end. Each wire has length 0.600 m and a circular cross section with radius 0.280 mm. What is the fundamental frequency of transverse standing waves for each wire?

15.64 •• A continuous succession of sinusoidal wave pulses are produced at one end of a very long string and travel along the length of the string. The wave has frequency 70.0 Hz, amplitude 5.00 mm, and wavelength 0.600 m. (a) How long does it take the wave to travel a distance of 8.00 m along the length of the string? (b) How long does it take a point on the string to travel a distance of 8.00 m, once the wave train has reached the point and set it into motion? (c) In parts (a) and (b), how does the time change if the amplitude is doubled?

15.65 ••• CALC Waves of Arbitrary Shape. (a) Explain why any wave described by a function of the form $y(x, t) = f(x - vt)$ moves in the $+x$ -direction with speed v . (b) Show that $y(x, t) = f(x - vt)$ satisfies the wave equation, no matter what the functional form of f . To do this, write $y(x, t) = f(u)$, where

$u = x - vt$. Then, to take partial derivatives of $y(x, t)$, use the chain rule:

$$\frac{\partial y(x, t)}{\partial t} = \frac{df(u)}{du} \frac{\partial u}{\partial t} = \frac{df(u)}{du} (-v)$$

$$\frac{\partial y(x, t)}{\partial x} = \frac{df(u)}{du} \frac{\partial u}{\partial x} = \frac{df(u)}{du}$$

(c) A wave pulse is described by the function $y(x, t) = De^{-(Bx - Ct)^2}$, where B , C , and D are all positive constants. What is the speed of this wave?

15.66 ••• CP A vertical, 1.20-m length of 18-gauge (diameter of 1.024 mm) copper wire has a 100.0-N ball hanging from it. (a) What is the wavelength of the third harmonic for this wire? (b) A 500.0-N ball now *replaces* the original ball. What is the change in the wavelength of the third harmonic caused by replacing the light ball with the heavy one? (*Hint*: See Table 11.1 for Young's modulus.)

15.67 • (a) Show that Eq. (15.25) can also be written as $P_{av} = \frac{1}{2} Fk\omega A^2$, where k is the wave number of the wave. (b) If the tension F in the string is quadrupled while the amplitude A is kept the same, how must k and ω each change to keep the average power constant? [*Hint*: Recall Eq. (15.6).]

15.68 ••• CALC Equation (15.7) for a sinusoidal wave can be made more general by including a phase angle ϕ , where $0 \leq \phi \leq 2\pi$ (in radians). Then the wave function $y(x, t)$ becomes

$$y(x, t) = A \cos(kx - \omega t + \phi)$$

(a) Sketch the wave as a function of x at $t = 0$ for $\phi = 0$, $\phi = \pi/4$, $\phi = \pi/2$, $\phi = 3\pi/4$, and $\phi = 3\pi/2$. (b) Calculate the transverse velocity $v_y = \partial y / \partial t$. (c) At $t = 0$, a particle on the string at $x = 0$ has displacement $y = A/\sqrt{2}$. Is this enough information to determine the value of ϕ ? In addition, if you are told that a particle at $x = 0$ is moving toward $y = 0$ at $t = 0$, what is the value of ϕ ? (d) Explain in general what you must know about the wave's behavior at a given instant to determine the value of ϕ .

15.69 ••• A sinusoidal transverse wave travels on a string. The string has length 8.00 m and mass 6.00 g. The wave speed is 30.0 m/s, and the wavelength is 0.200 m. (a) If the wave is to have an average power of 50.0 W, what must be the amplitude of the wave? (b) For this same string, if the amplitude and wavelength are the same as in part (a), what is the average power for the wave if the tension is increased such that the wave speed is doubled?

15.70 ••• CALC Energy in a Triangular Pulse. A triangular wave pulse on a taut string travels in the positive x -direction with speed v . The tension in the string is F , and the linear mass density of the string is μ . At $t = 0$, the shape of the pulse is given by

$$y(x, 0) = \begin{cases} 0 & \text{if } x < -L \\ h(L+x)/L & \text{for } -L < x < 0 \\ h(L-x)/L & \text{for } 0 < x < L \\ 0 & \text{for } x > L \end{cases}$$

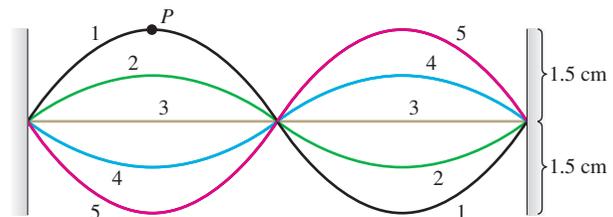
(a) Draw the pulse at $t = 0$. (b) Determine the wave function $y(x, t)$ at all times t . (c) Find the instantaneous power in the wave. Show that the power is zero except for $-L < (x - vt) < L$ and that in this interval the power is constant. Find the value of this constant power.

15.71 ••• CALC Instantaneous Power in a Wave. (a) Graph $y(x, t)$ as given by Eq. (15.7) as a function of x for a given time t (say, $t = 0$). On the same axes, make a graph of the instantaneous

power $P(x, t)$ as given by Eq. (15.23). (b) Explain the connection between the slope of the graph of $y(x, t)$ versus x and the value of $P(x, t)$. In particular, explain what is happening at points where $P = 0$, where there is no instantaneous energy transfer. (c) The quantity $P(x, t)$ always has the same sign. What does this imply about the direction of energy flow? (d) Consider a wave moving in the $-x$ -direction, for which $y(x, t) = A \cos(kx + \omega t)$. Calculate $P(x, t)$ for this wave, and make a graph of $y(x, t)$ and $P(x, t)$ as functions of x for a given time t (say, $t = 0$). What differences arise from reversing the direction of the wave?

15.72 •• A vibrating string 50.0 cm long is under a tension of 1.00 N. The results from five successive stroboscopic pictures are shown in Fig. P15.72. The strobe rate is set at 5000 flashes per minute, and observations reveal that the maximum displacement occurred at flashes 1 and 5 with no other maxima in between. (a) Find the period, frequency, and wavelength for the traveling waves on this string. (b) In what normal mode (harmonic) is the string vibrating? (c) What is the speed of the traveling waves on the string? (d) How fast is point P moving when the string is in (i) position 1 and (ii) position 3? (e) What is the mass of this string? (See Section 15.3.)

Figure P15.72



15.73 • Clothesline Nodes. Cousin Throckmorton is once again playing with the clothesline in Example 15.2 (Section 15.3). One end of the clothesline is attached to a vertical post. Throcky holds the other end loosely in his hand, so that the speed of waves on the clothesline is a relatively slow 0.720 m/s. He finds several frequencies at which he can oscillate his end of the clothesline so that a light clothespin 45.0 cm from the post doesn't move. What are these frequencies?

15.74 ••• CALC A guitar string is vibrating in its fundamental mode, with nodes at each end. The length of the segment of the string that is free to vibrate is 0.386 m. The maximum transverse acceleration of a point at the middle of the segment is 8.40×10^3 m/s² and the maximum transverse velocity is 3.80 m/s. (a) What is the amplitude of this standing wave? (b) What is the wave speed for the transverse traveling waves on this string?

15.75 ••• CALC A string that lies along the $+x$ -axis has a free end at $x = 0$. (a) By using steps similar to those used to derive Eq. (15.28), show that an incident traveling wave $y_1(x, t) = A \cos(kx + \omega t)$ gives rise to a standing wave $y(x, t) = 2A \cos \omega t \cos kx$. (b) Show that the standing wave has an antinode at its free end ($x = 0$). (c) Find the maximum displacement, maximum speed, and maximum acceleration of the free end of the string.

15.76 •• A string with both ends held fixed is vibrating in its third harmonic. The waves have a speed of 192 m/s and a frequency of 240 Hz. The amplitude of the standing wave at an antinode is 0.400 cm. (a) Calculate the amplitude at points on the string a distance of (i) 40.0 cm; (ii) 20.0 cm; and (iii) 10.0 cm from the left end of the string. (b) At each point in part (a), how much time does it take the string to go from its largest upward displacement to its largest downward displacement? (c) Calculate the maximum

transverse velocity and the maximum transverse acceleration of the string at each of the points in part (a).

15.77 ••• A uniform cylindrical steel wire, 55.0 cm long and 1.14 mm in diameter, is fixed at both ends. To what tension must it be adjusted so that, when vibrating in its first overtone, it produces the note D-sharp of frequency 311 Hz? Assume that it stretches an insignificant amount. (*Hint:* See Table 12.1.)

15.78 • **Holding Up Under Stress.** A string or rope will break apart if it is placed under too much tensile stress [Eq. (11.8)]. Thicker ropes can withstand more tension without breaking because the thicker the rope, the greater the cross-sectional area and the smaller the stress. One type of steel has density 7800 kg/m^3 and will break if the tensile stress exceeds $7.0 \times 10^8 \text{ N/m}^2$. You want to make a guitar string from 4.0 g of this type of steel. In use, the guitar string must be able to withstand a tension of 900 N without breaking. Your job is the following: (a) Determine the maximum length and minimum radius the string can have. (b) Determine the highest possible fundamental frequency of standing waves on this string, if the entire length of the string is free to vibrate.

15.79 ••• **Combining Standing Waves.** A guitar string of length L is plucked in such a way that the total wave produced is the sum of the fundamental and the second harmonic. That is, the standing wave is given by

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

where

$$y_1(x, t) = C \sin \omega_1 t \sin k_1 x$$

$$y_2(x, t) = C \sin \omega_2 t \sin k_2 x$$

with $\omega_1 = vk_1$ and $\omega_2 = vk_2$. (a) At what values of x are the nodes of y_1 ? (b) At what values of x are the nodes of y_2 ? (c) Graph the total wave at $t = 0$, $t = \frac{1}{8}f_1$, $t = \frac{1}{4}f_1$, $t = \frac{3}{8}f_1$, and $t = \frac{1}{2}f_1$. (d) Does the sum of the two standing waves y_1 and y_2 produce a standing wave? Explain.

15.80 •• **CP** When a massive aluminum sculpture is hung from a steel wire, the fundamental frequency for transverse standing waves on the wire is 250.0 Hz. The sculpture (but not the wire) is then completely submerged in water. (a) What is the new fundamental frequency? (*Hint:* See Table 12.1.) (b) Why is it a good approximation to treat the wire as being fixed at both ends?

15.81 ••• **CP** A large rock that weighs 164.0 N is suspended from the lower end of a thin wire that is 3.00 m long. The density of the rock is 3200 kg/m^3 . The mass of the wire is small enough that its effect on the tension in the wire can be neglected. The upper end of the wire is held fixed. When the rock is in air, the fundamental frequency for transverse standing waves on the wire is 42.0 Hz. When the rock is totally submerged in a liquid, with the top of the rock just below the surface, the fundamental frequency for the wire is 28.0 Hz. What is the density of the liquid?

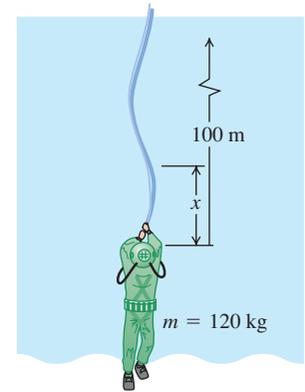
15.82 •• **Tuning an Instrument.** A musician tunes the C-string of her instrument to a fundamental frequency of 65.4 Hz. The vibrating portion of the string is 0.600 m long and has a mass of 14.4 g. (a) With what tension must the musician stretch it? (b) What percent increase in tension is needed to increase the frequency from 65.4 Hz to 73.4 Hz, corresponding to a rise in pitch from C to D?

15.83 ••• One type of steel has a density of $7.8 \times 10^3 \text{ kg/m}^3$ and a breaking stress of $7.0 \times 10^8 \text{ N/m}^2$. A cylindrical guitar string is to be made of 4.00 g of this steel. (a) What are the length and radius of the longest and thinnest string that can be placed under a tension of 900 N without breaking? (b) What is the highest fundamental frequency that this string could have?

CHALLENGE PROBLEMS

15.84 ••• **CP CALC** A deep-sea diver is suspended beneath the surface of Loch Ness by a 100-m-long cable that is attached to a boat on the surface (Fig. P15.84). The diver and his suit have a total mass of 120 kg and a volume of 0.0800 m^3 . The cable has a diameter of 2.00 cm and a linear mass density of $\mu = 1.10 \text{ kg/m}$. The diver thinks he sees something moving in the murky depths and jerks the end of the cable back and forth to send transverse waves up the cable as a signal to his companions in the boat. (a) What is the tension in the cable at its lower end, where it is attached to the diver? Do not forget to include the buoyant force that the water (density 1000 kg/m^3) exerts on him. (b) Calculate the tension in the cable a distance x above the diver. The buoyant force on the cable must be included in your calculation. (c) The speed of transverse waves on the cable is given by $v = \sqrt{F/\mu}$ (Eq. 15.13). The speed therefore varies along the cable, since the tension is not constant. (This expression neglects the damping force that the water exerts on the moving cable.) Integrate to find the time required for the first signal to reach the surface.

Figure P15.84



15.85 ••• **CALC** (a) Show that for a wave on a string, the kinetic energy per unit length of string is

$$u_k(x, t) = \frac{1}{2} \mu v_y^2(x, t) = \frac{1}{2} \mu \left(\frac{\partial y(x, t)}{\partial t} \right)^2$$

where μ is the mass per unit length. (b) Calculate $u_k(x, t)$ for a sinusoidal wave given by Eq. (15.7). (c) There is also elastic potential energy in the string, associated with the work required to deform and stretch the string. Consider a short segment of string at position x that has unstretched length Δx , as in Fig. 15.13. Ignoring the (small) curvature of the segment, its slope is $\partial y(x, t)/\partial x$. Assume that the displacement of the string from equilibrium is small, so that $\partial y/\partial x$ has a magnitude much less than unity. Show that the stretched length of the segment is approximately

$$\Delta x \left[1 + \frac{1}{2} \left(\frac{\partial y(x, t)}{\partial x} \right)^2 \right]$$

(*Hint:* Use the relationship $\sqrt{1+u} \approx 1 + \frac{1}{2}u$, valid for $|u| \ll 1$.) (d) The potential energy stored in the segment equals the work done by the string tension F (which acts along the string) to stretch the segment from its unstretched length Δx to the length calculated in part (c). Calculate this work and show that the potential energy per unit length of string is

$$u_p(x, t) = \frac{1}{2} F \left(\frac{\partial y(x, t)}{\partial x} \right)^2$$

(e) Calculate $u_p(x, t)$ for a sinusoidal wave given by Eq. (15.7). (f) Show that $u_k(x, t) = u_p(x, t)$, for all x and t . (g) Show $y(x, t)$, $u_k(x, t)$, and $u_p(x, t)$ as functions of x for $t = 0$ in one graph with all three functions on the same axes. Explain why u_k and u_p are maximum where y is zero, and vice versa. (h) Show that the instantaneous power in the wave, given by Eq. (15.22), is equal to the total energy per unit length multiplied by the wave speed v . Explain why this result is reasonable.

Answers

Chapter Opening Question ?

The power of a mechanical wave depends on its frequency and amplitude [see Eq. (15.25)].

Test Your Understanding Questions

15.1 Answer: (i) The “wave” travels horizontally from one spectator to the next along each row of the stadium, but the displacement of each spectator is vertically upward. Since the displacement is perpendicular to the direction in which the wave travels, the wave is transverse.

15.2 Answer: (iv) The speed of waves on a string, v , does not depend on the wavelength. We can rewrite the relationship $v = \lambda f$ as $f = v/\lambda$, which tells us that if the wavelength λ doubles, the frequency f becomes one-half as great.

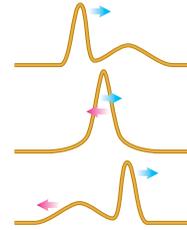
15.3 Answers: (a) $\frac{2}{8}T$, (b) $\frac{4}{8}T$, (c) $\frac{5}{8}T$ Since the wave is sinusoidal, each point on the string oscillates in simple harmonic motion (SHM). Hence we can apply all of the ideas from Chapter 14 about SHM to the wave depicted in Fig. 15.8. (a) A particle in SHM has its maximum speed when it is passing through the equilibrium position ($y = 0$ in Fig. 15.8). The particle at point A is moving upward through this position at $t = \frac{2}{8}T$. (b) In vertical SHM the greatest *upward* acceleration occurs when a particle is at its maximum *downward* displacement. This occurs for the particle at point B at $t = \frac{4}{8}T$. (c) A particle in vertical SHM has a *downward* acceleration when its displacement is *upward*. The particle at C has an upward displacement and is moving *downward* at $t = \frac{5}{8}T$.

15.4 Answer: (ii) The relationship $v = \sqrt{F/\mu}$ [Eq. (15.13)] says that the wave speed is greatest on the string with the smallest linear mass density. This is the thinnest string, which has the smallest amount of mass m and hence the smallest linear mass density $\mu = m/L$ (all strings are the same length).

15.5 Answer: (iii), (iv), (ii), (i) Equation (15.25) says that the average power in a sinusoidal wave on a string is $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$. All four strings are identical, so all have the

same mass, the same length, and the same linear mass density μ . The frequency f is the same for each wave, as is the angular frequency $\omega = 2\pi f$. Hence the average wave power for each string is proportional to the square root of the string tension F and the square of the amplitude A . Compared to string (i), the average power in each string is (ii) $\sqrt{4} = 2$ times greater; (iii) $4^2 = 16$ times greater; and (iv) $\sqrt{2}(2)^2 = 4\sqrt{2}$ times greater.

15.6 Answer:



15.7 Answers: yes, yes Doubling the frequency makes the wavelength half as large. Hence the spacing between nodes (equal to $\lambda/2$) is also half as large. There are nodes at all of the previous positions, but there is also a new node between every pair of old nodes.

15.8 Answers: $n = 1, 3, 5, \dots$ When you touch the string at its center, you are demanding that there be a node at the center. Hence only standing waves with a node at $x = L/2$ are allowed. From Figure 15.26 you can see that the normal modes $n = 1, 3, 5, \dots$ cannot be present.

Bridging Problem

Answers: (a) $F(r) = \frac{m\omega^2}{2L}(L^2 - r^2)$

(b) $v(r) = \omega\sqrt{\frac{L^2 - r^2}{2}}$

(c) $\frac{\pi}{\omega\sqrt{2}}$

SOUND AND HEARING

16



? Most people like to listen to music, but hardly anyone likes to listen to noise. What is the physical difference between musical sound and noise?

Of all the mechanical waves that occur in nature, the most important in our everyday lives are longitudinal waves in a medium—usually air—called *sound waves*. The reason is that the human ear is tremendously sensitive and can detect sound waves even of very low intensity. Besides their use in spoken communication, our ears allow us to pick up a myriad of cues about our environment, from the welcome sound of a meal being prepared to the warning sound of an approaching car. The ability to hear an unseen nocturnal predator was essential to the survival of our ancestors, so it is no exaggeration to say that we humans owe our existence to our highly evolved sense of hearing.

Up to this point we have described mechanical waves primarily in terms of displacement; however, a description of sound waves in terms of *pressure* fluctuations is often more appropriate, largely because the ear is primarily sensitive to changes in pressure. We'll study the relationships among displacement, pressure fluctuation, and intensity and the connections between these quantities and human sound perception.

When a source of sound or a listener moves through the air, the listener may hear a frequency different from the one emitted by the source. This is the Doppler effect, which has important applications in medicine and technology.

16.1 Sound Waves

The most general definition of **sound** is a longitudinal wave in a medium. Our main concern in this chapter is with sound waves in air, but sound can travel through any gas, liquid, or solid. You may be all too familiar with the propagation of sound through a solid if your neighbor's stereo speakers are right next to your wall.

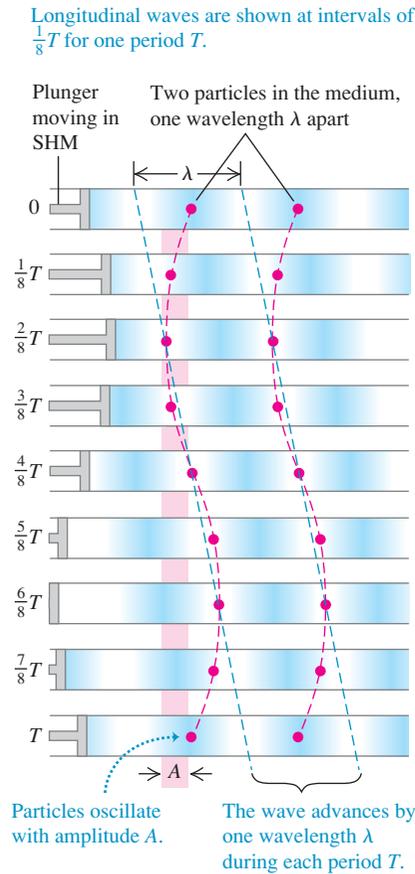
The simplest sound waves are sinusoidal waves, which have definite frequency, amplitude, and wavelength. The human ear is sensitive to waves in the frequency range from about 20 to 20,000 Hz, called the **audible range**, but we also use the

LEARNING GOALS

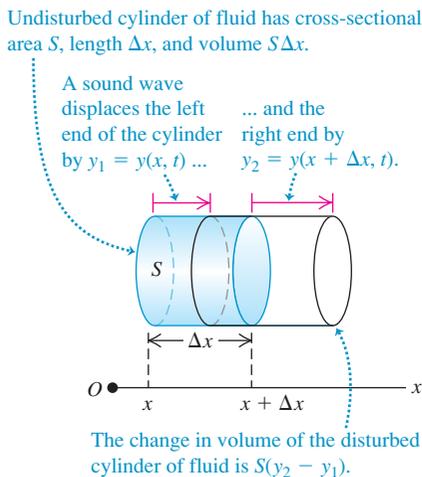
By studying this chapter, you will learn:

- How to describe a sound wave in terms of either particle displacements or pressure fluctuations.
- How to calculate the speed of sound waves in different materials.
- How to calculate the intensity of a sound wave.
- What determines the particular frequencies of sound produced by an organ or a flute.
- How resonance occurs in musical instruments.
- What happens when sound waves from different sources overlap.
- How to describe what happens when two sound waves of slightly different frequencies are combined.
- Why the pitch of a siren changes as it moves past you.

16.1 A sinusoidal longitudinal wave traveling to the right in a fluid. (Compare to Fig. 15.7.)



16.2 As a sound wave propagates along the x -axis, the left and right ends undergo different displacements y_1 and y_2 .



term “sound” for similar waves with frequencies above (**ultrasonic**) and below (**infrasonic**) the range of human hearing.

Sound waves usually travel out in all directions from the source of sound, with an amplitude that depends on the direction and distance from the source. We’ll return to this point in the next section. For now, we concentrate on the idealized case of a sound wave that propagates in the positive x -direction only. As we discussed in Section 15.3, such a wave is described by a wave function $y(x, t)$, which gives the instantaneous displacement y of a particle in the medium at position x at time t . If the wave is sinusoidal, we can express it using Eq. (15.7):

$$y(x, t) = A \cos(kx - \omega t) \quad \text{(sound wave propagating in the } +x\text{-direction)} \quad (16.1)$$

Remember that in a longitudinal wave the displacements are *parallel* to the direction of travel of the wave, so distances x and y are measured parallel to each other, not perpendicular as in a transverse wave. The amplitude A is the maximum displacement of a particle in the medium from its equilibrium position (Fig. 16.1). Hence A is also called the **displacement amplitude**.

Sound Waves As Pressure Fluctuations

Sound waves may also be described in terms of variations of *pressure* at various points. In a sinusoidal sound wave in air, the pressure fluctuates above and below atmospheric pressure p_a in a sinusoidal variation with the same frequency as the motions of the air particles. The human ear operates by sensing such pressure variations. A sound wave entering the ear canal exerts a fluctuating pressure on one side of the eardrum; the air on the other side of the eardrum, vented to the outside by the Eustachian tube, is at atmospheric pressure. The pressure difference on the two sides of the eardrum sets it into motion. Microphones and similar devices also usually sense pressure differences, not displacements, so it is very useful to develop a relationship between these two descriptions.

Let $p(x, t)$ be the instantaneous pressure fluctuation in a sound wave at any point x at time t . That is, $p(x, t)$ is the amount by which the pressure *differs* from normal atmospheric pressure p_a . Think of $p(x, t)$ as the *gauge pressure* defined in Section 12.2; it can be either positive or negative. The *absolute* pressure at a point is then $p_a + p(x, t)$.

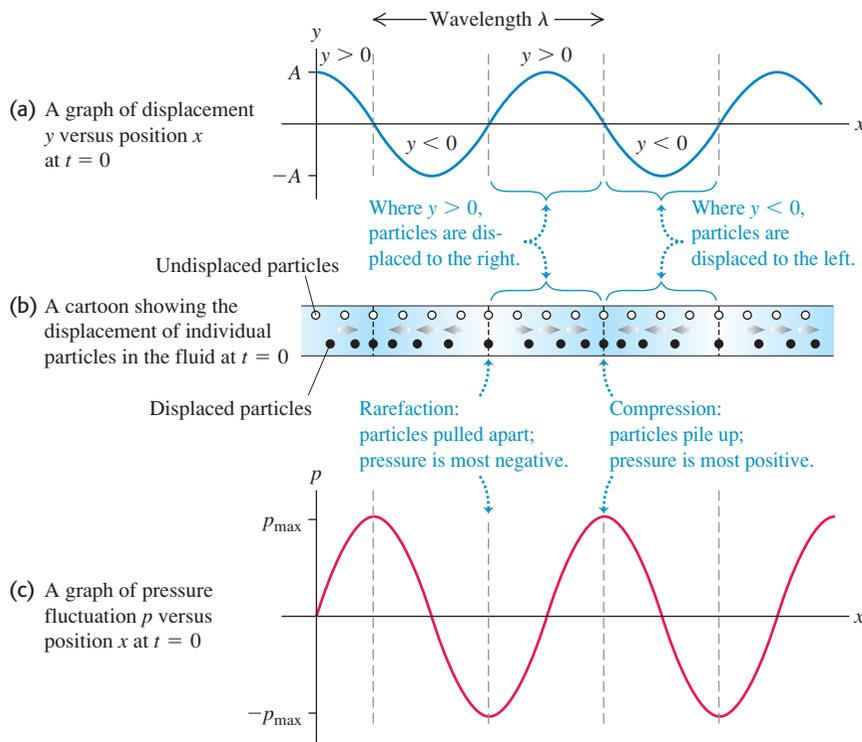
To see the connection between the pressure fluctuation $p(x, t)$ and the displacement $y(x, t)$ in a sound wave propagating in the $+x$ -direction, consider an imaginary cylinder of a wave medium (gas, liquid, or solid) with cross-sectional area S and axis along the direction of propagation (Fig. 16.2). When no sound wave is present, the cylinder has length Δx and volume $V = S\Delta x$, as shown by the shaded volume in Fig. 16.2. When a wave is present, at time t the end of the cylinder that is initially at x is displaced by $y_1 = y(x, t)$, and the end that is initially at $x + \Delta x$ is displaced by $y_2 = y(x + \Delta x, t)$; this is shown by the red lines. If $y_2 > y_1$, as shown in Fig. 16.2, the cylinder’s volume has increased, which causes a decrease in pressure. If $y_2 < y_1$, the cylinder’s volume has decreased and the pressure has increased. If $y_2 = y_1$, the cylinder is simply shifted to the left or right; there is no volume change and no pressure fluctuation. The pressure fluctuation depends on the *difference* between the displacements at neighboring points in the medium.

Quantitatively, the change in volume ΔV of the cylinder is

$$\Delta V = S(y_2 - y_1) = S[y(x + \Delta x, t) - y(x, t)]$$

In the limit as $\Delta x \rightarrow 0$, the fractional change in volume dV/V (volume change divided by original volume) is

$$\frac{dV}{V} = \lim_{\Delta x \rightarrow 0} \frac{S[y(x + \Delta x, t) - y(x, t)]}{S\Delta x} = \frac{\partial y(x, t)}{\partial x} \quad (16.2)$$



16.3 Three ways to describe a sound wave.

The fractional volume change is related to the pressure fluctuation by the bulk modulus B , which by definition [Eq. (11.13)] is $B = -p(x, t)/(dV/V)$ (see Section 11.4). Solving for $p(x, t)$, we have

$$p(x, t) = -B \frac{\partial y(x, t)}{\partial x} \quad (16.3)$$

The negative sign arises because when $\partial y(x, t)/\partial x$ is positive, the displacement is greater at $x + \Delta x$ than at x , corresponding to an increase in volume and a *decrease* in pressure.

When we evaluate $\partial y(x, t)/\partial x$ for the sinusoidal wave of Eq. (16.1), we find

$$p(x, t) = BkA \sin(kx - \omega t) \quad (16.4)$$

Figure 16.3 shows $y(x, t)$ and $p(x, t)$ for a sinusoidal sound wave at $t = 0$. It also shows how individual particles of the wave are displaced at this time. While $y(x, t)$ and $p(x, t)$ describe the same wave, these two functions are one-quarter cycle out of phase: At any time, the displacement is greatest where the pressure fluctuation is zero, and vice versa. In particular, note that the compressions (points of greatest pressure and density) and rarefactions (points of lowest pressure and density) are points of *zero* displacement.

CAUTION **Graphs of a sound wave** Keep in mind that the graphs in Fig. 16.3 show the wave at only *one* instant of time. Because the wave is propagating in the $+x$ -direction, as time goes by the wave patterns in the functions $y(x, t)$ and $p(x, t)$ move to the right at the wave speed $v = \omega/k$. Hence the positions of the compressions and rarefactions also move to the right at this same speed. The particles, by contrast, simply oscillate back and forth in simple harmonic motion as shown in Fig. 16.1. **|**

Equation (16.4) shows that the quantity BkA represents the maximum pressure fluctuation. We call this the **pressure amplitude**, denoted by p_{\max} :

$$p_{\max} = BkA \quad (\text{sinusoidal sound wave}) \quad (16.5)$$

The pressure amplitude is directly proportional to the displacement amplitude A , as we might expect, and it also depends on wavelength. Waves of shorter wavelength λ (larger wave number $k = 2\pi/\lambda$) have greater pressure variations for a given amplitude because the maxima and minima are squeezed closer together. A medium with a large value of bulk modulus B requires a relatively large pressure amplitude for a given displacement amplitude because large B means a less compressible medium; that is, greater pressure change is required for a given volume change.

Example 16.1 Amplitude of a sound wave in air

In a sinusoidal sound wave of moderate loudness, the maximum pressure variations are about 3.0×10^{-2} Pa above and below atmospheric pressure. Find the corresponding maximum displacement if the frequency is 1000 Hz. In air at normal atmospheric pressure and density, the speed of sound is 344 m/s and the bulk modulus is 1.42×10^5 Pa.

SOLUTION

IDENTIFY and SET UP: This problem involves the relationship between two ways of describing a sound wave: in terms of displacement and in terms of pressure. The target variable is the displacement amplitude A . We are given the pressure amplitude p_{\max} , wave speed v , frequency f , and bulk modulus B . Equation (16.5) relates the target variable A to p_{\max} . We use $\omega = vk$ [Eq. (15.6)] to

determine the wave number k from v and the angular frequency $\omega = 2\pi f$.

EXECUTE: From Eq. (15.6),

$$k = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{(2\pi \text{ rad})(1000 \text{ Hz})}{344 \text{ m/s}} = 18.3 \text{ rad/m}$$

Then from Eq. (16.5), the maximum displacement is

$$A = \frac{p_{\max}}{Bk} = \frac{3.0 \times 10^{-2} \text{ Pa}}{(1.42 \times 10^5 \text{ Pa})(18.3 \text{ rad/m})} = 1.2 \times 10^{-8} \text{ m}$$

EVALUATE: This displacement amplitude is only about $\frac{1}{100}$ the size of a human cell. The ear actually senses pressure fluctuations; it detects these minuscule displacements only indirectly.

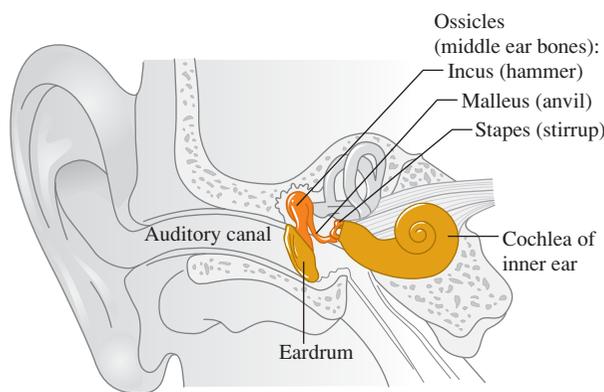
Example 16.2 Amplitude of a sound wave in the inner ear

A sound wave that enters the human ear sets the eardrum into oscillation, which in turn causes oscillation of the *ossicles*, a chain of three tiny bones in the middle ear (Fig. 16.4). The ossicles transmit this oscillation to the fluid (mostly water) in the inner ear; there the fluid motion disturbs hair cells that send nerve impulses to the brain with information about the sound. The area of the moving part of the eardrum is about 43 mm^2 , and that of the stapes (the smallest of the ossicles) where it connects to the inner ear is about 3.2 mm^2 . For the sound in Example 16.1, determine (a) the pressure amplitude and (b) the displacement amplitude of the wave in the fluid of the inner ear, in which the speed of sound is 1500 m/s.

SOLUTION

IDENTIFY and SET UP: Although the sound wave here travels in liquid rather than air, the same principles and relationships among the properties of the wave apply. We can neglect the mass of the tiny ossicles (about $58 \text{ mg} = 5.8 \times 10^{-5} \text{ kg}$), so the force they exert on the inner-ear fluid is the same as that exerted on the eardrum and ossicles by the incident sound wave. (In Chapters 4 and 5 we used the same idea to say that the tension is the same at either end of a massless rope.) Hence the pressure amplitude in the inner ear, $p_{\max(\text{inner ear})}$, is greater than in the outside air, $p_{\max(\text{air})}$, because the same force is exerted on a smaller area (the area of the stapes versus the area of the eardrum). Given $p_{\max(\text{inner ear})}$, we find the displacement amplitude $A_{\text{inner ear}}$ using Eq. (16.5).

16.4 The anatomy of the human ear. The middle ear is the size of a small marble; the ossicles (incus, malleus, and stapes) are the smallest bones in the human body.



EXECUTE: (a) From the area of the eardrum and the pressure amplitude in air found in Example 16.1, the maximum force exerted by the sound wave in air on the eardrum is $F_{\max} = p_{\max(\text{air})}S_{\text{eardrum}}$. Then

$$\begin{aligned} p_{\max(\text{inner ear})} &= \frac{F_{\max}}{S_{\text{stapes}}} = p_{\max(\text{air})} \frac{S_{\text{eardrum}}}{S_{\text{stapes}}} \\ &= (3.0 \times 10^{-2} \text{ Pa}) \frac{43 \text{ mm}^2}{3.2 \text{ mm}^2} = 0.40 \text{ Pa} \end{aligned}$$

(b) To find the maximum displacement $A_{\text{inner ear}}$, we use $A = p_{\text{max}}/Bk$ as in Example 16.1. The inner-ear fluid is mostly water, which has a much greater bulk modulus B than air. From Table 11.2 the compressibility of water (unfortunately also called k) is $45.8 \times 10^{-11} \text{ Pa}^{-1}$, so $B_{\text{fluid}} = 1/(45.8 \times 10^{-11} \text{ Pa}^{-1}) = 2.18 \times 10^9 \text{ Pa}$.

The wave in the inner ear has the same angular frequency ω as the wave in the air because the air, eardrum, ossicles, and inner-ear fluid all oscillate together (see Example 15.8 in Section 15.8). But because the wave speed v is greater in the inner ear than in the air (1500 m/s versus 344 m/s), the wave number $k = \omega/v$ is smaller. Using the value of ω from Example 16.1,

$$k_{\text{inner ear}} = \frac{\omega}{v_{\text{inner ear}}} = \frac{(2\pi \text{ rad})(1000 \text{ Hz})}{1500 \text{ m/s}} = 4.2 \text{ rad/m}$$

Putting everything together, we have

$$\begin{aligned} A_{\text{inner ear}} &= \frac{p_{\text{max (inner ear)}}}{B_{\text{fluid}} k_{\text{inner ear}}} = \frac{0.40 \text{ Pa}}{(2.18 \times 10^9 \text{ Pa})(4.2 \text{ rad/m})} \\ &= 4.4 \times 10^{-11} \text{ m} \end{aligned}$$

EVALUATE: In part (a) we see that the ossicles increase the pressure amplitude by a factor of $(43 \text{ mm}^2)/(3.2 \text{ mm}^2) = 13$. This amplification helps give the human ear its great sensitivity.

The displacement amplitude in the inner ear is even smaller than in the air. But *pressure* variations within the inner-ear fluid are what set the hair cells into motion, so what matters is that the pressure amplitude is larger in the inner ear than in the air.

Perception of Sound Waves

The physical characteristics of a sound wave are directly related to the perception of that sound by a listener. For a given frequency, the greater the pressure amplitude of a sinusoidal sound wave, the greater the perceived **loudness**. The relationship between pressure amplitude and loudness is not a simple one, and it varies from one person to another. One important factor is that the ear is not equally sensitive to all frequencies in the audible range. A sound at one frequency may seem louder than one of equal pressure amplitude at a different frequency. At 1000 Hz the minimum pressure amplitude that can be perceived with normal hearing is about $3 \times 10^{-5} \text{ Pa}$; to produce the same loudness at 200 Hz or 15,000 Hz requires about $3 \times 10^{-4} \text{ Pa}$. Perceived loudness also depends on the health of the ear. A loss of sensitivity at the high-frequency end usually happens naturally with age but can be further aggravated by excessive noise levels.

The frequency of a sound wave is the primary factor in determining the **pitch** of a sound, the quality that lets us classify the sound as “high” or “low.” The higher the frequency of a sound (within the audible range), the higher the pitch that a listener will perceive. Pressure amplitude also plays a role in determining pitch. When a listener compares two sinusoidal sound waves with the same frequency but different pressure amplitudes, the one with the greater pressure amplitude is usually perceived as louder but also as slightly lower in pitch.

Musical sounds have wave functions that are more complicated than a simple sine function. The pressure fluctuation in the sound wave produced by a clarinet is shown in Fig. 16.5a. The pattern is so complex because the column of air in a wind instrument like a clarinet vibrates at a fundamental frequency and at many harmonics at the same time. (In Section 15.8, we described this same behavior for a string that has been plucked, bowed, or struck. We’ll examine the physics of wind instruments in Section 16.5.) The sound wave produced in the surrounding air has a similar amount of each harmonic—that is, a similar *harmonic content*. Figure 16.5b shows the harmonic content of the sound of a clarinet. The mathematical process of translating a pressure–time graph like Fig. 16.5a into a graph of harmonic content like Fig. 16.5b is called *Fourier analysis*.

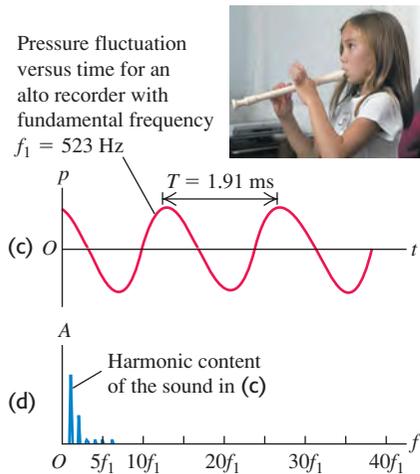
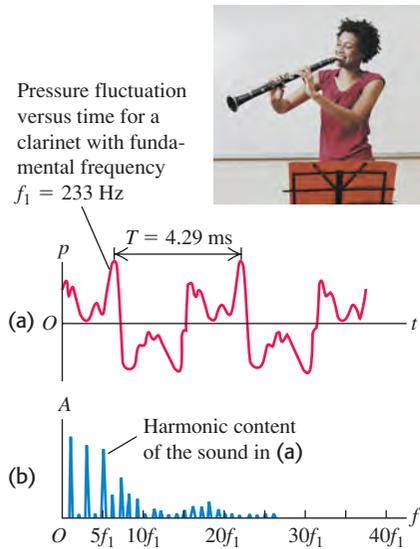
Two tones produced by different instruments might have the same fundamental frequency (and thus the same pitch) but sound different because of different harmonic content. The difference in sound is called *tone color*, *quality*, or **timbre** and is often described in subjective terms such as reedy, golden, round, mellow, and tinny. A tone that is rich in harmonics, like the clarinet tone in Figs. 16.5a and 16.5b, usually sounds thin and “stringy” or “reedy,” while a tone containing mostly a fundamental, like the alto recorder tone in Figs. 16.5c and 16.5d, is more mellow and flutelike. The same principle applies to the human voice, which is another example of a wind instrument; the vowels “a” and “e” sound different because of differences in harmonic content.

Application Hearing Loss from Amplified Sound

Due to exposure to highly amplified music, many young popular musicians have suffered permanent ear damage and have hearing typical of persons 65 years of age. Headphones for personal music players used at high volume pose similar threats to hearing. Be careful!



16.5 Different representations of the sound of (a), (b) a clarinet and (c), (d) an alto recorder. (Graphs adapted from R.E. Berg and D.G. Stork, *The Physics of Sound*, Prentice-Hall, 1982.)



Another factor in determining tone quality is the behavior at the beginning (*attack*) and end (*decay*) of a tone. A piano tone begins with a thump and then dies away gradually. A harpsichord tone, in addition to having different harmonic content, begins much more quickly with a click, and the higher harmonics begin before the lower ones. When the key is released, the sound also dies away much more rapidly with a harpsichord than with a piano. Similar effects are present in other musical instruments. With wind and string instruments the player has considerable control over the attack and decay of the tone, and these characteristics help to define the unique characteristics of each instrument.

Unlike the tones made by musical instruments or the vowels in human speech, **noise** is a combination of *all* frequencies, not just frequencies that are integer multiples of a fundamental frequency. (An extreme case is “white noise,” which contains equal amounts of all frequencies across the audible range.) Examples include the sound of the wind and the hissing sound you make in saying the consonant “s.”

Test Your Understanding of Section 16.1 You use an electronic signal generator to produce a sinusoidal sound wave in air. You then increase the frequency of the wave from 100 Hz to 400 Hz while keeping the pressure amplitude constant. What effect does this have on the displacement amplitude of the sound wave?
 (i) It becomes four times greater; (ii) it becomes twice as great; (iii) it is unchanged; (iv) it becomes $\frac{1}{2}$ as great; (v) it becomes $\frac{1}{4}$ as great.



16.2 Speed of Sound Waves

We found in Section 15.4 that the speed v of a transverse wave on a string depends on the string tension F and the linear mass density μ : $v = \sqrt{F/\mu}$. What, we may ask, is the corresponding expression for the speed of sound waves in a gas or liquid? On what properties of the medium does the speed depend?

We can make an educated guess about these questions by remembering a claim that we made in Section 15.4: For mechanical waves in general, the expression for the wave speed is of the form

$$v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$$

A sound wave in a bulk fluid causes compressions and rarefactions of the fluid, so the restoring-force term in the above expression must be related to how easy or difficult it is to compress the fluid. This is precisely what the bulk modulus B of the medium tells us. According to Newton’s second law, inertia is related to mass. The “massiveness” of a bulk fluid is described by its density, or mass per unit volume, ρ . (The corresponding quantity for a string is the mass per unit length, μ .) Hence we expect that the speed of sound waves should be of the form $v = \sqrt{B/\rho}$.

To check our guess, we’ll derive the speed of sound waves in a fluid in a pipe. This is a situation of some importance, since all musical wind instruments are fundamentally pipes in which a longitudinal wave (sound) propagates in a fluid (air) (Fig. 16.6). Human speech works on the same principle; sound waves propagate in your vocal tract, which is basically an air-filled pipe connected to the lungs at one end (your larynx) and to the outside air at the other end (your mouth). The steps in our derivation are completely parallel to those we used in Section 15.4 to find the speed of transverse waves, so you’ll find it useful to review that section.

Speed of Sound in a Fluid

Figure 16.7 shows a fluid (either liquid or gas) with density ρ in a pipe with cross-sectional area A . In the equilibrium state, the fluid is under a uniform

pressure p . In Fig. 16.7a the fluid is at rest. We take the x -axis along the length of the pipe. This is also the direction in which we make a longitudinal wave propagate, so the displacement y is also measured along the pipe, just as in Section 16.1 (see Fig. 16.2).

At time $t = 0$ we start the piston at the left end moving toward the right with constant speed v_y . This initiates a wave motion that travels to the right along the length of the pipe, in which successive sections of fluid begin to move and become compressed at successively later times.

Figure 16.7b shows the fluid at time t . All portions of fluid to the left of point P are moving to the right with speed v_y , and all portions to the right of P are still at rest. The boundary between the moving and stationary portions travels to the right with a speed equal to the speed of propagation or wave speed v . At time t the piston has moved a distance $v_y t$, and the boundary has advanced a distance vt . As with a transverse disturbance in a string, we can compute the speed of propagation from the impulse–momentum theorem.

The quantity of fluid set in motion in time t is the amount that originally occupied a section of the cylinder with length vt , cross-sectional area A , and volume vtA . The mass of this fluid is ρvtA , and its longitudinal momentum (that is, momentum along the length of the pipe) is

$$\text{Longitudinal momentum} = (\rho vtA)v_y$$

Next we compute the increase of pressure, Δp , in the moving fluid. The original volume of the moving fluid, Avt , has decreased by an amount $Av_y t$. From the definition of the bulk modulus B , Eq. (11.13) in Section 11.5,

$$B = \frac{-\text{Pressure change}}{\text{Fractional volume change}} = \frac{-\Delta p}{-Av_y t / Avt}$$

$$\Delta p = B \frac{v_y}{v}$$

The pressure in the moving fluid is $p + \Delta p$ and the force exerted on it by the piston is $(p + \Delta p)A$. The net force on the moving fluid (see Fig. 16.7b) is ΔpA , and the longitudinal impulse is

$$\text{Longitudinal impulse} = \Delta pAt = B \frac{v_y}{v} At$$

Because the fluid was at rest at time $t = 0$, the change in momentum up to time t is equal to the momentum at that time. Applying the impulse–momentum theorem (see Section 8.1), we find

$$B \frac{v_y}{v} At = \rho vtAv_y \quad (16.6)$$

When we solve this expression for v , we get

$$v = \sqrt{\frac{B}{\rho}} \quad (\text{speed of a longitudinal wave in a fluid}) \quad (16.7)$$

which agrees with our educated guess. Thus the speed of propagation of a longitudinal pulse in a fluid depends only on the bulk modulus B and the density ρ of the medium.

While we derived Eq. (16.7) for waves in a pipe, it also applies to longitudinal waves in a bulk fluid. Thus the speed of sound waves traveling in air or water is determined by this equation.

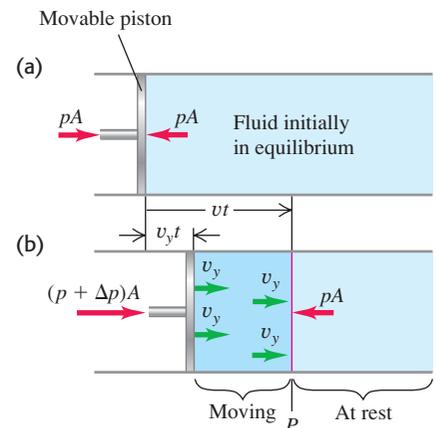
Speed of Sound in a Solid

When a longitudinal wave propagates in a *solid* rod or bar, the situation is somewhat different. The rod expands sideways slightly when it is compressed

16.6 When a wind instrument like this French horn is played, sound waves propagate through the air within the instrument's pipes. The properties of the sound that emerges from the large bell depend on the speed of these waves.



16.7 A sound wave propagating in a fluid confined to a tube. (a) Fluid in equilibrium. (b) A time t after the piston begins moving to the right at speed v_y , the fluid between the piston and point P is in motion. The speed of sound waves is v .



longitudinally, while a fluid in a pipe with constant cross section cannot move sideways. Using the same kind of reasoning that led us to Eq. (16.7), we can show that the speed of a longitudinal pulse in the rod is given by

$$v = \sqrt{\frac{Y}{\rho}} \quad (\text{speed of a longitudinal wave in a solid rod}) \quad (16.8)$$

Table 16.1 Speed of Sound in Various Bulk Materials

Material	Speed of Sound (m/s)
<i>Gases</i>	
Air (20°C)	344
Helium (20°C)	999
Hydrogen (20°C)	1330
<i>Liquids</i>	
Liquid helium (4 K)	211
Mercury (20°C)	1451
Water (0°C)	1402
Water (20°C)	1482
Water (100°C)	1543
<i>Solids</i>	
Aluminum	6420
Lead	1960
Steel	5941

where Y is Young's modulus, defined in Section 11.4.

CAUTION **Solid rods vs. bulk solids** Equation (16.8) applies only to a rod or bar whose sides are free to bulge and shrink a little as the wave travels. It does not apply to longitudinal waves in a *bulk* solid, since in these materials, sideways motion in any element of material is prevented by the surrounding material. The speed of longitudinal waves in a bulk solid depends on the density, the bulk modulus, and the *shear* modulus; a full discussion is beyond the scope of this book. **|**

As with the derivation for a transverse wave on a string, Eqs. (16.7) and (16.8) are valid for sinusoidal and other periodic waves, not just for the special case discussed here.

Table 16.1 lists the speed of sound in several bulk materials. Sound waves travel more slowly in lead than in aluminum or steel because lead has a lower bulk modulus and shear modulus and a higher density.

Example 16.3 Wavelength of sonar waves

A ship uses a sonar system (Fig. 16.8) to locate underwater objects. Find the speed of sound waves in water using Eq. (16.7), and find the wavelength of a 262-Hz wave.

SOLUTION

IDENTIFY and SET UP: Our target variables are the speed and wavelength of a sound wave in water. In Eq. (16.7), we use the density of water, $\rho = 1.00 \times 10^3 \text{ kg/m}^3$, and the bulk modulus of water, which we find from the compressibility (see Table 11.2). Given the speed and the frequency $f = 262 \text{ Hz}$, we find the wavelength from $v = f\lambda$.

EXECUTE: In Example 16.2, we used Table 11.2 to find $B = 2.18 \times 10^9 \text{ Pa}$. Then

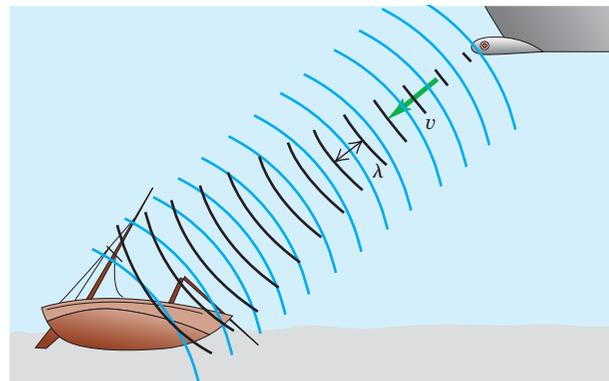
$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.18 \times 10^9 \text{ Pa}}{1.00 \times 10^3 \text{ kg/m}^3}} = 1480 \text{ m/s}$$

and

$$\lambda = \frac{v}{f} = \frac{1480 \text{ m/s}}{262 \text{ s}^{-1}} = 5.65 \text{ m}$$

EVALUATE: The calculated value of v agrees well with the value in Table 16.1. Water is denser than air (ρ is larger) but is also much

16.8 A sonar system uses underwater sound waves to detect and locate submerged objects.



more incompressible (B is much larger), and so the speed $v = \sqrt{B/\rho}$ is greater than the 344-m/s speed of sound in air at ordinary temperatures. The relationship $\lambda = v/f$ then says that a sound wave in water must have a longer wavelength than a wave of the same frequency in air. Indeed, we found in Example 15.1 (Section 15.2) that a 262-Hz sound wave in air has a wavelength of only 1.31 m.

Dolphins emit high-frequency sound waves (typically 100,000 Hz) and use the echoes for guidance and for hunting. The corresponding wavelength in water is 1.48 cm. With this high-frequency “sonar” system they can sense objects that are roughly as small as the wavelength (but not much smaller). *Ultrasonic imaging* is a medical technique that uses exactly the same physical principle; sound waves of very high frequency and very short wavelength, called *ultrasound*, are

scanned over the human body, and the “echoes” from interior organs are used to create an image. With ultrasound of frequency $5 \text{ MHz} = 5 \times 10^6 \text{ Hz}$, the wavelength in water (the primary constituent of the body) is 0.3 mm , and features as small as this can be discerned in the image. Ultrasound is used for the study of heart-valve action, detection of tumors, and prenatal examinations (Fig. 16.9). Ultrasound is more sensitive than x rays in distinguishing various kinds of tissues and does not have the radiation hazards associated with x rays.

Speed of Sound in a Gas

Most of the sound waves that we encounter on a daily basis propagate in air. To use Eq. (16.7) to find the speed of sound waves in air, we must keep in mind that the bulk modulus of a gas depends on the pressure of the gas: The greater the pressure applied to a gas to compress it, the more it resists further compression and hence the greater the bulk modulus. (That’s why specific values of the bulk modulus for gases are not given in Table 11.1.) The expression for the bulk modulus of a gas for use in Eq. (16.7) is

$$B = \gamma p_0 \quad (16.9)$$

where p_0 is the equilibrium pressure of the gas. The quantity γ (the Greek letter gamma) is called the *ratio of heat capacities*. It is a dimensionless number that characterizes the thermal properties of the gas. (We’ll learn more about this quantity in Chapter 19.) As an example, the ratio of heat capacities for air is $\gamma = 1.40$. At normal atmospheric pressure $p_0 = 1.013 \times 10^5 \text{ Pa}$, so $B = (1.40)(1.013 \times 10^5 \text{ Pa}) = 1.42 \times 10^5 \text{ Pa}$. This value is minuscule compared to the bulk modulus of a typical solid (see Table 11.1), which is approximately 10^{10} to 10^{11} Pa . This shouldn’t be surprising: It’s simply a statement that air is far easier to compress than steel.

The density ρ of a gas also depends on the pressure, which in turn depends on the temperature. It turns out that the ratio B/ρ for a given type of ideal gas does *not* depend on the pressure at all, only the temperature. From Eq. (16.7), this means that the speed of sound in a gas is fundamentally a function of temperature T :

$$v = \sqrt{\frac{\gamma RT}{M}} \quad (\text{speed of sound in an ideal gas}) \quad (16.10)$$

This expression incorporates several quantities that you may recognize from your study of ideal gases in chemistry and that we will study in Chapters 17, 18, and 19. The temperature T is the *absolute* temperature in kelvins (K), equal to the Celsius temperature plus 273.15; thus 20.00°C corresponds to $T = 293.15 \text{ K}$. The quantity M is the *molar mass*, or mass per mole of the substance of which the gas is composed. The *gas constant* R has the same value for all gases. The current best numerical value of R is

$$R = 8.314472(15) \text{ J/mol} \cdot \text{K}$$

which for practical calculations we can write as $8.314 \text{ J/mol} \cdot \text{K}$.

For any particular gas, γ , R , and M are constants, and the wave speed is proportional to the square root of the absolute temperature. We will see in Chapter 18 that Eq. (16.10) is almost identical to the expression for the average speed of molecules in an ideal gas. This shows that sound speeds and molecular speeds are closely related.

Example 16.4 Speed of sound in air

Find the speed of sound in air at $T = 20^\circ\text{C}$, and find the range of wavelengths in air to which the human ear (which can hear frequencies in the range of 20–20,000 Hz) is sensitive. The mean molar

mass for air (a mixture of mostly nitrogen and oxygen) is $M = 28.8 \times 10^{-3} \text{ kg/mol}$ and the ratio of heat capacities is $\gamma = 1.40$.

16.9 This three-dimensional image of a fetus in the womb was made using a sequence of ultrasound scans. Each individual scan reveals a two-dimensional “slice” through the fetus; many such slices were then combined digitally.



Continued

SOLUTION

IDENTIFY and SET UP: We use Eq. (16.10) to find the sound speed from γ , T , and M , and we use $v = f\lambda$ to find the wavelengths corresponding to the frequency limits. Note that in Eq. (16.10) temperature T must be expressed in kelvins, not Celsius degrees.

EXECUTE: At $T = 20^\circ\text{C} = 293\text{ K}$, we find

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.40)(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})}{28.8 \times 10^{-3}\text{ kg/mol}}} = 344\text{ m/s}$$

Using this value of v in $\lambda = v/f$, we find that at 20°C the frequency $f = 20\text{ Hz}$ corresponds to $\lambda = 17\text{ m}$ and $f = 20,000\text{ Hz}$ to $\lambda = 1.7\text{ cm}$.

EVALUATE: Our calculated value of v agrees with the measured sound speed at $T = 20^\circ\text{C}$ to within 0.3%.

In this discussion we have treated a gas as a continuous medium. A gas is actually composed of molecules in random motion, separated by distances that are large in comparison with their diameters. The vibrations that constitute a wave in a gas are superposed on the random thermal motion. At atmospheric pressure, a molecule travels an average distance of about 10^{-7} m between collisions, while the displacement amplitude of a faint sound may be only 10^{-9} m . We can think of a gas with a sound wave passing through as being comparable to a swarm of bees; the swarm as a whole oscillates slightly while individual insects move about through the swarm, apparently at random.

Test Your Understanding of Section 16.2 Mercury is 13.6 times denser than water. Based on Table 16.1, at 20°C which of these liquids has the greater bulk modulus? (i) mercury; (ii) water; (iii) both are about the same; (iv) not enough information is given to decide.



16.3 Sound Intensity

Traveling sound waves, like all other traveling waves, transfer energy from one region of space to another. We saw in Section 15.5 that a useful way to describe the energy carried by a sound wave is through the *wave intensity* I , equal to the time average rate at which energy is transported per unit area across a surface perpendicular to the direction of propagation. Let's see how to express the intensity of a sound wave in terms of the displacement amplitude A or pressure amplitude p_{max} .

Intensity and Displacement Amplitude

For simplicity, let us consider a sound wave propagating in the $+x$ -direction so that we can use our expressions from Section 16.1 for the displacement $y(x, t)$ and pressure fluctuation $p(x, t)$ —Eqs. (16.1) and (16.4), respectively. In Section 6.4 we saw that power equals the product of force and velocity [see Eq. (6.18)]. So the power per unit area in this sound wave equals the product of $p(x, t)$ (force per unit area) and the *particle* velocity $v_y(x, t)$. The particle velocity $v_y(x, t)$ is the velocity at time t of that portion of the wave medium at coordinate x . Using Eqs. (16.1) and (16.4), we find

$$\begin{aligned} v_y(x, t) &= \frac{\partial y(x, t)}{\partial t} = \omega A \sin(kx - \omega t) \\ p(x, t)v_y(x, t) &= [BkA \sin(kx - \omega t)][\omega A \sin(kx - \omega t)] \\ &= B\omega k A^2 \sin^2(kx - \omega t) \end{aligned}$$

CAUTION **Wave velocity vs. particle velocity** Remember that the velocity of the wave as a whole is *not* the same as the particle velocity. While the wave continues to move in the direction of propagation, individual particles in the wave medium merely slosh back and forth, as shown in Fig. 16.1. Furthermore, the maximum speed of a particle of the medium can be very different from the wave speed. **|**

The intensity is, by definition, the time average value of $p(x, t)v_y(x, t)$. For any value of x the average value of the function $\sin^2(kx - \omega t)$ over one period $T = 2\pi/\omega$ is $\frac{1}{2}$, so

$$I = \frac{1}{2}B\omega kA^2 \quad (16.11)$$

By using the relationships $\omega = vk$ and $v^2 = B/\rho$, we can transform Eq. (16.11) into the form

$$I = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2 \quad (\text{intensity of a sinusoidal sound wave}) \quad (16.12)$$

This equation shows why in a stereo system, a low-frequency woofer has to vibrate with much larger amplitude than a high-frequency tweeter to produce the same sound intensity.

Intensity and Pressure Amplitude

It is usually more useful to express I in terms of the pressure amplitude p_{\max} . Using Eq. (16.5) and the relationship $\omega = vk$, we find

$$I = \frac{\omega p_{\max}^2}{2Bk} = \frac{v p_{\max}^2}{2B} \quad (16.13)$$

By using the wave speed relationship $v^2 = B/\rho$, we can also write Eq. (16.13) in the alternative forms

$$I = \frac{p_{\max}^2}{2\rho v} = \frac{p_{\max}^2}{2\sqrt{\rho B}} \quad (\text{intensity of a sinusoidal sound wave}) \quad (16.14)$$

You should verify these expressions. Comparison of Eqs. (16.12) and (16.14) shows that sinusoidal sound waves of the same intensity but different frequency have different displacement amplitudes A but the *same* pressure amplitude p_{\max} . This is another reason it is usually more convenient to describe a sound wave in terms of pressure fluctuations, not displacement.

The *total* average power carried across a surface by a sound wave equals the product of the intensity at the surface and the surface area, if the intensity over the surface is uniform. The average total sound power emitted by a person speaking in an ordinary conversational tone is about 10^{-5} W, while a loud shout corresponds to about 3×10^{-2} W. If all the residents of New York City were to talk at the same time, the total sound power would be about 100 W, equivalent to the electric power requirement of a medium-sized light bulb. On the other hand, the power required to fill a large auditorium or stadium with loud sound is considerable (see Example 16.7.)

If the sound source emits waves in all directions equally, the intensity decreases with increasing distance r from the source according to the inverse-square law: The intensity is proportional to $1/r^2$. We discussed this law and its consequences in Section 15.5. If the sound goes predominantly in one direction, the inverse-square law does not apply and the intensity decreases with distance more slowly than $1/r^2$ (Fig. 16.10).

16.10 By cupping your hands like this, you direct the sound waves emerging from your mouth so that they don't propagate to the sides. Hence the intensity decreases with distance more slowly than the inverse-square law would predict, and you can be heard at greater distances.



The inverse-square relationship also does not apply indoors because sound energy can reach a listener by reflection from the walls and ceiling. Indeed, part of the architect's job in designing an auditorium is to tailor these reflections so that the intensity is as nearly uniform as possible over the entire auditorium.

Problem-Solving Strategy 16.1 Sound Intensity



IDENTIFY *the relevant concepts:* The relationships between the intensity and amplitude of a sound wave are straightforward. Other quantities are involved in these relationships, however, so it's particularly important to decide which is your target variable.

SET UP *the problem* using the following steps:

- Sort the physical quantities into categories. Wave properties include the displacement and pressure amplitudes A and p_{\max} and the frequency f , which can be determined from the angular frequency ω , the wave number k , or the wavelength λ . These quantities are related through the wave speed v , which is determined by properties of the medium (B and ρ for a liquid, and γ , T , and M for a gas).

- List the given quantities and identify the target variables. Find relationships that take you where you want to go.

EXECUTE *the solution:* Use your selected equations to solve for the target variables. Express the temperature in kelvins (Celsius temperature plus 273.15) to calculate the speed of sound in a gas.

EVALUATE *your answer:* If possible, use an alternative relationship to check your results.

Example 16.5 Intensity of a sound wave in air

Find the intensity of the sound wave in Example 16.1, with $p_{\max} = 3.0 \times 10^{-2}$ Pa. Assume the temperature is 20°C so that the density of air is $\rho = 1.20$ kg/m³ and the speed of sound is $v = 344$ m/s.

SOLUTION

IDENTIFY and SET UP: Our target variable is the intensity I of the sound wave. We are given the pressure amplitude p_{\max} of the wave as well as the density ρ and wave speed v for the medium. We can determine I from p_{\max} , ρ , and v using Eq. (16.14).

EXECUTE: From Eq. (16.14),

$$\begin{aligned} I &= \frac{p_{\max}^2}{2\rho v} = \frac{(3.0 \times 10^{-2} \text{ Pa})^2}{2(1.20 \text{ kg/m}^3)(344 \text{ m/s})} \\ &= 1.1 \times 10^{-6} \text{ J/(s} \cdot \text{m}^2) = 1.1 \times 10^{-6} \text{ W/m}^2 \end{aligned}$$

EVALUATE: This seems like a very low intensity, but it is well within the range of sound intensities encountered on a daily basis. A very loud sound wave at the threshold of pain has a pressure amplitude of about 30 Pa and an intensity of about 1 W/m². The pressure amplitude of the faintest sound wave that can be heard is about 3×10^{-5} Pa, and the corresponding intensity is about 10^{-12} W/m². (Try these values of p_{\max} in Eq. (16.14) to check that the corresponding intensities are as we have stated.)

Example 16.6 Same intensity, different frequencies

What are the pressure and displacement amplitudes of a 20-Hz sound wave with the same intensity as the 1000-Hz sound wave of Examples 16.1 and 16.5?

SOLUTION

IDENTIFY and SET UP: In Examples 16.1 and 16.5 we found that for a 1000-Hz sound wave with $p_{\max} = 3.0 \times 10^{-2}$ Pa, $A = 1.2 \times 10^{-8}$ m and $I = 1.1 \times 10^{-6}$ W/m². Our target variables are p_{\max} and A for a 20-Hz sound wave of the same intensity I . We can find these using Eqs. (16.14) and (16.12), respectively.

EXECUTE: We can rearrange Eqs. (16.14) and (16.12) as $p_{\max}^2 = 2I\sqrt{\rho B}$ and $\omega^2 A^2 = 2I/\sqrt{\rho B}$, respectively. These tell us that for a given sound intensity I in a given medium (constant ρ and B), the

quantities p_{\max} and ωA (or, equivalently, fA) are both *constants* that don't depend on frequency. From the first result we immediately have $p_{\max} = 3.0 \times 10^{-2}$ Pa for $f = 20$ Hz, the same as for $f = 1000$ Hz. If we write the second result as $f_{20}A_{20} = f_{1000}A_{1000}$, we have

$$\begin{aligned} A_{20} &= \left(\frac{f_{1000}}{f_{20}}\right)A_{1000} \\ &= \left(\frac{1000 \text{ Hz}}{20 \text{ Hz}}\right)(1.2 \times 10^{-8} \text{ m}) = 6.0 \times 10^{-7} \text{ m} = 0.60 \mu\text{m} \end{aligned}$$

EVALUATE: Our result reinforces the idea that pressure amplitude is a more convenient description of a sound wave and its intensity than displacement amplitude.

Example 16.7 “Play it loud!”

For an outdoor concert we want the sound intensity to be 1 W/m^2 at a distance of 20 m from the speaker array. If the sound intensity is uniform in all directions, what is the required acoustic power output of the array?

SOLUTION

IDENTIFY, SET UP, and EXECUTE: This example uses the definition of sound intensity as power per unit area. The total power is the target variable; the area in question is a hemisphere centered on the speaker array. We assume that the speakers are on the ground and

that none of the acoustic power is directed into the ground, so the acoustic power is uniform over a hemisphere 20 m in radius. The surface area of this hemisphere is $(\frac{1}{2})(4\pi)(20 \text{ m})^2$, or about 2500 m^2 . The required power is the product of this area and the intensity: $(1 \text{ W/m}^2)(2500 \text{ m}^2) = 2500 \text{ W} = 2.5 \text{ kW}$.

EVALUATE: The electrical power input to the speaker would need to be considerably greater than 2.5 kW, because speaker efficiency is not very high (typically a few percent for ordinary speakers, and up to 25% for horn-type speakers).

The Decibel Scale

Because the ear is sensitive over a broad range of intensities, a *logarithmic* intensity scale is usually used. The **sound intensity level** β of a sound wave is defined by the equation

$$\beta = (10 \text{ dB}) \log \frac{I}{I_0} \quad (\text{definition of sound intensity level}) \quad (16.15)$$

In this equation, I_0 is a reference intensity, chosen to be 10^{-12} W/m^2 , approximately the threshold of human hearing at 1000 Hz. Recall that “log” means the logarithm to base 10. Sound intensity levels are expressed in **decibels**, abbreviated dB. A decibel is $\frac{1}{10}$ of a *bel*, a unit named for Alexander Graham Bell (the inventor of the telephone). The bel is inconveniently large for most purposes, and the decibel is the usual unit of sound intensity level.

If the intensity of a sound wave equals I_0 or 10^{-12} W/m^2 , its sound intensity level is 0 dB. An intensity of 1 W/m^2 corresponds to 120 dB. Table 16.2 gives the sound intensity levels in decibels of some familiar sounds. You can use Eq. (16.15) to check the value of sound intensity level β given for each intensity in the table.

Because the ear is not equally sensitive to all frequencies in the audible range, some sound-level meters weight the various frequencies unequally. One such scheme leads to the so-called dBA scale; this scale deemphasizes the low and very high frequencies, where the ear is less sensitive than at midrange frequencies.

Table 16.2 Sound Intensity Levels from Various Sources (Representative Values)

Source or Description of Sound	Sound Intensity Level, β (dB)	Intensity, I (W/m^2)
Military jet aircraft 30 m away	140	10^2
Threshold of pain	120	1
Riveter	95	3.2×10^{-3}
Elevated train	90	10^{-3}
Busy street traffic	70	10^{-5}
Ordinary conversation	65	3.2×10^{-6}
Quiet automobile	50	10^{-7}
Quiet radio in home	40	10^{-8}
Average whisper	20	10^{-10}
Rustle of leaves	10	10^{-11}
Threshold of hearing at 1000 Hz	0	10^{-12}

Example 16.8 Temporary—or permanent—hearing loss

A 10-min exposure to 120-dB sound will temporarily shift your threshold of hearing at 1000 Hz from 0 dB up to 28 dB. Ten years of exposure to 92-dB sound will cause a *permanent* shift to 28 dB. What sound intensities correspond to 28 dB and 92 dB?

SOLUTION

IDENTIFY and SET UP: We are given two sound intensity levels β ; our target variables are the corresponding intensities. We can solve Eq. (16.15) to find the intensity I that corresponds to each value of β .

EXECUTE: We solve Eq. (16.15) for I by dividing both sides by 10 dB and using the relationship $10^{\log x} = x$:

$$I = I_0 10^{\beta/(10 \text{ dB})}$$

For $\beta = 28 \text{ dB}$ and $\beta = 92 \text{ dB}$, the exponents are $\beta/(10 \text{ dB}) = 2.8$ and 9.2 , respectively, so that

$$I_{28 \text{ dB}} = (10^{-12} \text{ W/m}^2) 10^{2.8} = 6.3 \times 10^{-10} \text{ W/m}^2$$

$$I_{92 \text{ dB}} = (10^{-12} \text{ W/m}^2) 10^{9.2} = 1.6 \times 10^{-3} \text{ W/m}^2$$

EVALUATE: If your answers are a factor of 10 too large, you may have entered 10×10^{-12} in your calculator instead of 1×10^{-12} . Be careful!

Example 16.9 A bird sings in a meadow

Consider an idealized bird (treated as a point source) that emits constant sound power, with intensity obeying the inverse-square law (Fig. 16.11). If you move twice the distance from the bird, by how many decibels does the sound intensity level drop?

SOLUTION

IDENTIFY and SET UP: The decibel scale is logarithmic, so the *difference* between two sound intensity levels (the target variable) corresponds to the *ratio* of the corresponding intensities, which is determined by the inverse-square law. We label the two points P_1 and P_2 (Fig. 16.11). We use Eq. (16.15), the definition of sound intensity level, at each point. We use Eq. (15.26), the inverse-square law, to relate the intensities at the two points.

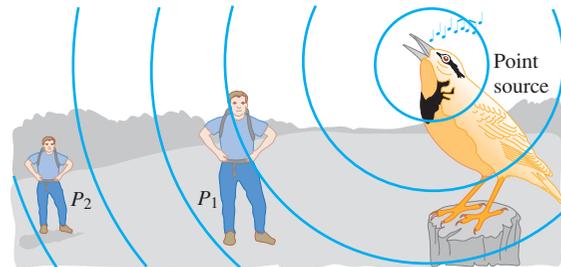
EXECUTE: The difference $\beta_2 - \beta_1$ between any two sound intensity levels is related to the corresponding intensities by

$$\begin{aligned} \beta_2 - \beta_1 &= (10 \text{ dB}) \left(\log \frac{I_2}{I_0} - \log \frac{I_1}{I_0} \right) \\ &= (10 \text{ dB}) [(\log I_2 - \log I_0) - (\log I_1 - \log I_0)] \\ &= (10 \text{ dB}) \log \frac{I_2}{I_1} \end{aligned}$$

For this inverse-square-law source, Eq. (15.26) yields $I_2/I_1 = r_1^2/r_2^2 = \frac{1}{4}$, so

$$\beta_2 - \beta_1 = (10 \text{ dB}) \log \frac{I_1}{I_2} = (10 \text{ dB}) \log \frac{1}{\frac{1}{4}} = -6.0 \text{ dB}$$

16.11 When you double your distance from a point source of sound, by how much does the sound intensity level decrease?



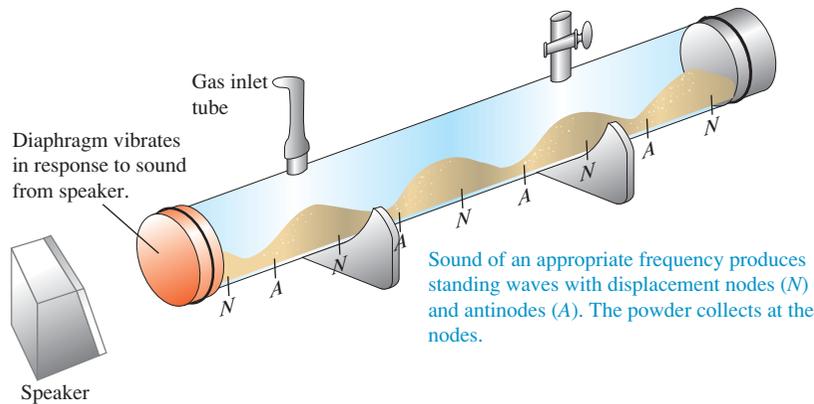
EVALUATE: Our result is negative, which tells us (correctly) that the sound intensity level is less at P_2 than at P_1 . The 6-dB difference doesn't depend on the sound intensity level at P_1 ; *any* doubling of the distance from an inverse-square-law source reduces the sound intensity level by 6 dB.

Note that the perceived *loudness* of a sound is not directly proportional to its intensity. For example, most people interpret an increase of 8 dB to 10 dB in sound intensity level (corresponding to increasing intensity by a factor of 6 to 10) as a doubling of loudness.

Test Your Understanding of Section 16.3 You double the intensity of a sound wave in air while leaving the frequency unchanged. (The pressure, density, and temperature of the air remain unchanged as well.) What effect does this have on the displacement amplitude, pressure amplitude, bulk modulus, sound speed, and sound intensity level?

16.4 Standing Sound Waves and Normal Modes

When longitudinal (sound) waves propagate in a fluid in a pipe with finite length, the waves are reflected from the ends in the same way that transverse waves on a string are reflected at its ends. The superposition of the waves traveling in opposite directions again forms a standing wave. Just as for transverse standing waves on a string (see Section 15.7), standing sound waves (normal modes) in a pipe can



16.12 Demonstrating standing sound waves using a Kundt's tube. The blue shading represents the density of the gas at an instant when the gas pressure at the displacement nodes is a maximum or a minimum.

be used to create sound waves in the surrounding air. This is the operating principle of the human voice as well as many musical instruments, including woodwinds, brasses, and pipe organs.

Transverse waves on a string, including standing waves, are usually described only in terms of the displacement of the string. But, as we have seen, sound waves in a fluid may be described either in terms of the displacement of the fluid or in terms of the pressure variation in the fluid. To avoid confusion, we'll use the terms **displacement node** and **displacement antinode** to refer to points where particles of the fluid have zero displacement and maximum displacement, respectively.

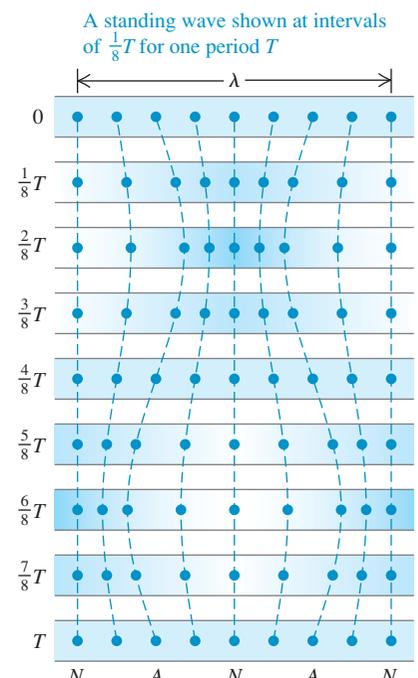
We can demonstrate standing sound waves in a column of gas using an apparatus called a Kundt's tube (Fig. 16.12). A horizontal glass tube a meter or so long is closed at one end and has a flexible diaphragm at the other end that can transmit vibrations. A nearby loudspeaker is driven by an audio oscillator and amplifier; this produces sound waves that force the diaphragm to vibrate sinusoidally with a frequency that we can vary. The sound waves within the tube are reflected at the other, closed end of the tube. We spread a small amount of light powder uniformly along the bottom of the tube. As we vary the frequency of the sound, we pass through frequencies at which the amplitude of the standing waves becomes large enough for the powder to be swept along the tube at those points where the gas is in motion. The powder therefore collects at the displacement nodes (where the gas is not moving). Adjacent nodes are separated by a distance equal to $\lambda/2$, and we can measure this distance. Given the wavelength, we can use this experiment to determine the wave speed: We read the frequency f from the oscillator dial, and we can then calculate the speed v of the waves from the relationship $v = \lambda f$.

Figure 16.13 shows the motions of nine different particles within a gas-filled tube in which there is a standing sound wave. A particle at a displacement node (N) does not move, while a particle at a displacement antinode (A) oscillates with maximum amplitude. Note that particles on opposite sides of a displacement node vibrate in opposite phase. When these particles approach each other, the gas between them is compressed and the pressure rises; when they recede from each other, there is an expansion and the pressure drops. Hence at a displacement *node* the gas undergoes the maximum amount of compression and expansion, and the variations in pressure and density above and below the average have their maximum value. By contrast, particles on opposite sides of a displacement *antinode* vibrate *in phase*; the distance between the particles is nearly constant, and there is *no* variation in pressure or density at a displacement antinode.

We use the term **pressure node** to describe a point in a standing sound wave at which the pressure and density do not vary and the term **pressure antinode** to describe a point at which the variations in pressure and density are greatest. Using these terms, we can summarize our observations about standing sound waves as follows:

A pressure node is always a displacement antinode, and a pressure antinode is always a displacement node.

16.13 In a standing sound wave, a displacement node N is a pressure antinode (a point where the pressure fluctuates the most) and a displacement antinode A is a pressure node (a point where the pressure does not fluctuate at all).



N = a displacement node = a pressure antinode
 A = a displacement antinode = a pressure node

Figure 16.12 depicts a standing sound wave at an instant at which the pressure variations are greatest; the blue shading shows that the density and pressure of the gas have their maximum and minimum values at the displacement nodes.

When reflection takes place at a *closed* end of a pipe (an end with a rigid barrier or plug), the displacement of the particles at this end must always be zero, analogous to a fixed end of a string. Thus a closed end of a pipe is a displacement node and a pressure antinode; the particles do not move, but the pressure variations are maximum. An *open* end of a pipe is a pressure node because it is open to the atmosphere, where the pressure is constant. Because of this, an open end is always a displacement *antinode*, in analogy to a free end of a string; the particles oscillate with maximum amplitude, but the pressure does not vary. (Strictly speaking, the pressure node actually occurs somewhat beyond an open end of a pipe. But if the diameter of the pipe is small in comparison to the wavelength, which is true for most musical instruments, this effect can safely be neglected.) Thus longitudinal waves in a column of fluid are reflected at the closed and open ends of a pipe in the same way that transverse waves in a string are reflected at fixed and free ends, respectively.

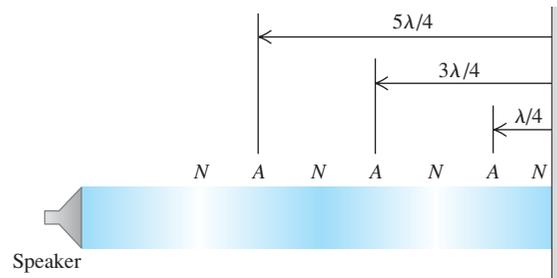
Conceptual Example 16.10 The sound of silence

A directional loudspeaker directs a sound wave of wavelength λ at a wall (Fig. 16.14). At what distances from the wall could you stand and hear no sound at all?

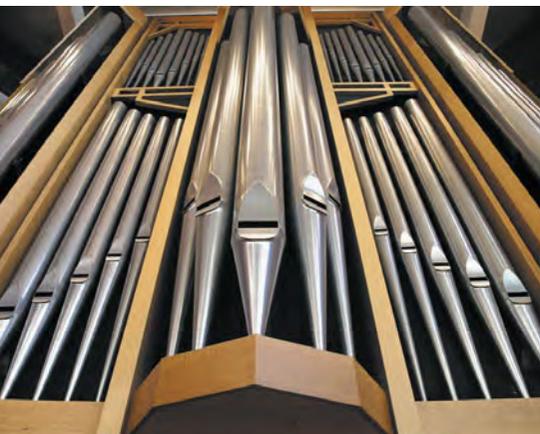
SOLUTION

Your ear detects pressure variations in the air; you will therefore hear no sound if your ear is at a *pressure node*, which is a displacement antinode. The wall is at a displacement node; the distance from any node to an adjacent antinode is $\lambda/4$, and the distance from one antinode to the next is $\lambda/2$ (Fig. 16.14). Hence the displacement antinodes (pressure nodes), at which no sound will be heard, are at distances $d = \lambda/4$, $d = \lambda/4 + \lambda/2 = 3\lambda/4$, $d = 3\lambda/4 + \lambda/2 = 5\lambda/4$, . . . from the wall. If the loudspeaker is not highly directional, this effect is hard to notice because of reflections of sound waves from the floor, ceiling, and other walls.

16.14 When a sound wave is directed at a wall, it interferes with the reflected wave to create a standing wave. The *N*'s and *A*'s are *displacement nodes* and *antinodes*.



16.15 Organ pipes of different sizes produce tones with different frequencies.



Organ Pipes and Wind Instruments

The most important application of standing sound waves is the production of musical tones by wind instruments. Organ pipes are one of the simplest examples (Fig. 16.15). Air is supplied by a blower, at a gauge pressure typically of the order of 10^3 Pa (10^{-2} atm), to the bottom end of the pipe (Fig. 16.16). A stream of air emerges from the narrow opening at the edge of the horizontal surface and is directed against the top edge of the opening, which is called the *mouth* of the pipe. The column of air in the pipe is set into vibration, and there is a series of possible normal modes, just as with the stretched string. The mouth always acts as an open end; thus it is a pressure node and a displacement antinode. The other end of the pipe (at the top in Fig. 16.16) may be either open or closed.

In Fig. 16.17, both ends of the pipe are open, so both ends are pressure nodes and displacement antinodes. An organ pipe that is open at both ends is called an *open pipe*. The fundamental frequency f_1 corresponds to a standing-wave pattern with a displacement antinode at each end and a displacement node in the middle (Fig. 16.17a). The distance between adjacent antinodes is always equal to one

half-wavelength, and in this case that is equal to the length L of the pipe; $\lambda/2 = L$. The corresponding frequency, obtained from the relationship $f = v/\lambda$, is

$$f_1 = \frac{v}{2L} \quad (\text{open pipe}) \quad (16.16)$$

Figures 16.17b and 16.17c show the second and third harmonics (first and second overtones); their vibration patterns have two and three displacement nodes, respectively. For these, a half-wavelength is equal to $L/2$ and $L/3$, respectively, and the frequencies are twice and three times the fundamental, respectively. That is, $f_2 = 2f_1$ and $f_3 = 3f_1$. For every normal mode of an open pipe the length L must be an integer number of half-wavelengths, and the possible wavelengths λ_n are given by

$$L = n\frac{\lambda_n}{2} \quad \text{or} \quad \lambda_n = \frac{2L}{n} \quad (n = 1, 2, 3, \dots) \quad (\text{open pipe}) \quad (16.17)$$

The corresponding frequencies f_n are given by $f_n = v/\lambda_n$, so all the normal-mode frequencies for a pipe that is open at both ends are given by

$$f_n = \frac{nv}{2L} \quad (n = 1, 2, 3, \dots) \quad (\text{open pipe}) \quad (16.18)$$

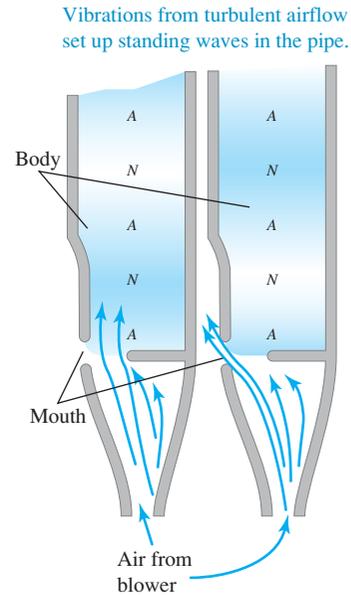
The value $n = 1$ corresponds to the fundamental frequency, $n = 2$ to the second harmonic (or first overtone), and so on. Alternatively, we can say

$$f_n = nf_1 \quad (n = 1, 2, 3, \dots) \quad (\text{open pipe}) \quad (16.19)$$

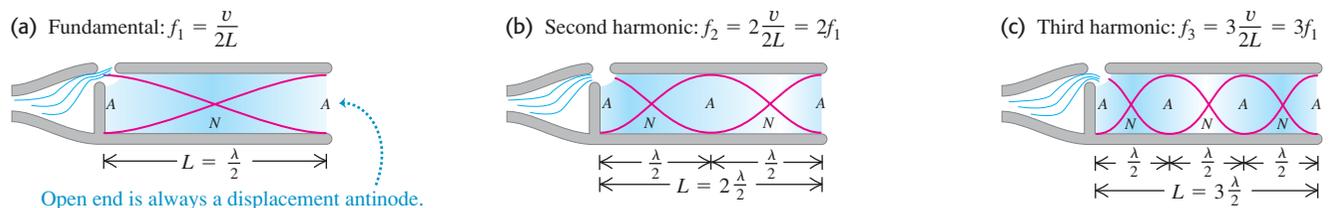
with f_1 given by Eq. (16.16).

Figure 16.18 shows a pipe that is open at the left end but closed at the right end. This is called a *stopped pipe*. The left (open) end is a displacement antinode (pressure node), but the right (closed) end is a displacement node (pressure antinode). The distance between a node and the adjacent antinode is always one quarter-wavelength. Figure 16.18a shows the lowest-frequency mode; the length

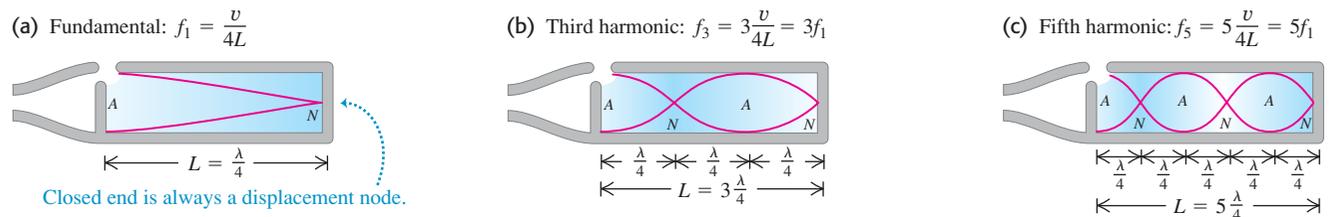
16.16 Cross sections of an organ pipe at two instants one half-period apart. The N 's and A 's are *displacement* nodes and antinodes; as the blue shading shows, these are points of maximum pressure variation and zero pressure variation, respectively.



16.17 A cross section of an open pipe showing the first three normal modes. The shading indicates the pressure variations. The red curves are graphs of the displacement along the pipe axis at two instants separated in time by one half-period. The N 's and A 's are the *displacement* nodes and antinodes; interchange these to show the *pressure* nodes and antinodes.



16.18 A cross section of a stopped pipe showing the first three normal modes as well as the *displacement* nodes and antinodes. Only odd harmonics are possible.



of the pipe is a quarter-wavelength ($L = \lambda_1/4$). The fundamental frequency is $f_1 = v/\lambda_1$, or

$$f_1 = \frac{v}{4L} \quad (\text{stopped pipe}) \quad (16.20)$$

This is one-half the fundamental frequency for an *open* pipe of the same length. In musical language, the *pitch* of a closed pipe is one octave lower (a factor of 2 in frequency) than that of an open pipe of the same length. Figure 16.18b shows the next mode, for which the length of the pipe is *three-quarters* of a wavelength, corresponding to a frequency $3f_1$. For Fig. 16.18c, $L = 5\lambda/4$ and the frequency is $5f_1$. The possible wavelengths are given by

$$L = n \frac{\lambda_n}{4} \quad \text{or} \quad \lambda_n = \frac{4L}{n} \quad (n = 1, 3, 5, \dots) \quad (\text{stopped pipe}) \quad (16.21)$$

The normal-mode frequencies are given by $f_n = v/\lambda_n$, or

$$f_n = \frac{nv}{4L} \quad (n = 1, 3, 5, \dots) \quad (\text{stopped pipe}) \quad (16.22)$$

or

$$f_n = nf_1 \quad (n = 1, 3, 5, \dots) \quad (\text{stopped pipe}) \quad (16.23)$$

with f_1 given by Eq. (16.20). We see that the second, fourth, and all *even* harmonics are missing. In a pipe that is closed at one end, the fundamental frequency is $f_1 = v/4L$, and only the odd harmonics in the series ($3f_1, 5f_1, \dots$) are possible.

A final possibility is a pipe that is closed at *both* ends, with displacement nodes and pressure antinodes at both ends. This wouldn't be of much use as a musical instrument because there would be no way for the vibrations to get out of the pipe.

Example 16.11 A tale of two pipes

On a day when the speed of sound is 345 m/s, the fundamental frequency of a particular stopped organ pipe is 220 Hz. (a) How long is this pipe? (b) The second *overtone* of this pipe has the same wavelength as the third *harmonic* of an *open* pipe. How long is the open pipe?

SOLUTION

IDENTIFY and SET UP: This problem uses the relationship between the length and normal-mode frequencies of open pipes (Fig. 16.17) and stopped pipes (Fig. 16.18). In part (a), we determine the length of the stopped pipe from Eq. (16.22). In part (b), we must determine the length of an open pipe, for which Eq. (16.18) gives the frequencies.

EXECUTE: (a) For a stopped pipe $f_1 = v/4L$, so

$$L_{\text{stopped}} = \frac{v}{4f_1} = \frac{345 \text{ m/s}}{4(220 \text{ s}^{-1})} = 0.392 \text{ m}$$

(b) The frequency of the second overtone of a stopped pipe (the *third* possible frequency) is $f_5 = 5f_1 = 5(220 \text{ Hz}) = 1100 \text{ Hz}$. If the wavelengths for the two pipes are the same, the frequencies are also the same. Hence the frequency of the third harmonic of the open pipe, which is at $3f_1 = 3(v/2L)$, equals 1100 Hz. Then

$$1100 \text{ Hz} = 3 \left(\frac{345 \text{ m/s}}{2L_{\text{open}}} \right) \quad \text{and} \quad L_{\text{open}} = 0.470 \text{ m}$$

EVALUATE: The 0.392-m stopped pipe has a fundamental frequency of 220 Hz; the *longer* (0.470-m) open pipe has a *higher* fundamental frequency, $(1100 \text{ Hz})/3 = 367 \text{ Hz}$. This is not a contradiction, as you can see if you compare Figs. 16.17a and 16.18a.

In an organ pipe in actual use, several modes are always present at once; the motion of the air is a superposition of these modes. This situation is analogous to a string that is struck or plucked, as in Fig. 15.28. Just as for a vibrating string, a complex standing wave in the pipe produces a traveling sound wave in the surrounding air with a harmonic content similar to that of the standing wave. A very

narrow pipe produces a sound wave rich in higher harmonics, which we hear as a thin and “stringy” tone; a fatter pipe produces mostly the fundamental mode, heard as a softer, more flutelike tone. The harmonic content also depends on the shape of the pipe’s mouth.

We have talked about organ pipes, but this discussion is also applicable to other wind instruments. The flute and the recorder are directly analogous. The most significant difference is that those instruments have holes along the pipe. Opening and closing the holes with the fingers changes the effective length L of the air column and thus changes the pitch. Any individual organ pipe, by comparison, can play only a single note. The flute and recorder behave as *open* pipes, while the clarinet acts as a *stopped* pipe (closed at the reed end, open at the bell).

Equations (16.18) and (16.22) show that the frequencies of any wind instrument are proportional to the speed of sound v in the air column inside the instrument. As Eq. (16.10) shows, v depends on temperature; it increases when temperature increases. Thus the pitch of all wind instruments rises with increasing temperature. An organ that has some of its pipes at one temperature and others at a different temperature is bound to sound out of tune.

Test Your Understanding of Section 16.4 If you connect a hose to one end of a metal pipe and blow compressed air into it, the pipe produces a musical tone. If instead you blow compressed helium into the pipe at the same pressure and temperature, will the pipe produce (i) the same tone, (ii) a higher-pitch tone, or (iii) a lower-pitch tone? 

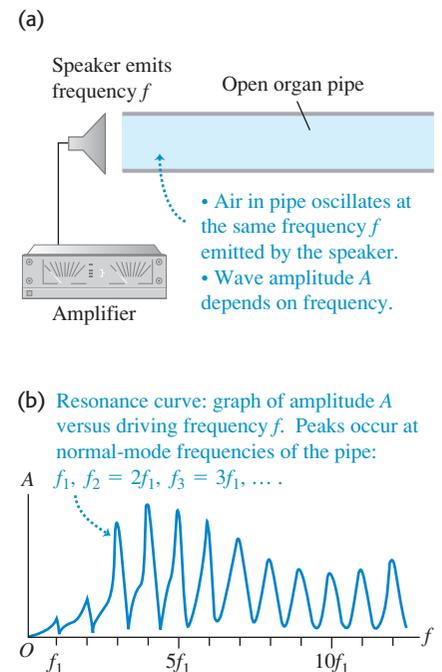
16.5 Resonance and Sound

Many mechanical systems have normal modes of oscillation. As we have seen, these include columns of air (as in an organ pipe) and stretched strings (as in a guitar; see Section 15.8). In each mode, every particle of the system oscillates with simple harmonic motion at the same frequency as the mode. Air columns and stretched strings have an infinite series of normal modes, but the basic concept is closely related to the simple harmonic oscillator, discussed in Chapter 14, which has only a single normal mode (that is, only one frequency at which it oscillates after being disturbed).

Suppose we apply a periodically varying force to a system that can oscillate. The system is then forced to oscillate with a frequency equal to the frequency of the applied force (called the *driving frequency*). This motion is called a *forced oscillation*. We talked about forced oscillations of the harmonic oscillator in Section 14.8, and we suggest that you review that discussion. In particular, we described the phenomenon of mechanical **resonance**. A simple example of resonance is pushing Cousin Throckmorton on a swing. The swing is a pendulum; it has only a single normal mode, with a frequency determined by its length. If we push the swing periodically with this frequency, we can build up the amplitude of the motion. But if we push with a very different frequency, the swing hardly moves at all.

Resonance also occurs when a periodically varying force is applied to a system with many normal modes. An example is shown in Fig. 16.19a. An open organ pipe is placed next to a loudspeaker that is driven by an amplifier and emits pure sinusoidal sound waves of frequency f , which can be varied by adjusting the amplifier. The air in the pipe is forced to vibrate with the same frequency f as the *driving force* provided by the loudspeaker. In general the amplitude of this motion is relatively small, and the air inside the pipe will not move in any of the normal-mode patterns shown in Fig. 16.17. But if the frequency f of the force is close to one of the normal-mode frequencies, the air in the pipe moves in the normal-mode pattern for that frequency, and the amplitude can become quite large. Figure 16.19b shows the amplitude of oscillation of the air

16.19 (a) The air in an open pipe is forced to oscillate at the same frequency as the sinusoidal sound waves coming from the loudspeaker. (b) The resonance curve of the open pipe graphs the amplitude of the standing sound wave in the pipe as a function of the driving frequency.



Application Resonance and the Sensitivity of the Ear

The auditory canal of the human ear (see Fig. 16.4) is an air-filled pipe open at one end and closed at the other (eardrum) end. The canal is about $2.5 \text{ cm} = 0.025 \text{ m}$ long, so it has a resonance at its fundamental frequency $f_1 = v/4L = (344 \text{ m/s})/[4(0.025 \text{ m})] = 3440 \text{ Hz}$. The resonance means that a sound at this frequency produces a strong oscillation of the eardrum. That's why your ear is most sensitive to sounds near 3440 Hz.



16.20 The frequency of the sound from this trumpet exactly matches one of the normal-mode frequencies of the goblet. The resonant vibrations of the goblet have such large amplitude that the goblet tears itself apart.



in the pipe as a function of the driving frequency f . The shape of this graph is called the **resonance curve** of the pipe; it has peaks where f equals the normal-mode frequencies of the pipe. The detailed shape of the resonance curve depends on the geometry of the pipe.

If the frequency of the force is precisely *equal* to a normal-mode frequency, the system is in resonance, and the amplitude of the forced oscillation is maximum. If there were no friction or other energy-dissipating mechanism, a driving force at a normal-mode frequency would continue to add energy to the system, and the amplitude would increase indefinitely. In such an idealized case the peaks in the resonance curve of Fig. 16.19b would be infinitely high. But in any real system there is always some dissipation of energy, or damping, as we discussed in Section 14.8; the amplitude of oscillation in resonance may be large, but it cannot be infinite.

The “sound of the ocean” you hear when you put your ear next to a large seashell is due to resonance. The noise of the outside air moving past the seashell is a mixture of sound waves of almost all audible frequencies, which forces the air inside the seashell to oscillate. The seashell behaves like an organ pipe, with a set of normal-mode frequencies; hence the inside air oscillates most strongly at those frequencies, producing the seashell’s characteristic sound. To hear a similar phenomenon, uncap a full bottle of your favorite beverage and blow across the open top. The noise is provided by your breath blowing across the top, and the “organ pipe” is the column of air inside the bottle above the surface of the liquid. If you take a drink and repeat the experiment, you will hear a lower tone because the “pipe” is longer and the normal-mode frequencies are lower.

Resonance also occurs when a stretched string is forced to oscillate (see Section 15.8). Suppose that one end of a stretched string is held fixed while the other is given a transverse sinusoidal motion with small amplitude, setting up standing waves. If the frequency of the driving mechanism is *not* equal to one of the normal-mode frequencies of the string, the amplitude at the antinodes is fairly small. However, if the frequency is equal to any one of the normal-mode frequencies, the string is in resonance, and the amplitude at the antinodes is very much larger than that at the driven end. The driven end is not precisely a node, but it lies much closer to a node than to an antinode when the string is in resonance. The photographs in Fig. 15.23 were made this way, with the left end of the string fixed and the right end oscillating vertically with small amplitude; large-amplitude standing waves resulted when the frequency of oscillation of the right end was equal to the fundamental frequency or to one of the first three overtones.

It is easy to demonstrate resonance with a piano. Push down the damper pedal (the right-hand pedal) so that the dampers are lifted and the strings are free to vibrate, and then sing a steady tone into the piano. When you stop singing, the piano seems to continue to sing the same note. The sound waves from your voice excite vibrations in the strings that have natural frequencies close to the frequencies (fundamental and harmonics) present in the note you sang.

A more spectacular example is a singer breaking a wine glass with her amplified voice. A good-quality wine glass has normal-mode frequencies that you can hear by tapping it. If the singer emits a loud note with a frequency corresponding exactly to one of these normal-mode frequencies, large-amplitude oscillations can build up and break the glass (Fig. 16.20).

Example 16.12 An organ–guitar duet

A stopped organ pipe is sounded near a guitar, causing one of the strings to vibrate with large amplitude. We vary the string tension until we find the maximum amplitude. The string is 80% as long as the pipe. If both pipe and string vibrate at their fundamental frequency, calculate the ratio of the wave speed on the string to the speed of sound in air.

SOLUTION

IDENTIFY and SET UP: The large response of the string is an example of resonance. It occurs because the organ pipe and the guitar string have the same fundamental frequency. Letting the subscripts a and s stand for the air in the pipe and the string, respectively, the

condition for resonance is $f_{1a} = f_{1s}$. Equation (16.20) gives the fundamental frequency for a stopped pipe, and Eq. (15.32) gives the fundamental frequency for a guitar string held at both ends. These expressions involve the wave speed in air (v_a) and on the string (v_s) and the lengths of the pipe and string. We are given that $L_s = 0.80L_a$; our target variable is the ratio v_s/v_a .

EXECUTE: From Eqs. (16.20) and (15.32), $f_{1a} = v_a/4L_a$ and $f_{1s} = v_s/2L_s$. These frequencies are equal, so

$$\frac{v_a}{4L_a} = \frac{v_s}{2L_s}$$

Substituting $L_s = 0.80L_a$ and rearranging, we get $v_s/v_a = 0.40$.

EVALUATE: As an example, if the speed of sound in air is 345 m/s, the wave speed on the string is $(0.40)(345 \text{ m/s}) = 138 \text{ m/s}$. Note that while the standing waves in the pipe and on the string have the same frequency, they have different *wavelengths* $\lambda = v/f$ because the two media have different wave speeds v . Which standing wave has the greater wavelength?

Test Your Understanding of Section 16.5 A stopped organ pipe of length L has a fundamental frequency of 220 Hz. For which of the following organ pipes will there be a resonance if a tuning fork of frequency 660 Hz is sounded next to the pipe? (There may be more than one correct answer.) (i) a stopped organ pipe of length L ; (ii) a stopped organ pipe of length $2L$; (iii) an open organ pipe of length L ; (iii) an open organ pipe of length $2L$.

16.6 Interference of Waves

Wave phenomena that occur when two or more waves overlap in the same region of space are grouped under the heading *interference*. As we have seen, standing waves are a simple example of an interference effect: Two waves traveling in opposite directions in a medium combine to produce a standing wave pattern with nodes and antinodes that do not move.

Figure 16.21 shows an example of another type of interference that involves waves that spread out in space. Two speakers, driven in phase by the same amplifier, emit identical sinusoidal sound waves with the same constant frequency. We place a microphone at point P in the figure, equidistant from the speakers. Wave crests emitted from the two speakers at the same time travel equal distances and arrive at point P at the same time; hence the waves arrive in phase, and there is constructive interference. The total wave amplitude at P is twice the amplitude from each individual wave, and we can measure this combined amplitude with the microphone.

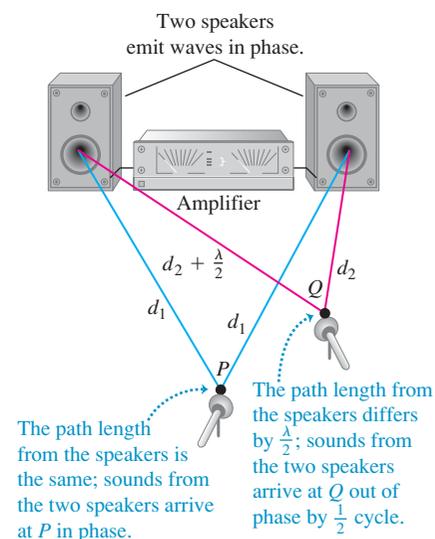
Now let's move the microphone to point Q , where the distances from the two speakers to the microphone differ by a half-wavelength. Then the two waves arrive a half-cycle out of step, or *out of phase*; a positive crest from one speaker arrives at the same time as a negative crest from the other. Destructive interference takes place, and the amplitude measured by the microphone is much *smaller* than when only one speaker is present. If the amplitudes from the two speakers are equal, the two waves cancel each other out completely at point Q , and the total amplitude there is zero.

CAUTION **Interference and traveling waves** Although this situation bears some resemblance to standing waves in a pipe, the total wave in Fig. 16.21 is a *traveling* wave, not a standing wave. To see why, recall that in a standing wave there is no net flow of energy in any direction. By contrast, in Fig. 16.21 there is an overall flow of energy from the speakers into the surrounding air; this is characteristic of a traveling wave. The interference between the waves from the two speakers simply causes the energy flow to be *channeled* into certain directions (for example, toward P) and away from other directions (for example, away from Q). You can see another difference between Fig. 16.21 and a standing wave by considering a point, such as Q , where destructive interference occurs. Such a point is *both* a displacement node *and* a pressure node because there is no wave at all at this point. Compare this to a standing wave, in which a pressure node is a displacement antinode, and vice versa.

MasteringPHYSICS

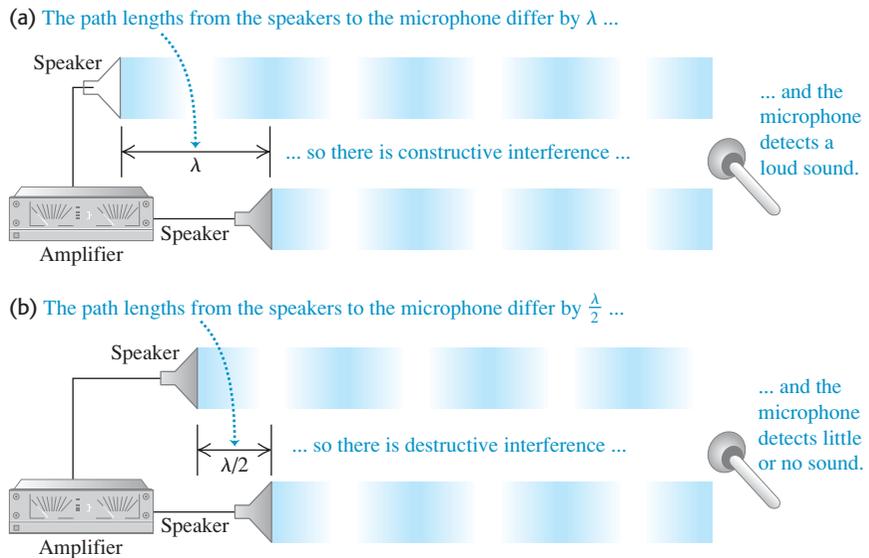
PhET: Sound
PhET: Wave Interference

16.21 Two speakers driven by the same amplifier. Constructive interference occurs at point P , and destructive interference occurs at point Q .



Constructive interference occurs wherever the distances traveled by the two waves differ by a whole number of wavelengths, $0, \lambda, 2\lambda, 3\lambda, \dots$; in all these cases the waves arrive at the microphone in phase (Fig. 16.22a). If the distances from the two speakers to the microphone differ by any half-integer number of wavelengths, $\lambda/2, 3\lambda/2, 5\lambda/2, \dots$, the waves arrive at the microphone out of phase and there will be destructive interference (Fig. 16.22b). In this case, little or no sound energy flows toward the microphone directly in front of the speakers. The energy is instead directed to the sides, where constructive interference occurs.

16.22 Two speakers driven by the same amplifier, emitting waves in phase. Only the waves directed toward the microphone are shown, and they are separated for clarity. (a) Constructive interference occurs when the path difference is $0, \lambda, 2\lambda, 3\lambda, \dots$ (b) Destructive interference occurs when the path difference is $\lambda/2, 3\lambda/2, 5\lambda/2, \dots$



Example 16.13 Loudspeaker interference

Two small loudspeakers, *A* and *B* (Fig. 16.23), are driven by the same amplifier and emit pure sinusoidal waves in phase. (a) For what frequencies does constructive interference occur at point *P*? (b) For what frequencies does destructive interference occur? The speed of sound is 350 m/s.

SOLUTION

IDENTIFY and SET UP: The nature of the interference at *P* depends on the difference *d* in path lengths from point *A* to *P* and from point *B* to *P*. We calculate the path lengths using the Pythagorean theorem. Constructive interference occurs when *d* equals a whole number of wavelengths, while destructive interference occurs

when *d* is a half-integer number of wavelengths. To find the corresponding frequencies, we use $v = f\lambda$.

EXECUTE: The distance from *A* to *P* is $[(2.00 \text{ m})^2 + (4.00 \text{ m})^2]^{1/2} = 4.47 \text{ m}$, and the distance from *B* to *P* is $[(1.00 \text{ m})^2 + (4.00 \text{ m})^2]^{1/2} = 4.12 \text{ m}$. The path difference is $d = 4.47 \text{ m} - 4.12 \text{ m} = 0.35 \text{ m}$.

(a) Constructive interference occurs when $d = 0, \lambda, 2\lambda, \dots$ or $d = 0, v/f, 2v/f, \dots = nv/f$. So the possible frequencies are

$$f_n = \frac{nv}{d} = n \frac{350 \text{ m/s}}{0.35 \text{ m}} \quad (n = 1, 2, 3, \dots)$$

$$= 1000 \text{ Hz}, 2000 \text{ Hz}, 3000 \text{ Hz}, \dots$$

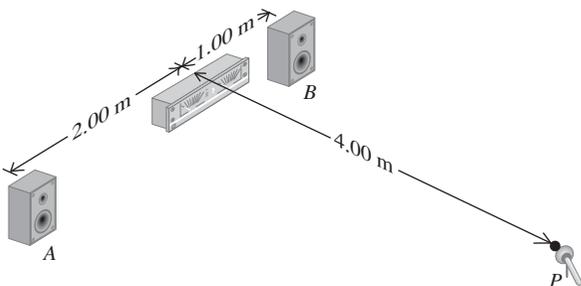
(b) Destructive interference occurs when $d = \lambda/2, 3\lambda/2, 5\lambda/2, \dots$ or $d = v/2f, 3v/2f, 5v/2f, \dots$. The possible frequencies are

$$f_n = \frac{nv}{2d} = n \frac{350 \text{ m/s}}{2(0.35 \text{ m})} \quad (n = 1, 3, 5, \dots)$$

$$= 500 \text{ Hz}, 1500 \text{ Hz}, 2500 \text{ Hz}, \dots$$

EVALUATE: As we increase the frequency, the sound at point *P* alternates between large and small (near zero) amplitudes, with maxima and minima at the frequencies given above. This effect may not be strong in an ordinary room because of reflections from the walls, floor, and ceiling. It is stronger outdoors and best in an anechoic chamber, which has walls that absorb almost all sound and thereby eliminate reflections.

16.23 What sort of interference occurs at *P*?



Interference effects are used to control noise from very loud sound sources such as gas-turbine power plants or jet engine test cells. The idea is to use additional sound sources that in some regions of space interfere destructively with the unwanted sound and cancel it out. Microphones in the controlled area feed signals back to the sound sources, which are continuously adjusted for optimum cancellation of noise in the controlled area.

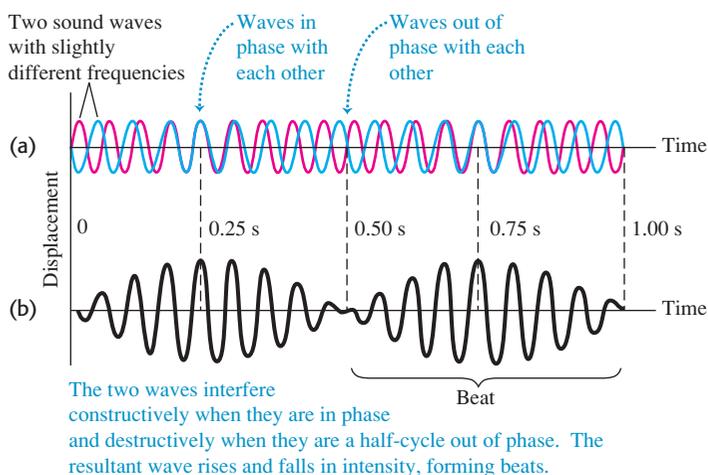
Test Your Understanding of Section 16.6 Suppose that speaker *A* in Fig. 16.23 emits a sinusoidal sound wave of frequency 500 Hz and speaker *B* emits a sinusoidal sound wave of frequency 1000 Hz. What sort of interference will there be between these two waves? (i) constructive interference at various points, including point *P*, and destructive interference at various other points; (ii) destructive interference at various points, including point *P*, and constructive interference at various points; (iii) neither (i) nor (ii).

16.7 Beats

In Section 16.6 we talked about *interference* effects that occur when two different waves with the same frequency overlap in the same region of space. Now let's look at what happens when we have two waves with equal amplitude but slightly different frequencies. This occurs, for example, when two tuning forks with slightly different frequencies are sounded together, or when two organ pipes that are supposed to have exactly the same frequency are slightly “out of tune.”

Consider a particular point in space where the two waves overlap. The displacements of the individual waves at this point are plotted as functions of time in Fig. 16.24a. The total length of the time axis represents 1 second, and the frequencies are 16 Hz (blue graph) and 18 Hz (red graph). Applying the principle of superposition, we add the two displacements at each instant of time to find the total displacement at that time. The result is the graph of Fig. 16.24b. At certain times the two waves are in phase; their maxima coincide and their amplitudes add. But because of their slightly different frequencies, the two waves cannot be in phase at all times. Indeed, at certain times (like $t = 0.50$ s in Fig. 16.24) the two waves are exactly *out of phase*. The two waves then cancel each other, and the total amplitude is zero.

The resultant wave in Fig. 16.24b looks like a single sinusoidal wave with a varying amplitude that goes from a maximum to zero and back. In this example the amplitude goes through two maxima and two minima in 1 second, so the frequency of this amplitude variation is 2 Hz. The amplitude variation causes variations of loudness called **beats**, and the frequency with which the loudness varies is called the **beat frequency**. In this example the beat frequency is the *difference*



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ActivPhysics 10.7: Beats and Beat Frequency

16.24 Beats are fluctuations in amplitude produced by two sound waves of slightly different frequency, here 16 Hz and 18 Hz. (a) Individual waves. (b) Resultant wave formed by superposition of the two waves. The beat frequency is $18 \text{ Hz} - 16 \text{ Hz} = 2 \text{ Hz}$.

of the two frequencies. If the beat frequency is a few hertz, we hear it as a waver or pulsation in the tone.

We can prove that the beat frequency is *always* the difference of the two frequencies f_a and f_b . Suppose f_a is larger than f_b ; the corresponding periods are T_a and T_b , with $T_a < T_b$. If the two waves start out in phase at time $t = 0$, they are again in phase when the first wave has gone through exactly one more cycle than the second. This happens at a value of t equal to T_{beat} , the *period* of the beat. Let n be the number of cycles of the first wave in time T_{beat} ; then the number of cycles of the second wave in the same time is $(n - 1)$, and we have the relationships

$$T_{\text{beat}} = nT_a \quad \text{and} \quad T_{\text{beat}} = (n - 1)T_b$$

Eliminating n between these two equations, we find

$$T_{\text{beat}} = \frac{T_a T_b}{T_b - T_a}$$

The reciprocal of the beat period is the beat *frequency*, $f_{\text{beat}} = 1/T_{\text{beat}}$, so

$$f_{\text{beat}} = \frac{T_b - T_a}{T_a T_b} = \frac{1}{T_a} - \frac{1}{T_b}$$

and finally

$$f_{\text{beat}} = f_a - f_b \quad (\text{beat frequency}) \quad (16.24)$$

As claimed, the beat frequency is the difference of the two frequencies. In using Eq. (16.24), remember that f_a is the higher frequency.

An alternative way to derive Eq. (16.24) is to write functions to describe the curves in Fig. 16.24a and then add them. Suppose that at a certain position the two waves are given by $y_a(t) = A \sin 2\pi f_a t$ and $y_b(t) = -A \sin 2\pi f_b t$. We use the trigonometric identity

$$\sin a - \sin b = 2 \sin \frac{1}{2}(a - b) \cos \frac{1}{2}(a + b)$$

We can then express the total wave $y(t) = y_a(t) + y_b(t)$ as

$$y_a(t) + y_b(t) = [2A \sin \frac{1}{2}(2\pi)(f_a - f_b)t] \cos \frac{1}{2}(2\pi)(f_a + f_b)t$$

The amplitude factor (the quantity in brackets) varies slowly with frequency $\frac{1}{2}(f_a - f_b)$. The cosine factor varies with a frequency equal to the *average* frequency $\frac{1}{2}(f_a + f_b)$. The *square* of the amplitude factor, which is proportional to the intensity that the ear hears, goes through two maxima and two minima per cycle. So the beat frequency f_{beat} that is heard is twice the quantity $\frac{1}{2}(f_a - f_b)$, or just $f_a - f_b$, in agreement with Eq. (16.24).

Beats between two tones can be heard up to a beat frequency of about 6 or 7 Hz. Two piano strings or two organ pipes differing in frequency by 2 or 3 Hz sound wavery and “out of tune,” although some organ stops contain two sets of pipes deliberately tuned to beat frequencies of about 1 to 2 Hz for a gently undulating effect. Listening for beats is an important technique in tuning all musical instruments.

At frequency differences greater than about 6 or 7 Hz, we no longer hear individual beats, and the sensation merges into one of *consonance* or *dissonance*, depending on the frequency ratio of the two tones. In some cases the ear perceives a tone called a *difference tone*, with a pitch equal to the beat frequency of the two tones. For example, if you listen to a whistle that produces sounds at 1800 Hz and 1900 Hz when blown, you will hear not only these tones but also a much lower 100-Hz tone.

The engines on multiengine propeller aircraft have to be synchronized so that the propeller sounds don’t cause annoying beats, which are heard as loud throbbing sounds (Fig. 16.25). On some planes this is done electronically; on others the pilot does it by ear, just like tuning a piano.

16.25 If the two propellers on this airplane are not precisely synchronized, the pilots, passengers, and listeners on the ground will hear beats.



Test Your Understanding of Section 16.7 One tuning fork vibrates at 440 Hz, while a second tuning fork vibrates at an unknown frequency. When both tuning forks are sounded simultaneously, you hear a tone that rises and falls in intensity three times per second. What is the frequency of the second tuning fork? (i) 434 Hz; (ii) 437 Hz; (iii) 443 Hz; (iv) 446 Hz; (v) either 434 Hz or 446 Hz; (vi) either 437 Hz or 443 Hz. **I**

16.8 The Doppler Effect

You've probably noticed that when a car approaches you with its horn sounding, the pitch seems to drop as the car passes. This phenomenon, first described by the 19th-century Austrian scientist Christian Doppler, is called the **Doppler effect**. When a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency. A similar effect occurs for light and radio waves; we'll return to this later in this section.

To analyze the Doppler effect for sound, we'll work out a relationship between the frequency shift and the velocities of source and listener relative to the medium (usually air) through which the sound waves propagate. To keep things simple, we consider only the special case in which the velocities of both source and listener lie along the line joining them. Let v_S and v_L be the velocity components along this line for the source and the listener, respectively, relative to the medium. We choose the positive direction for both v_S and v_L to be the direction from the listener L to the source S. The speed of sound relative to the medium, v , is always considered positive.

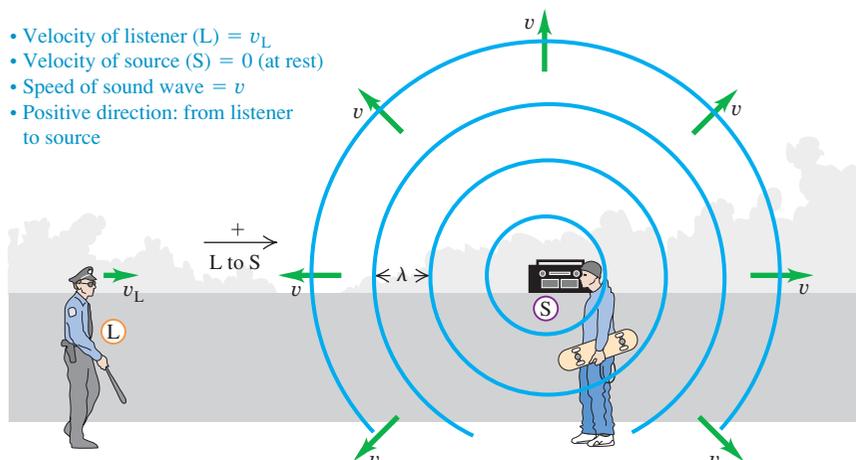
Moving Listener and Stationary Source

Let's think first about a listener L moving with velocity v_L toward a stationary source S (Fig. 16.26). The source emits a sound wave with frequency f_S and wavelength $\lambda = v/f_S$. The figure shows four wave crests, separated by equal distances λ . The wave crests approaching the moving listener have a speed of propagation *relative to the listener* of $(v + v_L)$. So the frequency f_L with which the crests arrive at the listener's position (that is, the frequency the listener hears) is

$$f_L = \frac{v + v_L}{\lambda} = \frac{v + v_L}{v/f_S} \quad (16.25)$$

or

$$f_L = \left(\frac{v + v_L}{v} \right) f_S = \left(1 + \frac{v_L}{v} \right) f_S \quad (\text{moving listener, stationary source}) \quad (16.26)$$



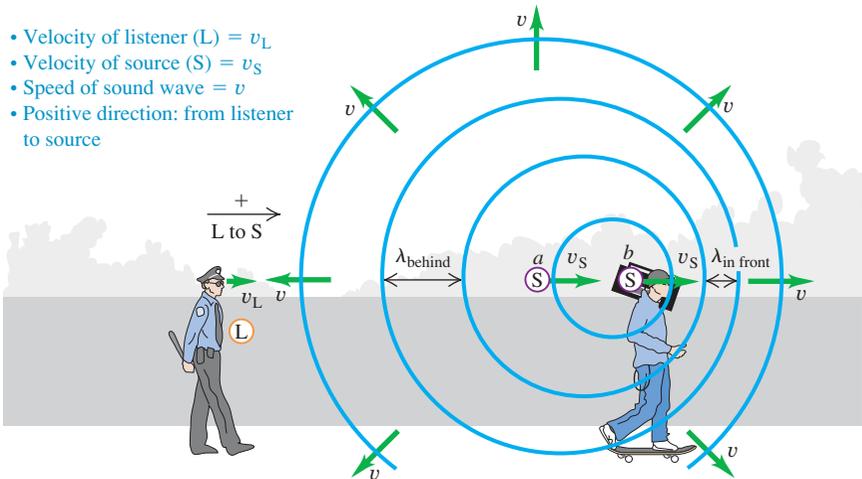
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ActivPhysics 10.8: Doppler Effect: Conceptual Introduction

ActivPhysics 10.9: Doppler Effect: Problems

16.26 A listener moving toward a stationary source hears a frequency that is higher than the source frequency. This is because the relative speed of listener and wave is greater than the wave speed v .

16.27 Wave crests emitted by a moving source are crowded together in front of the source (to the right of this source) and stretched out behind it (to the left of this source).



So a listener moving toward a source ($v_L > 0$), as in Fig. 16.26, hears a higher frequency (higher pitch) than does a stationary listener. A listener moving away from the source ($v_L < 0$) hears a lower frequency (lower pitch).

Moving Source and Moving Listener

Now suppose the source is also moving, with velocity v_S (Fig. 16.27). The wave speed relative to the wave medium (air) is still v ; it is determined by the properties of the medium and is not changed by the motion of the source. But the wavelength is no longer equal to v/f_S . Here's why. The time for emission of one cycle of the wave is the period $T = 1/f_S$. During this time, the wave travels a distance $vT = v/f_S$ and the source moves a distance $v_S T = v_S/f_S$. The wavelength is the distance between successive wave crests, and this is determined by the *relative* displacement of source and wave. As Fig. 16.27 shows, this is different in front of and behind the source. In the region to the right of the source in Fig. 16.27 (that is, in front of the source), the wavelength is

$$\lambda_{\text{in front}} = \frac{v}{f_S} - \frac{v_S}{f_S} = \frac{v - v_S}{f_S} \quad (\text{wavelength in front of a moving source}) \quad (16.27)$$

In the region to the left of the source (that is, behind the source), it is

$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S} \quad (\text{wavelength behind a moving source}) \quad (16.28)$$

The waves in front of and behind the source are compressed and stretched out, respectively, by the motion of the source.

To find the frequency heard by the listener behind the source, we substitute Eq. (16.28) into the first form of Eq. (16.25):

$$f_L = \frac{v + v_L}{\lambda_{\text{behind}}} = \frac{v + v_L}{(v + v_S)/f_S}$$

$$f_L = \frac{v + v_L}{v + v_S} f_S \quad (\text{Doppler effect, moving source and moving listener}) \quad (16.29)$$

This expresses the frequency f_L heard by the listener in terms of the frequency f_S of the source.

Although we derived it for the particular situation shown in Fig. 16.27, Eq. (16.29) includes *all* possibilities for motion of source and listener (relative to

the medium) along the line joining them. If the listener happens to be at rest in the medium, v_L is zero. When both source and listener are at rest or have the same velocity relative to the medium, $v_L = v_S$ and $f_L = f_S$. Whenever the direction of the source or listener velocity is opposite to the direction from the listener toward the source (which we have defined as positive), the corresponding velocity to be used in Eq. (16.29) is negative.

As an example, the frequency heard by a listener at rest ($v_L = 0$) is $f_L = [v/(v + v_S)]f_S$. If the source is moving toward the listener (in the negative direction), then $v_S < 0$, $f_L > f_S$, and the listener hears a higher frequency than that emitted by the source. If instead the source is moving away from the listener (in the positive direction), then $v_S > 0$, $f_L < f_S$, and the listener hears a lower frequency. This explains the change in pitch that you hear from the siren of an ambulance as it passes you (Fig. 16.28).

16.28 The Doppler effect explains why the siren on a fire engine or ambulance has a high pitch ($f_L > f_S$) when it is approaching you ($v_S < 0$) and a low pitch ($f_L < f_S$) when it is moving away ($v_S > 0$).



Problem-Solving Strategy 16.2 Doppler Effect

IDENTIFY the relevant concepts: The Doppler effect occurs whenever the source of waves, the wave detector (listener), or both are in motion.

SET UP the problem using the following steps:

1. Establish a coordinate system, with the positive direction from the listener toward the source. Carefully determine the signs of all relevant velocities. A velocity in the direction from the listener toward the source is positive; a velocity in the opposite direction is negative. All velocities must be measured relative to the air in which the sound travels.
2. Use consistent subscripts to identify the various quantities: S for source and L for listener.
3. Identify which unknown quantities are the target variables.

EXECUTE the solution as follows:

1. Use Eq. (16.29) to relate the frequencies at the source and the listener, the sound speed, and the velocities of the source and

the listener according to the sign convention of step 1. If the source is moving, you can find the wavelength measured by the listener using Eq. (16.27) or (16.28).

2. When a wave is reflected from a stationary or moving surface, solve the problem in two steps. In the first, the surface is the “listener”; the frequency with which the wave crests arrive at the surface is f_L . In the second, the surface is the “source,” emitting waves with this same frequency f_L . Finally, determine the frequency heard by a listener detecting this new wave.

EVALUATE your answer: Is the *direction* of the frequency shift reasonable? If the source and the listener are moving toward each other, $f_L > f_S$; if they are moving apart, $f_L < f_S$. If the source and the listener have no relative motion, $f_L = f_S$.



Example 16.14 Doppler effect I: Wavelengths

A police car’s siren emits a sinusoidal wave with frequency $f_S = 300$ Hz. The speed of sound is 340 m/s and the air is still. (a) Find the wavelength of the waves if the siren is at rest. (b) Find the wavelengths of the waves in front of and behind the siren if it is moving at 30 m/s.

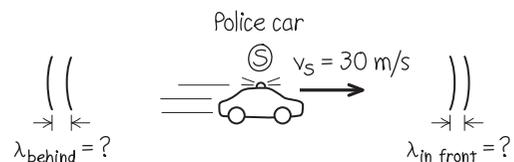
SOLUTION

IDENTIFY and SET UP: In part (a) there is no Doppler effect because neither source nor listener is moving with respect to the air; $v = \lambda f$ gives the wavelength. Figure 16.29 shows the situation in part (b): The source is in motion, so we find the wavelengths using Eqs. (16.27) and (16.28) for the Doppler effect.

EXECUTE: (a) When the source is at rest,

$$\lambda = \frac{v}{f_S} = \frac{340 \text{ m/s}}{300 \text{ Hz}} = 1.13 \text{ m}$$

16.29 Our sketch for this problem.



(b) From Eq. (16.27), in front of the siren

$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S} = \frac{340 \text{ m/s} - 30 \text{ m/s}}{300 \text{ Hz}} = 1.03 \text{ m}$$

From Eq. (16.28), behind the siren

$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S} = \frac{340 \text{ m/s} + 30 \text{ m/s}}{300 \text{ Hz}} = 1.23 \text{ m}$$

EVALUATE: The wavelength is shorter in front of the siren and longer behind it, as we expect.

Example 16.15 Doppler effect II: Frequencies

If a listener L is at rest and the siren in Example 16.14 is moving away from L at 30 m/s, what frequency does the listener hear?

SOLUTION

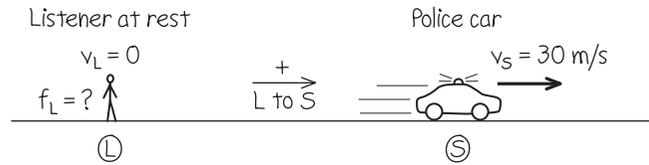
IDENTIFY and SET UP: Our target variable is the frequency f_L heard by a listener behind the moving source. Figure 16.30 shows the situation. We have $v_L = 0$ and $v_S = +30$ m/s (positive, since the velocity of the source is in the direction from listener to source).

EXECUTE: From Eq. (16.29),

$$f_L = \frac{v}{v + v_S} f_S = \frac{340 \text{ m/s}}{340 \text{ m/s} + 30 \text{ m/s}} (300 \text{ Hz}) = 276 \text{ Hz}$$

EVALUATE: The source and listener are moving apart, so $f_L < f_S$. Here's a check on our numerical result. From Example 16.14, the

16.30 Our sketch for this problem.



wavelength behind the source (where the listener in Fig. 16.30 is located) is 1.23 m. The wave speed relative to the stationary listener is $v = 340$ m/s even though the source is moving, so

$$f_L = \frac{v}{\lambda} = \frac{340 \text{ m/s}}{1.23 \text{ m}} = 276 \text{ Hz}$$

Example 16.16 Doppler effect III: A moving listener

If the siren is at rest and the listener is moving away from it at 30 m/s, what frequency does the listener hear?

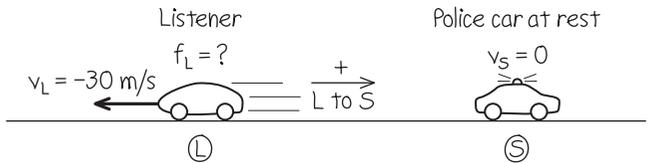
SOLUTION

IDENTIFY and SET UP: Again our target variable is f_L , but now L is in motion and S is at rest. Figure 16.31 shows the situation. The velocity of the listener is $v_L = -30$ m/s (negative, since the motion is in the direction from source to listener).

EXECUTE: From Eq. (16.29),

$$f_L = \frac{v + v_L}{v} f_S = \frac{340 \text{ m/s} + (-30 \text{ m/s})}{340 \text{ m/s}} (300 \text{ Hz}) = 274 \text{ Hz}$$

16.31 Our sketch for this problem.



EVALUATE: Again the source and listener are moving apart, so $f_L < f_S$. Note that the *relative velocity* of source and listener is the same as in Example 16.15, but the Doppler shift is different because v_S and v_L are different.

Example 16.17 Doppler effect IV: Moving source, moving listener

The siren is moving away from the listener with a speed of 45 m/s relative to the air, and the listener is moving toward the siren with a speed of 15 m/s relative to the air. What frequency does the listener hear?

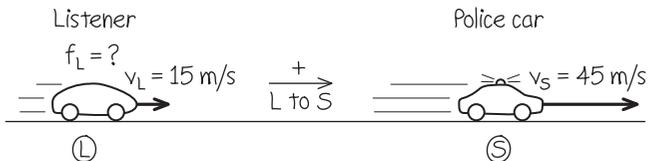
SOLUTION

IDENTIFY and SET UP: Now *both* L and S are in motion. Again our target variable is f_L . Both the source velocity $v_S = +45$ m/s and the listener's velocity $v_L = +15$ m/s are positive because both velocities are in the direction from listener to source.

EXECUTE: From Eq. (16.29),

$$f_L = \frac{v + v_L}{v + v_S} f_S = \frac{340 \text{ m/s} + 15 \text{ m/s}}{340 \text{ m/s} + 45 \text{ m/s}} (300 \text{ Hz}) = 277 \text{ Hz}$$

16.32 Our sketch for this problem.



EVALUATE: As in Examples 16.15 and 16.16, the source and listener again move away from each other at 30 m/s, so again $f_L < f_S$. But f_L is different in all three cases because the Doppler effect for sound depends on how the source and listener are moving relative to the *air*, not simply on how they move relative to each other.

Example 16.18 Doppler effect V: A double Doppler shift

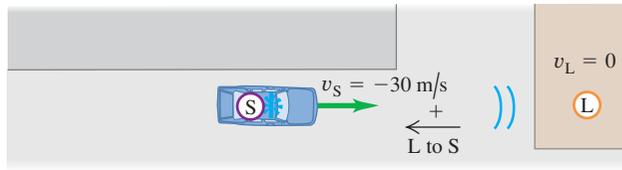
The police car is moving toward a warehouse at 30 m/s. What frequency does the driver hear reflected from the warehouse?

SOLUTION

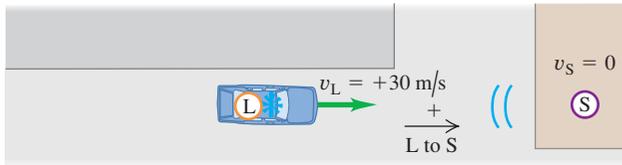
IDENTIFY: In this situation there are *two* Doppler shifts (Fig. 16.33). In the first shift, the warehouse is the stationary “listener.”

16.33 Two stages of the sound wave's motion from the police car to the warehouse and back to the police car.

(a) Sound travels from police car's siren (source S) to warehouse ("listener" L).



(b) Reflected sound travels from warehouse (source S) to police car (listener L).



The frequency of sound reaching the warehouse, which we call f_W , is greater than 300 Hz because the source is approaching. In the second shift, the warehouse acts as a source of sound with

frequency f_W , and the listener is the driver of the police car; she hears a frequency greater than f_W because she is approaching the source.

SET UP: To determine f_W , we use Eq. (16.29) with f_L replaced by f_W . For this part of the problem, $v_L = v_W = 0$ (the warehouse is at rest) and $v_S = -30$ m/s (the siren is moving in the negative direction from source to listener).

To determine the frequency heard by the driver (our target variable), we again use Eq. (16.29) but now with f_S replaced by f_W . For this second part of the problem, $v_S = 0$ because the stationary warehouse is the source and the velocity of the listener (the driver) is $v_L = +30$ m/s. (The listener's velocity is positive because it is in the direction from listener to source.)

EXECUTE: The frequency reaching the warehouse is

$$f_W = \frac{v}{v + v_S} f_S = \frac{340 \text{ m/s}}{340 \text{ m/s} + (-30 \text{ m/s})} (300 \text{ Hz}) = 329 \text{ Hz}$$

Then the frequency heard by the driver is

$$f_L = \frac{v + v_L}{v} f_W = \frac{340 \text{ m/s} + 30 \text{ m/s}}{340 \text{ m/s}} (329 \text{ Hz}) = 358 \text{ Hz}$$

EVALUATE: Because there are two Doppler shifts, the reflected sound heard by the driver has an even higher frequency than the sound heard by a stationary listener in the warehouse.

Doppler Effect for Electromagnetic Waves

In the Doppler effect for sound, the velocities v_L and v_S are always measured relative to the *air* or whatever medium we are considering. There is also a Doppler effect for *electromagnetic* waves in empty space, such as light waves or radio waves. In this case there is no medium that we can use as a reference to measure velocities, and all that matters is the *relative* velocity of source and receiver. (By contrast, the Doppler effect for sound does not depend simply on this relative velocity, as discussed in Example 16.17.)

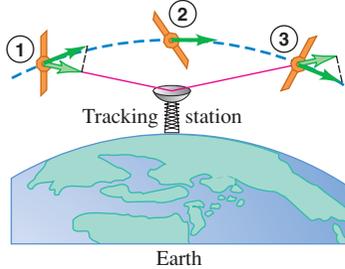
To derive the expression for the Doppler frequency shift for light, we have to use the special theory of relativity. We will discuss this in Chapter 37, but for now we quote the result without derivation. The wave speed is the speed of light, usually denoted by c , and it is the same for both source and receiver. In the frame of reference in which the receiver is at rest, the source is moving away from the receiver with velocity v . (If the source is *approaching* the receiver, v is negative.) The source frequency is again f_S . The frequency f_R measured by the receiver R (the frequency of arrival of the waves at the receiver) is then

$$f_R = \sqrt{\frac{c - v}{c + v}} f_S \quad (\text{Doppler effect for light}) \quad (16.30)$$

When v is positive, the source is moving directly *away* from the receiver and f_R is always *less* than f_S ; when v is negative, the source is moving directly *toward* the receiver and f_R is *greater* than f_S . The qualitative effect is the same as for sound, but the quantitative relationship is different.

A familiar application of the Doppler effect for radio waves is the radar device mounted on the side window of a police car to check other cars' speeds. The electromagnetic wave emitted by the device is reflected from a moving car, which acts as a moving source, and the wave reflected back to the device is Doppler-shifted in frequency. The transmitted and reflected signals are combined to produce beats, and the speed can be computed from the frequency of the beats. Similar techniques ("Doppler radar") are used to measure wind velocities in the atmosphere.

16.34 Change of velocity component along the line of sight of a satellite passing a tracking station. The frequency received at the tracking station changes from high to low as the satellite passes overhead.



The Doppler effect is also used to track satellites and other space vehicles. In Fig. 16.34 a satellite emits a radio signal with constant frequency f_S . As the satellite orbits past, it first approaches and then moves away from the receiver; the frequency f_R of the signal received on earth changes from a value greater than f_S to a value less than f_S as the satellite passes overhead.

Test Your Understanding of Section 16.8 You are at an outdoor concert with a wind blowing at 10 m/s from the performers toward you. Is the sound you hear Doppler-shifted? If so, is it shifted to lower or higher frequencies? **I**

16.9 Shock Waves

You may have experienced “sonic booms” caused by an airplane flying overhead faster than the speed of sound. We can see qualitatively why this happens from Fig. 16.35. Let v_S denote the *speed* of the airplane relative to the air, so that it is always positive. The motion of the airplane through the air produces sound; if v_S is less than the speed of sound v , the waves in front of the airplane are crowded together with a wavelength given by Eq. (16.27):

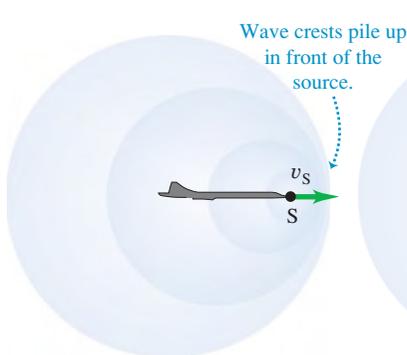
$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S}$$

As the speed v_S of the airplane approaches the speed of sound v , the wavelength approaches zero and the wave crests pile up on each other (Fig. 16.35a). The airplane must exert a large force to compress the air in front of it; by Newton’s third law, the air exerts an equally large force back on the airplane. Hence there is a large increase in aerodynamic drag (air resistance) as the airplane approaches the speed of sound, a phenomenon known as the “sound barrier.”

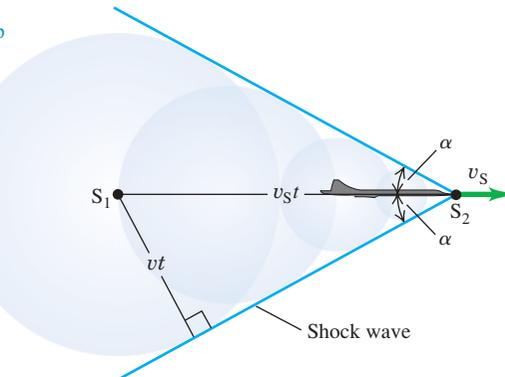
When v_S is greater in magnitude than v , the source of sound is **supersonic**, and Eqs. (16.27) and (16.29) for the Doppler effect no longer describe the sound wave in front of the source. Figure 16.35b shows a cross section of what happens. As the airplane moves, it displaces the surrounding air and produces sound. A series of wave crests is emitted from the nose of the airplane; each spreads out in a circle centered at the position of the airplane when it emitted the crest. After a time t the crest emitted from point S_1 has spread to a circle with radius vt , and the airplane has moved a greater distance $v_S t$ to position S_2 . You can see that the circular crests interfere constructively at points along the blue line that makes an angle α with

16.35 Wave crests around a sound source S moving (a) slightly slower than the speed of sound v and (b) faster than the sound speed v . (c) This photograph shows a T-38 jet airplane moving at 1.1 times the speed of sound. Separate shock waves are produced by the nose, wings, and tail. The angles of these waves vary because the air speeds up and slows down as it moves around the airplane, so the relative speed v_S of the airplane and air is different for shock waves produced at different points.

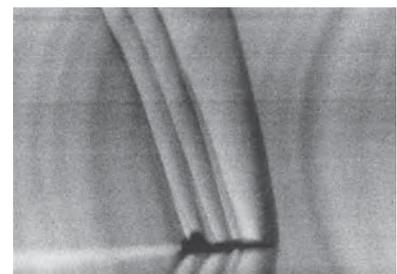
(a) Sound source S (airplane) moving at nearly the speed of sound



(b) Sound source moving faster than the speed of sound



(c) Shock waves around a supersonic airplane



the direction of the airplane velocity, leading to a very-large-amplitude wave crest along this line. This large-amplitude crest is called a **shock wave** (Fig. 16.35c).

From the right triangle in Fig. 16.35b we can see that the angle α is given by

$$\sin \alpha = \frac{vt}{v_S t} = \frac{v}{v_S} \quad (\text{shock wave}) \quad (16.31)$$

In this relationship, v_S is the *speed* of the source (the magnitude of its velocity) relative to the air and is always positive. The ratio v_S/v is called the **Mach number**. It is greater than unity for all supersonic speeds, and $\sin \alpha$ in Eq. (16.31) is the reciprocal of the Mach number. The first person to break the sound barrier was Capt. Chuck Yeager of the U.S. Air Force, flying the Bell X-1 at Mach 1.06 on October 14, 1947 (Fig. 16.36).

Shock waves are actually three-dimensional; a shock wave forms a *cone* around the direction of motion of the source. If the source (possibly a supersonic jet airplane or a rifle bullet) moves with constant velocity, the angle α is constant, and the shock-wave cone moves along with the source. It's the arrival of this shock wave that causes the sonic boom you hear after a supersonic airplane has passed by. The larger the airplane, the stronger the sonic boom; the shock wave produced at ground level by the (now retired) Concorde supersonic airliner flying at 12,000 m (40,000 ft) caused a sudden jump in air pressure of about 20 Pa. In front of the shock-wave cone, there is no sound. Inside the cone a stationary listener hears the Doppler-shifted sound of the airplane moving away.

CAUTION Shock waves We emphasize that a shock wave is produced *continuously* by any object that moves through the air at supersonic speed, not only at the instant that it “breaks the sound barrier.” The sound waves that combine to form the shock wave, as in Fig. 16.35b, are created by the motion of the object itself, not by any sound source that the object may carry. The cracking noises of a bullet and of the tip of a circus whip are due to their supersonic motion. A supersonic jet airplane may have very loud engines, but these do not cause the shock wave. Indeed, a space shuttle makes a very loud sonic boom when coming in for a landing; its engines are out of fuel at this point, so it is a supersonic glider. |

Shock waves have applications outside of aviation. They are used to break up kidney stones and gallstones without invasive surgery, using a technique with the impressive name *extracorporeal shock-wave lithotripsy*. A shock wave produced outside the body is focused by a reflector or acoustic lens so that as much of it as possible converges on the stone. When the resulting stresses in the stone exceed its tensile strength, it breaks into small pieces and can be eliminated. This technique requires accurate determination of the location of the stone, which may be done using ultrasonic imaging techniques (see Fig. 16.9).

Example 16.19 Sonic boom from a supersonic airplane

An airplane is flying at Mach 1.75 at an altitude of 8000 m, where the speed of sound is 320 m/s. How long after the plane passes directly overhead will you hear the sonic boom?

SOLUTION

IDENTIFY and SET UP: The shock wave forms a cone trailing backward from the airplane, so the problem is really asking for how much time elapses from when the airplane flies overhead to when the shock wave reaches you at point L (Fig. 16.37). During the time t (our target variable) since the airplane traveling at speed

v_S passed overhead, it has traveled a distance $v_S t$. Equation (16.31) gives the shock cone angle α ; we use trigonometry to solve for t .

EXECUTE: From Eq. (16.31) the angle α of the shock cone is

$$\alpha = \arcsin \frac{1}{1.75} = 34.8^\circ$$

The speed of the plane is the speed of sound multiplied by the Mach number:

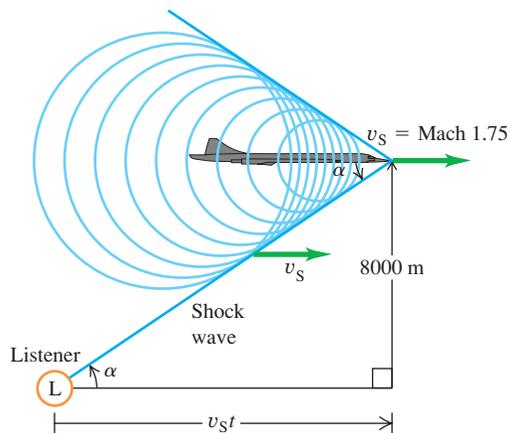
$$v_S = (1.75)(320 \text{ m/s}) = 560 \text{ m/s}$$

16.36 The first supersonic airplane, the Bell X-1, was shaped much like a 50-caliber bullet—which was known to be able to travel faster than sound.



Continued

16.37 You hear a sonic boom when the shock wave reaches you at L (*not* just when the plane breaks the sound barrier). A listener to the right of L has not yet heard the sonic boom but will shortly; a listener to the left of L has already heard the sonic boom.



From Fig. 16.37 we have

$$\tan \alpha = \frac{8000 \text{ m}}{v_S t}$$

$$t = \frac{8000 \text{ m}}{(560 \text{ m/s})(\tan 34.8^\circ)} = 20.5 \text{ s}$$

EVALUATE: You hear the boom 20.5 s after the airplane passes overhead, at which time it has traveled $(560 \text{ m/s})(20.5 \text{ s}) = 11.5 \text{ km}$ since it passed overhead. We have assumed that the speed of sound is the same at all altitudes, so that $\alpha = \arcsin v/v_S$ is constant and the shock wave forms a perfect cone. In fact, the speed of sound decreases with increasing altitude. How would this affect the value of t ?

Test Your Understanding of Section 16.9 What would you hear if you were directly behind (to the left of) the supersonic airplane in Fig. 16.37? (i) a sonic boom; (ii) the sound of the airplane, Doppler-shifted to higher frequencies; (iii) the sound of the airplane, Doppler-shifted to lower frequencies; (iv) nothing.

Sound waves: Sound consists of longitudinal waves in a medium. A sinusoidal sound wave is characterized by its frequency f and wavelength λ (or angular frequency ω and wave number k) and by its displacement amplitude A . The pressure amplitude p_{\max} is directly proportional to the displacement amplitude, the wave number, and the bulk modulus B of the wave medium. (See Examples 16.1 and 16.2.)

The speed of a sound wave in a fluid depends on the bulk modulus B and density ρ . If the fluid is an ideal gas, the speed can be expressed in terms of the temperature T , molar mass M , and ratio of heat capacities γ of the gas. The speed of longitudinal waves in a solid rod depends on the density and Young's modulus Y . (See Examples 16.3 and 16.4.)

$$p_{\max} = BkA \quad (16.5)$$

(sinusoidal sound wave)

$$v = \sqrt{\frac{B}{\rho}} \quad (16.7)$$

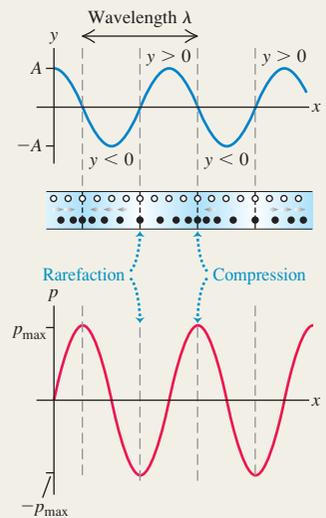
(longitudinal wave in a fluid)

$$v = \sqrt{\frac{\gamma RT}{M}} \quad (16.10)$$

(sound wave in an ideal gas)

$$v = \sqrt{\frac{Y}{\rho}} \quad (16.8)$$

(longitudinal wave in a solid rod)



Intensity and sound intensity level: The intensity I of a sound wave is the time average rate at which energy is transported by the wave, per unit area. For a sinusoidal wave, the intensity can be expressed in terms of the displacement amplitude A or the pressure amplitude p_{\max} . (See Examples 16.5–16.7.)

The sound intensity level β of a sound wave is a logarithmic measure of its intensity. It is measured relative to I_0 , an arbitrary intensity defined to be 10^{-12} W/m^2 . Sound intensity levels are expressed in decibels (dB). (See Examples 16.8 and 16.9.)

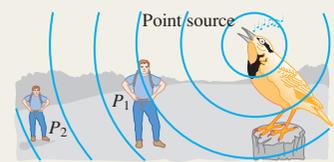
$$I = \frac{1}{2} \sqrt{\rho B} \omega^2 A^2 = \frac{p_{\max}^2}{2\rho v}$$

$$= \frac{p_{\max}^2}{2\sqrt{\rho B}} \quad (16.12), (16.14)$$

(intensity of a sinusoidal sound wave)

$$\beta = (10 \text{ dB}) \log \frac{I}{I_0} \quad (16.15)$$

(definition of sound intensity level)



Standing sound waves: Standing sound waves can be set up in a pipe or tube. A closed end is a displacement node and a pressure antinode; an open end is a displacement antinode and a pressure node. For a pipe of length L open at both ends, the normal-mode frequencies are integer multiples of the sound speed divided by $2L$. For a stopped pipe (one that is open at only one end), the normal-mode frequencies are the odd multiples of the sound speed divided by $4L$. (See Examples 16.10 and 16.11.)

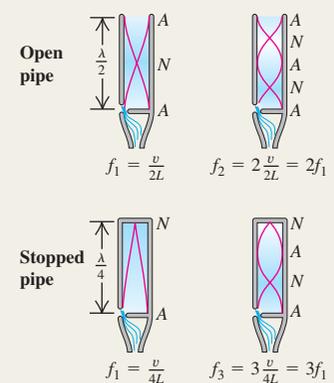
A pipe or other system with normal-mode frequencies can be driven to oscillate at any frequency. A maximum response, or resonance, occurs if the driving frequency is close to one of the normal-mode frequencies of the system. (See Example 16.12.)

$$f_n = \frac{nv}{2L} \quad (n = 1, 2, 3, \dots) \quad (16.18)$$

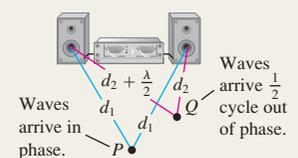
(open pipe)

$$f_n = \frac{nv}{4L} \quad (n = 1, 3, 5, \dots) \quad (16.22)$$

(stopped pipe)



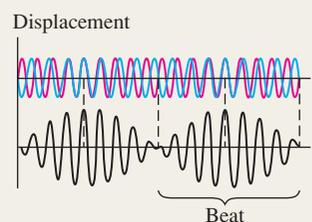
Interference: When two or more waves overlap in the same region of space, the resulting effects are called interference. The resulting amplitude can be either larger or smaller than the amplitude of each individual wave, depending on whether the waves are in phase (constructive interference) or out of phase (destructive interference). (See Example 16.13.)



Beats: Beats are heard when two tones with slightly different frequencies f_a and f_b are sounded together. The beat frequency f_{beat} is the difference between f_a and f_b .

$$f_{\text{beat}} = f_a - f_b \quad (16.24)$$

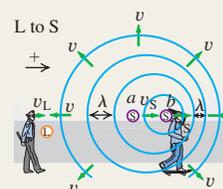
(beat frequency)



Doppler effect: The Doppler effect for sound is the frequency shift that occurs when there is motion of a source of sound, a listener, or both, relative to the medium. The source and listener frequencies f_S and f_L are related by the source and listener velocities v_S and v_L relative to the medium and to the speed of sound v . (See Examples 16.14–16.18.)

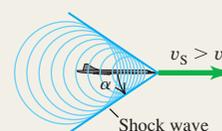
$$f_L = \frac{v + v_L}{v + v_S} f_S \quad (16.29)$$

(Doppler effect, moving source and moving listener)



Shock waves: A sound source moving with a speed v_S greater than the speed of sound v creates a shock wave. The wave front is a cone with angle α . (See Example 16.19.)

$$\sin \alpha = \frac{v}{v_S} \quad (\text{shock wave}) \quad (16.31)$$



BRIDGING PROBLEM

Loudspeaker Interference

Loudspeakers A and B are 7.00 m apart and vibrate in phase at 172 Hz. They radiate sound uniformly in all directions. Their acoustic power outputs are 8.00×10^{-4} W and 6.00×10^{-5} W, respectively. The air temperature is 20°C. (a) Determine the difference in phase of the two signals at a point C along the line joining A and B , 3.00 m from B and 4.00 m from A . (b) Determine the intensity and sound intensity level at C from speaker A alone (with B turned off) and from speaker B alone (with A turned off). (c) Determine the intensity and sound intensity level at C from both speakers together.

SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.



IDENTIFY and SET UP

- Sketch the situation and label the distances between A , B , and C .
- Choose the equations that relate power, distance from the source, intensity, pressure amplitude, and sound intensity level.
- Decide how you will determine the phase difference in part (a). Once you have found the phase difference, how can you use it to find the amplitude of the combined wave at C due to both sources?

- List the unknown quantities for each part of the problem and identify your target variables.

EXECUTE

- Determine the phase difference at point C .
- Find the intensity, sound intensity level, and pressure amplitude at C due to each speaker alone.
- Use your results from steps 5 and 6 to find the pressure amplitude at C due to both loudspeakers together.
- Use your result from step 7 to find the intensity and sound intensity level at C due to both loudspeakers together.

EVALUATE

- How do your results from part (c) for intensity and sound intensity level at C compare to those from part (b)? Does this make sense?
- What result would you have gotten in part (c) if you had (incorrectly) combined the *intensities* from A and B directly, rather than (correctly) combining the *pressure amplitudes* as you did in step 7?

Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q16.1 When sound travels from air into water, does the frequency of the wave change? The speed? The wavelength? Explain your reasoning.

Q16.2 The hero of a western movie listens for an oncoming train by putting his ear to the track. Why does this method give an earlier warning of the approach of a train than just listening in the usual way?

Q16.3 Would you expect the pitch (or frequency) of an organ pipe to increase or decrease with increasing temperature? Explain.

Q16.4 In most modern wind instruments the pitch is changed by using keys or valves to change the length of the vibrating air column. The bugle, however, has no valves or keys, yet it can play many notes. How might this be possible? Are there restrictions on what notes a bugle can play?

Q16.5 Symphonic musicians always “warm up” their wind instruments by blowing into them before a performance. What purpose does this serve?

Q16.6 In a popular and amusing science demonstration, a person inhales helium and then his voice becomes high and squeaky. Why does this happen? (*Warning*: Inhaling too much helium can cause unconsciousness or death.)

Q16.7 Lane dividers on highways sometimes have regularly spaced ridges or ripples. When the tires of a moving car roll along such a divider, a musical note is produced. Why? Explain how this phenomenon could be used to measure the car’s speed.

Q16.8 The tone quality of an acoustic guitar is different when the strings are plucked near the bridge (the lower end of the strings) than when they are plucked near the sound hole (close to the center of the strings). Why?

Q16.9 Which has a more direct influence on the loudness of a sound wave: the *displacement* amplitude or the *pressure* amplitude? Explain your reasoning.

Q16.10 If the pressure amplitude of a sound wave is halved, by what factor does the intensity of the wave decrease? By what factor must the pressure amplitude of a sound wave be increased in order to increase the intensity by a factor of 16? Explain.

Q16.11 Does the sound intensity level β obey the inverse-square law? Why?

Q16.12 A small fraction of the energy in a sound wave is absorbed by the air through which the sound passes. How does this modify the inverse-square relationship between intensity and distance from the source? Explain your reasoning.

Q16.13 A wire under tension and vibrating in its first overtone produces sound of wavelength λ . What is the new wavelength of the sound (in terms of λ) if the tension is doubled?

Q16.14 A small metal band is slipped onto one of the tines of a tuning fork. As this band is moved closer and closer to the end of the tine, what effect does this have on the wavelength and frequency of the sound the tine produces? Why?

Q16.15 An organist in a cathedral plays a loud chord and then releases the keys. The sound persists for a few seconds and gradually dies away. Why does it persist? What happens to the sound energy when the sound dies away?

Q16.16 Two vibrating tuning forks have identical frequencies, but one is stationary and the other is mounted at the rim of a rotating platform. What does a listener hear? Explain.

Q16.17 A large church has part of the organ in the front of the church and part in the back. A person walking rapidly down the aisle while both segments are playing at once reports that the two segments sound out of tune. Why?

Q16.18 A sound source and a listener are both at rest on the earth, but a strong wind is blowing from the source toward the listener. Is there a Doppler effect? Why or why not?

Q16.19 Can you think of circumstances in which a Doppler effect would be observed for surface waves in water? For elastic waves propagating in a body of water deep below the surface? If so, describe the circumstances and explain your reasoning. If not, explain why not.

Q16.20 Stars other than our sun normally appear featureless when viewed through telescopes. Yet astronomers can readily use the light from these stars to determine that they are rotating and even measure the speed of their surface. How do you think they can do this?

Q16.21 If you wait at a railroad crossing as a train approaches and passes, you hear a Doppler shift in its sound. But if you listen closely, you hear that the change in frequency is continuous; it does not suddenly go from one high frequency to another low frequency. Instead the frequency *smoothly* (but rather quickly) changes from high to low as the train passes. Why does this smooth change occur?

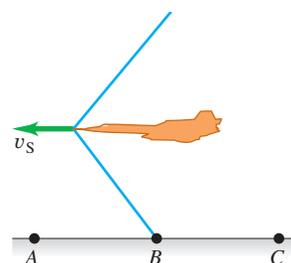
Q16.22 In case 1, a source of sound approaches a stationary observer at speed v . In case 2, the observer moves toward the stationary source at the same speed v . If the source is always producing the same frequency sound, will the observer hear the same frequency in both cases, since the relative speed is the same each time? Why or why not?

Q16.23 Does an aircraft make a sonic boom only at the instant its speed exceeds Mach 1? Explain your reasoning.

Q16.24 If you are riding in a supersonic aircraft, what do you hear? Explain your reasoning. In particular, do you hear a continuous sonic boom? Why or why not?

Q16.25 A jet airplane is flying at a constant altitude at a steady speed v_S greater than the speed of sound. Describe what observers at points A , B , and C hear at the instant shown in Fig. Q16.25, when the shock wave has just reached point B . Explain your reasoning.

Figure Q16.25



EXERCISES

Unless indicated otherwise, assume the speed of sound in air to be $v = 344$ m/s.

Section 16.1 Sound Waves

16.1 • Example 16.1 (Section 16.1) showed that for sound waves in air with frequency 1000 Hz, a displacement amplitude of

1.2×10^{-8} m produces a pressure amplitude of 3.0×10^{-2} Pa. (a) What is the wavelength of these waves? (b) For 1000-Hz waves in air, what displacement amplitude would be needed for the pressure amplitude to be at the pain threshold, which is 30 Pa? (c) For what wavelength and frequency will waves with a displacement amplitude of 1.2×10^{-8} m produce a pressure amplitude of 1.5×10^{-3} Pa?

16.2 • Example 16.1 (Section 16.1) showed that for sound waves in air with frequency 1000 Hz, a displacement amplitude of 1.2×10^{-8} m produces a pressure amplitude of 3.0×10^{-2} Pa. Water at 20°C has a bulk modulus of 2.2×10^9 Pa, and the speed of sound in water at this temperature is 1480 m/s. For 1000-Hz sound waves in 20°C water, what displacement amplitude is produced if the pressure amplitude is 3.0×10^{-2} Pa? Explain why your answer is much less than 1.2×10^{-8} m.

16.3 • Consider a sound wave in air that has displacement amplitude 0.0200 mm. Calculate the pressure amplitude for frequencies of (a) 150 Hz; (b) 1500 Hz; (c) 15,000 Hz. In each case compare the result to the pain threshold, which is 30 Pa.

16.4 • A loud factory machine produces sound having a displacement amplitude of $1.00 \mu\text{m}$, but the frequency of this sound can be adjusted. In order to prevent ear damage to the workers, the maximum pressure amplitude of the sound waves is limited to 10.0 Pa. Under the conditions of this factory, the bulk modulus of air is 1.42×10^5 Pa. What is the highest-frequency sound to which this machine can be adjusted without exceeding the prescribed limit? Is this frequency audible to the workers?

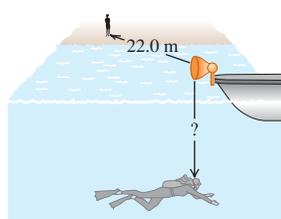
16.5 • BIO Ultrasound and Infrasound. (a) **Whale communication.** Blue whales apparently communicate with each other using sound of frequency 17 Hz, which can be heard nearly 1000 km away in the ocean. What is the wavelength of such a sound in seawater, where the speed of sound is 1531 m/s? (b) **Dolphin clicks.** One type of sound that dolphins emit is a sharp click of wavelength 1.5 cm in the ocean. What is the frequency of such clicks? (c) **Dog whistles.** One brand of dog whistles claims a frequency of 25 kHz for its product. What is the wavelength of this sound? (d) **Bats.** While bats emit a wide variety of sounds, one type emits pulses of sound having a frequency between 39 kHz and 78 kHz. What is the range of wavelengths of this sound? (e) **Sonograms.** Ultrasound is used to view the interior of the body, much as x rays are utilized. For sharp imagery, the wavelength of the sound should be around one-fourth (or less) the size of the objects to be viewed. Approximately what frequency of sound is needed to produce a clear image of a tumor that is 1.0 mm across if the speed of sound in the tissue is 1550 m/s?

Section 16.2 Speed of Sound Waves

16.6 • (a) In a liquid with density 1300 kg/m^3 , longitudinal waves with frequency 400 Hz are found to have wavelength 8.00 m. Calculate the bulk modulus of the liquid. (b) A metal bar with a length of 1.50 m has density 6400 kg/m^3 . Longitudinal sound waves take 3.90×10^{-4} s to travel from one end of the bar to the other. What is Young's modulus for this metal?

16.7 • A submerged scuba diver hears the sound of a boat horn directly above her on the surface of the lake. At the same time, a friend on dry land 22.0 m from the boat also hears the horn (Fig. E16.7). The horn is 1.2 m above the surface of the water.

Figure E16.7



What is the distance (labeled by “?” in Fig. E16.7) from the horn to the diver? Both air and water are at 20°C .

16.8 • At a temperature of 27.0°C , what is the speed of longitudinal waves in (a) hydrogen (molar mass 2.02 g/mol); (b) helium (molar mass 4.00 g/mol); (c) argon (molar mass 39.9 g/mol)? See Table 19.1 for values of γ . (d) Compare your answers for parts (a), (b), and (c) with the speed in air at the same temperature.

16.9 • An oscillator vibrating at 1250 Hz produces a sound wave that travels through an ideal gas at 325 m/s when the gas temperature is 22.0°C . For a certain experiment, you need to have the same oscillator produce sound of wavelength 28.5 cm in this gas. What should the gas temperature be to achieve this wavelength?

16.10 •• CALC (a) Show that the fractional change in the speed of sound (dv/v) due to a very small temperature change dT is given by $dv/v = \frac{1}{2}dT/T$. (Hint: Start with Eq. 16.10.) (b) The speed of sound in air at 20°C is found to be 344 m/s. Use the result in part (a) to find the change in the speed of sound for a 1.0°C change in air temperature.

16.11 •• An 80.0-m-long brass rod is struck at one end. A person at the other end hears two sounds as a result of two longitudinal waves, one traveling in the metal rod and the other traveling in the air. What is the time interval between the two sounds? (The speed of sound in air is 344 m/s; relevant information about brass can be found in Table 11.1 and Table 12.1.)

16.12 •• What must be the stress (F/A) in a stretched wire of a material whose Young's modulus is Y for the speed of longitudinal waves to equal 30 times the speed of transverse waves?

Section 16.3 Sound Intensity

16.13 •• BIO Energy Delivered to the Ear. Sound is detected when a sound wave causes the tympanic membrane (the eardrum) to vibrate. Typically, the diameter of this membrane is about 8.4 mm in humans. (a) How much energy is delivered to the eardrum each second when someone whispers (20 dB) a secret in your ear? (b) To comprehend how sensitive the ear is to very small amounts of energy, calculate how fast a typical 2.0-mg mosquito would have to fly (in mm/s) to have this amount of kinetic energy.

16.14 • Use information from Table 16.2 to answer the following questions about sound in air. At 20°C the bulk modulus for air is 1.42×10^5 Pa and its density is 1.20 kg/m^3 . At this temperature, what are the pressure amplitude (in Pa and atm) and the displacement amplitude (in m and nm) (a) for the softest sound a person can normally hear at 1000 Hz and (b) for the sound from a riveter at the same frequency? (c) How much energy per second does each wave deliver to a square 5.00 mm on a side?

16.15 •• Longitudinal Waves in Different Fluids. (a) A longitudinal wave propagating in a water-filled pipe has intensity $3.00 \times 10^{-6} \text{ W/m}^2$ and frequency 3400 Hz. Find the amplitude A and wavelength λ of the wave. Water has density 1000 kg/m^3 and bulk modulus 2.18×10^9 Pa. (b) If the pipe is filled with air at pressure 1.00×10^5 Pa and density 1.20 kg/m^3 , what will be the amplitude A and wavelength λ of a longitudinal wave with the same intensity and frequency as in part (a)? (c) In which fluid is the amplitude larger, water or air? What is the ratio of the two amplitudes? Why is this ratio so different from 1.00?

16.16 •• BIO Human Hearing. A fan at a rock concert is 30 m from the stage, and at this point the sound intensity level is 110 dB. (a) How much energy is transferred to her eardrums each second? (b) How fast would a 2.0-mg mosquito have to fly (in mm/s) to have this much kinetic energy? Compare the mosquito's speed with that found for the whisper in part (a) of Exercise 16.13.

16.17 • A sound wave in air at 20°C has a frequency of 150 Hz and a displacement amplitude of 5.00×10^{-3} mm. For this sound wave calculate the (a) pressure amplitude (in Pa); (b) intensity (in W/m^2); (c) sound intensity level (in decibels).

16.18 •• You live on a busy street, but as a music lover, you want to reduce the traffic noise. (a) If you install special sound-reflecting windows that reduce the sound intensity level (in dB) by 30 dB, by what fraction have you lowered the sound intensity (in W/m^2)? (b) If, instead, you reduce the intensity by half, what change (in dB) do you make in the sound intensity level?

16.19 • **BIO** For a person with normal hearing, the faintest sound that can be heard at a frequency of 400 Hz has a pressure amplitude of about 6.0×10^{-5} Pa. Calculate the (a) intensity; (b) sound intensity level; (c) displacement amplitude of this sound wave at 20°C.

16.20 •• The intensity due to a number of independent sound sources is the sum of the individual intensities. (a) When four quadruplets cry simultaneously, how many decibels greater is the sound intensity level than when a single one cries? (b) To increase the sound intensity level again by the same number of decibels as in part (a), how many more crying babies are required?

16.21 • **CP** A baby's mouth is 30 cm from her father's ear and 1.50 m from her mother's ear. What is the difference between the sound intensity levels heard by the father and by the mother?

16.22 •• The Sacramento City Council adopted a law to reduce the allowed sound intensity level of the much-despised leaf blowers from their current level of about 95 dB to 70 dB. With the new law, what is the ratio of the new allowed intensity to the previously allowed intensity?

16.23 •• **CP** At point A, 3.0 m from a small source of sound that is emitting uniformly in all directions, the sound intensity level is 53 dB. (a) What is the intensity of the sound at A? (b) How far from the source must you go so that the intensity is one-fourth of what it was at A? (c) How far must you go so that the sound intensity level is one-fourth of what it was at A? (d) Does intensity obey the inverse-square law? What about sound intensity level?

16.24 •• (a) If two sounds differ by 5.00 dB, find the ratio of the intensity of the louder sound to that of the softer one. (b) If one sound is 100 times as intense as another, by how much do they differ in sound intensity level (in decibels)? (c) If you increase the volume of your stereo so that the intensity doubles, by how much does the sound intensity level increase?

Section 16.4 Standing Sound Waves and Normal Modes

16.25 • Standing sound waves are produced in a pipe that is 1.20 m long. For the fundamental and first two overtones, determine the locations along the pipe (measured from the left end) of the displacement nodes and the pressure nodes if (a) the pipe is open at both ends and (b) the pipe is closed at the left end and open at the right end.

16.26 • The fundamental frequency of a pipe that is open at both ends is 594 Hz. (a) How long is this pipe? If one end is now closed, find (b) the wavelength and (c) the frequency of the new fundamental.

16.27 • **BIO The Human Voice.** The human vocal tract is a pipe that extends about 17 cm from the lips to the vocal folds (also called "vocal cords") near the middle of your throat. The vocal folds behave rather like the reed of a clarinet, and the vocal tract acts like a stopped pipe. Estimate the first three standing-wave frequencies of the vocal tract. Use $v = 344$ m/s. (The answers are only an estimate, since the position of lips and tongue affects the motion of air in the vocal tract.)

16.28 •• **BIO The Vocal Tract.** Many opera singers (and some pop singers) have a range of about $2\frac{1}{2}$ octaves or even greater. Suppose a soprano's range extends from A below middle C (frequency 220 Hz) up to E^b-flat above high C (frequency 1244 Hz). Although the vocal tract is quite complicated, we can model it as a resonating air column, like an organ pipe, that is open at the top and closed at the bottom. The column extends from the mouth down to the diaphragm in the chest cavity, and we can also assume that the lowest note is the fundamental. How long is this column of air if $v = 354$ m/s? Does your result seem reasonable, on the basis of observations of your own body?

16.29 •• A certain pipe produces a fundamental frequency of 262 Hz in air. (a) If the pipe is filled with helium at the same temperature, what fundamental frequency does it produce? (The molar mass of air is 28.8 g/mol, and the molar mass of helium is 4.00 g/mol.) (b) Does your answer to part (a) depend on whether the pipe is open or stopped? Why or why not?

16.30 • **Singing in the Shower.** A pipe closed at both ends can have standing waves inside of it, but you normally don't hear them because little of the sound can get out. But you *can* hear them if you are *inside* the pipe, such as someone singing in the shower. (a) Show that the wavelengths of standing waves in a pipe of length L that is closed at both ends are $\lambda_n = 2L/n$ and the frequencies are given by $f_n = nv/2L = nf_1$, where $n = 1, 2, 3, \dots$ (b) Modeling it as a pipe, find the frequency of the fundamental and the first two overtones for a shower 2.50 m tall. Are these frequencies audible?

Section 16.5 Resonance and Sound

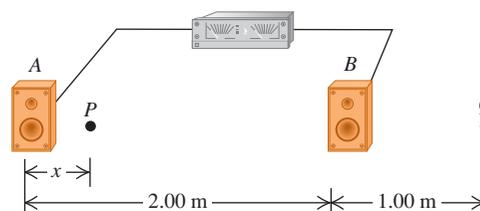
16.31 • You blow across the open mouth of an empty test tube and produce the fundamental standing wave of the air column inside the test tube. The speed of sound in air is 344 m/s and the test tube acts as a stopped pipe. (a) If the length of the air column in the test tube is 14.0 cm, what is the frequency of this standing wave? (b) What is the frequency of the fundamental standing wave in the air column if the test tube is half filled with water?

16.32 •• **CP** You have a stopped pipe of adjustable length close to a taut 85.0-cm, 7.25-g wire under a tension of 4110 N. You want to adjust the length of the pipe so that, when it produces sound at its fundamental frequency, this sound causes the wire to vibrate in its second *overtone* with very large amplitude. How long should the pipe be?

Section 16.6 Interference of Waves

16.33 • Two loudspeakers, A and B (Fig. E16.33), are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 2.00 m to the right of speaker A. Consider point Q along the extension of the line connecting the speakers, 1.00 m to the right of speaker B. Both speakers emit sound waves that travel directly from the speaker to point Q. (a) What is the lowest frequency for which *constructive* interference occurs at point Q? (b) What is the lowest frequency for which *destructive* interference occurs at point Q?

Figure E16.33



16.34 •• Two loudspeakers, *A* and *B* (see Fig. E16.33), are driven by the same amplifier and emit sinusoidal waves in phase. Speaker *B* is 2.00 m to the right of speaker *A*. The frequency of the sound waves produced by the loudspeakers is 206 Hz. Consider point *P* between the speakers and along the line connecting them, a distance *x* to the right of speaker *A*. Both speakers emit sound waves that travel directly from the speaker to point *P*. (a) For what values of *x* will *destructive* interference occur at point *P*? (b) For what values of *x* will *constructive* interference occur at point *P*? (c) Interference effects like those in parts (a) and (b) are almost never a factor in listening to home stereo equipment. Why not?

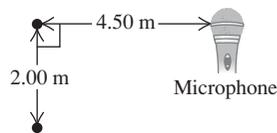
16.35 •• Two loudspeakers, *A* and *B*, are driven by the same amplifier and emit sinusoidal waves in phase. Speaker *B* is 12.0 m to the right of speaker *A*. The frequency of the waves emitted by each speaker is 688 Hz. You are standing between the speakers, along the line connecting them, and are at a point of constructive interference. How far must you walk toward speaker *B* to move to a point of destructive interference?

16.36 • Two loudspeakers, *A* and *B*, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves emitted by each speaker is 172 Hz. You are 8.00 m from *A*. What is the closest you can be to *B* and be at a point of destructive interference?

16.37 • Two loudspeakers, *A* and *B*, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves emitted by each speaker is 860 Hz. Point *P* is 12.0 m from *A* and 13.4 m from *B*. Is the interference at *P* constructive or destructive? Give the reasoning behind your answer.

16.38 •• Two small stereo speakers are driven in step by the same variable-frequency oscillator. Their sound is picked up by a microphone arranged as shown in Fig. E16.38. For what frequencies does their sound at the speakers produce (a) constructive interference and (b) destructive interference?

Figure E16.38



Section 16.7 Beats

16.39 •• Tuning a Violin. A violinist is tuning her instrument to concert A (440 Hz). She plays the note while listening to an electronically generated tone of exactly that frequency and hears a beat of frequency 3 Hz, which increases to 4 Hz when she tightens her violin string slightly. (a) What was the frequency of the note played by her violin when she heard the 3-Hz beat? (b) To get her violin perfectly tuned to concert A, should she tighten or loosen her string from what it was when she heard the 3-Hz beat?

16.40 •• Two guitarists attempt to play the same note of wavelength 6.50 cm at the same time, but one of the instruments is slightly out of tune and plays a note of wavelength 6.52 cm instead. What is the frequency of the beat these musicians hear when they play together?

16.41 •• Two organ pipes, open at one end but closed at the other, are each 1.14 m long. One is now lengthened by 2.00 cm. Find the frequency of the beat they produce when playing together in their fundamental.

16.42 •• Adjusting Airplane Motors. The motors that drive airplane propellers are, in some cases, tuned by using beats. The whirring motor produces a sound wave having the same frequency as the propeller. (a) If one single-bladed propeller is turning at 575 rpm and you hear a 2.0-Hz beat when you run the second propeller, what are the two possible frequencies (in rpm) of the second

propeller? (b) Suppose you increase the speed of the second propeller slightly and find that the beat frequency changes to 2.1 Hz. In part (a), which of the two answers was the correct one for the frequency of the second single-bladed propeller? How do you know?

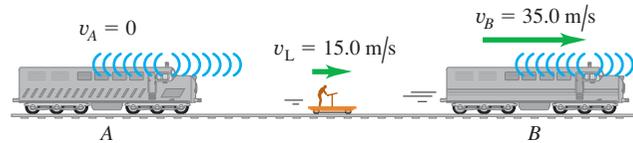
Section 16.8 The Doppler Effect

16.43 •• On the planet Arrakis a male ornithoid is flying toward his mate at 25.0 m/s while singing at a frequency of 1200 Hz. If the stationary female hears a tone of 1240 Hz, what is the speed of sound in the atmosphere of Arrakis?

16.44 •• In Example 16.18 (Section 16.8), suppose the police car is moving away from the warehouse at 20 m/s. What frequency does the driver of the police car hear reflected from the warehouse?

16.45 • Two train whistles, *A* and *B*, each have a frequency of 392 Hz. *A* is stationary and *B* is moving toward the right (away from *A*) at a speed of 35.0 m/s. A listener is between the two whistles and is moving toward the right with a speed of 15.0 m/s (Fig. E16.45). No wind is blowing. (a) What is the frequency from *A* as heard by the listener? (b) What is the frequency from *B* as heard by the listener? (c) What is the beat frequency detected by the listener?

Figure E16.45



16.46 • A railroad train is traveling at 25.0 m/s in still air. The frequency of the note emitted by the locomotive whistle is 400 Hz. What is the wavelength of the sound waves (a) in front of the locomotive and (b) behind the locomotive? What is the frequency of the sound heard by a stationary listener (c) in front of the locomotive and (d) behind the locomotive?

16.47 • A swimming duck paddles the water with its feet once every 1.6 s, producing surface waves with this period. The duck is moving at constant speed in a pond where the speed of surface waves is 0.32 m/s, and the crests of the waves ahead of the duck are spaced 0.12 m apart. (a) What is the duck's speed? (b) How far apart are the crests behind the duck?

16.48 • Moving Source vs. Moving Listener. (a) A sound source producing 1.00-kHz waves moves toward a stationary listener at one-half the speed of sound. What frequency will the listener hear? (b) Suppose instead that the source is stationary and the listener moves toward the source at one-half the speed of sound. What frequency does the listener hear? How does your answer compare to that in part (a)? Explain on physical grounds why the two answers differ.

16.49 • A car alarm is emitting sound waves of frequency 520 Hz. You are on a motorcycle, traveling directly away from the car. How fast must you be traveling if you detect a frequency of 490 Hz?

16.50 • A railroad train is traveling at 30.0 m/s in still air. The frequency of the note emitted by the train whistle is 262 Hz. What frequency is heard by a passenger on a train moving in the opposite direction to the first at 18.0 m/s and (a) approaching the first and (b) receding from the first?

16.51 • Two swift canaries fly toward each other, each moving at 15.0 m/s relative to the ground, each warbling a note of frequency 1750 Hz. (a) What frequency note does each bird hear from the

other one? (b) What wavelength will each canary measure for the note from the other one?

16.52 •• The siren of a fire engine that is driving northward at 30.0 m/s emits a sound of frequency 2000 Hz. A truck in front of this fire engine is moving northward at 20.0 m/s. (a) What is the frequency of the siren's sound that the fire engine's driver hears reflected from the back of the truck? (b) What wavelength would this driver measure for these reflected sound waves?

16.53 •• How fast (as a percentage of light speed) would a star have to be moving so that the frequency of the light we receive from it is 10.0% higher than the frequency of the light it is emitting? Would it be moving away from us or toward us? (Assume it is moving either directly away from us or directly toward us.)

16.54 • Extrasolar Planets. In the not-too-distant future, it should be possible to detect the presence of planets moving around other stars by measuring the Doppler shift in the infrared light they emit. If a planet is going around its star at 50.00 km/s while emitting infrared light of frequency 3.330×10^{14} Hz, what frequency light will be received from this planet when it is moving directly away from us? (Note: Infrared light is light having wavelengths longer than those of visible light.)

Section 16.9 Shock Waves

16.55 •• A jet plane flies overhead at Mach 1.70 and at a constant altitude of 950 m. (a) What is the angle α of the shock-wave cone? (b) How much time after the plane passes directly overhead do you hear the sonic boom? Neglect the variation of the speed of sound with altitude.

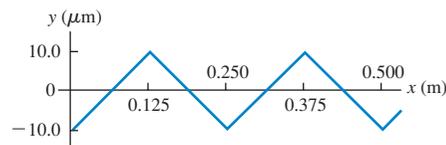
16.56 • The shock-wave cone created by the space shuttle at one instant during its reentry into the atmosphere makes an angle of 58.0° with its direction of motion. The speed of sound at this altitude is 331 m/s. (a) What is the Mach number of the shuttle at this instant, and (b) how fast (in m/s and in mi/h) is it traveling relative to the atmosphere? (c) What would be its Mach number and the angle of its shock-wave cone if it flew at the same speed but at low altitude where the speed of sound is 344 m/s?

PROBLEMS

16.57 ••• CP Two identical taut strings under the same tension F produce a note of the same fundamental frequency f_0 . The tension in one of them is now increased by a very small amount ΔF . (a) If they are played together in their fundamental, show that the frequency of the beat produced is $f_{\text{beat}} = f_0 (\Delta F / 2F)$. (b) Two identical violin strings, when in tune and stretched with the same tension, have a fundamental frequency of 440.0 Hz. One of the strings is retuned by increasing its tension. When this is done, 1.5 beats per second are heard when both strings are plucked simultaneously at their centers. By what percentage was the string tension changed?

16.58 •• CALC (a) Defend the following statement: "In a sinusoidal sound wave, the pressure variation given by Eq. (16.4) is greatest where the displacement given by Eq. (16.1) is zero." (b) For a sinusoidal sound wave given by Eq. (16.1) with amplitude $A = 10.0 \mu\text{m}$ and wavelength $\lambda = 0.250$ m, graph the displacement y and pressure fluctuation p as functions of x at time $t = 0$. Show at least two wavelengths of the wave on your graphs. (c) The displacement y in a nonsinusoidal sound wave is shown in Fig. P16.58 as a function of x for $t = 0$. Draw a graph showing the pressure fluctuation p in this wave as a function of x at $t = 0$. This sound wave has the same $10.0\text{-}\mu\text{m}$ amplitude as the wave in part (b). Does it have the same pressure amplitude? Why or why not? (d) Is the statement in part (a) necessarily true if the sound wave is not sinusoidal? Explain your reasoning.

Figure P16.58



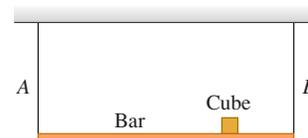
16.59 •• A soprano and a bass are singing a duet. While the soprano sings an A-sharp at 932 Hz, the bass sings an A-sharp but three octaves lower. In this concert hall, the density of air is 1.20 kg/m^3 and its bulk modulus is $1.42 \times 10^5 \text{ Pa}$. In order for their notes to have the same sound intensity level, what must be (a) the ratio of the pressure amplitude of the bass to that of the soprano and (b) the ratio of the displacement amplitude of the bass to that of the soprano? (c) What displacement amplitude (in m and in nm) does the soprano produce to sing her A-sharp at 72.0 dB?

16.60 •• CP The sound from a trumpet radiates uniformly in all directions in 20°C air. At a distance of 5.00 m from the trumpet the sound intensity level is 52.0 dB. The frequency is 587 Hz. (a) What is the pressure amplitude at this distance? (b) What is the displacement amplitude? (c) At what distance is the sound intensity level 30.0 dB?

16.61 ••• A Thermometer. Suppose you have a tube of length L containing a gas whose temperature you want to take, but you cannot get inside the tube. One end is closed, and the other end is open but a small speaker producing sound of variable frequency is at that end. You gradually increase the frequency of the speaker until the sound from the tube first becomes very loud. With further increase of the frequency, the loudness decreases but then gets very loud again at still higher frequencies. Call f_0 the lowest frequency at which the sound is very loud. (a) Show that the absolute temperature of this gas is given by $T = 16ML^2 f_0^2 / \gamma R$, where M is the molar mass of the gas, γ is the ratio of its heat capacities, and R is the ideal gas constant. (b) At what frequency above f_0 will the sound from the tube next reach a maximum in loudness? (c) How could you determine the speed of sound in this tube at temperature T ?

16.62 •• CP A uniform 165-N bar is supported horizontally by two identical wires A and B (Fig. P16.62). A small 185-N cube of lead is placed three-fourths of the way from A to B. The wires are each 75.0 cm long and have a mass of 5.50 g.

Figure P16.62



If both of them are simultaneously plucked at the center, what is the frequency of the beats that they will produce when vibrating in their fundamental?

16.63 • CP A person is playing a small flute 10.75 cm long, open at one end and closed at the other, near a taut string having a fundamental frequency of 600.0 Hz. If the speed of sound is 344.0 m/s, for which harmonics of the flute will the string resonate? In each case, which harmonic of the string is in resonance?

16.64 ••• CP A New Musical Instrument. You have designed a new musical instrument of very simple construction. Your design consists of a metal tube with length L and diameter $L/10$. You have stretched a string of mass per unit length μ across the open end of the tube. The other end of the tube is closed. To produce the musical effect you're looking for, you want the frequency of the third-harmonic standing wave on the string to be the same as the fundamental frequency for sound waves in the air column in the tube. The speed of sound waves in this air column is v_s . (a) What must

be the tension of the string to produce the desired effect? (b) What happens to the sound produced by the instrument if the tension is changed to twice the value calculated in part (a)? (c) For the tension calculated in part (a), what other harmonics of the string, if any, are in resonance with standing waves in the air column?

16.65 • An organ pipe has two successive harmonics with frequencies 1372 and 1764 Hz. (a) Is this an open or a stopped pipe? Explain. (b) What two harmonics are these? (c) What is the length of the pipe?

16.66 • **Longitudinal Standing Waves in a Solid.** Longitudinal standing waves can be produced in a solid rod by holding it at some point between the fingers of one hand and stroking it with the other hand. The rod oscillates with antinodes at both ends. (a) Why are the ends antinodes and not nodes? (b) The fundamental frequency can be obtained by stroking the rod while it is held at its center. Explain why this is the *only* place to hold the rod to obtain the fundamental. (c) Calculate the fundamental frequency of a steel rod of length 1.50 m (see Table 16.1). (d) What is the next possible standing-wave frequency of this rod? Where should the rod be held to excite a standing wave of this frequency?

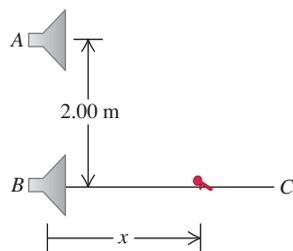
16.67 •• A long tube contains air at a pressure of 1.00 atm and a temperature of 77.0°C. The tube is open at one end and closed at the other by a movable piston. A tuning fork near the open end is vibrating with a frequency of 500 Hz. Resonance is produced when the piston is at distances 18.0, 55.5, and 93.0 cm from the open end. (a) From these measurements, what is the speed of sound in air at 77.0°C? (b) From the result of part (a), what is the value of γ ? (c) These data show that a displacement antinode is slightly outside of the open end of the tube. How far outside is it?

16.68 ••• The frequency of the note F_4 is 349 Hz. (a) If an organ pipe is open at one end and closed at the other, what length must it have for its fundamental mode to produce this note at 20.0°C? (b) At what air temperature will the frequency be 370 Hz, corresponding to a rise in pitch from F to F-sharp? (Ignore the change in length of the pipe due to the temperature change.)

16.69 • A standing wave with a frequency of 1100 Hz in a column of methane (CH_4) at 20.0°C produces nodes that are 0.200 m apart. What is the value of γ for methane? (The molar mass of methane is 16.0 g/mol.)

16.70 •• Two identical loudspeakers are located at points A and B, 2.00 m apart. The loudspeakers are driven by the same amplifier and produce sound waves with a frequency of 784 Hz. Take the speed of sound in air to be 344 m/s. A small microphone is moved out from point B along a line perpendicular to the line connecting A and B (line BC in Fig. P16.70). (a)

Figure P16.70



At what distances from B will there be *destructive* interference? (b) At what distances from B will there be *constructive* interference? (c) If the frequency is made low enough, there will be no positions along the line BC at which destructive interference occurs. How low must the frequency be for this to be the case?

16.71 • **Wagnerian Opera.** A man marries a great Wagnerian soprano but, alas, he discovers he cannot stand Wagnerian opera. In order to save his eardrums, the unhappy man decides he must silence his larklike wife for good. His plan is to tie her to the front of his car and send car and soprano speeding toward a brick wall.

her student days at the music conservatory. She realizes that this wall has a resonant frequency of 600 Hz, which means that if a continuous sound wave of this frequency hits the wall, it will fall down, and she will be saved to sing more Isolde. The car is heading toward the wall at a high speed of 30 m/s. (a) At what frequency must the soprano sing so that the wall will crumble? (b) What frequency will the soprano hear reflected from the wall just before it crumbles?

16.72 •• A bat flies toward a wall, emitting a steady sound of frequency 1.70 kHz. This bat hears its own sound plus the sound reflected by the wall. How fast should the bat fly in order to hear a beat frequency of 10.0 Hz?

16.73 •• **CP** A person leaning over a 125-m-deep well accidentally drops a siren emitting sound of frequency 2500 Hz. Just before this siren hits the bottom of the well, find the frequency and wavelength of the sound the person hears (a) coming directly from the siren and (b) reflected off the bottom of the well. (c) What beat frequency does this person perceive?

16.74 ••• **BIO Ultrasound in Medicine.** A 2.00-MHz sound wave travels through a pregnant woman's abdomen and is reflected from the fetal heart wall of her unborn baby. The heart wall is moving toward the sound receiver as the heart beats. The reflected sound is then mixed with the transmitted sound, and 72 beats per second are detected. The speed of sound in body tissue is 1500 m/s. Calculate the speed of the fetal heart wall at the instant this measurement is made.

16.75 •• The sound source of a ship's sonar system operates at a frequency of 22.0 kHz. The speed of sound in water (assumed to be at a uniform 20°C) is 1482 m/s. (a) What is the wavelength of the waves emitted by the source? (b) What is the difference in frequency between the directly radiated waves and the waves reflected from a whale traveling directly toward the ship at 4.95 m/s? The ship is at rest in the water.

16.76 • **CP** A police siren of frequency f_{siren} is attached to a vibrating platform. The platform and siren oscillate up and down in simple harmonic motion with amplitude A_p and frequency f_p . (a) Find the maximum and minimum sound frequencies that you would hear at a position directly above the siren. (b) At what point in the motion of the platform is the maximum frequency heard? The minimum frequency? Explain.

16.77 ••• **BIO** Horseshoe bats (genus *Rhinolophus*) emit sounds from their nostrils and then listen to the frequency of the sound reflected from their prey to determine the prey's speed. (The "horseshoe" that gives the bat its name is a depression around the nostrils that acts like a focusing mirror, so that the bat emits sound in a narrow beam like a flashlight.) A *Rhinolophus* flying at speed v_{bat} emits sound of frequency f_{bat} ; the sound it hears reflected from an insect flying toward it has a higher frequency f_{refl} . (a) Show that the speed of the insect is

$$v_{\text{insect}} = v \left[\frac{f_{\text{refl}}(v - v_{\text{bat}}) - f_{\text{bat}}(v + v_{\text{bat}})}{f_{\text{refl}}(v - v_{\text{bat}}) + f_{\text{bat}}(v + v_{\text{bat}})} \right]$$

where v is the speed of sound. (b) If $f_{\text{bat}} = 80.7$ kHz, $f_{\text{refl}} = 83.5$ kHz, and $v_{\text{bat}} = 3.9$ m/s, calculate the speed of the insect.

16.78 •• (a) Show that Eq. (16.30) can be written as

$$f_R = f_S \left(1 - \frac{v}{c} \right)^{1/2} \left(1 + \frac{v}{c} \right)^{-1/2}$$

(b) Use the binomial theorem to show that if $v \ll c$, this is approximately equal to

$$f_R = f_S \left(1 - \frac{v}{c} \right)$$

(c) A pilotless reconnaissance aircraft emits a radio signal with a frequency of 243 MHz. It is flying directly toward a test engineer on the ground. The engineer detects beats between the received signal and a local signal also of frequency 243 MHz. The beat frequency is 46.0 Hz. What is the speed of the aircraft? (Radio waves travel at the speed of light, $c = 3.00 \times 10^8$ m/s.)

16.79 •• Supernova! The gas cloud known as the Crab Nebula can be seen with even a small telescope. It is the remnant of a *supernova*, a cataclysmic explosion of a star. The explosion was seen on the earth on July 4, 1054 C.E. The streamers glow with the characteristic red color of heated hydrogen gas. In a laboratory on the earth, heated hydrogen produces red light with frequency 4.568×10^{14} Hz; the red light received from streamers in the Crab Nebula pointed toward the earth has frequency 4.586×10^{14} Hz. (a) Estimate the speed with which the outer edges of the Crab Nebula are expanding. Assume that the speed of the center of the nebula relative to the earth is negligible. (You may use the formulas derived in Problem 16.78. The speed of light is 3.00×10^8 m/s.) (b) Assuming that the expansion speed has been constant since the supernova explosion, estimate the diameter of the Crab Nebula. Give your answer in meters and in light-years. (c) The angular diameter of the Crab Nebula as seen from earth is about 5 arc minutes (1 arc minute = $\frac{1}{60}$ degree). Estimate the distance (in light-years) to the Crab Nebula, and estimate the year in which the supernova explosion actually took place.

16.80 •• CP A turntable 1.50 m in diameter rotates at 75 rpm. Two speakers, each giving off sound of wavelength 31.3 cm, are attached to the rim of the table at opposite ends of a diameter. A listener stands in front of the turntable. (a) What is the greatest beat frequency the listener will receive from this system? (b) Will the listener be able to distinguish individual beats?

16.81 •• A woman stands at rest in front of a large, smooth wall. She holds a vibrating tuning fork of frequency f_0 directly in front of her (between her and the wall). (a) The woman now runs toward the wall with speed v_w . She detects beats due to the interference between the sound waves reaching her directly from the fork and those reaching her after being reflected from the wall. How many beats per second will she detect? (*Note:* If the beat frequency is too large, the woman may have to use some instrumentation other than

her ears to detect and count the beats.) (b) If the woman instead runs away from the wall, holding the tuning fork at her back so it is between her and the wall, how many beats per second will she detect?

16.82 •• On a clear day you see a jet plane flying overhead. From the apparent size of the plane, you determine that it is flying at a constant altitude h . You hear the sonic boom at time T after the plane passes directly overhead. Show that if the speed of sound v is the same at all altitudes, the speed of the plane is

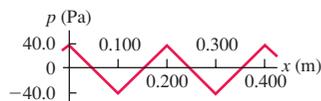
$$v_s = \frac{hv}{\sqrt{h^2 - v^2 T^2}}$$

(*Hint:* Trigonometric identities will be useful.)

CHALLENGE PROBLEMS

16.83 ••• CALC Figure P16.83 shows the pressure fluctuation p of a nonsinusoidal sound wave as a function of x for $t = 0$. The wave is traveling in the $+x$ -direction. (a) Graph the pressure fluctuation p as a function of t for $x = 0$. Show at least two cycles of oscillation. (b) Graph the displacement y in this sound wave as a function of x at $t = 0$. At $x = 0$, the displacement at $t = 0$ is zero. Show at least two wavelengths of the wave. (c) Graph the displacement y as a function of t for $x = 0$. Show at least two cycles of oscillation. (d) Calculate the maximum velocity and the maximum acceleration of an element of the air through which this sound wave is traveling. (e) Describe how the cone of a loudspeaker must move as a function of time to produce the sound wave in this problem.

Figure P16.83



16.84 ••• CP Longitudinal Waves on a Spring. A long spring such as a Slinky™ is often used to demonstrate longitudinal waves. (a) Show that if a spring that obeys Hooke's law has mass m , length L , and force constant k' , the speed of longitudinal waves on the spring is $v = L\sqrt{k'/m}$. (see Section 16.2). (b) Evaluate v for a spring with $m = 0.250$ kg, $L = 2.00$ m, and $k' = 1.50$ N/m.

Answers

Chapter Opening Question ?

Both musical sound and noise are made up of a combination of sinusoidal sound waves. The difference is that the frequencies of the sine waves in musical sound are all integer multiples of a fundamental frequency, while *all* frequencies are present in noise.

Test Your Understanding Questions

16.1 Answer: (v) From Eq. (16.5), the displacement amplitude is $A = p_{\max}/Bk$. The pressure amplitude p_{\max} and bulk modulus B remain the same, but the frequency f increases by a factor of 4. Hence the wave number $k = \omega/v = 2\pi f/v$ also increases by a factor of 4. Since A is inversely proportional to k , the displacement amplitude becomes $\frac{1}{4}$ as great. In other words, at higher frequency

a smaller maximum displacement is required to produce the same maximum pressure fluctuation.

16.2 Answer: (i) From Eq. (16.7), the speed of longitudinal waves (sound) in a fluid is $v = \sqrt{B/\rho}$. We can rewrite this to give an expression for the bulk modulus B in terms of the fluid density ρ and the sound speed v : $B = \rho v^2$. At 20°C the speed of sound in mercury is slightly less than in water (1451 m/s versus 1482 m/s), but the density of mercury is greater than that of water by a large factor (13.6). Hence the bulk modulus of mercury is greater than that of water by a factor of $(13.6)(1451/1482)^2 = 13.0$.

16.3 Answer: A and p_{\max} increase by a factor of $\sqrt{2}$, B and v are unchanged, β increases by 3.0 dB Equations (16.9) and (16.10) show that the bulk modulus B and sound speed v remain the same because the physical properties of the air are unchanged. From Eqs. (16.12) and (16.14), the intensity is proportional to the

square of the displacement amplitude or the square of the pressure amplitude. Hence doubling the intensity means that A and p_{\max} both increase by a factor of $\sqrt{2}$. Example 16.9 shows that multiplying the intensity by a factor of $2(I_2/I_1 = 2)$ corresponds to adding to the sound intensity level by $(10 \text{ dB}) \log(I_2/I_1) = (10 \text{ dB}) \log 2 = 3.0 \text{ dB}$.

16.4 Answer: (ii) Helium is less dense and has a lower molar mass than air, so sound travels faster in helium than in air. The normal-mode frequencies for a pipe are proportional to the sound speed v , so the frequency and hence the pitch increase when the air in the pipe is replaced with helium.

16.5 Answer: (i) and (iv) There will be a resonance if 660 Hz is one of the pipe's normal-mode frequencies. A stopped organ pipe has normal-mode frequencies that are odd multiples of its fundamental frequency [see Eq. (16.22) and Fig. 16.18]. Hence pipe (i), which has fundamental frequency 220 Hz, also has a normal-mode frequency of $3(220 \text{ Hz}) = 660 \text{ Hz}$. Pipe (ii) has twice the length of pipe (i); from Eq. (16.20), the fundamental frequency of a stopped pipe is inversely proportional to the length, so pipe (ii) has a fundamental frequency of $(\frac{1}{2})(220 \text{ Hz}) = 110 \text{ Hz}$. Its other normal-mode frequencies are 330 Hz, 550 Hz, 770 Hz, . . . , so a 660-Hz tuning fork will not cause resonance. Pipe (iii) is an open pipe of the same length as pipe (i), so its fundamental frequency is twice as great as for pipe (i) [compare Eqs. (16.16) and (16.20)], or $2(220 \text{ Hz}) = 440 \text{ Hz}$. Its other normal-mode frequencies are integer multiples of the fundamental frequency [see Eq. (16.19)], or 880 Hz, 1320 Hz, . . . , none of which match the 660-Hz frequency of the tuning fork. Pipe (iv) is also an open pipe but with twice the length of pipe (iii) [see Eq. (16.18)], so its normal-mode frequencies are one-half those of pipe (iii): 220 Hz, 440 Hz, 660 Hz, . . . , so the third harmonic will resonate with the tuning fork.

16.6 Answer: (iii) Constructive and destructive interference between two waves can occur only if the two waves have the same frequency. In this case the frequencies are different, so there are no points where the two waves always reinforce each other (constructive interference) or always cancel each other (destructive interference).

16.7 Answer: (vi) The beat frequency is 3 Hz, so the difference between the two tuning fork frequencies is also 3 Hz. Hence the second tuning fork vibrates at a frequency of either 443 Hz or 437 Hz. You can distinguish between the two possibilities by comparing the pitches of the two tuning forks sounded one at a time: The frequency is 437 Hz if the second tuning fork has a lower pitch and 443 Hz if it has a higher pitch.

16.8 Answer: no The air (the medium for sound waves) is moving from the source toward the listener. Hence, relative to the air, both the source and the listener are moving in the direction from listener to source. So both velocities are positive and $v_S = v_L = +10 \text{ m/s}$. The equality of these two velocities means that the numerator and the denominator in Eq. (16.29) are the same, so $f_L = f_S$ and there is no Doppler shift.

16.9 Answer: (iii) Figure 16.37 shows that there are sound waves inside the cone of the shock wave. Behind the airplane the wave crests are spread apart, just as they are behind the moving source in Fig. 16.27. Hence the waves that reach you have an increased wavelength and a lower frequency.

Bridging Problem

Answers: (a) $180^\circ = \pi \text{ rad}$

(b) A alone: $I = 3.98 \times 10^{-6} \text{ W/m}^2$, $\beta = 66.0 \text{ dB}$;
B alone: $I = 5.31 \times 10^{-7} \text{ W/m}^2$, $\beta = 57.2 \text{ dB}$

(c) $I = 1.60 \times 10^{-6} \text{ W/m}^2$, $\beta = 62.1 \text{ dB}$

TEMPERATURE AND HEAT

17



? At a steelworks, molten iron is heated to 1500° Celsius to remove impurities. Is it accurate to say that the molten iron contains heat?

Whether it's a sweltering summer day or a frozen midwinter night, your body needs to be kept at a nearly constant temperature. It has effective temperature-control mechanisms, but sometimes it needs help. On a hot day you wear less clothing to improve heat transfer from your body to the air and for better cooling by evaporation of perspiration. You drink cold beverages and may sit near a fan or in an air-conditioned room. On a cold day you wear more clothes or stay indoors where it's warm. When you're outside, you keep active and drink hot liquids to stay warm. The concepts in this chapter will help you understand the basic physics of keeping warm or cool.

The terms “temperature” and “heat” are often used interchangeably in everyday language. In physics, however, these two terms have very different meanings. In this chapter we'll define temperature in terms of how it's measured and see how temperature changes affect the dimensions of objects. We'll see that heat refers to energy transfer caused by temperature differences and learn how to calculate and control such energy transfers.

Our emphasis in this chapter is on the concepts of temperature and heat as they relate to *macroscopic* objects such as cylinders of gas, ice cubes, and the human body. In Chapter 18 we'll look at these same concepts from a *microscopic* viewpoint in terms of the behavior of individual atoms and molecules. These two chapters lay the groundwork for the subject of **thermodynamics**, the study of energy transformations involving heat, mechanical work, and other aspects of energy and how these transformations relate to the properties of matter. Thermodynamics forms an indispensable part of the foundation of physics, chemistry, and the life sciences, and its applications turn up in such places as car engines, refrigerators, biochemical processes, and the structure of stars. We'll explore the key ideas of thermodynamics in Chapters 19 and 20.

LEARNING GOALS

By studying this chapter, you will learn:

- The meaning of thermal equilibrium, and what thermometers really measure.
- How different types of thermometers function.
- The physics behind the absolute, or Kelvin, temperature scale.
- How the dimensions of an object change as a result of a temperature change.
- The meaning of heat, and how it differs from temperature.
- How to do calculations that involve heat flow, temperature changes, and changes of phase.
- How heat is transferred by conduction, convection, and radiation.

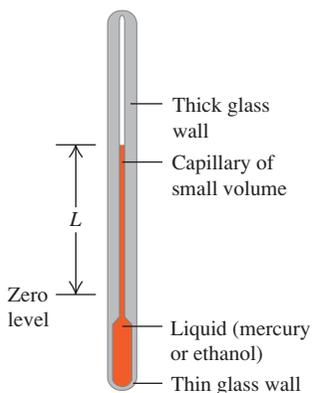
17.1 Temperature and Thermal Equilibrium

The concept of **temperature** is rooted in qualitative ideas of “hot” and “cold” based on our sense of touch. A body that feels hot usually has a higher temperature than a similar body that feels cold. That’s pretty vague, and the senses can be deceived. But many properties of matter that we can *measure* depend on temperature. The length of a metal rod, steam pressure in a boiler, the ability of a wire to conduct an electric current, and the color of a very hot glowing object—all these depend on temperature.

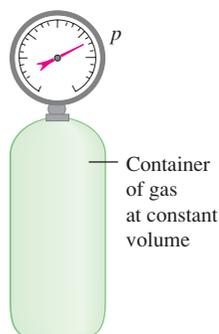
Temperature is also related to the kinetic energies of the molecules of a material. In general this relationship is fairly complex, so it’s not a good place to start in *defining* temperature. In Chapter 18 we will look at the relationship between temperature and the energy of molecular motion for an ideal gas. It is important to understand, however, that temperature and heat can be defined independently of any detailed molecular picture. In this section we’ll develop a *macroscopic* definition of temperature.

17.1 Two devices for measuring temperature.

(a) Changes in temperature cause the liquid’s volume to change.



(b) Changes in temperature cause the pressure of the gas to change.



To use temperature as a measure of hotness or coldness, we need to construct a temperature scale. To do this, we can use any measurable property of a system that varies with its “hotness” or “coldness.” Figure 17.1a shows a familiar system that is used to measure temperature. When the system becomes hotter, the colored liquid (usually mercury or ethanol) expands and rises in the tube, and the value of L increases. Another simple system is a quantity of gas in a constant-volume container (Fig. 17.1b). The pressure p , measured by the gauge, increases or decreases as the gas becomes hotter or colder. A third example is the electrical resistance R of a conducting wire, which also varies when the wire becomes hotter or colder. Each of these properties gives us a number (L , p , or R) that varies with hotness and coldness, so each property can be used to make a **thermometer**.

To measure the temperature of a body, you place the thermometer in contact with the body. If you want to know the temperature of a cup of hot coffee, you stick the thermometer in the coffee; as the two interact, the thermometer becomes hotter and the coffee cools off a little. After the thermometer settles down to a steady value, you read the temperature. The system has reached an *equilibrium* condition, in which the interaction between the thermometer and the coffee causes no further change in the system. We call this a state of **thermal equilibrium**.

If two systems are separated by an insulating material or **insulator** such as wood, plastic foam, or fiberglass, they influence each other more slowly. Camping coolers are made with insulating materials to delay the ice and cold food inside from warming up and attaining thermal equilibrium with the hot summer air outside. An *ideal insulator* is a material that permits no interaction at all between the two systems. It prevents the systems from attaining thermal equilibrium if they aren’t in thermal equilibrium at the start. An ideal insulator is just that, an idealization; real insulators, like those in camping coolers, aren’t ideal, so the contents of the cooler will warm up eventually.

The Zeroth Law of Thermodynamics

We can discover an important property of thermal equilibrium by considering three systems, A , B , and C , that initially are not in thermal equilibrium (Fig. 17.2). We surround them with an ideal insulating box so that they cannot interact with anything except each other. We separate systems A and B with an ideal insulating wall (the green slab in Fig. 17.2a), but we let system C interact with both systems A and B . This interaction is shown in the figure by a yellow slab representing a thermal **conductor**, a material that *permits* thermal interactions through it. We wait until thermal equilibrium is attained; then A and B are each in thermal equilibrium with C . But are they in thermal equilibrium *with each other*?

To find out, we separate system C from systems A and B with an ideal insulating wall (Fig. 17.2b), and then we replace the insulating wall between A and B

with a *conducting* wall that lets *A* and *B* interact. What happens? Experiment shows that *nothing* happens; there are no additional changes to *A* or *B*. We conclude:

If *C* is initially in thermal equilibrium with both *A* and *B*, then *A* and *B* are also in thermal equilibrium with each other. This result is called the zeroth law of thermodynamics.

(The importance of this law was recognized only after the first, second, and third laws of thermodynamics had been named. Since it is fundamental to all of them, the name “zeroth” seemed appropriate.)

Now suppose system *C* is a thermometer, such as the liquid-in-tube system of Fig. 17.1a. In Fig. 17.2a the thermometer *C* is in contact with both *A* and *B*. In thermal equilibrium, when the thermometer reading reaches a stable value, the thermometer measures the temperature of both *A* and *B*; hence *A* and *B* both have the *same* temperature. Experiment shows that thermal equilibrium isn’t affected by adding or removing insulators, so the reading of thermometer *C* wouldn’t change if it were in contact only with *A* or only with *B*. We conclude:

Two systems are in thermal equilibrium if and only if they have the same temperature.

This is what makes a thermometer useful; a thermometer actually measures *its own* temperature, but when a thermometer is in thermal equilibrium with another body, the temperatures must be equal. When the temperatures of two systems are different, they *cannot* be in thermal equilibrium.

Test Your Understanding of Section 17.1 You put a thermometer in a pot of hot water and record the reading. What temperature have you recorded? (i) the temperature of the water; (ii) the temperature of the thermometer; (iii) an equal average of the temperatures of the water and thermometer; (iv) a weighted average of the temperatures of the water and thermometer, with more emphasis on the temperature of the water; (v) a weighted average of the water and thermometer, with more emphasis on the temperature of the thermometer. 

17.2 Thermometers and Temperature Scales

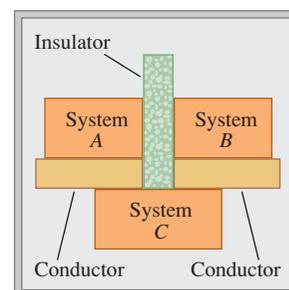
To make the liquid-in-tube device shown in Fig. 17.1a into a useful thermometer, we need to mark a scale on the tube wall with numbers on it. These numbers are arbitrary, and historically many different schemes have been used. Suppose we label the thermometer’s liquid level at the freezing temperature of pure water “zero” and the level at the boiling temperature “100,” and divide the distance between these two points into 100 equal intervals called *degrees*. The result is the **Celsius temperature scale** (formerly called the *centigrade* scale in English-speaking countries). The Celsius temperature for a state colder than freezing water is a negative number. The Celsius scale is used, both in everyday life and in science and industry, almost everywhere in the world.

Another common type of thermometer uses a *bimetallic strip*, made by bonding strips of two different metals together (Fig. 17.3a). When the temperature of the composite strip increases, one metal expands more than the other and the strip bends (Fig. 17.3b). This strip is usually formed into a spiral, with the outer end anchored to the thermometer case and the inner end attached to a pointer (Fig. 17.3c). The pointer rotates in response to temperature changes.

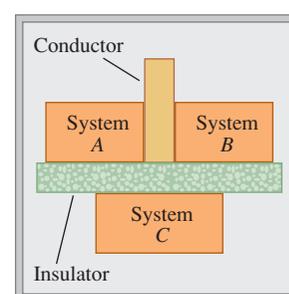
In a *resistance thermometer* the changing electrical resistance of a coil of fine wire, a carbon cylinder, or a germanium crystal is measured. Resistance thermometers are usually more precise than most other types.

17.2 The zeroth law of thermodynamics.

(a) If systems *A* and *B* are each in thermal equilibrium with system *C* ...

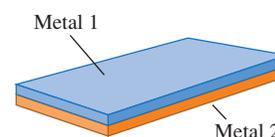


(b) ... then systems *A* and *B* are in thermal equilibrium with each other.

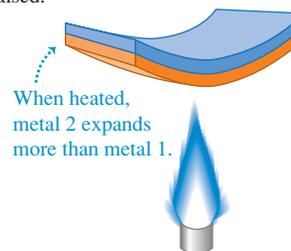


17.3 Use of a bimetallic strip as a thermometer.

(a) A bimetallic strip



(b) The strip bends when its temperature is raised.



(c) A bimetallic strip used in a thermometer



17.4 A temporal artery thermometer measures infrared radiation from the skin that overlies one of the important arteries in the head. Although the thermometer cover touches the skin, the infrared detector inside the cover does not.



Some thermometers work by detecting the amount of infrared radiation emitted by an object. (We'll see in Section 17.7 that *all* objects emit electromagnetic radiation, including infrared, as a consequence of their temperature.) A modern example is a *temporal artery thermometer* (Fig. 17.4). A nurse runs this over a patient's forehead in the vicinity of the temporal artery, and an infrared sensor in the thermometer measures the radiation from the skin. Tests show that this device gives more accurate values of body temperature than do oral or ear thermometers.

In the **Fahrenheit temperature scale**, still used in everyday life in the United States, the freezing temperature of water is 32°F (thirty-two degrees Fahrenheit) and the boiling temperature is 212°F, both at standard atmospheric pressure. There are 180 degrees between freezing and boiling, compared to 100 on the Celsius scale, so one Fahrenheit degree represents only $\frac{100}{180}$, or $\frac{5}{9}$, as great a temperature change as one Celsius degree.

To convert temperatures from Celsius to Fahrenheit, note that a Celsius temperature T_C is the number of Celsius degrees above freezing; the number of Fahrenheit degrees above freezing is $\frac{9}{5}$ of this. But freezing on the Fahrenheit scale is at 32°F, so to obtain the actual Fahrenheit temperature T_F , multiply the Celsius value by $\frac{9}{5}$ and then add 32°. Symbolically,

$$T_F = \frac{9}{5}T_C + 32^\circ \quad (17.1)$$

To convert Fahrenheit to Celsius, solve this equation for T_C :

$$T_C = \frac{5}{9}(T_F - 32^\circ) \quad (17.2)$$

In words, subtract 32° to get the number of Fahrenheit degrees above freezing, and then multiply by $\frac{5}{9}$ to obtain the number of Celsius degrees above freezing—that is, the Celsius temperature.

We don't recommend memorizing Eqs. (17.1) and (17.2). Instead, try to understand the reasoning that led to them so that you can derive them on the spot when you need them, checking your reasoning with the relationship $100^\circ\text{C} = 212^\circ\text{F}$.

It is useful to distinguish between an actual temperature and a temperature *interval* (a difference or change in temperature). An actual temperature of 20° is stated as 20°C (twenty degrees Celsius), and a temperature *interval* of 10° is 10 C° (ten Celsius degrees). A beaker of water heated from 20°C to 30°C undergoes a temperature change of 10 C°.

Application Mammalian Body Temperatures

Most mammals maintain body temperatures in the range from 36°C to 40°C (309 K to 313 K). A high metabolic rate warms the animal from within, and insulation (such as fur, feathers, and body fat) slows heat loss.



Test Your Understanding of Section 17.2 Which of the following types of thermometers have to be in thermal equilibrium with the object being measured in order to give accurate readings? (i) a bimetallic strip; (ii) a resistance thermometer; (iii) a temporal artery thermometer; (iv) both (i) and (ii); (v) all of (i), (ii), and (iii).



17.3 Gas Thermometers and the Kelvin Scale

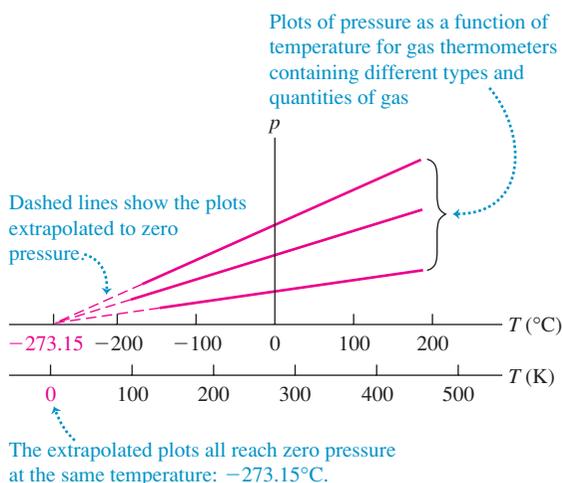
When we calibrate two thermometers, such as a liquid-in-tube system and a resistance thermometer, so that they agree at 0°C and 100°C, they may not agree exactly at intermediate temperatures. Any temperature scale defined in this way always depends somewhat on the specific properties of the material used. Ideally, we would like to define a temperature scale that *doesn't* depend on the properties of a particular material. To establish a truly material-independent scale, we first need to develop some principles of thermodynamics. We'll return to this fundamental problem in Chapter 20. Here we'll discuss a thermometer that comes close to the ideal, the *gas thermometer*.

17.5 (a) Using a constant-volume gas thermometer to measure temperature. (b) The greater the amount of gas in the thermometer, the higher the graph of pressure p versus temperature T .

(a) A constant-volume gas thermometer



(b) Graphs of pressure versus temperature at constant volume for three different types and quantities of gas



The principle of a gas thermometer is that the pressure of a gas at constant volume increases with temperature. A quantity of gas is placed in a constant-volume container (Fig. 17.5a), and its pressure is measured by one of the devices described in Section 12.2. To calibrate a constant-volume gas thermometer, we measure the pressure at two temperatures, say 0°C and 100°C , plot these points on a graph, and draw a straight line between them. Then we can read from the graph the temperature corresponding to any other pressure. Figure 17.5b shows the results of three such experiments, each using a different type and quantity of gas.

By extrapolating this graph, we see that there is a hypothetical temperature, -273.15°C , at which the absolute pressure of the gas would become zero. We might expect that this temperature would be different for different gases, but it turns out to be the *same* for many different gases (at least in the limit of very low gas density). We can't actually observe this zero-pressure condition. Gases liquefy and solidify at very low temperatures, and the proportionality of pressure to temperature no longer holds.

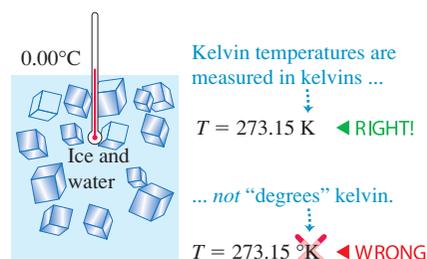
We use this extrapolated zero-pressure temperature as the basis for a temperature scale with its zero at this temperature. This is the **Kelvin temperature scale**, named for the British physicist Lord Kelvin (1824–1907). The units are the same size as those on the Celsius scale, but the zero is shifted so that $0\text{ K} = -273.15^\circ\text{C}$ and $273.15\text{ K} = 0^\circ\text{C}$; that is,

$$T_{\text{K}} = T_{\text{C}} + 273.15 \quad (17.3)$$

Figure 17.5b shows both the Celsius and Kelvin scales. A common room temperature, 20°C ($= 68^\circ\text{F}$), is $20 + 273.15$, or about 293 K.

CAUTION Never say “degrees kelvin” In SI nomenclature, “degree” is not used with the Kelvin scale; the temperature mentioned above is read “293 kelvins,” not “degrees kelvin” (Fig. 17.6). We capitalize Kelvin when it refers to the temperature scale; however, the *unit* of temperature is the *kelvin*, which is not capitalized (but is nonetheless abbreviated as a capital K).

17.6 Correct and incorrect uses of the Kelvin scale.



Example 17.1 Body temperature

You place a small piece of ice in your mouth. Eventually, the water all converts from ice at $T_1 = 32.00^\circ\text{F}$ to body temperature, $T_2 = 98.60^\circ\text{F}$. Express these temperatures in both Celsius degrees and kelvins, and find $\Delta T = T_2 - T_1$ in both cases.

SOLUTION

IDENTIFY and SET UP: Our target variables are stated above. We convert Fahrenheit temperatures to Celsius using Eq. (17.2), and Celsius temperatures to Kelvin using Eq. (17.3).

EXECUTE: From Eq. (17.2), $T_1 = 0.00^\circ\text{C}$ and $T_2 = 37.00^\circ\text{C}$; then $\Delta T = T_2 - T_1 = 37.00^\circ\text{C}$. To get the Kelvin temperatures, just add 273.15 to each Celsius temperature: $T_1 = 273.15\text{ K}$ and $T_2 = 310.15\text{ K}$. The temperature difference is $\Delta T = T_2 - T_1 = 37.00\text{ K}$.

EVALUATE: The Celsius and Kelvin scales have different zero points but the same size degrees. Therefore *any* temperature difference ΔT is the *same* on the Celsius and Kelvin scales. However, ΔT is *not* the same on the Fahrenheit scale; here, for example, $\Delta T = 66.60^\circ\text{F}$.

The Kelvin Scale and Absolute Temperature

The Celsius scale has two fixed points: the normal freezing and boiling temperatures of water. But we can define the Kelvin scale using a gas thermometer with only a single reference temperature. We define the ratio of any two temperatures T_1 and T_2 on the Kelvin scale as the ratio of the corresponding gas-thermometer pressures p_1 and p_2 :

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \quad (\text{constant-volume gas thermometer, } T \text{ in kelvins}) \quad (17.4)$$

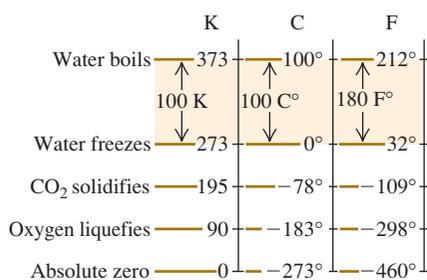
The pressure p is directly proportional to the Kelvin temperature, as shown in Fig. 17.5b. To complete the definition of T , we need only specify the Kelvin temperature of a single specific state. For reasons of precision and reproducibility, the state chosen is the *triple point* of water. This is the unique combination of temperature and pressure at which solid water (ice), liquid water, and water vapor can all coexist. It occurs at a temperature of 0.01°C and a water-vapor pressure of 610 Pa (about 0.006 atm). (This is the pressure of the *water*; it has nothing to do directly with the gas pressure in the *thermometer*.) The triple-point temperature T_{triple} of water is *defined* to have the value $T_{\text{triple}} = 273.16\text{ K}$, corresponding to 0.01°C . From Eq. (17.4), if p_{triple} is the pressure in a gas thermometer at temperature T_{triple} and p is the pressure at some other temperature T , then T is given on the Kelvin scale by

$$T = T_{\text{triple}} \frac{p}{p_{\text{triple}}} = (273.16\text{ K}) \frac{p}{p_{\text{triple}}} \quad (17.5)$$

Low-pressure gas thermometers using various gases are found to agree very closely, but they are large, bulky, and very slow to come to thermal equilibrium. They are used principally to establish high-precision standards and to calibrate other thermometers.

Figure 17.7 shows the relationships among the three temperature scales we have discussed. The Kelvin scale is called an **absolute temperature scale**, and its zero point ($T = 0\text{ K} = -273.15^\circ\text{C}$, the temperature at which $p = 0$ in Eq. (17.5)) is called **absolute zero**. At absolute zero a system of molecules (such as a quantity of a gas, a liquid, or a solid) has its *minimum* possible total energy (kinetic plus potential); because of quantum effects, however, it is *not* correct to say that all molecular motion ceases at absolute zero. To define more completely what we mean by absolute zero, we need to use the thermodynamic principles developed in the next several chapters. We will return to this concept in Chapter 20.

17.7 Relationships among Kelvin (K), Celsius (C), and Fahrenheit (F) temperature scales. Temperatures have been rounded off to the nearest degree.



Test Your Understanding of Section 17.3 Rank the following temperatures from highest to lowest: (i) 0.00°C ; (ii) 0.00°F ; (iii) 260.00 K ; (iv) 77.00 K ; (v) -180.00°C .

17.4 Thermal Expansion

Most materials expand when their temperatures increase. Rising temperatures make the liquid expand in a liquid-in-tube thermometer (Fig. 17.1a) and bend bimetallic strips (Fig. 17.3b). The decks of bridges need special joints and supports to allow for expansion. A completely filled and tightly capped bottle of water cracks when it is heated, but you can loosen a metal jar lid by running hot water over it. These are all examples of *thermal expansion*.

Linear Expansion

Suppose a rod of material has a length L_0 at some initial temperature T_0 . When the temperature changes by ΔT , the length changes by ΔL . Experiments show that if ΔT is not too large (say, less than 100 C° or so), ΔL is *directly proportional* to ΔT (Fig. 17.8a). If two rods made of the same material have the same temperature change, but one is twice as long as the other, then the *change* in its length is also twice as great. Therefore ΔL must also be proportional to L_0 (Fig. 17.8b). Introducing a proportionality constant α (which is different for different materials), we may express these relationships in an equation:

$$\Delta L = \alpha L_0 \Delta T \quad (\text{linear thermal expansion}) \quad (17.6)$$

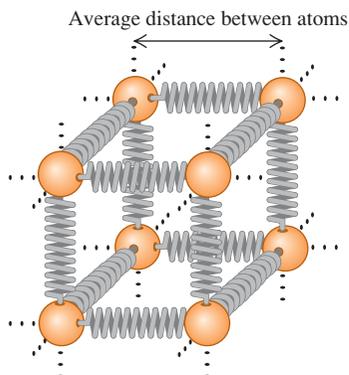
If a body has length L_0 at temperature T_0 , then its length L at a temperature $T = T_0 + \Delta T$ is

$$L = L_0 + \Delta L = L_0 + \alpha L_0 \Delta T = L_0(1 + \alpha \Delta T) \quad (17.7)$$

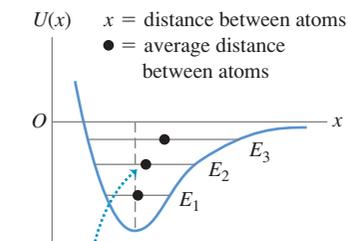
The constant α , which describes the thermal expansion properties of a particular material, is called the **coefficient of linear expansion**. The units of α are K^{-1} or $(\text{C}^\circ)^{-1}$. (Remember that a temperature *interval* is the same in the Kelvin and Celsius scales.) For many materials, every linear dimension changes according to Eq. (17.6) or (17.7). Thus L could be the thickness of a rod, the side length of a square sheet, or the diameter of a hole. Some materials, such as wood or single crystals, expand differently in different directions. We won't consider this complication.

We can understand thermal expansion qualitatively on a molecular basis. Picture the interatomic forces in a solid as springs, as in Fig. 17.9a. (We explored the analogy between spring forces and interatomic forces in Section 14.4.) Each atom vibrates about its equilibrium position. When the temperature increases, the energy and amplitude of the vibration also increase. The interatomic spring forces are not symmetrical about the equilibrium position; they usually behave like a spring that is easier to stretch than to compress. As a result, when the amplitude of vibration increases, the *average* distance between atoms also increases (Fig. 17.9b). As the atoms get farther apart, every dimension increases.

(a) A model of the forces between neighboring atoms in a solid



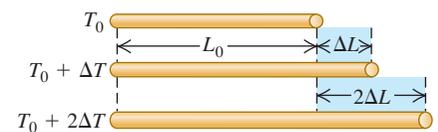
(b) A graph of the “spring” potential energy $U(x)$



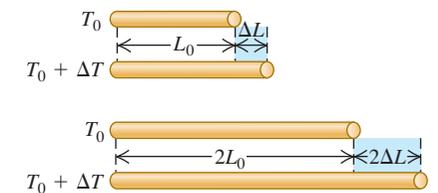
As energy increases from E_1 to E_2 to E_3 , average distance between atoms increases.

17.8 How the length of a rod changes with a change in temperature. (Length changes are exaggerated for clarity.)

(a) For moderate temperature changes, ΔL is directly proportional to ΔT .

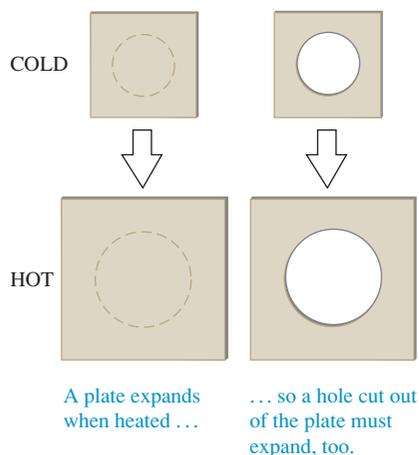


(b) ΔL is also directly proportional to L_0 .



17.9 (a) We can model atoms in a solid as being held together by “springs” that are easier to stretch than to compress. (b) A graph of the “spring” potential energy $U(x)$ versus distance x between neighboring atoms is *not* symmetrical (compare Fig. 14.20b). As the energy increases and the atoms oscillate with greater amplitude, the average distance increases.

17.10 When an object undergoes thermal expansion, any holes in the object expand as well. (The expansion is exaggerated.)



CAUTION Heating an object with a hole If a solid object has a hole in it, what happens to the size of the hole when the temperature of the object increases? A common misconception is that if the object expands, the hole will shrink because material expands into the hole. But the truth of the matter is that if the object expands, the hole will expand too (Fig. 17.10); as we stated above, *every* linear dimension of an object changes in the same way when the temperature changes. If you're not convinced, think of the atoms in Fig. 17.9a as outlining a cubical hole. When the object expands, the atoms move apart and the hole increases in size. The only situation in which a “hole” will fill in due to thermal expansion is when two separate objects expand and close the gap between them (Fig. 17.11). **I**

The direct proportionality expressed by Eq. (17.6) is not exact; it is *approximately* correct only for sufficiently small temperature changes. For a given material, α varies somewhat with the initial temperature T_0 and the size of the temperature interval. We'll ignore this complication here, however. Average values of α for several materials are listed in Table 17.1. Within the precision of these values we don't need to worry whether T_0 is 0°C or 20°C or some other temperature. Note that typical values of α are very small; even for a temperature change of 100°C , the fractional length change $\Delta L/L_0$ is only of the order of $\frac{1}{1000}$ for the metals in the table.

17.11 When this SR-71 aircraft is sitting on the ground, its wing panels fit together so loosely that fuel leaks out of the wings onto the ground. But once it is in flight at over three times the speed of sound, air friction heats the panels so much that they expand to make a perfect fit. (In-flight refueling makes up for the lost fuel.)



Volume Expansion

Increasing temperature usually causes increases in *volume* for both solid and liquid materials. Just as with linear expansion, experiments show that if the temperature change ΔT is not too great (less than 100°C or so), the increase in volume ΔV is approximately proportional to both the temperature change ΔT and the initial volume V_0 :

$$\Delta V = \beta V_0 \Delta T \quad (\text{volume thermal expansion}) \quad (17.8)$$

The constant β characterizes the volume expansion properties of a particular material; it is called the **coefficient of volume expansion**. The units of β are K^{-1} or $(^\circ\text{C})^{-1}$. As with linear expansion, β varies somewhat with temperature, and Eq. (17.8) is an approximate relationship that is valid only for small temperature changes. For many substances, β decreases at low temperatures. Several values of β in the neighborhood of room temperature are listed in Table 17.2. Note that the values for liquids are generally much larger than those for solids.

For solid materials there is a simple relationship between the volume expansion coefficient β and the linear expansion coefficient α . To derive this relationship, we consider a cube of material with side length L and volume $V = L^3$. At the initial temperature the values are L_0 and V_0 . When the temperature increases by dT , the side length increases by dL and the volume increases by an amount dV given by

$$dV = \frac{dV}{dL} dL = 3L^2 dL$$

Table 17.1 Coefficients of Linear Expansion

Material	α [K^{-1} or $(^\circ\text{C})^{-1}$]
Aluminum	2.4×10^{-5}
Brass	2.0×10^{-5}
Copper	1.7×10^{-5}
Glass	$0.4\text{--}0.9 \times 10^{-5}$
Invar (nickel–iron alloy)	0.09×10^{-5}
Quartz (fused)	0.04×10^{-5}
Steel	1.2×10^{-5}

Table 17.2 Coefficients of Volume Expansion

Solids	β [K^{-1} or $(^\circ\text{C})^{-1}$]	Liquids	β [K^{-1} or $(^\circ\text{C})^{-1}$]
Aluminum	7.2×10^{-5}	Ethanol	75×10^{-5}
Brass	6.0×10^{-5}	Carbon disulfide	115×10^{-5}
Copper	5.1×10^{-5}	Glycerin	49×10^{-5}
Glass	$1.2\text{--}2.7 \times 10^{-5}$	Mercury	18×10^{-5}
Invar	0.27×10^{-5}		
Quartz (fused)	0.12×10^{-5}		
Steel	3.6×10^{-5}		

Now we replace L and V by the initial values L_0 and V_0 . From Eq. (17.6), dL is

$$dL = \alpha L_0 dT$$

Since $V_0 = L_0^3$, this means that dV can also be expressed as

$$dV = 3L_0^2 \alpha L_0 dT = 3\alpha V_0 dT$$

This is consistent with the infinitesimal form of Eq. (17.8), $dV = \beta V_0 dT$, only if

$$\beta = 3\alpha \quad (17.9)$$

You should check this relationship for some of the materials listed in Tables 17.1 and 17.2.

Problem-Solving Strategy 17.1 Thermal Expansion



IDENTIFY *the relevant concepts:* Decide whether the problem involves changes in length (linear thermal expansion) or in volume (volume thermal expansion).

SET UP *the problem* using the following steps:

1. List the known and unknown quantities and identify the target variables.
2. Choose Eq. (17.6) for linear expansion and Eq. (17.8) for volume expansion.

EXECUTE *the solution* as follows:

1. Solve for the target variables. If you are given an initial temperature T_0 and must find a final temperature T corresponding to a

given length or volume change, find ΔT and calculate $T = T_0 + \Delta T$. Remember that the size of a hole in a material varies with temperature just as any other linear dimension, and that the volume of a hole (such as the interior of a container) varies just as that of the corresponding solid shape.

2. Maintain unit consistency. Both L_0 and ΔL (or V_0 and ΔV) must have the same units. If you use a value of α or β in K^{-1} or $(\text{C}^\circ)^{-1}$, then ΔT must be in either kelvins or Celsius degrees; from Example 17.1, the two scales are equivalent for temperature differences.

EVALUATE *your answer:* Check whether your results make sense.

Example 17.2 Length change due to temperature change

A surveyor uses a steel measuring tape that is exactly 50.000 m long at a temperature of 20°C . The markings on the tape are calibrated for this temperature. (a) What is the length of the tape when the temperature is 35°C ? (b) When it is 35°C , the surveyor uses the tape to measure a distance. The value that she reads off the tape is 35.794 m. What is the actual distance?

SOLUTION

IDENTIFY and SET UP: This problem concerns the linear expansion of a measuring tape. We are given the tape's initial length $L_0 = 50.000$ m at $T_0 = 20^\circ\text{C}$. In part (a) we use Eq. (17.6) to find the change ΔL in the tape's length at $T = 35^\circ\text{C}$, and use Eq. (17.7) to find L . (Table 17.1 gives the value of α for steel.) Since the tape expands, at 35°C the distance between two successive meter marks is greater than 1 m. Hence the actual distance in part (b) is *larger* than the distance read off the tape by a factor equal to the ratio of the tape's length L at 35°C to its length L_0 at 20°C .

EXECUTE: (a) The temperature change is $\Delta T = T - T_0 = 15^\circ\text{C}$; from Eqs. (17.6) and (17.7),

$$\begin{aligned} \Delta L &= \alpha L_0 \Delta T = (1.2 \times 10^{-5} \text{ K}^{-1})(50 \text{ m})(15 \text{ K}) \\ &= 9.0 \times 10^{-3} \text{ m} = 9.0 \text{ mm} \end{aligned}$$

$$L = L_0 + \Delta L = 50.000 \text{ m} + 0.009 \text{ m} = 50.009 \text{ m}$$

(b) Our result from part (a) shows that at 35°C , the slightly expanded tape reads a distance of 50.000 m when the true distance is 50.009 m. We can rewrite the algebra of part (a) as $L = L_0(1 + \alpha \Delta T)$; at 35°C , any true distance will be greater than the reading by the factor $50.009/50.000 = 1 + \alpha \Delta T = 1 + 1.8 \times 10^{-4}$. The true distance is therefore

$$(1 + 1.8 \times 10^{-4})(35.794 \text{ m}) = 35.800 \text{ m}$$

EVALUATE: Note that in part (a) we needed only two of the five significant figures of L_0 to compute ΔL to the same number of decimal places as L_0 . Our result shows that metals expand very little under moderate temperature changes. However, even the small difference $0.009 \text{ m} = 9 \text{ mm}$ found in part (b) between the scale reading and the true distance can be important in precision work.

Example 17.3 Volume change due to temperature change

A 200-cm^3 glass flask is filled to the brim with mercury at 20°C . How much mercury overflows when the temperature of the system

is raised to 100°C ? The coefficient of *linear* expansion of the glass is $0.40 \times 10^{-5} \text{ K}^{-1}$.

Continued

SOLUTION

IDENTIFY and SET UP: This problem involves the volume expansion of the glass and of the mercury. The amount of overflow depends on the *difference* between the volume changes ΔV for these two materials, both given by Eq. (17.8). The mercury will overflow if its coefficient of volume expansion β (given in Table 17.2) is greater than that of glass, which we find from Eq. (17.9) using the given value of α .

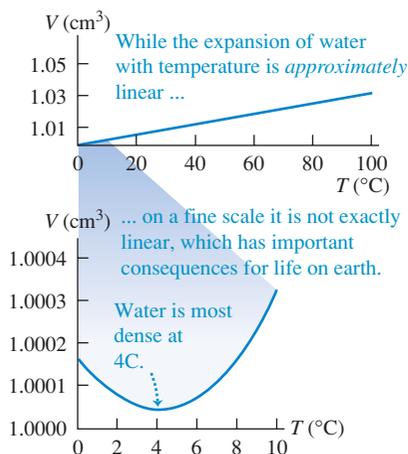
EXECUTE: From Table 17.2, $\beta_{\text{Hg}} = 18 \times 10^{-5} \text{K}^{-1}$. That is indeed greater than β_{glass} : From Eq. (17.9), $\beta_{\text{glass}} = 3\alpha_{\text{glass}} = 3(0.40 \times 10^{-5} \text{K}^{-1}) = 1.2 \times 10^{-5} \text{K}^{-1}$. The volume overflow is then

$$\begin{aligned}\Delta V_{\text{Hg}} - \Delta V_{\text{glass}} &= \beta_{\text{Hg}} V_0 \Delta T - \beta_{\text{glass}} V_0 \Delta T \\ &= V_0 \Delta T (\beta_{\text{Hg}} - \beta_{\text{glass}}) \\ &= (200 \text{ cm}^3)(80 \text{ C}^\circ)(18 \times 10^{-5} - 1.2 \times 10^{-5}) \\ &= 2.7 \text{ cm}^3\end{aligned}$$

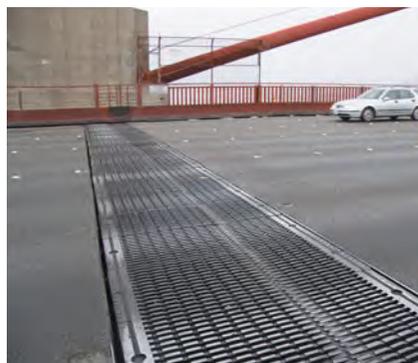
EVALUATE: This is basically how a mercury-in-glass thermometer works; the column of mercury inside a sealed tube rises as T increases because mercury expands faster than glass.

As Tables 17.1 and 17.2 show, glass has smaller coefficients of expansion α and β than do most metals. This is why you can use hot water to loosen a metal lid on a glass jar; the metal expands more than the glass does.

17.12 The volume of 1 gram of water in the temperature range from 0°C to 100°C . By 100°C the volume has increased to 1.034 cm^3 . If the coefficient of volume expansion were constant, the curve would be a straight line.



17.13 Expansion joints on bridges are needed to accommodate changes in length that result from thermal expansion.



Thermal Expansion of Water

Water, in the temperature range from 0°C to 4°C , *decreases* in volume with increasing temperature. In this range its coefficient of volume expansion is *negative*. Above 4°C , water expands when heated (Fig. 17.12). Hence water has its greatest density at 4°C . Water also expands when it freezes, which is why ice humps up in the middle of the compartments in an ice cube tray. By contrast, most materials contract when they freeze.

This anomalous behavior of water has an important effect on plant and animal life in lakes. A lake cools from the surface down; above 4°C , the cooled water at the surface flows to the bottom because of its greater density. But when the surface temperature drops below 4°C , the water near the surface is less dense than the warmer water below. Hence the downward flow ceases, and the water near the surface remains colder than that at the bottom. As the surface freezes, the ice floats because it is less dense than water. The water at the bottom remains at 4°C until nearly the entire lake is frozen. If water behaved like most substances, contracting continuously on cooling and freezing, lakes would freeze from the bottom up. Circulation due to density differences would continuously carry warmer water to the surface for efficient cooling, and lakes would freeze solid much more easily. This would destroy all plant and animal life that cannot withstand freezing. If water did not have this special property, the evolution of life would have taken a very different course.

Thermal Stress

If we clamp the ends of a rod rigidly to prevent expansion or contraction and then change the temperature, **thermal stresses** develop. The rod would like to expand or contract, but the clamps won't let it. The resulting stresses may become large enough to strain the rod irreversibly or even break it. (You may want to review the discussion of stress and strain in Section 11.4).

Engineers must account for thermal stress when designing structures. Concrete highways and bridge decks usually have gaps between sections, filled with a flexible material or bridged by interlocking teeth (Fig. 17.13), to permit expansion and contraction of the concrete. Long steam pipes have expansion joints or U-shaped sections to prevent buckling or stretching with temperature changes. If one end of a steel bridge is rigidly fastened to its abutment, the other end usually rests on rollers.

To calculate the thermal stress in a clamped rod, we compute the amount the rod *would* expand (or contract) if not held and then find the stress needed to compress (or stretch) it back to its original length. Suppose that a rod with length L_0 and cross-sectional area A is held at constant length while the temperature is reduced (negative ΔT), causing a tensile stress. The fractional change in length if the rod were free to contract would be

$$\left(\frac{\Delta L}{L_0}\right)_{\text{thermal}} = \alpha \Delta T \quad (17.10)$$

Both ΔL and ΔT are negative. The tension must increase by an amount F that is just enough to produce an equal and opposite fractional change in length $(\Delta L/L_0)_{\text{tension}}$. From the definition of Young's modulus, Eq. (11.10),

$$Y = \frac{F/A}{\Delta L/L_0} \quad \text{so} \quad \left(\frac{\Delta L}{L_0}\right)_{\text{tension}} = \frac{F}{AY} \quad (17.11)$$

If the length is to be constant, the *total* fractional change in length must be zero. From Eqs. (17.10) and (17.11), this means that

$$\left(\frac{\Delta L}{L_0}\right)_{\text{thermal}} + \left(\frac{\Delta L}{L_0}\right)_{\text{tension}} = \alpha \Delta T + \frac{F}{AY} = 0$$

Solving for the tensile stress F/A required to keep the rod's length constant, we find

$$\frac{F}{A} = -Y\alpha \Delta T \quad (\text{thermal stress}) \quad (17.12)$$

For a decrease in temperature, ΔT is negative, so F and F/A are positive; this means that a *tensile* force and stress are needed to maintain the length. If ΔT is positive, F and F/A are negative, and the required force and stress are *compressive*.

If there are temperature differences within a body, nonuniform expansion or contraction will result and thermal stresses can be induced. You can break a glass bowl by pouring very hot water into it; the thermal stress between the hot and cold parts of the bowl exceeds the breaking stress of the glass, causing cracks. The same phenomenon makes ice cubes crack when dropped into warm water. Heat-resistant glasses such as Pyrex™ have exceptionally low expansion coefficients and high strength.

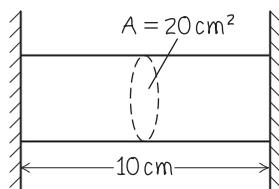
Example 17.4 Thermal stress

An aluminum cylinder 10 cm long, with a cross-sectional area of 20 cm², is used as a spacer between two steel walls. At 17.2°C it just slips between the walls. Calculate the stress in the cylinder and the total force it exerts on each wall when it warms to 22.3°C, assuming that the walls are perfectly rigid and a constant distance apart.

SOLUTION

IDENTIFY and SET UP: Figure 17.14 shows our sketch of the situation. Our target variables are the thermal stress F/A in the cylinder, whose cross-sectional area A is given, and the associated force F it

17.14 Our sketch for this problem.



exerts on the walls. We use Eq. (17.12) to relate F/A to the temperature change ΔT , and from that calculate F . (The length of the cylinder is irrelevant.) We find Young's modulus Y_{Al} and the coefficient of linear expansion α_{Al} from Tables 11.1 and 17.1, respectively.

EXECUTE: We have $Y_{\text{Al}} = 7.0 \times 10^{10}$ Pa and $\alpha_{\text{Al}} = 2.4 \times 10^{-5}$ K⁻¹, and $\Delta T = 22.3^\circ\text{C} - 17.2^\circ\text{C} = 5.1$ C° = 5.1 K. From Eq. (17.12), the stress is

$$\begin{aligned} \frac{F}{A} &= -Y_{\text{Al}}\alpha_{\text{Al}}\Delta T \\ &= -(7.0 \times 10^{10} \text{ Pa})(2.4 \times 10^{-5} \text{ K}^{-1})(5.1 \text{ K}) \\ &= -8.6 \times 10^6 \text{ Pa} = -1200 \text{ lb/in.}^2 \end{aligned}$$

The total force is the cross-sectional area times the stress:

$$\begin{aligned} F &= A\left(\frac{F}{A}\right) = (20 \times 10^{-4} \text{ m}^2)(-8.6 \times 10^6 \text{ Pa}) \\ &= -1.7 \times 10^4 \text{ N} = 1.9 \text{ tons} \end{aligned}$$

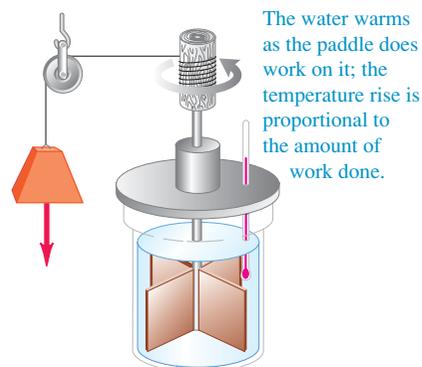
EVALUATE: The stress on the cylinder and the force it exerts on each wall are immense. Such thermal stresses must be accounted for in engineering.

Test Your Understanding of Section 17.4 In the bimetallic strip shown in Fig. 17.3a, metal 1 is copper. Which of the following materials could be used for metal 2? (There may be more than one correct answer). (i) steel; (ii) brass; (iii) aluminum. **I**

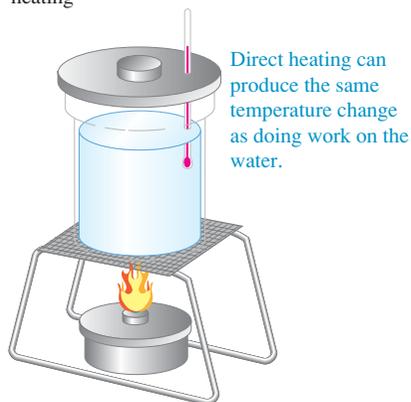
17.5 Quantity of Heat

17.15 The same temperature change of the same system may be accomplished by (a) doing work on it or (b) adding heat to it.

(a) Raising the temperature of water by doing work on it



(b) Raising the temperature of water by direct heating



17.16 The word “energy” is of Greek origin. This label on a can of Greek coffee shows that 100 milliliters of prepared coffee have an energy content (*ενέργεια*) of 9.6 kilojoules or 2.3 kilocalories.

ΔΙΑΤΡΟΦΙΚΕΣ ΠΛΗΡΟΦΟΡΙΕΣ	
Ανά 100ml ετοιμού προς κατανάλωση καφέ	
Ενέργεια	9,6 kJ / 2,3 kcal
Πρωτεΐνες	0,3 γρ.
Υδατάνθρακες	0,3 γρ.
Λιπαρά	0,004 γρ.

When you put a cold spoon into a cup of hot coffee, the spoon warms up and the coffee cools down as they approach thermal equilibrium. The interaction that causes these temperature changes is fundamentally a transfer of *energy* from one substance to another. Energy transfer that takes place solely because of a temperature difference is called *heat flow* or *heat transfer*, and energy transferred in this way is called **heat**.

An understanding of the relationship between heat and other forms of energy emerged during the 18th and 19th centuries. Sir James Joule (1818–1889) studied how water can be warmed by vigorous stirring with a paddle wheel (Fig. 17.15a). The paddle wheel adds energy to the water by doing *work* on it, and Joule found that *the temperature rise is directly proportional to the amount of work done*. The same temperature change can also be caused by putting the water in contact with some hotter body (Fig. 17.15b); hence this interaction must also involve an energy exchange. We will explore the relationship between heat and mechanical energy in Chapters 19 and 20.

CAUTION **Temperature vs. heat** It is absolutely essential for you to distinguish **?** between *temperature* and *heat*. Temperature depends on the physical state of a material and is a quantitative description of its hotness or coldness. In physics the term “heat” always refers to energy in transit from one body or system to another because of a temperature difference, never to the amount of energy contained within a particular system. We can change the temperature of a body by adding heat to it or taking heat away, or by adding or subtracting energy in other ways, such as mechanical work (Fig. 17.15a). If we cut a body in half, each half has the same temperature as the whole; but to raise the temperature of each half by a given interval, we add *half* as much heat as for the whole. **I**

We can define a *unit* of quantity of heat based on temperature changes of some specific material. The **calorie** (abbreviated cal) is defined as *the amount of heat required to raise the temperature of 1 gram of water from 14.5°C to 15.5°C*. The kilocalorie (kcal), equal to 1000 cal, is also used; a food-value calorie is actually a kilocalorie (Fig. 17.16). A corresponding unit of heat using Fahrenheit degrees and British units is the **British thermal unit**, or Btu. One Btu is the quantity of heat required to raise the temperature of 1 pound (weight) of water 1 F° from 63°F to 64°F.

Because heat is energy in transit, there must be a definite relationship between these units and the familiar mechanical energy units such as the joule. Experiments similar in concept to Joule’s have shown that

$$1 \text{ cal} = 4.186 \text{ J}$$

$$1 \text{ kcal} = 1000 \text{ cal} = 4186 \text{ J}$$

$$1 \text{ Btu} = 778 \text{ ft} \cdot \text{lb} = 252 \text{ cal} = 1055 \text{ J}$$

The calorie is not a fundamental SI unit. The International Committee on Weights and Measures recommends using the joule as the basic unit of energy in all forms, including heat. We will follow that recommendation in this book.

Specific Heat

We use the symbol Q for quantity of heat. When it is associated with an infinitesimal temperature change dT , we call it dQ . The quantity of heat Q required to increase the temperature of a mass m of a certain material from T_1 to T_2 is found to be approximately proportional to the temperature change $\Delta T = T_2 - T_1$. It is also proportional to the mass m of material. When you’re heating water to make tea, you need twice as much heat for two cups as for one if the temperature change is the same. The quantity of heat needed also depends on the nature of the material; raising the temperature of 1 kilogram of water by 1 C° requires 4190 J of heat, but only 910 J is needed to raise the temperature of 1 kilogram of aluminum by 1 C°.

Putting all these relationships together, we have

$$Q = mc \Delta T \quad (\text{heat required for temperature change } \Delta T \text{ of mass } m) \quad (17.13)$$

where c is a quantity, different for different materials, called the **specific heat** of the material. For an infinitesimal temperature change dT and corresponding quantity of heat dQ ,

$$dQ = mc dT \quad (17.14)$$

$$c = \frac{1}{m} \frac{dQ}{dT} \quad (\text{specific heat}) \quad (17.15)$$

In Eqs. (17.13), (17.14), and (17.15), Q (or dQ) and ΔT (or dT) can be either positive or negative. When they are positive, heat enters the body and its temperature increases; when they are negative, heat leaves the body and its temperature decreases.

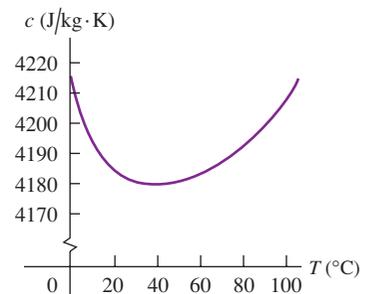
CAUTION **The definition of heat** Remember that dQ does not represent a change in the amount of heat *contained* in a body; this is a meaningless concept. Heat is always energy *in transit* as a result of a temperature difference. There is no such thing as “the amount of heat in a body.”

The specific heat of water is approximately

$$4190 \text{ J/kg} \cdot \text{K} \quad 1 \text{ cal/g} \cdot \text{C}^\circ \quad \text{or} \quad 1 \text{ Btu/lb} \cdot \text{F}^\circ$$

The specific heat of a material always depends somewhat on the initial temperature and the temperature interval. Figure 17.17 shows this dependence for water. In the problems and examples in this chapter we will usually ignore this small variation.

17.17 Specific heat of water as a function of temperature. The value of c varies by less than 1% between 0°C and 100°C .



Example 17.5 Feed a cold, starve a fever

During a bout with the flu an 80-kg man ran a fever of 39.0°C (102.2°F) instead of the normal body temperature of 37.0°C (98.6°F). Assuming that the human body is mostly water, how much heat is required to raise his temperature by that amount?

SOLUTION

IDENTIFY and SET UP: This problem uses the relationship among heat (the target variable), mass, specific heat, and temperature change. We use Eq. (17.13) to determine the required heat Q , with $m = 80 \text{ kg}$, $c = 4190 \text{ J/kg} \cdot \text{K}$ (for water), and $\Delta T = 39.0^\circ\text{C} - 37.0^\circ\text{C} = 2.0 \text{ C}^\circ = 2.0 \text{ K}$.

EXECUTE: From Eq. (17.13),

$$Q = mc \Delta T = (80 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(2.0 \text{ K}) = 6.7 \times 10^5 \text{ J}$$

EVALUATE: This corresponds to 160 kcal. In fact, the specific heat of the human body is about $3480 \text{ J/kg} \cdot \text{K}$, 83% that of water, because protein, fat, and minerals have lower specific heats. Hence a more accurate answer is $Q = 5.6 \times 10^5 \text{ J} = 133 \text{ kcal}$. Either result shows us that were it not for the body's temperature-regulating systems, taking in energy in the form of food would produce measurable changes in body temperature. (The elevated temperature of a person with the flu results from the body's extra activity in response to infection.)

Example 17.6 Overheating electronics

You are designing an electronic circuit element made of 23 mg of silicon. The electric current through it adds energy at the rate of $7.4 \text{ mW} = 7.4 \times 10^{-3} \text{ J/s}$. If your design doesn't allow any heat transfer out of the element, at what rate does its temperature increase? The specific heat of silicon is $705 \text{ J/kg} \cdot \text{K}$.

SOLUTION

IDENTIFY and SET UP: The energy added to the circuit element gives rise to a temperature increase, just as if heat were flowing into the element at the rate $dQ/dt = 7.4 \times 10^{-3} \text{ J/s}$. Our target variable is the rate of temperature change dT/dt . We can use Eq. (17.14),

which relates infinitesimal temperature changes dT to the corresponding heat dQ , to obtain an expression for dQ/dt in terms of dT/dt .

EXECUTE: We divide both sides of Eq. (17.14) by dt and rearrange:

$$\frac{dT}{dt} = \frac{dQ/dt}{mc} = \frac{7.4 \times 10^{-3} \text{ J/s}}{(23 \times 10^{-6} \text{ kg})(705 \text{ J/kg} \cdot \text{K})} = 0.46 \text{ K/s}$$

EVALUATE: At this rate of temperature rise (27 K/min), the circuit element would soon self-destruct. Heat transfer is an important design consideration in electronic circuit elements.

Molar Heat Capacity

Sometimes it's more convenient to describe a quantity of substance in terms of the number of *moles* n rather than the *mass* m of material. Recall from your study of chemistry that a mole of any pure substance always contains the same number of molecules. (We will discuss this point in more detail in Chapter 18.) The *molar mass* of any substance, denoted by M , is the mass per mole. (The quantity M is sometimes called *molecular weight*, but *molar mass* is preferable; the quantity depends on the mass of a molecule, not its weight.) For example, the molar mass of water is $18.0 \text{ g/mol} = 18.0 \times 10^{-3} \text{ kg/mol}$; 1 mole of water has a mass of $18.0 \text{ g} = 0.0180 \text{ kg}$. The total mass m of material is equal to the mass per mole M times the number of moles n :

$$m = nM \quad (17.16)$$

Replacing the mass m in Eq. (17.13) by the product nM , we find

$$Q = nMc \Delta T \quad (17.17)$$

The product Mc is called the **molar heat capacity** (or *molar specific heat*) and is denoted by C (capitalized). With this notation we rewrite Eq. (17.17) as

$$Q = nC \Delta T \quad (\text{heat required for temperature change of } n \text{ moles}) \quad (17.18)$$

Comparing to Eq. (17.15), we can express the molar heat capacity C (heat per mole per temperature change) in terms of the specific heat c (heat per mass per temperature change) and the molar mass M (mass per mole):

$$C = \frac{1}{n} \frac{dQ}{dT} = Mc \quad (\text{molar heat capacity}) \quad (17.19)$$

For example, the molar heat capacity of water is

$$C = Mc = (0.0180 \text{ kg/mol})(4190 \text{ J/kg} \cdot \text{K}) = 75.4 \text{ J/mol} \cdot \text{K}$$

Values of specific heat and molar heat capacity for several substances are given in Table 17.3. Note the remarkably large specific heat for water (Fig. 17.18).

CAUTION The meaning of “heat capacity” The term “heat capacity” is unfortunate because it gives the erroneous impression that a body *contains* a certain amount of heat. Remember, heat is energy in transit to or from a body, not the energy residing in the body. █

17.18 Water has a much higher specific heat than the glass or metals used to make cookware. This helps explain why it takes several minutes to boil water on a stove, even though the pot or kettle reaches a high temperature very quickly.



Table 17.3 Approximate Specific Heats and Molar Heat Capacities (Constant Pressure)

Substance	Specific Heat, c (J/kg · K)	Molar Mass, M (kg/mol)	Molar Heat Capacity, C (J/mol · K)
Aluminum	910	0.0270	24.6
Beryllium	1970	0.00901	17.7
Copper	390	0.0635	24.8
Ethanol	2428	0.0461	111.9
Ethylene glycol	2386	0.0620	148.0
Ice (near 0°C)	2100	0.0180	37.8
Iron	470	0.0559	26.3
Lead	130	0.207	26.9
Marble (CaCO ₃)	879	0.100	87.9
Mercury	138	0.201	27.7
Salt (NaCl)	879	0.0585	51.4
Silver	234	0.108	25.3
Water (liquid)	4190	0.0180	75.4

Precise measurements of specific heats and molar heat capacities require great experimental skill. Usually, a measured quantity of energy is supplied by an electric current in a heater wire wound around the specimen. The temperature change ΔT is measured with a resistance thermometer or thermocouple embedded in the specimen. This sounds simple, but great care is needed to avoid or compensate for unwanted heat transfer between the sample and its surroundings. Measurements for solid materials are usually made at constant atmospheric pressure; the corresponding values are called the *specific heat* and *molar heat capacity at constant pressure*, denoted by c_p and C_p . For a gas it is usually easier to keep the substance in a container with constant *volume*; the corresponding values are called the *specific heat* and *molar heat capacity at constant volume*, denoted by c_V and C_V . For a given substance, C_V and C_p are different. If the system can expand while heat is added, there is additional energy exchange through the performance of *work* by the system on its surroundings. If the volume is constant, the system does no work. For gases the difference between C_p and C_V is substantial. We will study heat capacities of gases in detail in Section 19.7.

The last column of Table 17.3 shows something interesting. The molar heat capacities for most elemental solids are about the same: about $25 \text{ J/mol} \cdot \text{K}$. This correlation, named the *rule of Dulong and Petit* (for its discoverers), forms the basis for a very important idea. The number of atoms in 1 mole is the same for all elemental substances. This means that on a *per atom* basis, about the same amount of heat is required to raise the temperature of each of these elements by a given amount, even though the *masses* of the atoms are very different. The heat required for a given temperature increase depends only on *how many* atoms the sample contains, not on the mass of an individual atom. We will see the reason the rule of Dulong and Petit works so well when we study the molecular basis of heat capacities in greater detail in Chapter 18.

Test Your Understanding of Section 17.5 You wish to raise the temperature of each of the following samples from 20°C to 21°C . Rank these in order of the amount of heat needed to do this, from highest to lowest. (i) 1 kilogram of mercury; (ii) 1 kilogram of ethanol; (iii) 1 mole of mercury; (iv) 1 mole of ethanol.



17.6 Calorimetry and Phase Changes

Calorimetry means “measuring heat.” We have discussed the energy transfer (heat) involved in temperature changes. Heat is also involved in *phase changes*, such as the melting of ice or boiling of water. Once we understand these additional heat relationships, we can analyze a variety of problems involving quantity of heat.

Phase Changes

We use the term **phase** to describe a specific state of matter, such as a solid, liquid, or gas. The compound H_2O exists in the *solid phase* as ice, in the *liquid phase* as water, and in the *gaseous phase* as steam. (These are also referred to as **states of matter**: the solid state, the liquid state, and the gaseous state.) A transition from one phase to another is called a **phase change** or *phase transition*. For any given pressure a phase change takes place at a definite temperature, usually accompanied by absorption or emission of heat and a change of volume and density.

A familiar example of a phase change is the melting of ice. When we add heat to ice at 0°C and normal atmospheric pressure, the temperature of the ice *does not* increase. Instead, some of it melts to form liquid water. If we add the heat slowly, to maintain the system very close to thermal equilibrium, the temperature remains at 0°C until all the ice is melted (Fig. 17.19). The effect of adding heat to this system is not to raise its temperature but to change its *phase* from solid to liquid.

To change 1 kg of ice at 0°C to 1 kg of liquid water at 0°C and normal atmospheric pressure requires $3.34 \times 10^5 \text{ J}$ of heat. The heat required per unit mass is

17.19 The surrounding air is at room temperature, but this ice–water mixture remains at 0°C until all of the ice has melted and the phase change is complete.



called the **heat of fusion** (or sometimes *latent heat of fusion*), denoted by L_f . For water at normal atmospheric pressure the heat of fusion is

$$L_f = 3.34 \times 10^5 \text{ J/kg} = 79.6 \text{ cal/g} = 143 \text{ Btu/lb}$$

More generally, to melt a mass m of material that has a heat of fusion L_f requires a quantity of heat Q given by

$$Q = mL_f$$

This process is *reversible*. To freeze liquid water to ice at 0°C , we have to *remove* heat; the magnitude is the same, but in this case, Q is negative because heat is removed rather than added. To cover both possibilities and to include other kinds of phase changes, we write

$$Q = \pm mL \quad (\text{heat transfer in a phase change}) \quad (17.20)$$

The plus sign (heat entering) is used when the material melts; the minus sign (heat leaving) is used when it freezes. The heat of fusion is different for different materials, and it also varies somewhat with pressure.

For any given material at any given pressure, the freezing temperature is the same as the melting temperature. At this unique temperature the liquid and solid phases (liquid water and ice, for example) can coexist in a condition called **phase equilibrium**.

We can go through this whole story again for *boiling* or *evaporation*, a phase transition between liquid and gaseous phases. The corresponding heat (per unit mass) is called the **heat of vaporization** L_v . At normal atmospheric pressure the heat of vaporization L_v for water is

$$L_v = 2.256 \times 10^6 \text{ J/kg} = 539 \text{ cal/g} = 970 \text{ Btu/lb}$$

That is, it takes $2.256 \times 10^6 \text{ J}$ to change 1 kg of liquid water at 100°C to 1 kg of water vapor at 100°C . By comparison, to raise the temperature of 1 kg of water from 0°C to 100°C requires $Q = mc \Delta T = (1.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{C}^\circ) \times (100 \text{ C}^\circ) = 4.19 \times 10^5 \text{ J}$, less than one-fifth as much heat as is required for vaporization at 100°C . This agrees with everyday kitchen experience; a pot of water may reach boiling temperature in a few minutes, but it takes a much longer time to completely evaporate all the water away.

Like melting, boiling is a reversible transition. When heat is removed from a gas at the boiling temperature, the gas returns to the liquid phase, or *condenses*, giving up to its surroundings the same quantity of heat (heat of vaporization) that was needed to vaporize it. At a given pressure the boiling and condensation temperatures are always the same; at this temperature the liquid and gaseous phases can coexist in phase equilibrium.

Both L_v and the boiling temperature of a material depend on pressure. Water boils at a lower temperature (about 95°C) in Denver than in Pittsburgh because Denver is at higher elevation and the average atmospheric pressure is lower. The heat of vaporization is somewhat greater at this lower pressure, about $2.27 \times 10^6 \text{ J/kg}$.

Table 17.4 lists heats of fusion and vaporization for some materials and their melting and boiling temperatures at normal atmospheric pressure. Very few *elements* have melting temperatures in the vicinity of ordinary room temperatures; one of the few is the metal gallium, shown in Fig. 17.20.

Figure 17.21 shows how the temperature varies when we add heat continuously to a specimen of ice with an initial temperature below 0°C (point *a*). The temperature rises until we reach the melting point (point *b*). As more heat is added, the temperature remains constant until all the ice has melted (point *c*). Then the temperature rises again until the boiling temperature is reached (point *d*). At that point the temperature again is constant until all the water is transformed into the vapor phase (point *e*). If the rate of heat input is constant, the line for the solid phase (ice) has a steeper slope than does the line for the liquid phase (water). Do you see why? (See Table 17.3.)

17.20 The metal gallium, shown here melting in a person's hand, is one of the few elements that melt in the vicinity of room temperature. Its melting temperature is 29.8°C , and its heat of fusion is $8.04 \times 10^4 \text{ J/kg}$.



Table 17.4 Heats of Fusion and Vaporization

Substance	Normal Melting Point		Heat of Fusion, L_f (J/kg)	Normal Boiling Point		Heat of Vaporization, L_v (J/kg)
	K	°C		K	°C	
Helium	*	*	*	4.216	-268.93	20.9×10^3
Hydrogen	13.84	-259.31	58.6×10^3	20.26	-252.89	452×10^3
Nitrogen	63.18	-209.97	25.5×10^3	77.34	-195.8	201×10^3
Oxygen	54.36	-218.79	13.8×10^3	90.18	-183.0	213×10^3
Ethanol	159	-114	104.2×10^3	351	78	854×10^3
Mercury	234	-39	11.8×10^3	630	357	272×10^3
Water	273.15	0.00	334×10^3	373.15	100.00	2256×10^3
Sulfur	392	119	38.1×10^3	717.75	444.60	326×10^3
Lead	600.5	327.3	24.5×10^3	2023	1750	871×10^3
Antimony	903.65	630.50	165×10^3	1713	1440	561×10^3
Silver	1233.95	960.80	88.3×10^3	2466	2193	2336×10^3
Gold	1336.15	1063.00	64.5×10^3	2933	2660	1578×10^3
Copper	1356	1083	134×10^3	1460	1187	5069×10^3

*A pressure in excess of 25 atmospheres is required to make helium solidify. At 1 atmosphere pressure, helium remains a liquid down to absolute zero.

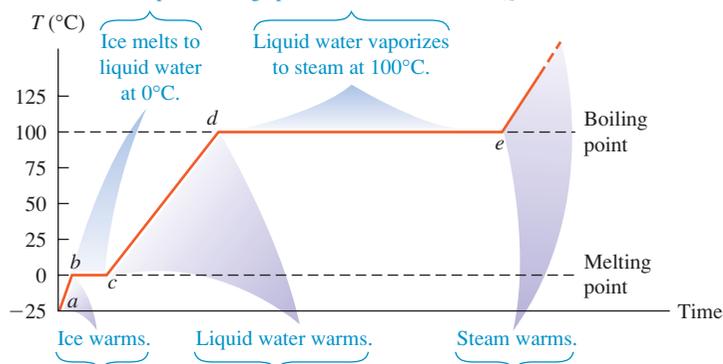
A substance can sometimes change directly from the solid to the gaseous phase. This process is called *sublimation*, and the solid is said to *sublime*. The corresponding heat is called the *heat of sublimation*, L_s . Liquid carbon dioxide cannot exist at a pressure lower than about 5×10^5 Pa (about 5 atm), and “dry ice” (solid carbon dioxide) sublimates at atmospheric pressure. Sublimation of water from frozen food causes freezer burn. The reverse process, a phase change from gas to solid, occurs when frost forms on cold bodies such as refrigerator cooling coils.

Very pure water can be cooled several degrees below the freezing temperature without freezing; the resulting unstable state is described as *supercooled*. When a small ice crystal is dropped in or the water is agitated, it crystallizes within a second or less. Supercooled water *vapor* condenses quickly into fog droplets when a disturbance, such as dust particles or ionizing radiation, is introduced. This principle is used in “seeding” clouds, which often contain supercooled water vapor, to cause condensation and rain.

A liquid can sometimes be *superheated* above its normal boiling temperature. Any small disturbance such as agitation causes local boiling with bubble formation.

Steam heating systems for buildings use a boiling–condensing process to transfer heat from the furnace to the radiators. Each kilogram of water that is

Phase of water changes. During these periods, temperature stays constant and the phase change proceeds as heat is added: $Q = +mL$.



Temperature of water changes. During these periods, temperature rises as heat is added: $Q = mc\Delta T$.

17.21 Graph of temperature versus time for a specimen of water initially in the solid phase (ice). Heat is added to the specimen at a constant rate. The temperature remains constant during each change of phase, provided that the pressure remains constant.

17.22 The water may be warm and it may be a hot day, but these children will feel cold when they first step out of the swimming pool. That's because as water evaporates from their skin, it removes the heat of vaporization from their bodies. To stay warm, they will need to dry off immediately.



turned to steam in the boiler absorbs over 2×10^6 J (the heat of vaporization L_v of water) from the boiler and gives it up when it condenses in the radiators. Boiling–condensing processes are also used in refrigerators, air conditioners, and heat pumps. We will discuss these systems in Chapter 20.

The temperature-control mechanisms of many warm-blooded animals make use of heat of vaporization, removing heat from the body by using it to evaporate water from the tongue (panting) or from the skin (sweating). Evaporative cooling enables humans to maintain normal body temperature in hot, dry desert climates where the air temperature may reach 55°C (about 130°F). The skin temperature may be as much as 30°C cooler than the surrounding air. Under these conditions a normal person may perspire several liters per day, and this lost water must be replaced. Old-time desert rats (such as one of the authors) state that in the desert, any canteen that holds less than a gallon should be viewed as a toy! Evaporative cooling also explains why you feel cold when you first step out of a swimming pool (Fig. 17.22).

Evaporative cooling is also used to cool buildings in hot, dry climates and to condense and recirculate “used” steam in coal-fired or nuclear-powered electric-generating plants. That’s what goes on in the large, tapered concrete towers that you see at such plants.

Chemical reactions such as combustion are analogous to phase changes in that they involve definite quantities of heat. Complete combustion of 1 gram of gasoline produces about 46,000 J or about 11,000 cal, so the **heat of combustion** L_c of gasoline is

$$L_c = 46,000 \text{ J/g} = 4.6 \times 10^7 \text{ J/kg}$$

Energy values of foods are defined similarly. When we say that a gram of peanut butter “contains 6 calories,” we mean that 6 kcal of heat (6000 cal or 25,000 J) is released when the carbon and hydrogen atoms in the peanut butter react with oxygen (with the help of enzymes) and are completely converted to CO_2 and H_2O . Not all of this energy is directly useful for mechanical work. We will study the *efficiency* of energy utilization in Chapter 20.

Heat Calculations

Let’s look at some examples of calorimetry calculations (calculations with heat). The basic principle is very simple: When heat flow occurs between two bodies that are isolated from their surroundings, the amount of heat lost by one body must equal the amount gained by the other. Heat is energy in transit, so this principle is really just conservation of energy. Calorimetry, dealing entirely with one conserved quantity, is in many ways the simplest of all physical theories!

Problem-Solving Strategy 17.2 Calorimetry Problems



IDENTIFY *the relevant concepts:* When heat flow occurs between two or more bodies that are isolated from their surroundings, the *algebraic sum* of the quantities of heat transferred to all the bodies is zero. We take a quantity of heat *added* to a body as *positive* and a quantity *leaving* a body as *negative*.

SET UP *the problem* using the following steps:

1. Identify the objects that exchange heat.
2. Each object may undergo a temperature change only, a phase change at constant temperature, or both. Use Eq. (17.13) for the heat transferred in a temperature change and Eq. (17.20) for the heat transferred in a phase change.
3. Consult Table 17.3 for values of specific heat or molar heat capacity and Table 17.4 for heats of fusion or vaporization.
4. List the known and unknown quantities and identify the target variables.

EXECUTE *the solution* as follows:

1. Use Eq. (17.13) and/or Eq. (17.20) and the energy-conservation relation $\sum Q = 0$ to solve for the target variables. Ensure that you use the correct algebraic signs for Q and ΔT terms, and that you correctly write $\Delta T = T_{\text{final}} - T_{\text{initial}}$ and not the reverse.
2. If a phase change occurs, you may not know in advance whether all, or only part, of the material undergoes a phase change. Make a reasonable guess; if that leads to an unreasonable result (such as a final temperature higher or lower than any initial temperature), the guess was wrong. Try again!

EVALUATE *your answer:* Double-check your calculations, and ensure that the results are physically sensible.

Example 17.7 A temperature change with no phase change

A camper pours 0.300 kg of coffee, initially in a pot at 70.0°C, into a 0.120-kg aluminum cup initially at 20.0°C. What is the equilibrium temperature? Assume that coffee has the same specific heat as water and that no heat is exchanged with the surroundings.

SOLUTION

IDENTIFY and SET UP: The target variable is the common final temperature T of the cup and coffee. No phase changes occur, so we need only Eq. (17.13). With subscripts C for coffee, W for water, and Al for aluminum, we have $T_{0C} = 70.0^\circ$ and $T_{0Al} = 20.0^\circ$; Table 17.3 gives $c_W = 4190 \text{ J/kg} \cdot \text{K}$ and $c_{Al} = 910 \text{ J/kg} \cdot \text{K}$.

EXECUTE: The (negative) heat gained by the coffee is $Q_C = m_{CC}c_W\Delta T_C$. The (positive) heat gained by the cup is $Q_{Al} = m_{Al}c_{Al}\Delta T_{Al}$. We set $Q_C + Q_{Al} = 0$ (see Problem-Solving Strategy 17.2) and substitute $\Delta T_C = T - T_{0C}$ and $\Delta T_{Al} = T - T_{0Al}$:

$$Q_C + Q_{Al} = m_{CC}c_W\Delta T_C + m_{Al}c_{Al}\Delta T_{Al} = 0$$

$$m_{CC}c_W(T - T_{0C}) + m_{Al}c_{Al}(T - T_{0Al}) = 0$$

Then we solve this expression for the final temperature T . A little algebra gives

$$T = \frac{m_{CC}c_W T_{0C} + m_{Al}c_{Al} T_{0Al}}{m_{CC}c_W + m_{Al}c_{Al}} = 66.0^\circ\text{C}$$

EVALUATE: The final temperature is much closer to the initial temperature of the coffee than to that of the cup; water has a much higher specific heat than aluminum, and we have more than twice as much mass of water. We can also find the quantities of heat by substituting the value $T = 66.0^\circ\text{C}$ back into the original equations. We find $Q_C = -5.0 \times 10^3 \text{ J}$ and $Q_{Al} = +5.0 \times 10^3 \text{ J}$. As expected, Q_C is negative: The coffee loses heat to the cup.

Example 17.8 Changes in both temperature and phase

A glass contains 0.25 kg of Omni-Cola (mostly water) initially at 25°C. How much ice, initially at -20°C , must you add to obtain a final temperature of 0°C with all the ice melted? Neglect the heat capacity of the glass.

SOLUTION

IDENTIFY and SET UP: The Omni-Cola and ice exchange heat. The cola undergoes a temperature change; the ice undergoes both a temperature change and a phase change from solid to liquid. We use subscripts C for cola, I for ice, and W for water. The target variable is the mass of ice, m_I . We use Eq. (17.13) to obtain an expression for the amount of heat involved in cooling the drink to $T = 0^\circ\text{C}$ and warming the ice to $T = 0^\circ\text{C}$, and Eq. (17.20) to obtain an expression for the heat required to melt the ice at 0°C . We have $T_{0C} = 25^\circ\text{C}$ and $T_{0I} = -20^\circ\text{C}$, Table 17.3 gives $c_W = 4190 \text{ J/kg} \cdot \text{K}$ and $c_I = 2100 \text{ J/kg} \cdot \text{K}$, and Table 17.4 gives $L_f = 3.34 \times 10^5 \text{ J/kg}$.

EXECUTE: From Eq. (17.13), the (negative) heat gained by the Omni-Cola is $Q_C = m_{CC}c_W\Delta T_C$. The (positive) heat gained by the ice in warming is $Q_I = m_Ic_I\Delta T_I$. The (positive) heat required to melt the ice is $Q_2 = m_I L_f$. We set $Q_C + Q_I + Q_2 = 0$, insert $\Delta T_C = T - T_{0C}$ and $\Delta T_I = T - T_{0I}$, and solve for m_I :

$$m_{CC}c_W\Delta T_C + m_Ic_I\Delta T_I + m_I L_f = 0$$

$$m_{CC}c_W(T - T_{0C}) + m_Ic_I(T - T_{0I}) + m_I L_f = 0$$

$$m_I[c_I(T - T_{0I}) + L_f] = -m_{CC}c_W(T - T_{0C})$$

$$m_I = m_C \frac{c_W(T_{0C} - T)}{c_I(T - T_{0I}) + L_f}$$

Substituting numerical values, we find that $m_I = 0.070 \text{ kg} = 70 \text{ g}$.

EVALUATE: Three or four medium-size ice cubes would make about 70 g, which seems reasonable given the 250 g of Omni-Cola to be cooled.

Example 17.9 What's cooking?

A hot copper pot of mass 2.0 kg (including its copper lid) is at a temperature of 150°C . You pour 0.10 kg of cool water at 25°C into the pot, then quickly replace the lid so no steam can escape. Find the final temperature of the pot and its contents, and determine the phase of the water (liquid, gas, or a mixture). Assume that no heat is lost to the surroundings.

SOLUTION

IDENTIFY and SET UP: The water and the pot exchange heat. Three outcomes are possible: (1) No water boils, and the final temperature T is less than 100°C ; (2) some water boils, giving a mixture of water and steam at 100°C ; or (3) all the water boils, giving 0.10 kg of steam at 100°C or greater. We use Eq. (17.13) for the heat transferred in a temperature change and Eq. (17.20) for the heat transferred in a phase change.

EXECUTE: First consider case (1), which parallels Example 17.8 exactly. The equation that states that the heat flow into the water equals the heat flow out of the pot is

$$Q_W + Q_{Cu} = m_Wc_W(T - T_{0W}) + m_{Cu}c_{Cu}(T - T_{0Cu}) = 0$$

Here we use subscripts W for water and Cu for copper, with $m_W = 0.10 \text{ kg}$, $m_{Cu} = 2.0 \text{ kg}$, $T_{0W} = 25^\circ\text{C}$, and $T_{0Cu} = 150^\circ\text{C}$. From Table 17.3, $c_W = 4190 \text{ J/kg} \cdot \text{K}$ and $c_{Cu} = 390 \text{ J/kg} \cdot \text{K}$. Solving for the final temperature T and substituting these values, we get

$$T = \frac{m_Wc_W T_{0W} + m_{Cu}c_{Cu} T_{0Cu}}{m_Wc_W + m_{Cu}c_{Cu}} = 106^\circ\text{C}$$

But this is above the boiling point of water, which contradicts our assumption that no water boils! So at least some of the water boils.

Continued

So consider case (2), in which the final temperature is $T = 100^\circ\text{C}$ and some unknown fraction x of the water boils, where (if this case is correct) x is greater than zero and less than or equal to 1. The (positive) amount of heat needed to vaporize this water is $xm_{\text{W}}L_{\text{v}}$. The energy-conservation condition $Q_{\text{W}} + Q_{\text{Cu}} = 0$ is then

$$m_{\text{W}}c_{\text{W}}(100^\circ\text{C} - T_{0\text{W}}) + xm_{\text{W}}L_{\text{v}} + m_{\text{Cu}}c_{\text{Cu}}(100^\circ\text{C} - T_{0\text{Cu}}) = 0$$

We solve for the target variable x :

$$x = \frac{-m_{\text{Cu}}c_{\text{Cu}}(100^\circ\text{C} - T_{0\text{Cu}}) - m_{\text{W}}c_{\text{W}}(100^\circ\text{C} - T_{0\text{W}})}{m_{\text{W}}L_{\text{v}}}$$

With $L_{\text{v}} = 2.256 \times 10^6 \text{ J}$ from Table 17.4, this yields $x = 0.034$. We conclude that the final temperature of the water and copper is 100°C and that $0.034(0.10 \text{ kg}) = 0.0034 \text{ kg} = 3.4 \text{ g}$ of the water is converted to steam at 100°C .

EVALUATE: Had x turned out to be greater than 1, case (3) would have held; all the water would have vaporized, and the final temperature would have been greater than 100°C . Can you show that this would have been the case if we had originally poured less than 15 g of 25°C water into the pot?

Example 17.10 Combustion, temperature change, and phase change

In a particular camp stove, only 30% of the energy released in burning gasoline goes to heating the water in a pot on the stove. How much gasoline must we burn to heat 1.00 L (1.00 kg) of water from 20°C to 100°C and boil away 0.25 kg of it?

SOLUTION

IDENTIFY and SET UP: All of the water undergoes a temperature change and part of it undergoes a phase change, from liquid to gas. We determine the heat required to cause both of these changes, and then use the 30% combustion efficiency to determine the amount of gasoline that must be burned (the target variable). We use Eqs. (17.13) and (17.20) and the idea of heat of combustion.

EXECUTE: To raise the temperature of the water from 20°C to 100°C requires

$$\begin{aligned} Q_1 &= mc \Delta T = (1.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(80 \text{ K}) \\ &= 3.35 \times 10^5 \text{ J} \end{aligned}$$

To boil 0.25 kg of water at 100°C requires

$$Q_2 = mL_{\text{v}} = (0.25 \text{ kg})(2.256 \times 10^6 \text{ J/kg}) = 5.64 \times 10^5 \text{ J}$$

The total energy needed is $Q_1 + Q_2 = 8.99 \times 10^5 \text{ J}$. This is 30% = 0.30 of the total heat of combustion, which is therefore $(8.99 \times 10^5 \text{ J})/0.30 = 3.00 \times 10^6 \text{ J}$. As we mentioned earlier, the combustion of 1 g of gasoline releases 46,000 J, so the mass of gasoline required is $(3.00 \times 10^6 \text{ J})/(46,000 \text{ J/g}) = 65 \text{ g}$, or a volume of about 0.09 L of gasoline.

EVALUATE: This result suggests the tremendous amount of energy released in burning even a small quantity of gasoline. Another 123 g of gasoline would be required to boil away the remaining water; can you prove this?

Test Your Understanding of Section 17.6 You take a block of ice at 0°C and add heat to it at a steady rate. It takes a time t to completely convert the block of ice to steam at 100°C . What do you have at time $t/2$? (i) all ice at 0°C ; (ii) a mixture of ice and water at 0°C ; (iii) water at a temperature between 0°C and 100°C ; (iv) a mixture of water and steam at 100°C .



MasteringPHYSICS

PhET: The Greenhouse Effect

17.7 Mechanisms of Heat Transfer

We have talked about *conductors* and *insulators*, materials that permit or prevent heat transfer between bodies. Now let's look in more detail at *rates* of energy transfer. In the kitchen you use a metal or glass pot for good heat transfer from the stove to whatever you're cooking, but your refrigerator is insulated with a material that *prevents* heat from flowing into the food inside the refrigerator. How do we describe the difference between these two materials?

The three mechanisms of heat transfer are conduction, convection, and radiation. *Conduction* occurs within a body or between two bodies in contact. *Convection* depends on motion of mass from one region of space to another. *Radiation* is heat transfer by electromagnetic radiation, such as sunshine, with no need for matter to be present in the space between bodies.

Conduction

If you hold one end of a copper rod and place the other end in a flame, the end you are holding gets hotter and hotter, even though it is not in direct contact with the flame. Heat reaches the cooler end by **conduction** through the material.

On the atomic level, the atoms in the hotter regions have more kinetic energy, on the average, than their cooler neighbors. They jostle their neighbors, giving them some of their energy. The neighbors jostle *their* neighbors, and so on through the material. The atoms themselves do not move from one region of material to another, but their energy does.

Most metals also use another, more effective mechanism to conduct heat. Within the metal, some electrons can leave their parent atoms and wander through the crystal lattice. These “free” electrons can rapidly carry energy from the hotter to the cooler regions of the metal, so metals are generally good conductors of heat. A metal rod at 20°C feels colder than a piece of wood at 20°C because heat can flow more easily from your hand into the metal. The presence of “free” electrons also causes most metals to be good electrical conductors.

Heat transfer occurs only between regions that are at different temperatures, and the direction of heat flow is always from higher to lower temperature. Figure 17.23a shows a rod of conducting material with cross-sectional area A and length L . The left end of the rod is kept at a temperature T_H and the right end at a lower temperature T_C , so heat flows from left to right. The sides of the rod are covered by an ideal insulator, so no heat transfer occurs at the sides.

When a quantity of heat dQ is transferred through the rod in a time dt , the rate of heat flow is dQ/dt . We call this rate the **heat current**, denoted by H . That is, $H = dQ/dt$. Experiments show that the heat current is proportional to the cross-sectional area A of the rod (Fig. 17.23b) and to the temperature difference $(T_H - T_C)$ and is inversely proportional to the rod length L (Fig. 17.23c). Introducing a proportionality constant k called the **thermal conductivity** of the material, we have

$$H = \frac{dQ}{dt} = kA \frac{T_H - T_C}{L} \quad (\text{heat current in conduction}) \quad (17.21)$$

The quantity $(T_H - T_C)/L$ is the temperature difference *per unit length*; it is called the magnitude of the **temperature gradient**. The numerical value of k depends on the material of the rod. Materials with large k are good conductors of heat; materials with small k are poor conductors, or insulators. Equation (17.21) also gives the heat current through a slab or through *any* homogeneous body with uniform cross section A perpendicular to the direction of flow; L is the length of the heat-flow path.

The units of heat current H are units of energy per time, or power; the SI unit of heat current is the watt ($1 \text{ W} = 1 \text{ J/s}$). We can find the units of k by solving Eq. (17.21) for k ; you can show that the SI units are $\text{W/m} \cdot \text{K}$. Some numerical values of k are given in Table 17.5.

The thermal conductivity of “dead” (that is, nonmoving) air is very small. A wool sweater keeps you warm because it traps air between the fibers. In fact, many insulating materials such as Styrofoam and fiberglass are mostly dead air.

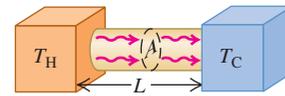
If the temperature varies in a nonuniform way along the length of the conducting rod, we introduce a coordinate x along the length and generalize the temperature gradient to be dT/dx . The corresponding generalization of Eq. (17.21) is

$$H = \frac{dQ}{dt} = -kA \frac{dT}{dx} \quad (17.22)$$

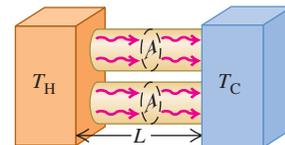
The negative sign shows that heat always flows in the direction of *decreasing* temperature.

17.23 Steady-state heat flow due to conduction in a uniform rod.

(a) Heat current H



(b) Doubling the cross-sectional area of the conductor doubles the heat current (H is proportional to A).



(c) Doubling the length of the conductor halves the heat current (H is inversely proportional to L).

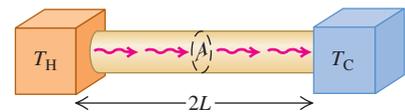


Table 17.5 Thermal Conductivities

Substance	k ($\text{W/m} \cdot \text{K}$)
<i>Metals</i>	
Aluminum	205.0
Brass	109.0
Copper	385.0
Lead	34.7
Mercury	8.3
Silver	406.0
Steel	50.2
<i>Solids (representative values)</i>	
Brick, insulating	0.15
Brick, red	0.6
Concrete	0.8
Cork	0.04
Felt	0.04
Fiberglass	0.04
Glass	0.8
Ice	1.6
Rock wool	0.04
Styrofoam	0.027
Wood	0.12–0.04
<i>Gases</i>	
Air	0.024
Argon	0.016
Helium	0.14
Hydrogen	0.14
Oxygen	0.023

Application Fur Versus Blubber

The fur of an arctic fox is a good thermal insulator because it traps air, which has a low thermal conductivity k . (The value $k = 0.04 \text{ W/m} \cdot \text{K}$ for fur is higher than for air, $k = 0.024 \text{ W/m} \cdot \text{K}$, because fur also includes solid hairs.) The layer of fat beneath a bowhead whale's skin, called blubber, has six times the thermal conductivity of fur ($k = 0.24 \text{ W/m} \cdot \text{K}$). So a 6-cm thickness of blubber ($L = 6 \text{ cm}$) is required to give the same insulation as 1 cm of fur.



For thermal insulation in buildings, engineers use the concept of **thermal resistance**, denoted by R . The thermal resistance R of a slab of material with area A is defined so that the heat current H through the slab is

$$H = \frac{A(T_H - T_C)}{R} \quad (17.23)$$

where T_H and T_C are the temperatures on the two sides of the slab. Comparing this with Eq. (17.21), we see that R is given by

$$R = \frac{L}{k} \quad (17.24)$$

where L is the thickness of the slab. The SI unit of R is $1 \text{ m}^2 \cdot \text{K}/\text{W}$. In the units used for commercial insulating materials in the United States, H is expressed in Btu/h, A is in ft^2 , and $T_H - T_C$ in $^\circ\text{F}$. (1 Btu/h = 0.293 W.) The units of R are then $\text{ft}^2 \cdot ^\circ\text{F} \cdot \text{h}/\text{Btu}$, though values of R are usually quoted without units; a 6-inch-thick layer of fiberglass has an R value of 19 (that is, $R = 19 \text{ ft}^2 \cdot ^\circ\text{F} \cdot \text{h}/\text{Btu}$), a 2-inch-thick slab of polyurethane foam has an R value of 12, and so on. Doubling the thickness doubles the R value. Common practice in new construction in severe northern climates is to specify R values of around 30 for exterior walls and ceilings. When the insulating material is in layers, such as a plastered wall, fiberglass insulation, and wood exterior siding, the R values are additive. Do you see why? (See Problem 17.108.)

Problem-Solving Strategy 17.3 Heat Conduction

IDENTIFY *the relevant concepts:* Heat conduction occurs whenever two objects at different temperatures are placed in contact.

SET UP *the problem* using the following steps:

1. Identify the direction of heat flow (from hot to cold). In Eq. (17.21), L is measured along this direction, and A is an area perpendicular to this direction. You can often approximate an irregular-shaped container with uniform wall thickness as a flat slab with the same thickness and total wall area.
2. List the known and unknown quantities and identify the target variable.

EXECUTE *the solution* as follows:

1. If heat flows through a single object, use Eq. (17.21) to solve for the target variable.
2. If the heat flows through two different materials in succession (in *series*), the temperature T at the interface between them is

intermediate between T_H and T_C , so that the temperature differences across the two materials are $(T_H - T)$ and $(T - T_C)$. In steady-state heat flow, the same heat must pass through both materials, so the heat current H must be the *same* in both materials.

3. If heat flows through two or more *parallel* paths, then the total heat current H is the sum of the currents H_1, H_2, \dots for the separate paths. An example is heat flow from inside a room to outside, both through the glass in a window and through the surrounding wall. In parallel heat flow the temperature difference is the same for each path, but L, A , and k may be different for each path.
4. Use consistent units. If k is expressed in $\text{W}/\text{m} \cdot \text{K}$, for example, use distances in meters, heat in joules, and T in kelvins.

EVALUATE *your answer:* Are the results physically reasonable?

Example 17.11 Conduction into a picnic cooler

A Styrofoam cooler (Fig. 17.24a) has total wall area (including the lid) of 0.80 m^2 and wall thickness 2.0 cm. It is filled with ice, water, and cans of Omni-Cola, all at 0°C . What is the rate of heat flow into the cooler if the temperature of the outside wall is 30°C ? How much ice melts in 3 hours?

SOLUTION

IDENTIFY and SET UP: The target variables are the heat current H and the mass m of ice melted. We use Eq. (17.21) to determine H and Eq. (17.20) to determine m .

EXECUTE: We assume that the total heat flow is the same as it would be through a flat Styrofoam slab of area 0.80 m^2 and thickness $2.0 \text{ cm} = 0.020 \text{ m}$ (Fig. 17.24b). We find k from Table 17.5. From Eq. (17.21),

$$\begin{aligned} H &= kA \frac{T_H - T_C}{L} = (0.027 \text{ W}/\text{m} \cdot \text{K})(0.80 \text{ m}^2) \frac{30^\circ\text{C} - 0^\circ\text{C}}{0.020 \text{ m}} \\ &= 32.4 \text{ W} = 32.4 \text{ J/s} \end{aligned}$$

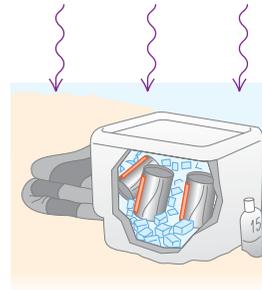
The total heat flow is $Q = Ht$, with $t = 3 \text{ h} = 10,800 \text{ s}$. From Table 17.4, the heat of fusion of ice is $L_f = 3.34 \times 10^5 \text{ J/kg}$, so from Eq. (17.20) the mass of ice that melts is

$$m = \frac{Q}{L_f} = \frac{(32.4 \text{ J/s})(10,800 \text{ s})}{3.34 \times 10^5 \text{ J/kg}} = 1.0 \text{ kg}$$

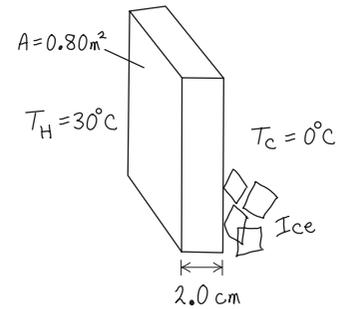
EVALUATE: The low heat current is a result of the low thermal conductivity of Styrofoam.

17.24 Conduction of heat across the walls of a Styrofoam cooler.

(a) A cooler at the beach



(b) Our sketch for this problem



Example 17.12 Conduction through two bars I

A steel bar 10.0 cm long is welded end to end to a copper bar 20.0 cm long. Each bar has a square cross section, 2.00 cm on a side. The free end of the steel bar is kept at 100°C by placing it in contact with steam, and the free end of the copper bar is kept at 0°C by placing it in contact with ice. Both bars are perfectly insulated on their sides. Find the steady-state temperature at the junction of the two bars and the total rate of heat flow through the bars.

SOLUTION

IDENTIFY and SET UP: Figure 17.25 shows the situation. The heat currents in these end-to-end bars must be the same (see Problem-Solving Strategy 17.3). We are given “hot” and “cold” temperatures $T_H = 100^\circ\text{C}$ and $T_C = 0^\circ\text{C}$. With subscripts S for steel and Cu for copper, we write Eq. (17.21) separately for the heat currents H_S and H_{Cu} and set the resulting expressions equal to each other.

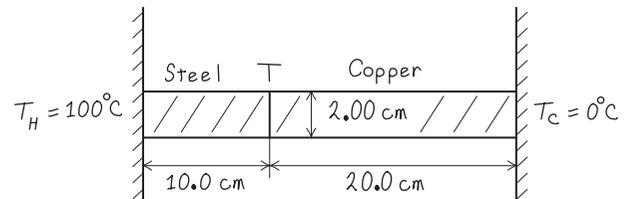
EXECUTE: Setting $H_S = H_{Cu}$, we have from Eq. (17.21)

$$H_S = k_S A \frac{T_H - T}{L_S} = H_{Cu} = k_{Cu} A \frac{T - T_C}{L_{Cu}}$$

We divide out the equal cross-sectional areas A and solve for T :

$$T = \frac{\frac{k_S T_H}{L_S} + \frac{k_{Cu} T_C}{L_{Cu}}}{\left(\frac{k_S}{L_S} + \frac{k_{Cu}}{L_{Cu}}\right)}$$

17.25 Our sketch for this problem.



Substituting $L_S = 10.0 \text{ cm}$ and $L_{Cu} = 20.0 \text{ cm}$, the given values of T_H and T_C , and the values of k_S and k_{Cu} from Table 17.5, we find $T = 20.7^\circ\text{C}$.

We can find the total heat current by substituting this value of T into either the expression for H_S or the one for H_{Cu} :

$$H_S = (50.2 \text{ W/m}\cdot\text{K})(0.0200 \text{ m})^2 \frac{100^\circ\text{C} - 20.7^\circ\text{C}}{0.100 \text{ m}} = 15.9 \text{ W}$$

$$H_{Cu} = (385 \text{ W/m}\cdot\text{K})(0.0200 \text{ m})^2 \frac{20.7^\circ\text{C}}{0.200 \text{ m}} = 15.9 \text{ W}$$

EVALUATE: Even though the steel bar is shorter, the temperature drop across it is much greater (from 100°C to 20.7°C) than across the copper bar (from 20.7°C to 0°C). That’s because steel is a much poorer conductor than copper.

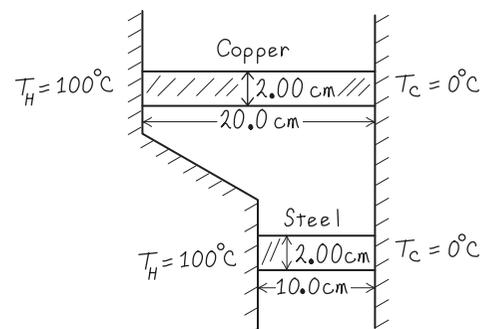
Example 17.13 Conduction through two bars II

Suppose the two bars of Example 17.12 are separated. One end of each bar is kept at 100°C and the other end of each bar is kept at 0°C . What is the *total* heat current in the two bars?

SOLUTION

IDENTIFY and SET UP: Figure 17.26 shows the situation. For each bar, $T_H - T_C = 100^\circ\text{C} - 0^\circ\text{C} = 100 \text{ K}$. The total heat current is the sum of the currents in the two bars, $H_S + H_{Cu}$.

17.26 Our sketch for this problem.



Continued

EXECUTE: We write the heat currents for the two rods individually, and then add them to get the total heat current:

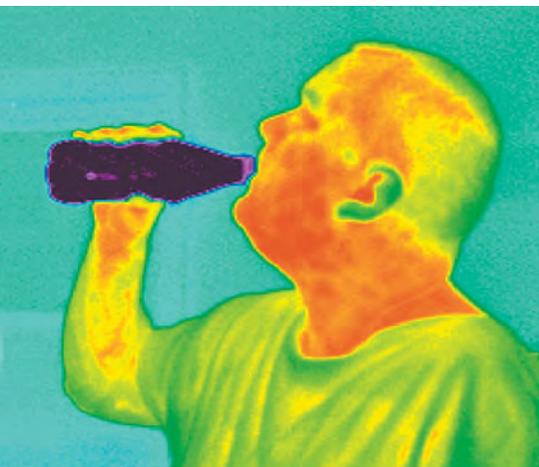
$$\begin{aligned} H &= H_S + H_{\text{Cu}} = k_S A \frac{T_H - T_C}{L_S} + k_{\text{Cu}} A \frac{T_H - T_C}{L_{\text{Cu}}} \\ &= (50.2 \text{ W/m} \cdot \text{K})(0.0200 \text{ m})^2 \frac{100 \text{ K}}{0.100 \text{ m}} \\ &\quad + (385 \text{ W/m} \cdot \text{K})(0.0200 \text{ m})^2 \frac{100 \text{ K}}{0.200 \text{ m}} \\ &= 20.1 \text{ W} + 77.0 \text{ W} = 97.1 \text{ W} \end{aligned}$$

EVALUATE: The heat flow in the copper bar is much greater than that in the steel bar, even though it is longer, because the thermal conductivity of copper is much larger. The total heat flow is greater than in Example 17.12 because the total cross section for heat flow is greater and because the full 100-K temperature difference appears across each bar.

17.27 A heating element in the tip of this submerged tube warms the surrounding water, producing a complex pattern of free convection.



17.28 This false-color infrared photograph reveals radiation emitted by various parts of the man's body. The strongest emission (colored red) comes from the warmest areas, while there is very little emission from the bottle of cold beverage.



Convection

Convection is the transfer of heat by mass motion of a fluid from one region of space to another. Familiar examples include hot-air and hot-water home heating systems, the cooling system of an automobile engine, and the flow of blood in the body. If the fluid is circulated by a blower or pump, the process is called *forced convection*; if the flow is caused by differences in density due to thermal expansion, such as hot air rising, the process is called *natural convection* or *free convection* (Fig. 17.27).

Free convection in the atmosphere plays a dominant role in determining the daily weather, and convection in the oceans is an important global heat-transfer mechanism. On a smaller scale, soaring hawks and glider pilots make use of thermal updrafts from the warm earth. The most important mechanism for heat transfer within the human body (needed to maintain nearly constant temperature in various environments) is *forced* convection of blood, with the heart serving as the pump.

Convective heat transfer is a very complex process, and there is no simple equation to describe it. Here are a few experimental facts:

1. The heat current due to convection is directly proportional to the surface area. This is the reason for the large surface areas of radiators and cooling fins.
2. The viscosity of fluids slows natural convection near a stationary surface, giving a surface film that on a vertical surface typically has about the same insulating value as 1.3 cm of plywood (R value = 0.7). Forced convection decreases the thickness of this film, increasing the rate of heat transfer. This is the reason for the “wind-chill factor”; you get cold faster in a cold wind than in still air with the same temperature.
3. The heat current due to convection is found to be approximately proportional to the $\frac{5}{4}$ power of the temperature difference between the surface and the main body of fluid.

Radiation

Radiation is the transfer of heat by electromagnetic waves such as visible light, infrared, and ultraviolet radiation. Everyone has felt the warmth of the sun’s radiation and the intense heat from a charcoal grill or the glowing coals in a fireplace. Most of the heat from these very hot bodies reaches you not by conduction or convection in the intervening air but by *radiation*. This heat transfer would occur even if there were nothing but vacuum between you and the source of heat.

Every body, even at ordinary temperatures, emits energy in the form of electromagnetic radiation. Around 20°C, nearly all the energy is carried by infrared waves with wavelengths much longer than those of visible light (see Figs. 17.4 and 17.28). As the temperature rises, the wavelengths shift to shorter values. At 800°C, a body emits enough visible radiation to appear “red-hot,” although even at this temperature most of the energy is carried by infrared waves. At 3000°C,

the temperature of an incandescent lamp filament, the radiation contains enough visible light that the body appears “white-hot.”

The rate of energy radiation from a surface is proportional to the surface area A and to the fourth power of the absolute (Kelvin) temperature T . The rate also depends on the nature of the surface; this dependence is described by a quantity e called the **emissivity**. A dimensionless number between 0 and 1, e represents the ratio of the rate of radiation from a particular surface to the rate of radiation from an equal area of an ideal radiating surface at the same temperature. Emissivity also depends somewhat on temperature. Thus the heat current $H = dQ/dt$ due to radiation from a surface area A with emissivity e at absolute temperature T can be expressed as

$$H = Ae\sigma T^4 \quad (\text{heat current in radiation}) \quad (17.25)$$

where σ is a fundamental physical constant called the **Stefan–Boltzmann constant**. This relationship is called the **Stefan–Boltzmann law** in honor of its late-19th-century discoverers. The current best numerical value of σ is

$$\sigma = 5.670400(40) \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$$

We invite you to check unit consistency in Eq. (17.25). Emissivity (e) is often larger for dark surfaces than for light ones. The emissivity of a smooth copper surface is about 0.3, but e for a dull black surface can be close to unity.

Example 17.14 Heat transfer by radiation

A thin, square steel plate, 10 cm on a side, is heated in a blacksmith’s forge to 800°C. If the emissivity is 0.60, what is the total rate of radiation of energy from the plate?

SOLUTION

IDENTIFY and SET UP: The target variable is H , the rate of emission of energy from the plate’s two surfaces. We use Eq. (17.25) to calculate H .

EXECUTE: The total surface area is $2(0.10 \text{ m})^2 = 0.020 \text{ m}^2$, and $T = 800^\circ\text{C} = 1073 \text{ K}$. Then Eq. (17.25) gives

$$\begin{aligned} H &= Ae\sigma T^4 \\ &= (0.020 \text{ m}^2)(0.60)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1073 \text{ K})^4 \\ &= 900 \text{ W} \end{aligned}$$

EVALUATE: The nearby blacksmith will easily feel the heat radiated from this plate.

Radiation and Absorption

While a body at absolute temperature T is radiating, its surroundings at temperature T_s are also radiating, and the body *absorbs* some of this radiation. If it is in thermal equilibrium with its surroundings, $T = T_s$ and the rates of radiation and absorption must be equal. For this to be true, the rate of absorption must be given in general by $H = Ae\sigma T_s^4$. Then the *net* rate of radiation from a body at temperature T with surroundings at temperature T_s is

$$H_{\text{net}} = Ae\sigma T^4 - Ae\sigma T_s^4 = Ae\sigma(T^4 - T_s^4) \quad (17.26)$$

In this equation a positive value of H means a net heat flow *out of* the body. Equation (17.26) shows that for radiation, as for conduction and convection, the heat current depends on the temperature *difference* between two bodies.

Example 17.15 Radiation from the human body

What is the total rate of radiation of energy from a human body with surface area 1.20 m^2 and surface temperature $30^\circ\text{C} = 303 \text{ K}$? If the surroundings are at a temperature of 20°C , what is the *net* rate of radiative heat loss from the body? The emissivity of the human body is very close to unity, irrespective of skin pigmentation.

SOLUTION

IDENTIFY and SET UP: We must consider both the radiation that the body emits and the radiation that it absorbs from its surroundings. Equation (17.25) gives the rate of radiation of energy from the body, and Eq. (17.26) gives the net rate of heat loss.

Continued

EXECUTE: Taking $e = 1$ in Eq. (17.25), we find that the body radiates at a rate

$$H = Ae\sigma T^4 = (1.20 \text{ m}^2)(1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(303 \text{ K})^4 = 574 \text{ W}$$

This loss is partly offset by absorption of radiation, which depends on the temperature of the surroundings. From Eq. (17.26), the *net* rate of radiative energy transfer is

$$\begin{aligned} H_{\text{net}} &= Ae\sigma(T^4 - T_s^4) \\ &= (1.20 \text{ m}^2)(1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(303 \text{ K})^4 - (293 \text{ K})^4] = 72 \text{ W} \end{aligned}$$

EVALUATE: The value of H_{net} is positive because the body is losing heat to its colder surroundings.

Applications of Radiation

Heat transfer by radiation is important in some surprising places. A premature baby in an incubator can be cooled dangerously by radiation if the walls of the incubator happen to be cold, even when the *air* in the incubator is warm. Some incubators regulate the air temperature by measuring the baby's skin temperature.

A body that is a good absorber must also be a good emitter. An ideal radiator, with an emissivity of unity, is also an ideal absorber, absorbing *all* of the radiation that strikes it. Such an ideal surface is called an ideal black body or simply a **blackbody**. Conversely, an ideal *reflector*, which absorbs *no* radiation at all, is also a very ineffective radiator.

This is the reason for the silver coatings on vacuum (“Thermos”) bottles, invented by Sir James Dewar (1842–1923). A vacuum bottle has double glass walls. The air is pumped out of the spaces between the walls; this eliminates nearly all heat transfer by conduction and convection. The silver coating on the walls reflects most of the radiation from the contents back into the container, and the wall itself is a very poor emitter. Thus a vacuum bottle can keep coffee or soup hot for several hours. The Dewar flask, used to store very cold liquefied gases, is exactly the same in principle.

Radiation, Climate, and Climate Change

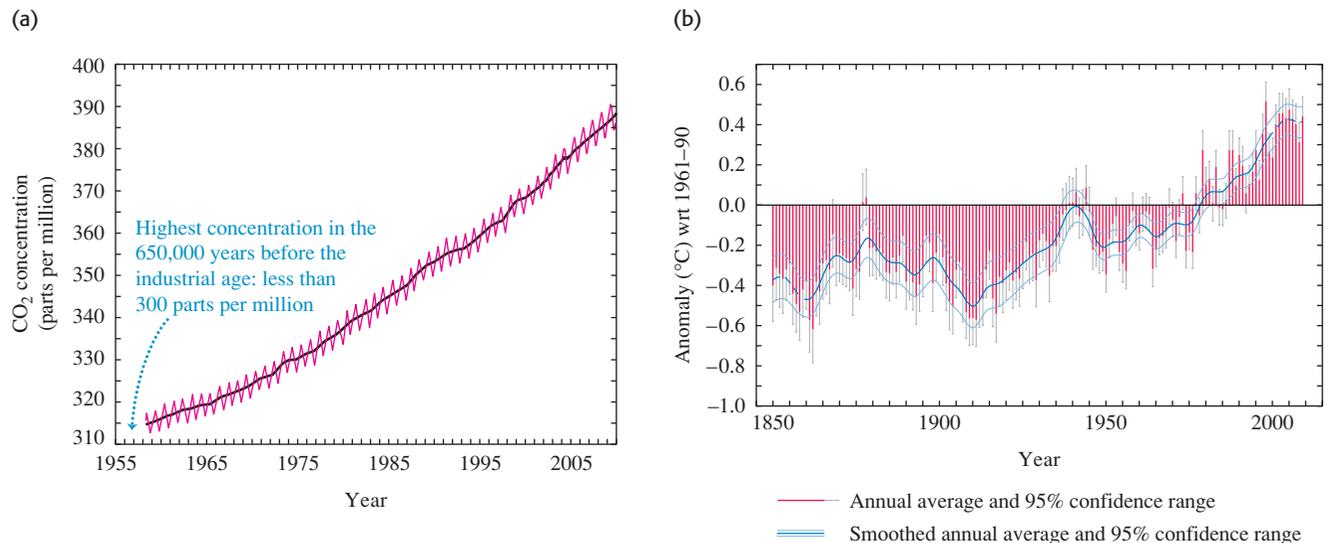
Our planet constantly absorbs radiation coming from the sun. In thermal equilibrium, the rate at which our planet absorbs solar radiation must equal the rate at which it emits radiation into space. The presence of an atmosphere on our planet has a significant effect on this equilibrium.

Most of the radiation emitted by the sun (which has a surface temperature of 5800 K) is in the visible part of the spectrum, to which our atmosphere is transparent. But the average surface temperature of the earth is only 287 K (14°C). Hence most of the radiation that our planet emits into space is infrared radiation, just like the radiation from the person shown in Fig. 17.28. However, our atmosphere is *not* completely transparent to infrared radiation. This is because our atmosphere contains carbon dioxide (CO₂), which is its fourth most abundant constituent (after nitrogen, oxygen, and argon). Molecules of CO₂ in the atmosphere have the property that they *absorb* some of the infrared radiation coming upward from the surface. They then re-radiate the absorbed energy, but some of the re-radiated energy is directed back down toward the surface instead of escaping into space. In order to maintain thermal equilibrium, the earth's surface must compensate for this by increasing its temperature T and hence its total rate of radiating energy (which is proportional to T^4). This phenomenon, called the **greenhouse effect**, makes our planet's surface temperature about 33°C higher than it would be if there were no atmospheric CO₂. If CO₂ were absent, the earth's average surface temperature would be below the freezing point of water, and life as we know it would be impossible.

While atmospheric CO₂ has a beneficial effect, too much of it can have extremely negative consequences. Measurements of air trapped in ancient Antarctic ice show that over the past 650,000 years CO₂ has constituted less than 300 parts per million of our atmosphere. Since the beginning of the industrial age,

however, the burning of fossil fuels such as coal and petroleum has elevated the atmospheric CO_2 concentration to unprecedented levels (Fig. 17.29a). As a consequence, since the 1950s the global average surface temperature has increased by 0.6°C and the earth has experienced the hottest years ever recorded (Fig. 17.29b). If we continue to consume fossil fuels at the same rate, by 2050 the atmospheric CO_2 concentration will reach 600 parts per million, well off the scale of Fig. 17.29a. The resulting temperature increase will have dramatic effects on climate around the world. In the polar regions massive quantities of ice will melt and run from solid land to the sea, thus raising ocean levels worldwide and threatening the homes and lives of hundreds of millions of people who live near the coast. Coping with these threats is one of the greatest challenges facing 21st-century civilization.

17.29 (a) The concentration of atmospheric CO_2 has increased by 22% since continuous measurements began in 1958. (The yearly variations are due to increased intake of CO_2 by plants in spring and summer.) (b) The increase in global average temperature since the beginning of the industrial era is a result of the increase in CO_2 concentration.



Test Your Understanding of Section 17.7 A room has one wall made of concrete, one wall made of copper, and one wall made of steel. All of the walls are the same size and at the same temperature of 20°C . Which wall feels coldest to the touch? (i) the concrete wall; (ii) the copper wall; (iii) the steel wall; (iv) all three walls feel equally cold.

Temperature and temperature scales: Two bodies in thermal equilibrium must have the same temperature. A conducting material between two bodies permits them to interact and come to thermal equilibrium; an insulating material impedes this interaction.

The Celsius and Fahrenheit temperature scales are based on the freezing ($0^\circ\text{C} = 32^\circ\text{F}$) and boiling ($100^\circ\text{C} = 212^\circ\text{F}$) temperatures of water. One Celsius degree equals $\frac{9}{5}$ Fahrenheit degrees. (See Example 17.1.)

The Kelvin scale has its zero at the extrapolated zero-pressure temperature for a gas thermometer, $-273.15^\circ\text{C} = 0\text{ K}$. In the gas-thermometer scale, the ratio of two temperatures T_1 and T_2 is defined to be equal to the ratio of the two corresponding gas-thermometer pressures p_1 and p_2 .

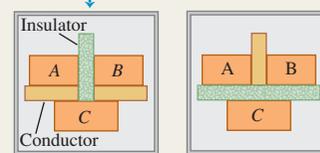
$$T_F = \frac{9}{5}T_C + 32^\circ \quad (17.1)$$

$$T_C = \frac{5}{9}(T_F - 32^\circ) \quad (17.2)$$

$$T_K = T_C + 273.15 \quad (17.3)$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \quad (17.4)$$

If systems A and B are each in thermal equilibrium with system C ...



... then systems A and B are in thermal equilibrium with each other.

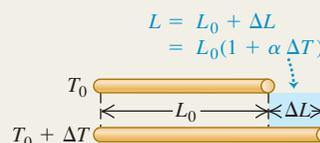
Thermal expansion and thermal stress: A temperature change ΔT causes a change in any linear dimension L_0 of a solid body. The change ΔL is approximately proportional to L_0 and ΔT . Similarly, a temperature change causes a change ΔV in the volume V_0 of any solid or liquid; ΔV is approximately proportional to V_0 and ΔT . The quantities α and β are the coefficients of linear expansion and volume expansion, respectively. For solids, $\beta = 3\alpha$. (See Examples 17.2 and 17.3.)

When a material is cooled or heated and held so it cannot contract or expand, it is under a tensile stress F/A . (See Example 17.4.)

$$\Delta L = \alpha L_0 \Delta T \quad (17.6)$$

$$\Delta V = \beta V_0 \Delta T \quad (17.8)$$

$$\frac{F}{A} = -Y\alpha \Delta T \quad (17.12)$$



Heat, phase changes, and calorimetry: Heat is energy in transit from one body to another as a result of a temperature difference. Equations (17.13) and (17.18) give the quantity of heat Q required to cause a temperature change ΔT in a quantity of material with mass m and specific heat c (alternatively, with number of moles n and molar heat capacity $C = Mc$, where M is the molar mass and $m = nM$). When heat is added to a body, Q is positive; when it is removed, Q is negative. (See Examples 17.5 and 17.6.)

To change a mass m of a material to a different phase at the same temperature (such as liquid to vapor), a quantity of heat given by Eq. (17.20) must be added or subtracted. Here L is the heat of fusion, vaporization, or sublimation.

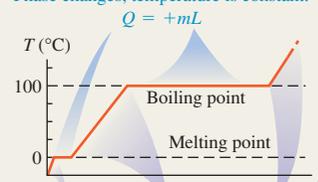
In an isolated system whose parts interact by heat exchange, the algebraic sum of the Q 's for all parts of the system must be zero. (See Examples 17.7–17.10.)

$$Q = mc \Delta T \quad (17.13)$$

$$Q = nC \Delta T \quad (17.18)$$

$$Q = \pm mL \quad (17.20)$$

Phase changes, temperature is constant:



Temperature rises, phase does not change: $Q = mc \Delta T$

Conduction, convection, and radiation: Conduction is the transfer of heat within materials without bulk motion of the materials. The heat current H depends on the area A through which the heat flows, the length L of the heat-flow path, the temperature difference ($T_H - T_C$), and the thermal conductivity k of the material. (See Examples 17.11–17.13.)

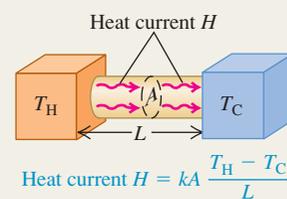
Convection is a complex heat-transfer process that involves mass motion from one region to another.

Radiation is energy transfer through electromagnetic radiation. The radiation heat current H depends on the surface area A , the emissivity e of the surface (a pure number between 0 and 1), and the Kelvin temperature T . Here σ is the Stefan–Boltzmann constant. The net radiation heat current H_{net} from a body at temperature T to its surroundings at temperature T_s depends on both T and T_s . (See Examples 17.14 and 17.15.)

$$H = \frac{dQ}{dt} = kA \frac{T_H - T_C}{L} \quad (17.21)$$

$$H = Ae\sigma T^4 \quad (17.25)$$

$$H_{\text{net}} = Ae\sigma(T^4 - T_s^4) \quad (17.26)$$



BRIDGING PROBLEM

Steady-State Heat Flow: Radiation and Conduction

One end of a solid cylindrical copper rod 0.200 m long and 0.0250 m in radius is inserted into a large block of solid hydrogen at its melting temperature, 13.84 K. The other end is blackened and exposed to thermal radiation from surrounding walls at 500.0 K. The sides of the rod are insulated, so no energy is lost or gained except at the ends of the rod. (a) When equilibrium is reached, what is the temperature of the blackened end? The thermal conductivity of copper at temperatures near 20 K is $1670 \text{ Wm} \cdot \text{K}$. (b) At what rate (in kg/h) does the solid hydrogen melt?

SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.



IDENTIFY and SET UP

1. Draw a sketch of the situation, showing all relevant dimensions.
2. List the known and unknown quantities, and identify the target variables.
3. In order for the rod to be in equilibrium, how must the radiation heat current from the walls into the blackened end of the rod compare to the conduction heat current from this end to the

other end and into the solid hydrogen? Use your answers to select the appropriate equations for part (a).

4. How does the heat current from the rod into the hydrogen determine the rate at which the hydrogen melts? (*Hint:* See Table 17.4.) Use your answer to select the appropriate equations for part (b).

EXECUTE

5. Solve for the temperature of the blackened end of the rod. (*Hint:* Since copper is an excellent conductor of heat at low temperature, you can assume that the temperature of the blackened end is only slightly higher than 13.84 K.)
6. Use your result from step 5 to find the rate at which the hydrogen melts.

EVALUATE

7. Is your result from step 5 consistent with the hint given in that step?
8. How would your results from steps 5 and 6 be affected if the rod had twice the radius?

Problems

For instructor-assigned homework, go to www.masteringphysics.com



•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q17.1 Explain why it would not make sense to use a full-size glass thermometer to measure the temperature of a thimbleful of hot water.

Q17.2 If you heat the air inside a rigid, sealed container until its Kelvin temperature doubles, the air pressure in the container will also double. Is the same thing true if you double the Celsius temperature of the air in the container? Explain.

Q17.3 Many automobile engines have cast-iron cylinders and aluminum pistons. What kinds of problems could occur if the engine gets too hot? (The coefficient of volume expansion of cast iron is approximately the same as that of steel.)

Q17.4 Why do frozen water pipes burst? Would a mercury thermometer break if the temperature went below the freezing temperature of mercury? Why or why not?

Q17.5 Two bodies made of the same material have the same external dimensions and appearance, but one is solid and the other is hollow. When their temperature is increased, is the overall volume expansion the same or different? Why?

Q17.6 The inside of an oven is at a temperature of 200°C (392°F). You can put your hand in the oven without injury as long as you don't touch anything. But since the air inside the oven is also at 200°C , why isn't your hand burned just the same?

Q17.7 A newspaper article about the weather states that "the temperature of a body measures how much heat the body contains." Is this description correct? Why or why not?

Q17.8 To raise the temperature of an object, must you add heat to it? If you add heat to an object, must you raise its temperature? Explain.

Q17.9 A student asserts that a suitable unit for specific heat is $1 \text{ m}^2/\text{s}^2 \cdot \text{C}^\circ$. Is she correct? Why or why not?

Q17.10 In some household air conditioners used in dry climates, air is cooled by blowing it through a water-soaked filter, evaporating some of the water. How does this cool the air? Would such a system work well in a high-humidity climate? Why or why not?

Q17.11 The units of specific heat c are $\text{J}/\text{kg} \cdot \text{K}$, but the units of heat of fusion L_f or heat of vaporization L_v are simply J/kg . Why do the units of L_f and L_v not include a factor of $(\text{K})^{-1}$ to account for a temperature change?

Q17.12 Why is a hot, humid day in the tropics generally more uncomfortable for human beings than a hot, dry day in the desert?

Q17.13 A piece of aluminum foil used to wrap a potato for baking in a hot oven can usually be handled safely within a few seconds after the potato is removed from the oven. The same is not true of the potato, however! Give two reasons for this difference.

Q17.14 Desert travelers sometimes keep water in a canvas bag. Some water seeps through the bag and evaporates. How does this cool the water inside the bag?

Q17.15 When you first step out of the shower, you feel cold. But as soon as you are dry you feel warmer, even though the room temperature does not change. Why?

Q17.16 The climate of regions adjacent to large bodies of water (like the Pacific and Atlantic coasts) usually features a narrower range of temperature than the climate of regions far from large bodies of water (like the prairies). Why?

Q17.17 When water is placed in ice-cube trays in a freezer, why doesn't the water freeze all at once when the temperature has reached 0°C ? In fact, the water freezes first in a layer adjacent to the sides of the tray. Why?

Q17.18 Before giving you an injection, a physician swabs your arm with isopropyl alcohol at room temperature. Why does this make your arm feel cold? (*Hint:* The reason is *not* the fear of the injection! The boiling point of isopropyl alcohol is 82.4°C .)

Q17.19 A cold block of metal feels colder than a block of wood at the same temperature. Why? A *hot* block of metal feels hotter than a block of wood at the same temperature. Again, why? Is there any temperature at which the two blocks feel equally hot or cold? What temperature is this?

Q17.20 A person pours a cup of hot coffee, intending to drink it five minutes later. To keep the coffee as hot as possible, should she put cream in it now or wait until just before she drinks it? Explain.

Q17.21 When a freshly baked apple pie has just been removed from the oven, the crust and filling are both at the same temperature. Yet if you sample the pie, the filling will burn your tongue but the crust will not. Why is there a difference? (*Hint:* The filling is moist while the crust is dry.)

Q17.22 Old-time kitchen lore suggests that things cook better (evenly and without burning) in heavy cast-iron pots. What desirable characteristics do such pots have?

Q17.23 In coastal regions in the winter, the temperature over the land is generally colder than the temperature over the nearby ocean; in the summer, the reverse is usually true. Explain. (*Hint:* The specific heat of soil is only 0.2–0.8 times as great as that of water.)

Q17.24 It is well known that a potato bakes faster if a large nail is stuck through it. Why? Does an aluminum nail work better than a steel one? Why or why not? (*Note:* Don't try this in a microwave oven!) There is also a gadget on the market to hasten the roasting of meat; it consists of a hollow metal tube containing a wick and some water. This is claimed to work much better than a solid metal rod. How does it work?

Q17.25 Glider pilots in the Midwest know that thermal updrafts are likely to occur in the vicinity of freshly plowed fields. Why?

Q17.26 Some folks claim that ice cubes freeze faster if the trays are filled with hot water, because hot water cools off faster than cold water. What do you think?

Q17.27 We're lucky that the earth isn't in thermal equilibrium with the sun (which has a surface temperature of 5800 K). But why aren't the two bodies in thermal equilibrium?

Q17.28 When energy shortages occur, magazine articles sometimes urge us to keep our homes at a constant temperature day and night to conserve fuel. They argue that when we turn down the heat at night, the walls, ceilings, and other areas cool off and must be reheated in the morning. So if we keep the temperature constant, these parts of the house will not cool off and will not have to be reheated. Does this argument make sense? Would we really save energy by following this advice?

EXERCISES

Section 17.2 Thermometers and Temperature Scales

17.1 • Convert the following Celsius temperatures to Fahrenheit: (a) -62.8°C , the lowest temperature ever recorded in North America (February 3, 1947, Snag, Yukon); (b) 56.7°C , the highest temperature ever recorded in the United States (July 10, 1913, Death Valley, California); (c) 31.1°C , the world's highest average annual temperature (Lugh Ferrandi, Somalia).

17.2 • **BIO Temperatures in Biomedicine.** (a) **Normal body temperature.** The average normal body temperature measured in the mouth is 310 K. What would Celsius and Fahrenheit thermometers read for this temperature? (b) **Elevated body temperature.** During very vigorous exercise, the body's temperature can go as high as 40°C . What would Kelvin and Fahrenheit thermometers read for this temperature? (c) **Temperature difference in the body.** The surface temperature of the body is normally about 7°C lower than the internal temperature. Express this temperature difference in kelvins and in Fahrenheit degrees. (d) **Blood storage.** Blood stored at 4.0°C lasts safely for about 3 weeks, whereas blood stored at -160°C lasts for 5 years. Express both temperatures on the Fahrenheit and Kelvin scales. (e) **Heat stroke.** If the body's temperature is above 105°F for a prolonged period, heat stroke can result. Express this temperature on the Celsius and Kelvin scales.

17.3 • (a) On January 22, 1943, the temperature in Spearfish, South Dakota, rose from -4.0°F to 45.0°F in just 2 minutes. What was the temperature change in Celsius degrees? (b) The temperature in Browning, Montana, was 44.0°F on January 23, 1916. The next day the temperature plummeted to -56°F . What was the temperature change in Celsius degrees?

Section 17.3 Gas Thermometers and the Kelvin Scale

17.4 • (a) Calculate the one temperature at which Fahrenheit and Celsius thermometers agree with each other. (b) Calculate the one temperature at which Fahrenheit and Kelvin thermometers agree with each other.

17.5 •• You put a bottle of soft drink in a refrigerator and leave it until its temperature has dropped 10.0 K. What is its temperature change in (a) $^{\circ}\text{F}$ and (b) $^{\circ}\text{C}$?

17.6 • Convert the following Kelvin temperatures to the Celsius and Fahrenheit scales: (a) the midday temperature at the surface of the moon (400 K); (b) the temperature at the tops of the clouds in the atmosphere of Saturn (95 K); (c) the temperature at the center of the sun (1.55×10^7 K).

17.7 • The pressure of a gas at the triple point of water is 1.35 atm. If its volume remains unchanged, what will its pressure be at the temperature at which CO_2 solidifies?

17.8 •• A gas thermometer registers an absolute pressure corresponding to 325 mm of mercury when in contact with water at the triple point. What pressure does it read when in contact with water at the normal boiling point?

17.9 •• **A Constant-Volume Gas Thermometer.** An experimenter using a gas thermometer found the pressure at the triple point of water (0.01°C) to be 4.80×10^4 Pa and the pressure at the normal boiling point (100°C) to be 6.50×10^4 Pa. (a) Assuming that the pressure varies linearly with temperature, use these two data points to find the Celsius temperature at which the gas pressure would be zero (that is, find the Celsius temperature of absolute zero). (b) Does the gas in this thermometer obey Eq. (17.4) precisely? If that equation were precisely obeyed and the pressure at 100°C were 6.50×10^4 Pa, what pressure would the experimenter have measured at 0.01°C ? (As we will learn in Section 18.1, Eq. (17.4) is accurate only for gases at very low density.)

17.10 • Like the Kelvin scale, the *Rankine scale* is an absolute temperature scale: Absolute zero is zero degrees Rankine (0°R). However, the units of this scale are the same size as those of the Fahrenheit scale rather than the Celsius scale. What is the numerical value of the triple-point temperature of water on the Rankine scale?

Section 17.4 Thermal Expansion

17.11 • The Humber Bridge in England has the world's longest single span, 1410 m. Calculate the change in length of the steel deck of the span when the temperature increases from -5.0°C to 18.0°C .

17.12 • One of the tallest buildings in the world is the Taipei 101 in Taiwan, at a height of 1671 feet. Assume that this height was measured on a cool spring day when the temperature was 15.5°C . You could use the building as a sort of giant thermometer on a hot summer day by carefully measuring its height. Suppose you do this and discover that the Taipei 101 is 0.471 foot taller than its official height. What is the temperature, assuming that the building is in thermal equilibrium with the air and that its entire frame is made of steel?

17.13 • A U.S. penny has a diameter of 1.9000 cm at 20.0°C . The coin is made of a metal alloy (mostly zinc) for which the coefficient of linear expansion is $2.6 \times 10^{-5} \text{ K}^{-1}$. What would its diameter be on a hot day in Death Valley (48.0°C)? On a cold night in the mountains of Greenland (-53°C)?

17.14 • **Ensuring a Tight Fit.** Aluminum rivets used in airplane construction are made slightly larger than the rivet holes and cooled by “dry ice” (solid CO_2) before being driven. If the diameter of a hole is 4.500 mm, what should be the diameter of a rivet at 23.0°C if its diameter is to equal that of the hole when the rivet is cooled to -78.0°C , the temperature of dry ice? Assume that the expansion coefficient remains constant at the value given in Table 17.1.

17.15 •• The outer diameter of a glass jar and the inner diameter of its iron lid are both 725 mm at room temperature (20.0°C). What will be the size of the difference in these diameters if the lid is briefly held under hot water until its temperature rises to 50.0°C , without changing the temperature of the glass?

17.16 •• A geodesic dome constructed with an aluminum framework is a nearly perfect hemisphere; its diameter measures 55.0 m on a winter day at a temperature of -15°C . How much more interior space does the dome have in the summer, when the temperature is 35°C ?

17.17 •• A copper cylinder is initially at 20.0°C . At what temperature will its volume be 0.150% larger than it is at 20.0°C ?

17.18 •• A steel tank is completely filled with 2.80 m^3 of ethanol when both the tank and the ethanol are at a temperature of 32.0°C . When the tank and its contents have cooled to 18.0°C , what additional volume of ethanol can be put into the tank?

17.19 •• A glass flask whose volume is 1000.00 cm^3 at 0.0°C is completely filled with mercury at this temperature. When flask and mercury are warmed to 55.0°C , 8.95 cm^3 of mercury overflow. If the coefficient of volume expansion of mercury is $18.0 \times 10^{-5} \text{ K}^{-1}$, compute the coefficient of volume expansion of the glass.

17.20 •• (a) If an area measured on the surface of a solid body is A_0 at some initial temperature and then changes by ΔA when the temperature changes by ΔT , show that

$$\Delta A = (2\alpha)A_0\Delta T$$

where α is the coefficient of linear expansion. (b) A circular sheet of aluminum is 55.0 cm in diameter at 15.0°C . By how much does the area of one side of the sheet change when the temperature increases to 27.5°C ?

17.21 •• A machinist bores a hole of diameter 1.35 cm in a steel plate at a temperature of 25.0°C . What is the cross-sectional area of the hole (a) at 25.0°C and (b) when the temperature of the plate is increased to 175°C ? Assume that the coefficient of linear expansion remains constant over this temperature range. (*Hint:* See Exercise 17.20.)

17.22 •• As a new mechanical engineer for Engines Inc., you have been assigned to design brass pistons to slide inside steel cylinders. The engines in which these pistons will be used will operate between 20.0°C and 150.0°C . Assume that the coefficients of expansion are constant over this temperature range. (a) If the piston just fits inside the chamber at 20.0°C , will the engines be able to run at higher temperatures? Explain. (b) If the cylindrical pistons are 25.000 cm in diameter at 20.0°C , what should be the minimum diameter of the cylinders at that temperature so the pistons will operate at 150.0°C ?

17.23 • (a) A wire that is 1.50 m long at 20.0°C is found to increase in length by 1.90 cm when warmed to 420.0°C . Compute its average coefficient of linear expansion for this temperature range. (b) The wire is stretched just taut (zero tension) at 420.0°C . Find the stress in the wire if it is cooled to 20.0°C without being allowed to contract. Young's modulus for the wire is $2.0 \times 10^{11} \text{ Pa}$.

17.24 •• A brass rod is 185 cm long and 1.60 cm in diameter. What force must be applied to each end of the rod to prevent it from contracting when it is cooled from 120.0°C to 10.0°C ?

17.25 •• Steel train rails are laid in 12.0-m-long segments placed end to end. The rails are laid on a winter day when their temperature is -2.0°C . (a) How much space must be left between adjacent rails if they are just to touch on a summer day when their temperature is 33.0°C ? (b) If the rails are originally laid in contact, what is the stress in them on a summer day when their temperature is 33.0°C ?

Section 17.5 Quantity of Heat

17.26 • In an effort to stay awake for an all-night study session, a student makes a cup of coffee by first placing a 200-W electric immersion heater in 0.320 kg of water. (a) How much heat must be added to the water to raise its temperature from 20.0°C to 80.0°C ? (b) How much time is required? Assume that all of the heater's power goes into heating the water.

17.27 •• An aluminum tea kettle with mass 1.50 kg and containing 1.80 kg of water is placed on a stove. If no heat is lost to the surroundings, how much heat must be added to raise the temperature from 20.0°C to 85.0°C ?

17.28 • **BIO Heat Loss During Breathing.** In very cold weather a significant mechanism for heat loss by the human body is energy expended in warming the air taken into the lungs with each breath. (a) On a cold winter day when the temperature is -20°C , what amount of heat is needed to warm to body temperature (37°C) the 0.50 L of air exchanged with each breath? Assume that the specific heat of air is $1020 \text{ J/kg}\cdot\text{K}$ and that 1.0 L of air has mass $1.3 \times 10^{-3} \text{ kg}$. (b) How much heat is lost per hour if the respiration rate is 20 breaths per minute?

17.29 • You are given a sample of metal and asked to determine its specific heat. You weigh the sample and find that its weight is 28.4 N. You carefully add $1.25 \times 10^4 \text{ J}$ of heat energy to the sample and find that its temperature rises 18.0°C . What is the sample's specific heat?

17.30 •• **On-Demand Water Heaters.** Conventional hot-water heaters consist of a tank of water maintained at a fixed temperature. The hot water is to be used when needed. The drawbacks are that energy is wasted because the tank loses heat when it is not in use and that you can run out of hot water if you use too much. Some utility companies are encouraging the use of *on-demand* water heaters (also known as *flash heaters*), which consist of heating units to heat the water as you use it. No water tank is involved, so no heat is wasted. A typical household shower flow rate is 2.5 gal/min

(9.46 L/min) with the tap water being heated from 50°F (10°C) to 120°F (49°C) by the on-demand heater. What rate of heat input (either electrical or from gas) is required to operate such a unit, assuming that all the heat goes into the water?

17.31 • BIO While running, a 70-kg student generates thermal energy at a rate of 1200 W. For the runner to maintain a constant body temperature of 37°C, this energy must be removed by perspiration or other mechanisms. If these mechanisms failed and the heat could not flow out of the student's body, for what amount of time could a student run before irreversible body damage occurred? (Note: Protein structures in the body are irreversibly damaged if body temperature rises to 44°C or higher. The specific heat of a typical human body is 3480 J/kg · K, slightly less than that of water. The difference is due to the presence of protein, fat, and minerals, which have lower specific heats.)

17.32 • CP While painting the top of an antenna 225 m in height, a worker accidentally lets a 1.00-L water bottle fall from his lunchbox. The bottle lands in some bushes at ground level and does not break. If a quantity of heat equal to the magnitude of the change in mechanical energy of the water goes into the water, what is its increase in temperature?

17.33 •• CP A crate of fruit with mass 35.0 kg and specific heat 3650 J/kg · K slides down a ramp inclined at 36.9° below the horizontal. The ramp is 8.00 m long. (a) If the crate was at rest at the top of the incline and has a speed of 2.50 m/s at the bottom, how much work was done on the crate by friction? (b) If an amount of heat equal to the magnitude of the work done by friction goes into the crate of fruit and the fruit reaches a uniform final temperature, what is its temperature change?

17.34 • CP A 25,000-kg subway train initially traveling at 15.5 m/s slows to a stop in a station and then stays there long enough for its brakes to cool. The station's dimensions are 65.0 m long by 20.0 m wide by 12.0 m high. Assuming all the work done by the brakes in stopping the train is transferred as heat uniformly to all the air in the station, by how much does the air temperature in the station rise? Take the density of the air to be 1.20 kg/m³ and its specific heat to be 1020 J/kg · K.

17.35 • CP A nail driven into a board increases in temperature. If we assume that 60% of the kinetic energy delivered by a 1.80-kg hammer with a speed of 7.80 m/s is transformed into heat that flows into the nail and does not flow out, what is the temperature increase of an 8.00-g aluminum nail after it is struck ten times?

17.36 • A technician measures the specific heat of an unidentified liquid by immersing an electrical resistor in it. Electrical energy is converted to heat transferred to the liquid for 120 s at a constant rate of 65.0 W. The mass of the liquid is 0.780 kg, and its temperature increases from 18.55°C to 22.54°C. (a) Find the average specific heat of the liquid in this temperature range. Assume that negligible heat is transferred to the container that holds the liquid and that no heat is lost to the surroundings. (b) Suppose that in this experiment heat transfer from the liquid to the container or surroundings cannot be ignored. Is the result calculated in part (a) an *overestimate* or an *underestimate* of the average specific heat? Explain.

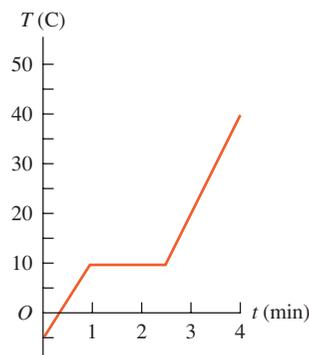
17.37 •• CP A 15.0-g bullet traveling horizontally at 865 m/s passes through a tank containing 13.5 kg of water and emerges with a speed of 534 m/s. What is the maximum temperature increase that the water could have as a result of this event?

Section 17.6 Calorimetry and Phase Changes

17.38 •• As a physicist, you put heat into a 500.0-g solid sample at the rate of 10.0 kJ/min, while recording its temperature as a

function of time. You plot your data and obtain the graph shown in Fig. E17.38. (a) What is the latent heat of fusion for this solid? (b) What are the specific heats of the liquid and solid states of the material?

Figure E17.38



17.39 •• A 500.0-g chunk of an unknown metal, which has been in boiling water for several minutes, is quickly dropped into an insulating Styrofoam beaker containing 1.00 kg of water at room temperature (20.0°C). After waiting and gently stirring for 5.00 minutes, you observe that the water's temperature has reached a constant value of 22.0°C. (a) Assuming that the Styrofoam absorbs a negligibly small amount of heat and that no heat was lost to the surroundings, what is the specific heat of the metal? (b) Which is more useful for storing thermal energy: this metal or an equal weight of water? Explain. (c) What if the heat absorbed by the Styrofoam actually is not negligible. How would the specific heat you calculated in part (a) be in error? Would it be too large, too small, or still correct? Explain.

17.40 • BIO Treatment for a Stroke. One suggested treatment for a person who has suffered a stroke is immersion in an ice-water bath at 0°C to lower the body temperature, which prevents damage to the brain. In one set of tests, patients were cooled until their internal temperature reached 32.0°C. To treat a 70.0-kg patient, what is the minimum amount of ice (at 0°C) you need in the bath so that its temperature remains at 0°C? The specific heat of the human body is 3480 J/kg · °C, and recall that normal body temperature is 37.0°C.

17.41 •• A copper pot with a mass of 0.500 kg contains 0.170 kg of water, and both are at a temperature of 20.0°C. A 0.250-kg block of iron at 85.0°C is dropped into the pot. Find the final temperature of the system, assuming no heat loss to the surroundings.

17.42 •• BIO Bicycling on a Warm Day. If the air temperature is the same as the temperature of your skin (about 30°C), your body cannot get rid of heat by transferring it to the air. In that case, it gets rid of the heat by evaporating water (sweat). During bicycling, a typical 70-kg person's body produces energy at a rate of about 500 W due to metabolism, 80% of which is converted to heat. (a) How many kilograms of water must the person's body evaporate in an hour to get rid of this heat? The heat of vaporization of water at body temperature is 2.42×10^6 J/kg. (b) The evaporated water must, of course, be replenished, or the person will dehydrate. How many 750-mL bottles of water must the bicyclist drink per hour to replenish the lost water? (Recall that the mass of a liter of water is 1.0 kg.)

17.43 •• BIO Overheating. (a) By how much would the body temperature of the bicyclist in the preceding problem increase in an hour if he were unable to get rid of the excess heat? (b) Is this

temperature increase large enough to be serious? To find out, how high a fever would it be equivalent to, in °F? (Recall that the normal internal body temperature is 98.6°F and the specific heat of the body is 3480 J/kg · °C.)

17.44 • In a container of negligible mass, 0.200 kg of ice at an initial temperature of -40.0°C is mixed with a mass m of water that has an initial temperature of 80.0°C . No heat is lost to the surroundings. If the final temperature of the system is 20.0°C , what is the mass m of the water that was initially at 80.0°C ?

17.45 • A 6.00-kg piece of solid copper metal at an initial temperature T is placed with 2.00 kg of ice that is initially at -20.0°C . The ice is in an insulated container of negligible mass and no heat is exchanged with the surroundings. After thermal equilibrium is reached, there is 1.20 kg of ice and 0.80 kg of liquid water. What was the initial temperature of the piece of copper?

17.46 • **BIO** Before going in for his annual physical, a 70.0-kg man whose body temperature is 37.0°C consumes an entire 0.355-L can of a soft drink (mostly water) at 12.0°C . (a) What will his body temperature be after equilibrium is attained? Ignore any heating by the man's metabolism. The specific heat of the man's body is 3480 J/kg · K. (b) Is the change in his body temperature great enough to be measured by a medical thermometer?

17.47 • **BIO** In the situation described in Exercise 17.46, the man's metabolism will eventually return the temperature of his body (and of the soft drink that he consumed) to 37.0°C . If his body releases energy at a rate of 7.00×10^3 kJ/day (the *basal metabolic rate*, or BMR), how long does this take? Assume that all of the released energy goes into raising the temperature.

17.48 • An ice-cube tray of negligible mass contains 0.350 kg of water at 18.0°C . How much heat must be removed to cool the water to 0.00°C and freeze it? Express your answer in joules, calories, and Btu.

17.49 • How much heat is required to convert 12.0 g of ice at -10.0°C to steam at 100.0°C ? Express your answer in joules, calories, and Btu.

17.50 • An open container holds 0.550 kg of ice at -15.0°C . The mass of the container can be ignored. Heat is supplied to the container at the constant rate of 800.0 J/min for 500.0 min. (a) After how many minutes does the ice *start* to melt? (b) After how many minutes, from the time when the heating is first started, does the temperature begin to rise above 0.0°C ? (c) Plot a curve showing the temperature as a function of the elapsed time.

17.51 • **CP** What must the initial speed of a lead bullet be at a temperature of 25.0°C so that the heat developed when it is brought to rest will be just sufficient to melt it? Assume that all the initial mechanical energy of the bullet is converted to heat and that no heat flows from the bullet to its surroundings. (Typical rifles have muzzle speeds that exceed the speed of sound in air, which is 347 m/s at 25.0°C .)

17.52 • **BIO Steam Burns Versus Water Burns.** What is the amount of heat input to your skin when it receives the heat released (a) by 25.0 g of steam initially at 100.0°C , when it is cooled to skin temperature (34.0°C)? (b) By 25.0 g of water initially at 100.0°C , when it is cooled to 34.0°C ? (c) What does this tell you about the relative severity of steam and hot water burns?

17.53 • **BIO “The Ship of the Desert.”** Camels require very little water because they are able to tolerate relatively large changes in their body temperature. While humans keep their body temperatures constant to within one or two Celsius degrees, a dehydrated camel permits its body temperature to drop to 34.0°C overnight and rise to 40.0°C during the day. To see how effective this mechanism is for saving water, calculate how many liters of water a 400-kg

camel would have to drink if it attempted to keep its body temperature at a constant 34.0°C by evaporation of sweat during the day (12 hours) instead of letting it rise to 40.0°C . (*Note:* The specific heat of a camel or other mammal is about the same as that of a typical human, 3480 J/kg · K. The heat of vaporization of water at 34°C is 2.42×10^6 J/kg.)

17.54 • **BIO** Evaporation of sweat is an important mechanism for temperature control in some warm-blooded animals. (a) What mass of water must evaporate from the skin of a 70.0-kg man to cool his body 1.00°C ? The heat of vaporization of water at body temperature (37°C) is 2.42×10^6 J/kg. The specific heat of a typical human body is 3480 J/kg · K (see Exercise 17.31). (b) What volume of water must the man drink to replenish the evaporated water? Compare to the volume of a soft-drink can (355 cm^3).

17.55 • **CP** An asteroid with a diameter of 10 km and a mass of 2.60×10^{15} kg impacts the earth at a speed of 32.0 km/s, landing in the Pacific Ocean. If 1.00% of the asteroid's kinetic energy goes to boiling the ocean water (assume an initial water temperature of 10.0°C), what mass of water will be boiled away by the collision? (For comparison, the mass of water contained in Lake Superior is about 2×10^{15} kg.)

17.56 • A laboratory technician drops a 0.0850-kg sample of unknown solid material, at a temperature of 100.0°C , into a calorimeter. The calorimeter can, initially at 19.0°C , is made of 0.150 kg of copper and contains 0.200 kg of water. The final temperature of the calorimeter can and contents is 26.1°C . Compute the specific heat of the sample.

17.57 • An insulated beaker with negligible mass contains 0.250 kg of water at a temperature of 75.0°C . How many kilograms of ice at a temperature of -20.0°C must be dropped into the water to make the final temperature of the system 40.0°C ?

17.58 • A glass vial containing a 16.0-g sample of an enzyme is cooled in an ice bath. The bath contains water and 0.120 kg of ice. The sample has specific heat 2250 J/kg · K; the glass vial has mass 6.00 g and specific heat 2800 J/kg · K. How much ice melts in cooling the enzyme sample from room temperature (19.5°C) to the temperature of the ice bath?

17.59 • A 4.00-kg silver ingot is taken from a furnace, where its temperature is 750.0°C , and placed on a large block of ice at 0.0°C . Assuming that all the heat given up by the silver is used to melt the ice, how much ice is melted?

17.60 • A copper calorimeter can with mass 0.100 kg contains 0.160 kg of water and 0.0180 kg of ice in thermal equilibrium at atmospheric pressure. If 0.750 kg of lead at a temperature of 255°C is dropped into the calorimeter can, what is the final temperature? Assume that no heat is lost to the surroundings.

17.61 • A vessel whose walls are thermally insulated contains 2.40 kg of water and 0.450 kg of ice, all at a temperature of 0.0°C . The outlet of a tube leading from a boiler in which water is boiling at atmospheric pressure is inserted into the water. How many grams of steam must condense inside the vessel (also at atmospheric pressure) to raise the temperature of the system to 28.0°C ? You can ignore the heat transferred to the container.

Section 17.7 Mechanisms of Heat Transfer

17.62 • Two rods, one made of brass and the other made of copper, are joined end to end. The length of the brass section is 0.200 m and the length of the copper section is 0.800 m. Each segment has cross-sectional area 0.00500 m^2 . The free end of the brass segment is in boiling water and the free end of the copper segment is in an ice and water mixture, in both cases under normal atmospheric pressure. The sides of the rods are insulated so there is no

heat loss to the surroundings. (a) What is the temperature of the point where the brass and copper segments are joined? (b) What mass of ice is melted in 5.00 min by the heat conducted by the composite rod?

17.63 • Suppose that the rod in Fig. 17.23a is made of copper, is 45.0 cm long, and has a cross-sectional area of 1.25 cm^2 . Let $T_H = 100.0^\circ\text{C}$ and $T_C = 0.0^\circ\text{C}$. (a) What is the final steady-state temperature gradient along the rod? (b) What is the heat current in the rod in the final steady state? (c) What is the final steady-state temperature at a point in the rod 12.0 cm from its left end?

17.64 •• One end of an insulated metal rod is maintained at 100.0°C , and the other end is maintained at 0.00°C by an ice–water mixture. The rod is 60.0 cm long and has a cross-sectional area of 1.25 cm^2 . The heat conducted by the rod melts 8.50 g of ice in 10.0 min. Find the thermal conductivity k of the metal.

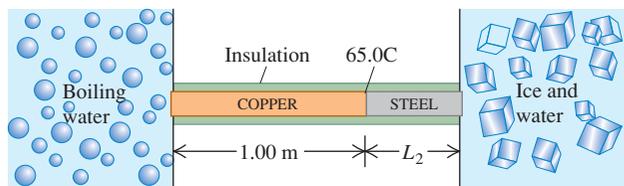
17.65 •• A carpenter builds an exterior house wall with a layer of wood 3.0 cm thick on the outside and a layer of Styrofoam insulation 2.2 cm thick on the inside wall surface. The wood has $k = 0.080 \text{ W/m}\cdot\text{K}$, and the Styrofoam has $k = 0.010 \text{ W/m}\cdot\text{K}$. The interior surface temperature is 19.0°C , and the exterior surface temperature is -10.0°C . (a) What is the temperature at the plane where the wood meets the Styrofoam? (b) What is the rate of heat flow per square meter through this wall?

17.66 • An electric kitchen range has a total wall area of 1.40 m^2 and is insulated with a layer of fiberglass 4.00 cm thick. The inside surface of the fiberglass has a temperature of 175°C , and its outside surface is at 35.0°C . The fiberglass has a thermal conductivity of $0.040 \text{ W/m}\cdot\text{K}$. (a) What is the heat current through the insulation, assuming it may be treated as a flat slab with an area of 1.40 m^2 ? (b) What electric-power input to the heating element is required to maintain this temperature?

17.67 • **BIO Conduction Through the Skin.** The blood plays an important role in removing heat from the body by bringing this heat directly to the surface where it can radiate away. Nevertheless, this heat must still travel through the skin before it can radiate away. We shall assume that the blood is brought to the bottom layer of skin at a temperature of 37.0°C and that the outer surface of the skin is at 30.0°C . Skin varies in thickness from 0.50 mm to a few millimeters on the palms and soles, so we shall assume an average thickness of 0.75 mm. A 165-lb, 6-ft-tall person has a surface area of about 2.0 m^2 and loses heat at a net rate of 75 W while resting. On the basis of our assumptions, what is the thermal conductivity of this person's skin?

17.68 • A long rod, insulated to prevent heat loss along its sides, is in perfect thermal contact with boiling water (at atmospheric pressure) at one end and with an ice–water mixture at the other (Fig. E17.68). The rod consists of a 1.00-m section of copper (one end in boiling water) joined end to end to a length L_2 of steel (one end in the ice–water mixture). Both sections of the rod have cross-sectional areas of 4.00 cm^2 . The temperature of the copper–steel junction is 65.0°C after a steady state has been set up. (a) How much heat per second flows from the boiling water to the ice–water mixture? (b) What is the length L_2 of the steel section?

Figure E17.68



17.69 • A pot with a steel bottom 8.50 mm thick rests on a hot stove. The area of the bottom of the pot is 0.150 m^2 . The water inside the pot is at 100.0°C , and 0.390 kg are evaporated every 3.00 min. Find the temperature of the lower surface of the pot, which is in contact with the stove.

17.70 •• You are asked to design a cylindrical steel rod 50.0 cm long, with a circular cross section, that will conduct 150.0 J/s from a furnace at 400.0°C to a container of boiling water under 1 atmosphere. What must the rod's diameter be?

17.71 •• A picture window has dimensions of $1.40 \text{ m} \times 2.50 \text{ m}$ and is made of glass 5.20 mm thick. On a winter day, the outside temperature is -20.0°C , while the inside temperature is a comfortable 19.5°C . (a) At what rate is heat being lost through the window by conduction? (b) At what rate would heat be lost through the window if you covered it with a 0.750-mm-thick layer of paper (thermal conductivity $0.0500 \text{ W/m}\cdot\text{K}$)?

17.72 • What is the rate of energy radiation per unit area of a blackbody at a temperature of (a) 273 K and (b) 2730 K?

17.73 • **Size of a Light-Bulb Filament.** The operating temperature of a tungsten filament in an incandescent light bulb is 2450 K, and its emissivity is 0.350. Find the surface area of the filament of a 150-W bulb if all the electrical energy consumed by the bulb is radiated by the filament as electromagnetic waves. (Only a fraction of the radiation appears as visible light.)

17.74 •• The emissivity of tungsten is 0.350. A tungsten sphere with radius 1.50 cm is suspended within a large evacuated enclosure whose walls are at 290.0 K. What power input is required to maintain the sphere at a temperature of 3000.0 K if heat conduction along the supports is neglected?

17.75 • **The Sizes of Stars.** The hot glowing surfaces of stars emit energy in the form of electromagnetic radiation. It is a good approximation to assume $e = 1$ for these surfaces. Find the radii of the following stars (assumed to be spherical): (a) Rigel, the bright blue star in the constellation Orion, which radiates energy at a rate of $2.7 \times 10^{32} \text{ W}$ and has surface temperature 11,000 K; (b) Procyon B (visible only using a telescope), which radiates energy at a rate of $2.1 \times 10^{23} \text{ W}$ and has surface temperature 10,000 K. (c) Compare your answers to the radius of the earth, the radius of the sun, and the distance between the earth and the sun. (Rigel is an example of a *supergiant* star, and Procyon B is an example of a *white dwarf* star.)

PROBLEMS

17.76 •• Suppose that a steel hoop could be constructed to fit just around the earth's equator at a temperature of 20.0°C . What would be the thickness of space between the hoop and the earth if the temperature of the hoop were increased by 0.500 C° ?

17.77 ••• You propose a new temperature scale with temperatures given in $^\circ\text{M}$. You define 0.0°M to be the normal melting point of mercury and 100.0° to be the normal boiling point of mercury. (a) What is the normal boiling point of water in $^\circ\text{M}$? (b) A temperature change of 10.0 M° corresponds to how many C° ?

17.78 • **CP, CALC** A 250-kg weight is hanging from the ceiling by a thin copper wire. In its fundamental mode, this wire vibrates at the frequency of concert A (440 Hz). You then increase the temperature of the wire by 40 C° . (a) By how much will the fundamental frequency change? Will it increase or decrease? (b) By what percentage will the speed of a wave on the wire change? (c) By what percentage will the wavelength of the fundamental standing wave change? Will it increase or decrease?

17.79 •• You are making pesto for your pasta and have a cylindrical measuring cup 10.0 cm high made of ordinary glass [$\beta = 2.7 \times 10^{-5} (\text{C}^\circ)^{-1}$] that is filled with olive oil [$\beta = 6.8 \times 10^{-4} (\text{C}^\circ)^{-1}$] to a height of 2.00 mm below the top of the cup. Initially, the cup and oil are at room temperature (22.0°C). You get a phone call and forget about the olive oil, which you inadvertently leave on the hot stove. The cup and oil heat up slowly and have a common temperature. At what temperature will the olive oil start to spill out of the cup?

17.80 •• A surveyor's 30.0-m steel tape is correct at a temperature of 20.0°C. The distance between two points, as measured by this tape on a day when its temperature is 5.00°C, is 25.970 m. What is the true distance between the points?

17.81 •• CP A Foucault pendulum consists of a brass sphere with a diameter of 35.0 cm suspended from a steel cable 10.5 m long (both measurements made at 20.0°C). Due to a design oversight, the swinging sphere clears the floor by a distance of only 2.00 mm when the temperature is 20.0°C. At what temperature will the sphere begin to brush the floor?

17.82 •• You pour 108 cm³ of ethanol, at a temperature of -10.0°C, into a graduated cylinder initially at 20.0°C, filling it to the very top. The cylinder is made of glass with a specific heat of 840 J/kg·K and a coefficient of volume expansion of $1.2 \times 10^{-5} \text{K}^{-1}$; its mass is 0.110 kg. The mass of the ethanol is 0.0873 kg. (a) What will be the final temperature of the ethanol, once thermal equilibrium is reached? (b) How much ethanol will overflow the cylinder before thermal equilibrium is reached?

17.83 •• A metal rod that is 30.0 cm long expands by 0.0650 cm when its temperature is raised from 0.0°C to 100.0°C. A rod of a different metal and of the same length expands by 0.0350 cm for the same rise in temperature. A third rod, also 30.0 cm long, is made up of pieces of each of the above metals placed end to end and expands 0.0580 cm between 0.0°C and 100.0°C. Find the length of each portion of the composite rod.

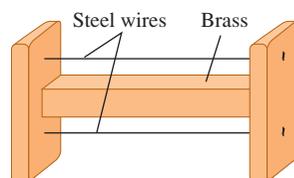
17.84 •• On a cool (4.0°C) Saturday morning, a pilot fills the fuel tanks of her Pitts S-2C (a two-seat aerobatic airplane) to their full capacity of 106.0 L. Before flying on Sunday morning, when the temperature is again 4.0°C, she checks the fuel level and finds only 103.4 L of gasoline in the tanks. She realizes that it was hot on Saturday afternoon, and that thermal expansion of the gasoline caused the missing fuel to empty out of the tank's vent. (a) What was the maximum temperature (in °C) reached by the fuel and the tank on Saturday afternoon? The coefficient of volume expansion of gasoline is $9.5 \times 10^{-4} \text{K}^{-1}$, and the tank is made of aluminum. (b) In order to have the maximum amount of fuel available for flight, when should the pilot have filled the fuel tanks?

17.85 •• (a) Equation (17.12) gives the stress required to keep the length of a rod constant as its temperature changes. Show that if the length is permitted to change by an amount ΔL when its temperature changes by ΔT , the stress is equal to

$$\frac{F}{A} = Y \left(\frac{\Delta L}{L_0} - \alpha \Delta T \right)$$

where F is the tension on the rod, L_0 is the original length of the rod, A its cross-sectional area, α its coefficient of linear expansion, and Y its Young's modulus. (b) A heavy brass bar has projections at its ends, as in Fig. P17.85. Two fine steel wires, fastened between the pro-

Figure P17.85



jections, are just taut (zero tension) when the whole system is at 20°C. What is the tensile stress in the steel wires when the temperature of the system is raised to 140°C? Make any simplifying assumptions you think are justified, but state what they are.

17.86 •• CP A metal wire, with density ρ and Young's modulus Y , is stretched between rigid supports. At temperature T , the speed of a transverse wave is found to be v_1 . When the temperature is increased to $T + \Delta T$, the speed decreases to $v_2 < v_1$. Determine the coefficient of linear expansion of the wire.

17.87 •• CP Out of Tune. The B-string of a guitar is made of steel (density 7800 kg/m³), is 63.5 cm long, and has diameter 0.406 mm. The fundamental frequency is $f = 247.0$ Hz. (a) Find the string tension. (b) If the tension F is changed by a small amount ΔF , the frequency f changes by a small amount Δf . Show that

$$\frac{\Delta f}{f} = \frac{\Delta F}{2F}$$

(c) The string is tuned to a fundamental frequency of 247.0 Hz when its temperature is 18.5°C. Strenuous playing can make the temperature of the string rise, changing its vibration frequency. Find Δf if the temperature of the string rises to 29.5°C. The steel string has a Young's modulus of 2.00×10^{11} Pa and a coefficient of linear expansion of $1.20 \times 10^{-5} (\text{C}^\circ)^{-1}$. Assume that the temperature of the body of the guitar remains constant. Will the vibration frequency rise or fall?

17.88 •• A steel rod 0.450 m long and an aluminum rod 0.250 m long, both with the same diameter, are placed end to end between rigid supports with no initial stress in the rods. The temperature of the rods is now raised by 60.0°C. What is the stress in each rod? (Hint: The length of the combined rod remains the same, but the lengths of the individual rods do not. See Problem 17.85.)

17.89 •• A steel ring with a 2.5000-in. inside diameter at 20.0°C is to be warmed and slipped over a brass shaft with a 2.5020-in. outside diameter at 20.0°C. (a) To what temperature should the ring be warmed? (b) If the ring and the shaft together are cooled by some means such as liquid air, at what temperature will the ring just slip off the shaft?

17.90 •• Bulk Stress Due to a Temperature Increase. (a) Prove that, if an object under pressure has its temperature raised but is not allowed to expand, the increase in pressure is

$$\Delta p = B\beta\Delta T$$

where the bulk modulus B and the average coefficient of volume expansion β are both assumed positive and constant. (b) What pressure is necessary to prevent a steel block from expanding when its temperature is increased from 20.0°C to 35.0°C?

17.91 •• A liquid is enclosed in a metal cylinder that is provided with a piston of the same metal. The system is originally at a pressure of 1.00 atm (1.013×10^5 Pa) and at a temperature of 30.0°C. The piston is forced down until the pressure on the liquid is increased by 50.0 atm, and then clamped in this position. Find the new temperature at which the pressure of the liquid is again 1.00 atm. Assume that the cylinder is sufficiently strong so that its volume is not altered by changes in pressure, but only by changes in temperature. Use the result derived in Problem 17.90. (Hint: See Section 11.4.)

Compressibility of liquid: $k = 8.50 \times 10^{-10} \text{Pa}^{-1}$

Coefficient of volume expansion of liquid: $\beta = 4.80 \times 10^{-4} \text{K}^{-1}$

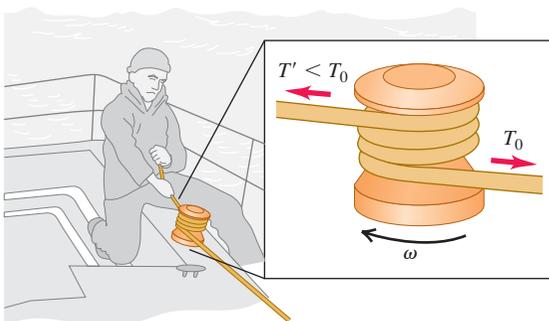
Coefficient of volume expansion of metal: $\beta = 3.90 \times 10^{-5} \text{K}^{-1}$

17.92 •• You cool a 100.0-g slug of red-hot iron (temperature 745°C) by dropping it into an insulated cup of negligible mass containing 85.0 g of water at 20.0°C. Assuming no heat exchange with the surroundings, (a) what is the final temperature of the water and (b) what is the final mass of the iron and the remaining water?

17.93 • CP Spacecraft Reentry. A spacecraft made of aluminum circles the earth at a speed of 7700 m/s. (a) Find the ratio of its kinetic energy to the energy required to raise its temperature from 0°C to 600°C. (The melting point of aluminum is 660°C. Assume a constant specific heat of 910 J/kg · K.) (b) Discuss the bearing of your answer on the problem of the reentry of a manned space vehicle into the earth's atmosphere.

17.94 • CP A capstan is a rotating drum or cylinder over which a rope or cord slides in order to provide a great amplification of the rope's tension while keeping both ends free (Fig. P17.94). Since the added tension in the rope is due to friction, the capstan generates thermal energy. (a) If the difference in tension between the two ends of the rope is 520.0 N and the capstan has a diameter of 10.0 cm and turns once in 0.900 s, find the rate at which thermal energy is generated. Why does the number of turns not matter? (b) If the capstan is made of iron and has mass 6.00 kg, at what rate does its temperature rise? Assume that the temperature in the capstan is uniform and that all the thermal energy generated flows into it.

Figure P17.94



17.95 •• CALC Debye's T^3 Law. At very low temperatures the molar heat capacity of rock salt varies with temperature according to Debye's T^3 law:

$$C = k \frac{T^3}{\Theta^3}$$

where $k = 1940 \text{ J/mol} \cdot \text{K}$ and $\Theta = 281 \text{ K}$. (a) How much heat is required to raise the temperature of 1.50 mol of rock salt from 10.0 K to 40.0 K? (Hint: Use Eq. (17.18) in the form $dQ = nC dT$ and integrate.) (b) What is the average molar heat capacity in this range? (c) What is the true molar heat capacity at 40.0 K?

17.96 •• CP A person of mass 70.0 kg is sitting in the bathtub. The bathtub is 190.0 cm by 80.0 cm; before the person got in, the water was 16.0 cm deep. The water is at a temperature of 37.0°C. Suppose that the water were to cool down spontaneously to form ice at 0.0°C, and that all the energy released was used to launch the hapless bather vertically into the air. How high would the bather go? (As you will see in Chapter 20, this event is allowed by energy conservation but is prohibited by the second law of thermodynamics.)

17.97 • Hot Air in a Physics Lecture. (a) A typical student listening attentively to a physics lecture has a heat output of 100 W. How much heat energy does a class of 90 physics students release into a lecture hall over the course of a 50-min lecture? (b) Assume that all the heat energy in part (a) is transferred to the 3200 m³ of air in the room. The air has specific heat 1020 J/kg · K and density 1.20 kg/m³. If none of the heat escapes and the air conditioning system is off, how much will the temperature of the air in the room rise during the 50-min lecture? (c) If the class is taking an exam,

the heat output per student rises to 280 W. What is the temperature rise during 50 min in this case?

17.98 ••• CALC The molar heat capacity of a certain substance varies with temperature according to the empirical equation

$$C = 29.5 \text{ J/mol} \cdot \text{K} + (8.20 \times 10^{-3} \text{ J/mol} \cdot \text{K}^2)T$$

How much heat is necessary to change the temperature of 3.00 mol of this substance from 27°C to 227°C? (Hint: Use Eq. (17.18) in the form $dQ = nC dT$ and integrate.)

17.99 ••• For your cabin in the wilderness, you decide to build a primitive refrigerator out of Styrofoam, planning to keep the interior cool with a block of ice that has an initial mass of 24.0 kg. The box has dimensions of 0.500 m × 0.800 m × 0.500 m. Water from melting ice collects in the bottom of the box. Suppose the ice block is at 0.00°C and the outside temperature is 21.0°C. If the top of the empty box is never opened and you want the interior of the box to remain at 5.00°C for exactly one week, until all the ice melts, what must be the thickness of the Styrofoam?

17.100 •• Hot Water Versus Steam Heating. In a household hot-water heating system, water is delivered to the radiators at 70.0°C (158.0°F) and leaves at 28.0°C (82.4°F). The system is to be replaced by a steam system in which steam at atmospheric pressure condenses in the radiators and the condensed steam leaves the radiators at 35.0°C (95.0°F). How many kilograms of steam will supply the same heat as was supplied by 1.00 kg of hot water in the first system?

17.101 ••• A copper calorimeter can with mass 0.446 kg contains 0.0950 kg of ice. The system is initially at 0.0°C. (a) If 0.0350 kg of steam at 100.0°C and 1.00 atm pressure is added to the can, what is the final temperature of the calorimeter can and its contents? (b) At the final temperature, how many kilograms are there of ice, how many of liquid water, and how many of steam?

17.102 • A Styrofoam bucket of negligible mass contains 1.75 kg of water and 0.450 kg of ice. More ice, from a refrigerator at −15.0°C, is added to the mixture in the bucket, and when thermal equilibrium has been reached, the total mass of ice in the bucket is 0.868 kg. Assuming no heat exchange with the surroundings, what mass of ice was added?

17.103 ••• In a container of negligible mass, 0.0400 kg of steam at 100°C and atmospheric pressure is added to 0.200 kg of water at 50.0°C. (a) If no heat is lost to the surroundings, what is the final temperature of the system? (b) At the final temperature, how many kilograms are there of steam and how many of liquid water?

17.104 •• BIO Mammal Insulation. Animals in cold climates often depend on *two* layers of insulation: a layer of body fat (of thermal conductivity 0.20 W/m · K) surrounded by a layer of air trapped inside fur or down. We can model a black bear (*Ursus americanus*) as a sphere 1.5 m in diameter having a layer of fat 4.0 cm thick. (Actually, the thickness varies with the season, but we are interested in hibernation, when the fat layer is thickest.) In studies of bear hibernation, it was found that the outer surface layer of the fur is at 2.7°C during hibernation, while the inner surface of the fat layer is at 31.0°C. (a) What is the temperature at the fat–inner fur boundary? (b) How thick should the air layer (contained within the fur) be so that the bear loses heat at a rate of 50.0 W?

17.105 ••• A worker pours 1.250 kg of molten lead at a temperature of 327.3°C into 0.5000 kg of water at a temperature of 75.00°C in an insulated bucket of negligible mass. Assuming no heat loss to the surroundings, calculate the mass of lead and water remaining in the bucket when the materials have reached thermal equilibrium.

17.106 •• One experimental method of measuring an insulating material's thermal conductivity is to construct a box of the material and measure the power input to an electric heater inside the box that maintains the interior at a measured temperature above the outside surface. Suppose that in such an apparatus a power input of 180 W is required to keep the interior surface of the box 65.0°C (about 120°F) above the temperature of the outer surface. The total area of the box is 2.18 m^2 , and the wall thickness is 3.90 cm . Find the thermal conductivity of the material in SI units.

17.107 •• Effect of a Window in a Door. A carpenter builds a solid wood door with dimensions $2.00\text{ m} \times 0.95\text{ m} \times 5.0\text{ cm}$. Its thermal conductivity is $k = 0.120\text{ W/m}\cdot\text{K}$. The air films on the inner and outer surfaces of the door have the same combined thermal resistance as an additional 1.8-cm thickness of solid wood. The inside air temperature is 20.0°C , and the outside air temperature is -8.0°C . (a) What is the rate of heat flow through the door? (b) By what factor is the heat flow increased if a window 0.500 m on a side is inserted in the door? The glass is 0.450 cm thick, and the glass has a thermal conductivity of $0.80\text{ W/m}\cdot\text{K}$. The air films on the two sides of the glass have a total thermal resistance that is the same as an additional 12.0 cm of glass.

17.108 • A wood ceiling with thermal resistance R_1 is covered with a layer of insulation with thermal resistance R_2 . Prove that the effective thermal resistance of the combination is $R = R_1 + R_2$.

17.109 •• Compute the ratio of the rate of heat loss through a single-pane window with area 0.15 m^2 to that for a double-pane window with the same area. The glass of a single pane is 4.2 mm thick, and the air space between the two panes of the double-pane window is 7.0 mm thick. The glass has thermal conductivity $0.80\text{ W/m}\cdot\text{K}$. The air films on the room and outdoor surfaces of either window have a combined thermal resistance of $0.15\text{ m}^2\cdot\text{K/W}$.

17.110 • Rods of copper, brass, and steel are welded together to form a Y-shaped figure. The cross-sectional area of each rod is 2.00 cm^2 . The free end of the copper rod is maintained at 100.0°C , and the free ends of the brass and steel rods at 0.0°C . Assume there is no heat loss from the surfaces of the rods. The lengths of the rods are: copper, 13.0 cm ; brass, 18.0 cm ; steel, 24.0 cm . (a) What is the temperature of the junction point? (b) What is the heat current in each of the three rods?

17.111 ••• CALC Time Needed for a Lake to Freeze Over. (a) When the air temperature is below 0°C , the water at the surface of a lake freezes to form an ice sheet. Why doesn't freezing occur throughout the entire volume of the lake? (b) Show that the thickness of the ice sheet formed on the surface of a lake is proportional to the square root of the time if the heat of fusion of the water freezing on the underside of the ice sheet is conducted through the sheet. (c) Assuming that the upper surface of the ice sheet is at -10°C and the bottom surface is at 0°C , calculate the time it will take to form an ice sheet 25 cm thick. (d) If the lake in part (c) is uniformly 40 m deep, how long would it take to freeze all the water in the lake? Is this likely to occur?

17.112 •• A rod is initially at a uniform temperature of 0°C throughout. One end is kept at 0°C , and the other is brought into contact with a steam bath at 100°C . The surface of the rod is insulated so that heat can flow only lengthwise along the rod. The cross-sectional area of the rod is 2.50 cm^2 , its length is 120 cm , its thermal conductivity is $380\text{ W/m}\cdot\text{K}$, its density is $1.00 \times 10^4\text{ kg/m}^3$, and its specific heat is $520\text{ J/kg}\cdot\text{K}$. Consider a short cylindrical element of the rod 1.00 cm in length. (a) If the temperature gradient at the cooler end of this element is 140°C/m , how many joules of heat energy flow across this end per second? (b) If the average temperature of the element is

increasing at the rate of what is the temperature gradient at the other end of the element?

17.113 •• A rustic cabin has a floor area of $3.50\text{ m} \times 3.00\text{ m}$. Its walls, which are 2.50 m tall, are made of wood (thermal conductivity $0.0600\text{ W/m}\cdot\text{K}$) 1.80 cm thick and are further insulated with 1.50 cm of a synthetic material. When the outside temperature is 2.00°C , it is found necessary to heat the room at a rate of 1.25 kW to maintain its temperature at 19.0°C . Calculate the thermal conductivity of the insulating material. Neglect the heat lost through the ceiling and floor. Assume the inner and outer surfaces of the wall have the same temperature as the air inside and outside the cabin.

17.114 • The rate at which radiant energy from the sun reaches the earth's upper atmosphere is about 1.50 kW/m^2 . The distance from the earth to the sun is $1.50 \times 10^{11}\text{ m}$, and the radius of the sun is $6.96 \times 10^8\text{ m}$. (a) What is the rate of radiation of energy per unit area from the sun's surface? (b) If the sun radiates as an ideal blackbody, what is the temperature of its surface?

17.115 ••• A Thermos for Liquid Helium. A physicist uses a cylindrical metal can 0.250 m high and 0.090 m in diameter to store liquid helium at 4.22 K ; at that temperature the heat of vaporization of helium is $2.09 \times 10^4\text{ J/kg}$. Completely surrounding the metal can are walls maintained at the temperature of liquid nitrogen, 77.3 K , with vacuum between the can and the surrounding walls. How much helium is lost per hour? The emissivity of the metal can is 0.200 . The only heat transfer between the metal can and the surrounding walls is by radiation.

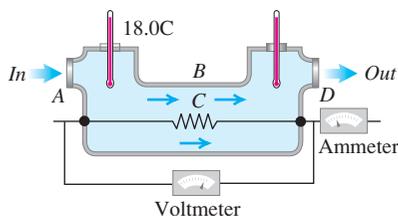
17.116 •• BIO Basal Metabolic Rate. The basal metabolic rate is the rate at which energy is produced in the body when a person is at rest. A 75-kg (165-lb) person of height 1.83 m (6 ft) has a body surface area of approximately 2.0 m^2 . (a) What is the net amount of heat this person could radiate per second into a room at 18°C (about 65°F) if his skin's surface temperature is 30°C ? (At such temperatures, nearly all the heat is infrared radiation, for which the body's emissivity is 1.0 , regardless of the amount of pigment.) (b) Normally, 80% of the energy produced by metabolism goes into heat, while the rest goes into things like pumping blood and repairing cells. Also normally, a person at rest can get rid of this excess heat just through radiation. Use your answer to part (a) to find this person's basal metabolic rate.

17.117 •• BIO Jogging in the Heat of the Day. You have probably seen people jogging in extremely hot weather and wondered Why? As we shall see, there are good reasons not to do this! When jogging strenuously, an average runner of mass 68 kg and surface area 1.85 m^2 produces energy at a rate of up to 1300 W , 80% of which is converted to heat. The jogger radiates heat, but actually absorbs more from the hot air than he radiates away. At such high levels of activity, the skin's temperature can be elevated to around 33°C instead of the usual 30°C . (We shall neglect conduction, which would bring even more heat into his body.) The only way for the body to get rid of this extra heat is by evaporating water (sweating). (a) How much heat per second is produced just by the act of jogging? (b) How much *net* heat per second does the runner gain just from radiation if the air temperature is 40.0°C (104°F)? (Remember that he radiates out, but the environment radiates back in.) (c) What is the *total* amount of excess heat this runner's body must get rid of per second? (d) How much water must the jogger's body evaporate every minute due to his activity? The heat of vaporization of water at body temperature is $2.42 \times 10^6\text{ J/kg}$. (e) How many 750-mL bottles of water must he drink after (or preferably before!) jogging for a half hour? Recall that a liter of water has a mass of 1.0 kg .

17.118 •• BIO Overheating While Jogging. (a) If the jogger in the preceding problem were not able to get rid of the excess heat, by how much would his body temperature increase above the normal 37°C in a half hour of jogging? The specific heat for a human is about $3500\text{ J/kg}\cdot\text{K}$. (b) How high a fever (in $^\circ\text{F}$) would this temperature increase be equivalent to? Is the increase large enough to be of concern? (Recall that normal body temperature is 98.6°F .)

17.119 •• An engineer is developing an electric water heater to provide a continuous supply of hot water. One trial design is shown in Fig. P17.119. Water is flowing at the rate of 0.500 kg/min , the inlet thermometer registers 18.0°C , the voltmeter reads 120 V , and the ammeter reads 15.0 A [corresponding to a power input of $(120\text{ V}) \times (15.0\text{ A}) = 1800\text{ W}$]. (a) When a steady state is finally reached, what is the reading of the outlet thermometer? (b) Why is it unnecessary to take into account the heat capacity mc of the apparatus itself?

Figure P17.119



17.120 • Food Intake of a Hamster. The energy output of an animal engaged in an activity is called the basal metabolic rate (BMR) and is a measure of the conversion of food energy into other forms of energy. A simple calorimeter to measure the BMR consists of an insulated box with a thermometer to measure the temperature of the air. The air has density 1.20 kg/m^3 and specific heat $1020\text{ J/kg}\cdot\text{K}$. A 50.0-g hamster is placed in a calorimeter that contains 0.0500 m^3 of air at room temperature. (a) When the hamster is running in a wheel, the temperature of the air in the calorimeter rises 1.60 C° per hour. How much heat does the running hamster generate in an hour? Assume that all this heat goes into the air in the calorimeter. You can ignore the heat that goes into the walls of the box and into the thermometer, and assume that no heat is lost to the surroundings. (b) Assuming that the hamster converts seed into heat with an efficiency of 10% and that hamster seed has a food energy value of 24 J/g , how many grams of seed must the hamster eat per hour to supply this energy?

17.121 •• The icecaps of Greenland and Antarctica contain about 1.75% of the total water (by mass) on the earth's surface; the oceans contain about 97.5%, and the other 0.75% is mainly groundwater. Suppose the icecaps, currently at an average temperature of about -30°C , somehow slid into the ocean and melted. What would be the resulting temperature decrease of the ocean? Assume that the average temperature of ocean water is currently 5.00°C .

17.122 •• Why Do the Seasons Lag? In the northern hemisphere, June 21 (the summer solstice) is both the longest day of the year and the day on which the sun's rays strike the earth most vertically, hence delivering the greatest amount of heat to the surface. Yet the hottest summer weather usually occurs about a month or so later. Let us see why this is the case. Because of the large specific heat of water, the oceans are slower to warm up than the land (and also slower to cool off in winter). In addition to perusing pertinent information in the tables included in this book, it is useful to know

that approximately two-thirds of the earth's surface is ocean composed of salt water having a specific heat of $3890\text{ J/kg}\cdot\text{K}$ and that the oceans, on the average, are 4000 m deep. Typically, an average of 1050 W/m^2 of solar energy falls on the earth's surface, and the oceans absorb essentially all of the light that strikes them. However, most of that light is absorbed in the upper 100 m of the surface. Depths below that do not change temperature seasonally. Assume that the sunlight falls on the surface for only 12 hours per day and that the ocean retains all the heat it absorbs. What will be the rise in temperature of the upper 100 m of the oceans during the month following the summer solstice? Does this seem to be large enough to be perceptible?

CHALLENGE PROBLEMS

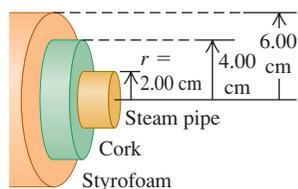
17.123 ••• CALC Suppose that both ends of the rod in Fig. 17.23a are kept at a temperature of 0°C , and that the initial temperature distribution along the rod is given by $T = (100^\circ\text{C}) \sin \pi x/L$, where x is measured from the left end of the rod. Let the rod be copper, with length $L = 0.100\text{ m}$ and cross-sectional area 1.00 cm^2 . (a) Show the initial temperature distribution in a diagram. (b) What is the final temperature distribution after a very long time has elapsed? (c) Sketch curves that you think would represent the temperature distribution at intermediate times. (d) What is the initial temperature gradient at the ends of the rod? (e) What is the initial heat current from the ends of the rod into the bodies making contact with its ends? (f) What is the initial heat current at the center of the rod? Explain. What is the heat current at this point at any later time? (g) What is the value of the *thermal diffusivity* $k/\rho c$ for copper, and in what unit is it expressed? (Here k is the thermal conductivity, $\rho = 8.9 \times 10^3\text{ kg/m}^3$ is the density, and c is the specific heat.) (h) What is the initial time rate of change of temperature at the center of the rod? (i) How much time would be required for the center of the rod to reach its final temperature if the temperature continued to decrease at this rate? (This time is called the *relaxation time* of the rod.) (j) From the graphs in part (c), would you expect the magnitude of the rate of temperature change at the midpoint to remain constant, increase, or decrease as a function of time? (k) What is the initial rate of change of temperature at a point in the rod 2.5 cm from its left end?

17.124 ••• CALC (a) A spherical shell has inner and outer radii a and b , respectively, and the temperatures at the inner and outer surfaces are T_2 and T_1 . The thermal conductivity of the material of which the shell is made is k . Derive an equation for the total heat current through the shell. (b) Derive an equation for the temperature variation within the shell in part (a); that is, calculate T as a function of r , the distance from the center of the shell. (c) A hollow cylinder has length L , inner radius a , and outer radius b , and the temperatures at the inner and outer surfaces are T_2 and T_1 . (The cylinder could represent an insulated hot-water pipe, for example.) The thermal conductivity of the material of which the cylinder is made is k . Derive an equation for the total heat current through the walls of the cylinder. (d) For the cylinder of part (c), derive an equation for the temperature variation inside the cylinder walls. (e) For the spherical shell of part (a) and the hollow cylinder of part (c), show that the equation for the total heat current in each case reduces to Eq. (17.21) for linear heat flow when the shell or cylinder is very thin.

17.125 ••• A steam pipe with a radius of 2.00 cm , carrying steam at 140°C , is surrounded by a cylindrical jacket with inner and outer radii 2.00 cm and 4.00 cm and made of a type of cork with thermal conductivity $4.00 \times 10^{-2}\text{ W/m}\cdot\text{K}$. This in turn is surrounded by

a cylindrical jacket made of a brand of Styrofoam with thermal conductivity $1.00 \times 10^{-2} \text{ W/m} \cdot \text{K}$ and having inner and outer radii 4.00 cm and 6.00 cm (Fig. P17.125). The outer surface of the Styrofoam is in contact with air at 15°C . Assume that this outer surface has a temperature of 15°C .

Figure P17.125



(a) What is the temperature at a radius of 4.00 cm, where the two insulating layers meet? (b) What is the total rate of transfer of heat out of a 2.00-m length of pipe? (*Hint:* Use the expression derived in part (c) of Challenge Problem 17.124.)

17.126 ••• CP Temperature Change in a Clock. A pendulum clock is designed to tick off one second on each side-to-side swing of the pendulum (two ticks per complete period). (a) Will a pendulum clock gain time in hot weather and lose it in cold, or the reverse? Explain your reasoning. (b) A particular pendulum clock keeps correct time at 20.0°C . The pendulum shaft is steel, and its mass can be ignored compared with that of the bob. What is the fractional change in the length of the shaft when it is cooled to 10.0°C ? (c) How many seconds per day will the clock gain or lose at 10.0°C ? (d) How closely must the temperature be controlled if the clock is not to gain or lose more than 1.00 s a day? Does the answer depend on the period of the pendulum?

Answers

Chapter Opening Question ?

No. By “heat” we mean energy that is in transit from one body to another as a result of temperature difference between the bodies. Bodies do not *contain* heat.

Test Your Understanding Questions

17.1 Answer: (ii) A liquid-in-tube thermometer actually measures its own temperature. If the thermometer stays in the hot water long enough, it will come to thermal equilibrium with the water and its temperature will be the same as that of the water.

17.2 Answer: (iv) Both a bimetallic strip and a resistance thermometer measure their own temperature. For this to be equal to the temperature of the object being measured, the thermometer and object must be in contact and in thermal equilibrium. A temporal artery thermometer detects the infrared radiation from a person’s skin, so there is no need for the detector and skin to be at the same temperature.

17.3 Answer: (i), (iii), (ii), (v), (iv) To compare these temperatures, convert them all to the Kelvin scale. For (i), the Kelvin temperature is $T_K = T_C + 273.15 = 0.00 + 273.15 = 273.15 \text{ K}$; for (ii), $T_C = \frac{5}{9}(T_F - 32^\circ) = \frac{5}{9}(0.00^\circ - 32^\circ) = -17.78^\circ\text{C}$ and $T_K = T_C + 273.15 = -17.78 + 273.15 = 255.37 \text{ K}$; for (iii), $T_K = 260.00 \text{ K}$; for (iv), $T_K = 77.00 \text{ K}$; and for (v), $T_K = T_C + 273.15 = -180.00 + 273.15 = 93.15 \text{ K}$.

17.4 Answer: (ii) and (iii) Metal 2 must expand more than metal 1 when heated and so must have a larger coefficient of linear expansion

17.127 ••• BIO A Walk in the Sun. Consider a poor lost soul walking at 5 km/h on a hot day in the desert, wearing only a bathing suit. This person’s skin temperature tends to rise due to four mechanisms: (i) energy is generated by metabolic reactions in the body at a rate of 280 W, and almost all of this energy is converted to heat that flows to the skin; (ii) heat is delivered to the skin by convection from the outside air at a rate equal to $k'A_{\text{skin}}(T_{\text{air}} - T_{\text{skin}})$, where k' is $54 \text{ J/h} \cdot \text{C}^\circ \cdot \text{m}^2$, the exposed skin area A_{skin} is 1.5 m^2 , the air temperature T_{air} is 47°C , and the skin temperature T_{skin} is 36°C ; (iii) the skin absorbs radiant energy from the sun at a rate of 1400 W/m^2 ; (iv) the skin absorbs radiant energy from the environment, which has temperature 47°C . (a) Calculate the net rate (in watts) at which the person’s skin is heated by all four of these mechanisms. Assume that the emissivity of the skin is $e = 1$ and that the skin temperature is initially 36°C . Which mechanism is the most important? (b) At what rate (in L/h) must perspiration evaporate from this person’s skin to maintain a constant skin temperature? (The heat of vaporization of water at 36°C is $2.42 \times 10^6 \text{ J/kg}$.) (c) Suppose instead the person is protected by light-colored clothing ($e \approx 0$) so that the exposed skin area is only 0.45 m^2 . What rate of perspiration is required now? Discuss the usefulness of the traditional clothing worn by desert peoples.

α . From Table 17.1, brass and aluminum have larger values of α than copper, but steel does not.

17.5 Answer: (ii), (i), (iv), (iii) For (i) and (ii), the relevant quantity is the specific heat c of the substance, which is the amount of heat required to raise the temperature of 1 kilogram of that substance by 1 K (1 C°). From Table 17.3, these values are (i) 138 J for mercury and (ii) 2428 J for ethanol. For (iii) and (iv) we need the molar heat capacity C , which is the amount of heat required to raise the temperature of 1 mole of that substance by 1 C° . Again from Table 17.3, these values are (iii) 27.7 J for mercury and (iv) 111.9 J for ethanol. (The ratio of molar heat capacities is different from the ratio of the specific heats because a mole of mercury and a mole of ethanol have different masses.)

17.6 Answer: (iv) In time t the system goes from point b to point e in Fig. 17.21. According to this figure, at time $t/2$ (halfway along the horizontal axis from b to e), the system is at 100°C and is still boiling; that is, it is a mixture of liquid and gas. This says that most of the heat added goes into boiling the water.

17.7 Answer: (ii) When you touch one of the walls, heat flows from your hand to the lower-temperature wall. The more rapidly heat flows from your hand, the colder you will feel. Equation (17.21) shows that the rate of heat flow is proportional to the thermal conductivity k . From Table 17.5, copper has a much higher thermal conductivity ($385.0 \text{ W/m} \cdot \text{K}$) than steel ($50.2 \text{ W/m} \cdot \text{K}$) or concrete ($0.8 \text{ W/m} \cdot \text{K}$), and so the copper wall feels the coldest.

Bridging Problem

Answers: (a) 14.26 K **(b)** 0.427 kg/h

18 THERMAL PROPERTIES OF MATTER

LEARNING GOALS

By studying this chapter, you will learn:

- How to relate the pressure, volume, and temperature of a gas.
- How the interactions between the molecules of a substance determine the properties of the substance.
- How the pressure and temperature of a gas are related to the kinetic energy of its molecules.
- How the heat capacities of a gas reveal whether its molecules are rotating or vibrating.
- What determines whether a substance is a gas, a liquid, or a solid.



? The higher the temperature of a gas, the greater the average kinetic energy of its molecules. How much faster are molecules moving in the air above a frying pan (100°C) than in the surrounding kitchen air (25°C)?

The kitchen is a great place to learn about how the properties of matter depend on temperature. When you boil water in a tea kettle, the increase in temperature produces steam that whistles out of the spout at high pressure. If you forget to poke holes in a potato before baking it, the high-pressure steam produced inside the potato can cause it to explode messily. Water vapor in the air can condense into droplets of liquid on the sides of a glass of ice water; if the glass is just out of the freezer, frost will form on the sides as water vapor changes to a solid.

These examples show the relationships among the large-scale or *macroscopic* properties of a substance, such as pressure, volume, temperature, and mass. But we can also describe a substance using a *microscopic* perspective. This means investigating small-scale quantities such as the masses, speeds, kinetic energies, and momenta of the individual molecules that make up a substance.

The macroscopic and microscopic descriptions are intimately related. For example, the (microscopic) forces that occur when air molecules strike a solid surface (such as your skin) cause (macroscopic) atmospheric pressure. To produce standard atmospheric pressure of 1.01×10^5 Pa, 10^{32} molecules strike your skin every day with an average speed of over 1700 km/h (1000 mi/h)!

In this chapter we'll begin our study of the thermal properties of matter by looking at some macroscopic aspects of matter in general. We'll pay special attention to the *ideal gas*, one of the simplest types of matter to understand. Using our knowledge of momentum and kinetic energy, we'll relate the macroscopic properties of an ideal gas to the microscopic behavior of its individual molecules. We'll also use microscopic ideas to understand the heat capacities of both gases and solids. Finally, we'll take a look at the various phases of matter—gas, liquid, and solid—and the conditions under which each occurs.

18.1 Equations of State

The conditions in which a particular material exists are described by physical quantities such as pressure, volume, temperature, and amount of substance. For example, a tank of oxygen in a welding outfit has a pressure gauge and a label stating its volume. We could add a thermometer and place the tank on a scale to determine its mass. These variables describe the *state* of the material and are called **state variables**.

The volume V of a substance is usually determined by its pressure p , temperature T , and amount of substance, described by the mass m_{total} or number of moles n . (We are calling the total mass of a substance m_{total} because later in the chapter we will use m for the mass of one molecule.) Ordinarily, we can't change one of these variables without causing a change in another. When the tank of oxygen gets hotter, the pressure increases. If the tank gets too hot, it explodes.

In a few cases the relationship among p , V , T , and m (or n) is simple enough that we can express it as an equation called the **equation of state**. When it's too complicated for that, we can use graphs or numerical tables. Even then, the relationship among the variables still exists; we call it an equation of state even when we don't know the actual equation.

Here's a simple (though approximate) equation of state for a solid material. The temperature coefficient of volume expansion β (see Section 17.4) is the fractional volume change $\Delta V/V_0$ per unit temperature change, and the compressibility k (see Section 11.4) is the negative of the fractional volume change $\Delta V/V_0$ per unit pressure change. If a certain amount of material has volume V_0 when the pressure is p_0 and the temperature is T_0 , the volume V at slightly differing pressure p and temperature T is approximately

$$V = V_0[1 + \beta(T - T_0) - k(p - p_0)] \quad (18.1)$$

(There is a negative sign in front of the term $k(p - p_0)$ because an *increase* in pressure causes a *decrease* in volume.)

The Ideal-Gas Equation

Another simple equation of state is the one for an *ideal gas*. Figure 18.1 shows an experimental setup to study the behavior of a gas. The cylinder has a movable piston to vary the volume, the temperature can be varied by heating, and we can pump any desired amount of any gas into the cylinder. We then measure the pressure, volume, temperature, and amount of gas. Note that *pressure* refers both to the force per unit area exerted by the cylinder on the gas and to the force per unit area exerted by the gas on the cylinder; by Newton's third law, these must be equal.

It is usually easiest to describe the amount of gas in terms of the number of moles n , rather than the mass. (We did this when we defined molar heat capacity in Section 17.5.) The **molar mass** M of a compound (sometimes called *molecular weight*) is the mass per mole, and the total mass m_{total} of a given quantity of that compound is the number of moles n times the mass per mole M :

$$m_{\text{total}} = nM \quad (\text{total mass, number of moles, and molar mass}) \quad (18.2)$$

Hence if we know the number of moles of gas in the cylinder, we can determine the mass of gas using Eq. (18.2).

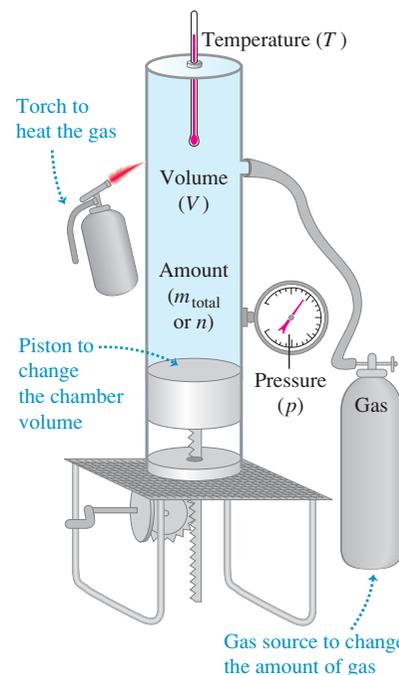
Measurements of the behavior of various gases lead to three conclusions:

1. The volume V is proportional to the number of moles n . If we double the number of moles, keeping pressure and temperature constant, the volume doubles.

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ActivPhysics 8.4: State Variables and Ideal Gas Law

18.1 A hypothetical setup for studying the behavior of gases. By heating the gas, varying the volume with a movable piston, and adding more gas, we can control the gas pressure p , volume V , temperature T , and number of moles n .

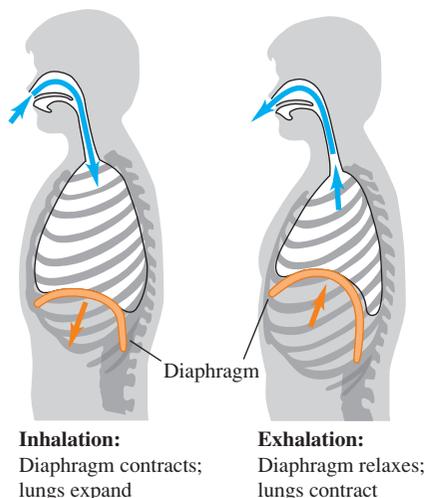


18.2 The ideal-gas equation $pV = nRT$ gives a good description of the air inside an inflated vehicle tire, where the pressure is about 3 atmospheres and the temperature is much too high for nitrogen or oxygen to liquefy. As the tire warms (T increases), the volume V changes only slightly but the pressure p increases.



Application Respiration and the Ideal-Gas Equation

To breathe, you rely on the ideal-gas equation $pV = nRT$. Contraction of the dome-shaped diaphragm muscle increases the volume V of the thoracic cavity (which encloses the lungs), decreasing its pressure p . The lowered pressure causes the lungs to expand and fill with air. [The temperature T is kept constant.] When you exhale, the diaphragm relaxes, allowing the lungs to contract and expel the air.



- The volume varies *inversely* with the absolute pressure p . If we double the pressure while holding the temperature T and number of moles n constant, the gas compresses to one-half of its initial volume. In other words, $pV = \text{constant}$ when n and T are constant.
- The pressure is proportional to the *absolute* temperature. If we double the absolute temperature, keeping the volume and number of moles constant, the pressure doubles. In other words, $p = (\text{constant})T$ when n and V are constant.

These three relationships can be combined neatly into a single equation, called the **ideal-gas equation**:

$$pV = nRT \quad (\text{ideal-gas equation}) \quad (18.3)$$

where R is a proportionality constant. An **ideal gas** is one for which Eq. (18.3) holds precisely for *all* pressures and temperatures. This is an idealized model; it works best at very low pressures and high temperatures, when the gas molecules are far apart and in rapid motion. It is reasonably good (within a few percent) at moderate pressures (such as a few atmospheres) and at temperatures well above those at which the gas liquefies (Fig. 18.2).

We might expect that the constant R in the ideal-gas equation would have different values for different gases, but it turns out to have the same value for *all* gases, at least at sufficiently high temperature and low pressure. It is called the **gas constant** (or *ideal-gas constant*). The numerical value of R depends on the units of p , V , and T . In SI units, in which the unit of p is Pa ($1 \text{ Pa} = 1 \text{ N/m}^2$) and the unit of V is m^3 , the current best numerical value of R is

$$R = 8.314472(15) \text{ J/mol} \cdot \text{K}$$

or $R = 8.314 \text{ J/mol} \cdot \text{K}$ to four significant figures. Note that the units of pressure times volume are the same as the units of work or energy (for example, N/m^2 times m^3); that's why R has units of energy per mole per unit of absolute temperature. In chemical calculations, volumes are often expressed in liters (L) and pressures in atmospheres (atm). In this system, to four significant figures,

$$R = 0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}$$

We can express the ideal-gas equation, Eq. (18.3), in terms of the mass m_{total} of gas, using $m_{\text{total}} = nM$ from Eq. (18.2):

$$pV = \frac{m_{\text{total}}}{M} RT \quad (18.4)$$

From this we can get an expression for the density $\rho = m_{\text{total}}/V$ of the gas:

$$\rho = \frac{pM}{RT} \quad (18.5)$$

CAUTION **Density vs. pressure** When using Eq. (18.5), be certain that you distinguish between the Greek letter ρ (rho) for density and the letter p for pressure. **|**

For a *constant mass* (or constant number of moles) of an ideal gas the product nR is constant, so the quantity pV/T is also constant. If the subscripts 1 and 2 refer to any two states of the same mass of a gas, then

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} = \text{constant} \quad (\text{ideal gas, constant mass}) \quad (18.6)$$

Notice that you don't need the value of R to use this equation.

We used the proportionality of pressure to absolute temperature in Chapter 17 to define a temperature scale in terms of pressure in a constant-volume gas thermometer. That may make it seem that the pressure–temperature relationship in the ideal-gas equation, Eq. (18.3), is just a result of the way we define temperature. But the equation also tells us what happens when we change the volume or the amount of substance. Also, we’ll see in Chapter 20 that the gas-thermometer scale corresponds closely to a temperature scale that does *not* depend on the properties of any particular material. For now, consider Eq. (18.6) as being based on this genuinely material-independent temperature scale.

Problem-Solving Strategy 18.1 Ideal Gases



IDENTIFY *the relevant concepts:* Unless the problem states otherwise, you can use the ideal-gas equation to find quantities related to the state of a gas, such as pressure p , volume V , temperature T , and/or number of moles n .

SET UP *the problem* using the following steps:

1. List the known and unknown quantities. Identify the target variables.
2. If the problem concerns only one state of the system, use Eq. (18.3), $pV = nRT$.
3. Use Eq. (18.5), $\rho = pM/RT$, as an alternative to Eq. (18.3) if the problem involves the density ρ rather than n and V .
4. In problems that concern two states (call them 1 and 2) of the same amount of gas, if all but one of the six quantities p_1 , p_2 , V_1 , V_2 , T_1 , and T_2 are known, use Eq. (18.6), $p_1V_1/T_1 = p_2V_2/T_2 = \text{constant}$. Otherwise, use Eq. (18.3) or Eq. (18.5) as appropriate.

EXECUTE *the solution* as follows:

1. Use consistent units. (SI units are entirely consistent.) The problem statement may make one system of units more con-

venient than others. Make appropriate unit conversions, such as from atmospheres to pascals or from liters to cubic meters.

2. You may have to convert between mass m_{total} and number of moles n , using $m_{\text{total}} = Mn$, where M is the molar mass. If you use Eq. (18.4), you *must* use the same mass units for m_{total} and M . So if M is in grams per mole (the usual units for molar mass), then m_{total} must also be in grams. To use m_{total} in kilograms, you must convert M to kg/mol. For example, the molar mass of oxygen is 32 g/mol or 32×10^{-3} kg/mol.
3. Remember that in the ideal-gas equations, T is always an *absolute* (Kelvin) temperature and p is always an absolute (not gauge) pressure.
4. Solve for the target variables.

EVALUATE *your answer:* Do your results make physical sense? Use benchmarks, such as the result of Example 18.1 below that a mole of an ideal gas at 1 atmosphere pressure occupies a volume of 22.4 liters.

Example 18.1 Volume of an ideal gas at STP

What is the volume of a container that holds exactly 1 mole of an ideal gas at *standard temperature and pressure* (STP), defined as $T = 0^\circ\text{C} = 273.15\text{ K}$ and $p = 1\text{ atm} = 1.013 \times 10^5\text{ Pa}$?

SOLUTION

IDENTIFY and SET UP: This problem involves the properties of a single state of an ideal gas, so we use Eq. (18.3). We are given the pressure p , temperature T , and number of moles n ; our target variable is the corresponding volume V .

EXECUTE: From Eq. (18.3), using R in $\text{J/mol} \cdot \text{K}$,

$$V = \frac{nRT}{p} = \frac{(1\text{ mol})(8.314\text{ J/mol} \cdot \text{K})(273.15\text{ K})}{1.013 \times 10^5\text{ Pa}} = 0.0224\text{ m}^3 = 22.4\text{ L}$$

EVALUATE: At STP, 1 mole of an ideal gas occupies 22.4 L. This is the volume of a cube 0.282 m (11.1 in.) on a side, or of a sphere 0.350 m (13.8 in.) in diameter.

Example 18.2 Compressing gas in an automobile engine

In an automobile engine, a mixture of air and vaporized gasoline is compressed in the cylinders before being ignited. A typical engine has a compression ratio of 9.00 to 1; that is, the gas in the cylinders is compressed to $\frac{1}{9.00}$ of its original volume (Fig. 18.3). The intake

and exhaust valves are closed during the compression, so the quantity of gas is constant. What is the final temperature of the compressed gas if its initial temperature is 27°C and the initial and final pressures are 1.00 atm and 21.7 atm, respectively?

Continued

SOLUTION

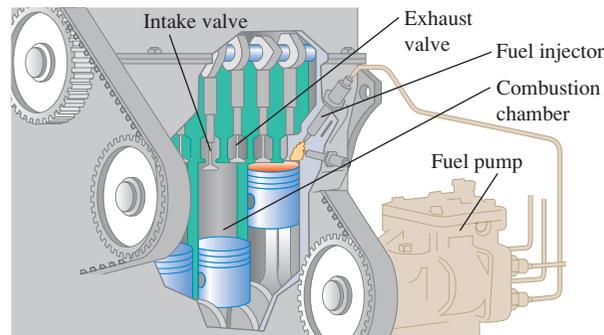
IDENTIFY and SET UP: We must compare two states of the same quantity of ideal gas, so we use Eq. (18.6). In the uncompressed state, $p_1 = 1.00$ atm and $T_1 = 27^\circ\text{C} = 300$ K. In the compressed state 2, $p_2 = 21.7$ atm. The cylinder volumes are not given, but we have $V_1 = 9.00V_2$. The temperature T_2 of the compressed gas is the target variable.

EXECUTE: We solve Eq. (18.6) for T_2 :

$$T_2 = T_1 \frac{p_2 V_2}{p_1 V_1} = (300 \text{ K}) \frac{(21.7 \text{ atm}) V_2}{(1.00 \text{ atm})(9.00 V_2)} = 723 \text{ K} = 450^\circ\text{C}$$

EVALUATE: This is the temperature of the air–gasoline mixture *before* the mixture is ignited; when burning starts, the temperature becomes higher still.

18.3 Cutaway of an automobile engine. While the air–gasoline mixture is being compressed prior to ignition, the intake and exhaust valves are both in the closed (up) position.

**Example 18.3** Mass of air in a scuba tank

An “empty” aluminum scuba tank contains 11.0 L of air at 21°C and 1 atm. When the tank is filled rapidly from a compressor, the air temperature is 42°C and the gauge pressure is 2.10×10^7 Pa. What mass of air was added? (Air is about 78% nitrogen, 21% oxygen, and 1% miscellaneous; its average molar mass is 28.8 g/mol = 28.8×10^{-3} kg/mol.)

SOLUTION

IDENTIFY and SET UP: Our target variable is the difference $m_2 - m_1$ between the masses present at the end (state 2) and at the beginning (state 1). We are given the molar mass M of air, so we can use Eq. (18.2) to find the target variable if we know the number of moles present in states 1 and 2. We determine n_1 and n_2 by applying Eq. (18.3) to each state individually.

EXECUTE: We convert temperatures to the Kelvin scale by adding 273 and convert the pressure to absolute by adding 1.013×10^5 Pa.

The tank’s volume is hardly affected by the increased temperature and pressure, so $V_2 = V_1$. From Eq. (18.3), the numbers of moles in the empty tank (n_1) and the full tank (n_2) are

$$n_1 = \frac{p_1 V_1}{RT_1} = \frac{(1.013 \times 10^5 \text{ Pa})(11.0 \times 10^{-3} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(294 \text{ K})} = 0.46 \text{ mol}$$

$$n_2 = \frac{p_2 V_2}{RT_2} = \frac{(2.11 \times 10^7 \text{ Pa})(11.0 \times 10^{-3} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(315 \text{ K})} = 88.6 \text{ mol}$$

We added $n_2 - n_1 = 88.6 \text{ mol} - 0.46 \text{ mol} = 88.1 \text{ mol}$ to the tank. From Eq. (18.2), the added mass is $M(n_2 - n_1) = (28.8 \times 10^{-3} \text{ kg/mol})(88.1 \text{ mol}) = 2.54 \text{ kg}$.

EVALUATE: The added mass is not insubstantial: You could certainly use a scale to determine whether the tank was empty or full.

Example 18.4 Variation of atmospheric pressure with elevation

Find the variation of atmospheric pressure with elevation in the earth’s atmosphere. Assume that at all elevations, $T = 0^\circ\text{C}$ and $g = 9.80 \text{ m/s}^2$.

SOLUTION

IDENTIFY and SET UP: As the elevation y increases, both the atmospheric pressure p and the density ρ decrease. Hence we have *two* unknown functions of y ; to solve for them, we need two independent equations. One is the ideal-gas equation, Eq. (18.5), which is expressed in terms of p and ρ . The other is Eq. (12.4), the relationship that we found in Section 12.2 among p , ρ , and y in a fluid in equilibrium: $dp/dy = -\rho g$. We are told to assume that g and T are the same at all elevations; we also assume that the atmosphere has the same chemical composition, and hence the same molar mass M , at all heights. We combine the two equations and solve for $p(y)$.

EXECUTE: We substitute $\rho = pM/RT$ into $dp/dy = -\rho g$, separate variables, and integrate, letting p_1 be the pressure at elevation y_1 and p_2 be the pressure at y_2 :

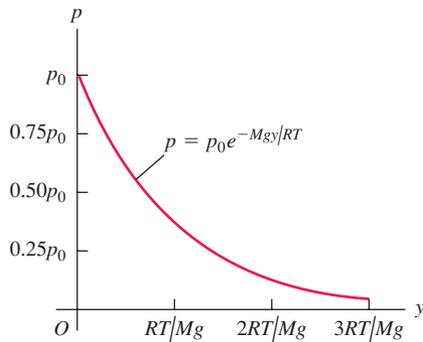
$$\begin{aligned} \frac{dp}{dy} &= -\frac{pM}{RT}g \\ \int_{p_1}^{p_2} \frac{dp}{p} &= -\frac{Mg}{RT} \int_{y_1}^{y_2} dy \\ \ln \frac{p_2}{p_1} &= -\frac{Mg}{RT}(y_2 - y_1) \\ \frac{p_2}{p_1} &= e^{-Mg(y_2 - y_1)/RT} \end{aligned}$$

Now let $y_1 = 0$ be at sea level and let the pressure at that point be $p_0 = 1.013 \times 10^5$ Pa. Then the pressure p at any height y is

$$p = p_0 e^{-Mgy/RT}$$

EVALUATE: According to our calculation, the pressure decreases exponentially with elevation. The graph in Fig. 18.4 shows that the slope dp/dy becomes less negative with greater elevation. That result makes sense, since $dp/dy = -\rho g$ and the density also

18.4 The variation of atmospheric pressure p with elevation y , assuming a constant temperature T .



decreases with elevation. At the summit of Mount Everest, where $y = 8863$ m,

$$\frac{Mgy}{RT} = \frac{(28.8 \times 10^{-3} \text{ kg/mol})(9.80 \text{ m/s}^2)(8863 \text{ m})}{(8.314 \text{ J/mol} \cdot \text{K})(273 \text{ K})} = 1.10$$

$$p = (1.013 \times 10^5 \text{ Pa})e^{-1.10} = 0.337 \times 10^5 \text{ Pa} = 0.33 \text{ atm}$$

The assumption of constant temperature isn't realistic, and g decreases a little with increasing elevation (see Challenge Problem 18.92). Even so, this example shows why mountaineers need to carry oxygen on Mount Everest. It also shows why jet airliners, which typically fly at altitudes of 8000 to 12,000 m, *must* have pressurized cabins for passenger comfort and health.

The van der Waals Equation

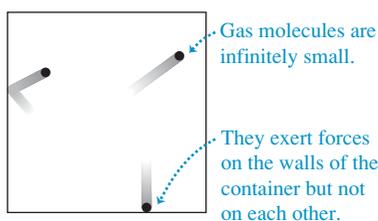
The ideal-gas equation, Eq. (18.3), can be obtained from a simple molecular model that ignores the volumes of the molecules themselves and the attractive forces between them (Fig. 18.5a). We'll examine that model in Section 18.3. Meanwhile, we mention another equation of state, the **van der Waals equation**, that makes approximate corrections for these two omissions (Fig. 18.5b). This equation was developed by the 19th-century Dutch physicist J. D. van der Waals; the interaction between atoms that we discussed in Section 14.4 was named the *van der Waals interaction* after him. The van der Waals equation is

$$\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT \quad (18.7)$$

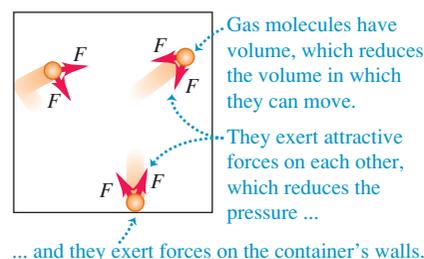
The constants a and b are empirical constants, different for different gases. Roughly speaking, b represents the volume of a mole of molecules; the total volume of the molecules is then nb , and the volume remaining in which the molecules can move is $V - nb$. The constant a depends on the attractive intermolecular forces, which reduce the pressure of the gas for given values of n , V , and T by *pulling* the molecules together as they *push* on the walls of the container. The decrease in pressure is proportional to the number of molecules per unit volume in a layer near the wall (which are exerting the pressure on the wall) and is also proportional to the number per unit volume in the next layer beyond the wall (which are doing the attracting). Hence the decrease in pressure due to intermolecular forces is proportional to n^2/V^2 .

When n/V is small (that is, when the gas is *dilute*), the average distance between molecules is large, the corrections in the van der Waals equation become insignificant, and Eq. (18.7) reduces to the ideal-gas equation. As an example, for carbon dioxide gas (CO_2) the constants in the van der Waals equation are $a = 0.364 \text{ J} \cdot \text{m}^3/\text{mol}^2$ and $b = 4.27 \times 10^{-5} \text{ m}^3/\text{mol}$. We found in Example 18.1 that 1 mole of an ideal gas at $T = 0^\circ\text{C} = 273.15 \text{ K}$ and $p = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ occupies a volume $V = 0.0224 \text{ m}^3$; according to Eq. (18.7),

(a) An idealized model of a gas



(b) A more realistic model of a gas



18.5 A gas as modeled by (a) the ideal-gas equation and (b) the van der Waals equation.

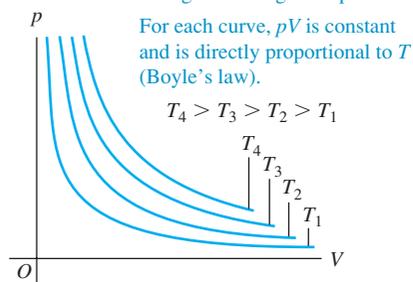
1 mole of CO_2 occupying this volume at this temperature would be at a pressure 532 Pa less than 1 atm, a difference of only 0.5% from the ideal-gas value.

pV -Diagrams

We could in principle represent the p - V - T relationship graphically as a *surface* in a three-dimensional space with coordinates p , V , and T . This representation sometimes helps us grasp the overall behavior of the substance, but ordinary two-dimensional graphs are usually more convenient. One of the most useful of these is a set of graphs of pressure as a function of volume, each for a particular constant temperature. Such a diagram is called a **pV -diagram**. Each curve, representing behavior at a specific temperature, is called an **isotherm**, or a pV -*isotherm*.

18.6 Isotherms, or constant-temperature curves, for a constant amount of an ideal gas. The highest temperature is T_4 ; the lowest is T_1 . This is a graphical representation of the ideal-gas equation of state.

Each curve represents pressure as a function of volume for an ideal gas at a single temperature.



18.7 A pV -diagram for a nonideal gas, showing isotherms for temperatures above and below the critical temperature T_c . The liquid-vapor equilibrium region is shown as a green shaded area. At still lower temperatures the material might undergo phase transitions from liquid to solid or from gas to solid; these are not shown in this diagram.

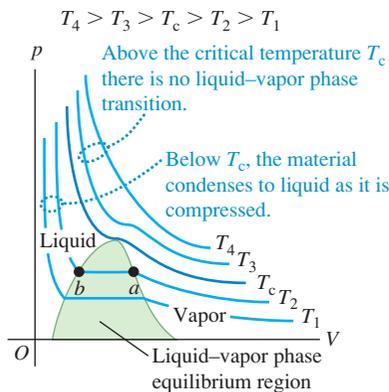


Figure 18.6 shows pV -isotherms for a constant amount of an ideal gas. Since $p = nRT/V$ from Eq. (18.3), along an isotherm (constant T) the pressure p is inversely proportional to the volume V and the isotherms are hyperbolic curves.

Figure 18.7 shows a pV -diagram for a material that *does not* obey the ideal-gas equation. At temperatures below T_c the isotherms develop flat regions in which we can compress the material (that is, reduce the volume V) without increasing the pressure p . Observation shows that the gas is *condensing* from the vapor (gas) to the liquid phase. The flat parts of the isotherms in the shaded area of Fig. 18.7 represent conditions of liquid-vapor *phase equilibrium*. As the volume decreases, more and more material goes from vapor to liquid, but the pressure does not change. (To keep the temperature constant during condensation, we have to remove the heat of vaporization, discussed in Section 17.6.)

When we compress such a gas at a constant temperature T_2 in Fig. 18.7, it is vapor until point *a* is reached. Then it begins to liquefy; as the volume decreases further, more material liquefies, and *both* the pressure and the temperature remain constant. At point *b*, all the material is in the liquid state. After this, any further compression requires a very rapid rise of pressure, because liquids are in general much less compressible than gases. At a lower constant temperature T_1 , similar behavior occurs, but the condensation begins at lower pressure and greater volume than at the constant temperature T_2 . At temperatures greater than T_c , *no* phase transition occurs as the material is compressed; at the highest temperatures, such as T_4 , the curves resemble the ideal-gas curves of Fig. 18.6. We call T_c the *critical temperature* for this material. In Section 18.6 we'll discuss what happens to the phase of the gas above the critical temperature.

We will use pV -diagrams often in the next two chapters. We will show that the *area* under a pV -curve (whether or not it is an isotherm) represents the *work* done by the system during a volume change. This work, in turn, is directly related to heat transfer and changes in the *internal energy* of the system.

Test Your Understanding of Section 18.1 Rank the following ideal gases in order from highest to lowest number of moles: (i) pressure 1 atm, volume 1 L, and temperature 300 K; (ii) pressure 2 atm, volume 1 L, and temperature 300 K; (iii) pressure 1 atm, volume 2 L, and temperature 300 K; (iv) pressure 1 atm, volume 1 L, and temperature 600 K; (v) pressure 2 atm, volume 1 L, and temperature 600 K. MP

18.2 Molecular Properties of Matter

We have studied several properties of matter in bulk, including elasticity, density, surface tension, heat capacities, and equations of state. Now we want to look in more detail at the relationship of bulk behavior to *molecular* structure. We begin with a general discussion of the molecular structure of matter. Then in the next two sections we develop the kinetic-molecular model of an ideal gas, obtaining from this molecular model the equation of state and an expression for heat capacity.

Molecules and Intermolecular Forces

Any specific chemical compound is made up of identical **molecules**. The smallest molecules contain one atom each and are of the order of 10^{-10} m in size; the largest contain many atoms and are at least 10,000 times larger. In gases the molecules move nearly independently; in liquids and solids they are held together by intermolecular forces. These forces arise from interactions among the electrically charged particles that make up the molecules. Gravitational forces between molecules are negligible in comparison with electrical forces.

The interaction of two *point* electric charges is described by a force (repulsive for like charges, attractive for unlike charges) with a magnitude proportional to $1/r^2$, where r is the distance between the points. We will study this relationship, called *Coulomb's law*, in Chapter 21. Molecules are *not* point charges but complex structures containing both positive and negative charge, and their interactions are more complex. The force between molecules in a gas varies with the distance r between molecules somewhat as shown in Fig. 18.8, where a positive F_r corresponds to a repulsive force and a negative F_r to an attractive force. When molecules are far apart, the intermolecular forces are very small and usually attractive. As a gas is compressed and its molecules are brought closer together, the attractive forces increase. The intermolecular force becomes zero at an equilibrium spacing r_0 , corresponding roughly to the spacing between molecules in the liquid and solid states. In liquids and solids, relatively large pressures are needed to compress the substance appreciably. This shows that at molecular distances slightly *less* than the equilibrium spacing, the forces become *repulsive* and relatively large.

Figure 18.8 also shows the potential energy as a function of r . This function has a *minimum* at r_0 , where the force is zero. The two curves are related by $F_r(r) = -dU/dr$, as we showed in Section 7.4. Such a potential-energy function is often called a **potential well**. A molecule at rest at a distance r_0 from a second molecule would need an additional energy $|U_0|$, the “depth” of the potential well, to “escape” to an indefinitely large value of r .

Molecules are always in motion; their kinetic energies usually increase with temperature. At very low temperatures the average kinetic energy of a molecule may be much *less* than the depth of the potential well. The molecules then condense into the liquid or solid phase with average intermolecular spacings of about r_0 . But at higher temperatures the average kinetic energy becomes larger than the depth $|U_0|$ of the potential well. Molecules can then escape the intermolecular force and become free to move independently, as in the gaseous phase of matter.

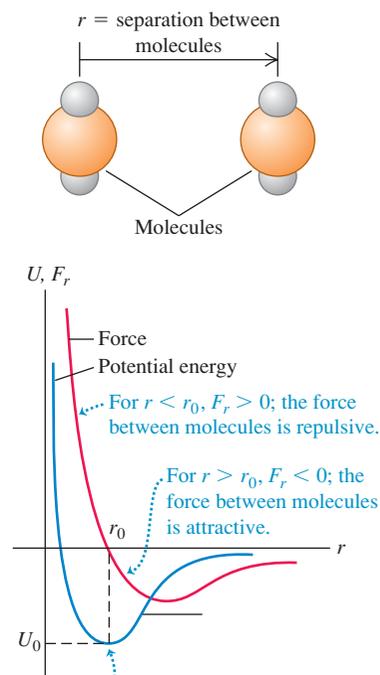
In *solids*, molecules vibrate about more or less fixed points. In a crystalline solid these points are arranged in a *crystal lattice*. Figure 18.9 shows the cubic crystal structure of sodium chloride, and Fig. 18.10 shows a scanning tunneling microscope image of individual silicon atoms on the surface of a crystal.

The vibration of molecules in a solid about their equilibrium positions may be nearly simple harmonic if the potential well is approximately parabolic in shape at distances close to r_0 . (We discussed this kind of simple harmonic motion in Section 14.4.) But if the potential-energy curve rises more gradually for $r > r_0$ than for $r < r_0$, as in Fig. 18.8, the average position shifts to larger r with increasing amplitude. As we pointed out in Section 17.4, this is the basis of thermal expansion.

In a *liquid*, the intermolecular distances are usually only slightly greater than in the solid phase of the same substance, but the molecules have much greater freedom of movement. Liquids show regularity of structure only in the immediate neighborhood of a few molecules.

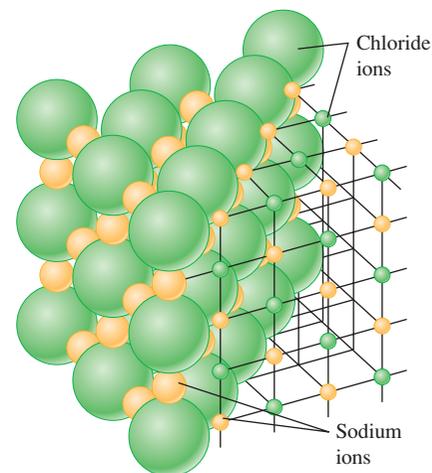
The molecules of a *gas* are usually widely separated and so have only very small attractive forces. A gas molecule moves in a straight line until it collides with another molecule or with a wall of the container. In molecular terms, an *ideal gas* is a gas whose molecules exert *no* attractive forces on each other (see Fig. 18.5a) and therefore have no *potential* energy.

18.8 How the force between molecules and their potential energy of interaction depend on their separation r .

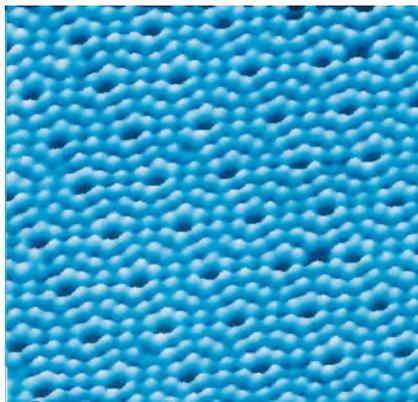


At a separation $r = r_0$, the potential energy of the two molecules is minimum and the force between the molecules is zero.

18.9 Schematic representation of the cubic crystal structure of sodium chloride (ordinary salt).



18.10 A scanning tunneling microscope image of the surface of a silicon crystal. The area shown is only 9.0 nm (9.0×10^{-9} m) across. Each blue “bead” is an individual silicon atom; you can clearly see how these atoms are arranged in a (nearly) perfect array of hexagons.



At low temperatures, most common substances are in the solid phase. As the temperature rises, a substance melts and then vaporizes. From a molecular point of view, these transitions are in the direction of increasing molecular kinetic energy. Thus temperature and molecular kinetic energy are closely related.

Moles and Avogadro's Number

We have used the mole as a measure of quantity of substance. One **mole** of any pure chemical element or compound contains a definite number of molecules, the same number for all elements and compounds. The official SI definition is:

One mole is the amount of substance that contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12.

In our discussion, the “elementary entities” are molecules. (In a monatomic substance such as carbon or helium, each molecule is a single atom.) Atoms of a given element may occur in any of several isotopes, which are chemically identical but have different atomic masses; “carbon-12” is a specific isotope of carbon.

The number of molecules in a mole is called **Avogadro's number**, denoted by N_A . The current best numerical value of N_A is

$$N_A = 6.02214179(30) \times 10^{23} \text{ molecules/mol (Avogadro's number)}$$

The *molar mass* M of a compound is the mass of 1 mole. It is equal to the mass m of a single molecule multiplied by Avogadro's number:

$$M = N_A m \quad (\text{molar mass, Avogadro's number, and mass of a molecule}) \quad (18.8)$$

When the molecule consists of a single atom, the term *atomic mass* is often used instead of molar mass or molecular weight.

Example 18.5 Atomic and molecular mass

Find the mass of a single hydrogen atom and of a single oxygen molecule.

SOLUTION

IDENTIFY and SET UP: This problem involves the relationship between the mass of a molecule or atom (our target variable) and the corresponding molar mass M . We use Eq. (18.8) in the form $m = M/N_A$ and the values of the atomic masses from the periodic table of the elements (see Appendix D).

EXECUTE: For atomic hydrogen the atomic mass (molar mass) is $M_H = 1.008$ g/mol, so the mass m_H of a single hydrogen atom is

$$m_H = \frac{1.008 \text{ g/mol}}{6.022 \times 10^{23} \text{ atoms/mol}} = 1.674 \times 10^{-24} \text{ g/atom}$$

For oxygen the atomic mass is 16.0 g/mol, so for the diatomic (two-atom) oxygen molecule the molar mass is 32.0 g/mol. Then the mass of a single oxygen molecule is

$$m_{O_2} = \frac{32.0 \text{ g/mol}}{6.022 \times 10^{23} \text{ molecules/mol}} = 53.1 \times 10^{-24} \text{ g/molecule}$$

EVALUATE: We note that the values in Appendix D are for the *average* atomic masses of a natural sample of each element. Such a sample may contain several *isotopes* of the element, each with a different atomic mass. Natural samples of hydrogen and oxygen are almost entirely made up of just one isotope.

Test Your Understanding of Section 18.2 Suppose you could adjust the value of r_0 for the molecules of a certain chemical compound (Fig. 18.8) by turning a dial. If you doubled the value of r_0 , the density of the solid form of this compound would become (i) twice as great; (ii) four times as great; (iii) eight times as great; (iv) $\frac{1}{2}$ as great; (v) $\frac{1}{4}$ as great; (vi) $\frac{1}{8}$ as great.



18.3 Kinetic-Molecular Model of an Ideal Gas

The goal of any molecular theory of matter is to understand the *macroscopic* properties of matter in terms of its atomic or molecular structure and behavior. Once we have this understanding, we can design materials to have specific desired properties. Theories have led to the development of high-strength steels, semiconductor materials for electronic devices, and countless other materials essential to contemporary technology.

In this and the following sections we will consider a simple molecular model of an ideal gas. This *kinetic-molecular model* represents the gas as a large number of particles bouncing around in a closed container. In this section we use the kinetic-molecular model to understand how the ideal-gas equation of state, Eq. (18.3), is related to Newton's laws. In the following section we use the kinetic-molecular model to predict the molar heat capacity of an ideal gas. We'll go on to elaborate the model to include "particles" that are not points but have a finite size.

Our discussion of the kinetic-molecular model has several steps, and you may need to go over them several times. Don't get discouraged!

Here are the assumptions of our model:

1. A container with volume V contains a very large number N of identical molecules, each with mass m .
2. The molecules behave as point particles that are small compared to the size of the container and to the average distance between molecules.
3. The molecules are in constant motion. Each molecule collides occasionally with a wall of the container. These collisions are perfectly elastic.
4. The container walls are rigid and infinitely massive and do not move.

CAUTION **Molecules vs. moles** Make sure you don't confuse N , the number of molecules in the gas, with n , the number of moles. The number of molecules is equal to the number of moles multiplied by Avogadro's number: $N = nN_A$.

Collisions and Gas Pressure

During collisions the molecules exert *forces* on the walls of the container; this is the origin of the *pressure* that the gas exerts. In a typical collision (Fig. 18.11) the velocity component parallel to the wall is unchanged, and the component perpendicular to the wall reverses direction but does not change in magnitude.

Our program is first to determine the *number* of collisions that occur per unit time for a certain area A of wall. Then we find the total momentum change associated with these collisions and the force needed to cause this momentum change. From this we can determine the pressure, which is force per unit area, and compare the result to the ideal-gas equation. We'll find a direct connection between the temperature of the gas and the kinetic energy of the gas molecules.

To begin, we will assume that all molecules in the gas have the same *magnitude* of x -velocity, $|v_x|$. This isn't right, but making this temporary assumption helps to clarify the basic ideas. We will show later that this assumption isn't really necessary.

As shown in Fig. 18.11, for each collision the x -component of velocity changes from $-|v_x|$ to $+|v_x|$. So the x -component of momentum changes from $-m|v_x|$ to $+m|v_x|$, and the *change* in the x -component of momentum is $m|v_x| - (-m|v_x|) = 2m|v_x|$.

If a molecule is going to collide with a given wall area A during a small time interval dt , then at the beginning of dt it must be within a distance $|v_x| dt$ from the wall (Fig. 18.12) and it must be headed toward the wall. So the number of molecules that collide with A during dt is equal to the number of molecules within a cylinder with base area A and length $|v_x| dt$ that have their x -velocity aimed

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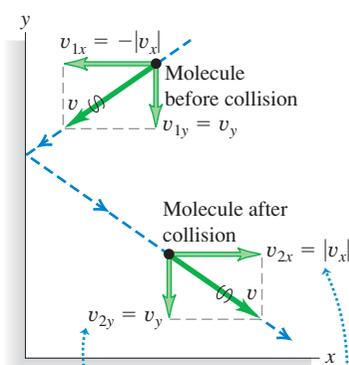
PhET: Balloons & Buoyancy

PhET: Friction

PhET: Gas Properties

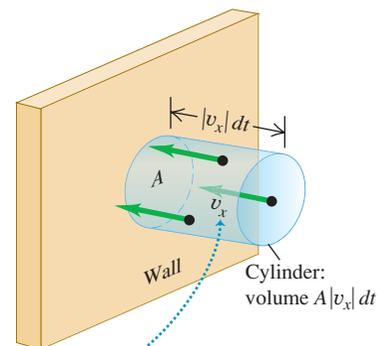
ActivPhysics 8.1: Characteristics of a Gas

18.11 Elastic collision of a molecule with an idealized container wall.



- Velocity component parallel to the wall (y -component) does not change.
- Velocity component perpendicular to the wall (x -component) reverses direction.
- Speed v does not change.

18.12 For a molecule to strike the wall in area A during a time interval dt , the molecule must be headed for the wall and be within the shaded cylinder of length $|v_x| dt$ at the beginning of the interval.



All molecules are assumed to have the same magnitude $|v_x|$ of x -velocity.

toward the wall. The volume of such a cylinder is $A|v_x| dt$. Assuming that the number of molecules per unit volume (N/V) is uniform, the *number* of molecules in this cylinder is $(N/V)(A|v_x| dt)$. On the average, half of these molecules are moving toward the wall and half are moving away from it. So the number of collisions with A during dt is

$$\frac{1}{2} \left(\frac{N}{V} \right) (A|v_x| dt)$$

For the system of all molecules in the gas, the total momentum change dP_x during dt is the *number* of collisions multiplied by $2m|v_x|$:

$$dP_x = \frac{1}{2} \left(\frac{N}{V} \right) (A|v_x| dt) (2m|v_x|) = \frac{NAmv_x^2 dt}{V} \quad (18.9)$$

(We are using capital P for total momentum and small p for pressure. Be careful!) We wrote v_x^2 rather than $|v_x|^2$ in the final expression because the square of the absolute value of a number is equal to the square of that number. The *rate* of change of momentum component P_x is

$$\frac{dP_x}{dt} = \frac{NAmv_x^2}{V} \quad (18.10)$$

According to Newton's second law, this rate of change of momentum equals the force exerted by the wall area A on the gas molecules. From Newton's *third* law this is equal and opposite to the force exerted *on* the wall *by* the molecules. Pressure p is the magnitude of the force exerted on the wall per unit area, and we obtain

$$p = \frac{F}{A} = \frac{Nmv_x^2}{V} \quad (18.11)$$

The pressure exerted by the gas depends on the number of molecules per volume (N/V), the mass m per molecule, and the speed of the molecules.

Pressure and Molecular Kinetic Energies

We mentioned that $|v_x|$ is really *not* the same for all the molecules. But we could have sorted the molecules into groups having the same $|v_x|$ within each group, then added up the resulting contributions to the pressure. The net effect of all this is just to replace v_x^2 in Eq. (18.11) by the *average* value of v_x^2 , which we denote by $(v_x^2)_{\text{av}}$. We can relate $(v_x^2)_{\text{av}}$ to the *speeds* of the molecules. The speed v of a molecule is related to the velocity components v_x , v_y , and v_z by

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

We can average this relation over all molecules:

$$(v^2)_{\text{av}} = (v_x^2)_{\text{av}} + (v_y^2)_{\text{av}} + (v_z^2)_{\text{av}}$$

But there is no real difference in our model between the x -, y -, and z -directions. (Molecular speeds are very fast in a typical gas, so the effects of gravity are negligibly small.) It follows that $(v_x^2)_{\text{av}}$, $(v_y^2)_{\text{av}}$, and $(v_z^2)_{\text{av}}$ must all be *equal*. Hence $(v^2)_{\text{av}}$ is equal to $3(v_x^2)_{\text{av}}$ and

$$(v_x^2)_{\text{av}} = \frac{1}{3}(v^2)_{\text{av}}$$

so Eq. (18.11) becomes

$$pV = \frac{1}{3}Nm(v^2)_{\text{av}} = \frac{1}{3}N\left[\frac{1}{2}m(v^2)_{\text{av}}\right] \quad (18.12)$$

We notice that $\frac{1}{2}m(v^2)_{\text{av}}$ is the average translational kinetic energy of a single molecule. The product of this and the total number of molecules N equals the

total random kinetic energy K_{tr} of translational motion of all the molecules. (The notation K_{tr} reminds us that this is the energy of *translational* motion. There may also be energies associated with molecular rotation and vibration.) The product pV equals two-thirds of the total translational kinetic energy:

$$pV = \frac{2}{3}K_{\text{tr}} \quad (18.13)$$

Now we compare this with the ideal-gas equation,

$$pV = nRT$$

which is based on experimental studies of gas behavior. For the two equations to agree, we must have

$$K_{\text{tr}} = \frac{3}{2}nRT \quad (\text{average translational kinetic energy of } n \text{ moles of ideal gas}) \quad (18.14)$$

This remarkably simple result shows that K_{tr} is *directly proportional* to the absolute temperature T (Fig. 18.13).

The average translational kinetic energy of a single molecule is the total translational kinetic energy K_{tr} of all molecules divided by the number of molecules, N :

$$\frac{K_{\text{tr}}}{N} = \frac{1}{2}m(v^2)_{\text{av}} = \frac{3nRT}{2N}$$

Also, the total number of molecules N is the number of moles n multiplied by Avogadro's number N_A , so

$$N = nN_A \quad \frac{n}{N} = \frac{1}{N_A}$$

and

$$\frac{K_{\text{tr}}}{N} = \frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}\left(\frac{R}{N_A}\right)T \quad (18.15)$$

The ratio R/N_A occurs frequently in molecular theory. It is called the **Boltzmann constant**, k :

$$\begin{aligned} k &= \frac{R}{N_A} = \frac{8.314 \text{ J/mol} \cdot \text{K}}{6.022 \times 10^{23} \text{ molecules/mol}} \\ &= 1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K} \end{aligned}$$

(The current best numerical value of k is $1.3806504(24) \times 10^{-23}$ J/molecule \cdot K). In terms of k we can rewrite Eq. (18.15) as

$$\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT \quad (\text{average translational kinetic energy of a gas molecule}) \quad (18.16)$$

This shows that the average translational kinetic energy *per molecule* depends only on the temperature, not on the pressure, volume, or kind of molecule. We can obtain the average translational kinetic energy *per mole* by multiplying Eq. (18.16) by Avogadro's number and using the relation $M = N_A m$:

$$N_A \frac{1}{2}m(v^2)_{\text{av}} = \frac{1}{2}M(v^2)_{\text{av}} = \frac{3}{2}RT \quad (\text{average translational kinetic energy per mole of gas}) \quad (18.17)$$

The translational kinetic energy of a mole of an ideal gas depends only on T .

18.13 Summer air (top) is warmer than winter air (bottom); that is, the average translational kinetic energy of air molecules is greater in summer.



Finally, it is sometimes convenient to rewrite the ideal-gas equation on a molecular basis. We use $N = N_A n$ and $R = N_A k$ to obtain this alternative form:

$$pV = NkT \quad (18.18)$$

This shows that we can think of the Boltzmann constant k as a gas constant on a “per-molecule” basis instead of the usual “per-mole” basis for R .

Molecular Speeds

From Eqs. (18.16) and (18.17) we can obtain expressions for the square root of $(v^2)_{\text{av}}$, called the **root-mean-square speed** (or **rms speed**) v_{rms} :

$$v_{\text{rms}} = \sqrt{(v^2)_{\text{av}}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} \quad \begin{array}{l} \text{(root-mean-square speed} \\ \text{of a gas molecule)} \end{array} \quad (18.19)$$

It might seem more natural to characterize molecular speeds by their *average* value rather than by v_{rms} , but we see that v_{rms} follows more directly from Eqs. (18.16) and (18.17). To compute the rms speed, we square each molecular speed, add, divide by the number of molecules, and take the square root; v_{rms} is the *root* of the *mean* of the *squares*. Example 18.7 illustrates this procedure.

Equations (18.16) and (18.19) show that at a given temperature T , gas molecules of different mass m have the same average kinetic energy but different root-mean-square speeds. On average, the nitrogen molecules ($M = 28 \text{ g/mol}$) in the air around you are moving faster than are the oxygen molecules ($M = 32 \text{ g/mol}$). Hydrogen molecules ($M = 2 \text{ g/mol}$) are fastest of all; this is why there is hardly any hydrogen in the earth’s atmosphere, despite its being the most common element in the universe (Fig. 18.14). A sizable fraction of any H_2 molecules in the atmosphere would have speeds greater than the earth’s escape speed of $1.12 \times 10^4 \text{ m/s}$ (calculated in Example 13.5 in Section 13.3) and would escape into space. The heavier, slower-moving gases cannot escape so easily, which is why they predominate in our atmosphere.

The assumption that individual molecules undergo perfectly elastic collisions with the container wall is actually a little too simple. More detailed investigation has shown that in most cases, molecules actually adhere to the wall for a short time and then leave again with speeds that are characteristic of the temperature of the wall. However, the gas and the wall are ordinarily in thermal equilibrium and have the same temperature. So there is no net energy transfer between gas and wall, and this discovery does not alter the validity of our conclusions.

18.14 While hydrogen is a desirable fuel for vehicles, it is only a trace constituent of our atmosphere (0.00005% by volume). Hence hydrogen fuel has to be generated by electrolysis of water, which is itself an energy-intensive process.



Problem-Solving Strategy 18.2 Kinetic-Molecular Theory



IDENTIFY the relevant concepts: Use the results of the kinetic-molecular model to relate the macroscopic properties of a gas, such as temperature and pressure, to microscopic properties, such as molecular speeds.

SET UP the problem using the following steps:

1. List knowns and unknowns; identify the target variables.
2. Choose appropriate equation(s) from among Eqs. (18.14), (18.16), and (18.19).

EXECUTE the solution as follows: Maintain consistency in units. Note especially the following:

1. The usual units for molar mass M are grams per mole; these units are often omitted in tables. In equations such as Eq. (18.19), when you use SI units you must express M in kilograms per

mole. For example, for oxygen $M_{\text{O}_2} = 32 \text{ g/mol} = 32 \times 10^{-3} \text{ kg/mol}$.

2. Are you working on a “per-molecule” basis (with m , N , and k) or a “per-mole” basis (with M , n , and R)? To check units, think of N as having units of “molecules”; then m has units of mass per molecule, and k has units of joules per molecule per kelvin. Similarly, n has units of moles; then M has units of mass per mole and R has units of joules per mole per kelvin.
3. Remember that T is always *absolute* (Kelvin) temperature.

EVALUATE your answer: Are your answers reasonable? Here’s a benchmark: Typical molecular speeds at room temperature are several hundred meters per second.

Example 18.6 Molecular kinetic energy and v_{rms}

(a) What is the average translational kinetic energy of an ideal-gas molecule at 27°C ? (b) What is the total random translational kinetic energy of the molecules in 1 mole of this gas? (c) What is the root-mean-square speed of oxygen molecules at this temperature?

SOLUTION

IDENTIFY and SET UP: This problem involves the translational kinetic energy of an ideal gas on a per-molecule and per-mole basis, as well as the root-mean-square molecular speed v_{rms} . We are given $T = 27^\circ\text{C} = 300\text{ K}$ and $n = 1\text{ mol}$; we use the molecular mass m for oxygen. We use Eq. (18.16) to determine the average kinetic energy of a molecule, Eq. (18.14) to find the total molecular kinetic energy K_{tr} of 1 mole, and Eq. (18.19) to find v_{rms} .

EXECUTE: (a) From Eq. (18.16),

$$\begin{aligned}\frac{1}{2}m(v^2)_{\text{av}} &= \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23}\text{ J/K})(300\text{ K}) \\ &= 6.21 \times 10^{-21}\text{ J}\end{aligned}$$

(b) From Eq. (18.14), the kinetic energy of one mole is

$$K_{\text{tr}} = \frac{3}{2}nRT = \frac{3}{2}(1\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(300\text{ K}) = 3740\text{ J}$$

(c) We found the mass per molecule m and molar mass M of molecular oxygen in Example 18.5. Using Eq. (18.19), we can calculate v_{rms} in two ways:

$$\begin{aligned}v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23}\text{ J/K})(300\text{ K})}{5.31 \times 10^{-26}\text{ kg}}} \\ &= 484\text{ m/s} = 1740\text{ km/h} = 1080\text{ mi/h} \\ v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}{32.0 \times 10^{-3}\text{ kg/mol}}} = 484\text{ m/s}\end{aligned}$$

EVALUATE: The answer in part (a) does not depend on the mass of the molecule. We can check our result in part (b) by noting that the translational kinetic energy per mole must be equal to the product of the average translational kinetic energy per molecule from part (a) and Avogadro's number N_A : $K_{\text{tr}} = (6.022 \times 10^{23}\text{ molecules})(6.21 \times 10^{-21}\text{ J/molecule}) = 3740\text{ J}$.

Example 18.7 Calculating rms and average speeds

Five gas molecules chosen at random are found to have speeds of 500, 600, 700, 800, and 900 m/s. What is the rms speed? What is the average speed?

SOLUTION

IDENTIFY and SET UP: We use the definitions of the root mean square and the average of a collection of numbers. To find v_{rms} , we square each speed, find the average (mean) of the squares, and take the square root of the result. We find v_{av} as usual.

EXECUTE: The average value of v^2 and the resulting v_{rms} for the five molecules are

$$\begin{aligned}(v^2)_{\text{av}} &= \frac{500^2 + 600^2 + 700^2 + 800^2 + 900^2}{5}\text{ m}^2/\text{s}^2 \\ &= 5.10 \times 10^5\text{ m}^2/\text{s}^2\end{aligned}$$

$$v_{\text{rms}} = \sqrt{(v^2)_{\text{av}}} = 714\text{ m/s}$$

The average speed v_{av} is

$$v_{\text{av}} = \frac{500 + 600 + 700 + 800 + 900}{5}\text{ m/s} = 700\text{ m/s}$$

EVALUATE: In general v_{rms} and v_{av} are *not* the same. Roughly speaking, v_{rms} gives greater weight to the higher speeds than does v_{av} .

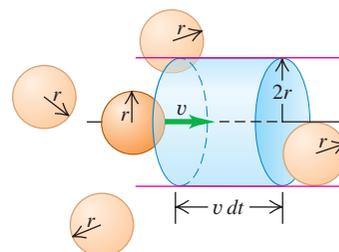
Collisions Between Molecules

We have ignored the possibility that two gas molecules might collide. If they are really points, they *never* collide. But consider a more realistic model in which the molecules are rigid spheres with radius r . How often do they collide with other molecules? How far do they travel, on average, between collisions? We can get approximate answers from the following rather primitive model.

Consider N spherical molecules with radius r in a volume V . Suppose only one molecule is moving. When it collides with another molecule, the distance between centers is $2r$. Suppose we draw a cylinder with radius $2r$, with its axis parallel to the velocity of the molecule (Fig. 18.15). The moving molecule collides with any other molecule whose center is inside this cylinder. In a short time dt a molecule with speed v travels a distance $v dt$; during this time it collides with any molecule that is in the cylindrical volume of radius $2r$ and length $v dt$. The volume of the cylinder is $4\pi r^2 v dt$. There are N/V molecules per unit volume, so the number dN with centers in this cylinder is

$$dN = 4\pi r^2 v dt N/V$$

18.15 In a time dt a molecule with radius r will collide with any other molecule within a cylindrical volume of radius $2r$ and length $v dt$.



Thus the number of collisions *per unit time* is

$$\frac{dN}{dt} = \frac{4\pi r^2 v N}{V}$$

This result assumes that only one molecule is moving. The analysis is quite a bit more involved when all the molecules move at once. It turns out that in this case the collisions are more frequent, and the above equation has to be multiplied by a factor of $\sqrt{2}$:

$$\frac{dN}{dt} = \frac{4\pi \sqrt{2} r^2 v N}{V}$$

The average time t_{mean} between collisions, called the *mean free time*, is the reciprocal of this expression:

$$t_{\text{mean}} = \frac{V}{4\pi \sqrt{2} r^2 v N} \quad (18.20)$$

The average distance traveled between collisions is called the **mean free path**, denoted by λ (the Greek letter lambda). In our simple model, this is just the molecule's speed v multiplied by t_{mean} :

$$\lambda = v t_{\text{mean}} = \frac{V}{4\pi \sqrt{2} r^2 N} \quad (\text{mean free path of a gas molecule}) \quad (18.21)$$

18.16 If you try to walk through a crowd, your mean free path—the distance you can travel on average without running into another person—depends on how large the people are and how closely they are spaced.



The mean free path is inversely proportional to the number of molecules per unit volume (N/V) and inversely proportional to the cross-sectional area πr^2 of a molecule; the more molecules there are and the larger the molecule, the shorter the mean distance between collisions (Fig. 18.16). Note that the mean free path *does not* depend on the speed of the molecule.

We can express Eq. (18.21) in terms of macroscopic properties of the gas, using the ideal-gas equation in the form of Eq. (18.18), $pV = NkT$. We find

$$\lambda = \frac{kT}{4\pi \sqrt{2} r^2 p} \quad (18.22)$$

If the temperature is increased at constant pressure, the gas expands, the average distance between molecules increases, and λ increases. If the pressure is increased at constant temperature, the gas compresses and λ decreases.

Example 18.8 Calculating mean free path

(a) Estimate the mean free path of a molecule of air at 27°C and 1 atm. Model the molecules as spheres with radius $r = 2.0 \times 10^{-10}$ m. (b) Estimate the mean free time of an oxygen molecule with $v = v_{\text{rms}}$ at 27°C and 1 atm.

SOLUTION

IDENTIFY and SET UP: This problem uses the concepts of mean free path and mean free time (our target variables). We use Eq. (18.22) to determine the mean free path λ . We then use the basic relationship $\lambda = v t_{\text{mean}}$ in Eq. (18.21), with $v = v_{\text{rms}}$, to find the mean free time t_{mean} .

EXECUTE: (a) From Eq. (18.22),

$$\begin{aligned} \lambda &= \frac{kT}{4\pi \sqrt{2} r^2 p} = \frac{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{4\pi \sqrt{2} (2.0 \times 10^{-10} \text{ m})^2 (1.01 \times 10^5 \text{ Pa})} \\ &= 5.8 \times 10^{-8} \text{ m} \end{aligned}$$

(b) From Example 18.6, for oxygen at 27°C the root-mean-square speed is $v_{\text{rms}} = 484$ m/s, so the mean free time for a molecule with this speed is

$$t_{\text{mean}} = \frac{\lambda}{v} = \frac{5.8 \times 10^{-8} \text{ m}}{484 \text{ m/s}} = 1.2 \times 10^{-10} \text{ s}$$

This molecule undergoes about 10^{10} collisions per second!

EVALUATE: Note that from Eqs. (18.21) and (18.22) the mean free path doesn't depend on the molecule's speed, but the mean free time does. Slower molecules have a longer average time interval t_{mean} between collisions than do fast ones, but the average distance λ between collisions is the same no matter what the molecule's speed. Our answer to part (a) says that the molecule doesn't go far between collisions, but the mean free path is still several hundred times the molecular radius r .

Test Your Understanding of Section 18.3 Rank the following gases in order from (a) highest to lowest rms speed of molecules and (b) highest to lowest average translational kinetic energy of a molecule: (i) oxygen ($M = 32.0$ g/mol) at 300 K; (ii) nitrogen ($M = 28.0$ g/mol) at 300 K; (iii) oxygen at 330 K; (iv) nitrogen at 330 K.



18.4 Heat Capacities

When we introduced the concept of heat capacity in Section 17.5, we talked about ways to *measure* the specific heat or molar heat capacity of a particular material. Now we'll see how to *predict* these on theoretical grounds.

Heat Capacities of Gases

The basis of our analysis is that heat is *energy* in transit. When we add heat to a substance, we are increasing its molecular energy. In this discussion the volume of the gas will remain constant so that we don't have to worry about energy transfer through mechanical work. If we were to let the gas expand, it would do work by pushing on moving walls of its container, and this additional energy transfer would have to be included in our calculations. We'll return to this more general case in Chapter 19. For now, with the volume held constant, we are concerned with C_V , the molar heat capacity *at constant volume*.

In the simple kinetic-molecular model of Section 18.3 the molecular energy consists only of the translational kinetic energy K_{tr} of the pointlike molecules. This energy is directly proportional to the absolute temperature T , as shown by Eq. (18.14), $K_{tr} = \frac{3}{2}nRT$. When the temperature changes by a small amount dT , the corresponding change in kinetic energy is

$$dK_{tr} = \frac{3}{2}nR dT \quad (18.23)$$

From the definition of molar heat capacity at constant volume, C_V (see Section 17.5), we also have

$$dQ = nC_V dT \quad (18.24)$$

where dQ is the heat input needed for a temperature change dT . Now if K_{tr} represents the total molecular energy, as we have assumed, then dQ and dK_{tr} must be *equal* (Fig. 18.17). From Eqs. (18.23) and (18.24), this says

$$nC_V dT = \frac{3}{2}nR dT$$

$$C_V = \frac{3}{2}R \quad (\text{ideal gas of point particles}) \quad (18.25)$$

This surprisingly simple result says that the molar heat capacity at constant volume is $3R/2$ for *any* gas whose molecules can be represented as points.

Does Eq. (18.25) agree with measured values of molar heat capacities? In SI units, Eq. (18.25) gives

$$C_V = \frac{3}{2}(8.314 \text{ J/mol} \cdot \text{K}) = 12.47 \text{ J/mol} \cdot \text{K}$$

For comparison, Table 18.1 gives measured values of C_V for several gases. We see that for *monatomic* gases our prediction is right on the money, but that it is way off for diatomic and polyatomic gases.

This comparison tells us that our point-molecule model is good enough for monatomic gases but that for diatomic and polyatomic molecules we need something more sophisticated. For example, we can picture a diatomic molecule as

18.17 (a) A fixed volume V of a monatomic ideal gas. (b) When an amount of heat dQ is added to the gas, the total translational kinetic energy increases by $dK_{tr} = dQ$ and the temperature increases by $dT = dQ/nC_V$.

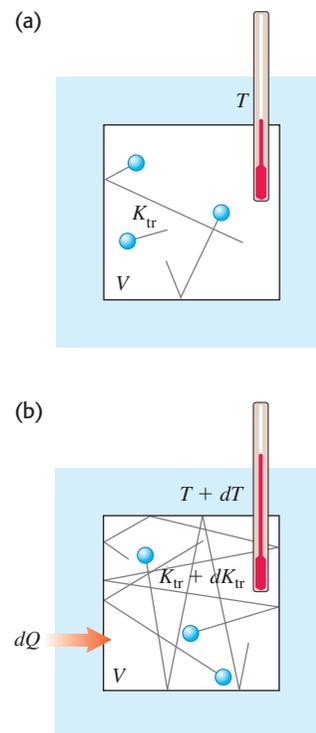
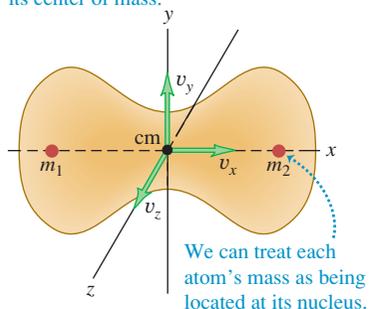


Table 18.1 Molar Heat Capacities of Gases

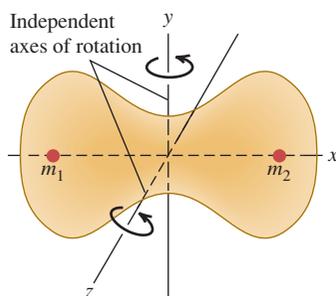
Type of Gas	Gas	C_V (J/mol · K)
Monatomic	He	12.47
	Ar	12.47
Diatomic	H ₂	20.42
	N ₂	20.76
	O ₂	20.85
	CO	20.85
Polyatomic	CO ₂	28.46
	SO ₂	31.39
	H ₂ S	25.95

18.18 Motions of a diatomic molecule.

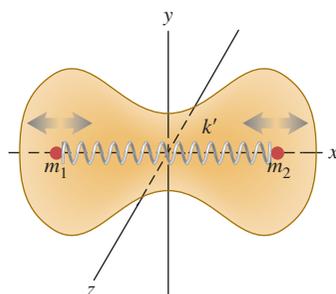
(a) **Translational motion.** The molecule moves as a whole; its velocity may be described as the x -, y -, and z -velocity components of its center of mass.



(b) **Rotational motion.** The molecule rotates about its center of mass. This molecule has two independent axes of rotation.



(c) **Vibrational motion.** The molecule oscillates as though the nuclei were connected by a spring.



two point masses, like a little elastic dumbbell, with an interaction force between the atoms of the kind shown in Fig. 18.8. Such a molecule can have additional kinetic energy associated with *rotation* about axes through its center of mass. The atoms may also have *vibrating* motion along the line joining them, with additional kinetic and potential energies. Figure 18.18 shows these possibilities.

When heat flows into a *monatomic* gas at constant volume, *all* of the added energy goes into an increase in random *translational* molecular kinetic energy. Equation (18.23) shows that this gives rise to an increase in temperature. But when the temperature is increased by the same amount in a *diatomic* or *polyatomic* gas, additional heat is needed to supply the increased rotational and vibrational energies. Thus polyatomic gases have *larger* molar heat capacities than monatomic gases, as Table 18.1 shows.

But how do we know how much energy is associated with each additional kind of motion of a complex molecule, compared to the translational kinetic energy? The new principle that we need is called the principle of **equipartition of energy**. It can be derived from sophisticated statistical-mechanics considerations; that derivation is beyond our scope, and we will treat the principle as an axiom.

The principle of equipartition of energy states that each velocity component (either linear or angular) has, on average, an associated kinetic energy per molecule of $\frac{1}{2}kT$, or one-half the product of the Boltzmann constant and the absolute temperature. The number of velocity components needed to describe the motion of a molecule completely is called the number of **degrees of freedom**. For a monatomic gas, there are three degrees of freedom (for the velocity components v_x , v_y , and v_z); this gives a total average kinetic energy per molecule of $3(\frac{1}{2}kT)$, consistent with Eq. (18.16).

For a *diatomic* molecule there are two possible axes of rotation, perpendicular to each other and to the molecule's axis. (We don't include rotation about the molecule's own axis because in ordinary collisions there is no way for this rotational motion to change.) If we assign five degrees of freedom to a diatomic molecule, the average total kinetic energy per molecule is $\frac{5}{2}kT$ instead of $\frac{3}{2}kT$. The total kinetic energy of n moles is $K_{\text{total}} = nN_A(\frac{5}{2}kT) = \frac{5}{2}n(kN_A)T = \frac{5}{2}nRT$, and the molar heat capacity (at constant volume) is

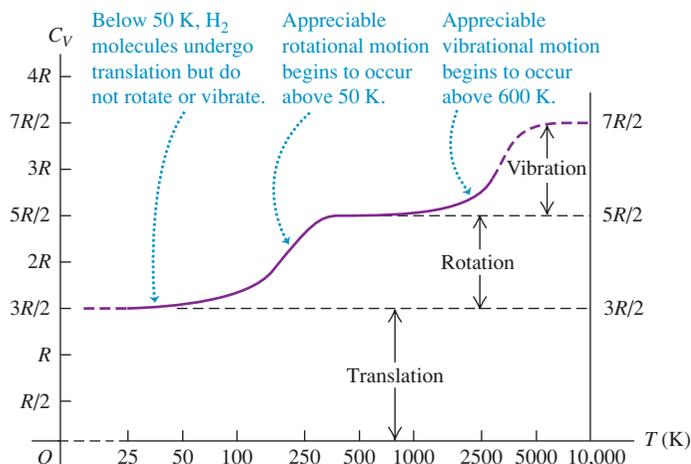
$$C_V = \frac{5}{2}R \quad (\text{diatomic gas, including rotation}) \quad (18.26)$$

In SI units,

$$C_V = \frac{5}{2}(8.314 \text{ J/mol} \cdot \text{K}) = 20.79 \text{ J/mol} \cdot \text{K}$$

This agrees within a few percent with the measured values for diatomic gases given in Table 18.1.

Vibrational motion can also contribute to the heat capacities of gases. Molecular bonds are not rigid; they can stretch and bend, and the resulting vibrations lead to additional degrees of freedom and additional energies. For most diatomic gases, however, vibrational motion does *not* contribute appreciably to heat capacity. The reason for this is a little subtle and involves some concepts of quantum mechanics. Briefly, vibrational energy can change only in finite steps. If the energy change of the first step is much larger than the energy possessed by most molecules, then nearly all the molecules remain in the minimum-energy state of motion. In that case, changing the temperature does not change their average vibrational energy appreciably, and the vibrational degrees of freedom are said to be "frozen out." In more complex molecules the gaps between permitted energy levels are sometimes much smaller, and then vibration *does* contribute to heat capacity. The rotational energy of a molecule also changes by finite steps, but they are usually much



18.19 Experimental values of C_V , the molar heat capacity at constant volume, for hydrogen gas (H_2). The temperature is plotted on a logarithmic scale.

smaller; the “freezing out” of rotational degrees of freedom occurs only in rare instances, such as for the hydrogen molecule below about 100 K.

In Table 18.1 the large values of C_V for some polyatomic molecules show the contributions of vibrational energy. In addition, a molecule with three or more atoms that are not in a straight line has three, not two, rotational degrees of freedom.

From this discussion we expect heat capacities to be temperature-dependent, generally increasing with increasing temperature. Figure 18.19 is a graph of the temperature dependence of C_V for hydrogen gas (H_2), showing the temperatures at which the rotational and vibrational energies begin to contribute.

Heat Capacities of Solids

We can carry out a similar heat-capacity analysis for a crystalline solid. Consider a crystal consisting of N identical atoms (a *monatomic solid*). Each atom is bound to an equilibrium position by interatomic forces. The elasticity of solid materials shows us that these forces must permit stretching and bending of the bonds. We can think of a crystal as an array of atoms connected by little springs (Fig. 18.20). Each atom can *vibrate* about its equilibrium position.

Each atom has three degrees of freedom, corresponding to its three components of velocity. According to the equipartition principle, each atom has an average kinetic energy of $\frac{1}{2}kT$ for each degree of freedom. In addition, each atom has *potential* energy associated with the elastic deformation. For a simple harmonic oscillator (discussed in Chapter 14) it is not hard to show that the average kinetic energy of an atom is *equal* to its average potential energy. In our model of a crystal, each atom is essentially a three-dimensional harmonic oscillator; it can be shown that the equality of average kinetic and potential energies also holds here, provided that the “spring” forces obey Hooke’s law.

Thus we expect each atom to have an average kinetic energy $\frac{3}{2}kT$ and an average potential energy $\frac{3}{2}kT$, or an average total energy $3kT$ per atom. If the crystal contains N atoms or n moles, its total energy is

$$E_{\text{total}} = 3NkT = 3nRT \quad (18.27)$$

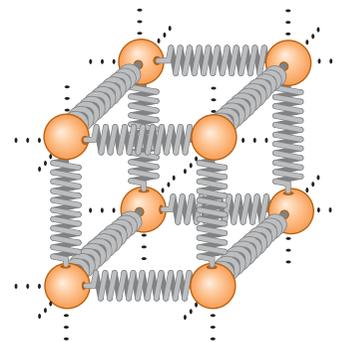
From this we conclude that the molar heat capacity of a crystal should be

$$C_V = 3R \quad (\text{ideal monatomic solid}) \quad (18.28)$$

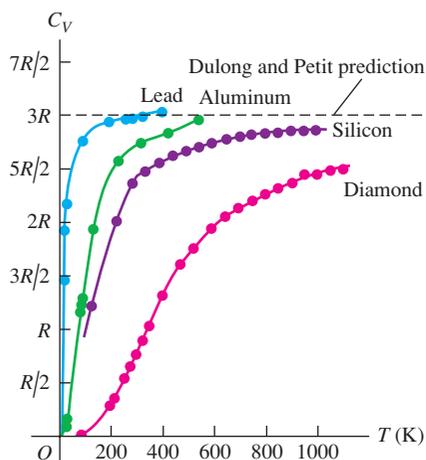
In SI units,

$$C_V = (3)(8.314 \text{ J/mol} \cdot \text{K}) = 24.9 \text{ J/mol} \cdot \text{K}$$

18.20 To visualize the forces between neighboring atoms in a crystal, envision every atom as being attached to its neighbors by springs.



18.21 Experimental values of C_V for lead, aluminum, silicon, and diamond. At high temperatures, C_V for each solid approaches about $3R$, in agreement with the rule of Dulong and Petit. At low temperatures, C_V is much less than $3R$.



This is the **rule of Dulong and Petit**, which we encountered as an *empirical* finding in Section 17.5: Elemental solids all have molar heat capacities of about $25 \text{ J/mol} \cdot \text{K}$. Now we have *derived* this rule from kinetic theory. The agreement is only approximate, to be sure, but considering the very simple nature of our model, it is quite significant.

At low temperatures, the heat capacities of most solids *decrease* with decreasing temperature (Fig. 18.21) for the same reason that vibrational degrees of freedom of molecules are frozen out at low temperatures. At very low temperatures the quantity kT is much *smaller* than the smallest energy step the vibrating atoms can take. Hence most of the atoms remain in their lowest energy states because the next higher energy level is out of reach. The average vibrational energy per atom is then *less* than $3kT$, and the heat capacity per molecule is *less* than $3k$. At higher temperatures when kT is *large* in comparison to the minimum energy step, the equipartition principle holds, and the total heat capacity is $3k$ per molecule or $3R$ per mole as the Dulong and Petit rule predicts. Quantitative understanding of the temperature variation of heat capacities was one of the triumphs of quantum mechanics during its initial development in the 1920s.

Test Your Understanding of Section 18.4 A cylinder with a fixed volume contains hydrogen gas (H_2) at 25 K. You then add heat to the gas at a constant rate until its temperature reaches 500 K. Does the temperature of the gas increase at a constant rate? Why or why not? If not, does the temperature increase most rapidly near the beginning or near the end of this process?

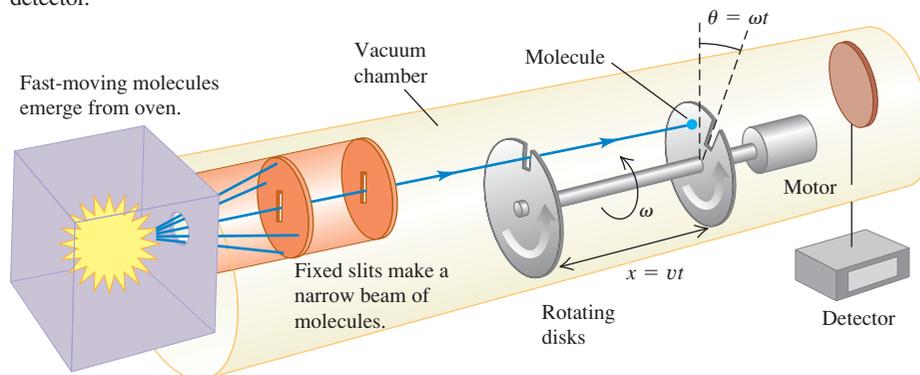
18.5 Molecular Speeds

As we mentioned in Section 18.3, the molecules in a gas don't all have the same speed. Figure 18.22 shows one experimental scheme for measuring the distribution of molecular speeds. A substance is vaporized in a hot oven; molecules of the vapor escape through an aperture in the oven wall and into a vacuum chamber. A series of slits blocks all molecules except those in a narrow beam, which is aimed at a pair of rotating disks. A molecule passing through the slit in the first disk is blocked by the second disk unless it arrives just as the slit in the second disk is lined up with the beam. The disks function as a speed selector that passes only molecules within a certain narrow speed range. This range can be varied by changing the disk rotation speed, and we can measure how many molecules lie within each of various speed ranges.

To describe the results of such measurements, we define a function $f(v)$ called a *distribution function*. If we observe a total of N molecules, the number dN having speeds in the range between v and $v + dv$ is given by

$$dN = Nf(v) dv \quad (18.29)$$

18.22 A molecule with a speed v passes through the slit in the first rotating disk. When the molecule reaches the second rotating disk, the disks have rotated through the offset angle θ . If $v = \omega x/\theta$, the molecule passes through the slit in the second rotating disk and reaches the detector.



We can also say that the *probability* that a randomly chosen molecule will have a speed in the interval v to $v + dv$ is $f(v) dv$. Hence $f(v)$ is the probability per unit speed *interval*; it is *not* equal to the probability that a molecule has speed exactly equal to v . Since a probability is a pure number, $f(v)$ has units of reciprocal speed (s/m).

Figure 18.23a shows distribution functions for three different temperatures. At each temperature the height of the curve for any value of v is proportional to the number of molecules with speeds near v . The peak of the curve represents the *most probable speed* v_{mp} for the corresponding temperature. As the temperature increases, the average molecular kinetic energy increases, and so the peak of $f(v)$ shifts to higher and higher speeds.

Figure 18.23b shows that the area under a curve between any two values of v represents the fraction of all the molecules having speeds in that range. Every molecule must have *some* value of v , so the integral of $f(v)$ over all v must be unity for any T .

If we know $f(v)$, we can calculate the most probable speed v_{mp} , the average speed v_{av} , and the rms speed v_{rms} . To find v_{mp} , we simply find the point where $df/dv = 0$; this gives the value of the speed where the curve has its peak. To find v_{av} , we take the number $Nf(v) dv$ having speeds in each interval dv , multiply each number by the corresponding speed v , add all these products (by integrating over all v from zero to infinity), and finally divide by N . That is,

$$v_{\text{av}} = \int_0^{\infty} v f(v) dv \quad (18.30)$$

The rms speed is obtained similarly; the average of v^2 is given by

$$(v^2)_{\text{av}} = \int_0^{\infty} v^2 f(v) dv \quad (18.31)$$

and v_{rms} is the square root of this.

The Maxwell–Boltzmann Distribution

The function $f(v)$ describing the actual distribution of molecular speeds is called the **Maxwell–Boltzmann distribution**. It can be derived from statistical-mechanics considerations, but that derivation is beyond our scope. Here is the result:

$$f(v) = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT} \quad (\text{Maxwell–Boltzmann distribution}) \quad (18.32)$$

We can also express this function in terms of the translational kinetic energy of a molecule, which we denote by ϵ ; that is, $\epsilon = \frac{1}{2}mv^2$. We invite you to verify that when this is substituted into Eq. (18.32), the result is

$$f(v) = \frac{8\pi}{m} \left(\frac{m}{2\pi kT} \right)^{3/2} \epsilon e^{-\epsilon/kT} \quad (18.33)$$

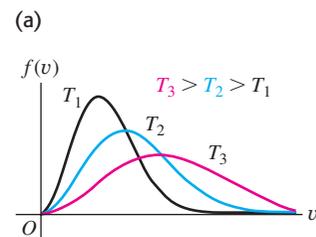
This form shows that the exponent in the Maxwell–Boltzmann distribution function is $-\epsilon/kT$ and that the shape of the curve is determined by the relative magnitude of ϵ and kT at any point. We leave it to you (see Exercise 18.48) to prove that the *peak* of each curve occurs where $\epsilon = kT$, corresponding to a most probable speed v_{mp} given by

$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}} \quad (18.34)$$

To find the average speed, we substitute Eq. (18.32) into Eq. (18.30) and carry out the integration, making a change of variable $v^2 = x$ and then integrating by parts. The result is

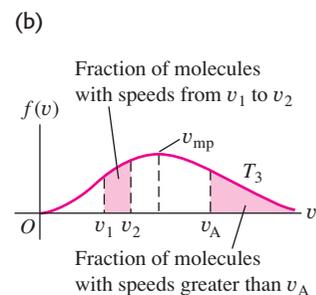
$$v_{\text{av}} = \sqrt{\frac{8kT}{\pi m}} \quad (18.35)$$

18.23 (a) Curves of the Maxwell–Boltzmann distribution function $f(v)$ for three temperatures. (b) The shaded areas under the curve represent the fractions of molecules within certain speed ranges. The most probable speed v_{mp} for a given temperature is at the peak of the curve.



As temperature increases:

- the curve flattens.
- the maximum shifts to higher speeds.



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Finally, to find the rms speed, we substitute Eq. (18.32) into Eq. (18.31). Evaluating the resulting integral takes some mathematical acrobatics, but we can find it in a table of integrals. The result is

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \quad (18.36)$$

This result agrees with Eq. (18.19); it *must* agree if the Maxwell–Boltzmann distribution is to be consistent with the equipartition principle and our other kinetic-theory calculations.

Table 18.2 shows the fraction of all the molecules in an ideal gas that have speeds *less than* various multiples of v_{rms} . These numbers were obtained by numerical integration; they are the same for all ideal gases.

The distribution of molecular speeds in liquids is similar, although not identical, to that for gases. We can understand the vapor pressure of a liquid and the phenomenon of boiling on this basis. Suppose a molecule must have a speed at least as great as v_A in Fig. 18.23b to escape from the surface of a liquid into the adjacent vapor. The number of such molecules, represented by the area under the “tail” of each curve (to the right of v_A), increases rapidly with temperature. Thus the rate at which molecules can escape is strongly temperature-dependent. This process is balanced by another one in which molecules in the vapor phase collide inelastically with the surface and are trapped back into the liquid phase. The number of molecules suffering this fate per unit time is proportional to the pressure in the vapor phase. Phase equilibrium between liquid and vapor occurs when these two competing processes proceed at exactly the same rate. So if the molecular speed distributions are known for various temperatures, we can make a theoretical prediction of vapor pressure as a function of temperature. When liquid evaporates, it’s the high-speed molecules that escape from the surface. The ones that are left have less energy on average; this gives us a molecular view of evaporative cooling.

Rates of chemical reactions are often strongly temperature-dependent, and the reason is contained in the Maxwell–Boltzmann distribution. When two reacting molecules collide, the reaction can occur only when the molecules are close enough for the electric-charge distributions of their electrons to interact strongly. This requires a minimum energy, called the *activation energy*, and thus a certain minimum molecular speed. Figure 18.23a shows that the number of molecules in the high-speed tail of the curve increases rapidly with temperature. Thus we expect the rate of any reaction that depends on an activation energy to increase rapidly with temperature.

Test Your Understanding of Section 18.5 A quantity of gas containing N molecules has a speed distribution function $f(v)$. How many molecules have speeds between v_1 and $v_2 > v_1$? (i) $\int_0^{v_2} f(v) dv - \int_0^{v_1} f(v) dv$; (ii) $N[\int_0^{v_2} f(v) dv - \int_0^{v_1} f(v) dv]$; (iii) $\int_0^{v_1} f(v) dv - \int_0^{v_2} f(v) dv$; (iv) $N[\int_0^{v_1} f(v) dv - \int_0^{v_2} f(v) dv]$; (v) none of these.



Table 18.2 Fractions of Molecules in an Ideal Gas with Speeds Less than Various Multiples of v/v_{rms}

v/v_{rms}	Fraction
0.20	0.011
0.40	0.077
0.60	0.218
0.80	0.411
1.00	0.608
1.20	0.771
1.40	0.882
1.60	0.947
1.80	0.979
2.00	0.993

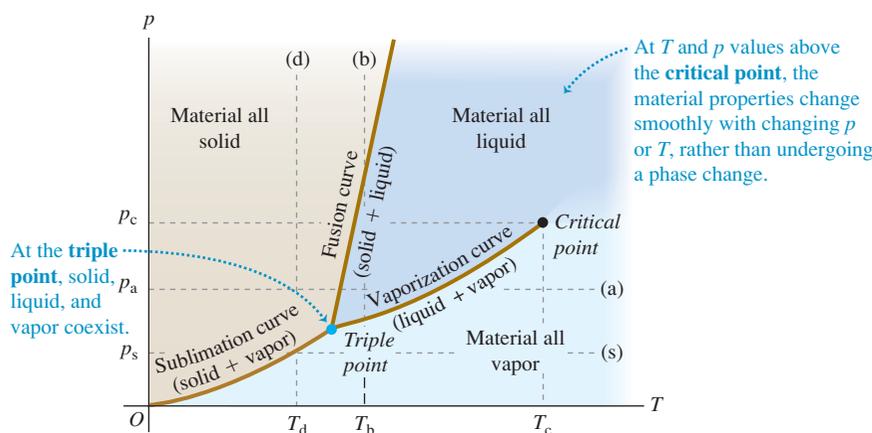
Application Activation Energy and Moth Activity

This hawkmoth of genus *Manduca* cannot fly if the temperature of its muscles is below 29°C. The reason is that the enzyme-catalyzed reactions that power aerobic metabolism and enable muscle action require a minimum molecular energy (activation energy). Just like the molecules in an ideal gas, at low temperatures very few of the molecules involved in these reactions have high energy. As the temperature increases, more molecules have the required minimum energy and the reactions take place at a greater rate. Above 29°C, enough power is generated to allow the hawkmoth to fly.



18.6 Phases of Matter

An ideal gas is the simplest system to analyze from a molecular viewpoint because we ignore the interactions between molecules. But those interactions are the very thing that makes matter condense into the liquid and solid phases under some conditions. So it’s not surprising that theoretical analysis of liquid and solid structure and behavior is a lot more complicated than that for gases. We won’t try to go far here with a microscopic picture, but we can talk in general about phases of matter, phase equilibrium, and phase transitions.



In Section 17.6 we learned that each phase is stable only in certain ranges of temperature and pressure. A transition from one phase to another ordinarily requires **phase equilibrium** between the two phases, and for a given pressure this occurs at only one specific temperature. We can represent these conditions on a graph with axes p and T , called a **phase diagram**; Fig. 18.24 shows an example. Each point on the diagram represents a pair of values of p and T .

Only a single phase can exist at each point in Fig. 18.24, except for points on the solid lines, where two phases can coexist in phase equilibrium. The fusion curve separates the solid and liquid areas and represents possible conditions of solid-liquid phase equilibrium. The vaporization curve separates the liquid and vapor areas, and the sublimation curve separates the solid and vapor areas. All three curves meet at the **triple point**, the only condition under which all three phases can coexist (Fig. 18.25). In Section 17.3 we used the triple-point temperature of water to define the Kelvin temperature scale. Table 18.3 gives triple-point data for several substances.

If we add heat to a substance at a constant pressure p_a , it goes through a series of states represented by the horizontal line (a) in Fig. 18.24. The melting and boiling temperatures at this pressure are the temperatures at which the line intersects the fusion and vaporization curves, respectively. When the pressure is p_s , constant-pressure heating transforms a substance from solid directly to vapor. This process is called *sublimation*; the intersection of line (s) with the sublimation curve gives the temperature T_s at which it occurs for a pressure p_s . At any pressure less than the triple-point pressure, no liquid phase is possible. The triple-point pressure for carbon dioxide is 5.1 atm. At normal atmospheric pressure, solid carbon dioxide (“dry ice”) undergoes sublimation; there is no liquid phase at this pressure.

Line (b) in Fig. 18.24 represents compression at a constant temperature T_b . The material passes from vapor to liquid and then to solid at the points where line (b) crosses the vaporization curve and fusion curve, respectively. Line (d) shows constant-temperature compression at a lower temperature T_d ; the material passes from vapor to solid at the point where line (d) crosses the sublimation curve.

We saw in the pV -diagram of Fig. 18.7 that a liquid-vapor phase transition occurs only when the temperature and pressure are less than those at the point lying at the top of the green shaded area labeled “Liquid-vapor phase equilibrium region.” This point corresponds to the endpoint at the top of the vaporization curve in Fig. 18.24. It is called the **critical point**, and the corresponding values of p and T are called the critical pressure and temperature, p_c and T_c . A gas at a pressure *above* the critical pressure does not separate into two phases when it is cooled at constant pressure (along a horizontal line above the critical point in Fig. 18.24). Instead, its properties change gradually and continuously from those we ordinarily associate with a gas (low density, large compressibility) to those of a liquid (high density, small compressibility) *without a phase transition*.

18.24 A typical pT phase diagram, showing regions of temperature and pressure at which the various phases exist and where phase changes occur.

18.25 Atmospheric pressure on earth is higher than the triple-point pressure of water (see line (a) in Fig. 18.24). Depending on the temperature, water can exist as a vapor (in the atmosphere), as a liquid (in the ocean), or as a solid (like the iceberg shown here).



Table 18.3 Triple-Point Data

Substance	Temperature (K)	Pressure (Pa)
Hydrogen	13.80	0.0704×10^5
Deuterium	18.63	0.171×10^5
Neon	24.56	0.432×10^5
Nitrogen	63.18	0.125×10^5
Oxygen	54.36	0.00152×10^5
Ammonia	195.40	0.0607×10^5
Carbon dioxide	216.55	5.17×10^5
Sulfur dioxide	197.68	0.00167×10^5
Water	273.16	0.00610×10^5

You can understand this by thinking about liquid-phase transitions at successively higher points on the vaporization curve. As we approach the critical point, the *differences* in physical properties (such as density and compressibility) between the liquid and vapor phases become smaller. Exactly *at* the critical point they all become zero, and at this point the distinction between liquid and vapor disappears. The heat of vaporization also grows smaller as we approach the critical point, and it too becomes zero at the critical point.

For nearly all familiar materials the critical pressures are much greater than atmospheric pressure, so we don't observe this behavior in everyday life. For example, the critical point for water is at 647.4 K and 221.2×10^5 Pa (about 218 atm or 3210 psi). But high-pressure steam boilers in electric generating plants regularly run at pressures and temperatures well above the critical point.

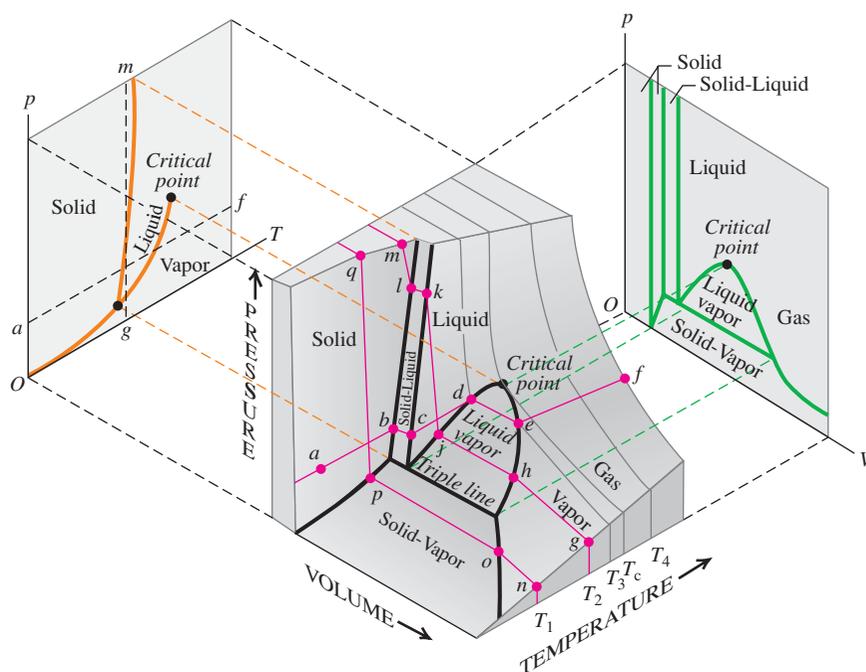
Many substances can exist in more than one solid phase. A familiar example is carbon, which exists as noncrystalline soot and crystalline graphite and diamond. Water is another example; at least eight types of ice, differing in crystal structure and physical properties, have been observed at very high pressures.

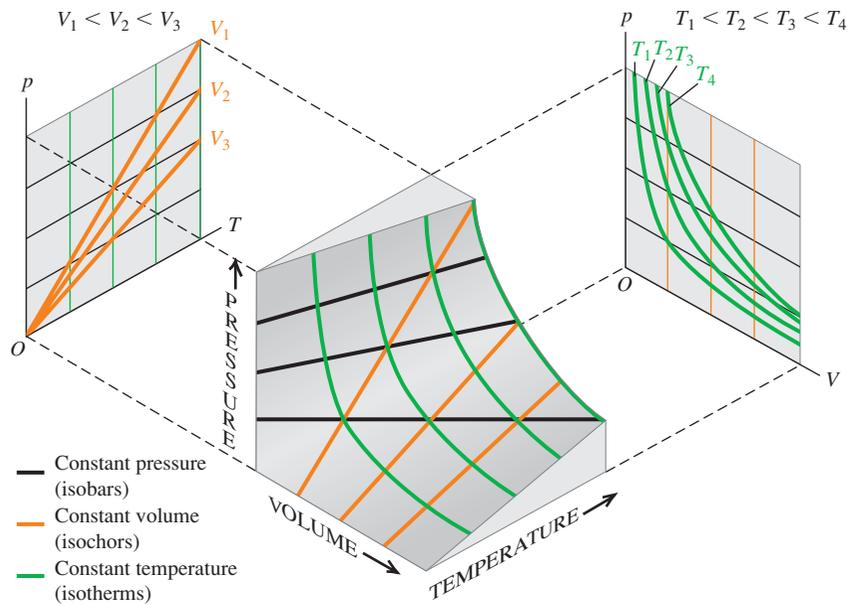
pVT-Surfaces

We remarked in Section 18.1 that the equation of state of any material can be represented graphically as a surface in a three-dimensional space with coordinates p , V , and T . Visualizing such a surface can add to our understanding of the behavior of materials at various temperatures and pressures. Figure 18.26 shows a typical *pVT*-surface. The light lines represent *pV*-isotherms; projecting them onto the *pV*-plane gives a diagram similar to Fig. 18.7. The *pV*-isotherms represent contour lines on the *pVT*-surface, just as contour lines on a topographic map represent the elevation (the third dimension) at each point. The projections of the edges of the surface onto the *pT*-plane give the *pT* phase diagram of Fig. 18.24.

Line *abcdef* in Fig. 18.26 represents constant-pressure heating, with melting along *bc* and vaporization along *de*. Note the volume changes that occur as T increases along this line. Line *ghijklm* corresponds to an isothermal (constant temperature) compression, with liquefaction along *hj* and solidification along *kl*. Between these, segments *gh* and *jk* represent isothermal compression with increase in pressure; the pressure increases are much greater in the liquid region

18.26 A *pVT*-surface for a substance that expands on melting. Projections of the boundaries on the surface onto the *pT*- and *pV*-planes are also shown.





18.27 A pVT -surface for an ideal gas. At the left, each red line corresponds to a certain constant volume; at the right, each green line corresponds to a certain constant temperature.

jk and the solid region lm than in the vapor region gh . Finally, line $nopq$ represents isothermal solidification directly from vapor, as in the formation of snowflakes or frost.

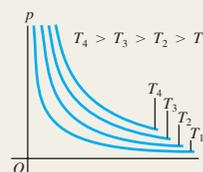
Figure 18.27 shows the much simpler pVT -surface for a substance that obeys the ideal-gas equation of state under all conditions. The projections of the constant-temperature curves onto the pV -plane correspond to the curves of Fig. 18.6, and the projections of the constant-volume curves onto the pT -plane show that pressure is directly proportional to absolute temperature.

Test Your Understanding of Section 18.6 The average atmospheric pressure on Mars is 6.0×10^2 Pa. Could there be lakes or rivers of liquid water on Mars today? What about in the past, when the atmospheric pressure is thought to have been substantially greater than today? I

CHAPTER 18 SUMMARY

Equations of state: The pressure p , volume V , and absolute temperature T of a given quantity of a substance are related by an equation of state. This relationship applies only for equilibrium states, in which p and T are uniform throughout the system. The ideal-gas equation of state, Eq. (18.3), involves the number of moles n and a constant R that is the same for all gases. (See Examples 18.1–18.4.)

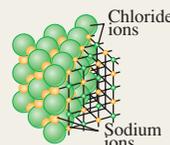
$$pV = nRT \quad (18.3)$$



Molecular properties of matter: The molar mass M of a pure substance is the mass per mole. The mass m_{total} of a quantity of substance equals M multiplied by the number of moles n . Avogadro's number N_A is the number of molecules in a mole. The mass m of an individual molecule is M divided by N_A . (See Example 18.5.)

$$m_{\text{total}} = nM \quad (18.2)$$

$$M = N_A m \quad (18.8)$$



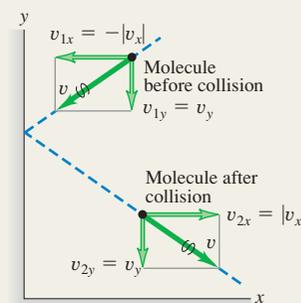
Kinetic-molecular model of an ideal gas: In an ideal gas, the total translational kinetic energy of the gas as a whole (K_{tr}) and the average translational kinetic energy per molecule [$\frac{1}{2}m(v^2)_{\text{av}}$] are proportional to the absolute temperature T , and the root-mean-square speed of molecules is proportional to the square root of T . These expressions involve the Boltzmann constant $k = R/N_A$. (See Examples 18.6 and 18.7.) The mean free path λ of molecules in an ideal gas depends on the number of molecules per volume (N/V) and the molecular radius r . (See Example 18.8.)

$$K_{\text{tr}} = \frac{3}{2}nRT \quad (18.14)$$

$$\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT \quad (18.16)$$

$$v_{\text{rms}} = \sqrt{(v^2)_{\text{av}}} = \sqrt{\frac{3kT}{m}} \quad (18.19)$$

$$\lambda = vI_{\text{mean}} = \frac{V}{4\pi\sqrt{2}r^2N} \quad (18.21)$$

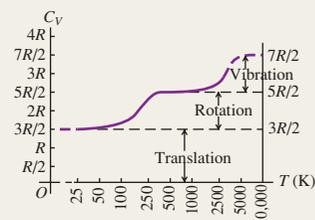


Heat capacities: The molar heat capacity at constant volume C_V is a simple multiple of the gas constant R for certain idealized cases: an ideal monatomic gas [Eq. (18.25)]; an ideal diatomic gas including rotational energy [Eq. (18.26)]; and an ideal monatomic solid [Eq. (18.28)]. Many real systems are approximated well by these idealizations.

$$C_V = \frac{3}{2}R \quad (\text{monatomic gas}) \quad (18.25)$$

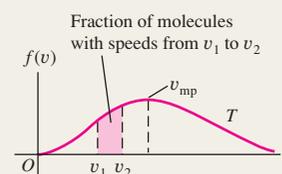
$$C_V = \frac{5}{2}R \quad (\text{diatomic gas}) \quad (18.26)$$

$$C_V = 3R \quad (\text{monatomic solid}) \quad (18.28)$$

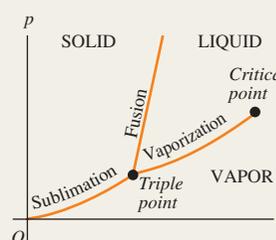


Molecular speeds: The speeds of molecules in an ideal gas are distributed according to the Maxwell–Boltzmann distribution $f(v)$. The quantity $f(v) dv$ describes what fraction of the molecules have speeds between v and $v + dv$.

$$f(v) = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT} \quad (18.32)$$



Phases of matter: Ordinary matter exists in the solid, liquid, and gas phases. A phase diagram shows conditions under which two phases can coexist in phase equilibrium. All three phases can coexist at the triple point. The vaporization curve ends at the critical point, above which the distinction between the liquid and gas phases disappears.



BRIDGING PROBLEM

Gas on Jupiter's Moon Europa

An astronaut visiting Jupiter's satellite Europa leaves a canister of 1.20 mol of nitrogen gas (28.0 g/mol) at 25.0°C on the satellite's surface. Europa has no significant atmosphere, and the acceleration due to gravity at its surface is 1.30 m/s². The canister springs a leak, allowing molecules to escape from a small hole. (a) What is the maximum height (in km) above Europa's surface that is reached by a nitrogen molecule whose speed equals the rms speed? Assume that the molecule is shot straight up out of the hole in the canister, and ignore the variation in g with altitude. (b) The escape speed from Europa is 2025 m/s. Can any of the nitrogen molecules escape from Europa and into space?

SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.



IDENTIFY and SET UP

1. Draw a sketch of the situation, showing all relevant dimensions.
2. Make a list of the unknown quantities, and decide which are the target variables.

3. How will you find the rms speed of the nitrogen molecules? What principle will you use to find the maximum height that a molecule with this speed can reach?
4. Does the rms speed of molecules in an ideal gas represent the maximum speed of the molecules? If not, what is the maximum speed?

EXECUTE

5. Solve for the rms speed. Use this to calculate the maximum height that a molecule with this speed can reach.
6. Use your result from step 5 to answer the question in part (b).

EVALUATE

7. Do your results depend on the amount of gas in the container? Why or why not?
8. How would your results from steps 5 and 6 be affected if the gas cylinder were instead left on Jupiter's satellite Ganymede, which has higher surface gravity than Europa and a higher escape speed? Like Europa, Ganymede has no significant atmosphere.

Problems

For instructor-assigned homework, go to www.masteringphysics.com



•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q18.1 Section 18.1 states that ordinarily, pressure, volume, and temperature cannot change individually without one affecting the others. Yet when a liquid evaporates, its volume changes, even though its pressure and temperature are constant. Is this inconsistent? Why or why not?

Q18.2 In the ideal-gas equation, could an equivalent Celsius temperature be used instead of the Kelvin one if an appropriate numerical value of the constant R is used? Why or why not?

Q18.3 On a chilly morning you can “see your breath.” Can you really? What are you actually seeing? Does this phenomenon depend on the temperature of the air, the humidity, or both? Explain.

Q18.4 When a car is driven some distance, the air pressure in the tires increases. Why? Should you let out some air to reduce the pressure? Why or why not?

Q18.5 The coolant in an automobile radiator is kept at a pressure higher than atmospheric pressure. Why is this desirable? The radiator cap will release coolant when the gauge pressure of the coolant reaches a certain value, typically 15 lb/in.² or so. Why not just seal the system completely?

Q18.6 Unwrapped food placed in a freezer experiences dehydration, known as “freezer burn.” Why?

Q18.7 “Freeze-drying” food involves the same process as “freezer burn,” referred to in Discussion Question Q18.6. For freeze-drying, the food is usually frozen first, and then placed in a vacuum chamber and irradiated with infrared radiation. What is the purpose of the vacuum? The radiation? What advantages might freeze-drying have in comparison to ordinary drying?

Q18.8 A group of students drove from their university (near sea level) up into the mountains for a skiing weekend. Upon arriving at the slopes, they discovered that the bags of potato chips they had brought for snacks had all burst open. What caused this to happen?

Q18.9 How does evaporation of perspiration from your skin cool your body?

Q18.10 A rigid, perfectly insulated container has a membrane dividing its volume in half. One side contains a gas at an absolute temperature T_0 and pressure p_0 , while the other half is completely empty. Suddenly a small hole develops in the membrane, allowing the gas to leak out into the other half until it eventually occupies twice its original volume. In terms of T_0 and p_0 , what will be the new temperature and pressure of the gas when it is distributed equally in both halves of the container? Explain your reasoning.

Q18.11 (a) Which has more atoms: a kilogram of hydrogen or a kilogram of lead? Which has more mass? (b) Which has more atoms: a mole of hydrogen or a mole of lead? Which has more mass? Explain your reasoning.

Q18.12 Use the concepts of the kinetic-molecular model to explain: (a) why the pressure of a gas in a rigid container increases as heat is added to the gas and (b) why the pressure of a gas increases as we compress it, even if we do not change its temperature.

Q18.13 The proportions of various gases in the earth's atmosphere change somewhat with altitude. Would you expect the proportion of oxygen at high altitude to be greater or less than at sea level compared to the proportion of nitrogen? Why?

Q18.14 Comment on the following statement: *When two gases are mixed, if they are to be in thermal equilibrium, they must have the*

same average molecular speed. Is the statement correct? Why or why not?

Q18.15 The kinetic-molecular model contains a hidden assumption about the temperature of the container walls. What is this assumption? What would happen if this assumption were not valid?

Q18.16 The temperature of an ideal gas is directly proportional to the average kinetic energy of its molecules. If a container of ideal gas is moving past you at 2000 m/s, is the temperature of the gas higher than if the container was at rest? Explain your reasoning.

Q18.17 If the pressure of an ideal monatomic gas is increased while the number of moles is kept constant, what happens to the average translational kinetic energy of one atom of the gas? Is it possible to change *both* the volume and the pressure of an ideal gas and keep the average translational kinetic energy of the atoms constant? Explain.

Q18.18 In deriving the ideal-gas equation from the kinetic-molecular model, we ignored potential energy due to the earth's gravity. Is this omission justified? Why or why not?

Q18.19 The derivation of the ideal-gas equation included the assumption that the number of molecules is very large, so that we could compute the average force due to many collisions. However, the ideal-gas equation holds accurately only at low pressures, where the molecules are few and far between. Is this inconsistent? Why or why not?

Q18.20 A gas storage tank has a small leak. The pressure in the tank drops more quickly if the gas is hydrogen or helium than if it is oxygen. Why?

Q18.21 Consider two specimens of ideal gas at the same temperature. Specimen A has the same total mass as specimen B, but the molecules in specimen A have greater molar mass than they do in specimen B. In which specimen is the total kinetic energy of the gas greater? Does your answer depend on the molecular structure of the gases? Why or why not?

Q18.22 The temperature of an ideal monatomic gas is increased from 25°C to 50°C. Does the average translational kinetic energy of each gas atom double? Explain. If your answer is no, what would the final temperature be if the average translational kinetic energy was doubled?

Q18.23 If the root-mean-square speed of the atoms of an ideal gas is to be doubled, by what factor must the Kelvin temperature of the gas be increased? Explain.

Q18.24 (a) If you apply the same amount of heat to 1.00 mol of an ideal monatomic gas and 1.00 mol of an ideal diatomic gas, which one (if any) will increase more in temperature? (b) Physically, why do diatomic gases have a greater molar heat capacity than monatomic gases?

Q18.25 The discussion in Section 18.4 concluded that all ideal diatomic gases have the same heat capacity C_V . Does this mean that it takes the same amount of heat to raise the temperature of 1.0 g of each one by 1.0 K? Explain your reasoning.

Q18.26 In a gas that contains N molecules, is it accurate to say that the number of molecules with speed v is equal to $f(v)$? Is it accurate to say that this number is given by $Nf(v)$? Explain your answers.

Q18.27 Imagine a special air filter placed in a window of a house. The tiny holes in the filter allow only air molecules moving faster than a certain speed to exit the house, and allow only air molecules moving slower than that speed to enter the house from outside. What effect would this filter have on the temperature inside the house? (It turns out that the second law of thermodynamics—which we will discuss in Chapter 20—tells us that such a wonderful air filter would be impossible to make.)

Q18.28 A beaker of water at room temperature is placed in an enclosure, and the air pressure in the enclosure is slowly reduced. When the air pressure is reduced sufficiently, the water begins to boil. The temperature of the water does not rise when it boils; in fact, the temperature *drops* slightly. Explain these phenomena.

Q18.29 Ice is slippery to walk on, and especially slippery if you wear ice skates. What does this tell you about how the melting temperature of ice depends on pressure? Explain.

Q18.30 Hydrothermal vents are openings in the ocean floor that discharge very hot water. The water emerging from one such vent off the Oregon coast, 2400 m below the surface, has a temperature of 279°C. Despite its high temperature, the water doesn't boil. Why not?

Q18.31 The dark areas on the moon's surface are called *maria*, Latin for "seas," and were once thought to be bodies of water. In fact, the maria are not "seas" at all, but plains of solidified lava. Given that there is no atmosphere on the moon, how can you explain the absence of liquid water on the moon's surface?

Q18.32 In addition to the normal cooking directions printed on the back of a box of rice, there are also "high-altitude directions." The only difference is that the "high-altitude directions" suggest increasing the cooking time and using a greater volume of boiling water in which to cook the rice. Why should the directions depend on the altitude in this way?

EXERCISES

Section 18.1 Equations of State

18.1 • A 20.0-L tank contains 4.86×10^{-4} kg of helium at 18.0°C. The molar mass of helium is 4.00 g/mol. (a) How many moles of helium are in the tank? (b) What is the pressure in the tank, in pascals and in atmospheres?

18.2 •• Helium gas with a volume of 2.60 L, under a pressure of 0.180 atm and at a temperature of 41.0°C, is warmed until both pressure and volume are doubled. (a) What is the final temperature? (b) How many grams of helium are there? The molar mass of helium is 4.00 g/mol.

18.3 • A cylindrical tank has a tight-fitting piston that allows the volume of the tank to be changed. The tank originally contains 0.110 m³ of air at a pressure of 0.355 atm. The piston is slowly pulled out until the volume of the gas is increased to 0.390 m³. If the temperature remains constant, what is the final value of the pressure?

18.4 • A 3.00-L tank contains air at 3.00 atm and 20.0°C. The tank is sealed and cooled until the pressure is 1.00 atm. (a) What is the temperature then in degrees Celsius? Assume that the volume of the tank is constant. (b) If the temperature is kept at the value found in part (a) and the gas is compressed, what is the volume when the pressure again becomes 3.00 atm?

18.5 • **Planetary Atmospheres.** (a) Calculate the density of the atmosphere at the surface of Mars (where the pressure is 650 Pa and the temperature is typically 253 K, with a CO₂ atmosphere), Venus (with an average temperature of 730 K and pressure of 92 atm, with a CO₂ atmosphere), and Saturn's moon Titan (where the pressure is 1.5 atm and the temperature is -178°C , with a N₂ atmosphere). (b) Compare each of these densities with that of the earth's atmosphere, which is 1.20 kg/m³. Consult the periodic chart in Appendix D to determine molar masses.

18.6 •• You have several identical balloons. You experimentally determine that a balloon will break if its volume exceeds 0.900 L. The pressure of the gas inside the balloon equals air pressure (1.00 atm). (a) If the air inside the balloon is at a constant temperature of

22.0°C and behaves as an ideal gas, what mass of air can you blow into one of the balloons before it bursts? (b) Repeat part (a) if the gas is helium rather than air.

18.7 •• A Jaguar XK8 convertible has an eight-cylinder engine. At the beginning of its compression stroke, one of the cylinders contains 499 cm³ of air at atmospheric pressure (1.01×10^5 Pa) and a temperature of 27.0°C. At the end of the stroke, the air has been compressed to a volume of 46.2 cm³ and the gauge pressure has increased to 2.72×10^6 Pa. Compute the final temperature.

18.8 •• A welder using a tank of volume 0.0750 m³ fills it with oxygen (molar mass 32.0 g/mol) at a gauge pressure of 3.00×10^5 Pa and temperature of 37.0°C. The tank has a small leak, and in time some of the oxygen leaks out. On a day when the temperature is 22.0°C, the gauge pressure of the oxygen in the tank is 1.80×10^5 Pa. Find (a) the initial mass of oxygen and (b) the mass of oxygen that has leaked out.

18.9 •• A large cylindrical tank contains 0.750 m³ of nitrogen gas at 27°C and 7.50×10^3 Pa (absolute pressure). The tank has a tight-fitting piston that allows the volume to be changed. What will be the pressure if the volume is decreased to 0.480 m³ and the temperature is increased to 157°C?

18.10 • An empty cylindrical canister 1.50 m long and 90.0 cm in diameter is to be filled with pure oxygen at 22.0°C to store in a space station. To hold as much gas as possible, the absolute pressure of the oxygen will be 21.0 atm. The molar mass of oxygen is 32.0 g/mol. (a) How many moles of oxygen does this canister hold? (b) For someone lifting this canister, by how many kilograms does this gas increase the mass to be lifted?

18.11 • The gas inside a balloon will always have a pressure nearly equal to atmospheric pressure, since that is the pressure applied to the outside of the balloon. You fill a balloon with helium (a nearly ideal gas) to a volume of 0.600 L at a temperature of 19.0°C. What is the volume of the balloon if you cool it to the boiling point of liquid nitrogen (77.3 K)?

18.12 • **Deviations from the Ideal-Gas Equation.** For carbon dioxide gas (CO₂), the constants in the van der Waals equation are $a = 0.364 \text{ J} \cdot \text{m}^3/\text{mol}^2$ and $b = 4.27 \times 10^{-5} \text{ m}^3/\text{mol}$. (a) If 1.00 mol of CO₂ gas at 350 K is confined to a volume of 400 cm³, find the pressure of the gas using the ideal-gas equation and the van der Waals equation. (b) Which equation gives a lower pressure? Why? What is the percentage difference of the van der Waals equation result from the ideal-gas equation result? (c) The gas is kept at the same temperature as it expands to a volume of 4000 cm³. Repeat the calculations of parts (a) and (b). (d) Explain how your calculations show that the van der Waals equation is equivalent to the ideal-gas equation if n/V is small.

18.13 •• If a certain amount of ideal gas occupies a volume V at STP on earth, what would be its volume (in terms of V) on Venus, where the temperature is 1003°C and the pressure is 92 atm?

18.14 • A diver observes a bubble of air rising from the bottom of a lake (where the absolute pressure is 3.50 atm) to the surface (where the pressure is 1.00 atm). The temperature at the bottom is 4.0°C, and the temperature at the surface is 23.0°C. (a) What is the ratio of the volume of the bubble as it reaches the surface to its volume at the bottom? (b) Would it be safe for the diver to hold his breath while ascending from the bottom of the lake to the surface? Why or why not?

18.15 • A metal tank with volume 3.10 L will burst if the absolute pressure of the gas it contains exceeds 100 atm. (a) If 11.0 mol of an ideal gas is put into the tank at a temperature of 23.0°C, to what temperature can the gas be warmed before the tank ruptures? You can ignore the thermal expansion of the tank. (b) Based on your

answer to part (a), is it reasonable to ignore the thermal expansion of the tank? Explain.

18.16 • Three moles of an ideal gas are in a rigid cubical box with sides of length 0.200 m. (a) What is the force that the gas exerts on each of the six sides of the box when the gas temperature is 20.0°C? (b) What is the force when the temperature of the gas is increased to 100.0°C?

18.17 • With the assumptions of Example 18.4 (Section 18.1), at what altitude above sea level is air pressure 90% of the pressure at sea level?

18.18 • Make the same assumptions as in Example 18.4 (Section 18.1). How does the percentage decrease in air pressure in going from sea level to an altitude of 100 m compare to that when going from sea level to an altitude of 1000 m? If your second answer is not 10 times your first answer, explain why.

18.19 •• (a) Calculate the mass of nitrogen present in a volume of 3000 cm³ if the temperature of the gas is 22.0°C and the absolute pressure of 2.00×10^{-13} atm is a partial vacuum easily obtained in laboratories. (b) What is the density (in kg/m³) of the N₂?

18.20 •• With the assumption that the air temperature is a uniform 0.0°C (as in Example 18.4), what is the density of the air at an altitude of 1.00 km as a percentage of the density at the surface?

18.21 • At an altitude of 11,000 m (a typical cruising altitude for a jet airliner), the air temperature is -56.5°C and the air density is 0.364 kg/m³. What is the pressure of the atmosphere at that altitude? (*Note:* The temperature at this altitude is not the same as at the surface of the earth, so the calculation of Example 18.4 in Section 18.1 doesn't apply.)

Section 18.2 Molecular Properties of Matter

18.22 • A large organic molecule has a mass of 1.41×10^{-21} kg. What is the molar mass of this compound?

18.23 •• Suppose you inherit 3.00 mol of gold from your uncle (an eccentric chemist) at a time when this metal is selling for \$14.75 per gram. Consult the periodic table in Appendix D and Table 12.1. (a) To the nearest dollar, what is this gold worth? (b) If you have your gold formed into a spherical nugget, what is its diameter?

18.24 •• Modern vacuum pumps make it easy to attain pressures of the order of 10^{-13} atm in the laboratory. Consider a volume of air and treat the air as an ideal gas. (a) At a pressure of 9.00×10^{-14} atm and an ordinary temperature of 300.0 K, how many molecules are present in a volume of 1.00 cm³? (b) How many molecules would be present at the same temperature but at 1.00 atm instead?

18.25 •• The Lagoon Nebula (Fig. E18.25) is a cloud of hydrogen gas located 3900 light-years from the earth. The cloud is about 45 light-years in diameter and glows because of its high temperature of 7500 K. (The gas is raised to this temperature by the stars that

Figure E18.25



lie within the nebula.) The cloud is also very thin; there are only 80 molecules per cubic centimeter. (a) Find the gas pressure (in atmospheres) in the Lagoon Nebula. Compare it to the laboratory pressure referred to in Exercise 18.24. (b) Science-fiction films sometimes show starships being buffeted by turbulence as they fly through gas clouds such as the Lagoon Nebula. Does this seem realistic? Why or why not?

18.26 •• In a gas at standard conditions, what is the length of the side of a cube that contains a number of molecules equal to the population of the earth (about 6×10^9 people)?

18.27 • How many moles are in a 1.00-kg bottle of water? How many molecules? The molar mass of water is 18.0 g/mol.

18.28 •• How Close Together Are Gas Molecules? Consider an ideal gas at 27°C and 1.00 atm pressure. To get some idea how close these molecules are to each other, on the average, imagine them to be uniformly spaced, with each molecule at the center of a small cube. (a) What is the length of an edge of each cube if adjacent cubes touch but do not overlap? (b) How does this distance compare with the diameter of a typical molecule? (c) How does their separation compare with the spacing of atoms in solids, which typically are about 0.3 nm apart?

18.29 •• Consider 5.00 mol of liquid water. (a) What volume is occupied by this amount of water? The molar mass of water is 18.0 g/mol. (b) Imagine the molecules to be, on average, uniformly spaced, with each molecule at the center of a small cube. What is the length of an edge of each small cube if adjacent cubes touch but don't overlap? (c) How does this distance compare with the diameter of a molecule?

Section 18.3 Kinetic-Molecular Model of an Ideal Gas

18.30 • A flask contains a mixture of neon (Ne), krypton (Kr), and radon (Rn) gases. Compare (a) the average kinetic energies of the three types of atoms and (b) the root-mean-square speeds. (*Hint:* The periodic table in Appendix D shows the molar mass (in g/mol) of each element under the chemical symbol for that element.)

18.31 • Gaseous Diffusion of Uranium. (a) A process called *gaseous diffusion* is often used to separate isotopes of uranium—that is, atoms of the elements that have different masses, such as ^{235}U and ^{238}U . The only gaseous compound of uranium at ordinary temperatures is uranium hexafluoride, UF_6 . Speculate on how $^{235}\text{UF}_6$ and $^{238}\text{UF}_6$ molecules might be separated by diffusion. (b) The molar masses for $^{235}\text{UF}_6$ and $^{238}\text{UF}_6$ molecules are 0.349 kg/mol and 0.352 kg/mol, respectively. If uranium hexafluoride acts as an ideal gas, what is the ratio of the root-mean-square speed of $^{235}\text{UF}_6$ molecules to that of $^{238}\text{UF}_6$ molecules if the temperature is uniform?

18.32 • The ideas of average and root-mean-square value can be applied to any distribution. A class of 150 students had the following scores on a 100-point quiz:

Score	Number of Students
10	11
20	12
30	24
40	15
50	19
60	10
70	12
80	20
90	17
100	10

(a) Find the average score for the class. (b) Find the root-mean-square score for the class.

18.33 • We have two equal-size boxes, *A* and *B*. Each box contains gas that behaves as an ideal gas. We insert a thermometer into each box and find that the gas in box *A* is at a temperature of 50°C while the gas in box *B* is at 10°C. This is all we know about the gas in the boxes. Which of the following statements *must* be true? Which *could* be true? (a) The pressure in *A* is higher than in *B*. (b) There are more molecules in *A* than in *B*. (c) *A* and *B* do not contain the same type of gas. (d) The molecules in *A* have more average kinetic energy per molecule than those in *B*. (e) The molecules in *A* are moving faster than those in *B*. Explain the reasoning behind your answers.

18.34 • A container with volume 1.48 L is initially evacuated. Then it is filled with 0.226 g of N_2 . Assume that the pressure of the gas is low enough for the gas to obey the ideal-gas law to a high degree of accuracy. If the root-mean-square speed of the gas molecules is 182 m/s, what is the pressure of the gas?

18.35 •• (a) A deuteron, ${}^2_1\text{H}$, is the nucleus of a hydrogen isotope and consists of one proton and one neutron. The plasma of deuterons in a nuclear fusion reactor must be heated to about 300 million K. What is the rms speed of the deuterons? Is this a significant fraction of the speed of light ($c = 3.0 \times 10^8$ m/s)? (b) What would the temperature of the plasma be if the deuterons had an rms speed equal to $0.10c$?

18.36 • Martian Climate. The atmosphere of Mars is mostly CO_2 (molar mass 44.0 g/mol) under a pressure of 650 Pa, which we shall assume remains constant. In many places the temperature varies from 0.0°C in summer to -100°C in winter. Over the course of a Martian year, what are the ranges of (a) the rms speeds of the CO_2 molecules and (b) the density (in mol/m^3) of the atmosphere?

18.37 •• (a) Oxygen (O_2) has a molar mass of 32.0 g/mol. What is the average translational kinetic energy of an oxygen molecule at a temperature of 300 K? (b) What is the average value of the square of its speed? (c) What is the root-mean-square speed? (d) What is the momentum of an oxygen molecule traveling at this speed? (e) Suppose an oxygen molecule traveling at this speed bounces back and forth between opposite sides of a cubical vessel 0.10 m on a side. What is the average force the molecule exerts on one of the walls of the container? (Assume that the molecule's velocity is perpendicular to the two sides that it strikes.) (f) What is the average force per unit area? (g) How many oxygen molecules traveling at this speed are necessary to produce an average pressure of 1 atm? (h) Compute the number of oxygen molecules that are actually contained in a vessel of this size at 300 K and atmospheric pressure. (i) Your answer for part (h) should be three times as large as the answer for part (g). Where does this discrepancy arise?

18.38 •• Calculate the mean free path of air molecules at a pressure of 3.50×10^{-13} atm and a temperature of 300 K. (This pressure is readily attainable in the laboratory; see Exercise 18.24.) As in Example 18.8, model the air molecules as spheres of radius 2.0×10^{-10} m.

18.39 •• At what temperature is the root-mean-square speed of nitrogen molecules equal to the root-mean-square speed of hydrogen molecules at 20.0°C? (*Hint:* The periodic table in Appendix D shows the molar mass (in g/mol) of each element under the chemical symbol for that element. The molar mass of H_2 is twice the molar mass of hydrogen atoms, and similarly for N_2 .)

18.40 • Smoke particles in the air typically have masses of the order of 10^{-16} kg. The Brownian motion (rapid, irregular movement) of these particles, resulting from collisions with air molecules, can

be observed with a microscope. (a) Find the root-mean-square speed of Brownian motion for a particle with a mass of 3.00×10^{-16} kg in air at 300 K. (b) Would the root-mean-square speed be different if the particle were in hydrogen gas at the same temperature? Explain.

Section 18.4 Heat Capacities

18.41 • (a) How much heat does it take to increase the temperature of 2.50 mol of a diatomic ideal gas by 50.0 K near room temperature if the gas is held at constant volume? (b) What is the answer to the question in part (a) if the gas is monatomic rather than diatomic?

18.42 •• Perfectly rigid containers each hold n moles of ideal gas, one being hydrogen (H_2) and other being neon (Ne). If it takes 300 J of heat to increase the temperature of the hydrogen by 2.50°C, by how many degrees will the same amount of heat raise the temperature of the neon?

18.43 •• (a) Compute the specific heat at constant volume of nitrogen (N_2) gas, and compare it with the specific heat of liquid water. The molar mass of N_2 is 28.0 g/mol. (b) You warm 1.00 kg of water at a constant volume of 1.00 L from 20.0°C to 30.0°C in a kettle. For the same amount of heat, how many kilograms of 20.0°C air would you be able to warm to 30.0°C? What volume (in liters) would this air occupy at 20.0°C and a pressure of 1.00 atm? Make the simplifying assumption that air is 100% N_2 .

18.44 •• (a) Calculate the specific heat at constant volume of water vapor, assuming the nonlinear triatomic molecule has three translational and three rotational degrees of freedom and that vibrational motion does not contribute. The molar mass of water is 18.0 g/mol. (b) The actual specific heat of water vapor at low pressures is about 2000 J/kg · K. Compare this with your calculation and comment on the actual role of vibrational motion.

18.45 •• (a) Use Eq. 18.28 to calculate the specific heat at constant volume of aluminum in units of J/kg · K. Consult the periodic table in Appendix D. (b) Compare the answer in part (a) with the value given in Table 17.3. Try to explain any disagreement between these two values.

Section 18.5 Molecular Speeds

18.46 • For a gas of nitrogen molecules (N_2), what must the temperature be if 94.7% of all the molecules have speeds less than (a) 1500 m/s; (b) 1000 m/s; (c) 500 m/s? Use Table 18.2. The molar mass of N_2 is 28.0 g/mol.

18.47 • For diatomic carbon dioxide gas (CO_2 , molar mass 44.0 g/mol) at $T = 300$ K, calculate (a) the most probable speed v_{mp} ; (b) the average speed v_{av} ; (c) the root-mean-square speed v_{rms} .

18.48 • **CALC** Prove that $f(v)$ as given by Eq. (18.33) is maximum for $\epsilon = kT$. Use this result to obtain Eq. (18.34).

Section 18.6 Phases of Matter

18.49 • Solid water (ice) is slowly warmed from a very low temperature. (a) What minimum external pressure p_1 must be applied to the solid if a melting phase transition is to be observed? Describe the sequence of phase transitions that occur if the applied pressure p is such that $p < p_1$. (b) Above a certain maximum pressure p_2 , no boiling transition is observed. What is this pressure? Describe the sequence of phase transitions that occur if $p_1 < p < p_2$.

18.50 • Puffy cumulus clouds, which are made of water droplets, occur at lower altitudes in the atmosphere. Wispy cirrus clouds,

which are made of ice crystals, occur only at higher altitudes. Find the altitude y (measured from sea level) above which only cirrus clouds can occur. On a typical day and at altitudes less than 11 km, the temperature at an altitude y is given by $T = T_0 - \alpha y$, where $T_0 = 15.0^\circ\text{C}$ and $\alpha = 6.0^\circ\text{C}/1000$ m.

18.51 • The atmosphere of the planet Mars is 95.3% carbon dioxide (CO_2) and about 0.03% water vapor. The atmospheric pressure is only about 600 Pa, and the surface temperature varies from -30°C to -100°C . The polar ice caps contain both CO_2 ice and water ice. Could there be liquid CO_2 on the surface of Mars? Could there be liquid water? Why or why not?

18.52 • A physics lecture room has a volume of 216 m^3 . (a) For a pressure of 1.00 atm and a temperature of 27.0°C, use the ideal-gas law to estimate the number of air molecules in the room. Assume all the air is N_2 . (b) Calculate the particle density—that is, the number of N_2 molecules per cubic centimeter. (c) Calculate the mass of the air in the room.

PROBLEMS

18.53 •• **CP BIO The Effect of Altitude on the Lungs.** (a) Calculate the *change* in air pressure you will experience if you climb a 1000-m mountain, assuming that the temperature and air density do not change over this distance and that they were 22°C and 1.2 kg/ m^3 , respectively, at the bottom of the mountain. (Note that the result of Example 18.4 doesn't apply, since the expression derived in that example accounts for the variation of air density with altitude and we are told to ignore that in this problem.) (b) If you took a 0.50-L breath at the foot of the mountain and managed to hold it until you reached the top, what would be the volume of this breath when you exhaled it there?

18.54 •• **CP BIO The Bends.** If deep-sea divers rise to the surface too quickly, nitrogen bubbles in their blood can expand and prove fatal. This phenomenon is known as the *bends*. If a scuba diver rises quickly from a depth of 25 m in Lake Michigan (which is fresh water), what will be the volume at the surface of an N_2 bubble that occupied 1.0 mm^3 in his blood at the lower depth? Does it seem that this difference is large enough to be a problem? (Assume that the pressure difference is due only to the changing water pressure, not to any temperature difference, an assumption that is reasonable, since we are warm-blooded creatures.)

18.55 ••• **CP** A hot-air balloon stays aloft because hot air at atmospheric pressure is less dense than cooler air at the same pressure. If the volume of the balloon is 500.0 m^3 and the surrounding air is at 15.0°C, what must the temperature of the air in the balloon be for it to lift a total load of 290 kg (in addition to the mass of the hot air)? The density of air at 15.0°C and atmospheric pressure is 1.23 kg/ m^3 .

18.56 •• (a) Use Eq. (18.1) to estimate the change in the volume of a solid steel sphere of volume 11 L when the temperature and pressure increase from 21°C and 1.013×10^5 Pa to 42°C and 2.10×10^7 Pa. (*Hint:* Consult Chapters 11 and 17 to determine the values of β and k .) (b) In Example 18.3 the change in volume of an 11-L steel scuba tank was ignored. Was this a good approximation? Explain.

18.57 ••• A cylinder 1.00 m tall with inside diameter 0.120 m is used to hold propane gas (molar mass 44.1 g/mol) for use in a barbecue. It is initially filled with gas until the gauge pressure is 1.30×10^6 Pa and the temperature is 22.0°C. The temperature of the gas remains constant as it is partially emptied out of the tank, until the gauge pressure is 2.50×10^5 Pa. Calculate the mass of propane that has been used.

18.58 • CP During a test dive in 1939, prior to being accepted by the U.S. Navy, the submarine *Squalus* sank at a point where the depth of water was 73.0 m. The temperature at the surface was 27.0°C, and at the bottom it was 7.0°C. The density of seawater is 1030 kg/m³. (a) A diving bell was used to rescue 33 trapped crewmen from the *Squalus*. The diving bell was in the form of a circular cylinder 2.30 m high, open at the bottom and closed at the top. When the diving bell was lowered to the bottom of the sea, to what height did water rise within the diving bell? (*Hint*: You may ignore the relatively small variation in water pressure between the bottom of the bell and the surface of the water within the bell.) (b) At what gauge pressure must compressed air have been supplied to the bell while on the bottom to expel all the water from it?

18.59 • Atmosphere of Titan. Titan, the largest satellite of Saturn, has a thick nitrogen atmosphere. At its surface, the pressure is 1.5 earth-atmospheres and the temperature is 94 K. (a) What is the surface temperature in °C? (b) Calculate the surface density in Titan's atmosphere in molecules per cubic meter. (c) Compare the density of Titan's surface atmosphere to the density of earth's atmosphere at 22°C. Which body has denser atmosphere?

18.60 • Pressure on Venus. At the surface of Venus the average temperature is a balmy 460°C due to the greenhouse effect (global warming!), the pressure is 92 earth-atmospheres, and the acceleration due to gravity is 0.894g_{earth}. The atmosphere is nearly all CO₂ (molar mass 44.0 g/mol) and the temperature remains remarkably constant. We shall assume that the temperature does not change at all with altitude. (a) What is the atmospheric pressure 1.00 km above the surface of Venus? Express your answer in Venus-atmospheres and earth-atmospheres. (b) What is the root-mean-square speed of the CO₂ molecules at the surface of Venus and at an altitude of 1.00 km?

18.61 •• An automobile tire has a volume of 0.0150 m³ on a cold day when the temperature of the air in the tire is 5.0°C and atmospheric pressure is 1.02 atm. Under these conditions the gauge pressure is measured to be 1.70 atm (about 25 lb/in.²). After the car is driven on the highway for 30 min, the temperature of the air in the tires has risen to 45.0°C and the volume has risen to 0.0159 m³. What then is the gauge pressure?

18.62 •• A flask with a volume of 1.50 L, provided with a stopcock, contains ethane gas (C₂H₆) at 300 K and atmospheric pressure (1.013 × 10⁵ Pa). The molar mass of ethane is 30.1 g/mol. The system is warmed to a temperature of 490 K, with the stopcock open to the atmosphere. The stopcock is then closed, and the flask is cooled to its original temperature. (a) What is the final pressure of the ethane in the flask? (b) How many grams of ethane remain in the flask?

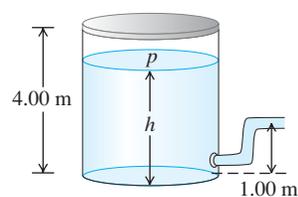
18.63 •• CP A balloon whose volume is 750 m³ is to be filled with hydrogen at atmospheric pressure (1.01 × 10⁵ Pa). (a) If the hydrogen is stored in cylinders with volumes of 1.90 m³ at a gauge pressure of 1.20 × 10⁶ Pa, how many cylinders are required? Assume that the temperature of the hydrogen remains constant. (b) What is the total weight (in addition to the weight of the gas) that can be supported by the balloon if the gas in the balloon and the surrounding air are both at 15.0°C? The molar mass of hydrogen (H₂) is 2.02 g/mol. The density of air at 15.0°C and atmospheric pressure is 1.23 kg/m³. See Chapter 12 for a discussion of buoyancy. (c) What weight could be supported if the balloon were filled with helium (molar mass 4.00 g/mol) instead of hydrogen, again at 15.0°C?

18.64 •• A vertical cylindrical tank contains 1.80 mol of an ideal gas under a pressure of 0.500 atm at 20.0°C. The round part of the tank has a radius of 10.0 cm, and the gas is supporting a piston that

can move up and down in the cylinder without friction. There is a vacuum above the piston. (a) What is the mass of this piston? (b) How tall is the column of gas that is supporting the piston?

18.65 •• CP A large tank of water has a hose connected to it, as shown in Fig. P18.65. The tank is sealed at the top and has compressed air between the water surface and the top. When the water height h has the value 3.50 m, the absolute pressure p of the compressed air is 4.20 × 10⁵ Pa. Assume that the air

Figure P18.65



above the water expands at constant temperature, and take the atmospheric pressure to be 1.00 × 10⁵ Pa. (a) What is the speed with which water flows out of the hose when $h = 3.50$ m? (b) As water flows out of the tank, h decreases. Calculate the speed of flow for $h = 3.00$ m and for $h = 2.00$ m. (c) At what value of h does the flow stop?

18.66 •• BIO A person at rest inhales 0.50 L of air with each breath at a pressure of 1.00 atm and a temperature of 20.0°C. The inhaled air is 21.0% oxygen. (a) How many oxygen molecules does this person inhale with each breath? (b) Suppose this person is now resting at an elevation of 2000 m but the temperature is still 20.0°C. Assuming that the oxygen percentage and volume per inhalation are the same as stated above, how many oxygen molecules does this person now inhale with each breath? (c) Given that the body still requires the same number of oxygen molecules per second as at sea level to maintain its functions, explain why some people report “shortness of breath” at high elevations.

18.67 •• BIO How Many Atoms Are You? Estimate the number of atoms in the body of a 50-kg physics student. Note that the human body is mostly water, which has molar mass 18.0 g/mol, and that each water molecule contains three atoms.

18.68 • The size of an oxygen molecule is about 2.0×10^{-10} m. Make a rough estimate of the pressure at which the finite volume of the molecules should cause noticeable deviations from ideal-gas behavior at ordinary temperatures ($T = 300$ K).

18.69 •• You have two identical containers, one containing gas A and the other gas B. The masses of these molecules are $m_A = 3.34 \times 10^{-27}$ kg and $m_B = 5.34 \times 10^{-26}$ kg. Both gases are under the same pressure and are at 10.0°C. (a) Which molecules (A or B) have greater translational kinetic energy per molecule and rms speeds? (b) Now you want to raise the temperature of only one of these containers so that both gases will have the same rms speed. For which gas should you raise the temperature? (c) At what temperature will you accomplish your goal? (d) Once you have accomplished your goal, which molecules (A or B) now have greater average translational kinetic energy per molecule?

18.70 • Insect Collisions. A cubical cage 1.25 m on each side contains 2500 angry bees, each flying randomly at 1.10 m/s. We can model these insects as spheres 1.50 cm in diameter. On the average, (a) how far does a typical bee travel between collisions, (b) what is the average time between collisions, and (c) how many collisions per second does a bee make?

18.71 •• You blow up a spherical balloon to a diameter of 50.0 cm until the absolute pressure inside is 1.25 atm and the temperature is 22.0°C. Assume that all the gas is N₂, of molar mass 28.0 g/mol. (a) Find the mass of a single N₂ molecule. (b) How much translational kinetic energy does an average N₂ molecule have? (c) How many N₂ molecules are in this balloon? (d) What is the total translational kinetic energy of all the molecules in the balloon?

18.72 • CP (a) Compute the increase in gravitational potential energy for a nitrogen molecule (molar mass 28.0 g/mol) for an increase in elevation of 400 m near the earth's surface. (b) At what temperature is this equal to the average kinetic energy of a nitrogen molecule? (c) Is it possible that a nitrogen molecule near sea level where $T = 15.0^\circ\text{C}$ could rise to an altitude of 400 m? Is it likely that it could do so without hitting any other molecules along the way? Explain.

18.73 •• CP, CALC The Lennard-Jones Potential. A commonly used potential-energy function for the interaction of two molecules (see Fig. 18.8) is the Lennard-Jones 6-12 potential:

$$U(r) = U_0 \left[\left(\frac{R_0}{r} \right)^{12} - 2 \left(\frac{R_0}{r} \right)^6 \right]$$

where r is the distance between the centers of the molecules and U_0 and R_0 are positive constants. The corresponding force $F(r)$ is given in Eq. (14.26). (a) Graph $U(r)$ and $F(r)$ versus r . (b) Let r_1 be the value of r at which $U(r) = 0$, and let r_2 be the value of r at which $F(r) = 0$. Show the locations of r_1 and r_2 on your graphs of $U(r)$ and $F(r)$. Which of these values represents the equilibrium separation between the molecules? (c) Find the values of r_1 and r_2 in terms of R_0 , and find the ratio r_1/r_2 . (d) If the molecules are located a distance r_2 apart [as calculated in part (c)], how much work must be done to pull them apart so that $r \rightarrow \infty$?

18.74 • (a) What is the total random translational kinetic energy of 5.00 L of hydrogen gas (molar mass 2.016 g/mol) with pressure 1.01×10^5 Pa and temperature 300 K? (*Hint:* Use the procedure of Problem 18.71 as a guide.) (b) If the tank containing the gas is placed on a swift jet moving at 300.0 m/s, by what percentage is the *total* kinetic energy of the gas increased? (c) Since the kinetic energy of the gas molecules is greater when it is on the jet, does this mean that its temperature has gone up? Explain.

18.75 • The speed of propagation of a sound wave in air at 27°C is about 350 m/s. Calculate, for comparison, (a) v_{rms} for nitrogen molecules and (b) the rms value of v_x at this temperature. The molar mass of nitrogen (N_2) is 28.0 g/mol.

18.76 • Hydrogen on the Sun. The surface of the sun has a temperature of about 5800 K and consists largely of hydrogen atoms. (a) Find the rms speed of a hydrogen atom at this temperature. (The mass of a single hydrogen atom is 1.67×10^{-27} kg.) (b) The escape speed for a particle to leave the gravitational influence of the sun is given by $(2GM/R)^{1/2}$, where M is the sun's mass, R its radius, and G the gravitational constant (see Example 13.5 of Section 13.3). Use the data in Appendix F to calculate this escape speed. (c) Can appreciable quantities of hydrogen escape from the sun? Can *any* hydrogen escape? Explain.

18.77 •• CP (a) Show that a projectile with mass m can “escape” from the surface of a planet if it is launched vertically upward with a kinetic energy greater than mgR_p , where g is the acceleration due to gravity at the planet's surface and R_p is the planet's radius. Ignore air resistance. (See Problem 18.76.) (b) If the planet in question is the earth, at what temperature does the average translational kinetic energy of a nitrogen molecule (molar mass 28.0 g/mol) equal that required to escape? What about a hydrogen molecule (molar mass 2.02 g/mol)? (c) Repeat part (b) for the moon, for which $g = 1.63 \text{ m/s}^2$ and $R_p = 1740 \text{ km}$. (d) While the earth and the moon have similar average surface temperatures, the moon has essentially no atmosphere. Use your results from parts (b) and (c) to explain why.

18.78 • Planetary Atmospheres. (a) The temperature near the top of Jupiter's multicolored cloud layer is about 140 K. The temperature at the top of the earth's troposphere, at an altitude of about

20 km, is about 220 K. Calculate the rms speed of hydrogen molecules in both these environments. Give your answers in m/s and as a fraction of the escape speed from the respective planet (see Problem 18.76). (b) Hydrogen gas (H_2) is a rare element in the earth's atmosphere. In the atmosphere of Jupiter, by contrast, 89% of all molecules are H_2 . Explain why, using your results from part (a). (c) Suppose an astronomer claims to have discovered an oxygen (O_2) atmosphere on the asteroid Ceres. How likely is this? Ceres has a mass equal to 0.014 times the mass of the moon, a density of 2400 kg/m^3 , and a surface temperature of about 200 K.

18.79 •• (a) For what mass of molecule or particle is v_{rms} equal to 1.00 mm/s at 300 K? (b) If the particle is an ice crystal, how many molecules does it contain? The molar mass of water is 18.0 g/mol. (c) Calculate the diameter of the particle if it is a spherical piece of ice. Would it be visible to the naked eye?

18.80 •• In describing the heat capacities of solids in Section 18.4, we stated that the potential energy $U = \frac{1}{2}kx^2$ of a harmonic oscillator averaged over one period of the motion is equal to the kinetic energy $K = \frac{1}{2}mv^2$ averaged over one period. Prove this result using Eqs. (14.13) and (14.15) for the position and velocity of a simple harmonic oscillator. For simplicity, assume that the initial position and velocity make the phase angle ϕ equal to zero. (*Hint:* Use the trigonometric identities $\cos^2(\theta) = [1 + \cos(2\theta)]/2$ and $\sin^2(\theta) = [1 - \cos(2\theta)]/2$. What is the average value of $\cos(2\omega t)$ over one period?)

18.81 •• It is possible to make crystalline solids that are only one layer of atoms thick. Such “two-dimensional” crystals can be created by depositing atoms on a very flat surface. (a) If the atoms in such a two-dimensional crystal can move only within the plane of the crystal, what will be its molar heat capacity near room temperature? Give your answer as a multiple of R and in J/mol · K. (b) At very low temperatures, will the molar heat capacity of a two-dimensional crystal be greater than, less than, or equal to the result you found in part (a)? Explain why.

18.82 •• (a) Calculate the total *rotational* kinetic energy of the molecules in 1.00 mol of a diatomic gas at 300 K. (b) Calculate the moment of inertia of an oxygen molecule (O_2) for rotation about either the y - or z -axis shown in Fig. 18.18b. Treat the molecule as two massive points (representing the oxygen atoms) separated by a distance of 1.21×10^{-10} m. The molar mass of oxygen *atoms* is 16.0 g/mol. (c) Find the rms angular velocity of rotation of an oxygen molecule about either the y - or z -axis shown in Fig. 18.18b. How does your answer compare to the angular velocity of a typical piece of rapidly rotating machinery (10,000 rev/min)?

18.83 • For each polyatomic gas in Table 18.1, compute the value of the molar heat capacity at constant volume, C_V , on the assumption that there is no vibrational energy. Compare with the measured values in the table, and compute the fraction of the total heat capacity that is due to vibration for each of the three gases. (*Note:* CO_2 is linear; SO_2 and H_2S are not. Recall that a linear polyatomic molecule has two rotational degrees of freedom, and a nonlinear molecule has three.)

18.84 •• CALC (a) Show that $\int_0^\infty f(v) dv = 1$, where $f(v)$ is the Maxwell-Boltzmann distribution of Eq. (18.32). (b) In terms of the physical definition of $f(v)$, explain why the integral in part (a) *must* have this value.

18.85 •• CALC Calculate the integral in Eq. (18.31), $\int_0^\infty v^2 f(v) dv$, and compare this result to $(v^2)_{\text{av}}$ as given by Eq. (18.16). (*Hint:* You may use the tabulated integral

$$\int_0^\infty x^{2n} e^{-\alpha x^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} \alpha^n} \sqrt{\frac{\pi}{\alpha}}$$

where n is a positive integer and α is a positive constant.)

18.86 •• CALC Calculate the integral in Eq. (18.30), $\int_0^\infty v f(v) dv$, and compare this result to v_{av} as given by Eq. (18.35). (*Hint:* Make the change of variable $v^2 = x$ and use the tabulated integral

$$\int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$$

where n is a positive integer and α is a positive constant.)

18.87 •• CALC (a) Explain why in a gas of N molecules, the number of molecules having speeds in the *finite* interval v to $v + \Delta v$ is $\Delta N = N \int_v^{v+\Delta v} f(v) dv$. (b) If Δv is small, then $f(v)$ is approximately constant over the interval and $\Delta N \approx N f(v) \Delta v$. For oxygen gas (O_2 , molar mass 32.0 g/mol) at $T = 300$ K, use this approximation to calculate the number of molecules with speeds within $\Delta v = 20$ m/s of v_{mp} . Express your answer as a multiple of N . (c) Repeat part (b) for speeds within $\Delta v = 20$ m/s of $7v_{mp}$. (d) Repeat parts (b) and (c) for a temperature of 600 K. (e) Repeat parts (b) and (c) for a temperature of 150 K. (f) What do your results tell you about the shape of the distribution as a function of temperature? Do your conclusions agree with what is shown in Fig. 18.23?

18.88 • Meteorology. The *vapor pressure* is the pressure of the vapor phase of a substance when it is in equilibrium with the solid or liquid phase of the substance. The *relative humidity* is the partial pressure of water vapor in the air divided by the vapor pressure of water at that same temperature, expressed as a percentage. The air is saturated when the humidity is 100%. (a) The vapor pressure of water at 20.0°C is 2.34×10^3 Pa. If the air temperature is 20.0°C and the relative humidity is 60%, what is the partial pressure of water vapor in the atmosphere (that is, the pressure due to water vapor alone)? (b) Under the conditions of part (a), what is the mass of water in 1.00 m³ of air? (The molar mass of water is 18.0 g/mol. Assume that water vapor can be treated as an ideal gas.)

18.89 • The Dew Point. The vapor pressure of water (see Problem 18.88) decreases as the temperature decreases. If the amount of water vapor in the air is kept constant as the air is cooled, a temperature is reached, called the *dew point*, at which the partial pressure and vapor pressure coincide and the vapor is saturated. If the air is cooled further, vapor condenses to liquid until the partial pressure again equals the vapor pressure at that temperature. The temperature in a room is 30.0°C. A meteorologist cools a metal can by gradually adding cold water. When the can temperature reaches 16.0°C, water droplets form on its outside surface. What is the relative humidity of the 30.0°C air in the room? The table lists the vapor pressure of water at various temperatures:

Temperature (°C)	Vapor Pressure (Pa)
10.0	1.23×10^3
12.0	1.40×10^3
14.0	1.60×10^3
16.0	1.81×10^3
18.0	2.06×10^3
20.0	2.34×10^3
22.0	2.65×10^3
24.0	2.99×10^3
26.0	3.36×10^3
28.0	3.78×10^3
30.0	4.25×10^3

18.90 ••• Altitude at Which Clouds Form. On a spring day in the midwestern United States, the air temperature at the surface is

28.0°C. Puffy cumulus clouds form at an altitude where the air temperature equals the dew point (see Problem 18.89). If the air temperature decreases with altitude at a rate of 0.6 C°/100 m, at approximately what height above the ground will clouds form if the relative humidity at the surface is 35% and 80%? (*Hint:* Use the table in Problem 18.89.)

CHALLENGE PROBLEMS

18.91 ••• CP Dark Nebulae and the Interstellar Medium.

The dark area in Fig. P18.91 that appears devoid of stars is a *dark nebula*, a cold gas cloud in interstellar space that contains enough material to block out light from the stars behind it. A typical dark nebula is about 20 light-years in diameter and contains about 50 hydrogen atoms per cubic centimeter (monatomic hydrogen, *not* H₂) at a temperature of about 20 K. (A light-year is the distance light travels in vacuum in one year and is equal to 9.46×10^{15} m.) (a) Estimate the mean free path for a hydrogen atom in a dark nebula. The radius of a hydrogen atom is 5.0×10^{-11} m. (b) Estimate the rms speed of a hydrogen atom and the mean free time (the average time between collisions for a given atom). Based on this result, do you think that atomic collisions, such as those leading to H₂ molecule formation, are very important in determining the composition of the nebula? (c) Estimate the pressure inside a dark nebula. (d) Compare the rms speed of a hydrogen atom to the escape speed at the surface of the nebula (assumed spherical). If the space around the nebula were a vacuum, would such a cloud be stable or would it tend to evaporate? (e) The stability of dark nebulae is explained by the presence of the *interstellar medium* (ISM), an even thinner gas that permeates space and in which the dark nebulae are embedded. Show that for dark nebulae to be in equilibrium with the ISM, the numbers of atoms per volume (N/V) and the temperatures (T) of dark nebulae and the ISM must be related by

$$\frac{(N/V)_{\text{nebula}}}{(N/V)_{\text{ISM}}} = \frac{T_{\text{ISM}}}{T_{\text{nebula}}}$$

(f) In the vicinity of the sun, the ISM contains about 1 hydrogen atom per 200 cm³. Estimate the temperature of the ISM in the vicinity of the sun. Compare to the temperature of the sun's surface, about 5800 K. Would a spacecraft coasting through interstellar space burn up? Why or why not?

Figure P18.91



18.92 ••• CALC Earth's Atmosphere. In the *troposphere*, the part of the atmosphere that extends from earth's surface to an altitude

of about 11 km, the temperature is not uniform but decreases with increasing elevation. (a) Show that if the temperature variation is approximated by the linear relationship

$$T = T_0 - \alpha y$$

where T_0 is the temperature at the earth's surface and T is the temperature at height y , the pressure p at height y is given by

$$\ln\left(\frac{p}{p_0}\right) = \frac{Mg}{R\alpha} \ln\left(\frac{T_0 - \alpha y}{T_0}\right)$$

where p_0 is the pressure at the earth's surface and M is the molar mass for air. The coefficient α is called the lapse rate of temperature. It varies with atmospheric conditions, but an average value is about $0.6\text{ C}^\circ/100\text{ m}$. (b) Show that the above result reduces to the result of Example 18.4 (Section 18.1) in the limit that $\alpha \rightarrow 0$. (c) With $\alpha = 0.6\text{ C}^\circ/100\text{ m}$, calculate p for $y = 8863\text{ m}$ and compare your answer to the result of Example 18.4. Take $T_0 = 288\text{ K}$ and $p_0 = 1.00\text{ atm}$.

Answers

Chapter Opening Question ?

From Eq. (18.19), the root-mean-square speed of a gas molecule is proportional to the square root of the absolute temperature T . The temperature range we're considering is from $(25 + 273.15)\text{ K} = 298\text{ K}$ to $(100 + 273.15)\text{ K} = 373\text{ K}$. Hence the speeds increase by a factor of $\sqrt{(373\text{ K})/(298\text{ K})} = 1.12$; that is, there is a 12% increase. While 100°C feels far warmer than 25°C , the difference in molecular speeds is relatively small.

Test Your Understanding Questions

18.1 Answer: (ii) and (iii) (tie), (i) and (v) (tie), (iv) We can rewrite the ideal-gas equation, Eq. (18.3), as $n = pV/RT$. This tells us that the number of moles n is proportional to the pressure and volume and inversely proportional to the absolute temperature. Hence, compared to (i), the number of moles in each case is (ii) $(2)(1)/(1) = 2$ times as much, (iii) $(1)(2)/(1) = 2$ times as much, (iv) $(1)(1)/(2) = \frac{1}{2}$ as much, and (v) $(2)(1)/(2) = 1$ time as much (that is, equal).

18.2 Answer: (vi) The value of r_0 determines the equilibrium separation of the molecules in the solid phase, so doubling r_0 means that the separation doubles as well. Hence a solid cube of this compound might grow from 1 cm on a side to 2 cm on a side. The volume would then be $2^3 = 8$ times larger, and the density (mass divided by volume) would be $\frac{1}{8}$ as great.

18.3 Answers: (a) (iv), (ii), (iii), (i); (b) (iii) and (iv) (tie), (i) and (ii) (tie) (a) Equation (18.19) tells us that $v_{\text{rms}} = \sqrt{3RT/M}$, so the rms speed is proportional to the square root of the ratio of absolute temperature T to molar mass M . Compared to (i) oxygen at 300 K, v_{rms} in the other cases is (ii) $\sqrt{(32.0\text{ g/mol})/(28.0\text{ g/mol})} = 1.07$ times faster, (iii) $\sqrt{(330\text{ K})/(300\text{ K})} = 1.05$ times faster, and (iv) $\sqrt{(330\text{ K})(32.0\text{ g/mol})/(300\text{ K})(28.0\text{ g/mol})} = 1.12$ times faster. (b) From Eq. (18.16), the average translational kinetic energy per molecule is $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT$, which is directly

18.93 ••• In Example 18.7 (Section 18.3) we saw that $v_{\text{rms}} > v_{\text{av}}$. It is not difficult to show that this is *always* the case. (The only exception is when the particles have the same speed, in which case $v_{\text{rms}} = v_{\text{av}}$.) (a) For two particles with speeds v_1 and v_2 , show that $v_{\text{rms}} \geq v_{\text{av}}$, regardless of the numerical values of v_1 and v_2 . Then show that $v_{\text{rms}} > v_{\text{av}}$ if $v_1 \neq v_2$. (b) Suppose that for a collection of N particles you know that $v_{\text{rms}} > v_{\text{av}}$. Another particle, with speed u , is added to the collection of particles. If the new rms and average speeds are denoted as v'_{rms} and v'_{av} , show that

$$v'_{\text{rms}} = \sqrt{\frac{Nv_{\text{rms}}^2 + u^2}{N + 1}} \quad \text{and} \quad v'_{\text{av}} = \frac{Nv_{\text{av}} + u}{N + 1}$$

(c) Use the expressions in part (b) to show that $v'_{\text{rms}} > v'_{\text{av}}$ regardless of the numerical value of u . (d) Explain why your results for (a) and (c) together show that $v_{\text{rms}} > v_{\text{av}}$ for any collection of particles if the particles do not all have the same speed.

proportional to T and independent of M . We have $T = 300\text{ K}$ for cases (i) and (ii) and $T = 330\text{ K}$ for cases (iii) and (iv), so $\frac{1}{2}m(v^2)_{\text{av}}$ has equal values for cases (iii) and (iv) and equal (but smaller) values for cases (i) and (ii).

18.4 Answers: no, near the beginning Adding a small amount of heat dQ to the gas changes the temperature by dT , where $dQ = nC_V dT$ from Eq. (18.24). Figure 18.19 shows that C_V for H_2 varies with temperature between 25 K and 500 K, so a given amount of heat gives rise to different amounts of temperature change during the process. Hence the temperature will *not* increase at a constant rate. The temperature change $dT = dQ/nC_V$ is inversely proportional to C_V , so the temperature increases most rapidly at the beginning of the process when the temperature is lowest and C_V is smallest (see Fig. 18.19).

18.5 Answer: (ii) Figure 18.23b shows that the *fraction* of molecules with speeds between v_1 and v_2 equals the area under the curve of $f(v)$ versus v from $v = v_1$ to $v = v_2$. This is equal to the integral $\int_{v_1}^{v_2} f(v) dv$, which in turn is equal to the difference between the integrals $\int_0^{v_2} f(v) dv$ (the fraction of molecules with speeds from 0 to v_2) and $\int_0^{v_1} f(v) dv$ (the fraction of molecules with speeds from 0 to the slower speed v_1). The *number* of molecules with speeds from v_1 to v_2 equals the fraction of molecules in this speed range multiplied by N , the total number of molecules.

18.6 Answers: no, yes The triple-point pressure of water from Table 18.3 is $6.10 \times 10^2\text{ Pa}$. The present-day pressure on Mars is just less than this value, corresponding to the line labeled p_s in Fig. 18.24. Hence liquid water cannot exist on the present-day Martian surface, and there are no rivers or lakes. Planetary scientists conclude that liquid water could have and almost certainly did exist on Mars in the past, when the atmosphere was thicker.

Bridging Problem

Answers: (a) 102 km (b) yes

19 THE FIRST LAW OF THERMODYNAMICS

LEARNING GOALS

By studying this chapter, you will learn:

- How to represent heat transfer and work done in a thermodynamic process.
- How to calculate the work done by a thermodynamic system when its volume changes.
- What is meant by a path between thermodynamic states.
- How to use the first law of thermodynamics to relate heat transfer, work done, and internal energy change.
- How to distinguish among adiabatic, isochoric, isobaric, and isothermal processes.
- How we know that the internal energy of an ideal gas depends only on its temperature.
- The difference between molar heat capacities at constant volume and at constant pressure, and how to use these quantities in calculations.
- How to analyze adiabatic processes in an ideal gas.

19.1 The popcorn in the pot is a thermodynamic system. In the thermodynamic process shown here, heat is added to the system, and the system does work on its surroundings to lift the lid of the pot.



? A steam locomotive operates using the first law of thermodynamics: Water is heated and boils, and the expanding steam does work to propel the locomotive. Would it be possible for the steam to propel the locomotive by doing work as it *condenses*?

Every time you drive a car, turn on an air conditioner, or cook a meal, you reap the practical benefits of *thermodynamics*, the study of relationships involving heat, mechanical work, and other aspects of energy and energy transfer. For example, in a car engine heat is generated by the chemical reaction of oxygen and vaporized gasoline in the engine's cylinders. The heated gas pushes on the pistons within the cylinders, doing mechanical work that is used to propel the car. This is an example of a *thermodynamic process*.

The first law of thermodynamics, central to the understanding of such processes, is an extension of the principle of conservation of energy. It broadens this principle to include energy exchange by both heat transfer and mechanical work and introduces the concept of the *internal energy* of a system. Conservation of energy plays a vital role in every area of physical science, and the first law has extremely broad usefulness. To state energy relationships precisely, we need the concept of a *thermodynamic system*. We'll discuss *heat* and *work* as two means of transferring energy into or out of such a system.

19.1 Thermodynamic Systems

We have studied energy transfer through mechanical work (Chapter 6) and through heat transfer (Chapters 17 and 18). Now we are ready to combine and generalize these principles.

We always talk about energy transfer to or from some specific *system*. The system might be a mechanical device, a biological organism, or a specified quantity of material, such as the refrigerant in an air conditioner or steam expanding in a turbine. In general, a **thermodynamic system** is any collection of objects that is convenient to regard as a unit, and that may have the potential to exchange energy with its surroundings. A familiar example is a quantity of popcorn kernels in a pot with a lid. When the pot is placed on a stove, energy is added to the popcorn

by conduction of heat. As the popcorn pops and expands, it does work as it exerts an upward force on the lid and moves it through a displacement (Fig. 19.1). The *state* of the popcorn changes in this process, since the volume, temperature, and pressure of the popcorn all change as it pops. A process such as this one, in which there are changes in the state of a thermodynamic system, is called a **thermodynamic process**.

In mechanics we used the concept of *system* with free-body diagrams and with conservation of energy and momentum. For *thermodynamic* systems, as for all others, it is essential to define clearly at the start exactly what is and is not included in the system. Only then can we describe unambiguously the energy transfers into and out of that system. For instance, in our popcorn example we defined the system to include the popcorn but not the pot, lid, or stove.

Thermodynamics has its roots in many practical problems other than popping popcorn (Fig. 19.2). The gasoline engine in an automobile, the jet engines in an airplane, and the rocket engines in a launch vehicle use the heat of combustion of their fuel to perform mechanical work in propelling the vehicle. Muscle tissue in living organisms metabolizes chemical energy in food and performs mechanical work on the organism's surroundings. A steam engine or steam turbine uses the heat of combustion of coal or other fuel to perform mechanical work such as driving an electric generator or pulling a train.

Signs for Heat and Work in Thermodynamics

We describe the energy relationships in any thermodynamic process in terms of the quantity of heat Q added *to* the system and the work W done *by* the system. Both Q and W may be positive, negative, or zero (Fig. 19.3). A positive value of Q represents heat flow *into* the system, with a corresponding input of energy to it; negative Q represents heat flow *out of* the system. A positive value of W represents work done *by* the system against its surroundings, such as work done by an expanding gas, and hence corresponds to energy *leaving* the system. Negative W , such as work done during compression of a gas in which work is done *on the gas* by its surroundings, represents energy *entering* the system. We will use these conventions consistently in the examples in this chapter and the next.

CAUTION Be careful with the sign of work W Note that our sign rule for work is *opposite* to the one we used in mechanics, in which we always spoke of the work done by the forces acting *on* a body. In thermodynamics it is usually more convenient to call W the work done *by* the system so that when a system expands, the pressure, volume change, and work are all positive. Take care to use the sign rules for work and heat consistently!

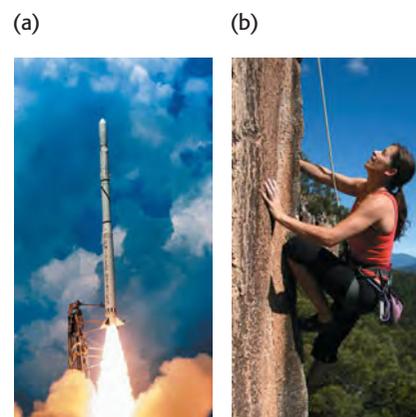
Test Your Understanding of Section 19.1 In Example 17.8 (Section 17.6), what is the sign of Q for the coffee? For the aluminum cup? If a block slides along a horizontal surface with friction, what is the sign of W for the block?

19.2 Work Done During Volume Changes

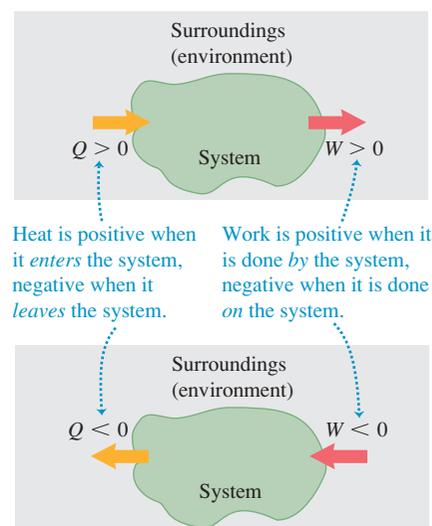
A simple but common example of a thermodynamic system is a quantity of gas enclosed in a cylinder with a movable piston. Internal-combustion engines, steam engines, and compressors in refrigerators and air conditioners all use some version of such a system. In the next several sections we will use the gas-in-cylinder system to explore several kinds of processes involving energy transformations.

We'll use a microscopic viewpoint, based on the kinetic and potential energies of individual molecules in a material, to develop intuition about thermodynamic quantities. But it is important to understand that the central principles of thermodynamics can be treated in a completely *macroscopic* way, without reference to microscopic models. Indeed, part of the great power and generality of thermodynamics is that it does *not* depend on details of the structure of matter.

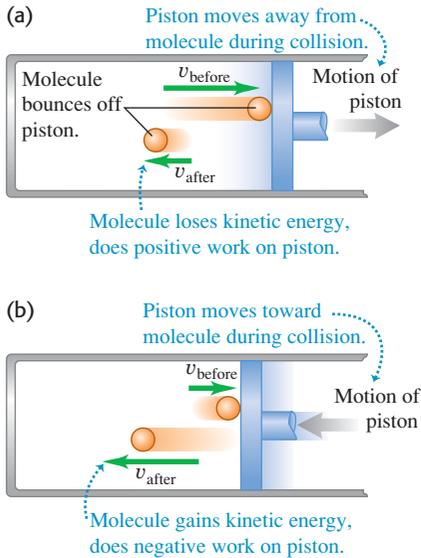
19.2 (a) A rocket engine uses the heat of combustion of its fuel to do work propelling the launch vehicle. (b) Humans and other biological organisms are more complicated systems than we can analyze fully in this book, but the same basic principles of thermodynamics apply to them.



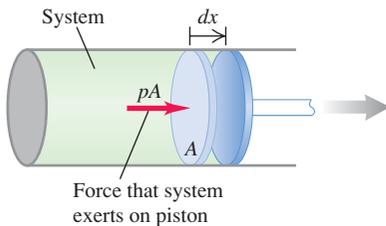
19.3 A thermodynamic system may exchange energy with its surroundings (environment) by means of heat, work, or both. Note the sign conventions for Q and W .



19.4 A molecule striking a piston (a) does positive work if the piston is moving away from the molecule and (b) does negative work if the piston is moving toward the molecule. Hence a gas does positive work when it expands as in (a) but does negative work when it compresses as in (b).



19.5 The infinitesimal work done by the system during the small expansion dx is $dW = pA dx$.



First we consider the *work* done by the system during a volume change. When a gas expands, it pushes outward on its boundary surfaces as they move outward. Hence an expanding gas always does positive work. The same thing is true of any solid or fluid material that expands under pressure, such as the popcorn in Fig. 19.1.

We can understand the work done by a gas in a volume change by considering the molecules that make up the gas. When one such molecule collides with a stationary surface, it exerts a momentary force on the wall but does no work because the wall does not move. But if the surface is moving, like a piston in a gasoline engine, the molecule *does* do work on the surface during the collision. If the piston in Fig. 19.4a moves to the right, so that the volume of the gas increases, the molecules that strike the piston exert a force through a distance and do *positive* work on the piston. If the piston moves toward the left as in Fig. 19.4b, so the volume of the gas decreases, then positive work is done *on* the molecule during the collision. Hence the gas molecules do *negative* work on the piston.

Figure 19.5 shows a system whose volume can change (a gas, liquid, or solid) in a cylinder with a movable piston. Suppose that the cylinder has cross-sectional area A and that the pressure exerted by the system at the piston face is p . The total force F exerted by the system on the piston is $F = pA$. When the piston moves out an infinitesimal distance dx , the work dW done by this force is

$$dW = F dx = pA dx$$

But

$$A dx = dV$$

where dV is the infinitesimal change of volume of the system. Thus we can express the work done by the system in this infinitesimal volume change as

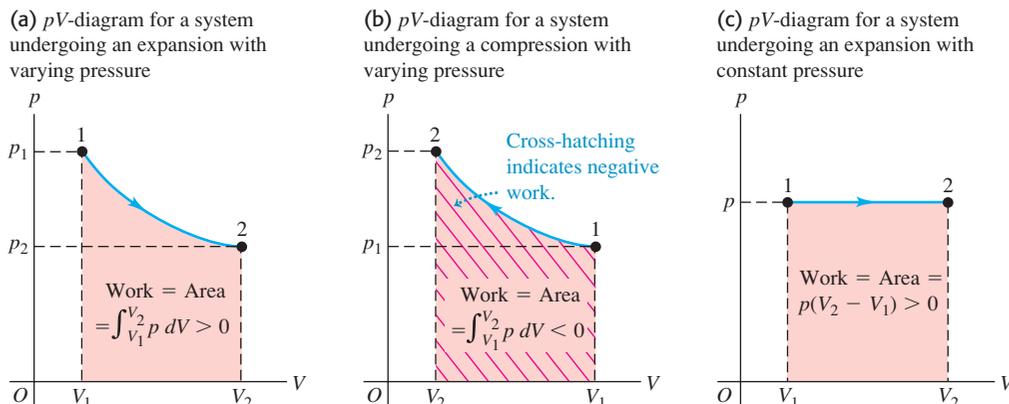
$$dW = p dV \tag{19.1}$$

In a finite change of volume from V_1 to V_2 ,

$$W = \int_{V_1}^{V_2} p dV \quad (\text{work done in a volume change}) \tag{19.2}$$

In general, the pressure of the system may vary during the volume change. For example, this is the case in the cylinders of an automobile engine as the pistons move back and forth. To evaluate the integral in Eq. (19.2), we have to know how the pressure varies as a function of volume. We can represent this relationship as a graph of p as a function of V (a pV -diagram, described at the end of Section 18.1). Figure 19.6 a shows a simple example. In this figure, Eq. (19.2) is represented

19.6 The work done equals the area under the curve on a pV -diagram.



graphically as the *area* under the curve of p versus V between the limits V_1 and V_2 . (In Section 6.3 we used a similar interpretation of the work done by a force F as the area under the curve of F versus x between the limits x_1 and x_2 .)

According to the rule we stated in Section 19.1, work is *positive* when a system *expands*. In an expansion from state 1 to state 2 in Fig. 19.6a, the area under the curve and the work are positive. A *compression* from 1 to 2 in Fig. 19.6b gives a *negative* area; when a system is compressed, its volume decreases and it does *negative* work on its surroundings (see also Fig. 19.4b).

CAUTION Be careful with subscripts 1 and 2 When using Eq. (19.2), always remember that V_1 is the *initial* volume and V_2 is the *final* volume. That's why the labels 1 and 2 are reversed in Fig. 19.6b compared to Fig. 19.6a, even though both processes move between the same two thermodynamic states. 

If the pressure p remains constant while the volume changes from V_1 to V_2 (Fig. 19.6c), the work done by the system is

$$W = p(V_2 - V_1) \quad \text{(work done in a volume change at constant pressure)} \quad (19.3)$$

MasteringPHYSICS
ActivPhysics 8.5: Work Done By a Gas

In any process in which the volume is *constant*, the system does no work because there is no displacement.

Example 19.1 Isothermal expansion of an ideal gas

As an ideal gas undergoes an *isothermal* (constant-temperature) expansion at temperature T , its volume changes from V_1 to V_2 . How much work does the gas do?

SOLUTION

IDENTIFY and SET UP: The ideal-gas equation, Eq. (18.3), tells us that if the temperature T of n moles of an ideal gas is constant, the quantity $pV = nRT$ is also constant: p and V are inversely related. If V changes, p changes as well, so we *cannot* use Eq. (19.3) to calculate the work done. Instead we must use Eq. (19.2). To evaluate the integral in Eq. (19.2) we must know p as a function of V ; for this we use Eq. (18.3).

EXECUTE: From Eq. (18.3),

$$p = \frac{nRT}{V}$$

We substitute this into the integral of Eq. (19.2), take the constant factor nRT outside, and evaluate the integral:

$$\begin{aligned} W &= \int_{V_1}^{V_2} p \, dV \\ &= nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln \frac{V_2}{V_1} \quad \text{(ideal gas, isothermal process)} \end{aligned}$$

We can rewrite this expression for W in terms of p_1 and p_2 . Because $pV = nRT$ is constant,

$$p_1 V_1 = p_2 V_2 \quad \text{or} \quad \frac{V_2}{V_1} = \frac{p_1}{p_2}$$

so

$$W = nRT \ln \frac{p_1}{p_2} \quad \text{(ideal gas, isothermal process)}$$

EVALUATE: We check our result by noting that in an expansion $V_2 > V_1$ and the ratio V_2/V_1 is greater than 1. The logarithm of a number greater than 1 is positive, so $W > 0$, as it should be. As an additional check, look at our second expression for W : In an isothermal expansion the volume increases and the pressure drops, so $p_2 < p_1$, the ratio $p_1/p_2 > 1$, and $W = nRT \ln(p_1/p_2)$ is again positive.

These results also apply to an isothermal *compression* of a gas, for which $V_2 < V_1$ and $p_2 > p_1$.

Test Your Understanding of Section 19.2 A quantity of ideal gas undergoes an expansion that increases its volume from V_1 to $V_2 = 2V_1$. The final pressure of the gas is p_2 . Does the gas do more work on its surroundings if the expansion is at constant *pressure* or at constant *temperature*? (i) constant pressure; (ii) constant temperature; (iii) the same amount of work is done in both cases; (iv) not enough information is given to decide.



19.3 Paths Between Thermodynamic States

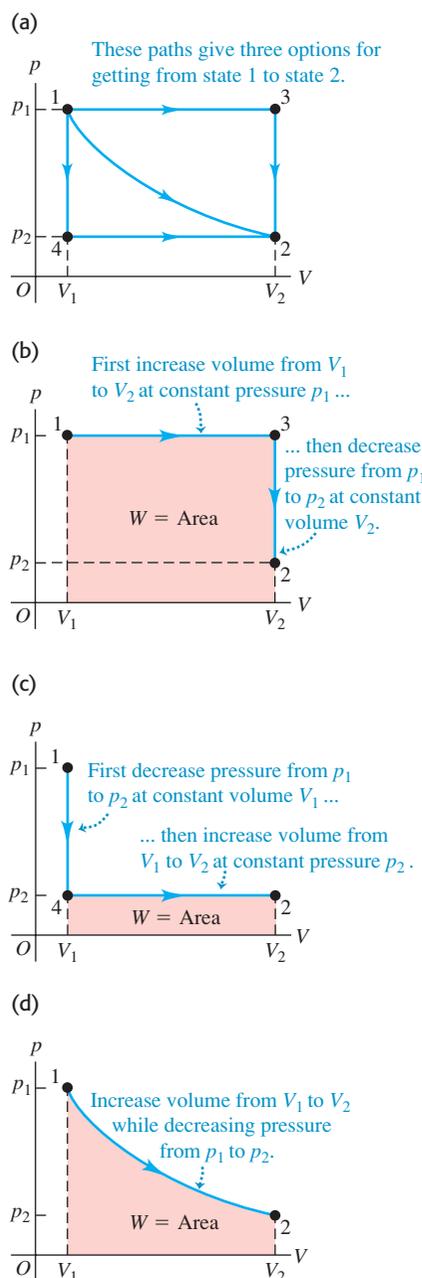
We've seen that if a thermodynamic process involves a change in volume, the system undergoing the process does work (either positive or negative) on its surroundings. Heat also flows into or out of the system during the process if there is a temperature difference between the system and its surroundings. Let's now examine how the work done by and the heat added to the system during a thermodynamic process depend on the details of how the process takes place.

Work Done in a Thermodynamic Process

When a thermodynamic system changes from an initial state to a final state, it passes through a series of intermediate states. We call this series of states a **path**. There are always infinitely many different possibilities for these intermediate states. When they are all equilibrium states, the path can be plotted on a pV -diagram (Fig. 19.7a). Point 1 represents an initial state with pressure p_1 and volume V_1 , and point 2 represents a final state with pressure p_2 and volume V_2 . To pass from state 1 to state 2, we could keep the pressure constant at p_1 while the system expands to volume V_2 (point 3 in Fig. 19.7b), then reduce the pressure to p_2 (probably by decreasing the temperature) while keeping the volume constant at V_2 (to point 2 on the diagram). The work done by the system during this process is the area under the line $1 \rightarrow 3$; no work is done during the constant-volume process $3 \rightarrow 2$. Or the system might traverse the path $1 \rightarrow 4 \rightarrow 2$ (Fig. 19.7c); in that case the work is the area under the line $4 \rightarrow 2$, since no work is done during the constant-volume process $1 \rightarrow 4$. The smooth curve from 1 to 2 is another possibility (Fig. 19.7d), and the work for this path is different from that for either of the other paths.

We conclude that *the work done by the system depends not only on the initial and final states, but also on the intermediate states—that is, on the path*. Furthermore, we can take the system through a series of states forming a closed loop, such as $1 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$. In this case the final state is the same as the initial state, but the total work done by the system is *not* zero. (In fact, it is represented on the graph by the area enclosed by the loop; can you prove that? See Exercise 19.7.) It follows that it doesn't make sense to talk about the amount of work *contained* in a system. In a particular state, a system may have definite values of the state coordinates p , V , and T , but it wouldn't make sense to say that it has a definite value of W .

19.7 The work done by a system during a transition between two states depends on the path chosen.



Heat Added in a Thermodynamic Process

Like work, the *heat* added to a thermodynamic system when it undergoes a change of state depends on the path from the initial state to the final state. Here's an example. Suppose we want to change the volume of a certain quantity of an ideal gas from 2.0 L to 5.0 L while keeping the temperature constant at $T = 300$ K. Figure 19.8 shows two different ways in which we can do this. In Fig. 19.8a the gas is contained in a cylinder with a piston, with an initial volume of 2.0 L. We let the gas expand slowly, supplying heat from the electric heater to keep the temperature at 300 K. After expanding in this slow, controlled, isothermal manner, the gas reaches its final volume of 5.0 L; it absorbs a definite amount of heat in the process.

Figure 19.8b shows a different process leading to the same final state. The container is surrounded by insulating walls and is divided by a thin, breakable partition into two compartments. The lower part has volume 2.0 L and the upper part has volume 3.0 L. In the lower compartment we place the same amount of the same gas as in Fig. 19.8a, again at $T = 300$ K. The initial state is the same as before. Now we break the partition; the gas undergoes a rapid, uncontrolled expansion, with no heat passing through the insulating walls. The final volume is 5.0 L, the same as in Fig. 19.8a. The gas does no work during this expansion

because it doesn't push against anything that moves. This uncontrolled expansion of a gas into vacuum is called a **free expansion**; we will discuss it further in Section 19.6.

Experiments have shown that when an ideal gas undergoes a free expansion, there is no temperature change. Therefore the final state of the gas is the same as in Fig. 19.8a. The intermediate states (pressures and volumes) during the transition from state 1 to state 2 are entirely different in the two cases; Figs. 19.8a and 19.8b represent *two different paths* connecting the *same states* 1 and 2. For the path in Fig. 19.8b, *no heat is transferred into the system*, and the system does no work. Like work, *heat depends not only on the initial and final states but also on the path*.

Because of this path dependence, it would not make sense to say that a system "contains" a certain quantity of heat. To see this, suppose we assign an arbitrary value to the "heat in a body" in some reference state. Then presumably the "heat in the body" in some other state would equal the heat in the reference state plus the heat added when the body goes to the second state. But that's ambiguous, as we have just seen; the heat added depends on the *path* we take from the reference state to the second state. We are forced to conclude that there is *no* consistent way to define "heat in a body"; it is not a useful concept.

While it doesn't make sense to talk about "work in a body" or "heat in a body," it *does* make sense to speak of the amount of *internal energy* in a body. This important concept is our next topic.

Test Your Understanding of Section 19.3 The system described in Fig. 19.7a undergoes four different thermodynamic processes. Each process is represented in a pV -diagram as a straight line from the initial state to the final state. (These processes are different from those shown in the pV -diagrams of Fig. 19.7.) Rank the processes in order of the amount of work done by the system, from the most positive to the most negative. (i) $1 \rightarrow 2$; (ii) $2 \rightarrow 1$; (iii) $3 \rightarrow 4$; (iv) $4 \rightarrow 3$.

19.4 Internal Energy and the First Law of Thermodynamics

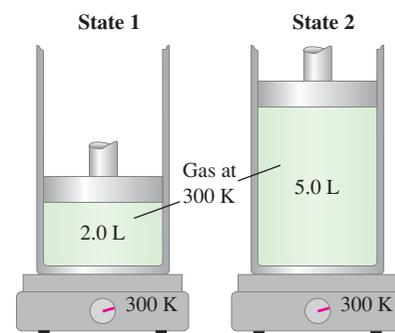
Internal energy is one of the most important concepts in thermodynamics. In Section 7.3, when we discussed energy changes for a body sliding with friction, we stated that warming a body increased its internal energy and that cooling the body decreased its internal energy. But what *is* internal energy? We can look at it in various ways; let's start with one based on the ideas of mechanics. Matter consists of atoms and molecules, and these are made up of particles having kinetic and potential energies. We *tentatively* define the **internal energy** of a system as the sum of the kinetic energies of all of its constituent particles, plus the sum of all the potential energies of interaction among these particles.

CAUTION **Is it internal?** Note that internal energy does *not* include potential energy arising from the interaction between the system and its surroundings. If the system is a glass of water, placing it on a high shelf increases the gravitational potential energy arising from the interaction between the glass and the earth. But this has no effect on the interaction between the molecules of the water, and so the internal energy of the water does not change.

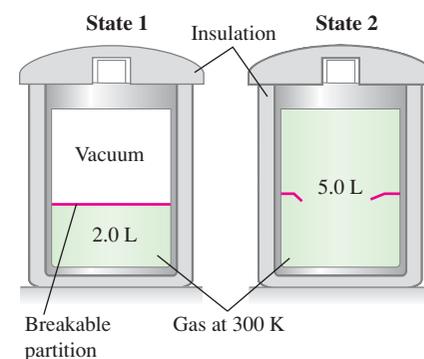
We use the symbol U for internal energy. (We used this same symbol in our study of mechanics to represent potential energy. You may have to remind yourself occasionally that U has a different meaning in thermodynamics.) During a change of state of the system, the internal energy may change from an initial value U_1 to a final value U_2 . We denote the change in internal energy as $\Delta U = U_2 - U_1$.

19.8 (a) Slow, controlled isothermal expansion of a gas from an initial state 1 to a final state 2 with the same temperature but lower pressure. (b) Rapid, uncontrolled expansion of the same gas starting at the same state 1 and ending at the same state 2.

(a) System does work on piston; hot plate adds heat to system ($W > 0$ and $Q > 0$).



(b) System does no work; no heat enters or leaves system ($W = 0$ and $Q = 0$).

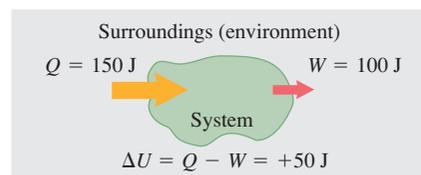


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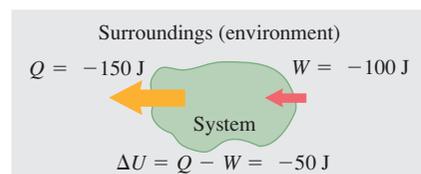
ActivPhysics 8.6: Heat, Internal Energy, and First Law of Thermodynamics

19.9 In a thermodynamic process, the internal energy U of a system may (a) increase ($\Delta U > 0$), (b) decrease ($\Delta U < 0$), or (c) remain the same ($\Delta U = 0$).

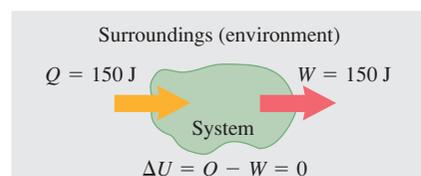
(a) More heat is added to system than system does work: Internal energy of system increases.



(b) More heat flows out of system than work is done: Internal energy of system decreases.

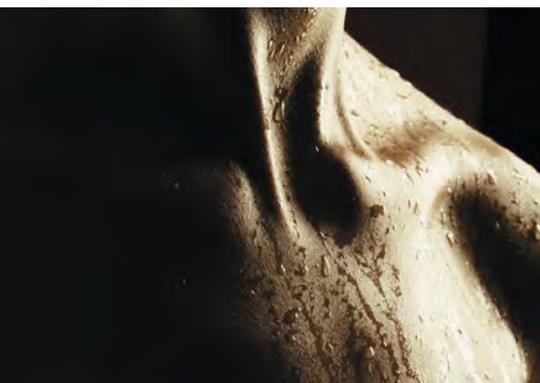


(c) Heat added to system equals work done by system: Internal energy of system unchanged.



Application The First Law of Exercise Thermodynamics

Your body is a thermodynamic system. When you exercise, your body does work (such as the work done to lift your body as a whole in a push-up). Hence $W > 0$. Your body also warms up during exercise; by perspiration and other means the body rids itself of this heat, so $Q < 0$. Since Q is negative and W is positive, $\Delta U = Q - W < 0$ and the body's internal energy decreases. That's why exercise helps you lose weight: It uses up some of the internal energy stored in your body in the form of fat.



When we add a quantity of heat Q to a system and the system does no work during the process (so $W = 0$), the internal energy increases by an amount equal to Q ; that is, $\Delta U = Q$. When a system does work W by expanding against its surroundings and no heat is added during the process, energy leaves the system and the internal energy decreases: W is positive, Q is zero, and $\Delta U = -W$. When both heat transfer and work occur, the *total* change in internal energy is

$$U_2 - U_1 = \Delta U = Q - W \quad (\text{first law of thermodynamics}) \quad (19.4)$$

We can rearrange this to the form

$$Q = \Delta U + W \quad (19.5)$$

The message of Eq. (19.5) is that in general, when heat Q is added to a system, some of this added energy remains within the system, changing its internal energy by an amount ΔU ; the remainder leaves the system again as the system does work W against its surroundings. Because W and Q may be positive, negative, or zero, ΔU can be positive, negative, or zero for different processes (Fig. 19.9).

Equation (19.4) or (19.5) is the **first law of thermodynamics**. It is a generalization of the principle of conservation of energy to include energy transfer through heat as well as mechanical work. As you will see in later chapters, this principle can be extended to ever-broader classes of phenomena by identifying additional forms of energy and energy transfer. In every situation in which it seems that the total energy in all known forms is not conserved, it has been possible to identify a new form of energy such that the total energy, including the new form, *is* conserved. There is energy associated with electric fields, with magnetic fields, and, according to the theory of relativity, even with mass itself.

Understanding the First Law of Thermodynamics

At the beginning of this discussion we tentatively defined internal energy in terms of microscopic kinetic and potential energies. This has drawbacks, however. Actually *calculating* internal energy in this way for any real system would be hopelessly complicated. Furthermore, this definition isn't an *operational* one because it doesn't describe how to determine internal energy from physical quantities that we can measure directly.

So let's look at internal energy in another way. Starting over, we define the *change* in internal energy ΔU during any change of a system as the quantity given by Eq. (19.4), $\Delta U = Q - W$. This *is* an operational definition because we can measure Q and W . It does not define U itself, only ΔU . This is not a shortcoming because we can *define* the internal energy of a system to have a specified value in some reference state, and then use Eq. (19.4) to define the internal energy in any other state. This is analogous to our treatment of potential energy in Chapter 7, in which we arbitrarily defined the potential energy of a mechanical system to be zero at a certain position.

This new definition trades one difficulty for another. If we define ΔU by Eq. (19.4), then when the system goes from state 1 to state 2 by two different paths, how do we know that ΔU is the same for the two paths? We have already seen that Q and W are, in general, *not* the same for different paths. If ΔU , which equals $Q - W$, is also path dependent, then ΔU is ambiguous. If so, the concept of internal energy of a system is subject to the same criticism as the erroneous concept of quantity of heat in a system, as we discussed at the end of Section 19.3.

The only way to answer this question is through *experiment*. For various materials we measure Q and W for various changes of state and various paths to learn whether ΔU is or is not path dependent. The results of many such investigations are clear and unambiguous: While Q and W depend on the path, $\Delta U = Q - W$ is *independent of path*. The change in internal energy of a system

during any thermodynamic process depends only on the initial and final states, not on the path leading from one to the other.

Experiment, then, is the ultimate justification for believing that a thermodynamic system in a specific state has a unique internal energy that depends only on that state. An equivalent statement is that the internal energy U of a system is a function of the state coordinates p , V , and T (actually, any two of these, since the three variables are related by the equation of state).

To say that the first law of thermodynamics, given by Eq. (19.4) or (19.5), represents conservation of energy for thermodynamic processes is correct, as far as it goes. But an important *additional* aspect of the first law is the fact that internal energy depends only on the state of a system (Fig. 19.10). In changes of state, the change in internal energy is independent of the path.

All this may seem a little abstract if you are satisfied to think of internal energy as microscopic mechanical energy. There's nothing wrong with that view, and we will make use of it at various times during our discussion. But in the interest of precise *operational* definitions, internal energy, like heat, can and must be defined in a way that is independent of the detailed microscopic structure of the material.

Cyclic Processes and Isolated Systems

Two special cases of the first law of thermodynamics are worth mentioning. A process that eventually returns a system to its initial state is called a *cyclic process*. For such a process, the final state is the same as the initial state, and so the *total* internal energy change must be zero. Then

$$U_2 = U_1 \quad \text{and} \quad Q = W$$

If a net quantity of work W is done by the system during this process, an equal amount of energy must have flowed into the system as heat Q . But there is no reason either Q or W individually has to be zero (Fig. 19.11).

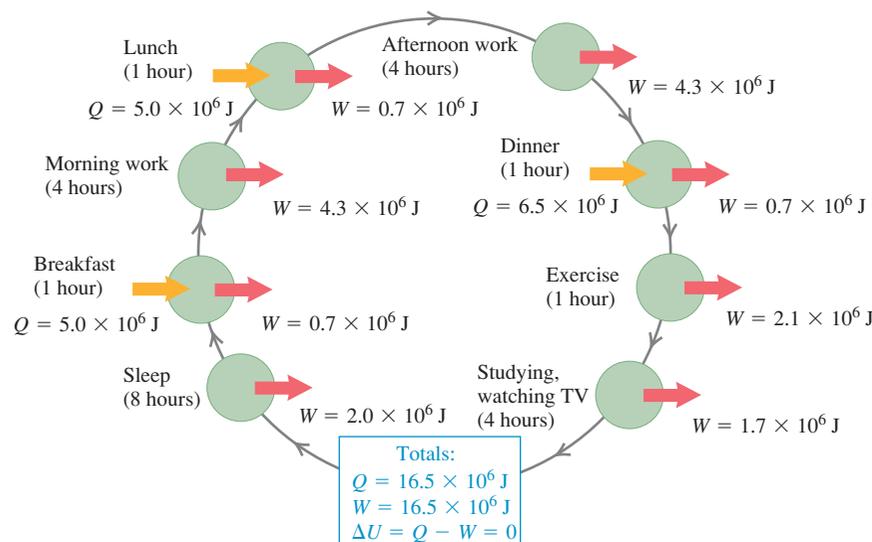
Another special case occurs in an *isolated system*, one that does no work on its surroundings and has no heat flow to or from its surroundings. For any process taking place in an isolated system,

$$W = Q = 0$$

and therefore

$$U_2 = U_1 = \Delta U = 0$$

In other words, *the internal energy of an isolated system is constant.*



19.10 The internal energy of a cup of coffee depends on just its thermodynamic state—how much water and ground coffee it contains, and what its temperature is. It does not depend on the history of how the coffee was prepared—that is, the thermodynamic path that led to its current state.



19.11 Every day, your body (a thermodynamic system) goes through a cyclic thermodynamic process like this one. Heat Q is added by metabolizing food, and your body does work W in breathing, walking, and other activities. If you return to the same state at the end of the day, $Q = W$ and the net change in your internal energy is zero.

Problem-Solving Strategy 19.1 The First Law of Thermodynamics



IDENTIFY *the relevant concepts:* The first law of thermodynamics is the statement of the law of conservation of energy in its most general form. You can apply it to *any* thermodynamic process in which the internal energy of a system changes, heat flows into or out of the system, and/or work is done by or on the system.

SET UP *the problem* using the following steps:

1. Define the thermodynamic system to be considered.
2. If the thermodynamic process has more than one step, identify the initial and final states for each step.
3. List the known and unknown quantities and identify the target variables.
4. Confirm that you have enough equations. You can apply the first law, $\Delta U = Q - W$, just once to each step in a thermodynamic process, so you will often need additional equations. These may include Eq. (19.2), $W = \int_{V_1}^{V_2} p dV$, which gives the work W done in a volume change, and the equation of state of the material that makes up the thermodynamic system (for an ideal gas, $pV = nRT$).

EXECUTE *the solution* as follows:

1. Be sure to use consistent units. If p is in Pa and V in m^3 , then W is in joules. If a heat capacity is given in terms of calories,

convert it to joules. When you use $n = m_{\text{total}}/M$ to relate total mass m_{total} to number of moles n , remember that if m_{total} is in kilograms, M must be in *kilograms* per mole; M is usually tabulated in *grams* per mole.

2. The internal energy change ΔU in any thermodynamic process or series of processes is independent of the path, whether the substance is an ideal gas or not. If you can calculate ΔU for *any* path between given initial and final states, you know ΔU for *every possible path* between those states; you can then relate the various energy quantities for any of those other paths.
3. In a process comprising several steps, tabulate Q , W , and ΔU for each step, with one line per step and with the Q 's, W 's, and ΔU 's forming columns (see Example 19.4). You can apply the first law to each line, and you can add each column and apply the first law to the sums. Do you see why?
4. Using steps 1–3, solve for the target variables.

EVALUATE *your answer:* Check your results for reasonableness. Ensure that each of your answers has the correct algebraic sign. A positive Q means that heat flows *into* the system; a negative Q means that heat flows *out of* the system. A positive W means that work is done *by* the system on its environment; a negative W means that work is done *on* the system by its environment.

Example 19.2 Working off your dessert

You propose to climb several flights of stairs to work off the energy you took in by eating a 900-calorie hot fudge sundae. How high must you climb? Assume that your mass is 60.0 kg.

SOLUTION

IDENTIFY and SET UP: The thermodynamic system is your body. You climb the stairs to make the final state of the system the same as the initial state (no fatter, no leaner). There is therefore no net change in internal energy: $\Delta U = 0$. Eating the hot fudge sundae corresponds to a heat flow into your body, and you do work climbing the stairs. We can relate these quantities using the first law of thermodynamics. We are given that $Q = 900$ food calories (900 kcal) of heat flow into your body. The work you must do to raise your mass m a height h is $W = mgh$; our target variable is h .

EXECUTE: From the first law of thermodynamics, $\Delta U = 0 = Q - W$, so $W = mgh = Q$. Hence you must climb to height $h = Q/mg$. First convert units: $Q = (900 \text{ kcal})(4186 \text{ J/1 kcal}) = 3.77 \times 10^6 \text{ J}$. Then

$$h = \frac{Q}{mg} = \frac{3.77 \times 10^6 \text{ J}}{(60.0 \text{ kg})(9.80 \text{ m/s}^2)} = 6410 \text{ m}$$

EVALUATE: We have unrealistically assumed 100% efficiency in the conversion of food energy into mechanical work. You would in fact have to climb considerably *less* than 6140 m (about 21,000 ft).

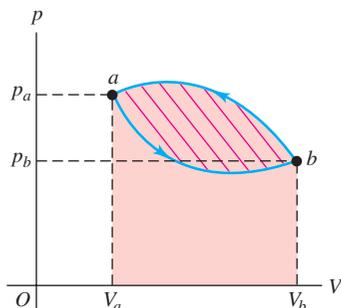
Example 19.3 A cyclic process

Figure 19.12 shows a pV -diagram for a *cyclic* process in which the initial and final states of some thermodynamic system are the same. As shown, the state of the system starts at point a and proceeds counterclockwise in the pV -diagram to point b , then back to a ; the total work is $W = -500 \text{ J}$. (a) Why is the work negative? (b) Find the change in internal energy and the heat added during this process.

SOLUTION

IDENTIFY and SET UP: We must relate the change in internal energy, the heat added, and the work done in a thermodynamic process. Hence we can apply the first law of thermodynamics. The process is cyclic, and it has two steps: $a \rightarrow b$ via the lower curve in Fig. 19.12 and $b \rightarrow a$ via the upper curve. We are asked only about the *entire* cyclic process $a \rightarrow b \rightarrow a$.

19.12 The net work done by the system in the process aba is -500 J. What would it have been if the process had proceeded clockwise in this pV -diagram?



EXECUTE: (a) The work done in any step equals the area under the curve in the pV -diagram, with the area taken as positive if $V_2 > V_1$

and negative if $V_2 < V_1$; this rule yields the signs that result from the actual integrations in Eq. (19.2), $W = \int_{V_1}^{V_2} p dV$. The area under the lower curve $a \rightarrow b$ is therefore positive, but it is smaller than the absolute value of the (negative) area under the upper curve $b \rightarrow a$. Therefore the net area (the area enclosed by the path, shown with red stripes) and the net work W are negative. In other words, 500 J more work is done *on* the system than *by* the system in the complete process.

(b) In any cyclic process, $\Delta U = 0$, so $Q = W$. Here, that means $Q = -500$ J; that is, 500 J of heat flows *out* of the system.

EVALUATE:

In cyclic processes, the total work is positive if the process goes clockwise around the pV -diagram representing the cycle, and negative if the process goes counterclockwise (as here).

Example 19.4 Comparing thermodynamic processes

The pV -diagram of Fig. 19.13 shows a series of thermodynamic processes. In process ab , 150 J of heat is added to the system; in process bd , 600 J of heat is added. Find (a) the internal energy change in process ab ; (b) the internal energy change in process abd (shown in light blue); and (c) the total heat added in process acd (shown in dark blue).

SOLUTION

IDENTIFY and SET UP: In each process we use $\Delta U = Q - W$ to determine the desired quantity. We are given $Q_{ab} = +150$ J and $Q_{bd} = +600$ J (both values are positive because heat is *added* to the system). Our target variables are (a) ΔU_{ab} , (b) ΔU_{abd} , and (c) Q_{acd} .

EXECUTE: (a) No volume change occurs during process ab , so the system does no work: $W_{ab} = 0$ and so $\Delta U_{ab} = Q_{ab} = 150$ J.

(b) Process bd is an expansion at constant pressure, so from Eq. (19.3),

$$\begin{aligned} W_{bd} &= p(V_2 - V_1) \\ &= (8.0 \times 10^4 \text{ Pa})(5.0 \times 10^{-3} \text{ m}^3 - 2.0 \times 10^{-3} \text{ m}^3) \\ &= 240 \text{ J} \end{aligned}$$

The total work for the two-step process abd is then

$$W_{abd} = W_{ab} + W_{bd} = 0 + 240 \text{ J} = 240 \text{ J}$$

and the total heat is

$$Q_{abd} = Q_{ab} + Q_{bd} = 150 \text{ J} + 600 \text{ J} = 750 \text{ J}$$

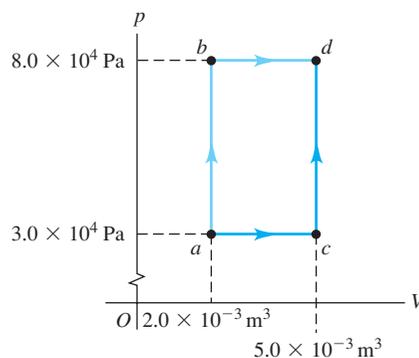
Applying Eq. (19.4) to abd , we then have

$$\Delta U_{abd} = Q_{abd} - W_{abd} = 750 \text{ J} - 240 \text{ J} = 510 \text{ J}$$

(c) Because ΔU is *independent of the path* from a to d , the internal energy change is the same for path acd as for path abd :

$$\Delta U_{acd} = \Delta U_{abd} = 510 \text{ J}$$

19.13 A pV -diagram showing the various thermodynamic processes.



The total work for path acd is

$$\begin{aligned} W_{acd} &= W_{ac} + W_{cd} = p(V_2 - V_1) + 0 \\ &= (3.0 \times 10^4 \text{ Pa})(5.0 \times 10^{-3} \text{ m}^3 - 2.0 \times 10^{-3} \text{ m}^3) \\ &= 90 \text{ J} \end{aligned}$$

Now we apply Eq. (19.5) to process acd :

$$Q_{acd} = \Delta U_{acd} + W_{acd} = 510 \text{ J} + 90 \text{ J} = 600 \text{ J}$$

We tabulate the quantities above:

Step	Q	W	$\Delta U = Q - W$	Step	Q	W	$\Delta U = Q - W$
ab	150 J	0 J	150 J	ac	?	90 J	?
bd	600 J	240 J	360 J	cd	?	0 J	?
abd	750 J	240 J	510 J	acd	600 J	90 J	510 J

EVALUATE: Be sure that you understand how each entry in the table above was determined. Although ΔU is the same (510 J) for abd and acd , W (240 J versus 90 J) and Q (750 J versus 600 J) are quite different. Although we couldn't find Q or ΔU for processes ac and cd , we could analyze the composite process acd by comparing it with process abd , which has the same initial and final states and for which we have more information.

Example 19.5 Thermodynamics of boiling water

One gram of water (1 cm^3) becomes 1671 cm^3 of steam when boiled at a constant pressure of 1 atm ($1.013 \times 10^5 \text{ Pa}$). The heat of vaporization at this pressure is $L_v = 2.256 \times 10^6 \text{ J/kg}$. Compute (a) the work done by the water when it vaporizes and (b) its increase in internal energy.

SOLUTION

IDENTIFY and SET UP: The heat added causes the system (water) to change phase from liquid to vapor. We can analyze this process using the first law of thermodynamics, which holds for thermodynamic processes of *all* kinds. The water is boiled at constant pressure, so we can use Eq. (19.3) to calculate the work W done by the vaporizing water as it expands. We are given the mass of water and the heat of vaporization, so we can use Eq. (17.20), $Q = mL_v$, to calculate the heat Q added to the water. We can then find the internal energy change using Eq. (19.4), $\Delta U = Q - W$.

EXECUTE: (a) From Eq. (19.3), the water does work

$$\begin{aligned} W &= p(V_2 - V_1) \\ &= (1.013 \times 10^5 \text{ Pa})(1671 \times 10^{-6} \text{ m}^3 - 1 \times 10^{-6} \text{ m}^3) \\ &= 169 \text{ J} \end{aligned}$$

(b) From Eq. (17.20), the heat added to the water is

$$Q = mL_v = (10^{-3} \text{ kg})(2.256 \times 10^6 \text{ J/kg}) = 2256 \text{ J}$$

Then from Eq. (19.4),

$$\Delta U = Q - W = 2256 \text{ J} - 169 \text{ J} = 2087 \text{ J}$$

EVALUATE: To vaporize 1 g of water, we must add 2256 J of heat, most of which (2087 J) remains in the system as an increase in internal energy. The remaining 169 J leaves the system as the system expands from liquid to vapor and does work against the surroundings. (The increase in internal energy is associated mostly with the attractive intermolecular forces that hold the molecules together in the liquid state. The associated potential energies are greater after work has been done to pull the molecules apart, forming the vapor state. It's like increasing gravitational potential energy by pulling an elevator farther from the center of the earth.)

Infinitesimal Changes of State

In the preceding examples the initial and final states differ by a finite amount. Later we will consider *infinitesimal* changes of state in which a small amount of heat dQ is added to the system, the system does a small amount of work dW , and its internal energy changes by an amount dU . For such a process we state the first law in differential form as

$$dU = dQ - dW \quad \text{(first law of thermodynamics, infinitesimal process)} \quad (19.6)$$

For the systems we will discuss, the work dW is given by $dW = p dV$, so we can also state the first law as

$$dU = dQ - p dV \quad (19.7)$$

Test Your Understanding of Section 19.4 Rank the following thermodynamic processes according to the change in internal energy in each process, from most positive to most negative. (i) As you do 250 J of work on a system, it transfers 250 J of heat to its surroundings; (ii) as you do 250 J of work on a system, it absorbs 250 J of heat from its surroundings; (iii) as a system does 250 J of work on you, it transfers 250 J of heat to its surroundings; (iv) as a system does 250 J of work on you, it absorbs 250 J of heat from its surroundings.

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ActivPhysics 8.4: State Variables and Ideal Gas Law

19.5 Kinds of Thermodynamic Processes

In this section we describe four specific kinds of thermodynamic processes that occur often in practical situations. These can be summarized briefly as “no heat transfer” or *adiabatic*, “constant volume” or *isochoric*, “constant pressure” or *isobaric*, and “constant temperature” or *isothermal*. For some of these processes we can use a simplified form of the first law of thermodynamics.

Adiabatic Process

An **adiabatic process** (pronounced “ay-dee-ah-bat-ic”) is defined as one with no heat transfer into or out of a system; $Q = 0$. We can prevent heat flow either by surrounding the system with thermally insulating material or by carrying out the process so quickly that there is not enough time for appreciable heat flow. From the first law we find that for every adiabatic process,

$$U_2 - U_1 = \Delta U = -W \quad (\text{adiabatic process}) \quad (19.8)$$

When a system expands adiabatically, W is positive (the system does work on its surroundings), so ΔU is negative and the internal energy decreases. When a system is *compressed* adiabatically, W is negative (work is done on the system by its surroundings) and U increases. In many (but not all) systems an increase of internal energy is accompanied by a rise in temperature, and a decrease in internal energy by a drop in temperature (Fig. 19.14).

The compression stroke in an internal-combustion engine is an approximately adiabatic process. The temperature rises as the air–fuel mixture in the cylinder is compressed. The expansion of the burned fuel during the power stroke is also an approximately adiabatic expansion with a drop in temperature. In Section 19.8 we’ll consider adiabatic processes in an ideal gas.

Isochoric Process

An **isochoric process** (pronounced “eye-so-kor-ic”) is a *constant-volume* process. When the volume of a thermodynamic system is constant, it does no work on its surroundings. Then $W = 0$ and

$$U_2 - U_1 = \Delta U = Q \quad (\text{isochoric process}) \quad (19.9)$$

In an isochoric process, all the energy added as heat remains in the system as an increase in internal energy. Heating a gas in a closed constant-volume container is an example of an isochoric process. The processes *ab* and *cd* in Example 19.4 are also examples of isochoric processes. (Note that there are types of work that do not involve a volume change. For example, we can do work on a fluid by stirring it. In some literature, “isochoric” is used to mean that no work of any kind is done.)

Isobaric Process

An **isobaric process** (pronounced “eye-so-bear-ic”) is a *constant-pressure* process. In general, none of the three quantities ΔU , Q , and W is zero in an isobaric process, but calculating W is easy nonetheless. From Eq. (19.3),

$$W = p(V_2 - V_1) \quad (\text{isobaric process}) \quad (19.10)$$

Example 19.5 concerns an isobaric process, boiling water at constant pressure (Fig. 19.15).

Isothermal Process

An **isothermal process** is a *constant-temperature* process. For a process to be isothermal, any heat flow into or out of the system must occur slowly enough that thermal equilibrium is maintained. In general, none of the quantities ΔU , Q , or W is zero in an isothermal process.

In some special cases the internal energy of a system depends *only* on its temperature, not on its pressure or volume. The most familiar system having this special property is an ideal gas, as we’ll discuss in the next section. For such systems, if the temperature is constant, the internal energy is also constant; $\Delta U = 0$ and $Q = W$. That is, any energy entering the system as heat Q must leave it again as work W done by the system. Example 19.1, involving an ideal gas, is an example of an isothermal process in which U is also constant. For most systems other than ideal gases, the internal energy depends on pressure as well as temperature, so U may vary even when T is constant.

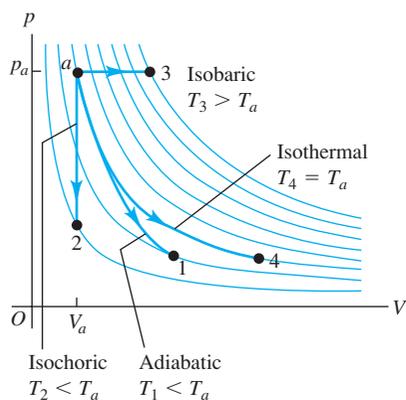
19.14 When the cork is popped on a bottle of champagne, the pressurized gases inside the bottle expand rapidly and do work on the outside air ($W > 0$). There is no time for the gases to exchange heat with their surroundings, so the expansion is adiabatic ($Q = 0$). Hence the internal energy of the expanding gases decreases ($\Delta U = -W < 0$) and their temperature drops. This makes water vapor condense and form a miniature cloud.



19.15 Most cooking involves isobaric processes. That’s because the air pressure above a saucepan or frying pan, or inside a microwave oven, remains essentially constant while the food is being heated.



19.16 Four different processes for a constant amount of an ideal gas, all starting at state a . For the adiabatic process, $Q = 0$; for the isochoric process, $W = 0$; and for the isothermal process, $\Delta U = 0$. The temperature increases only during the isobaric expansion.



19.17 The partition is broken (or removed) to start the free expansion of gas into the vacuum region.

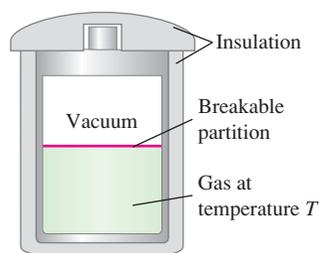


Figure 19.16 shows a pV -diagram for these four processes for a constant amount of an ideal gas. The path followed in an adiabatic process (a to 1) is called an **adiabat**. A vertical line (constant volume) is an **isochor**, a horizontal line (constant pressure) is an **isobar**, and a curve of constant temperature (shown as light blue lines in Fig. 19.16) is an **isotherm**.

Test Your Understanding of Section 19.5 Which of the processes in Fig. 19.7 are isochoric? Which are isobaric? Is it possible to tell if any of the processes are isothermal or adiabatic?

19.6 Internal Energy of an Ideal Gas

We now show that for an ideal gas, the internal energy U depends only on temperature, not on pressure or volume. Let's think again about the free-expansion experiment described in Section 19.3. A thermally insulated container with rigid walls is divided into two compartments by a partition (Fig. 19.17). One compartment has a quantity of an ideal gas and the other is evacuated.

When the partition is removed or broken, the gas expands to fill both parts of the container. The gas does no work on its surroundings because the walls of the container don't move, and there is no heat flow through the insulation. So both Q and W are zero and the internal energy U is constant. This is true of any substance, whether it is an ideal gas or not.

Does the *temperature* change during a free expansion? Suppose it *does* change, while the internal energy stays the same. In that case we have to conclude that the internal energy depends on both the temperature and the volume or on both the temperature and the pressure, but certainly not on the temperature alone. But if T is constant during a free expansion, for which we know that U is constant even though both p and V change, then we have to conclude that U depends only on T , not on p or V .

Many experiments have shown that when a low-density gas undergoes a free expansion, its temperature *does not* change. Such a gas is essentially an ideal gas. The conclusion is:

The internal energy of an ideal gas depends only on its temperature, not on its pressure or volume.

This property, in addition to the ideal-gas equation of state, is part of the ideal-gas model. Make sure you understand that U depends only on T for an ideal gas, for we will make frequent use of this fact.

For nonideal gases, some temperature change occurs during free expansions, even though the internal energy is constant. This shows that the internal energy cannot depend *only* on temperature; it must depend on pressure as well. From the microscopic viewpoint, in which internal energy U is the sum of the kinetic and potential energies for all the particles that make up the system, this is not surprising. Nonideal gases usually have attractive intermolecular forces, and when molecules move farther apart, the associated potential energies increase. If the total internal energy is constant, the kinetic energies must decrease. Temperature is directly related to molecular *kinetic* energy, and for such a gas a free expansion is usually accompanied by a *drop* in temperature.

Test Your Understanding of Section 19.6 Is the internal energy of a solid likely to be independent of its volume, as is the case for an ideal gas? Explain your reasoning. (*Hint*: See Fig. 18.20.)

19.7 Heat Capacities of an Ideal Gas

We defined specific heat and molar heat capacity in Section 17.5. We also remarked at the end of that section that the specific heat or molar heat capacity of a substance depends on the conditions under which the heat is added. It is usually easiest to measure the heat capacity of a gas in a closed container under constant-volume conditions. The corresponding heat capacity is the **molar heat capacity at constant volume**, denoted by C_V . Heat capacity measurements for solids and liquids are usually carried out in the atmosphere under constant atmospheric pressure, and we call the corresponding heat capacity the **molar heat capacity at constant pressure**, C_p . If neither p nor V is constant, we have an infinite number of possible heat capacities.

Let's consider C_V and C_p for an ideal gas. To measure C_V , we raise the temperature of an ideal gas in a rigid container with constant volume, neglecting its thermal expansion (Fig. 19.18a). To measure C_p , we let the gas expand just enough to keep the pressure constant as the temperature rises (Fig. 19.18b).

Why should these two molar heat capacities be different? The answer lies in the first law of thermodynamics. In a constant-volume temperature increase, the system does no work, and the change in internal energy ΔU equals the heat added Q . In a constant-pressure temperature increase, on the other hand, the volume *must* increase; otherwise, the pressure (given by the ideal-gas equation of state, $p = nRT/V$) could not remain constant. As the material expands, it does an amount of work W . According to the first law,

$$Q = \Delta U + W \quad (19.11)$$

For a given temperature increase, the internal energy change ΔU of an ideal gas has the same value no matter what the process (remember that the internal energy of an ideal gas depends only on temperature, not on pressure or volume). Equation (19.11) then shows that the heat input for a constant-pressure process must be *greater* than that for a constant-volume process because additional energy must be supplied to account for the work W done during the expansion. So C_p is greater than C_V for an ideal gas. The pV -diagram in Fig. 19.19 shows this relationship. For air, C_p is 40% greater than C_V .

For a very few substances (one of which is water between 0°C and 4°C) the volume *decreases* during heating. In this case, W is negative, the heat input is *less* than in the constant-volume case, and C_p is *less* than C_V .

Relating C_p and C_V for an Ideal Gas

We can derive a simple relationship between C_p and C_V for an ideal gas. First consider the constant-volume process. We place n moles of an ideal gas at temperature T in a constant-volume container. We place it in thermal contact with a hotter body; an infinitesimal quantity of heat dQ flows into the gas, and its temperature increases by an infinitesimal amount dT . By the definition of C_V , the molar heat capacity at constant volume,

$$dQ = nC_V dT \quad (19.12)$$

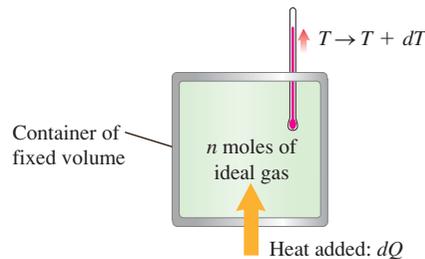
The pressure increases during this process, but the gas does no work ($dW = 0$) because the volume is constant. The first law in differential form, Eq. (19.6), is $dQ = dU + dW$. Since $dW = 0$, $dQ = dU$ and Eq. (19.12) can also be written as

$$dU = nC_V dT \quad (19.13)$$

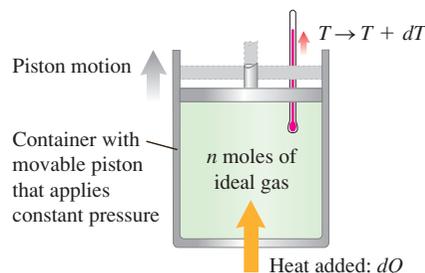
Now consider a constant-pressure process with the same temperature change dT . We place the same gas in a cylinder with a piston that we can allow to move just enough to maintain constant pressure, as shown in Fig. 19.18b. Again we bring the system into contact with a hotter body. As heat flows into

19.18 Measuring the molar heat capacity of an ideal gas (a) at constant volume and (b) at constant pressure.

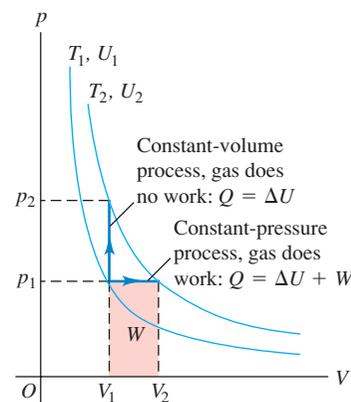
(a) Constant volume: $dQ = nC_V dT$



(b) Constant pressure: $dQ = nC_p dT$



19.19 Raising the temperature of an ideal gas from T_1 to T_2 by a constant-volume or a constant-pressure process. For an ideal gas, U depends only on T , so ΔU is the same for both processes. But for the constant-pressure process, more heat Q must be added to both increase U and do work W . Hence $C_p > C_V$.





ActivPhysics 8.7: Heat Capacity

ActivPhysics 8.8: Isochoric Process

ActivPhysics 8.9: Isobaric Process

ActivPhysics 8.10: Isothermal Process

the gas, it expands at constant pressure and does work. By the definition of C_p , the molar heat capacity at constant pressure, the amount of heat dQ entering the gas is

$$dQ = nC_p dT \quad (19.14)$$

The work dW done by the gas in this constant-pressure process is

$$dW = p dV$$

We can also express dW in terms of the temperature change dT by using the ideal-gas equation of state, $pV = nRT$. Because p is constant, the change in V is proportional to the change in T :

$$dW = p dV = nR dT \quad (19.15)$$

Now we substitute Eqs. (19.14) and (19.15) into the first law, $dQ = dU + dW$. We obtain

$$nC_p dT = dU + nR dT \quad (19.16)$$

Now here comes the crux of the calculation. The internal energy change dU for the constant-pressure process is again given by Eq. (19.13), $dU = nC_V dT$, even though now the volume is not constant. Why is this so? Recall the discussion of Section 19.6; one of the special properties of an ideal gas is that its internal energy depends *only* on temperature. Thus the *change* in internal energy during any process must be determined only by the temperature change. If Eq. (19.13) is valid for an ideal gas for one particular kind of process, it must be valid for an ideal gas for *every* kind of process with the same dT . So we may replace dU in Eq. (19.16) by $nC_V dT$:

$$nC_p dT = nC_V dT + nR dT$$

When we divide each term by the common factor $n dT$, we get

$$C_p = C_V + R \quad (\text{molar heat capacities of an ideal gas}) \quad (19.17)$$

As we predicted, the molar heat capacity of an ideal gas at constant pressure is *greater* than the molar heat capacity at constant volume; the difference is the gas constant R . (Of course, R must be expressed in the same units as C_p and C_V , such as $\text{J/mol} \cdot \text{K}$.)

We have used the ideal-gas model to derive Eq. (19.17), but it turns out to be obeyed to within a few percent by many real gases at moderate pressures. Measured values of C_p and C_V are given in Table 19.1 for several real gases at low pressures; the difference in most cases is approximately $R = 8.314 \text{ J/mol} \cdot \text{K}$.

The table also shows that the molar heat capacity of a gas is related to its molecular structure, as we discussed in Section 18.4. In fact, the first two columns of Table 19.1 are the same as Table 18.1.

Table 19.1 Molar Heat Capacities of Gases at Low Pressure

Type of Gas	Gas	C_V (J/mol · K)	C_p (J/mol · K)	$C_p - C_V$ (J/mol · K)	$\gamma = C_p/C_V$
Monatomic	He	12.47	20.78	8.31	1.67
	Ar	12.47	20.78	8.31	1.67
Diatomic	H ₂	20.42	28.74	8.32	1.41
	N ₂	20.76	29.07	8.31	1.40
	O ₂	20.85	29.17	8.31	1.40
	CO	20.85	29.16	8.31	1.40
Polyatomic	CO ₂	28.46	36.94	8.48	1.30
	SO ₂	31.39	40.37	8.98	1.29
	H ₂ S	25.95	34.60	8.65	1.33

The Ratio of Heat Capacities

The last column of Table 19.1 lists the values of the dimensionless **ratio of heat capacities**, C_p/C_V , denoted by γ (the Greek letter gamma):

$$\gamma = \frac{C_p}{C_V} \quad (\text{ratio of heat capacities}) \quad (19.18)$$

(This is sometimes called the “ratio of specific heats.”) For gases, C_p is always greater than C_V and γ is always greater than unity. This quantity plays an important role in *adiabatic* processes for an ideal gas, which we will study in the next section.

We can use our kinetic-theory discussion of the molar heat capacity of an ideal gas (see Section 18.4) to predict values of γ . As an example, an ideal monatomic gas has $C_V = \frac{3}{2}R$. From Eq. (19.17),

$$C_p = C_V + R = \frac{3}{2}R + R = \frac{5}{2}R$$

so

$$\gamma = \frac{C_p}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3} = 1.67$$

As Table 19.1 shows, this agrees well with values of γ computed from measured heat capacities. For most diatomic gases near room temperature, $C_V = \frac{5}{2}R$, $C_p = C_V + R = \frac{7}{2}R$, and

$$\gamma = \frac{C_p}{C_V} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5} = 1.40$$

also in good agreement with measured values.

Here’s a final reminder: For an ideal gas the internal energy change in *any* process is given by $\Delta U = nC_V \Delta T$, *whether the volume is constant or not*. This relationship, which comes in handy in the following example, holds for other substances *only* when the volume is constant.

Example 19.6 Cooling your room

A typical dorm room or bedroom contains about 2500 moles of air. Find the change in the internal energy of this much air when it is cooled from 35.0°C to 26.0°C at a constant pressure of 1.00 atm. Treat the air as an ideal gas with $\gamma = 1.400$.

SOLUTION

IDENTIFY and SET UP: Our target variable is the change in the internal energy ΔU of an ideal gas in a constant-pressure process. We are given the number of moles, the temperature change, and the value of γ for air. We use Eq. (19.13), $\Delta U = nC_V \Delta T$, which gives the internal energy change for an ideal gas in *any* process, *whether the volume is constant or not*. [See the discussion following Eq. (19.16).] We use Eqs. (19.17) and (19.18) to find C_V .

EXECUTE: From Eqs. (19.17) and (19.18),

$$\gamma = \frac{C_p}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V}$$

$$C_V = \frac{R}{\gamma - 1} = \frac{8.314 \text{ J/mol} \cdot \text{K}}{1.400 - 1} = 20.79 \text{ J/mol} \cdot \text{K}$$

Then from Eq. (19.13),

$$\begin{aligned} \Delta U &= nC_V \Delta T \\ &= (2500 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(26.0^\circ\text{C} - 35.0^\circ\text{C}) \\ &= -4.68 \times 10^5 \text{ J} \end{aligned}$$

EVALUATE: To cool 2500 moles of air from 35.0°C to 26.0°C, a room air conditioner must extract this much internal energy from the air and transfer it to the air outside. In Chapter 20 we’ll discuss how this is done.

Test Your Understanding of Section 19.7 You want to cool a storage cylinder containing 10 moles of compressed gas from 30°C to 20°C. For which kind of gas would this be easiest? (i) a monatomic gas; (ii) a diatomic gas; (iii) a polyatomic gas; (iv) it would be equally easy for all of these.

19.8 Adiabatic Processes for an Ideal Gas

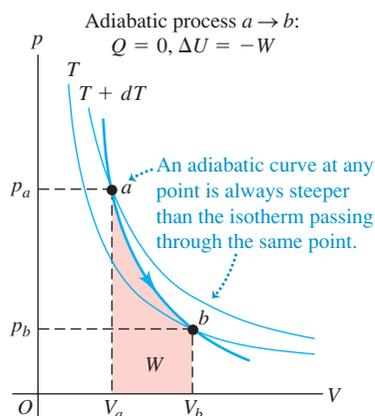
An adiabatic process, defined in Section 19.5, is a process in which no heat transfer takes place between a system and its surroundings. Zero heat transfer is an idealization, but a process is approximately adiabatic if the system is well insulated or if the process takes place so quickly that there is not enough time for appreciable heat flow to occur.

In an adiabatic process, $Q = 0$, so from the first law, $\Delta U = -W$. An adiabatic process for an ideal gas is shown in the pV -diagram of Fig. 19.20. As the gas expands from volume V_a to V_b , it does positive work, so its internal energy decreases and its temperature drops. If point a , representing the initial state, lies on an isotherm at temperature $T + dT$, then point b for the final state is on a different isotherm at a lower temperature T . For an ideal gas an adiabatic curve (adiabat) at any point is always steeper than the isotherm passing through the same point. For an adiabatic compression from V_b to V_a the situation is reversed and the temperature rises.

The air in the output hoses of air compressors used in gasoline stations, in paint-spraying equipment, and to fill scuba tanks is always warmer than the air entering the compressor; this is because the compression is rapid and hence approximately adiabatic. Adiabatic cooling occurs when you open a bottle of your favorite carbonated beverage. The gas just above the beverage surface expands rapidly in a nearly adiabatic process; the temperature of the gas drops so much that water vapor in the gas condenses, forming a miniature cloud (see Fig. 19.14).

CAUTION “Heating” and “cooling” without heat Keep in mind that when we talk about “adiabatic heating” and “adiabatic cooling,” we really mean “raising the temperature” and “lowering the temperature,” respectively. In an adiabatic process, the temperature change is due to work done by or on the system; there is *no* heat flow at all.

19.20 A pV -diagram of an adiabatic ($Q = 0$) process for an ideal gas. As the gas expands from V_a to V_b , it does positive work W on its environment, its internal energy decreases ($\Delta U = -W < 0$), and its temperature drops from $T + dT$ to T . (An adiabatic process is also shown in Fig. 19.16.)



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ActivPhysics 8.11: Adiabatic Process

Adiabatic Ideal Gas: Relating V , T , and p

We can derive a relationship between volume and temperature changes for an infinitesimal adiabatic process in an ideal gas. Equation (19.13) gives the internal energy change dU for any process for an ideal gas, adiabatic or not, so we have $dU = nC_V dT$. Also, the work done by the gas during the process is given by $dW = p dV$. Then, since $dU = -dW$ for an adiabatic process, we have

$$nC_V dT = -p dV \quad (19.19)$$

To obtain a relationship containing only the volume V and temperature T , we eliminate p using the ideal-gas equation in the form $p = nRT/V$. Substituting this into Eq. (19.19) and rearranging, we get

$$nC_V dT = -\frac{nRT}{V} dV$$

$$\frac{dT}{T} + \frac{R}{C_V} \frac{dV}{V} = 0$$

The coefficient R/C_V can be expressed in terms of $\gamma = C_p/C_V$. We have

$$\frac{R}{C_V} = \frac{C_p - C_V}{C_V} = \frac{C_p}{C_V} - 1 = \gamma - 1$$

$$\frac{dT}{T} + (\gamma - 1)\frac{dV}{V} = 0 \quad (19.20)$$

Because γ is always greater than unity for a gas, $(\gamma - 1)$ is always positive. This means that in Eq. (19.20), dV and dT always have opposite signs. An adiabatic *expansion* of an ideal gas ($dV > 0$) always occurs with a *drop* in temperature ($dT < 0$), and an adiabatic *compression* ($dV < 0$) always occurs with a *rise* in temperature ($dT > 0$); this confirms our earlier prediction.

For finite changes in temperature and volume we integrate Eq. (19.20), obtaining

$$\ln T + (\gamma - 1) \ln V = \text{constant}$$

$$\ln T + \ln V^{\gamma-1} = \text{constant}$$

$$\ln(TV^{\gamma-1}) = \text{constant}$$

and finally,

$$TV^{\gamma-1} = \text{constant} \quad (19.21)$$

Thus for an initial state (T_1, V_1) and a final state (T_2, V_2) ,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \quad (\text{adiabatic process, ideal gas}) \quad (19.22)$$

Because we have used the ideal-gas equation in our derivation of Eqs. (19.21) and (19.22), the T 's must always be *absolute* (Kelvin) temperatures.

We can also convert Eq. (19.21) into a relationship between pressure and volume by eliminating T , using the ideal-gas equation in the form $T = pV/nR$. Substituting this into Eq. (19.21), we find

$$\frac{pV}{nR} V^{\gamma-1} = \text{constant}$$

or, because n and R are constant,

$$pV^\gamma = \text{constant} \quad (19.23)$$

For an initial state (p_1, V_1) and a final state (p_2, V_2) , Eq. (19.23) becomes

$$p_1 V_1^\gamma = p_2 V_2^\gamma \quad (\text{adiabatic process, ideal gas}) \quad (19.24)$$

We can also calculate the *work* done by an ideal gas during an adiabatic process. We know that $Q = 0$ and $W = -\Delta U$ for *any* adiabatic process. For an ideal gas, $\Delta U = nC_V(T_2 - T_1)$. If the number of moles n and the initial and final temperatures T_1 and T_2 are known, we have simply

$$W = nC_V(T_1 - T_2) \quad (\text{adiabatic process, ideal gas}) \quad (19.25)$$

We may also use $pV = nRT$ in this equation to obtain

$$W = \frac{C_V}{R}(p_1 V_1 - p_2 V_2) = \frac{1}{\gamma - 1}(p_1 V_1 - p_2 V_2) \quad (\text{adiabatic process, ideal gas}) \quad (19.26)$$

(We used the result $C_V = R/(\gamma - 1)$ from Example 19.6.) If the process is an expansion, the temperature drops, T_1 is greater than T_2 , $p_1 V_1$ is greater than $p_2 V_2$, and the work is *positive*, as we should expect. If the process is a compression, the work is negative.

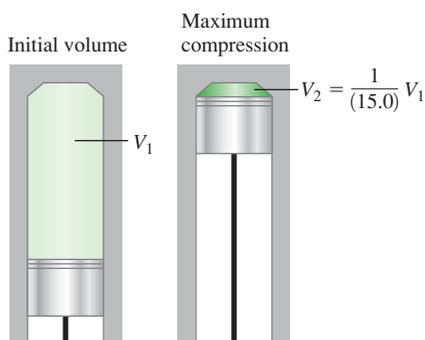
Throughout this analysis of adiabatic processes we have used the ideal-gas equation of state, which is valid only for *equilibrium* states. Strictly speaking, our results are valid only for a process that is fast enough to prevent appreciable heat exchange with the surroundings (so that $Q = 0$ and the process is adiabatic), yet slow enough that the system does not depart very much from thermal and mechanical equilibrium. Even when these conditions are not strictly satisfied, though, Eqs. (19.22), (19.24), and (19.26) give useful approximate results.

Example 19.7 Adiabatic compression in a diesel engine

The compression ratio of a diesel engine is 15.0 to 1; that is, air in a cylinder is compressed to $\frac{1}{15.0}$ of its initial volume (Fig. 19.21). (a) If the initial pressure is 1.01×10^5 Pa and the initial temperature is 27°C (300 K), find the final pressure and the temperature after adiabatic compression. (b) How much work does the gas do during the compression if the initial volume of the cylinder is $1.00 \text{ L} = 1.00 \times 10^{-3} \text{ m}^3$? Use the values $C_V = 20.8 \text{ J/mol} \cdot \text{K}$ and $\gamma = 1.400$ for air.

SOLUTION

IDENTIFY and SET UP: This problem involves the adiabatic compression of an ideal gas, so we can use the ideas of this section. In part (a) we are given the initial pressure and temperature $p_1 = 1.01 \times 10^5$ Pa and $T_1 = 300$ K; the ratio of initial and final volumes is $V_1/V_2 = 15.0$. We use Eq. (19.22) to find the final temperature T_2 and Eq. (19.24) to find the final pressure p_2 . In part (b) our target variable is W , the work done by the gas during the adiabatic compression. We use Eq. (19.26) to calculate W .

19.21 Adiabatic compression of air in a cylinder of a diesel engine.

EXECUTE: (a) From Eqs. (19.22) and (19.24),

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (300 \text{ K})(15.0)^{0.40} = 886 \text{ K} = 613^\circ\text{C}$$

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma = (1.01 \times 10^5 \text{ Pa})(15.0)^{1.40} \\ = 44.8 \times 10^5 \text{ Pa} = 44 \text{ atm}$$

(b) From Eq. (19.26), the work done is

$$W = \frac{1}{\gamma - 1} (p_1 V_1 - p_2 V_2)$$

Using $V_1/V_2 = 15.0$, this becomes

$$W = \frac{1}{1.400 - 1} \left[(1.01 \times 10^5 \text{ Pa})(1.00 \times 10^{-3} \text{ m}^3) \right. \\ \left. - (44.8 \times 10^5 \text{ Pa}) \left(\frac{1.00 \times 10^{-3} \text{ m}^3}{15.0} \right) \right] \\ = -494 \text{ J}$$

EVALUATE: If the compression had been isothermal, the final pressure would have been 15.0 atm. Because the temperature also increases during an adiabatic compression, the final pressure is much greater. When fuel is injected into the cylinders near the end of the compression stroke, the high temperature of the air attained during compression causes the fuel to ignite spontaneously without the need for spark plugs.

We can check our result in part (b) using Eq. (19.25). The number of moles of gas in the cylinder is

$$n = \frac{p_1 V_1}{RT_1} = \frac{(1.01 \times 10^5 \text{ Pa})(1.00 \times 10^{-3} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 0.0405 \text{ mol}$$

Then Eq. (19.25) gives

$$W = nC_V(T_1 - T_2) \\ = (0.0405 \text{ mol})(20.8 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 886 \text{ K}) \\ = -494 \text{ J}$$

The work is negative because the gas is compressed.

Test Your Understanding of Section 19.8 You have four samples of ideal gas, each of which contains the same number of moles of gas and has the same initial temperature, volume, and pressure. You compress each sample to one-half of its initial volume. Rank the four samples in order from highest to lowest value of the final pressure. (i) a monatomic gas compressed isothermally; (ii) a monatomic gas compressed adiabatically; (iii) a diatomic gas compressed isothermally; (iv) a diatomic gas compressed adiabatically.

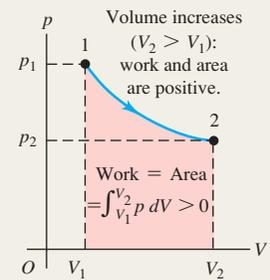


Heat and work in thermodynamic processes: A thermodynamic system has the potential to exchange energy with its surroundings by heat transfer or by mechanical work. When a system at pressure p changes volume from V_1 to V_2 , it does an amount of work W given by the integral of p with respect to volume. If the pressure is constant, the work done is equal to p times the change in volume. A negative value of W means that work is done on the system. (See Example 19.1.)

$$W = \int_{V_1}^{V_2} p \, dV \quad (19.2)$$

$$W = p(V_2 - V_1) \quad (19.3)$$

(constant pressure only)



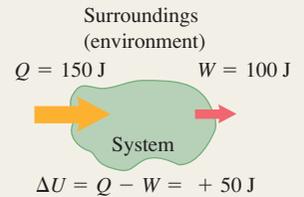
In any thermodynamic process, the heat added to the system and the work done by the system depend not only on the initial and final states, but also on the path (the series of intermediate states through which the system passes).

The first law of thermodynamics: The first law of thermodynamics states that when heat Q is added to a system while the system does work W , the internal energy U changes by an amount equal to $Q - W$. This law can also be expressed for an infinitesimal process. (See Examples 19.2, 19.3, and 19.5.)

$$\Delta U = Q - W \quad (19.4)$$

$$dU = dQ - dW \quad (19.6)$$

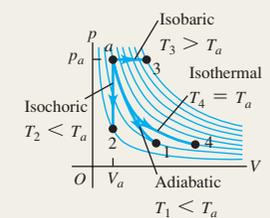
(infinitesimal process)



The internal energy of any thermodynamic system depends only on its state. The change in internal energy in any process depends only on the initial and final states, not on the path. The internal energy of an isolated system is constant. (See Example 19.4.)

Important kinds of thermodynamic processes:

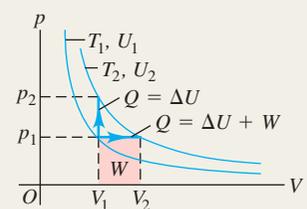
- Adiabatic process: No heat transfer into or out of a system; $Q = 0$.
- Isochoric process: Constant volume; $W = 0$.
- Isobaric process: Constant pressure; $W = p(V_2 - V_1)$.
- Isothermal process: Constant temperature.



Thermodynamics of ideal gases: The internal energy of an ideal gas depends only on its temperature, not on its pressure or volume. For other substances the internal energy generally depends on both pressure and temperature.

$$C_p = C_v + R \quad (19.17)$$

$$\gamma = \frac{C_p}{C_v} \quad (19.18)$$



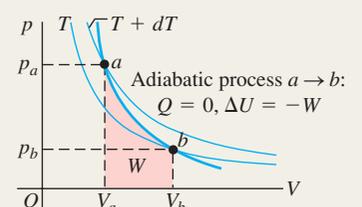
The molar heat capacities C_v and C_p of an ideal gas differ by R , the ideal-gas constant. The dimensionless ratio of heat capacities, C_p/C_v , is denoted by γ . (See Example 19.6.)

Adiabatic processes in ideal gases: For an adiabatic process for an ideal gas, the quantities $TV^{\gamma-1}$ and pV^γ are constant. The work done by an ideal gas during an adiabatic expansion can be expressed in terms of the initial and final values of temperature, or in terms of the initial and final values of pressure and volume. (See Example 19.7.)

$$W = nC_v(T_1 - T_2) \quad (19.25)$$

$$= \frac{C_v}{R}(p_1V_1 - p_2V_2) \quad (19.25)$$

$$= \frac{1}{\gamma - 1}(p_1V_1 - p_2V_2) \quad (19.26)$$



BRIDGING PROBLEM

Work Done by a Van Der Waals Gas

The van der Waals equation of state, an approximate representation of the behavior of gases at high pressure, is given by Eq. (18.7): $[p + (an^2/V^2)](V - nb) = nRT$, where a and b are constants having different values for different gases. (In the special case of $a = b = 0$, this is the ideal-gas equation.) (a) Calculate the work done by a gas with this equation of state in an isothermal expansion from V_1 to V_2 . (b) For ethane gas (C_2H_6), $a = 0.554 \text{ J} \cdot \text{m}^3/\text{mol}^2$ and $b = 6.38 \times 10^{-5} \text{ m}^3/\text{mol}$. Calculate the work W done by 1.80 mol of ethane when it expands from $2.00 \times 10^{-3} \text{ m}^3$ to $4.00 \times 10^{-3} \text{ m}^3$ at a constant temperature of 300 K. Do the calculation using (i) the van der Waals equation of state and (ii) the ideal-gas equation of state. (c) For which equation of state is W larger? Why should this be so?

SOLUTION GUIDE

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IDENTIFY and SET UP

1. Review the discussion of the van der Waals equation of state in Section 18.1. What is the significance of the quantities a and b ?
2. Decide how to find the work done by an expanding gas whose pressure p does not depend on V in the same way as for an ideal gas. (*Hint:* See Section 19.2.)
3. How will you find the work done by an expanding ideal gas?

EXECUTE

4. Find the general expression for the work done by a van der Waals gas as it expands from volume V_1 to volume V_2 . (*Hint:* If you set $a = b = 0$ in your result, it should reduce to the expression for the work done by an expanding ideal gas.)
5. Use your result from step 4 to solve part (b) for ethane treated as a van der Waals gas.
6. Use the formula you chose in step 3 to solve part (b) for ethane treated as an ideal gas.

EVALUATE

7. Is the difference between W for the two equations of state large enough to be significant?
8. Does the term with a in the van der Waals equation of state increase or decrease the amount of work done? What about the term with b ? Which one is more important for the ethane in this problem?

Problems

For instructor-assigned homework, go to www.masteringphysics.com



•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q19.1 For the following processes, is the work done by the system (defined as the expanding or contracting gas) on the environment positive or negative? (a) expansion of the burned gasoline–air mixture in the cylinder of an automobile engine; (b) opening a bottle of champagne; (c) filling a scuba tank with compressed air; (d) partial crumpling of a sealed, empty water bottle, as you drive from the mountains down to sea level.

Q19.2 It is not correct to say that a body contains a certain amount of heat, yet a body can transfer heat to another body. How can a body give away something it does not have in the first place?

Q19.3 In which situation must you do more work: inflating a balloon at sea level or inflating the same balloon to the same volume at the summit of Mt. McKinley? Explain in terms of pressure and volume change.

Q19.4 If you are told the initial and final states of a system and the associated change in internal energy, can you determine whether the internal energy change was due to work or to heat transfer? Explain.

Q19.5 Discuss the application of the first law of thermodynamics to a mountaineer who eats food, gets warm and perspires a lot during a climb, and does a lot of mechanical work in raising herself to the summit. The mountaineer also gets warm during the descent. Is the source of this energy the same as the source during the ascent?

Q19.6 When ice melts at 0°C , its volume decreases. Is the internal energy change greater than, less than, or equal to the heat added? How can you tell?

Q19.7 You hold an inflated balloon over a hot-air vent in your house and watch it slowly expand. You then remove it and let it cool back to room temperature. During the expansion, which was larger: the heat added to the balloon or the work done by the air inside it? Explain. (Assume that air is an ideal gas.) Once the balloon has returned to room temperature, how does the net heat gained or lost by the air inside it compare to the net work done on or by the surrounding air?

Q19.8 You bake chocolate chip cookies and put them, still warm, in a container with a loose (not airtight) lid. What kind of process does the air inside the container undergo as the cookies gradually cool to room temperature (isothermal, isochoric, adiabatic, isobaric, or some combination)? Explain your answer.

Q19.9 Imagine a gas made up entirely of negatively charged electrons. Like charges repel, so the electrons exert repulsive forces on each other. Would you expect that the temperature of such a gas would rise, fall, or stay the same in a free expansion? Why?

Q19.10 There are a few materials that contract when their temperature is increased, such as water between 0°C and 4°C . Would you expect C_p for such materials to be greater or less than C_v ? Explain?

Q19.11 When you blow on the back of your hand with your mouth wide open, your breath feels warm. But if you partially close your mouth to form an “o” and then blow on your hand, your breath feels cool. Why?

Q19.12 An ideal gas expands while the pressure is kept constant. During this process, does heat flow into the gas or out of the gas? Justify your answer.

Q19.13 A liquid is irregularly stirred in a well-insulated container and thereby undergoes a rise in temperature. Regard the liquid as the system. Has heat been transferred? How can you tell? Has work been done? How can you tell? Why is it important that the stirring is irregular? What is the sign of ΔU ? How can you tell?

Q19.14 When you use a hand pump to inflate the tires of your bicycle, the pump gets warm after a while. Why? What happens to the temperature of the air in the pump as you compress it? Why does this happen? When you raise the pump handle to draw outside air into the pump, what happens to the temperature of the air taken in? Again, why does this happen?

Q19.15 In the carburetor of an aircraft or automobile engine, air flows through a relatively small aperture and then expands. In cool, foggy weather, ice sometimes forms in this aperture even though the outside air temperature is above freezing. Why?

Q19.16 On a sunny day, large “bubbles” of air form on the sun-warmed earth, gradually expand, and finally break free to rise through the atmosphere. Soaring birds and glider pilots are fond of using these “thermals” to gain altitude easily. This expansion is essentially an adiabatic process. Why?

Q19.17 The prevailing winds on the Hawaiian island of Kauai blow from the northeast. The winds cool as they go up the slope of Mt. Waialeale (elevation 1523 m), causing water vapor to condense and rain to fall. There is much more precipitation at the summit than at the base of the mountain. In fact, Mt. Waialeale is the rainiest spot on earth, averaging 11.7 m of rainfall a year. But what makes the winds cool?

Q19.18 Applying the same considerations as in Question Q19.17, explain why the island of Niihau, a few kilometers to the southwest of Kauai, is almost a desert and farms there need to be irrigated.

Q19.19 In a constant-volume process, $dU = nC_V dT$. But in a constant-pressure process, it is *not* true that $dU = nC_p dT$. Why not?

Q19.20 When a gas surrounded by air is compressed adiabatically, its temperature rises even though there is no heat input to the gas. Where does the energy come from to raise the temperature?

Q19.21 When a gas expands adiabatically, it does work on its surroundings. But if there is no heat input to the gas, where does the energy come from to do the work?

Q19.22 The gas used in separating the two uranium isotopes ^{235}U and ^{238}U has the formula UF_6 . If you added heat at equal rates to a mole of UF_6 gas and a mole of H_2 gas, which one’s temperature would you expect to rise faster? Explain.

EXERCISES

Section 19.2 Work Done During Volume Changes and Section 19.3 Paths Between Thermodynamic States

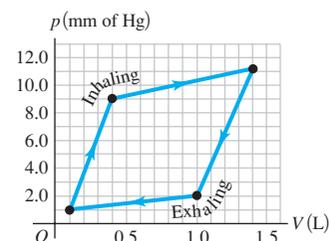
19.1 • Two moles of an ideal gas are heated at constant pressure from $T = 27^\circ\text{C}$ to $T = 107^\circ\text{C}$. (a) Draw a pV -diagram for this process. (b) Calculate the work done by the gas.

19.2 • Six moles of an ideal gas are in a cylinder fitted at one end with a movable piston. The initial temperature of the gas is 27.0°C and the pressure is constant. As part of a machine design project, calculate the final temperature of the gas after it has done 2.40×10^3 J of work.

19.3 • • **CALC** Two moles of an ideal gas are compressed in a cylinder at a constant temperature of 65.0°C until the original pressure has tripled. (a) Sketch a pV -diagram for this process. (b) Calculate the amount of work done.

19.4 • • **BIO Work Done by the Lungs.** The graph in Fig. E19.4 shows a pV -diagram of the air in a human lung when a person is inhaling and then exhaling a deep breath. Such graphs, obtained in clinical practice, are normally somewhat curved, but we have modeled one as a set of straight lines of the same general shape. (*Important:* The pressure shown is the *gauge* pressure, *not* the absolute pressure.)

Figure E19.4



(a) How many joules of *net* work does this person’s lung do during one complete breath? (b) The process illustrated here is somewhat different from those we have been studying, because the pressure change is due to changes in the amount of gas in the lung, not to temperature changes. (Think of your own breathing. Your lungs do not expand because they’ve gotten hot.) If the temperature of the air in the lung remains a reasonable 20°C , what is the maximum number of moles in this person’s lung during a breath?

19.5 • • **CALC** During the time 0.305 mol of an ideal gas undergoes an isothermal compression at 22.0°C , 468 J of work is done on it by the surroundings. (a) If the final pressure is 1.76 atm, what was the initial pressure? (b) Sketch a pV -diagram for the process.

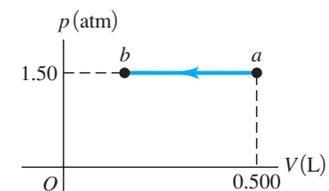
19.6 • • A gas undergoes two processes. In the first, the volume remains constant at 0.200 m³ and the pressure increases from 2.00×10^5 Pa to 5.00×10^5 Pa. The second process is a compression to a volume of 0.120 m³ at a constant pressure of 5.00×10^5 Pa. (a) In a pV -diagram, show both processes. (b) Find the total work done by the gas during both processes.

19.7 • **Work Done in a Cyclic Process.** (a) In Fig. 19.7a, consider the closed loop $1 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$. This is a *cyclic* process in which the initial and final states are the same. Find the total work done by the system in this cyclic process, and show that it is equal to the area enclosed by the loop. (b) How is the work done for the process in part (a) related to the work done if the loop is traversed in the opposite direction, $1 \rightarrow 4 \rightarrow 2 \rightarrow 3 \rightarrow 1$? Explain.

Section 19.4 Internal Energy and the First Law of Thermodynamics

19.8 • • Figure E19.8 shows a pV -diagram for an ideal gas in which its absolute temperature at b is one-fourth of its absolute temperature at a . (a) What volume does this gas occupy at point b ? (b) How many joules of work was done by or on the gas in this process?

Figure E19.8



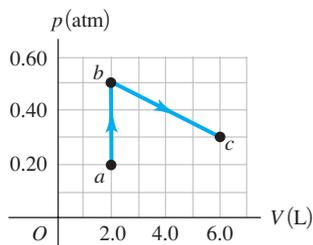
Was it done by or on the gas? (c) Did the internal energy of the gas increase or decrease from a to b ? How do you know? (d) Did heat enter or leave the gas from a to b ? How do you know?

19.9 • A gas in a cylinder expands from a volume of 0.110 m³ to 0.320 m³. Heat flows into the gas just rapidly enough to keep the pressure constant at 1.65×10^5 Pa during the expansion. The total heat added is 1.15×10^5 J. (a) Find the work done by the gas. (b) Find the change in internal energy of the gas. (c) Does it matter whether the gas is ideal? Why or why not?

19.10 • Five moles of an ideal monatomic gas with an initial temperature of 127°C expand and, in the process, absorb 1200 J of heat and do 2100 J of work. What is the final temperature of the gas?

19.11 • The process abc shown in the pV -diagram in Fig. E19.11 involves 0.0175 mole of an ideal gas. (a) What was the lowest temperature the gas reached in this process? Where did it occur? (b) How much work was done by or on the gas from a to b ? From b to c ? (c) If 215 J of heat was put into the gas during abc , how many of those joules went into internal energy?

Figure E19.11



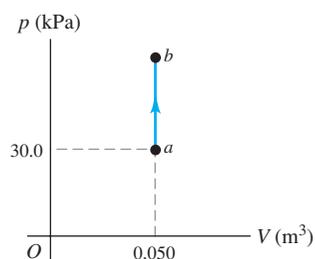
19.12 • A gas in a cylinder is held at a constant pressure of $1.80 \times 10^5\text{ Pa}$ and is cooled and compressed from 1.70 m^3 to 1.20 m^3 . The internal energy of the gas decreases by $1.40 \times 10^5\text{ J}$. (a) Find the work done by the gas. (b) Find the absolute value $|Q|$ of the heat flow into or out of the gas, and state the direction of the heat flow. (c) Does it matter whether the gas is ideal? Why or why not?

19.13 • **BIO Doughnuts: Breakfast of Champions!** A typical doughnut contains 2.0 g of protein, 17.0 g of carbohydrates, and 7.0 g of fat. The average food energy values of these substances are 4.0 kcal/g for protein and carbohydrates and 9.0 kcal/g for fat. (a) During heavy exercise, an average person uses energy at a rate of 510 kcal/h . How long would you have to exercise to “work off” one doughnut? (b) If the energy in the doughnut could somehow be converted into the kinetic energy of your body as a whole, how fast could you move after eating the doughnut? Take your mass to be 60 kg , and express your answer in m/s and in km/h .

19.14 • **Boiling Water at High Pressure.** When water is boiled at a pressure of 2.00 atm , the heat of vaporization is $2.20 \times 10^6\text{ J/kg}$ and the boiling point is 120°C . At this pressure, 1.00 kg of water has a volume of $1.00 \times 10^{-3}\text{ m}^3$, and 1.00 kg of steam has a volume of 0.824 m^3 . (a) Compute the work done when 1.00 kg of steam is formed at this temperature. (b) Compute the increase in internal energy of the water.

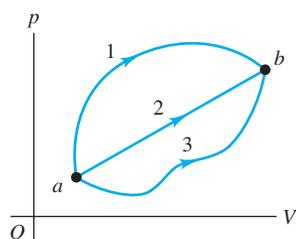
19.15 • An ideal gas is taken from a to b on the pV -diagram shown in Fig. E19.15. During this process, 700 J of heat is added and the pressure doubles. (a) How much work is done by or on the gas? Explain. (b) How does the temperature of the gas at a compare to its temperature at b ? Be specific. (c) How does the internal energy of the gas at a compare to the internal energy at b ? Again, be specific and explain.

Figure E19.15



19.16 • A system is taken from state a to state b along the three paths shown in Fig. E19.16. (a) Along which path is the work done by the system the greatest? The least? (b) If $U_b > U_a$, along which path is

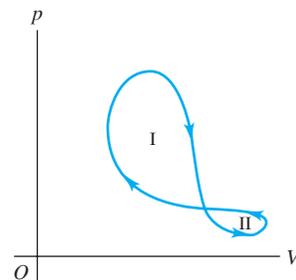
Figure E19.16



the absolute value $|Q|$ of the heat transfer the greatest? For this path, is heat absorbed or liberated by the system?

19.17 • A thermodynamic system undergoes a cyclic process as shown in Fig. E19.17. The cycle consists of two closed loops: I and II. (a) Over one complete cycle, does the system do positive or negative work? (b) In each of loops I and II, is the net work done by the system positive or negative? (c) Over one complete cycle, does heat flow into or out of the system? (d) In each of loops I and II, does heat flow into or out of the system?

Figure E19.17



Section 19.5 Kinds of Thermodynamic Processes, Section 19.6 Internal Energy of an Ideal Gas, and Section 19.7 Heat Capacities of an Ideal Gas

19.18 • During an isothermal compression of an ideal gas, 335 J of heat must be removed from the gas to maintain constant temperature. How much work is done by the gas during the process?

19.19 • A cylinder contains 0.250 mol of carbon dioxide (CO_2) gas at a temperature of 27.0°C . The cylinder is provided with a frictionless piston, which maintains a constant pressure of 1.00 atm on the gas. The gas is heated until its temperature increases to 127.0°C . Assume that the CO_2 may be treated as an ideal gas. (a) Draw a pV -diagram for this process. (b) How much work is done by the gas in this process? (c) On what is this work done? (d) What is the change in internal energy of the gas? (e) How much heat was supplied to the gas? (f) How much work would have been done if the pressure had been 0.50 atm ?

19.20 • A cylinder contains 0.0100 mol of helium at $T = 27.0^\circ\text{C}$. (a) How much heat is needed to raise the temperature to 67.0°C while keeping the volume constant? Draw a pV -diagram for this process. (b) If instead the pressure of the helium is kept constant, how much heat is needed to raise the temperature from 27.0°C to 67.0°C ? Draw a pV -diagram for this process. (c) What accounts for the difference between your answers to parts (a) and (b)? In which case is more heat required? What becomes of the additional heat? (d) If the gas is ideal, what is the change in its internal energy in part (a)? In part (b)? How do the two answers compare? Why?

19.21 • In an experiment to simulate conditions inside an automobile engine, 0.185 mol of air at a temperature of 780 K and a pressure of $3.00 \times 10^6\text{ Pa}$ is contained in a cylinder of volume 40.0 cm^3 . Then 645 J of heat is transferred to the cylinder. (a) If the volume of the cylinder is constant while the heat is added, what is the final temperature of the air? Assume that the air is essentially nitrogen gas, and use the data in Table 19.1 even though the pressure is not low. Draw a pV -diagram for this process. (b) If instead the volume of the cylinder is allowed to increase while the pressure remains constant, find the final temperature of the air. Draw a pV -diagram for this process.

19.22 • When a quantity of monatomic ideal gas expands at a constant pressure of $4.00 \times 10^4\text{ Pa}$, the volume of the gas increases from $2.00 \times 10^{-3}\text{ m}^3$ to $8.00 \times 10^{-3}\text{ m}^3$. What is the change in the internal energy of the gas?

19.23 • Heat Q flows into a monatomic ideal gas, and the volume increases while the pressure is kept constant. What fraction of the heat energy is used to do the expansion work of the gas?

19.24 • Three moles of an ideal monatomic gas expands at a constant pressure of 2.50 atm; the volume of the gas changes from $3.20 \times 10^{-2} \text{ m}^3$ to $4.50 \times 10^{-2} \text{ m}^3$. (a) Calculate the initial and final temperatures of the gas. (b) Calculate the amount of work the gas does in expanding. (c) Calculate the amount of heat added to the gas. (d) Calculate the change in internal energy of the gas.

19.25 • A cylinder with a movable piston contains 3.00 mol of N_2 gas (assumed to behave like an ideal gas). (a) The N_2 is heated at constant volume until 1557 J of heat have been added. Calculate the change in temperature. (b) Suppose the same amount of heat is added to the N_2 , but this time the gas is allowed to expand while remaining at constant pressure. Calculate the temperature change. (c) In which case, (a) or (b), is the final internal energy of the N_2 higher? How do you know? What accounts for the difference between the two cases?

19.26 • Propane gas (C_3H_8) behaves like an ideal gas with $\gamma = 1.127$. Determine the molar heat capacity at constant volume and the molar heat capacity at constant pressure.

19.27 • **CALC** The temperature of 0.150 mol of an ideal gas is held constant at 77.0°C while its volume is reduced to 25.0% of its initial volume. The initial pressure of the gas is 1.25 atm. (a) Determine the work done by the gas. (b) What is the change in its internal energy? (c) Does the gas exchange heat with its surroundings? If so, how much? Does the gas absorb or liberate heat?

19.28 • An experimenter adds 970 J of heat to 1.75 mol of an ideal gas to heat it from 10.0°C to 25.0°C at constant pressure. The gas does +223 J of work during the expansion. (a) Calculate the change in internal energy of the gas. (b) Calculate γ for the gas.

Section 19.8 Adiabatic Processes for an Ideal Gas

19.29 • A monatomic ideal gas that is initially at a pressure of $1.50 \times 10^5 \text{ Pa}$ and has a volume of 0.0800 m^3 is compressed adiabatically to a volume of 0.0400 m^3 . (a) What is the final pressure? (b) How much work is done by the gas? (c) What is the ratio of the final temperature of the gas to its initial temperature? Is the gas heated or cooled by this compression?

19.30 • In an adiabatic process for an ideal gas, the pressure decreases. In this process does the internal energy of the gas increase or decrease? Explain your reasoning.

19.31 • Two moles of carbon monoxide (CO) start at a pressure of 1.2 atm and a volume of 30 liters. The gas is then compressed adiabatically to $\frac{1}{3}$ this volume. Assume that the gas may be treated as ideal. What is the change in the internal energy of the gas? Does the internal energy increase or decrease? Does the temperature of the gas increase or decrease during this process? Explain.

19.32 • The engine of a Ferrari F355 F1 sports car takes in air at 20.0°C and 1.00 atm and compresses it adiabatically to 0.0900 times the original volume. The air may be treated as an ideal gas with $\gamma = 1.40$. (a) Draw a pV -diagram for this process. (b) Find the final temperature and pressure.

19.33 • During an adiabatic expansion the temperature of 0.450 mol of argon (Ar) drops from 50.0°C to 10.0°C . The argon may be treated as an ideal gas. (a) Draw a pV -diagram for this process. (b) How much work does the gas do? (c) What is the change in internal energy of the gas?

19.34 • A player bounces a basketball on the floor, compressing it to 80.0% of its original volume. The air (assume it is essentially N_2 gas) inside the ball is originally at a temperature of 20.0°C and a pressure of 2.00 atm. The ball's inside diameter is 23.9 cm. (a) What temperature does the air in the ball reach at its maximum compression? Assume the compression is adiabatic and treat the gas as ideal. (b) By how much does the internal energy of the air change between the ball's original state and its maximum compression?

19.35 • On a warm summer day, a large mass of air (atmospheric pressure $1.01 \times 10^5 \text{ Pa}$) is heated by the ground to a temperature of 26.0°C and then begins to rise through the cooler surrounding air. (This can be treated approximately as an adiabatic process; why?) Calculate the temperature of the air mass when it has risen to a level at which atmospheric pressure is only $0.850 \times 10^5 \text{ Pa}$. Assume that air is an ideal gas, with $\gamma = 1.40$. (This rate of cooling for dry, rising air, corresponding to roughly 1°C per 100 m of altitude, is called the *dry adiabatic lapse rate*.)

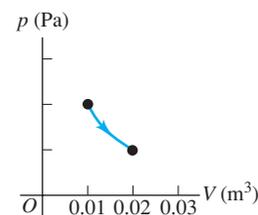
19.36 • A cylinder contains 0.100 mol of an ideal monatomic gas. Initially the gas is at a pressure of $1.00 \times 10^5 \text{ Pa}$ and occupies a volume of $2.50 \times 10^{-3} \text{ m}^3$. (a) Find the initial temperature of the gas in kelvins. (b) If the gas is allowed to expand to twice the initial volume, find the final temperature (in kelvins) and pressure of the gas if the expansion is (i) isothermal; (ii) isobaric; (iii) adiabatic.

PROBLEMS

19.37 • One mole of ideal gas is slowly compressed to one-third of its original volume. In this compression, the work done on the gas has magnitude 600 J. For the gas, $C_p = 7R/2$. (a) If the process is isothermal, what is the heat flow Q for the gas? Does heat flow into or out of the gas? (b) If the process is isobaric, what is the change in internal energy of the gas? Does the internal energy increase or decrease?

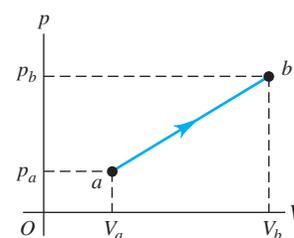
19.38 • **CALC** Figure P19.38 shows the pV -diagram for an isothermal expansion of 1.50 mol of an ideal gas, at a temperature of 15.0°C . (a) What is the change in internal energy of the gas? Explain. (b) Calculate the work done by (or on) the gas and the heat absorbed (or released) by the gas during the expansion.

Figure P19.38



19.39 • A quantity of air is taken from state a to state b along a path that is a straight line in the pV -diagram (Fig. P19.39).

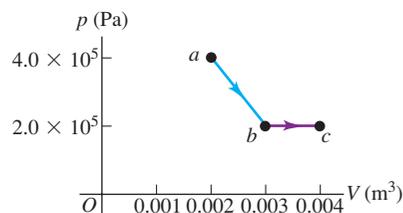
Figure P19.39



(a) In this process, does the temperature of the gas increase, decrease, or stay the same? Explain. (b) If $V_a = 0.0700 \text{ m}^3$, $V_b = 0.1100 \text{ m}^3$, $p_a = 1.00 \times 10^5 \text{ Pa}$, and $p_b = 1.40 \times 10^5 \text{ Pa}$, what is the work W done by the gas in this process? Assume that the gas may be treated as ideal.

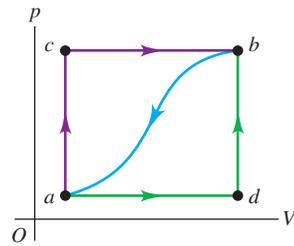
19.40 • One-half mole of an ideal gas is taken from state a to state c , as shown in Fig. P19.40. (a) Calculate the final temperature of the gas. (b) Calculate the work done on (or by) the gas as it moves from state a to state c . (c) Does heat leave the system or enter the system during this process? How much heat? Explain.

Figure P19.40



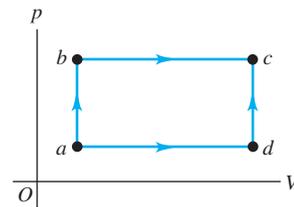
19.41 •• When a system is taken from state a to state b in Fig. P19.41 along the path acb , 90.0 J of heat flows into the system and 60.0 J of work is done by the system. (a) How much heat flows into the system along path adb if the work done by the system is 15.0 J? (b) When the system is returned from b to a along the curved path, the absolute value of the work done by the system is 35.0 J. Does the system absorb or liberate heat? How much heat? (c) If $U_a = 0$ and $U_d = 8.0$ J, find the heat absorbed in the processes ad and db .

Figure P19.41



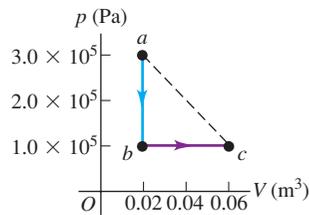
19.42 • A thermodynamic system is taken from state a to state c in Fig. P19.42 along either path abc or path adc . Along path abc , the work W done by the system is 450 J. Along path adc , W is 120 J. The internal energies of each of the four states shown in the figure are $U_a = 150$ J, $U_b = 240$ J, $U_c = 680$ J, and $U_d = 330$ J. Calculate the heat flow Q for each of the four processes ab , bc , ad , and dc . In each process, does the system absorb or liberate heat?

Figure P19.42



19.43 • A volume of air (assumed to be an ideal gas) is first cooled without changing its volume and then expanded without changing its pressure, as shown by the path abc in Fig. P19.43. (a) How does the final temperature of the gas compare with its initial temperature? (b) How much heat does the air exchange with its surroundings during the process abc ? Does the air absorb heat or release heat during this process? Explain. (c) If the air instead expands from state a to state c by the straight-line path shown, how much heat does it exchange with its surroundings?

Figure P19.43



19.44 • Three moles of argon gas (assumed to be an ideal gas) originally at a pressure of 1.50×10^4 Pa and a volume of 0.0280 m³ are first heated and expanded at constant pressure to a volume of 0.0435 m³, then heated at constant volume until the pressure reaches 3.50×10^4 Pa, then cooled and compressed at constant pressure until the volume is again 0.0280 m³, and finally cooled at constant volume until the pressure drops to its original value of 1.50×10^4 Pa. (a) Draw the pV -diagram for this cycle. (b) Calculate the total work done by (or on) the gas during the cycle. (c) Calculate the net heat exchanged with the surroundings. Does the gas gain or lose heat overall?

19.45 •• Two moles of an ideal monatomic gas go through the cycle abc . For the complete cycle, 800 J of heat flows out of the gas. Process ab is at constant pressure, and process bc is at constant volume. States a and b have temperatures $T_a = 200$ K and $T_b = 300$ K. (a) Sketch the pV -diagram for the cycle. (b) What is the work W for the process ca ?

19.46 •• Three moles of an ideal gas are taken around the cycle acb shown in Fig. P19.46. For this gas, $C_p = 29.1$ J/mol·K. Process ac is at constant pressure, process ba is at constant volume,

and process cb is adiabatic. The temperatures of the gas in states a , c , and b are $T_a = 300$ K, $T_c = 492$ K, and $T_b = 600$ K. Calculate the total work W for the cycle.

Figure P19.46

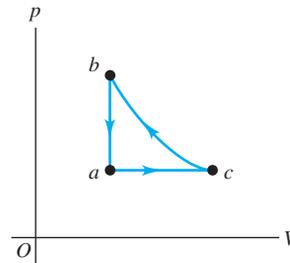
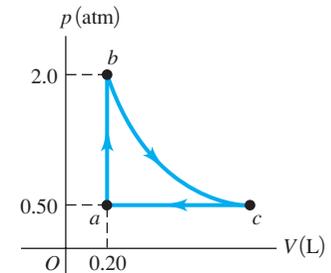


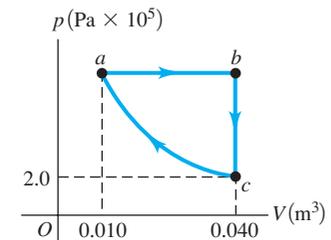
Figure P19.47



19.47 •• Figure P19.47 shows a pV -diagram for 0.0040 mole of ideal H_2 gas. The temperature of the gas does not change during segment bc . (a) What volume does this gas occupy at point c ? (b) Find the temperature of the gas at points a , b , and c . (c) How much heat went into or out of the gas during segments ab , ca , and bc ? Indicate whether the heat has gone into or out of the gas. (d) Find the change in the internal energy of this hydrogen during segments ab , bc , and ca . Indicate whether the internal energy increased or decreased during each of these segments.

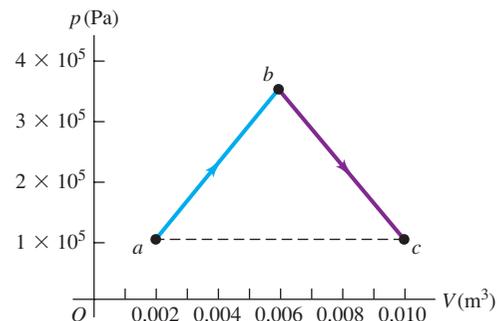
19.48 •• The graph in Fig. P19.48 shows a pV -diagram for 3.25 moles of ideal helium (He) gas. Part ca of this process is isothermal. (a) Find the pressure of the He at point a . (b) Find the temperature of the He at points a , b , and c . (c) How much heat entered or left the He during segments ab , bc , and ca ? In each segment, did the heat enter or leave? (d) By how much did the internal energy of the He change from a to b , from b to c , and from c to a ? Indicate whether this energy increased or decreased.

Figure P19.48



19.49 •• (a) One-third of a mole of He gas is taken along the path abc shown as the solid line in Fig. P19.49. Assume that the gas may be treated as ideal. How much heat is transferred into or out of the gas? (b) If the gas instead went from state a to state c along the horizontal dashed line in Fig. P19.49, how much heat would be transferred into or out of the gas? (c) How does Q in part (b) compare with Q in part (a)? Explain.

Figure P19.49



19.50 •• Two moles of helium are initially at a temperature of 27.0°C and occupy a volume of 0.0300 m³. The helium first

expands at constant pressure until its volume has doubled. Then it expands adiabatically until the temperature returns to its initial value. Assume that the helium can be treated as an ideal gas. (a) Draw a diagram of the process in the pV -plane. (b) What is the total heat supplied to the helium in the process? (c) What is the total change in internal energy of the helium? (d) What is the total work done by the helium? (e) What is the final volume of the helium?

19.51 •• Starting with 2.50 mol of N_2 gas (assumed to be ideal) in a cylinder at 1.00 atm and 20.0°C , a chemist first heats the gas at constant volume, adding 1.52×10^4 J of heat, then continues heating and allows the gas to expand at constant pressure to twice its original volume. (a) Calculate the final temperature of the gas. (b) Calculate the amount of work done by the gas. (c) Calculate the amount of heat added to the gas while it was expanding. (d) Calculate the change in internal energy of the gas for the whole process.

19.52 •• Nitrogen gas in an expandable container is cooled from 50.0°C to 10.0°C with the pressure held constant at 3.00×10^5 Pa. The total heat liberated by the gas is 2.50×10^4 J. Assume that the gas may be treated as ideal. (a) Find the number of moles of gas. (b) Find the change in internal energy of the gas. (c) Find the work done by the gas. (d) How much heat would be liberated by the gas for the same temperature change if the volume were constant?

19.53 • In a certain process, 2.15×10^5 J of heat is liberated by a system, and at the same time the system contracts under a constant external pressure of 9.50×10^5 Pa. The internal energy of the system is the same at the beginning and end of the process. Find the change in volume of the system. (The system is *not* an ideal gas.)

19.54 • CALC A cylinder with a frictionless, movable piston like that shown in Fig. 19.5 contains a quantity of helium gas. Initially the gas is at a pressure of 1.00×10^5 Pa, has a temperature of 300 K, and occupies a volume of 1.50 L. The gas then undergoes two processes. In the first, the gas is heated and the piston is allowed to move to keep the temperature equal to 300 K. This continues until the pressure reaches 2.50×10^4 Pa. In the second process, the gas is compressed at constant pressure until it returns to its original volume of 1.50 L. Assume that the gas may be treated as ideal. (a) In a pV -diagram, show both processes. (b) Find the volume of the gas at the end of the first process, and find the pressure and temperature at the end of the second process. (c) Find the total work done by the gas during both processes. (d) What would you have to do to the gas to return it to its original pressure and temperature?

19.55 •• CP A Thermodynamic Process in a Liquid. A chemical engineer is studying the properties of liquid methanol (CH_3OH). She uses a steel cylinder with a cross-sectional area of 0.0200 m² and containing 1.20×10^{-2} m³ of methanol. The cylinder is equipped with a tightly fitting piston that supports a load of 3.00×10^4 N. The temperature of the system is increased from 20.0°C to 50.0°C . For methanol, the coefficient of volume expansion is 1.20×10^{-3} K⁻¹, the density is 791 kg/m³, and the specific heat at constant pressure is $c_p = 2.51 \times 10^3$ J/kg · K. You can ignore the expansion of the steel cylinder. Find (a) the increase in volume of the methanol; (b) the mechanical work done by the methanol against the 3.00×10^4 N force; (c) the amount of heat added to the methanol; (d) the change in internal energy of the methanol. (e) Based on your results, explain whether there is any substantial difference between the specific heats c_p (at constant pressure) and c_V (at constant volume) for methanol under these conditions.

19.56 • CP A Thermodynamic Process in a Solid. A cube of copper 2.00 cm on a side is suspended by a string. (The physical

properties of copper are given in Tables 14.1, 17.2, and 17.3.) The cube is heated with a burner from 20.0°C to 90.0°C . The air surrounding the cube is at atmospheric pressure (1.01×10^5 Pa). Find (a) the increase in volume of the cube; (b) the mechanical work done by the cube to expand against the pressure of the surrounding air; (c) the amount of heat added to the cube; (d) the change in internal energy of the cube. (e) Based on your results, explain whether there is any substantial difference between the specific heats c_p (at constant pressure) and c_V (at constant volume) for copper under these conditions.

19.57 • BIO A Thermodynamic Process in an Insect. The African bombardier beetle (*Stenaptinus insignis*) can emit a jet of

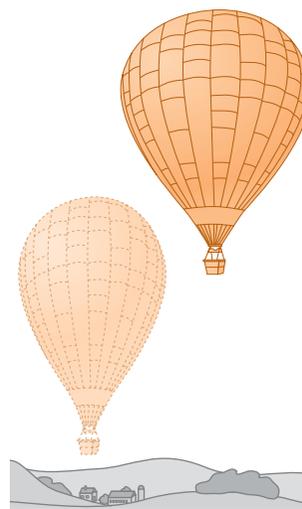
Figure P19.57



defensive spray from the movable tip of its abdomen (Fig. P19.57). The beetle's body has reservoirs of two different chemicals; when the beetle is disturbed, these chemicals are combined in a reaction chamber, producing a compound that is warmed from 20°C to 100°C by the heat of reaction. The high pressure produced allows the compound to be sprayed out at speeds up to 19 m/s (68 km/h), scaring away predators of all kinds. (The beetle shown in the figure is 2 cm long.) Calculate the heat of reaction of the two chemicals (in J/kg). Assume that the specific heat of the two chemicals and the spray is the same as that of water, 4.19×10^3 J/kg · K, and that the initial temperature of the chemicals is 20°C .

19.58 •• High-Altitude Research. A large research balloon containing 2.00×10^3 m³ of helium gas at 1.00 atm and a temper-

Figure P19.58



ature of 15.0°C rises rapidly from ground level to an altitude at which the atmospheric pressure is only 0.900 atm (Fig. P19.58). Assume the helium behaves like an ideal gas and the balloon's ascent is too rapid to permit much heat exchange with the surrounding air. (a) Calculate the volume of the gas at the higher altitude. (b) Calculate the temperature of the gas at the higher altitude. (c) What is the change in internal energy of the helium as the balloon rises to the higher altitude?

19.59 ••• Chinook. During certain seasons strong winds called chinooks blow from the west across the eastern slopes of the Rockies and downhill into Denver and nearby areas. Although the mountains are cool, the wind in Denver is very hot; within a few minutes after the chinook wind arrives, the temperature can climb 20°C ("chinook" is a Native American word meaning "snow eater"). Similar winds occur in the Alps (called foehns) and in southern California (called Santa Anas). (a) Explain why the temperature of the chinook wind rises as it descends the slopes. Why is it important that the wind be fast moving? (b) Suppose a strong wind is blowing toward Denver (elevation 1630 m) from Grays Peak (80 km west of Denver, at an elevation of 4350 m), where the air pressure is

5.60×10^4 Pa and the air temperature is -15.0°C . The temperature and pressure in Denver before the wind arrives are 2.0°C and 8.12×10^4 Pa. By how many Celsius degrees will the temperature in Denver rise when the chinook arrives?

19.60 •• A certain ideal gas has molar heat capacity at constant volume C_V . A sample of this gas initially occupies a volume V_0 at pressure p_0 and absolute temperature T_0 . The gas expands isobarically to a volume $2V_0$ and then expands further adiabatically to a final volume $4V_0$. (a) Draw a pV -diagram for this sequence of processes. (b) Compute the total work done by the gas for this sequence of processes. (c) Find the final temperature of the gas. (d) Find the absolute value $|Q|$ of the total heat flow into or out of the gas for this sequence of processes, and state the direction of heat flow.

19.61 ••• An air pump has a cylinder 0.250 m long with a movable piston. The pump is used to compress air from the atmosphere (at absolute pressure 1.01×10^5 Pa) into a very large tank at 4.20×10^5 Pa gauge pressure. (For air, $C_V = 20.8$ J/mol \cdot K.) (a) The piston begins the compression stroke at the open end of the cylinder. How far down the length of the cylinder has the piston moved when air first begins to flow from the cylinder into the tank? Assume that the compression is adiabatic. (b) If the air is taken into the pump at 27.0°C , what is the temperature of the compressed air? (c) How much work does the pump do in putting 20.0 mol of air into the tank?

19.62 •• Engine Turbochargers and Intercoolers. The power output of an automobile engine is directly proportional to the mass of air that can be forced into the volume of the engine's cylinders to react chemically with gasoline. Many cars have a *turbocharger*, which compresses the air before it enters the engine, giving a greater mass of air per volume. This rapid, essentially adiabatic compression also heats the air. To compress it further, the air then passes through an *intercooler* in which the air exchanges heat with its surroundings at essentially constant pressure. The air is then drawn into the cylinders. In a typical installation, air is taken into the turbocharger at atmospheric pressure (1.01×10^5 Pa), density $\rho = 1.23$ kg/m³, and temperature 15.0°C . It is compressed adiabatically to 1.45×10^5 Pa. In the intercooler, the air is cooled to the original temperature of 15.0°C at a constant pressure of 1.45×10^5 Pa. (a) Draw a pV -diagram for this sequence of processes. (b) If the volume of one of the engine's cylinders is 575 cm³, what mass of air exiting from the intercooler will fill the cylinder at 1.45×10^5 Pa? Compared to the power output of an engine that takes in air at 1.01×10^5 Pa at 15.0°C , what percentage increase in power is obtained by using the turbocharger and intercooler? (c) If the intercooler is not used, what mass of air exiting from the turbocharger will fill the cylinder at 1.45×10^5 Pa? Compared to the power output of an engine that takes in air at 1.01×10^5 Pa at 15.0°C , what percentage increase in power is obtained by using the turbocharger alone?

19.63 • A monatomic ideal gas expands slowly to twice its original volume, doing 300 J of work in the process. Find the heat added to the gas and the change in internal energy of the gas if the process is (a) isothermal; (b) adiabatic; (c) isobaric.

19.64 •• CALC A cylinder with a piston contains 0.250 mol of oxygen at 2.40×10^5 Pa and 355 K. The oxygen may be treated as an ideal gas. The gas first expands isobarically to twice its original volume. It is then compressed isothermally back to its original volume, and finally it is cooled isochorically to its original pressure. (a) Show the series of processes on a pV -diagram. (b) Compute the temperature during the isothermal compression. (c) Compute the maximum pressure. (d) Compute the total work done by the piston on the gas during the series of processes.

19.65 • Use the conditions and processes of Problem 19.64 to compute (a) the work done by the gas, the heat added to it, and its internal energy change during the initial expansion; (b) the work done, the heat added, and the internal energy change during the final cooling; (c) the internal energy change during the isothermal compression.

19.66 •• CALC A cylinder with a piston contains 0.150 mol of nitrogen at 1.80×10^5 Pa and 300 K. The nitrogen may be treated as an ideal gas. The gas is first compressed isobarically to half its original volume. It then expands adiabatically back to its original volume, and finally it is heated isochorically to its original pressure. (a) Show the series of processes in a pV -diagram. (b) Compute the temperatures at the beginning and end of the adiabatic expansion. (c) Compute the minimum pressure.

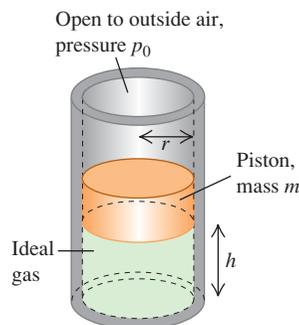
19.67 • Use the conditions and processes of Problem 19.66 to compute (a) the work done by the gas, the heat added to it, and its internal energy change during the initial compression; (b) the work done by the gas, the heat added to it, and its internal energy change during the adiabatic expansion; (c) the work done, the heat added, and the internal energy change during the final heating.

19.68 • Comparing Thermodynamic Processes. In a cylinder, 1.20 mol of an ideal monatomic gas, initially at 3.60×10^5 Pa and 300 K, expands until its volume triples. Compute the work done by the gas if the expansion is (a) isothermal; (b) adiabatic; (c) isobaric. (d) Show each process in a pV -diagram. In which case is the absolute value of the work done by the gas greatest? Least? (e) In which case is the absolute value of the heat transfer greatest? Least? (f) In which case is the absolute value of the change in internal energy of the gas greatest? Least?

CHALLENGE PROBLEMS

19.69 ••• CP Oscillations of a Piston. A vertical cylinder of radius r contains a quantity of ideal gas and is fitted with a piston with mass m that is free to move (Fig. P19.69). The piston and the walls of the cylinder are frictionless, and the entire cylinder is placed in a constant-temperature bath. The outside air pressure is p_0 . In equilibrium, the piston sits at a height h above the bottom of the cylinder. (a) Find the absolute pressure of the gas trapped below the piston when in equilibrium. (b) The piston is pulled up by a small distance and released. Find the net force acting on the piston when its base is a distance $h + y$ above the bottom of the cylinder, where y is much less than h . (c) After the piston is displaced from equilibrium and released, it oscillates up and down. Find the frequency of these small oscillations. If the displacement is not small, are the oscillations simple harmonic? How can you tell?

Figure P19.69



Answers

Chapter Opening Question ?

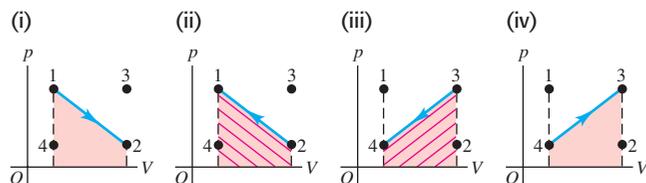
No. The work done by a gas as its volume changes from V_1 to V_2 is equal to the integral $\int p dV$ between those two volume limits. If the volume of the gas contracts, the final volume V_2 is less than the initial volume V_1 and the gas does negative work. Propelling the locomotive requires that the gas do positive work, so the gas doesn't contribute to propulsion while contracting.

Test Your Understanding Questions

19.1 Answers: negative, positive, positive Heat flows out of the coffee, so $Q_{\text{coffee}} < 0$; heat flows into the aluminum cup, so $Q_{\text{aluminum}} > 0$. In mechanics, we would say that negative work is done *on* the block, since the surface exerts a force on the block that opposes the block's motion. But in thermodynamics we use the opposite convention and say that $W > 0$, which means that positive work is done *by* the block on the surface.

19.2 Answer: (ii) The work done in an expansion is represented by the area under the curve of pressure p versus volume V . In an isothermal expansion the pressure decreases as the volume increases, so the pV -diagram looks like Fig. 19.6a and the work done equals the shaded area under the blue curve from point 1 to point 2. If, however, the expansion is at constant pressure, the curve of p versus V would be the same as the dashed horizontal line at pressure p_2 in Fig. 19.6a. The area under this dashed line is smaller than the area under the blue curve for an isothermal expansion, so less work is done in the constant-pressure expansion than in the isothermal expansion.

19.3 Answer: (i) and (iv) (tie), (ii) and (iii) (tie) The accompanying figure shows the pV -diagrams for each of the four processes. The trapezoidal area under the curve, and hence the absolute value of the work, is the same in all four cases. In cases (i) and (iv) the volume increases, so the system does positive work as it expands against its surroundings. In cases (ii) and (iii) the volume decreases, so the system does negative work (shown by cross-hatching) as the surroundings push inward on it.



19.4 Answer: (ii), (i) and (iv) (tie), (iii) In the expression $\Delta U = Q - W$, Q is the heat *added* to the system and W is the

work done *by* the system. If heat is transferred from the system to its surroundings, Q is negative; if work is done on the system, W is negative. Hence we have (i) $Q = -250 \text{ J}$, $W = -250 \text{ J}$, $\Delta U = -250 \text{ J} - (-250 \text{ J}) = 0$; (ii) $Q = 250 \text{ J}$, $W = -250 \text{ J}$, $\Delta U = 250 \text{ J} - (-250 \text{ J}) = 500 \text{ J}$; (iii) $Q = -250 \text{ J}$, $W = 250 \text{ J}$, $\Delta U = -250 \text{ J} - 250 \text{ J} = -500 \text{ J}$; and (iv) $Q = 250 \text{ J}$, $W = 250 \text{ J}$, $\Delta U = 250 \text{ J} - 250 \text{ J} = 0$.

19.5 Answers: 1 \rightarrow 4 and 3 \rightarrow 2 are isochoric; 1 \rightarrow 3 and 4 \rightarrow 2 are isobaric; no In a pV -diagram like those shown in Fig. 19.7, isochoric processes are represented by vertical lines (lines of constant volume) and isobaric processes are represented by horizontal lines (lines of constant pressure). The process 1 \rightarrow 2 in Fig. 19.7 is shown as a curved line, which superficially resembles the adiabatic and isothermal processes for an ideal gas in Fig. 19.16. Without more information we can't tell whether process 1 \rightarrow 2 is isothermal, adiabatic, or neither.

19.6 Answer: no Using the model of a solid in Fig. 18.20, we can see that the internal energy of a solid *does* depend on its volume. Compressing the solid means compressing the "springs" between the atoms, thereby increasing their stored potential energy and hence the internal energy of the solid.

19.7 Answer: (i) For a given number of moles n and a given temperature change ΔT , the amount of heat that must be transferred out of a fixed volume of air is $Q = nC_V\Delta T$. Hence the amount of heat transfer required is least for the gas with the smallest value of C_V . From Table 19.1, C_V is smallest for monatomic gases.

19.8 Answer: (ii), (iv), (i) and (iii) (tie) Samples (i) and (iii) are compressed isothermally, so $pV = \text{constant}$. The volume of each sample decreases to one-half of its initial value, so the final pressure is twice the initial pressure. Samples (ii) and (iv) are compressed adiabatically, so $pV^\gamma = \text{constant}$ and the pressure increases by a factor of 2^γ . Sample (ii) is a monatomic gas for which $\gamma = \frac{5}{3}$, so its final pressure is $2^{5/3} = 3.17$ times greater than the initial pressure. Sample (iv) is a diatomic gas for which $\gamma = \frac{7}{5}$, so its final pressure is greater than the initial pressure by a factor of $2^{7/5} = 2.64$.

Bridging Problem

Answers: (a) $W = nRT \ln \left[\frac{V_2 - nb}{V_1 - nb} \right] + an^2 \left[\frac{1}{V_2} - \frac{1}{V_1} \right]$

(b) (i) $W = 2.80 \times 10^3 \text{ J}$, (ii) $W = 3.11 \times 10^3 \text{ J}$

(c) Ideal gas, for which there is no attraction between molecules

20

THE SECOND LAW OF THERMODYNAMICS

LEARNING GOALS

By studying this chapter, you will learn:

- What determines whether a thermodynamic process is reversible or irreversible.
- What a heat engine is, and how to calculate its efficiency.
- The physics of internal-combustion engines.
- How refrigerators and heat engines are related, and how to analyze the performance of a refrigerator.
- How the second law of thermodynamics sets limits on the efficiency of engines and the performance of refrigerators.
- How to do calculations involving the idealized Carnot cycle for engines and refrigerators.
- What is meant by entropy, and how to use this concept to analyze thermodynamic processes.



? The second law of thermodynamics tells us that heat naturally flows from a hot body (such as molten lava, shown here flowing into the ocean in Hawaii) to a cold one (such as ocean water, which is heated to make steam). Is it ever possible for heat to flow from a cold body to a hot one?

Many thermodynamic processes proceed naturally in one direction but not the opposite. For example, heat by itself always flows from a hot body to a cooler body, never the reverse. Heat flow from a cool body to a hot body would not violate the first law of thermodynamics; energy would be conserved. But it doesn't happen in nature. Why not? As another example, note that it is easy to convert mechanical energy completely into heat; this happens every time we use a car's brakes to stop it. In the reverse direction, there are plenty of devices that convert heat *partially* into mechanical energy. (An automobile engine is an example.) But no one has ever managed to build a machine that converts heat *completely* into mechanical energy. Again, why not?

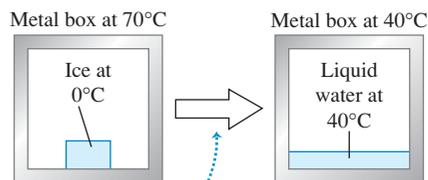
The answer to both of these questions has to do with the *directions* of thermodynamic processes and is called the *second law of thermodynamics*. This law places fundamental limitations on the efficiency of an engine or a power plant. It also places limitations on the minimum energy input needed to operate a refrigerator. So the second law is directly relevant for many important practical problems.

We can also state the second law in terms of the concept of *entropy*, a quantitative measure of the degree of disorder or randomness of a system. The idea of entropy helps explain why ink mixed with water never spontaneously unmixes and why we never observe a host of other seemingly possible processes.

20.1 Directions of Thermodynamic Processes

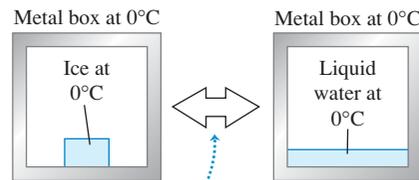
Thermodynamic processes that occur in nature are all **irreversible processes**. These are processes that proceed spontaneously in one direction but not the other (Fig. 20.1a). The flow of heat from a hot body to a cooler body is irreversible, as is the free expansion of a gas discussed in Sections 19.3 and 19.6. Sliding a book across a table converts mechanical energy into heat by friction;

(a) A block of ice melts *irreversibly* when we place it in a hot (70°C) metal box.



Heat flows from the box into the ice and water, never the reverse.

(b) A block of ice at 0°C can be melted *reversibly* if we put it in a 0°C metal box.



By infinitesimally raising or lowering the temperature of the box, we can make heat flow into the ice to melt it or make heat flow out of the water to refreeze it.

20.1 Reversible and irreversible processes.

this process is irreversible, for no one has ever observed the reverse process (in which a book initially at rest on the table would spontaneously start moving and the table and book would cool down). Our main topic for this chapter is the *second law of thermodynamics*, which determines the preferred direction for such processes.

Despite this preferred direction for every natural process, we can think of a class of idealized processes that *would* be reversible. A system that undergoes such an idealized **reversible process** is always very close to being in thermodynamic equilibrium within itself and with its surroundings. Any change of state that takes place can then be reversed by making only an infinitesimal change in the conditions of the system. For example, we can reverse heat flow between two bodies whose temperatures differ only infinitesimally by making only a very small change in one temperature or the other (Fig. 20.1b).

Reversible processes are thus **equilibrium processes**, with the system always in thermodynamic equilibrium. Of course, if a system were *truly* in thermodynamic equilibrium, no change of state would take place. Heat would not flow into or out of a system with truly uniform temperature throughout, and a system that is truly in mechanical equilibrium would not expand and do work against its surroundings. A reversible process is an idealization that can never be precisely attained in the real world. But by making the temperature gradients and the pressure differences in the substance very small, we can keep the system very close to equilibrium states and make the process nearly reversible.

By contrast, heat flow with finite temperature difference, free expansion of a gas, and conversion of work to heat by friction are all *irreversible* processes; no small change in conditions could make any of them go the other way. They are also all *nonequilibrium* processes, in that the system is not in thermodynamic equilibrium at any point until the end of the process.

Disorder and Thermodynamic Processes

There is a relationship between the direction of a process and the *disorder* or *randomness* of the resulting state. For example, imagine a thousand names written on file cards and arranged in alphabetical order. Throw the alphabetized stack of cards into the air, and they will likely come down in a random, disordered state. In the free expansion of a gas discussed in Sections 19.3 and 19.6, the air is more disordered after it has expanded into the entire box than when it was confined in one side, just as your clothes are more disordered when scattered all over your floor than when confined to your closet.

Similarly, macroscopic kinetic energy is energy associated with organized, coordinated motions of many molecules, but heat transfer involves changes in energy of random, disordered molecular motion. Therefore conversion of mechanical energy into heat involves an increase of randomness or disorder.

In the following sections we will introduce the second law of thermodynamics by considering two broad classes of devices: *heat engines*, which are partly

successful in converting heat into work, and *refrigerators*, which are partly successful in transporting heat from cooler to hotter bodies.

Test Your Understanding of Section 20.1 Your left and right hands are normally at the same temperature, just like the metal box and ice in Fig. 20.1b. Is rubbing your hands together to warm them (i) a reversible process or (ii) an irreversible process? **I**

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ActivPhysics 8.12: Cyclic Process—Strategies

ActivPhysics 8.13: Cyclic Process—Problems

20.2 All motorized vehicles other than purely electric vehicles use heat engines for propulsion. (Hybrid vehicles use their internal-combustion engine to help charge the batteries for the electric motor.)



20.2 Heat Engines

The essence of our technological society is the ability to use sources of energy other than muscle power. Sometimes, mechanical energy is directly available; water power and wind power are examples. But most of our energy comes from the burning of fossil fuels (coal, oil, and gas) and from nuclear reactions. They supply energy that is transferred as *heat*. This is directly useful for heating buildings, for cooking, and for chemical processing, but to operate a machine or propel a vehicle, we need *mechanical* energy.

Thus it's important to know how to take heat from a source and convert as much of it as possible into mechanical energy or work. This is what happens in gasoline engines in automobiles, jet engines in airplanes, steam turbines in electric power plants, and many other systems. Closely related processes occur in the animal kingdom; food energy is “burned” (that is, carbohydrates combine with oxygen to yield water, carbon dioxide, and energy) and partly converted to mechanical energy as an animal’s muscles do work on its surroundings.

Any device that transforms heat partly into work or mechanical energy is called a **heat engine** (Fig. 20.2). Usually, a quantity of matter inside the engine undergoes inflow and outflow of heat, expansion and compression, and sometimes change of phase. We call this matter the **working substance** of the engine. In internal-combustion engines, such as those used in automobiles, the working substance is a mixture of air and fuel; in a steam turbine it is water.

The simplest kind of engine to analyze is one in which the working substance undergoes a **cyclic process**, a sequence of processes that eventually leaves the substance in the same state in which it started. In a steam turbine the water is recycled and used over and over. Internal-combustion engines do not use the same air over and over, but we can still analyze them in terms of cyclic processes that approximate their actual operation.

Hot and Cold Reservoirs

All heat engines *absorb* heat from a source at a relatively high temperature, perform some mechanical work, and *discard* or *reject* some heat at a lower temperature. As far as the engine is concerned, the discarded heat is wasted. In internal-combustion engines the waste heat is that discarded in the hot exhaust gases and the cooling system; in a steam turbine it is the heat that must flow out of the used steam to condense and recycle the water.

When a system is carried through a cyclic process, its initial and final internal energies are equal. For any cyclic process, the first law of thermodynamics requires that

$$U_2 - U_1 = 0 = Q - W \quad \text{so} \quad Q = W$$

That is, the net heat flowing into the engine in a cyclic process equals the net work done by the engine.

When we analyze heat engines, it helps to think of two bodies with which the working substance of the engine can interact. One of these, called the *hot reservoir*, represents the heat source; it can give the working substance large amounts of heat at a constant temperature T_H without appreciably changing its own

temperature. The other body, called the *cold reservoir*, can absorb large amounts of discarded heat from the engine at a constant lower temperature T_C . In a steam-turbine system the flames and hot gases in the boiler are the hot reservoir, and the cold water and air used to condense and cool the used steam are the cold reservoir.

We denote the quantities of heat transferred from the hot and cold reservoirs as Q_H and Q_C , respectively. A quantity of heat Q is positive when heat is transferred *into* the working substance and is negative when heat leaves the working substance. Thus in a heat engine, Q_H is positive but Q_C is negative, representing heat *leaving* the working substance. This sign convention is consistent with the rules we stated in Section 19.1; we will continue to use those rules here. For clarity, we'll often state the relationships in terms of the absolute values of the Q 's and W 's because absolute values are always positive.

Energy-Flow Diagrams and Efficiency

We can represent the energy transformations in a heat engine by the *energy-flow diagram* of Fig. 20.3. The engine itself is represented by the circle. The amount of heat Q_H supplied to the engine by the hot reservoir is proportional to the width of the incoming "pipeline" at the top of the diagram. The width of the outgoing pipeline at the bottom is proportional to the magnitude $|Q_C|$ of the heat rejected in the exhaust. The branch line to the right represents the portion of the heat supplied that the engine converts to mechanical work, W .

When an engine repeats the same cycle over and over, Q_H and Q_C represent the quantities of heat absorbed and rejected by the engine *during one cycle*; Q_H is positive, and Q_C is negative. The *net* heat Q absorbed per cycle is

$$Q = Q_H + Q_C = |Q_H| - |Q_C| \quad (20.1)$$

The useful output of the engine is the net work W done by the working substance. From the first law,

$$W = Q = Q_H + Q_C = |Q_H| - |Q_C| \quad (20.2)$$

Ideally, we would like to convert *all* the heat Q_H into work; in that case we would have $Q_H = W$ and $Q_C = 0$. Experience shows that this is impossible; there is always some heat wasted, and Q_C is *never zero*. We define the **thermal efficiency** of an engine, denoted by e , as the quotient

$$e = \frac{W}{Q_H} \quad (20.3)$$

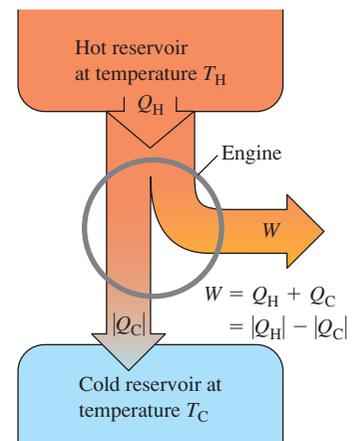
The thermal efficiency e represents the fraction of Q_H that is converted to work. To put it another way, e is what you get divided by what you pay for. This is always less than unity, an all-too-familiar experience! In terms of the flow diagram of Fig. 20.3, the most efficient engine is one for which the branch pipeline representing the work output is as wide as possible and the exhaust pipeline representing the heat thrown away is as narrow as possible.

When we substitute the two expressions for W given by Eq. (20.2) into Eq. (20.3), we get the following equivalent expressions for e :

$$e = \frac{W}{Q_H} = 1 + \frac{Q_C}{Q_H} = 1 - \left| \frac{Q_C}{Q_H} \right| \quad (\text{thermal efficiency of an engine}) \quad (20.4)$$

Note that e is a quotient of two energy quantities and thus is a pure number, without units. Of course, we must always express W , Q_H , and Q_C in the same units.

20.3 Schematic energy-flow diagram for a heat engine.



Application Biological Efficiency

Although a biological organism is not a heat engine, the concept of efficiency still applies: Here e is the ratio of the work done to the energy that was used to do that work. To exercise on a stationary bike, your body must first convert the chemical-bond energy in glucose to chemical-bond energy in ATP (adenosine triphosphate), then convert energy from ATP into motion of your leg muscles, and finally convert muscular motion into motion of the pedals. The overall efficiency of this entire process is only about 25%. The remaining 75% of the energy liberated from glucose goes into heating your body.



Problem-Solving Strategy 20.1 Heat Engines



Problems involving heat engines are, fundamentally, problems in the first law of thermodynamics. You should review Problem-Solving Strategy 19.1 (Section 19.4).

IDENTIFY *the relevant concepts:* A heat engine is any device that converts heat partially to work, as shown schematically in Fig. 20.3. We will see in Section 20.4 that a refrigerator is essentially a heat engine running in reverse, so many of the same concepts apply.

SET UP *the problem* as suggested in Problem-Solving Strategy 19.1. Use Eq. (20.4) if the thermal efficiency of the engine is relevant. Sketch an energy-flow diagram like Fig. 20.3.

EXECUTE *the solution* as follows:

1. Be careful with the sign conventions for W and the various Q 's. W is positive when the system expands and does work; W is

negative when the system is compressed and work is done on it. Each Q is positive if it represents heat entering the system and is negative if it represents heat leaving the system. When you know that a quantity is negative, such as Q_C in the above discussion, it sometimes helps to write it as $Q_C = -|Q_C|$.

2. Power is work per unit time ($P = W/t$), and rate of heat transfer (heat current) H is heat transfer per unit time ($H = Q/t$). In problems involving these concepts it helps to ask, "What is W or Q in one second (or one hour)?"
3. Keeping steps 1 and 2 in mind, solve for the target variables.

EVALUATE *your answer:* Use the first law of thermodynamics to check your results. Pay particular attention to algebraic signs.

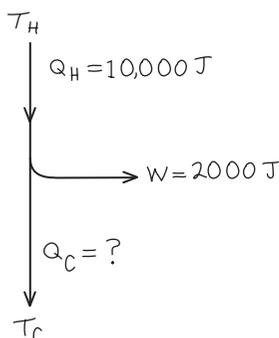
Example 20.1 Analyzing a heat engine

A gasoline truck engine takes in 10,000 J of heat and delivers 2000 J of mechanical work per cycle. The heat is obtained by burning gasoline with heat of combustion $L_c = 5.0 \times 10^4$ J/g. (a) What is the thermal efficiency of this engine? (b) How much heat is discarded in each cycle? (c) If the engine goes through 25 cycles per second, what is its power output in watts? In horsepower? (d) How much gasoline is burned in each cycle? (e) How much gasoline is burned per second? Per hour?

SOLUTION

IDENTIFY and SET UP: This problem concerns a heat engine, so we can use the ideas of this section. Figure 20.4 is our energy-flow diagram for one cycle. In each cycle the engine does $W = 2000$ J of work and takes in heat $Q_H = 10,000$ J. We use Eq. (20.4), in the form $e = W/Q_H$, to find the thermal efficiency. We use Eq. (20.2) to find the amount of heat Q_C rejected per cycle. The heat of combustion tells us how much gasoline must be burned per cycle and hence per unit time. The power output is the time rate at which the work W is done.

20.4 Our sketch for this problem.



EXECUTE: (a) From Eq. (20.4), the thermal efficiency is

$$e = \frac{W}{Q_H} = \frac{2000 \text{ J}}{10,000 \text{ J}} = 0.20 = 20\%$$

(b) From Eq. (20.2), $W = Q_H + Q_C$, so

$$Q_C = W - Q_H = 2000 \text{ J} - 10,000 \text{ J} = -8000 \text{ J}$$

That is, 8000 J of heat leaves the engine during each cycle.

(c) The power P equals the work per cycle multiplied by the number of cycles per second:

$$\begin{aligned} P &= (2000 \text{ J/cycle})(25 \text{ cycles/s}) = 50,000 \text{ W} = 50 \text{ kW} \\ &= (50,000 \text{ W}) \frac{1 \text{ hp}}{746 \text{ W}} = 67 \text{ hp} \end{aligned}$$

(d) Let m be the mass of gasoline burned during each cycle. Then $Q_H = mL_c$ and

$$m = \frac{Q_H}{L_c} = \frac{10,000 \text{ J}}{5.0 \times 10^4 \text{ J/g}} = 0.20 \text{ g}$$

(e) The mass of gasoline burned per second equals the mass per cycle multiplied by the number of cycles per second:

$$(0.20 \text{ g/cycle})(25 \text{ cycles/s}) = 5.0 \text{ g/s}$$

The mass burned per hour is

$$(5.0 \text{ g/s}) \frac{3600 \text{ s}}{1 \text{ h}} = 18,000 \text{ g/h} = 18 \text{ kg/h}$$

EVALUATE: An efficiency of 20% is fairly typical for cars and trucks if W includes only the work delivered to the wheels. We can check the mass burned per hour by expressing it in miles per gallon ("mileage"). The density of gasoline is about 0.70 g/cm^3 , so this is about $25,700 \text{ cm}^3$, 25.7 L, or 6.8 gallons of gasoline per hour. If the truck is traveling at 55 mi/h (88 km/h), this represents fuel consumption of 8.1 miles/gallon (3.4 km/L). This is a fairly typical mileage for large trucks.

Test Your Understanding of Section 20.2 Rank the following heat engines in order from highest to lowest thermal efficiency. (i) an engine that in one cycle absorbs 5000 J of heat and rejects 4500 J of heat; (ii) an engine that in one cycle absorbs 25,000 J of heat and does 2000 J of work; (iii) an engine that in one cycle does 400 J of work and rejects 2800 J of heat. 

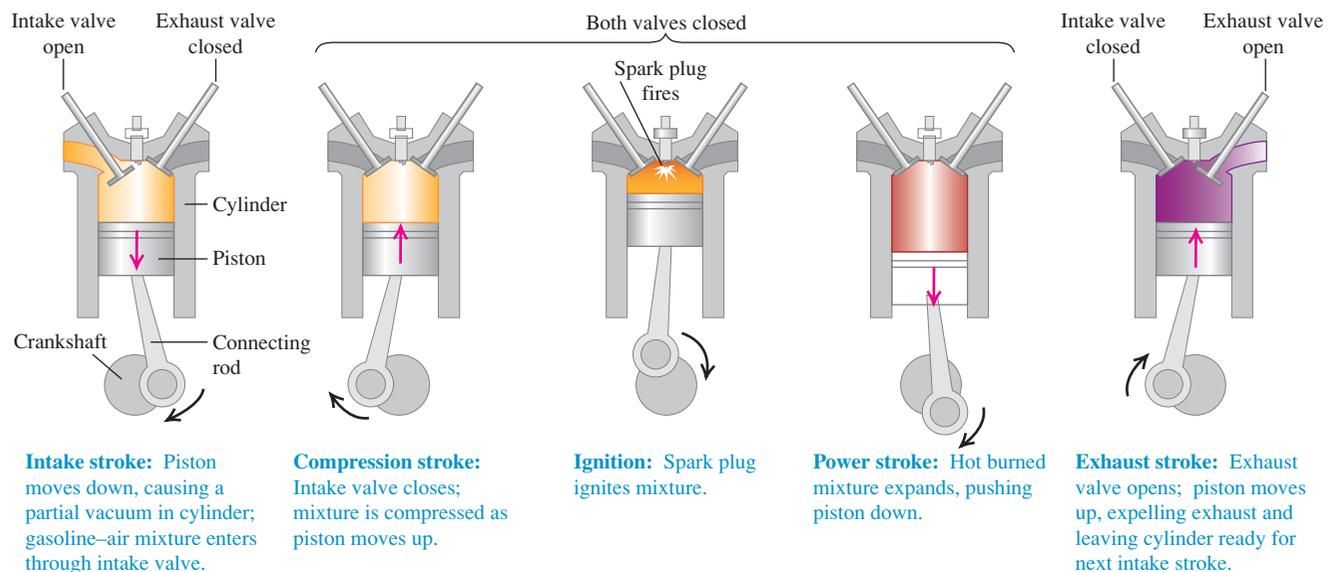
20.3 Internal-Combustion Engines

The gasoline engine, used in automobiles and many other types of machinery, is a familiar example of a heat engine. Let's look at its thermal efficiency. Figure 20.5 shows the operation of one type of gasoline engine. First a mixture of air and gasoline vapor flows into a cylinder through an open intake valve while the piston descends, increasing the volume of the cylinder from a minimum of V (when the piston is all the way up) to a maximum of rV (when it is all the way down). The quantity r is called the **compression ratio**; for present-day automobile engines its value is typically 8 to 10. At the end of this *intake stroke*, the intake valve closes and the mixture is compressed, approximately adiabatically, to volume V during the *compression stroke*. The mixture is then ignited by the spark plug, and the heated gas expands, approximately adiabatically, back to volume rV , pushing on the piston and doing work; this is the *power stroke*. Finally, the exhaust valve opens, and the combustion products are pushed out (during the *exhaust stroke*), leaving the cylinder ready for the next intake stroke.

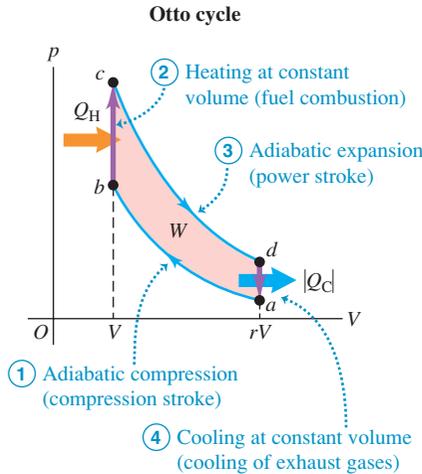
The Otto Cycle

Figure 20.6 is a pV -diagram for an idealized model of the thermodynamic processes in a gasoline engine. This model is called the **Otto cycle**. At point a the gasoline–air mixture has entered the cylinder. The mixture is compressed adiabatically to point b and is then ignited. Heat Q_H is added to the system by the burning gasoline along line bc , and the power stroke is the adiabatic expansion to d . The gas is cooled to the temperature of the outside air along line da ; during this process, heat $|Q_C|$ is rejected. This gas leaves the engine as exhaust and does not enter the engine again. But since an equivalent amount of gasoline and air enters, we may consider the process to be cyclic.

20.5 Cycle of a four-stroke internal-combustion engine.



20.6 The pV -diagram for the Otto cycle, an idealized model of the thermodynamic processes in a gasoline engine. 



We can calculate the efficiency of this idealized cycle. Processes bc and da are constant-volume, so the heats Q_H and Q_C are related simply to the temperatures:

$$Q_H = nC_V(T_c - T_b) > 0$$

$$Q_C = nC_V(T_a - T_d) < 0$$

The thermal efficiency is given by Eq. (20.4). Inserting the above expressions and cancelling out the common factor nC_V , we find

$$e = \frac{Q_H + Q_C}{Q_H} = \frac{T_c - T_b + T_a - T_d}{T_c - T_b} \quad (20.5)$$

To simplify this further, we use the temperature–volume relationship for adiabatic processes for an ideal gas, Eq. (19.22). For the two adiabatic processes ab and cd ,

$$T_a(rV)^{\gamma-1} = T_bV^{\gamma-1} \quad \text{and} \quad T_d(rV)^{\gamma-1} = T_cV^{\gamma-1}$$

We divide each of these equations by the common factor $V^{\gamma-1}$ and substitute the resulting expressions for T_b and T_c back into Eq. (20.5). The result is

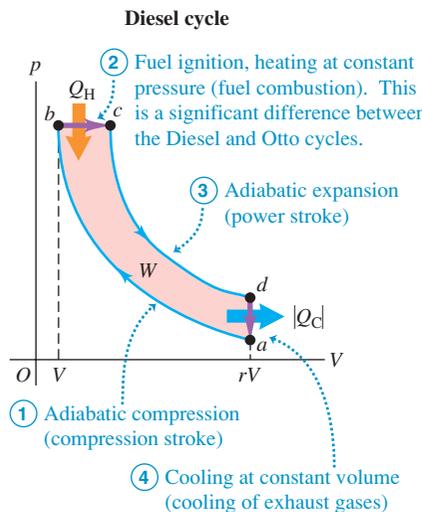
$$e = \frac{T_dr^{\gamma-1} - T_ar^{\gamma-1} + T_a - T_d}{T_dr^{\gamma-1} - T_ar^{\gamma-1}} = \frac{(T_d - T_a)(r^{\gamma-1} - 1)}{(T_d - T_a)r^{\gamma-1}}$$

Dividing out the common factor $(T_d - T_a)$, we get

$$e = 1 - \frac{1}{r^{\gamma-1}} \quad (\text{thermal efficiency in Otto cycle}) \quad (20.6)$$

The thermal efficiency given by Eq. (20.6) is always less than unity, even for this idealized model. With $r = 8$ and $\gamma = 1.4$ (the value for air) the theoretical efficiency is $e = 0.56$, or 56%. The efficiency can be increased by increasing r . However, this also increases the temperature at the end of the adiabatic compression of the air–fuel mixture. If the temperature is too high, the mixture explodes spontaneously during compression instead of burning evenly after the spark plug ignites it. This is called *pre-ignition* or *detonation*; it causes a knocking sound and can damage the engine. The octane rating of a gasoline is a measure of its antiknock qualities. The maximum practical compression ratio for high-octane, or “premium,” gasoline is about 10 to 13.

20.7 The pV -diagram for the idealized Diesel cycle. 



The Otto cycle is a highly idealized model. It assumes that the mixture behaves as an ideal gas; it neglects friction, turbulence, loss of heat to cylinder walls, and many other effects that reduce the efficiency of an engine. Efficiencies of real gasoline engines are typically around 35%.

The Diesel Cycle

The Diesel engine is similar in operation to the gasoline engine. The most important difference is that there is no fuel in the cylinder at the beginning of the compression stroke. A little before the beginning of the power stroke, the injectors start to inject fuel directly into the cylinder, just fast enough to keep the pressure approximately constant during the first part of the power stroke. Because of the high temperature developed during the adiabatic compression, the fuel ignites spontaneously as it is injected; no spark plugs are needed.

Figure 20.7 shows the idealized **Diesel cycle**. Starting at point a , air is compressed adiabatically to point b , heated at constant pressure to point c , expanded adiabatically to point d , and cooled at constant volume to point a . Because there is no fuel in the cylinder during most of the compression stroke, pre-ignition cannot occur, and the compression ratio r can be much higher than for a gasoline engine. This improves efficiency and ensures reliable ignition when the fuel is injected (because of the high temperature reached during the

adiabatic compression). Values of r of 15 to 20 are typical; with these values and $\gamma = 1.4$, the theoretical efficiency of the idealized Diesel cycle is about 0.65 to 0.70. As with the Otto cycle, the efficiency of any actual engine is substantially less than this. While Diesel engines are very efficient, they must be built to much tighter tolerances than gasoline engines and the fuel-injection system requires careful maintenance.

Test Your Understanding of Section 20.3 For an Otto-cycle engine with cylinders of a fixed size and a fixed compression ratio, which of the following aspects of the pV -diagram in Fig. 20.6 would change if you doubled the amount of fuel burned per cycle? (There may be more than one correct answer.) (i) the vertical distance between points b and c ; (ii) the vertical distance between points a and d ; (iii) the horizontal distance between points b and a .

20.4 Refrigerators

We can think of a **refrigerator** as a heat engine operating in reverse. A heat engine takes heat from a hot place and gives off heat to a colder place. A refrigerator does the opposite; it takes heat from a cold place (the inside of the refrigerator) and gives it off to a warmer place (usually the air in the room where the refrigerator is located). A heat engine has a net *output* of mechanical work; the refrigerator requires a net *input* of mechanical work. Using the sign conventions from Section 20.2, for a refrigerator Q_C is positive but both W and Q_H are negative; hence $|W| = -W$ and $|Q_H| = -Q_H$.

Figure 20.8 shows an energy-flow diagram for a refrigerator. From the first law for a cyclic process,

$$Q_H + Q_C - W = 0 \quad \text{or} \quad -Q_H = Q_C - W$$

or, because both Q_H and W are negative,

$$|Q_H| = |Q_C| + |W| \quad (20.7)$$

Thus, as the diagram shows, the heat $|Q_H|$ leaving the working substance and given to the hot reservoir is always *greater* than the heat Q_C taken from the cold reservoir. Note that the absolute-value relationship

$$|Q_H| = |Q_C| + |W| \quad (20.8)$$

is valid for both heat engines and refrigerators.

From an economic point of view, the best refrigeration cycle is one that removes the greatest amount of heat $|Q_C|$ from the inside of the refrigerator for the least expenditure of mechanical work, $|W|$. The relevant ratio is therefore $|Q_C|/|W|$; the larger this ratio, the better the refrigerator. We call this ratio the **coefficient of performance**, denoted by K . From Eq. (20.8), $|W| = |Q_H| - |Q_C|$, so

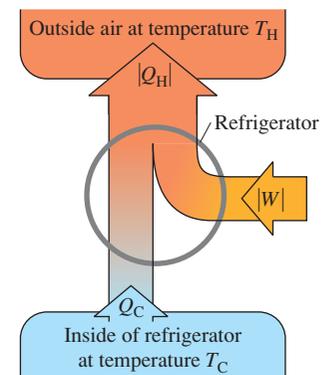
$$K = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|} \quad (\text{coefficient of performance of a refrigerator}) \quad (20.9)$$

As always, we measure Q_H , Q_C , and W all in the same energy units; K is then a dimensionless number.

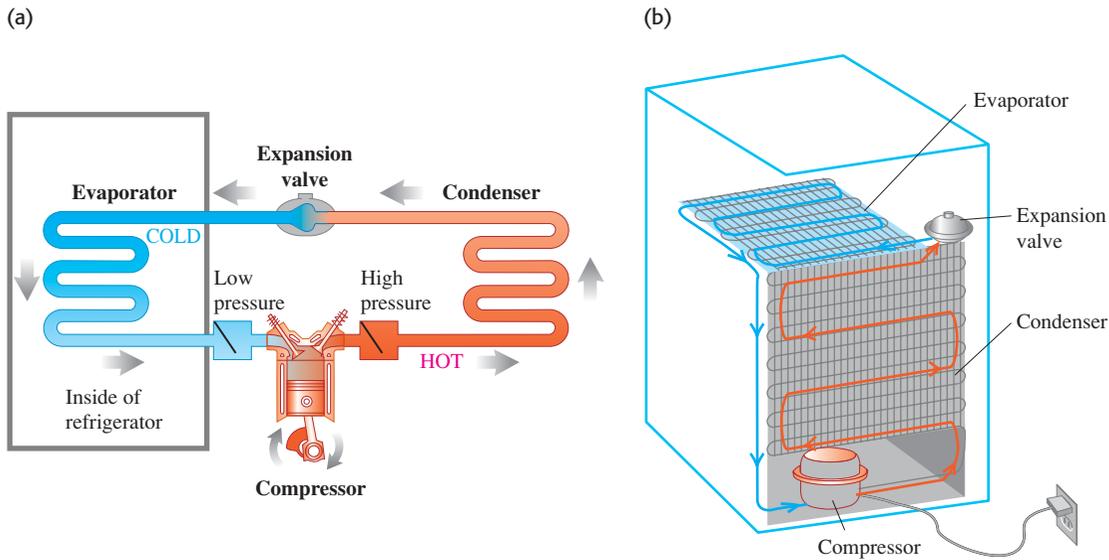
Practical Refrigerators

The principles of the common refrigeration cycle are shown schematically in Fig. 20.9a. The fluid “circuit” contains a refrigerant fluid (the working substance). The left side of the circuit (including the cooling coils inside the refrigerator) is at low temperature and low pressure; the right side (including the condenser coils outside the refrigerator) is at high temperature and high pressure. Ordinarily, both sides contain liquid and vapor in phase equilibrium.

20.8 Schematic energy-flow diagram of a refrigerator.



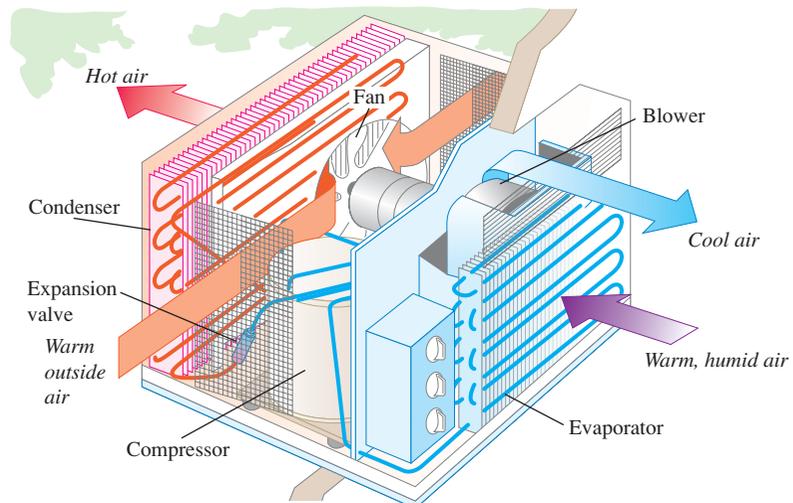
20.9 (a) Principle of the mechanical refrigeration cycle. (b) How the key elements are arranged in a practical refrigerator.



The compressor takes in fluid, compresses it adiabatically, and delivers it to the condenser coil at high pressure. The fluid temperature is then higher than that of the air surrounding the condenser, so the refrigerant gives off heat $|Q_H|$ and partially condenses to liquid. The fluid then expands adiabatically into the evaporator at a rate controlled by the expansion valve. As the fluid expands, it cools considerably, enough that the fluid in the evaporator coil is colder than its surroundings. It absorbs heat $|Q_C|$ from its surroundings, cooling them and partially vaporizing. The fluid then enters the compressor to begin another cycle. The compressor, usually driven by an electric motor (Fig. 20.9b), requires energy input and does work $|W|$ on the working substance during each cycle.

An air conditioner operates on exactly the same principle. In this case the refrigerator box becomes a room or an entire building. The evaporator coils are inside, the condenser is outside, and fans circulate air through these (Fig. 20.10). In large installations the condenser coils are often cooled by water. For air conditioners the quantities of greatest practical importance are the *rate* of heat removal (the heat current H from the region being cooled) and the *power* input $P = W/t$

20.10 An air conditioner works on the same principle as a refrigerator.



to the compressor. If heat $|Q_C|$ is removed in time t , then $H = |Q_C|/t$. Then we can express the coefficient of performance as

$$K = \frac{|Q_C|}{|W|} = \frac{Ht}{Pt} = \frac{H}{P}$$

Typical room air conditioners have heat removal rates H of 5000 to 10,000 Btu/h, or about 1500–3000 W, and require electric power input of about 600 to 1200 W. Typical coefficients of performance are about 3; the actual values depend on the inside and outside temperatures.

A variation on this theme is the **heat pump**, used to heat buildings by cooling the outside air. It functions like a refrigerator turned inside out. The evaporator coils are outside, where they take heat from cold air, and the condenser coils are inside, where they give off heat to the warmer air. With proper design, the heat $|Q_H|$ delivered to the inside per cycle can be considerably greater than the work $|W|$ required to get it there.

Work is *always* needed to transfer heat from a colder to a hotter body. **?** Heat flows spontaneously from hotter to colder, and to reverse this flow **?** requires the addition of work from the outside. Experience shows that it is impossible to make a refrigerator that transports heat from a colder body to a hotter body without the addition of work. If no work were needed, the coefficient of performance would be infinite. We call such a device a *workless refrigerator*; it is a mythical beast, like the unicorn and the free lunch.

Test Your Understanding of Section 20.4 Can you cool your house by leaving the refrigerator door open? **I**

20.5 The Second Law of Thermodynamics

Experimental evidence suggests strongly that it is *impossible* to build a heat engine that converts heat completely to work—that is, an engine with 100% thermal efficiency. This impossibility is the basis of one statement of the **second law of thermodynamics**, as follows:

It is impossible for any system to undergo a process in which it absorbs heat from a reservoir at a single temperature and converts the heat completely into mechanical work, with the system ending in the same state in which it began.

We will call this the “engine” statement of the second law. (It is also known to physicists as the *Kelvin–Planck statement* of this law.)

The basis of the second law of thermodynamics is the difference between the nature of internal energy and that of macroscopic mechanical energy. In a moving body the molecules have random motion, but superimposed on this is a coordinated motion of every molecule in the direction of the body’s velocity. The kinetic energy associated with this *coordinated* macroscopic motion is what we call the kinetic energy of the moving body. The kinetic and potential energies associated with the *random* motion constitute the internal energy.

When a body sliding on a surface comes to rest as a result of friction, the organized motion of the body is converted to random motion of molecules in the body and in the surface. Since we cannot control the motions of individual molecules, we cannot convert this random motion completely back to organized motion. We can convert *part* of it, and this is what a heat engine does.

If the second law were *not* true, we could power an automobile or run a power plant by cooling the surrounding air. Neither of these impossibilities violates the *first* law of thermodynamics. The second law, therefore, is not a deduction from the first but stands by itself as a separate law of nature. The first law denies the possibility of creating or destroying energy; the second law limits the *availability* of energy and the ways in which it can be used and converted.

Restating the Second Law

Our analysis of refrigerators in Section 20.4 forms the basis for an alternative statement of the second law of thermodynamics. Heat flows spontaneously from hotter to colder bodies, never the reverse. A refrigerator does take heat from a colder to a hotter body, but its operation requires an input of mechanical energy or work. Generalizing this observation, we state:

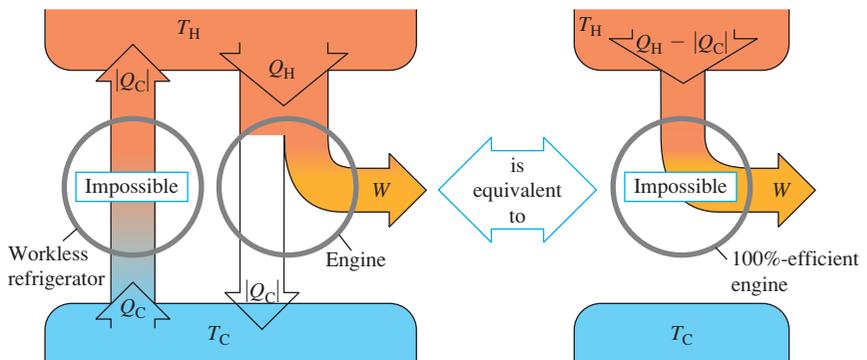
It is impossible for any process to have as its sole result the transfer of heat from a cooler to a hotter body.

We'll call this the “refrigerator” statement of the second law. (It is also known as the *Clausius statement*.) It may not seem to be very closely related to the “engine” statement. In fact, though, the two statements are completely equivalent. For example, if we could build a workless refrigerator, violating the second or “refrigerator” statement of the second law, we could use it in conjunction with a heat engine, pumping the heat rejected by the engine back to the hot reservoir to be reused. This composite machine (Fig. 20.11a) would violate the “engine” statement of the second law because its net effect would be to take a net quantity of heat $Q_H - |Q_C|$ from the hot reservoir and convert it completely to work W .

Alternatively, if we could make an engine with 100% thermal efficiency, in violation of the first statement, we could run it using heat from the hot reservoir and use the work output to drive a refrigerator that pumps heat from the cold reservoir to the hot (Fig. 20.11b). This composite device would violate the “refrigerator” statement because its net effect would be to take heat Q_C from the

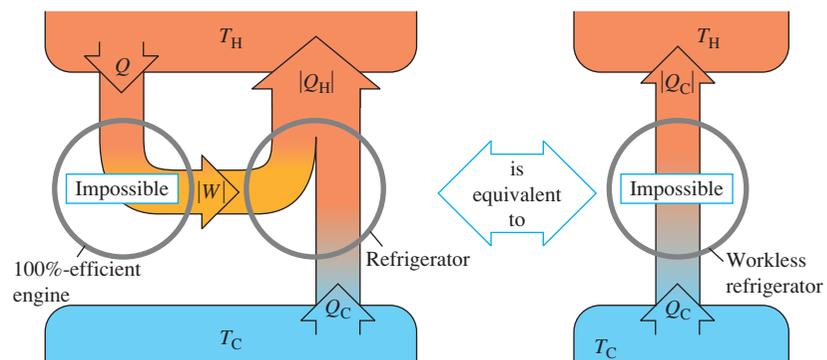
20.11 Energy-flow diagrams showing that the two forms of the second law are equivalent.

(a) The “engine” statement of the second law of thermodynamics



If a workless refrigerator were possible, it could be used in conjunction with an ordinary heat engine to form a 100%-efficient engine, converting heat $Q_H - |Q_C|$ completely to work.

(b) The “refrigerator” statement of the second law of thermodynamics



If a 100%-efficient engine were possible, it could be used in conjunction with an ordinary refrigerator to form a workless refrigerator, transferring heat Q_C from the cold to the hot reservoir with no input of work.

cold reservoir and deliver it to the hot reservoir without requiring any input of work. Thus any device that violates one form of the second law can be used to make a device that violates the other form. If violations of the first form are impossible, so are violations of the second!

The conversion of work to heat and the heat flow from hot to cold across a finite temperature gradient are *irreversible* processes. The “engine” and “refrigerator” statements of the second law state that these processes can be only partially reversed. We could cite other examples. Gases naturally flow from a region of high pressure to a region of low pressure; gases and miscible liquids left by themselves always tend to mix, not to unmix. The second law of thermodynamics is an expression of the inherent one-way aspect of these and many other irreversible processes. Energy conversion is an essential aspect of all plant and animal life and of human technology, so the second law of thermodynamics is of fundamental importance.

Test Your Understanding of Section 20.5 Would a 100%-efficient engine (Fig. 20.11a) violate the *first* law of thermodynamics? What about a workless refrigerator (Fig. 20.11b)?

20.6 The Carnot Cycle

According to the second law, no heat engine can have 100% efficiency. How great an efficiency *can* an engine have, given two heat reservoirs at temperatures T_H and T_C ? This question was answered in 1824 by the French engineer Sadi Carnot (1796–1832), who developed a hypothetical, idealized heat engine that has the maximum possible efficiency consistent with the second law. The cycle of this engine is called the **Carnot cycle**.

To understand the rationale of the Carnot cycle, we return to *reversibility* and its relationship to directions of thermodynamic processes. Conversion of work to heat is an irreversible process; the purpose of a heat engine is a *partial* reversal of this process, the conversion of heat to work with as great an efficiency as possible. For maximum heat-engine efficiency, therefore, *we must avoid all irreversible processes* (Fig. 20.12).

Heat flow through a finite temperature drop is an irreversible process. Therefore, during heat transfer in the Carnot cycle there must be *no* finite temperature difference. When the engine takes heat from the hot reservoir at temperature T_H , the working substance of the engine must also be at T_H ; otherwise, irreversible heat flow would occur. Similarly, when the engine discards heat to the cold reservoir at T_C , the engine itself must be at T_C . That is, every process that involves heat transfer must be *isothermal* at either T_H or T_C .

Conversely, in any process in which the temperature of the working substance of the engine is intermediate between T_H and T_C , there must be *no* heat transfer between the engine and either reservoir because such heat transfer could not be reversible. Therefore any process in which the temperature T of the working substance changes must be *adiabatic*.

The bottom line is that every process in our idealized cycle must be either isothermal or adiabatic. In addition, thermal and mechanical equilibrium must be maintained at all times so that each process is completely reversible.

Steps of the Carnot Cycle

The Carnot cycle consists of two reversible isothermal and two reversible adiabatic processes. Figure 20.13 shows a Carnot cycle using as its working substance an ideal gas in a cylinder with a piston. It consists of the following steps:

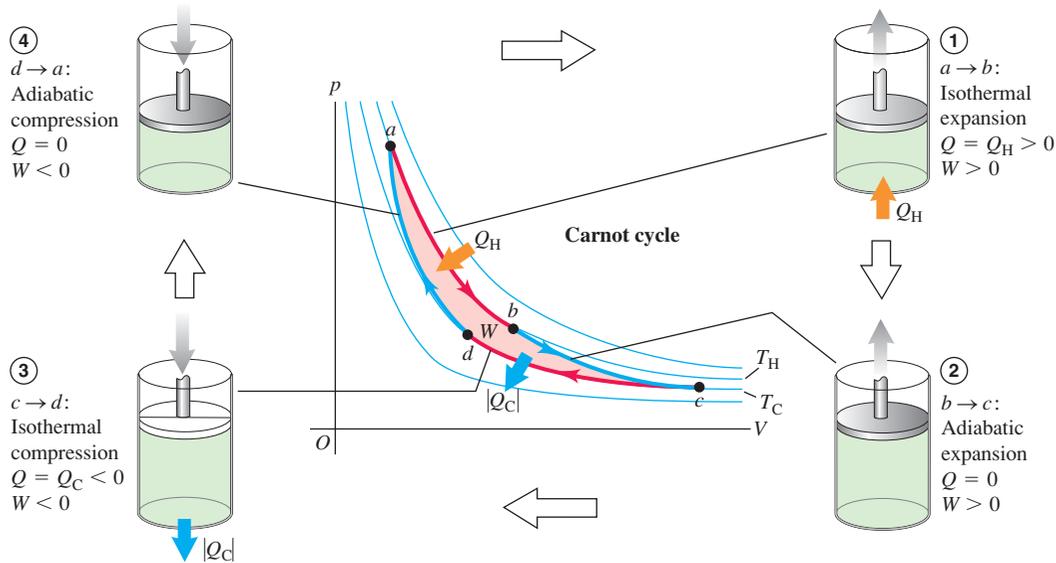
1. The gas expands isothermally at temperature T_H , absorbing heat Q_H (*ab*).
2. It expands adiabatically until its temperature drops to T_C (*bc*).
3. It is compressed isothermally at T_C , rejecting heat $|Q_C|$ (*cd*).
4. It is compressed adiabatically back to its initial state at temperature T_H (*da*).

20.12 The temperature of the firebox of a steam engine is much higher than the temperature of water in the boiler, so heat flows irreversibly from firebox to water. Carnot’s quest to understand the efficiency of steam engines led him to the idea that an ideal engine would involve only *reversible* processes.



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20.13 The Carnot cycle for an ideal gas. The light blue lines in the pV -diagram are isotherms (curves of constant temperature) and the dark blue lines are adiabats (curves of zero heat flow).



We can calculate the thermal efficiency e of a Carnot engine in the special case shown in Fig. 20.13 in which the working substance is an *ideal gas*. To carry out this calculation, we will first find the ratio Q_C/Q_H of the quantities of heat transferred in the two isothermal processes and then use Eq. (20.4) to find e .

For an ideal gas the internal energy U depends only on temperature and is thus constant in any isothermal process. For the isothermal expansion ab , $\Delta U_{ab} = 0$ and Q_H is equal to the work W_{ab} done by the gas during its isothermal expansion at temperature T_H . We calculated this work in Example 19.1 (Section 19.2); using that result, we have

$$Q_H = W_{ab} = nRT_H \ln \frac{V_b}{V_a} \quad (20.10)$$

Similarly,

$$Q_C = W_{cd} = nRT_C \ln \frac{V_d}{V_c} = -nRT_C \ln \frac{V_c}{V_d} \quad (20.11)$$

Because V_d is less than V_c , Q_C is negative ($Q_C = -|Q_C|$); heat flows out of the gas during the isothermal compression at temperature T_C .

The ratio of the two quantities of heat is thus

$$\frac{Q_C}{Q_H} = - \left(\frac{T_C}{T_H} \right) \frac{\ln(V_c/V_d)}{\ln(V_b/V_a)} \quad (20.12)$$

This can be simplified further by use of the temperature–volume relationship for an adiabatic process, Eq. (19.22). We find for the two adiabatic processes:

$$T_H V_b^{\gamma-1} = T_C V_c^{\gamma-1} \quad \text{and} \quad T_H V_a^{\gamma-1} = T_C V_d^{\gamma-1}$$

Dividing the first of these by the second, we find

$$\frac{V_b^{\gamma-1}}{V_a^{\gamma-1}} = \frac{V_c^{\gamma-1}}{V_d^{\gamma-1}} \quad \text{and} \quad \frac{V_b}{V_a} = \frac{V_c}{V_d}$$

Thus the two logarithms in Eq. (20.12) are equal, and that equation reduces to

$$\frac{Q_C}{Q_H} = -\frac{T_C}{T_H} \quad \text{or} \quad \frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H} \quad (\text{heat transfer in a Carnot engine}) \quad (20.13)$$

The ratio of the heat rejected at T_C to the heat absorbed at T_H is just equal to the ratio T_C/T_H . Then from Eq. (20.4) the efficiency of the Carnot engine is

$$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = \frac{T_H - T_C}{T_H} \quad (\text{efficiency of a Carnot engine}) \quad (20.14)$$

This simple result says that the efficiency of a Carnot engine depends only on the temperatures of the two heat reservoirs. The efficiency is large when the temperature *difference* is large, and it is very small when the temperatures are nearly equal. The efficiency can never be exactly unity unless $T_C = 0$; we'll see later that this, too, is impossible.

CAUTION Use Kelvin temperature in Carnot calculations In all calculations involving the Carnot cycle, you must make sure that you use *absolute* (Kelvin) temperatures only. That's because Eqs. (20.10) through (20.14) come from the ideal-gas equation $pV = nRT$, in which T is absolute temperature. **|**

Example 20.2 Analyzing a Carnot engine I

A Carnot engine takes 2000 J of heat from a reservoir at 500 K, does some work, and discards some heat to a reservoir at 350 K. How much work does it do, how much heat is discarded, and what is its efficiency?

SOLUTION

IDENTIFY and SET UP: This problem involves a Carnot engine, so we can use the ideas of this section and those of Section 20.2 (which apply to heat engines of all kinds). Figure 20.14 shows the energy-flow diagram. We have $Q_H = 2000$ J, $T_H = 500$ K, and $T_C = 350$ K. We use Eq. (20.13) to find Q_C , and then use the first law of thermodynamics as given by Eq. (20.2) to find W . We find the efficiency e from T_C and T_H using Eq. (20.14).

EXECUTE: From Eq. (20.13),

$$Q_C = -Q_H \frac{T_C}{T_H} = -(2000 \text{ J}) \frac{350 \text{ K}}{500 \text{ K}} = -1400 \text{ J}$$

Then from Eq. (20.2), the work done is

$$W = Q_H + Q_C = 2000 \text{ J} + (-1400 \text{ J}) = 600 \text{ J}$$

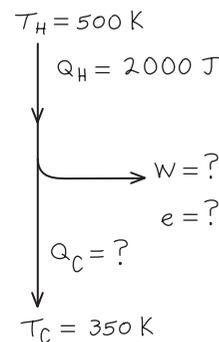
From Eq. (20.14), the thermal efficiency is

$$e = 1 - \frac{T_C}{T_H} = 1 - \frac{350 \text{ K}}{500 \text{ K}} = 0.30 = 30\%$$

EVALUATE: The negative sign of Q_C is correct: It shows that 1400 J of heat flows *out* of the engine and into the cold reservoir. We can check our result for e by using the basic definition of thermal efficiency, Eq. (20.3):

$$e = \frac{W}{Q_H} = \frac{600 \text{ J}}{2000 \text{ J}} = 0.30 = 30\%$$

20.14 Our sketch for this problem.



Example 20.3 Analyzing a Carnot engine II

Suppose 0.200 mol of an ideal diatomic gas ($\gamma = 1.40$) undergoes a Carnot cycle between 227°C and 27°C , starting at $p_a = 10.0 \times 10^5$ Pa at point a in the pV -diagram of Fig. 20.13. The volume doubles during the isothermal expansion step $a \rightarrow b$. (a) Find

the pressure and volume at points a , b , c , and d . (b) Find Q , W , and ΔU for each step and for the entire cycle. (c) Find the efficiency directly from the results of part (b), and compare with the value calculated from Eq. (20.14).

Continued

SOLUTION

IDENTIFY and SET UP: This problem involves the properties of the Carnot cycle and those of an ideal gas. We are given the number of moles n and the pressure and temperature at point a (which is at the higher of the two reservoir temperatures); we can find the volume at a using the ideal-gas equation $pV = nRT$. We then find the pressure and volume at points b , c , and d from the known doubling of volume in step $a \rightarrow b$, from equations given in this section, and from $pV = nRT$. In each step we use Eqs. (20.10) and (20.11) to find the heat flow and work done and Eq. (19.13) to find the internal energy change.

EXECUTE: (a) With $T_H = (227 + 273.15) \text{ K} = 500 \text{ K}$ and $T_C = (27 + 273.15) \text{ K} = 300 \text{ K}$, $pV = nRT$ yields

$$V_a = \frac{nRT_H}{p_a} = \frac{(0.200 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(500 \text{ K})}{10.0 \times 10^5 \text{ Pa}} = 8.31 \times 10^{-4} \text{ m}^3$$

The volume doubles during the isothermal expansion $a \rightarrow b$:

$$V_b = 2V_a = 2(8.31 \times 10^{-4} \text{ m}^3) = 16.6 \times 10^{-4} \text{ m}^3$$

Because the expansion $a \rightarrow b$ is isothermal, $p_a V_a = p_b V_b$, so

$$p_b = \frac{p_a V_a}{V_b} = 5.00 \times 10^5 \text{ Pa}$$

For the adiabatic expansion $b \rightarrow c$, we use the equation $T_H V_b^{\gamma-1} = T_C V_c^{\gamma-1}$ that follows Eq. (20.12) as well as the ideal-gas equation:

$$\begin{aligned} V_c &= V_b \left(\frac{T_H}{T_C} \right)^{1/(\gamma-1)} = (16.6 \times 10^{-4} \text{ m}^3) \left(\frac{500 \text{ K}}{300 \text{ K}} \right)^{2.5} \\ &= 59.6 \times 10^{-4} \text{ m}^3 \\ p_c &= \frac{nRT_C}{V_c} = \frac{(0.200 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{59.6 \times 10^{-4} \text{ m}^3} \\ &= 0.837 \times 10^5 \text{ Pa} \end{aligned}$$

For the adiabatic compression $d \rightarrow a$ we have $T_C V_d^{\gamma-1} = T_H V_a^{\gamma-1}$ and so

$$\begin{aligned} V_d &= V_a \left(\frac{T_H}{T_C} \right)^{1/(\gamma-1)} = (8.31 \times 10^{-4} \text{ m}^3) \left(\frac{500 \text{ K}}{300 \text{ K}} \right)^{2.5} \\ &= 29.8 \times 10^{-4} \text{ m}^3 \\ p_d &= \frac{nRT_C}{V_d} = \frac{(0.200 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{29.8 \times 10^{-4} \text{ m}^3} \\ &= 1.67 \times 10^5 \text{ Pa} \end{aligned}$$

(b) For the isothermal expansion $a \rightarrow b$, $\Delta U_{ab} = 0$. From Eq. (20.10),

$$\begin{aligned} W_{ab} &= Q_H = nRT_H \ln \frac{V_b}{V_a} \\ &= (0.200 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(500 \text{ K})(\ln 2) = 576 \text{ J} \end{aligned}$$

For the adiabatic expansion $b \rightarrow c$, $Q_{bc} = 0$. From the first law of thermodynamics, $\Delta U_{bc} = Q_{bc} - W_{bc} = -W_{bc}$; the work W_{bc} done by the gas in this adiabatic expansion equals the negative of the change in internal energy of the gas. From Eq. (19.13) we have $\Delta U = nC_V \Delta T$, where $\Delta T = T_C - T_H$. Using $C_V = 20.8 \text{ J/mol} \cdot \text{K}$ for an ideal diatomic gas, we find

$$\begin{aligned} W_{bc} &= -\Delta U_{bc} = -nC_V(T_C - T_H) = nC_V(T_H - T_C) \\ &= (0.200 \text{ mol})(20.8 \text{ J/mol} \cdot \text{K})(500 \text{ K} - 300 \text{ K}) = 832 \text{ J} \end{aligned}$$

For the isothermal compression $c \rightarrow d$, $\Delta U_{cd} = 0$; Eq. (20.11) gives

$$\begin{aligned} W_{cd} &= Q_C = nRT_C \ln \frac{V_d}{V_c} \\ &= (0.200 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K}) \left(\ln \frac{29.8 \times 10^{-4} \text{ m}^3}{59.6 \times 10^{-4} \text{ m}^3} \right) \\ &= -346 \text{ J} \end{aligned}$$

For the adiabatic compression $d \rightarrow a$, $Q_{da} = 0$ and

$$\begin{aligned} W_{da} &= -\Delta U_{da} = -nC_V(T_H - T_C) = nC_V(T_C - T_H) \\ &= (0.200 \text{ mol})(20.8 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 500 \text{ K}) = -832 \text{ J} \end{aligned}$$

We can tabulate these results as follows:

Process	Q	W	ΔU
$a \rightarrow b$	576 J	576 J	0
$b \rightarrow c$	0	832 J	-832 J
$c \rightarrow d$	-346 J	-346 J	0
$d \rightarrow a$	0	-832 J	832 J
Total	230 J	230 J	0

(c) From the above table, $Q_H = 576 \text{ J}$ and the total work is 230 J. Thus

$$e = \frac{W}{Q_H} = \frac{230 \text{ J}}{576 \text{ J}} = 0.40 = 40\%$$

We can compare this to the result from Eq. (20.14),

$$e = \frac{T_H - T_C}{T_H} = \frac{500 \text{ K} - 300 \text{ K}}{500 \text{ K}} = 0.40 = 40\%$$

EVALUATE: The table in part (b) shows that for the entire cycle $Q = W$ and $\Delta U = 0$, just as we would expect: In a complete cycle, the *net* heat input is used to do work, and there is zero net change in the internal energy of the system. Note also that the quantities of work in the two adiabatic processes are negatives of each other. Can you show from the analysis leading to Eq. (20.13) that this must *always* be the case in a Carnot cycle?

The Carnot Refrigerator

Because each step in the Carnot cycle is reversible, the *entire cycle* may be reversed, converting the engine into a refrigerator. The coefficient of performance of the Carnot refrigerator is obtained by combining the general definition of K , Eq. (20.9), with Eq. (20.13) for the Carnot cycle. We first rewrite Eq. (20.9) as

$$K = \frac{|Q_C|}{|Q_H| - |Q_C|} = \frac{|Q_C|/|Q_H|}{1 - |Q_C|/|Q_H|}$$

Then we substitute Eq. (20.13), $|Q_C|/|Q_H| = T_C/T_H$, into this expression. The result is

$$K_{\text{Carnot}} = \frac{T_C}{T_H - T_C} \quad (\text{coefficient of performance of a Carnot refrigerator}) \quad (20.15)$$

When the temperature difference $T_H - T_C$ is small, K is much larger than unity; in this case a lot of heat can be “pumped” from the lower to the higher temperature with only a little expenditure of work. But the greater the temperature difference, the smaller the value of K and the more work is required to transfer a given quantity of heat.

Example 20.4 Analyzing a Carnot refrigerator

If the cycle described in Example 20.3 is run backward as a refrigerator, what is its coefficient of performance?

SOLUTION

IDENTIFY and SET UP: This problem uses the ideas of Section 20.3 (for refrigerators in general) and the above discussion of Carnot refrigerators. Equation (20.9) gives the coefficient of performance K of any refrigerator in terms of the heat Q_C extracted from the cold reservoir per cycle and the work W that must be done per cycle.

EXECUTE: In Example 20.3 we found that in one cycle the Carnot engine rejects heat $Q_C = -346$ J to the cold reservoir and does work $W = 230$ J. When run in reverse as a refrigerator, the system

extracts heat $Q_C = +346$ J from the cold reservoir while requiring a work input of $W = -230$ J. From Eq. (20.9),

$$K = \frac{|Q_C|}{|W|} = \frac{346 \text{ J}}{230 \text{ J}} = 1.50$$

Because this is a Carnot cycle, we can also use Eq. (20.15):

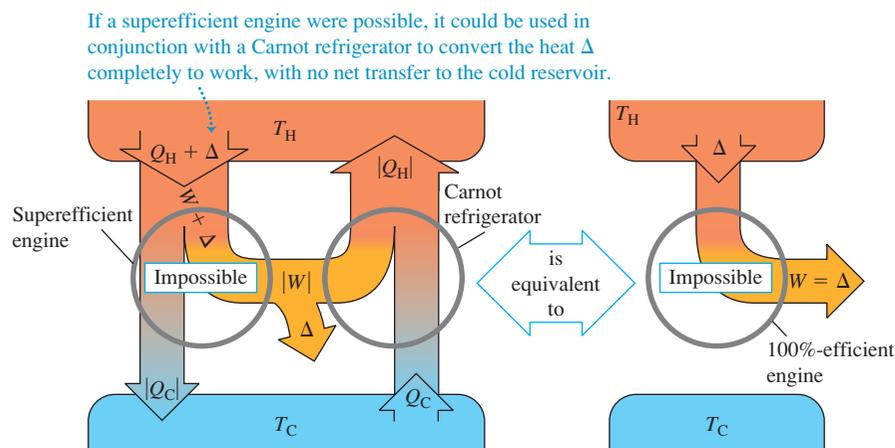
$$K = \frac{T_C}{T_H - T_C} = \frac{300 \text{ K}}{500 \text{ K} - 300 \text{ K}} = 1.50$$

EVALUATE: Equations (20.14) and (20.15) show that e and K for a Carnot cycle depend only on T_H and T_C , and we don't need to calculate Q and W . For cycles containing irreversible processes, however, these two equations are not valid, and more detailed calculations are necessary.

The Carnot Cycle and the Second Law

We can prove that **no engine can be more efficient than a Carnot engine operating between the same two temperatures.** The key to the proof is the above observation that since each step in the Carnot cycle is reversible, the *entire cycle* may be reversed. Run backward, the engine becomes a refrigerator. Suppose we have an engine that is more efficient than a Carnot engine (Fig. 20.15). Let the Carnot engine, run backward as a refrigerator by negative work $-|W|$, take

20.15 Proving that the Carnot engine has the highest possible efficiency. A “superefficient” engine (more efficient than a Carnot engine) combined with a Carnot refrigerator could convert heat completely into work with no net heat transfer to the cold reservoir. This would violate the second law of thermodynamics.



in heat Q_C from the cold reservoir and expel heat $|Q_H|$ to the hot reservoir. The superefficient engine expels heat $|Q_C|$, but to do this, it takes in a greater amount of heat $Q_H + \Delta$. Its work output is then $W + \Delta$, and the net effect of the two machines together is to take a quantity of heat Δ and convert it completely into work. This violates the engine statement of the second law. We could construct a similar argument that a superefficient engine could be used to violate the refrigerator statement of the second law. Note that we don't have to assume that the superefficient engine is reversible. In a similar way we can show that *no refrigerator can have a greater coefficient of performance than a Carnot refrigerator operating between the same two temperatures.*

Thus the statement that no engine can be more efficient than a Carnot engine is yet another equivalent statement of the second law of thermodynamics. It also follows directly that **all Carnot engines operating between the same two temperatures have the same efficiency, irrespective of the nature of the working substance.** Although we derived Eq. (20.14) for a Carnot engine using an ideal gas as its working substance, it is in fact valid for *any* Carnot engine, no matter what its working substance.

20.16 To maximize efficiency, the temperatures inside a jet engine are made as high as possible. Exotic ceramic materials are used that can withstand temperatures in excess of 1000°C without melting or becoming soft.



Equation (20.14), the expression for the efficiency of a Carnot engine, sets an upper limit to the efficiency of a real engine such as a steam turbine. To maximize this upper limit and the actual efficiency of the real engine, the designer must make the intake temperature T_H as high as possible and the exhaust temperature T_C as low as possible (Fig. 20.16).

The exhaust temperature cannot be lower than the lowest temperature available for cooling the exhaust. For a steam turbine at an electric power plant, T_C may be the temperature of river or lake water; then we want the boiler temperature T_H to be as high as possible. The vapor pressures of all liquids increase rapidly with temperature, so we are limited by the mechanical strength of the boiler. At 500°C the vapor pressure of water is about 240×10^5 Pa (235 atm); this is about the maximum practical pressure in large present-day steam boilers.

The Kelvin Temperature Scale

In Chapter 17 we expressed the need for a temperature scale that doesn't depend on the properties of any particular material. We can now use the Carnot cycle to define such a scale. The thermal efficiency of a Carnot engine operating between two heat reservoirs at temperatures T_H and T_C is independent of the nature of the working substance and depends only on the temperatures. From Eq. (20.4), this thermal efficiency is

$$e = \frac{Q_H + Q_C}{Q_H} = 1 + \frac{Q_C}{Q_H}$$

Therefore the ratio Q_C/Q_H is the same for *all* Carnot engines operating between two given temperatures T_H and T_C .

Kelvin proposed that we *define* the ratio of the temperatures, T_C/T_H , to be equal to the magnitude of the ratio Q_C/Q_H of the quantities of heat absorbed and rejected:

$$\frac{T_C}{T_H} = \frac{|Q_C|}{|Q_H|} = -\frac{Q_C}{Q_H} \quad (\text{definition of Kelvin temperature}) \quad (20.16)$$

Equation (20.16) looks identical to Eq. (20.13), but there is a subtle and crucial difference. The temperatures in Eq. (20.13) are based on an ideal-gas thermometer, as defined in Section 17.3, while Eq. (20.16) *defines* a temperature scale based on the Carnot cycle and the second law of thermodynamics and is independent of the behavior of any particular substance. Thus the **Kelvin temperature scale** is truly *absolute*. To complete the definition of the Kelvin scale, we assign, as in Section 17.3, the arbitrary value of 273.16 K to the temperature of the triple point of water. When a substance is taken around a Carnot cycle, the

ratio of the heats absorbed and rejected, $|Q_H|/|Q_C|$, is equal to the ratio of the temperatures of the reservoirs *as expressed on the gas-thermometer scale* defined in Section 17.3. Since the triple point of water is chosen to be 273.16 K in both scales, it follows that *the Kelvin and ideal-gas scales are identical*.

The zero point on the Kelvin scale is called **absolute zero**. At absolute zero a system has its *minimum* possible total internal energy (kinetic plus potential). Because of quantum effects, however, it is *not* true that at $T = 0$, all molecular motion ceases. There are theoretical reasons for believing that absolute zero cannot be attained experimentally, although temperatures below 10^{-7} K have been achieved. The more closely we approach absolute zero, the more difficult it is to get closer. One statement of the *third law of thermodynamics* is that it is impossible to reach absolute zero in a finite number of thermodynamic steps.

Test Your Understanding of Section 20.6 An inventor looking for financial support comes to you with an idea for a gasoline engine that runs on a novel type of thermodynamic cycle. His design is made entirely of copper and is air-cooled. He claims that the engine will be 85% efficient. Should you invest in this marvelous new engine? (*Hint:* See Table 17.4.)

20.7 Entropy

The second law of thermodynamics, as we have stated it, is not an equation or a quantitative relationship but rather a statement of *impossibility*. However, the second law *can* be stated as a quantitative relationship with the concept of *entropy*, the subject of this section.

We have talked about several processes that proceed naturally in the direction of increasing disorder. Irreversible heat flow increases disorder because the molecules are initially sorted into hotter and cooler regions; this sorting is lost when the system comes to thermal equilibrium. Adding heat to a body increases its disorder because it increases average molecular speeds and therefore the randomness of molecular motion. Free expansion of a gas increases its disorder because the molecules have greater randomness of position after the expansion than before. Figure 20.17 shows another process in which disorder increases.

Entropy and Disorder

Entropy provides a *quantitative* measure of disorder. To introduce this concept, let's consider an infinitesimal isothermal expansion of an ideal gas. We add heat dQ and let the gas expand just enough to keep the temperature constant. Because the internal energy of an ideal gas depends only on its temperature, the internal energy is also constant; thus from the first law, the work dW done by the gas is equal to the heat dQ added. That is,

$$dQ = dW = p dV = \frac{nRT}{V} dV \quad \text{so} \quad \frac{dV}{V} = \frac{dQ}{nRT}$$

The gas is more disordered after the expansion than before: The molecules are moving in a larger volume and have more randomness of position. Thus the fractional volume change dV/V is a measure of the increase in disorder, and the above equation shows that it is proportional to the quantity dQ/T . We introduce the symbol S for the entropy of the system, and we define the infinitesimal entropy change dS during an infinitesimal reversible process at absolute temperature T as

$$dS = \frac{dQ}{T} \quad (\text{infinitesimal reversible process}) \quad (20.17)$$

If a total amount of heat Q is added during a reversible isothermal process at absolute temperature T , the total entropy change $\Delta S = S_2 - S_1$ is given by

$$\Delta S = S_2 - S_1 = \frac{Q}{T} \quad (\text{reversible isothermal process}) \quad (20.18)$$

20.17 When firecrackers explode, disorder increases: The neatly packaged chemicals within each firecracker are dispersed in all directions, and the stored chemical energy is converted to random kinetic energy of the fragments.



Entropy has units of energy divided by temperature; the SI unit of entropy is 1 J/K.

We can see how the quotient Q/T is related to the increase in disorder. Higher temperature means greater randomness of motion. If the substance is initially cold, with little molecular motion, adding heat Q causes a substantial fractional increase in molecular motion and randomness. But if the substance is already hot, the same quantity of heat adds relatively little to the greater molecular motion already present. So Q/T is an appropriate characterization of the increase in randomness or disorder when heat flows into a system.

Example 20.5 Entropy change in melting

What is the change of entropy of 1 kg of ice that is melted reversibly at 0°C and converted to water at 0°C ? The heat of fusion of water is $L_f = 3.34 \times 10^5 \text{ J/kg}$.

SOLUTION

IDENTIFY and SET UP: The melting occurs at a constant temperature $T = 0^\circ\text{C} = 273 \text{ K}$, so this is an *isothermal* reversible process. We can calculate the added heat Q required to melt the ice, then calculate the entropy change ΔS using Eq. (20.18).

EXECUTE: The heat needed to melt the ice is $Q = mL_f = 3.34 \times 10^5 \text{ J}$. Then from Eq. (20.18),

$$\Delta S = S_2 - S_1 = \frac{Q}{T} = \frac{3.34 \times 10^5 \text{ J}}{273 \text{ K}} = 1.22 \times 10^3 \text{ J/K}$$

EVALUATE: This entropy increase corresponds to the increase in disorder when the water molecules go from the highly ordered state of a crystalline solid to the much more disordered state of a liquid. In *any* isothermal reversible process, the entropy change equals the heat transferred divided by the absolute temperature. When we refreeze the water, Q has the opposite sign, and the entropy change is $\Delta S = -1.22 \times 10^3 \text{ J/K}$. The water molecules rearrange themselves into a crystal to form ice, so disorder and entropy both decrease.

Entropy in Reversible Processes

We can generalize the definition of entropy change to include *any* reversible process leading from one state to another, whether it is isothermal or not. We represent the process as a series of infinitesimal reversible steps. During a typical step, an infinitesimal quantity of heat dQ is added to the system at absolute temperature T . Then we sum (integrate) the quotients dQ/T for the entire process; that is,

$$\Delta S = \int_1^2 \frac{dQ}{T} \quad (\text{entropy change in a reversible process}) \quad (20.19)$$

The limits 1 and 2 refer to the initial and final states.

Because entropy is a measure of the disorder of a system in any specific state, it must depend only on the current state of the system, not on its past history. (We will verify this later.) When a system proceeds from an initial state with entropy S_1 to a final state with entropy S_2 , the change in entropy $\Delta S = S_2 - S_1$ defined by Eq. (20.19) does not depend on the path leading from the initial to the final state but is the same for *all possible* processes leading from state 1 to state 2. Thus the entropy of a system must also have a definite value for any given state of the system. *Internal energy*, introduced in Chapter 19, also has this property, although entropy and internal energy are very different quantities.

Since entropy is a function only of the state of a system, we can also compute entropy changes in *irreversible* (nonequilibrium) processes for which Eqs. (20.17) and (20.19) are not applicable. We simply invent a path connecting the given initial and final states that *does* consist entirely of reversible equilibrium processes and compute the total entropy change for that path. It is not the actual path, but the entropy change must be the same as for the actual path.

As with internal energy, the above discussion does not tell us how to calculate entropy itself, but only the change in entropy in any given process. Just as with internal energy, we may arbitrarily assign a value to the entropy of a system in a specified reference state and then calculate the entropy of any other state with reference to this.

Example 20.6 Entropy change in a temperature change

One kilogram of water at 0°C is heated to 100°C. Compute its change in entropy. Assume that the specific heat of water is constant at 4190 J/kg · K over this temperature range.

SOLUTION

IDENTIFY and SET UP: The entropy change of the water depends only on the initial and final states of the system, no matter whether the process is reversible or irreversible. We can imagine a reversible process in which the water temperature is increased in a sequence of infinitesimal steps dT . We can use Eq. (20.19) to integrate over all these steps and calculate the entropy change for such a reversible process. (Heating the water on a stove whose cooking surface is maintained at 100°C would be an irreversible process. The entropy change would be the same, however.)

EXECUTE: From Eq. (17.14) the heat required to carry out each infinitesimal step is $dQ = mc dT$. Substituting this into Eq. (20.19) and integrating, we find

$$\begin{aligned}\Delta S = S_2 - S_1 &= \int_1^2 \frac{dQ}{T} = \int_{T_1}^{T_2} mc \frac{dT}{T} = mc \ln \frac{T_2}{T_1} \\ &= (1.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \left(\ln \frac{373 \text{ K}}{273 \text{ K}} \right) \\ &= 1.31 \times 10^3 \text{ J/K}\end{aligned}$$

EVALUATE: The entropy change is positive, as it must be for a process in which the system absorbs heat. Our assumption about the specific heat is a pretty good one, since c for water increases by only 1% between 0°C and 100°C.

CAUTION When $\Delta S = Q/T$ can (and cannot) be used In solving this problem you might be tempted to avoid doing an integral by using the simpler expression in Eq. (20.18), $\Delta S = Q/T$. This would be incorrect, however, because Eq. (20.18) is applicable only to *isothermal* processes, and the initial and final temperatures in our example are *not* the same. The *only* correct way to find the entropy change in a process with different initial and final temperatures is to use Eq. (20.19). **|**

Conceptual Example 20.7 Entropy change in a reversible adiabatic process

A gas expands adiabatically and reversibly. What is its change in entropy?

SOLUTION

In an adiabatic process, no heat enters or leaves the system. Hence $dQ = 0$ and there is *no* change in entropy in this reversible

process: $\Delta S = 0$. Every *reversible* adiabatic process is a constant-entropy process. (That's why such processes are also called *isentropic* processes.) The increase in disorder resulting from the gas occupying a greater volume is exactly balanced by the decrease in disorder associated with the lowered temperature and reduced molecular speeds.

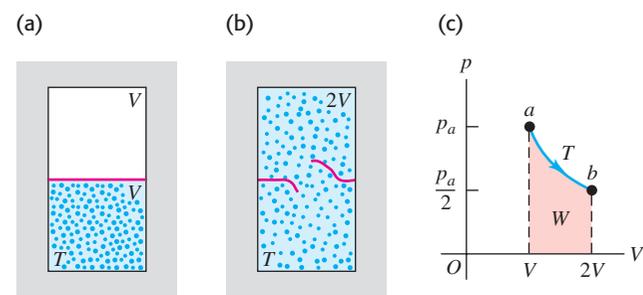
Example 20.8 Entropy change in a free expansion

A partition divides a thermally insulated box into two compartments, each of volume V (Fig. 20.18). Initially, one compartment contains n moles of an ideal gas at temperature T , and the other compartment is evacuated. We break the partition and the gas expands, filling both compartments. What is the entropy change in this free-expansion process?

SOLUTION

IDENTIFY and SET UP: For this process, $Q = 0$, $W = 0$, $\Delta U = 0$, and therefore (because the system is an ideal gas) $\Delta T = 0$. We might think that the entropy change is zero because there is no heat exchange. But Eq. (20.19) can be used to calculate entropy changes for *reversible* processes only; this free expansion is *not* reversible, and there *is* an entropy change. As we mentioned at the beginning of this section, entropy increases in a free expansion because the positions of the molecules are more random than before the expansion. To calculate ΔS , we recall that the entropy change depends only on the initial and final states. We can devise a

20.18 (a, b) Free expansion of an insulated ideal gas. (c) The free-expansion process doesn't pass through equilibrium states from a to b . However, the entropy change $S_b - S_a$ can be calculated by using the isothermal path shown or *any* reversible path from a to b .



reversible process having the same endpoints as this free expansion, and in general we can then use Eq. (20.19) to calculate its entropy change, which will be the same as for the free expansion.

Continued

An appropriate reversible process is an *isothermal* expansion from V to $2V$ at temperature T , which allows us to use the simpler Eq. (20.18) to calculate ΔS . The gas does work W during this expansion, so an equal amount of heat Q must be supplied to keep the internal energy constant.

EXECUTE: We saw in Example 19.1 that the work done by n moles of ideal gas in an isothermal expansion from V_1 to V_2 is $W = nRT \ln(V_2/V_1)$. With $V_1 = V$ and $V_2 = 2V$, we have

$$Q = W = nRT \ln \frac{2V}{V} = nRT \ln 2$$

From Eq. (20.18), the entropy change is

$$\Delta S = \frac{Q}{T} = nR \ln 2$$

EVALUATE: For 1 mole, $\Delta S = (1 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(\ln 2) = 5.76 \text{ J/K}$. The entropy change is positive, as we predicted. The factor $(\ln 2)$ in our answer is a result of the volume having increased by a factor of 2, from V to $2V$. Can you show that if the volume increases in a free expansion from V to xV , where x is an arbitrary number, the entropy change is $\Delta S = nR \ln x$?

Example 20.9 Entropy and the Carnot cycle

For the Carnot engine in Example 20.2 (Section 20.6), what is the total entropy change during one cycle?

SOLUTION

IDENTIFY and SET UP: All four steps in the Carnot cycle (see Fig. 20.13) are reversible, so we can use our expressions for the entropy change ΔS in a reversible process. We find ΔS for each step and add them to get ΔS for the complete cycle.

EXECUTE: There is no entropy change during the adiabatic expansion $b \rightarrow c$ or the adiabatic compression $d \rightarrow a$. During the isothermal expansion $a \rightarrow b$ at $T_H = 500 \text{ K}$, the engine takes in 2000 J of heat, and from Eq. (20.18),

$$\Delta S_H = \frac{Q_H}{T_H} = \frac{2000 \text{ J}}{500 \text{ K}} = 4.0 \text{ J/K}$$

During the isothermal compression $c \rightarrow d$ at $T_C = 350 \text{ K}$, the engine gives off 1400 J of heat, and

$$\Delta S_C = \frac{Q_C}{T_C} = \frac{-1400 \text{ J}}{350 \text{ K}} = -4.0 \text{ J/K}$$

The total entropy change in the engine during one cycle is $\Delta S_{\text{tot}} = \Delta S_H + \Delta S_C = 4.0 \text{ J/K} + (-4.0 \text{ J/K}) = 0$.

EVALUATE: The result $\Delta S_{\text{total}} = 0$ tells us that when the Carnot engine completes a cycle, it has the same entropy as it did at the beginning of the cycle. We'll explore this result in the next subsection.

What is the total entropy change of the engine's *environment* during this cycle? During the reversible isothermal expansion $a \rightarrow b$, the hot (500 K) reservoir gives off 2000 J of heat, so its entropy change is $(-2000 \text{ J})/(500 \text{ K}) = -4.0 \text{ J/K}$. During the reversible isothermal compression $c \rightarrow d$, the cold (350 K) reservoir absorbs 1400 J of heat, so its entropy change is $(+1400 \text{ J})/(350 \text{ K}) = +4.0 \text{ J/K}$. Thus the hot and cold reservoirs each have an entropy change, but the sum of these changes—that is, the total entropy change of the system's environment—is zero.

These results apply to the special case of the Carnot cycle, for which *all* of the processes are reversible. In this case we find that the total entropy change of the system and the environment together is zero. We will see that if the cycle includes irreversible processes (as is the case for the Otto and Diesel cycles of Section 20.3), the total entropy change of the system and the environment *cannot* be zero, but rather must be positive.

Entropy in Cyclic Processes

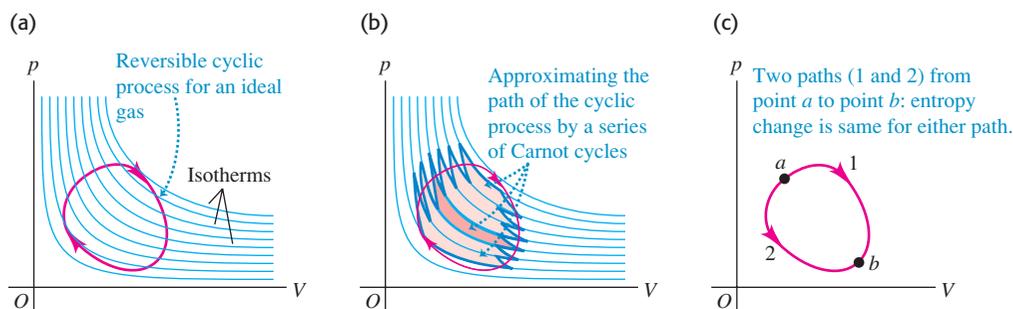
Example 20.9 showed that the total entropy change for a cycle of a particular Carnot engine, which uses an ideal gas as its working substance, is zero. This result follows directly from Eq. (20.13), which we can rewrite as

$$\frac{Q_H}{T_H} + \frac{Q_C}{T_C} = 0 \quad (20.20)$$

The quotient Q_H/T_H equals ΔS_H , the entropy change of the engine that occurs at $T = T_H$. Likewise, Q_C/T_C equals ΔS_C , the (negative) entropy change of the engine that occurs at $T = T_C$. Hence Eq. (20.20) says that $\Delta S_H + \Delta S_C = 0$; that is, there is zero net entropy change in one cycle.

What about Carnot engines that use a different working substance? According to the second law, *any* Carnot engine operating between given temperatures T_H and T_C has the same efficiency $e = 1 - T_C/T_H$ [Eq. (20.14)]. Combining this expression for e with Eq. (20.4), $e = 1 + Q_C/Q_H$, just reproduces Eq. (20.20). So Eq. (20.20) is valid for any Carnot engine working between these temperatures, whether its working substance is an ideal gas or not. We conclude that *the total entropy change in one cycle of any Carnot engine is zero*.

20.19 (a) A reversible cyclic process for an ideal gas is shown as a red closed path on a pV -diagram. Several ideal-gas isotherms are shown in blue. (b) We can approximate the path in (a) by a series of long, thin Carnot cycles; one of these is highlighted in gold. The total entropy change is zero for each Carnot cycle and for the actual cyclic process. (c) The entropy change between points a and b is independent of the path.



This result can be generalized to show that the total entropy change during *any* reversible cyclic process is zero. A reversible cyclic process appears on a pV -diagram as a closed path (Fig. 20.19a). We can approximate such a path as closely as we like by a sequence of isothermal and adiabatic processes forming parts of many long, thin Carnot cycles (Fig. 20.19b). The total entropy change for the full cycle is the sum of the entropy changes for each small Carnot cycle, each of which is zero. So **the total entropy change during any reversible cycle is zero:**

$$\int \frac{dQ}{T} = 0 \quad (\text{reversible cyclic process}) \quad (20.21)$$

It follows that when a system undergoes a reversible process leading from any state a to any other state b , *the entropy change of the system is independent of the path* (Fig. 20.19c). If the entropy change for path 1 were different from the change for path 2, the system could be taken along path 1 and then backward along path 2 to the starting point, with a nonzero net change in entropy. This would violate the conclusion that the total entropy change in such a cyclic process must be zero. Because the entropy change in such processes is independent of path, we conclude that in any given state, the system has a definite value of entropy that depends only on the state, not on the processes that led to that state.

Entropy in Irreversible Processes

In an idealized, reversible process involving only equilibrium states, the total entropy change of the system and its surroundings is zero. But all *irreversible* processes involve an increase in entropy. Unlike energy, *entropy is not a conserved quantity*. The entropy of an isolated system *can* change, but as we shall see, it can never decrease. The free expansion of a gas, described in Example 20.8, is an irreversible process in an isolated system in which there is an entropy increase.



Application Entropy Changes in a Living Organism

When a kitten or other growing animal eats, it takes ordered chemical energy from the food and uses it to make new cells that are even more highly ordered. This process alone lowers entropy. But most of the energy in the food is either excreted in the animal's feces or used to generate heat, processes that lead to a large increase in entropy. So while the entropy of the animal alone decreases, the *total* entropy of animal plus food *increases*.

Example 20.10 Entropy change in an irreversible process

Suppose 1.00 kg of water at 100°C is placed in thermal contact with 1.00 kg of water at 0°C . What is the total change in entropy? Assume that the specific heat of water is constant at $4190 \text{ J/kg} \cdot \text{K}$ over this temperature range.

SOLUTION

IDENTIFY and SET UP: This process involves irreversible heat flow because of the temperature differences. There are equal

masses of 0°C water and 100°C water, so the final temperature is the average of these two temperatures: $50^\circ\text{C} = 323 \text{ K}$. Although the processes are irreversible, we can calculate the entropy changes for the (initially) hot water and the (initially) cold water by assuming that the process occurs reversibly. As in Example 20.6, we must use Eq. (20.19) to calculate ΔS for each substance because the temperatures are not constant.

Continued

EXECUTE: The entropy changes of the hot water (subscript H) and the cold water (subscript C) are

$$\begin{aligned}\Delta S_H &= mc \int_{T_1}^{T_2} \frac{dT}{T} = (1.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \int_{373 \text{ K}}^{323 \text{ K}} \frac{dT}{T} \\ &= (4190 \text{ J/K}) \left(\ln \frac{323 \text{ K}}{373 \text{ K}} \right) = -603 \text{ J/K} \\ \Delta S_C &= (4190 \text{ J/K}) \left(\ln \frac{323 \text{ K}}{273 \text{ K}} \right) = +705 \text{ J/K}\end{aligned}$$

The *total* entropy change of the system is

$$\Delta S_{\text{tot}} = \Delta S_H + \Delta S_C = (-603 \text{ J/K}) + 705 \text{ J/K} = +102 \text{ J/K}$$

EVALUATE: An irreversible heat flow in an isolated system is accompanied by an increase in entropy. We could reach the same

end state by mixing the hot and cold water, which is also an irreversible process; the total entropy change, which depends only on the initial and final states of the system, would again be 102 J/K.

Note that the entropy of the system increases *continuously* as the two quantities of water come to equilibrium. For example, the first 4190 J of heat transferred cools the hot water to 99°C and warms the cold water to 1°C. The net change in entropy for this step is approximately

$$\Delta S = \frac{-4190 \text{ J}}{373 \text{ K}} + \frac{4190 \text{ J}}{273 \text{ K}} = +4.1 \text{ J/K}$$

Can you show in a similar way that the net entropy change is positive for *any* one-degree temperature change leading to the equilibrium condition?

Entropy and the Second Law

The results of Example 20.10 about the flow of heat from a higher to a lower temperature are characteristic of *all* natural (that is, irreversible) processes. When we include the entropy changes of all the systems taking part in the process, the increases in entropy are always greater than the decreases. In the special case of a *reversible* process, the increases and decreases are equal. Hence we can state the general principle: **When all systems taking part in a process are included, the entropy either remains constant or increases.** In other words: **No process is possible in which the total entropy decreases, when all systems taking part in the process are included.** This is an alternative statement of the second law of thermodynamics in terms of entropy. Thus it is equivalent to the “engine” and “refrigerator” statements discussed earlier. Figure 20.20 shows a specific example of this general principle.

The increase of entropy in every natural, irreversible process measures the increase of disorder or randomness in the universe associated with that process. Consider again the example of mixing hot and cold water (Example 20.10). We *might* have used the hot and cold water as the high- and low-temperature reservoirs of a heat engine. While removing heat from the hot water and giving heat to the cold water, we could have obtained some mechanical work. But once the hot and cold water have been mixed and have come to a uniform temperature, this opportunity to convert heat to mechanical work is lost irretrievably. The lukewarm water will never *unmix* itself and separate into hotter and colder portions. No decrease in *entropy* occurs when the hot and cold water are mixed. What has been lost is the *opportunity* to convert part of the heat from the hot water into mechanical work. Hence when entropy increases, energy becomes less *available*, and the universe becomes more random or “run down.”

20.20 The mixing of colored ink and water starts from a state of relative order (low entropy) in which each fluid is separate and distinct from the other. The final state after mixing is more disordered (has greater entropy). Spontaneous unmixing of the ink and water, a process in which there would be a net decrease in entropy, is never observed.



Test Your Understanding of Section 20.7 Suppose 2.00 kg of water at 50°C spontaneously changes temperature, so that half of the water cools to 0°C while the other half spontaneously warms to 100°C. (All of the water remains liquid, so it doesn't freeze or boil.) What would be the entropy change of the water? Is this process possible? (*Hint:* See Example 20.10.)

20.8 Microscopic Interpretation of Entropy

We described in Section 19.4 how the internal energy of a system could be calculated, at least in principle, by adding up all the kinetic energies of its constituent particles and all the potential energies of interaction among the particles. This is called a *microscopic* calculation of the internal energy. We can also make a microscopic calculation of the entropy S of a system. Unlike energy, however, entropy is not something that belongs to each individual particle or pair of particles in the system. Rather, entropy is a measure of the disorder of the system as a whole. To see how to calculate entropy microscopically, we first have to introduce the idea of *macroscopic* and *microscopic* states.

Suppose you toss N identical coins on the floor, and half of them show heads and half show tails. This is a description of the large-scale or **macroscopic state** of the system of N coins. A description of the **microscopic state** of the system includes information about each individual coin: Coin 1 was heads, coin 2 was tails, coin 3 was tails, and so on. There can be many microscopic states that correspond to the same macroscopic description. For instance, with $N = 4$ coins there are six possible states in which half are heads and half are tails (Fig. 20.21). The number of microscopic states grows rapidly with increasing N ; for $N = 100$ there are $2^{100} = 1.27 \times 10^{30}$ microscopic states, of which 1.01×10^{29} are half heads and half tails.

The least probable outcomes of the coin toss are the states that are either all heads or all tails. It is certainly possible that you could throw 100 heads in a row, but don't bet on it; the probability of doing this is only 1 in 1.27×10^{30} . The most probable outcome of tossing N coins is that half are heads and half are tails. The reason is that this *macroscopic* state has the greatest number of corresponding *microscopic* states, as Fig. 20.21 shows.

To make the connection to the concept of entropy, note that N coins that are all heads constitute a completely ordered macroscopic state; the description "all heads" completely specifies the state of each one of the N coins. The same is true if the coins are all tails. But the macroscopic description "half heads, half tails" by itself tells you very little about the state (heads or tails) of each individual coin. We say that the system is *disordered* because we know so little about its microscopic state. Compared to the state "all heads" or "all tails," the state "half heads, half tails" has a much greater number of possible microscopic states, much greater disorder, and hence much greater entropy (which is a quantitative measure of disorder).

Now instead of N coins, consider a mole of an ideal gas containing Avogadro's number of molecules. The macroscopic state of this gas is given by its pressure p , volume V , and temperature T ; a description of the microscopic state involves stating the position and velocity for each molecule in the gas. At a given pressure, volume, and temperature, the gas may be in any one of an astronomically large number of microscopic states, depending on the positions and velocities of its 6.02×10^{23} molecules. If the gas undergoes a free expansion into a greater volume, the range of possible positions increases, as does the number of possible microscopic states. The system becomes more disordered, and the entropy increases as calculated in Example 20.8 (Section 20.7).

We can draw the following general conclusion: **For any system, the most probable macroscopic state is the one with the greatest number of corresponding microscopic states, which is also the macroscopic state with the greatest disorder and the greatest entropy.**

20.21 All possible microscopic states of four coins. There can be several possible microscopic states for each macroscopic state.

Macroscopic state	Corresponding microscopic states
Four heads	
Three heads, one tails	
Two heads, two tails	
One heads, three tails	
Four tails	

Calculating Entropy: Microscopic States

Let w represent the number of possible microscopic states for a given macroscopic state. (For the four coins shown in Fig. 20.21 the state of four heads has $w = 1$, the state of three heads and one tails has $w = 4$, and so on.) Then the entropy S of a macroscopic state can be shown to be given by

$$S = k \ln w \quad (\text{microscopic expression for entropy}) \quad (20.22)$$

where $k = R/N_A$ is the Boltzmann constant (gas constant per molecule) introduced in Section 18.3. As Eq. (20.22) shows, increasing the number of possible microscopic states w increases the entropy S .

What matters in a thermodynamic process is not the absolute entropy S but the *difference* in entropy between the initial and final states. Hence an equally valid and useful definition would be $S = k \ln w + C$, where C is a constant, since C cancels in any calculation of an entropy difference between two states. But it's convenient to set this constant equal to zero and use Eq. (20.22). With this choice, since the smallest possible value of w is unity, the smallest possible value of S for any system is $k \ln 1 = 0$. Entropy can *never* be negative.

In practice, calculating w is a difficult task, so Eq. (20.22) is typically used only to calculate the absolute entropy S of certain special systems. But we can use this relationship to calculate *differences* in entropy between one state and another. Consider a system that undergoes a thermodynamic process that takes it from macroscopic state 1, for which there are w_1 possible microscopic states, to macroscopic state 2, with w_2 associated microscopic states. The change in entropy in this process is

$$\Delta S = S_2 - S_1 = k \ln w_2 - k \ln w_1 = k \ln \frac{w_2}{w_1} \quad (20.23)$$

The *difference* in entropy between the two macroscopic states depends on the *ratio* of the numbers of possible microscopic states.

As the following example shows, using Eq. (20.23) to calculate a change in entropy from one macroscopic state to another gives the same results as considering a reversible process connecting those two states and using Eq. (20.19).

Example 20.11 A microscopic calculation of entropy change

Use Eq. (20.23) to calculate the entropy change in the free expansion of n moles of gas at temperature T described in Example 20.8 (Fig. 20.22).

SOLUTION

IDENTIFY and SET UP: We are asked to calculate the entropy change using the number of microstates in the initial and final macroscopic states (Figs. 20.22a and b). When the partition is broken, no work is done, so the velocities of the molecules are unaffected. But each molecule now has twice as much volume in which it can move and hence has twice the number of possible positions. This is all we need to calculate the entropy change using Eq. (20.23).

EXECUTE: Let w_1 be the number of microscopic states of the system as a whole when the gas occupies volume V (Fig. 20.22a). The number of molecules is $N = nN_A$, and each of these N molecules has twice as many possible states after the partition is broken. Hence the number w_2 of microscopic states when the gas occupies volume $2V$ (Fig. 20.22b) is greater by a factor of 2^N ; that is, $w_2 = 2^N w_1$. The change in entropy in this process is

$$\Delta S = k \ln \frac{w_2}{w_1} = k \ln \frac{2^N w_1}{w_1} = k \ln 2^N = Nk \ln 2$$

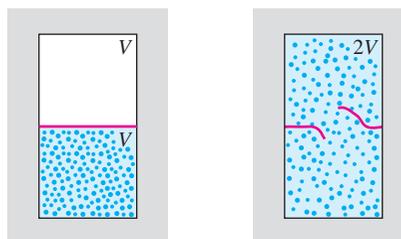
Since $N = nN_A$ and $k = R/N_A$, this becomes

$$\Delta S = (nN_A)(R/N_A) \ln 2 = nR \ln 2$$

EVALUATE: We found the same result as in Example 20.8, but without any reference to the thermodynamic path taken.

20.22 In a free expansion of N molecules in which the volume doubles, the number of possible microscopic states increases by 2^N .

- (a) Gas occupies volume V ; number of microstates = w_1 .
 (b) Gas occupies volume $2V$; number of microstates = $w_2 = 2^N w_1$.



Microscopic States and the Second Law

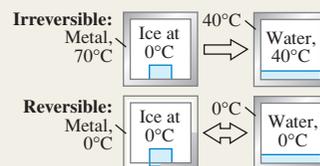
The relationship between entropy and the number of microscopic states gives us new insight into the entropy statement of the second law of thermodynamics: that the entropy of a closed system can never decrease. From Eq. (20.22) this means that a closed system can never spontaneously undergo a process that decreases the number of possible microscopic states.

An example of such a forbidden process would be if all of the air in your room spontaneously moved to one half of the room, leaving a vacuum in the other half. Such a “free compression” would be the reverse of the free expansion of Examples 20.8 and 20.11. This would decrease the number of possible microscopic states by a factor of 2^N . Strictly speaking, this process is not impossible! The probability of finding a given molecule in one half of the room is $\frac{1}{2}$, so the probability of finding all of the molecules in one half of the room at once is $(\frac{1}{2})^N$. (This is exactly the same as the probability of having a tossed coin come up heads N times in a row.) This probability is *not* zero. But lest you worry about suddenly finding yourself gasping for breath in the evacuated half of your room, consider that a typical room might hold 1000 moles of air, and so $N = 1000N_A = 6.02 \times 10^{26}$ molecules. The probability of all the molecules being in the same half of the room is therefore $(\frac{1}{2})^{6.02 \times 10^{26}}$. Expressed as a decimal, this number has more than 10^{26} zeros to the right of the decimal point!

Because the probability of such a “free compression” taking place is so vanishingly small, it has almost certainly never occurred anywhere in the universe since the beginning of time. We conclude that for all practical purposes the second law of thermodynamics is never violated.

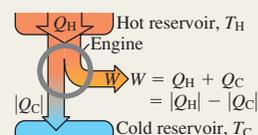
Test Your Understanding of Section 20.8 A quantity of N molecules of an ideal gas initially occupies volume V . The gas then expands to volume $2V$. The number of microscopic states of the gas increases in this expansion. Under which of the following circumstances will this number increase the most? (i) if the expansion is reversible and isothermal; (ii) if the expansion is reversible and adiabatic; (iii) the number will change by the same amount for both circumstances. 

Reversible and irreversible processes: A reversible process is one whose direction can be reversed by an infinitesimal change in the conditions of the process, and in which the system is always in or very close to thermal equilibrium. All other thermodynamic processes are irreversible.



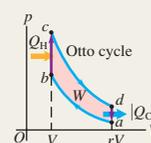
Heat engines: A heat engine takes heat Q_H from a source, converts part of it to work W , and discards the remainder $|Q_C|$ at a lower temperature. The thermal efficiency e of a heat engine measures how much of the absorbed heat is converted to work. (See Example 20.1.)

$$e = \frac{W}{Q_H} = 1 + \frac{Q_C}{Q_H} = 1 - \left| \frac{Q_C}{Q_H} \right| \quad (20.4)$$



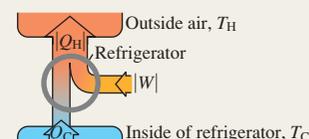
The Otto cycle: A gasoline engine operating on the Otto cycle has a theoretical maximum thermal efficiency e that depends on the compression ratio r and the ratio of heat capacities γ of the working substance.

$$e = 1 - \frac{1}{r^{\gamma-1}} \quad (20.6)$$

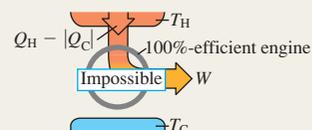


Refrigerators: A refrigerator takes heat Q_C from a colder place, has a work input $|W|$, and discards heat $|Q_H|$ at a warmer place. The effectiveness of the refrigerator is given by its coefficient of performance K .

$$K = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|} \quad (20.9)$$

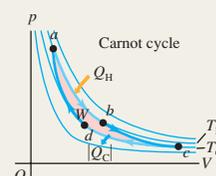


The second law of thermodynamics: The second law of thermodynamics describes the directionality of natural thermodynamic processes. It can be stated in several equivalent forms. The *engine* statement is that no cyclic process can convert heat completely into work. The *refrigerator* statement is that no cyclic process can transfer heat from a colder place to a hotter place with no input of mechanical work.



The Carnot cycle: The Carnot cycle operates between two heat reservoirs at temperatures T_H and T_C and uses only reversible processes. Its thermal efficiency depends only on T_H and T_C . An additional equivalent statement of the second law is that no engine operating between the same two temperatures can be more efficient than a Carnot engine. (See Examples 20.2 and 20.3.)

$$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = \frac{T_H - T_C}{T_H} \quad (20.14)$$



A Carnot engine run backward is a Carnot refrigerator. Its coefficient of performance depends only on T_H and T_C . Another form of the second law states that no refrigerator operating between the same two temperatures can have a larger coefficient of performance than a Carnot refrigerator. (See Example 20.4.)

$$K_{\text{Carnot}} = \frac{T_C}{T_H - T_C} \quad (20.15)$$

Entropy: Entropy is a quantitative measure of the disorder of a system. The entropy change in any reversible process depends on the amount of heat flow and the absolute temperature T . Entropy depends only on the state of the system, and the change in entropy between given initial and final states is the same for all processes leading from one state to the other. This fact can be used to find the entropy change in an irreversible process. (See Examples 20.5–20.10.)

$$\Delta S = \int_1^2 \frac{dQ}{T} \quad (20.19)$$

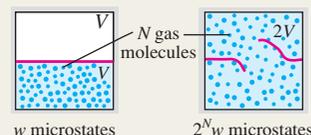
(reversible process)



An important statement of the second law of thermodynamics is that the entropy of an isolated system may increase but can never decrease. When a system interacts with its surroundings, the total entropy change of system and surroundings can never decrease. When the interaction involves only reversible processes, the total entropy is constant and $\Delta S = 0$; when there is any irreversible process, the total entropy increases and $\Delta S > 0$.

Entropy and microscopic states: When a system is in a particular macroscopic state, the particles that make up the system may be in any of w possible microscopic states. The greater the number w , the greater the entropy. (See Example 20.11.)

$$S = k \ln w \quad (20.22)$$



BRIDGING PROBLEM

Entropy Changes: Cold Ice in Hot Water

An insulated container of negligible mass holds 0.600 kg of water at 45.0°C. You put a 0.0500-kg ice cube at -15.0°C in the water. (a) Calculate the final temperature of the water once the ice has melted. (b) Calculate the change in entropy of the system.

SOLUTION GUIDE

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IDENTIFY and SET UP

1. Make a list of the known and unknown quantities, and identify the target variables.
2. How will you find the final temperature of the ice–water mixture? How will you decide whether or not all the ice melts?
3. Once you find the final temperature of the mixture, how will you determine the changes in entropy of (i) the ice initially at -15.0°C and (ii) the water initially at 45.0°C ?

EXECUTE

4. Use the methods of Chapter 17 to calculate the final temperature T . (*Hint:* First assume that all of the ice melts, then write

an equation which says that the heat that flows into the ice equals the heat that flows out of the water. If your assumption is correct, the final temperature that you calculate will be greater than 0°C . If your assumption is incorrect, the final temperature will be 0°C or less, which means that some ice remains. You'll then need to redo the calculation to account for this.)

5. Use your result from step 4 to calculate the entropy changes of the ice and the water. (*Hint:* You must include the heat flow associated with temperature changes, as in Example 20.6, as well as the heat flow associated with the change of phase.)
6. Find the total change in entropy of the system.

EVALUATE

7. Do the signs of the entropy changes make sense? Why or why not?

Problems

For instructor-assigned homework, go to www.masteringphysics.com



•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q20.1 A pot is half-filled with water, and a lid is placed on it, forming a tight seal so that no water vapor can escape. The pot is heated on a stove, forming water vapor inside the pot. The heat is then turned off and the water vapor condenses back to liquid. Is this cycle reversible or irreversible? Why?

Q20.2 Give two examples of reversible processes and two examples of irreversible processes in purely mechanical systems, such as blocks sliding on planes, springs, pulleys, and strings. Explain what makes each process reversible or irreversible.

Q20.3 What irreversible processes occur in a gasoline engine? Why are they irreversible?

Q20.4 Suppose you try to cool the kitchen of your house by leaving the refrigerator door open. What happens? Why? Would the result be the same if you left open a picnic cooler full of ice? Explain the reason for any differences.

Q20.5 A member of the U.S. Congress proposed a scheme to produce energy as follows. Water molecules (H_2O) are to be broken apart to produce hydrogen and oxygen. The hydrogen is then burned (that is, combined with oxygen), releasing energy in the process. The only product of this combustion is water, so there is no pollution. In light of the second law of thermodynamics, what do you think of this energy-producing scheme?

Q20.6 Is it a violation of the second law of thermodynamics to convert mechanical energy completely into heat? To convert heat completely into work? Explain your answers.

Q20.7 Imagine a special air filter placed in a window of a house. The tiny holes in the filter allow only air molecules moving faster than a certain speed to exit the house, and allow only air molecules moving slower than that speed to enter the house from outside. Explain why such an air filter would cool the house, and why the second law of thermodynamics makes building such a filter an impossible task.

Q20.8 An electric motor has its shaft coupled to that of an electric generator. The motor drives the generator, and some current from the generator is used to run the motor. The excess current is used to light a home. What is wrong with this scheme?

Q20.9 When a wet cloth is hung up in a hot wind in the desert, it is cooled by evaporation to a temperature that may be 20°C or so below that of the air. Discuss this process in light of the second law of thermodynamics.

Q20.10 Compare the pV -diagram for the Otto cycle in Fig. 20.6 with the diagram for the Carnot heat engine in Fig. 20.13. Explain some of the important differences between the two cycles.

Q20.11 If no real engine can be as efficient as a Carnot engine operating between the same two temperatures, what is the point of developing and using Eq. (20.14)?

Q20.12 The efficiency of heat engines is high when the temperature difference between the hot and cold reservoirs is large. Refrigerators, on the other hand, work better when the temperature difference is small. Thinking of the mechanical refrigeration cycle shown in Fig. 20.9, explain in physical terms why it takes less work to remove heat from the working substance if the two reservoirs (the inside of the refrigerator and the outside air) are at nearly the same temperature, than if the outside air is much warmer than the interior of the refrigerator.

Q20.13 What would be the efficiency of a Carnot engine operating with $T_H = T_C$? What would be the efficiency if $T_C = 0\text{ K}$ and T_H were any temperature above 0 K ? Interpret your answers.

Q20.14 Real heat engines, like the gasoline engine in a car, always have some friction between their moving parts, although lubricants keep the friction to a minimum. Would a heat engine with completely frictionless parts be 100% efficient? Why or why not? Does the answer depend on whether or not the engine runs on the Carnot cycle? Again, why or why not?

Q20.15 Does a refrigerator full of food consume more power if the room temperature is 20°C than if it is 15°C ? Or is the power consumption the same? Explain your reasoning.

Q20.16 In Example 20.4, a Carnot refrigerator requires a work input of only 230 J to extract 346 J of heat from the cold reservoir. Doesn't this discrepancy imply a violation of the law of conservation of energy? Explain why or why not.

Q20.17 Explain why each of the following processes is an example of increasing disorder or randomness: mixing hot and cold

water; free expansion of a gas; irreversible heat flow; developing heat by mechanical friction. Are entropy increases involved in all of these? Why or why not?

Q20.18 The free expansion of a gas is an adiabatic process and so no heat is transferred. No work is done, so the internal energy does not change. Thus, $Q/T = 0$, yet the disorder of the system and thus its entropy have increased after the expansion. Why does Eq. (20.19) not apply to this situation?

Q20.19 Are the earth and sun in thermal equilibrium? Are there entropy changes associated with the transmission of energy from the sun to the earth? Does radiation differ from other modes of heat transfer with respect to entropy changes? Explain your reasoning.

Q20.20 Discuss the entropy changes involved in the preparation and consumption of a hot fudge sundae.

Q20.21 If you run a movie film backward, it is as if the direction of time were reversed. In the time-reversed movie, would you see processes that violate conservation of energy? Conservation of linear momentum? Would you see processes that violate the second law of thermodynamics? In each case, if law-breaking processes could occur, give some examples.

Q20.22 BIO Some critics of biological evolution claim that it violates the second law of thermodynamics, since evolution involves simple life forms developing into more complex and more highly ordered organisms. Explain why this is not a valid argument against evolution.

Q20.23 BIO A growing plant creates a highly complex and organized structure out of simple materials such as air, water, and trace minerals. Does this violate the second law of thermodynamics? Why or why not? What is the plant's ultimate source of energy? Explain your reasoning.

EXERCISES

Section 20.2 Heat Engines

20.1 • A diesel engine performs 2200 J of mechanical work and discards 4300 J of heat each cycle. (a) How much heat must be supplied to the engine in each cycle? (b) What is the thermal efficiency of the engine?

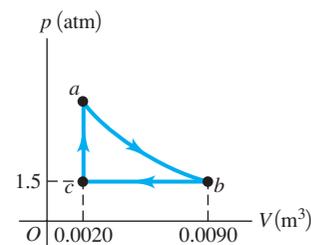
20.2 • An aircraft engine takes in 9000 J of heat and discards 6400 J each cycle. (a) What is the mechanical work output of the engine during one cycle? (b) What is the thermal efficiency of the engine?

20.3 • A Gasoline Engine. A gasoline engine takes in $1.61 \times 10^4\text{ J}$ of heat and delivers 3700 J of work per cycle. The heat is obtained by burning gasoline with a heat of combustion of $4.60 \times 10^4\text{ J/g}$. (a) What is the thermal efficiency? (b) How much heat is discarded in each cycle? (c) What mass of fuel is burned in each cycle? (d) If the engine goes through 60.0 cycles per second, what is its power output in kilowatts? In horsepower?

20.4 • A gasoline engine has a power output of 180 kW (about 241 hp). Its thermal efficiency is 28.0%. (a) How much heat must be supplied to the engine per second? (b) How much heat is discarded by the engine per second?

20.5 •• The pV -diagram in Fig. E20.5 shows a cycle of a heat engine that uses 0.250 mole of an ideal gas having $\gamma = 1.40$. The curved part ab

Figure E20.5



of the cycle is adiabatic. (a) Find the pressure of the gas at point a . (b) How much heat enters this gas per cycle, and where does it happen? (c) How much heat leaves this gas in a cycle, and where does it occur? (d) How much work does this engine do in a cycle? (e) What is the thermal efficiency of the engine?

Section 20.3 Internal-Combustion Engines

20.6 • (a) Calculate the theoretical efficiency for an Otto-cycle engine with $\gamma = 1.40$ and $r = 9.50$. (b) If this engine takes in 10,000 J of heat from burning its fuel, how much heat does it discard to the outside air?

20.7 •• The Otto-cycle engine in a Mercedes-Benz SLK230 has a compression ratio of 8.8. (a) What is the ideal efficiency of the engine? Use $\gamma = 1.40$. (b) The engine in a Dodge Viper GT2 has a slightly higher compression ratio of 9.6. How much increase in the ideal efficiency results from this increase in the compression ratio?

Section 20.4 Refrigerators

20.8 • The coefficient of performance $K = H/P$ is a dimensionless quantity. Its value is independent of the units used for H and P , as long as the same units, such as watts, are used for both quantities. However, it is common practice to express H in Btu/h and P in watts. When these mixed units are used, the ratio H/P is called the energy efficiency rating (EER). If a room air conditioner has a coefficient of performance $K = 3.0$, what is its EER?

20.9 • A refrigerator has a coefficient of performance of 2.10. In each cycle it absorbs 3.40×10^4 J of heat from the cold reservoir. (a) How much mechanical energy is required each cycle to operate the refrigerator? (b) During each cycle, how much heat is discarded to the high-temperature reservoir?

20.10 • A room air conditioner has a coefficient of performance of 2.9 on a hot day and uses 850 W of electrical power. (a) How many joules of heat does the air conditioner remove from the room in one minute? (b) How many joules of heat does the air conditioner deliver to the hot outside air in one minute? (c) Explain why your answers to parts (a) and (b) are not the same.

20.11 •• A refrigerator has a coefficient of performance of 2.25, runs on an input of 95 W of electrical power, and keeps its inside compartment at 5°C. If you put a dozen 1.0-L plastic bottles of water at 31°C into this refrigerator, how long will it take for them to be cooled down to 5°C? (Ignore any heat that leaves the plastic.)

20.12 •• A freezer has a coefficient of performance of 2.40. The freezer is to convert 1.80 kg of water at 25.0°C to 1.80 kg of ice at -5.0°C in one hour. (a) What amount of heat must be removed from the water at 25.0°C to convert it to ice at -5.0°C ? (b) How much electrical energy is consumed by the freezer during this hour? (c) How much wasted heat is delivered to the room in which the freezer sits?

Section 20.6 The Carnot Cycle

20.13 • A Carnot engine whose high-temperature reservoir is at 620 K takes in 550 J of heat at this temperature in each cycle and gives up 335 J to the low-temperature reservoir. (a) How much mechanical work does the engine perform during each cycle? (b) What is the temperature of the low-temperature reservoir? (c) What is the thermal efficiency of the cycle?

20.14 • A Carnot engine is operated between two heat reservoirs at temperatures of 520 K and 300 K. (a) If the engine receives 6.45 kJ of heat energy from the reservoir at 520 K in each cycle, how many joules per cycle does it discard to the reservoir at 300 K? (b) How much mechanical work is performed by the engine during each cycle? (c) What is the thermal efficiency of the engine?

20.15 • A Carnot engine has an efficiency of 59% and performs 2.5×10^4 J of work in each cycle. (a) How much heat does the engine extract from its heat source in each cycle? (b) Suppose the engine exhausts heat at room temperature (20.0°C). What is the temperature of its heat source?

20.16 •• An ice-making machine operates in a Carnot cycle. It takes heat from water at 0.0°C and rejects heat to a room at 24.0°C. Suppose that 85.0 kg of water at 0.0°C are converted to ice at 0.0°C. (a) How much heat is discharged into the room? (b) How much energy must be supplied to the device?

20.17 • A Carnot refrigerator is operated between two heat reservoirs at temperatures of 320 K and 270 K. (a) If in each cycle the refrigerator receives 415 J of heat energy from the reservoir at 270 K, how many joules of heat energy does it deliver to the reservoir at 320 K? (b) If the refrigerator completes 165 cycles each minute, what power input is required to operate it? (c) What is the coefficient of performance of the refrigerator?

20.18 •• A certain brand of freezer is advertised to use 730 kW·h of energy per year. (a) Assuming the freezer operates for 5 hours each day, how much power does it require while operating? (b) If the freezer keeps its interior at a temperature of -5.0°C in a 20.0°C room, what is its theoretical maximum performance coefficient? (c) What is the theoretical maximum amount of ice this freezer could make in an hour, starting with water at 20.0°C?

20.19 •• A Carnot heat engine has a thermal efficiency of 0.600, and the temperature of its hot reservoir is 800 K. If 3000 J of heat is rejected to the cold reservoir in one cycle, what is the work output of the engine during one cycle?

20.20 •• A Carnot heat engine uses a hot reservoir consisting of a large amount of boiling water and a cold reservoir consisting of a large tub of ice and water. In 5 minutes of operation, the heat rejected by the engine melts 0.0400 kg of ice. During this time, how much work W is performed by the engine?

20.21 •• You design an engine that takes in 1.50×10^4 J of heat at 650 K in each cycle and rejects heat at a temperature of 350 K. The engine completes 240 cycles in 1 minute. What is the theoretical maximum power output of your engine, in horsepower?

Section 20.7 Entropy

20.22 • A 4.50-kg block of ice at 0.00°C falls into the ocean and melts. The average temperature of the ocean is 3.50°C, including all the deep water. By how much does the melting of this ice change the entropy of the world? Does it make it larger or smaller? (*Hint:* Do you think that the ocean will change temperature appreciably as the ice melts?)

20.23 • A sophomore with nothing better to do adds heat to 0.350 kg of ice at 0.0°C until it is all melted. (a) What is the change in entropy of the water? (b) The source of heat is a very massive body at a temperature of 25.0°C. What is the change in entropy of this body? (c) What is the total change in entropy of the water and the heat source?

20.24 • **CALC** You decide to take a nice hot bath but discover that your thoughtless roommate has used up most of the hot water. You fill the tub with 270 kg of 30.0°C water and attempt to warm it further by pouring in 5.00 kg of boiling water from the stove. (a) Is this a reversible or an irreversible process? Use physical reasoning to explain. (b) Calculate the final temperature of the bath water. (c) Calculate the net change in entropy of the system (bath water + boiling water), assuming no heat exchange with the air or the tub itself.

20.25 •• A 15.0-kg block of ice at 0.0°C melts to liquid water at 0.0°C inside a large room that has a temperature of 20.0°C. Treat

the ice and the room as an isolated system, and assume that the room is large enough for its temperature change to be ignored. (a) Is the melting of the ice reversible or irreversible? Explain, using simple physical reasoning without resorting to any equations. (b) Calculate the net entropy change of the system during this process. Explain whether or not this result is consistent with your answer to part (a).

20.26 •• CALC You make tea with 0.250 kg of 85.0°C water and let it cool to room temperature (20.0°C) before drinking it. (a) Calculate the entropy change of the water while it cools. (b) The cooling process is essentially isothermal for the air in your kitchen. Calculate the change in entropy of the air while the tea cools, assuming that all the heat lost by the water goes into the air. What is the total entropy change of the system tea + air?

20.27 • Three moles of an ideal gas undergo a reversible isothermal compression at 20.0°C. During this compression, 1850 J of work is done on the gas. What is the change of entropy of the gas?

20.28 •• What is the change in entropy of 0.130 kg of helium gas at the normal boiling point of helium when it all condenses isothermally to 1.00 L of liquid helium? (*Hint:* See Table 17.4 in Section 17.6.)

20.29 • (a) Calculate the change in entropy when 1.00 kg of water at 100°C is vaporized and converted to steam at 100°C (see Table 17.4). (b) Compare your answer to the change in entropy when 1.00 kg of ice is melted at 0°C, calculated in Example 20.5 (Section 20.7). Is the change in entropy greater for melting or for vaporization? Interpret your answer using the idea that entropy is a measure of the randomness of a system.

20.30 • (a) Calculate the change in entropy when 1.00 mol of water (molecular mass 18.0 g/mol) at 100°C evaporates to form water vapor at 100°C. (b) Repeat the calculation of part (a) for 1.00 mol of liquid nitrogen, 1.00 mol of silver, and 1.00 mol of mercury when each is vaporized at its normal boiling point. (See Table 17.4 for the heats of vaporization, and Appendix D for the molar masses. Note that the nitrogen molecule is N₂.) (c) Your results in parts (a) and (b) should be in relatively close agreement. (This is called the *rule of Dreyer and Trouton*.) Explain why this should be so, using the idea that entropy is a measure of the randomness of a system.

20.31 •• A 10.0-L gas tank containing 3.20 moles of ideal He gas at 20.0°C is placed inside a completely evacuated, insulated bell jar of volume 35.0 L. A small hole in the tank allows the He to leak out into the jar until the gas reaches a final equilibrium state with no more leakage. (a) What is the change in entropy of this system due to the leaking of the gas? (b) Is the process reversible or irreversible? How do you know?

Section 20.8 Microscopic Interpretation of Entropy

20.32 • A box is separated by a partition into two parts of equal volume. The left side of the box contains 500 molecules of nitrogen gas; the right side contains 100 molecules of oxygen gas. The two gases are at the same temperature. The partition is punctured, and equilibrium is eventually attained. Assume that the volume of the box is large enough for each gas to undergo a free expansion and not change temperature. (a) On average, how many molecules of each type will there be in either half of the box? (b) What is the change in entropy of the system when the partition is punctured? (c) What is the probability that the molecules will be found in the same distribution as they were before the partition was punctured—that is, 500 nitrogen molecules in the left half and 100 oxygen molecules in the right half?

20.33 • CALC Two moles of an ideal gas occupy a volume V . The gas expands isothermally and reversibly to a volume $3V$. (a) Is the velocity distribution changed by the isothermal expansion? Explain. (b) Use Eq. (20.23) to calculate the change in entropy of the gas. (c) Use Eq. (20.18) to calculate the change in entropy of the gas. Compare this result to that obtained in part (b).

20.34 • CALC A lonely party balloon with a volume of 2.40 L and containing 0.100 mol of air is left behind to drift in the temporarily uninhabited and depressurized International Space Station. Sunlight coming through a porthole heats and explodes the balloon, causing the air in it to undergo a free expansion into the empty station, whose total volume is 425 m³. Calculate the entropy change of the air during the expansion.

PROBLEMS

20.35 •• CP An ideal Carnot engine operates between 500°C and 100°C with a heat input of 250 J per cycle. (a) How much heat is delivered to the cold reservoir in each cycle? (b) What minimum number of cycles is necessary for the engine to lift a 500-kg rock through a height of 100 m?

20.36 • You are designing a Carnot engine that has 2 mol of CO₂ as its working substance; the gas may be treated as ideal. The gas is to have a maximum temperature of 527°C and a maximum pressure of 5.00 atm. With a heat input of 400 J per cycle, you want 300 J of useful work. (a) Find the temperature of the cold reservoir. (b) For how many cycles must this engine run to melt completely a 10.0-kg block of ice originally at 0.0°C, using only the heat rejected by the engine?

20.37 •• CP A certain heat engine operating on a Carnot cycle absorbs 150 J of heat per cycle at its hot reservoir at 135°C and has a thermal efficiency of 22.0%. (a) How much work does this engine do per cycle? (b) How much heat does the engine waste each cycle? (c) What is the temperature of the cold reservoir? (d) By how much does the engine change the entropy of the world each cycle? (e) What mass of water could this engine pump per cycle from a well 35.0 m deep?

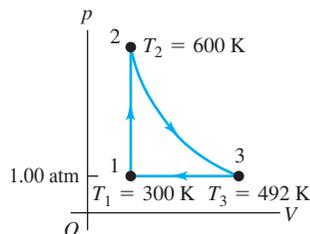
20.38 •• BIO Entropy of Metabolism. An average sleeping person metabolizes at a rate of about 80 W by digesting food or burning fat. Typically, 20% of this energy goes into bodily functions, such as cell repair, pumping blood, and other uses of mechanical energy, while the rest goes to heat. Most people get rid of all this excess heat by transferring it (by conduction and the flow of blood) to the surface of the body, where it is radiated away. The normal internal temperature of the body (where the metabolism takes place) is 37°C, and the skin is typically 7°C cooler. By how much does the person's entropy change per second due to this heat transfer?

20.39 •• BIO Entropy Change from Digesting Fat. Digesting fat produces 9.3 food calories per gram of fat, and typically 80% of this energy goes to heat when metabolized. (One food calorie is 1000 calories and therefore equals 4186 J.) The body then moves all this heat to the surface by a combination of thermal conductivity and motion of the blood. The internal temperature of the body (where digestion occurs) is normally 37°C, and the surface is usually about 30°C. By how much do the digestion and metabolism of a 2.50-g pat of butter change your body's entropy? Does it increase or decrease?

20.40 • A heat engine takes 0.350 mol of a diatomic ideal gas around the cycle shown in the pV -diagram of Fig. P20.40. Process 1 → 2 is at constant volume, process 2 → 3 is adiabatic, and

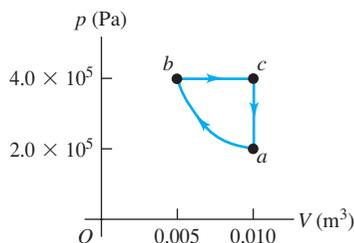
process $3 \rightarrow 1$ is at a constant pressure of 1.00 atm. The value of γ for this gas is 1.40. (a) Find the pressure and volume at points 1, 2, and 3. (b) Calculate Q , W , and ΔU for each of the three processes. (c) Find the net work done by the gas in the cycle. (d) Find the net heat flow into the engine in one cycle. (e) What is the thermal efficiency of the engine? How does this compare to the efficiency of a Carnot-cycle engine operating between the same minimum and maximum temperatures T_1 and T_2 ?

Figure P20.40



20.41 •• CALC You build a heat engine that takes 1.00 mol of an ideal diatomic gas through the cycle shown in Fig. P20.41. (a) Show that segment ab is an isothermal compression. (b) During which segment(s) of the cycle is heat absorbed by the gas? During which segment(s) is heat rejected?

Figure P20.41

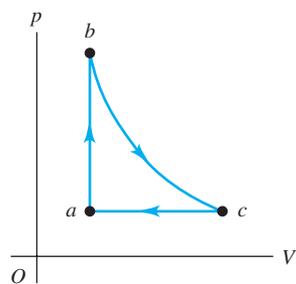


How do you know? (c) Calculate the temperature at points a , b , and c . (d) Calculate the net heat exchanged with the surroundings and the net work done by the engine in one cycle. (e) Calculate the thermal efficiency of the engine.

20.42 • Heat Pump. A heat pump is a heat engine run in reverse. In winter it pumps heat from the cold air outside into the warmer air inside the building, maintaining the building at a comfortable temperature. In summer it pumps heat from the cooler air inside the building to the warmer air outside, acting as an air conditioner. (a) If the outside temperature in winter is -5.0°C and the inside temperature is 17.0°C , how many joules of heat will the heat pump deliver to the inside for each joule of electrical energy used to run the unit, assuming an ideal Carnot cycle? (b) Suppose you have the option of using electrical resistance heating rather than a heat pump. How much electrical energy would you need in order to deliver the same amount of heat to the inside of the house as in part (a)? Consider a Carnot heat pump delivering heat to the inside of a house to maintain it at 68°F . Show that the heat pump delivers less heat for each joule of electrical energy used to operate the unit as the outside temperature decreases. Notice that this behavior is opposite to the dependence of the efficiency of a Carnot heat engine on the difference in the reservoir temperatures. Explain why this is so.

20.43 • CALC A heat engine operates using the cycle shown in Fig. P20.43. The working substance is 2.00 mol of helium gas, which reaches a maximum temperature of 327°C . Assume the helium can be treated as an ideal gas. Process bc is isothermal. The pressure in states a and c is 1.00×10^5 Pa, and the pressure in state b is 3.00×10^5 Pa. (a) How much heat enters the gas and how much

Figure P20.43



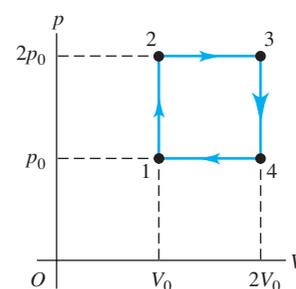
leaves the gas each cycle? (b) How much work does the engine do each cycle, and what is its efficiency? (c) Compare this engine's efficiency with the maximum possible efficiency attainable with the hot and cold reservoirs used by this cycle.

20.44 • CP As a budding mechanical engineer, you are called upon to design a Carnot engine that has 2.00 mol of a monatomic ideal gas as its working substance and operates from a high-temperature reservoir at 500°C . The engine is to lift a 15.0-kg weight 2.00 m per cycle, using 500 J of heat input. The gas in the engine chamber can have a minimum volume of 5.00 L during the cycle. (a) Draw a pV -diagram for this cycle. Show in your diagram where heat enters and leaves the gas. (b) What must be the temperature of the cold reservoir? (c) What is the thermal efficiency of the engine? (d) How much heat energy does this engine waste per cycle? (e) What is the maximum pressure that the gas chamber will have to withstand?

20.45 ••• An experimental power plant at the Natural Energy Laboratory of Hawaii generates electricity from the temperature gradient of the ocean. The surface and deep-water temperatures are 27°C and 6°C , respectively. (a) What is the maximum theoretical efficiency of this power plant? (b) If the power plant is to produce 210 kW of power, at what rate must heat be extracted from the warm water? At what rate must heat be absorbed by the cold water? Assume the maximum theoretical efficiency. (c) The cold water that enters the plant leaves it at a temperature of 10°C . What must be the flow rate of cold water through the system? Give your answer in kg/h and in L/h.

20.46 •• What is the thermal efficiency of an engine that operates by taking n moles of diatomic ideal gas through the cycle $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$ shown in Fig. P20.46?

Figure P20.46



20.47 • CALC A cylinder contains oxygen at a pressure of 2.00 atm. The volume is 4.00 L, and the temperature is 300 K. Assume that the oxygen may be treated as an ideal gas. The oxygen is carried through the following processes:

- (i) Heated at constant pressure from the initial state (state 1) to state 2, which has $T = 450$ K.
- (ii) Cooled at constant volume to 250 K (state 3).
- (iii) Compressed at constant temperature to a volume of 4.00 L (state 4).
- (iv) Heated at constant volume to 300 K, which takes the system back to state 1.

(a) Show these four processes in a pV -diagram, giving the numerical values of p and V in each of the four states. (b) Calculate Q and W for each of the four processes. (c) Calculate the net work done by the oxygen in the complete cycle. (d) What is the efficiency of this device as a heat engine? How does this compare to the efficiency of a Carnot-cycle engine operating between the same minimum and maximum temperatures of 250 K and 450 K?

20.48 •• CP BIO Human Entropy. A person who has skin of surface area 1.85 m² and temperature 30.0°C is resting in an insulated room where the ambient air temperature is 20.0°C . In this state, a person gets rid of excess heat by radiation. By how much does the person change the entropy of the air in this room each second?

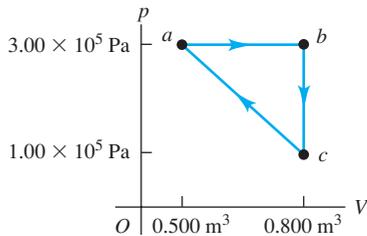
(Recall that the room radiates back into the person and that the emissivity of the skin is 1.00.)

20.49 • CP BIO A Human Engine. You decide to use your body as a Carnot heat engine. The operating gas is in a tube with one end in your mouth (where the temperature is 37.0°C) and the other end at the surface of your skin, at 30.0°C . (a) What is the maximum efficiency of such a heat engine? Would it be a very useful engine? (b) Suppose you want to use this human engine to lift a 2.50-kg box from the floor to a tabletop 1.20 m above the floor. How much must you increase the gravitational potential energy, and how much heat input is needed to accomplish this? (c) If your favorite candy bar has 350 food calories (1 food calorie = 4186 J) and 80% of the food energy goes into heat, how many of these candy bars must you eat to lift the box in this way?

20.50 • CP Entropy Change Due to the Sun. Our sun radiates from a surface at 5800 K (with an emissivity of 1.0) into the near-vacuum of space, which is at a temperature of 3 K. (a) By how much does our sun change the entropy of the universe every second? (Consult Appendix F.) (b) Is the process reversible or irreversible? Is your answer to part (a) consistent with this conclusion? Explain.

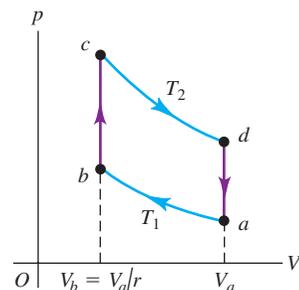
20.51 • A monatomic ideal gas is taken around the cycle shown in Fig. P20.51 in the direction shown in the figure. The path for process $c \rightarrow a$ is a straight line in the pV -diagram. (a) Calculate Q , W , and ΔU for each process $a \rightarrow b$, $b \rightarrow c$, and $c \rightarrow a$. (b) What are Q , W , and ΔU for one complete cycle? (c) What is the efficiency of the cycle?

Figure P20.51



20.52 • CALC A Stirling-Cycle Engine. The *Stirling cycle* is similar to the Otto cycle, except that the compression and expansion of the gas are done at constant temperature, not adiabatically as in the Otto cycle. The Stirling cycle is used in *external* combustion engines (in fact, burning fuel is not necessary; any way of producing a temperature difference will do—solar, geothermal, ocean temperature gradient, etc.), which means that the gas inside the cylinder is not used in the combustion process. Heat is supplied by burning fuel steadily outside the cylinder, instead of explosively inside the cylinder as in the Otto cycle. For this reason Stirling-cycle engines are quieter than Otto-cycle engines, since there are no intake and exhaust valves (a major source of engine noise). While small Stirling engines are used for a variety of purposes, Stirling engines for automobiles have not been successful because they are larger, heavier, and more expensive than conventional automobile engines. In the cycle, the working fluid goes through the following sequence of steps (Fig. P20.52):

Figure P20.52



- (i) Compressed isothermally at temperature T_1 from the initial state a to state b , with a compression ratio r .
- (ii) Heated at constant volume to state c at temperature T_2 .
- (iii) Expanded isothermally at T_2 to state d .
- (iv) Cooled at constant volume back to the initial state a .

Assume that the working fluid is n moles of an ideal gas (for which C_V is independent of temperature). (a) Calculate Q , W , and ΔU for each of the processes $a \rightarrow b$, $b \rightarrow c$, $c \rightarrow d$, and $d \rightarrow a$. (b) In the Stirling cycle, the heat transfers in the processes $b \rightarrow c$ and $d \rightarrow a$ do not involve external heat sources but rather use *regeneration*: The same substance that transfers heat to the gas inside the cylinder in the process $b \rightarrow c$ also absorbs heat back from the gas in the process $d \rightarrow a$. Hence the heat transfers $Q_{b \rightarrow c}$ and $Q_{d \rightarrow a}$ do not play a role in determining the efficiency of the engine. Explain this last statement by comparing the expressions for $Q_{b \rightarrow c}$ and $Q_{d \rightarrow a}$ calculated in part (a). (c) Calculate the efficiency of a Stirling-cycle engine in terms of the temperatures T_1 and T_2 . How does this compare to the efficiency of a Carnot-cycle engine operating between these same two temperatures? (Historically, the Stirling cycle was devised before the Carnot cycle.) Does this result violate the second law of thermodynamics? Explain. Unfortunately, actual Stirling-cycle engines cannot achieve this efficiency due to problems with the heat-transfer processes and pressure losses in the engine.

20.53 • A Carnot engine operates between two heat reservoirs at temperatures T_H and T_C . An inventor proposes to increase the efficiency by running one engine between T_H and an intermediate temperature T' and a second engine between T' and T_C , using as input the heat expelled by the first engine. Compute the efficiency of this composite system, and compare it to that of the original engine.

20.54 ••• A typical coal-fired power plant generates 1000 MW of usable power at an overall thermal efficiency of 40%. (a) What is the rate of heat input to the plant? (b) The plant burns anthracite coal, which has a heat of combustion of 2.65×10^7 J/kg. How much coal does the plant use per day, if it operates continuously? (c) At what rate is heat ejected into the cool reservoir, which is the nearby river? (d) The river's temperature is 18.0°C before it reaches the power plant and 18.5°C after it has received the plant's waste heat. Calculate the river's flow rate, in cubic meters per second. (e) By how much does the river's entropy increase each second?

20.55 • Automotive Thermodynamics. A Volkswagen Passat has a six-cylinder Otto-cycle engine with compression ratio $r = 10.6$. The diameter of each cylinder, called the *bore* of the engine, is 82.5 mm. The distance that the piston moves during the compression in Fig. 20.5, called the *stroke* of the engine, is 86.4 mm. The initial pressure of the air–fuel mixture (at point a in Fig. 20.6) is 8.50×10^4 Pa, and the initial temperature is 300 K (the same as the outside air). Assume that 200 J of heat is added to each cylinder in each cycle by the burning gasoline, and that the gas has $C_V = 20.5$ J/mol·K and $\gamma = 1.40$. (a) Calculate the total work done in one cycle in each cylinder of the engine, and the heat released when the gas is cooled to the temperature of the outside air. (b) Calculate the volume of the air–fuel mixture at point a in the cycle. (c) Calculate the pressure, volume, and temperature of the gas at points b , c , and d in the cycle. In a pV -diagram, show the numerical values of p , V , and T for each of the four states. (d) Compare the efficiency of this engine with the efficiency of a Carnot-cycle engine operating between the same maximum and minimum temperatures.

20.56 • An air conditioner operates on 800 W of power and has a performance coefficient of 2.80 with a room temperature of 21.0°C and an outside temperature of 35.0°C. (a) Calculate the rate of heat removal for this unit. (b) Calculate the rate at which heat is discharged to the outside air. (c) Calculate the total entropy change in the room if the air conditioner runs for 1 hour. Calculate the total entropy change in the outside air for the same time period. (d) What is the net change in entropy for the system (room + outside air)?

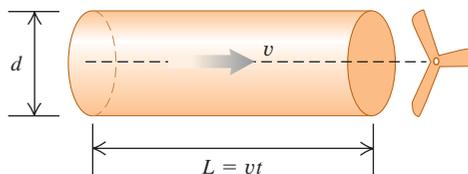
20.57 •• CALC Unavailable Energy. The discussion of entropy and the second law that follows Example 20.10 (Section 20.7) says that the increase in entropy in an irreversible process is associated with energy becoming less available. Consider a Carnot cycle that uses a low-temperature reservoir with Kelvin temperature T_c . This is a true reservoir—that is, large enough not to change temperature when it accepts heat from the engine. Let the engine accept heat from an object of temperature T' , where $T' > T_c$. The object is of finite size, so it cools as heat is extracted from it. The engine continues to operate until $T' = T_c$. (a) Show that the total magnitude of heat rejected to the low-temperature reservoir is $T_c |\Delta S_h|$, where ΔS_h is the change in entropy of the high-temperature reservoir. (b) Apply the result of part (a) to 1.00 kg of water initially at a temperature of 373 K as the heat source for the engine and $T_c = 273$ K. How much total mechanical work can be performed by the engine until it stops? (c) Repeat part (b) for 2.00 kg of water at 323 K. (d) Compare the amount of work that can be obtained from the energy in the water of Example 20.10 before and after it is mixed. Discuss whether your result shows that energy has become less available.

20.58 ••• CP The maximum power that can be extracted by a wind turbine from an air stream is approximately

$$P = kd^2v^3$$

where d is the blade diameter, v is the wind speed, and the constant $k = 0.5 \text{ W} \cdot \text{s}^3/\text{m}^5$. (a) Explain the dependence of P on d and on v by considering a cylinder of air that passes over the turbine blades in time t (Fig. P20.58). This cylinder has diameter d , length $L = vt$, and density ρ . (b) The Mod-5B wind turbine at Kahaku on the Hawaiian island of Oahu has a blade diameter of 97 m (slightly longer than a football field) and sits atop a 58-m tower. It can produce 3.2 MW of electric power. Assuming 25% efficiency, what wind speed is required to produce this amount of power? Give your answer in m/s and in km/h. (c) Commercial wind turbines are commonly located in or downwind of mountain passes. Why?

Figure **P20.58**



20.59 •• CALC (a) For the Otto cycle shown in Fig. 20.6, calculate the changes in entropy of the gas in each of the constant-volume processes $b \rightarrow c$ and $d \rightarrow a$ in terms of the temperatures T_a , T_b , T_c , and T_d and the number of moles n and the heat capacity C_V of the

gas. (b) What is the total entropy change in the engine during one cycle? (*Hint:* Use the relationships between T_a and T_b and between T_d and T_c .) (c) The processes $b \rightarrow c$ and $d \rightarrow a$ occur irreversibly in a real Otto engine. Explain how can this be reconciled with your result in part (b).

20.60 •• CALC A TS-Diagram. (a) Graph a Carnot cycle, plotting Kelvin temperature vertically and entropy horizontally. This is called a temperature–entropy diagram, or *TS*-diagram. (b) Show that the area under any curve representing a reversible path in a temperature–entropy diagram represents the heat absorbed by the system. (c) Derive from your diagram the expression for the thermal efficiency of a Carnot cycle. (d) Draw a temperature–entropy diagram for the Stirling cycle described in Problem 20.52. Use this diagram to relate the efficiencies of the Carnot and Stirling cycles.

20.61 • A physics student immerses one end of a copper rod in boiling water at 100°C and the other end in an ice–water mixture at 0°C. The sides of the rod are insulated. After steady-state conditions have been achieved in the rod, 0.120 kg of ice melts in a certain time interval. For this time interval, find (a) the entropy change of the boiling water; (b) the entropy change of the ice–water mixture; (c) the entropy change of the copper rod; (d) the total entropy change of the entire system.

20.62 •• CALC To heat 1 cup of water (250 cm³) to make coffee, you place an electric heating element in the cup. As the water temperature increases from 20°C to 78°C, the temperature of the heating element remains at a constant 120°C. Calculate the change in entropy of (a) the water; (b) the heating element; (c) the system of water and heating element. (Make the same assumption about the specific heat of water as in Example 20.10 in Section 20.7, and ignore the heat that flows into the ceramic coffee cup itself.) (d) Is this process reversible or irreversible? Explain.

20.63 •• CALC An object of mass m_1 , specific heat c_1 , and temperature T_1 is placed in contact with a second object of mass m_2 , specific heat c_2 , and temperature $T_2 > T_1$. As a result, the temperature of the first object increases to T and the temperature of the second object decreases to T' . (a) Show that the entropy increase of the system is

$$\Delta S = m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \frac{T'}{T_2}$$

and show that energy conservation requires that

$$m_1 c_1 (T - T_1) = m_2 c_2 (T_2 - T')$$

(b) Show that the entropy change ΔS , considered as a function of T , is a *maximum* if $T = T'$, which is just the condition of thermodynamic equilibrium. (c) Discuss the result of part (b) in terms of the idea of entropy as a measure of disorder.

CHALLENGE PROBLEM

20.64 ••• Consider a Diesel cycle that starts (at point a in Fig. 20.7) with air at temperature T_a . The air may be treated as an ideal gas. (a) If the temperature at point c is T_c , derive an expression for the efficiency of the cycle in terms of the compression ratio r . (b) What is the efficiency if $T_a = 300$ K, $T_c = 950$ K, $\gamma = 1.40$, and $r = 21.0$?

Answers

Chapter Opening Question ?

Yes. That's what a refrigerator does: It makes heat flow from the cold interior of the refrigerator to the warm outside. The second law of thermodynamics says that heat cannot *spontaneously* flow from a cold body to a hot one. A refrigerator has a motor that does work on the system to *force* the heat to flow in that way.

Test Your Understanding Questions

20.1 Answer: (ii) Like sliding a book across a table, rubbing your hands together uses friction to convert mechanical energy into heat. The (impossible) reverse process would involve your hands spontaneously getting colder, with the released energy forcing your hands to move rhythmically back and forth!

20.2 Answer: (iii), (i), (ii) From Eq. (20.4) the efficiency is $e = W/Q_H$, and from Eq. (20.2) $W = Q_H + Q_C = |Q_H| - |Q_C|$. For engine (i) $Q_H = 5000 \text{ J}$ and $Q_C = -4500 \text{ J}$, so $W = 5000 \text{ J} + (-4500 \text{ J}) = 500 \text{ J}$ and $e = (500 \text{ J})/(5000 \text{ J}) = 0.100$. For engine (ii) $Q_H = 25,000 \text{ J}$ and $W = 2000 \text{ J}$, so $e = (2000 \text{ J})/(25,000 \text{ J}) = 0.080$. For engine (iii) $W = 400 \text{ J}$ and $Q_C = -2800 \text{ J}$, so $Q_H = W - Q_C = 400 \text{ J} - (-2800 \text{ J}) = 3200 \text{ J}$ and $e = (400 \text{ J})/(3200 \text{ J}) = 0.125$.

20.3 Answers: (i), (ii) Doubling the amount of fuel burned per cycle means that Q_H is doubled, so the resulting pressure increase from b to c in Fig. 20.6 is greater. The compression ratio and hence the efficiency remain the same, so $|Q_C|$ (the amount of heat rejected to the environment) must increase by the same factor as Q_H . Hence the pressure drop from d to a in Fig. 20.6 is also greater. The volume V and the compression ratio r don't change, so the horizontal dimensions of the pV -diagram don't change.

20.4 Answer: no A refrigerator uses an input of work to transfer heat from one system (the refrigerator's interior) to another system (its exterior, which includes the house in which the refrigerator is installed). If the door is open, these two systems are really the *same* system and will eventually come to the same temperature. By the first law of thermodynamics, all of the work input to the refrigerator motor will be converted into heat and the temperature in your house will actually *increase*. To cool the house you need a system that will transfer heat from it to the outside world, such as an air conditioner or heat pump.

20.5 Answers: no, no Both the 100%-efficient engine of Fig. 20.11a and the workless refrigerator of Fig. 20.11b return to the

same state at the end of a cycle as at the beginning, so the net change in internal energy of each system is zero ($\Delta U = 0$). For the 100%-efficient engine, the net heat flow into the engine equals the net work done, so $Q = W$, $Q - W = 0$, and the first law ($\Delta U = Q - W$) is obeyed. For the workless refrigerator, no net work is done (so $W = 0$) and as much heat flows into it as out (so $Q = 0$), so again $Q - W = 0$ and $\Delta U = Q - W$ in accordance with the first law. It is the *second* law of thermodynamics that tells us that both the 100%-efficient engine and the workless refrigerator are impossible.

20.6 Answer: no The efficiency can be no better than that of a Carnot engine running between the same two temperature limits, $e_{\text{Carnot}} = 1 - (T_C/T_H)$ [Eq. (20.14)]. The temperature T_C of the cold reservoir for this air-cooled engine is about 300 K (ambient temperature), and the temperature T_H of the hot reservoir cannot exceed the melting point of copper, 1356 K (see Table 17.4). Hence the maximum possible Carnot efficiency is $e = 1 - (300 \text{ K})/(1356 \text{ K}) = 0.78$, or 78%. The temperature of any real engine would be less than this, so it would be impossible for the inventor's engine to attain 85% efficiency. You should invest your money elsewhere.

20.7 Answers: -102 J/K, no The process described is exactly the opposite of the process used in Example 20.10. The result violates the second law of thermodynamics, which states that the entropy of an isolated system cannot decrease.

20.8 Answer: (i) For case (i), we saw in Example 20.8 (Section 20.7) that for an ideal gas, the entropy change in a free expansion is the same as in an isothermal expansion. From Eq. (20.23), this implies that the ratio of the number of microscopic states after and before the expansion, w_2/w_1 , is also the same for these two cases. From Example 20.11, $w_2/w_1 = 2^N$, so the number of microscopic states increases by a factor 2^N . For case (ii), in a reversible expansion the entropy change is $\Delta S = \int dQ/T = 0$; if the expansion is adiabatic there is no heat flow, so $\Delta S = 0$. From Eq. (20.23), $w_2/w_1 = 1$ and there is *no* change in the number of microscopic states. The difference is that in an adiabatic expansion the temperature drops and the molecules move more slowly, so they have fewer microscopic states available to them than in an isothermal expansion.

Bridging Problem

Answers: (a) 34.83°C **(b)** +12.1 J/K

ELECTRIC CHARGE AND ELECTRIC FIELD

21



? Water makes life possible: The cells of your body could not function without water in which to dissolve essential biological molecules. What electrical properties of water make it such a good solvent?

In Chapter 5 we mentioned the four kinds of fundamental forces. To this point the only one of these forces that we have examined in any detail is gravity. Now we are ready to examine the force of *electromagnetism*, which encompasses both electricity and magnetism. Electromagnetic phenomena will occupy our attention for most of the remainder of this book.

Electromagnetic interactions involve particles that have a property called *electric charge*, an attribute that is as fundamental as mass. Just as objects with mass are accelerated by gravitational forces, so electrically charged objects are accelerated by electric forces. The shock you feel when you scuff your shoes across a carpet and then reach for a metal doorknob is due to charged particles leaping between your finger and the doorknob. Electric currents are simply streams of charged particles flowing within wires in response to electric forces. Even the forces that hold atoms together to form solid matter, and that keep the atoms of solid objects from passing through each other, are fundamentally due to electric interactions between the charged particles within atoms.

We begin our study of electromagnetism in this chapter by examining the nature of electric charge. We'll find that charge is quantized and obeys a conservation principle. When charges are at rest in our frame of reference, they exert *electrostatic* forces on each other. These forces are of tremendous importance in chemistry and biology and have many technological applications. Electrostatic forces are governed by a simple relationship known as *Coulomb's law* and are most conveniently described by using the concept of *electric field*. In later chapters we'll expand our discussion to include electric charges in motion. This will lead us to an understanding of magnetism and, remarkably, of the nature of light.

While the key ideas of electromagnetism are conceptually simple, applying them to practical problems will make use of many of your mathematical skills, especially your knowledge of geometry and integral calculus. For this reason you may find this chapter and those that follow to be more mathematically demanding

LEARNING GOALS

By studying this chapter, you will learn:

- The nature of electric charge, and how we know that electric charge is conserved.
- How objects become electrically charged.
- How to use Coulomb's law to calculate the electric force between charges.
- The distinction between electric force and electric field.
- How to calculate the electric field due to a collection of charges.
- How to use the idea of electric field lines to visualize and interpret electric fields.
- How to calculate the properties of electric dipoles.

than earlier chapters. The reward for your extra effort will be a deeper understanding of principles that are at the heart of modern physics and technology.

21.1 Electric Charge

The ancient Greeks discovered as early as 600 B.C. that after they rubbed amber with wool, the amber could attract other objects. Today we say that the amber has acquired a net **electric charge**, or has become *charged*. The word “electric” is derived from the Greek word *elektron*, meaning amber. When you scuff your shoes across a nylon carpet, you become electrically charged, and you can charge a comb by passing it through dry hair.

Plastic rods and fur (real or fake) are particularly good for demonstrating **electrostatics**, the interactions between electric charges that are at rest (or nearly so). After we charge both plastic rods in Fig. 21.1a by rubbing them with the piece of fur, we find that the rods repel each other.

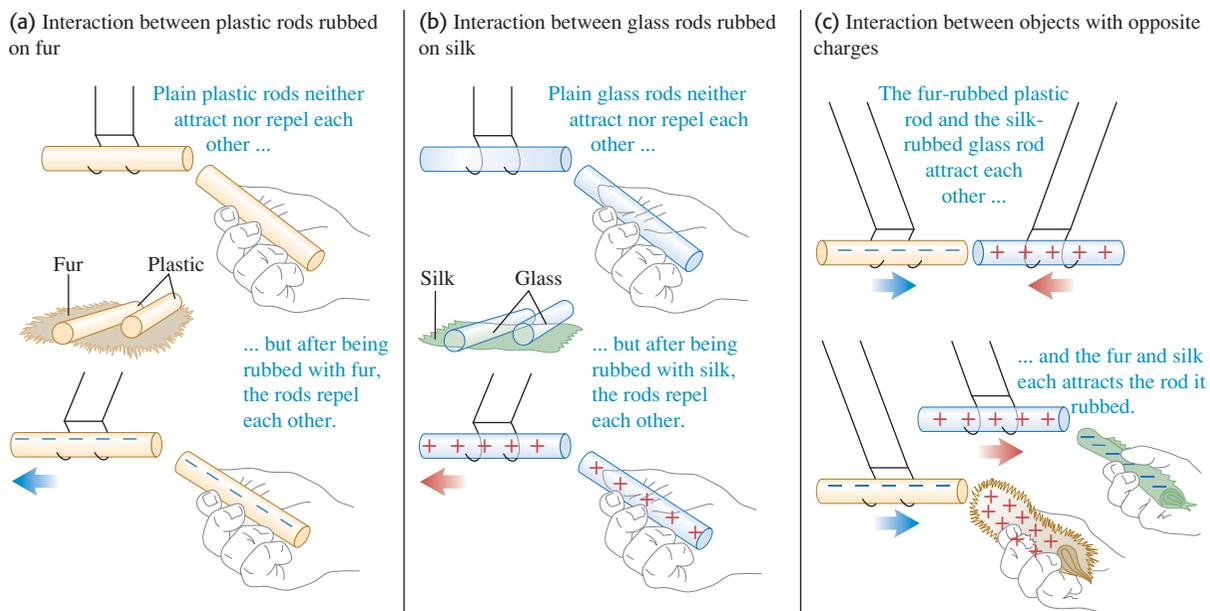
When we rub glass rods with silk, the glass rods also become charged and repel each other (Fig. 21.1b). But a charged plastic rod *attracts* a charged glass rod; furthermore, the plastic rod and the fur attract each other, and the glass rod and the silk attract each other (Fig. 21.1c).

These experiments and many others like them have shown that there are exactly two kinds of electric charge: the kind on the plastic rod rubbed with fur and the kind on the glass rod rubbed with silk. Benjamin Franklin (1706–1790) suggested calling these two kinds of charge *negative* and *positive*, respectively, and these names are still used. The plastic rod and the silk have negative charge; the glass rod and the fur have positive charge.

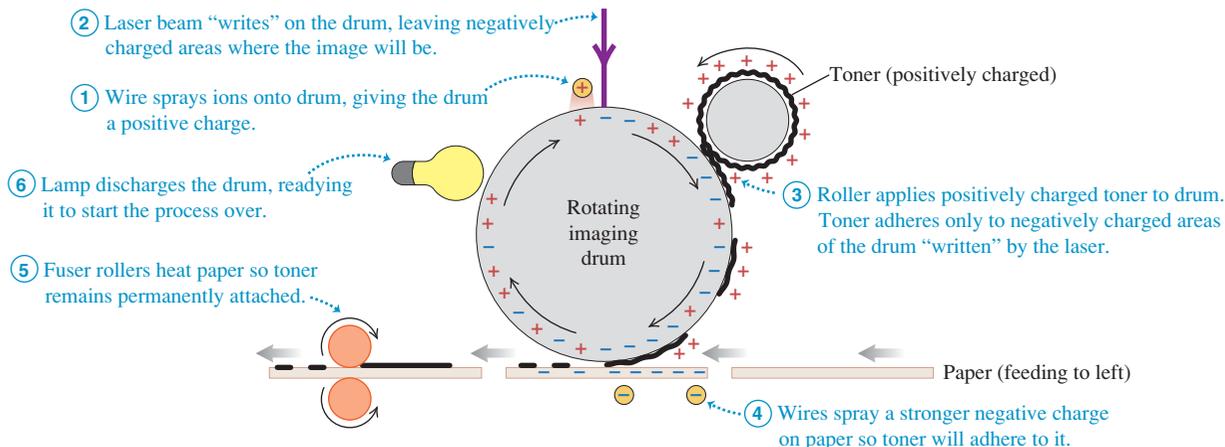
Two positive charges or two negative charges repel each other. A positive charge and a negative charge attract each other.

CAUTION **Electric attraction and repulsion** The attraction and repulsion of two charged objects are sometimes summarized as “Like charges repel, and opposite charges attract.” But keep in mind that the phrase “like charges” does *not* mean that the two charges are exactly identical, only that both charges have the same algebraic *sign* (both positive or both negative). “Opposite charges” means that both objects have an electric charge, and those charges have different signs (one positive and the other negative). ▮

21.1 Experiments in electrostatics. (a) Negatively charged objects repel each other. (b) Positively charged objects repel each other. (c) Positively charged objects and negatively charged objects attract each other.



21.2 Schematic diagram of the operation of a laser printer.



One application of forces between charged bodies is in a laser printer (Fig. 21.2). The printer’s light-sensitive imaging drum is given a positive charge. As the drum rotates, a laser beam shines on selected areas of the drum, leaving those areas with a *negative* charge. Positively charged particles of toner adhere only to the areas of the drum “written” by the laser. When a piece of paper is placed in contact with the drum, the toner particles stick to the paper and form an image.

Electric Charge and the Structure of Matter

When you charge a rod by rubbing it with fur or silk as in Fig. 21.1, there is no visible change in the appearance of the rod. What, then, actually happens to the rod when you charge it? To answer this question, we must look more closely at the structure of atoms, the building blocks of ordinary matter.

The structure of atoms can be described in terms of three particles: the negatively charged **electron**, the positively charged **proton**, and the uncharged **neutron** (Fig. 21.3). The proton and neutron are combinations of other entities called *quarks*, which have charges of $\pm\frac{1}{3}$ and $\pm\frac{2}{3}$ times the electron charge. Isolated quarks have not been observed, and there are theoretical reasons to believe that it is impossible in principle to observe a quark in isolation.

The protons and neutrons in an atom make up a small, very dense core called the **nucleus**, with dimensions of the order of 10^{-15} m. Surrounding the nucleus are the electrons, extending out to distances of the order of 10^{-10} m from the nucleus. If an atom were a few kilometers across, its nucleus would be the size of a tennis ball. The negatively charged electrons are held within the atom by the attractive electric forces exerted on them by the positively charged nucleus. (The protons and neutrons are held within stable atomic nuclei by an attractive interaction, called the *strong nuclear force*, that overcomes the electric repulsion of the protons. The strong nuclear force has a short range, and its effects do not extend far beyond the nucleus.)

The masses of the individual particles, to the precision that they are presently known, are

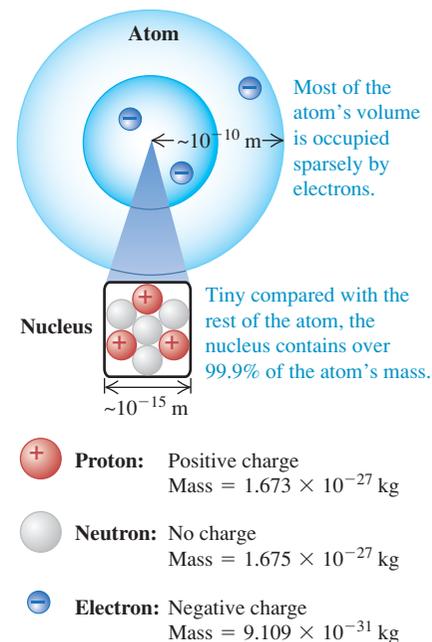
$$\text{Mass of electron} = m_e = 9.10938215(45) \times 10^{-31} \text{ kg}$$

$$\text{Mass of proton} = m_p = 1.672621637(83) \times 10^{-27} \text{ kg}$$

$$\text{Mass of neutron} = m_n = 1.674927211(84) \times 10^{-27} \text{ kg}$$

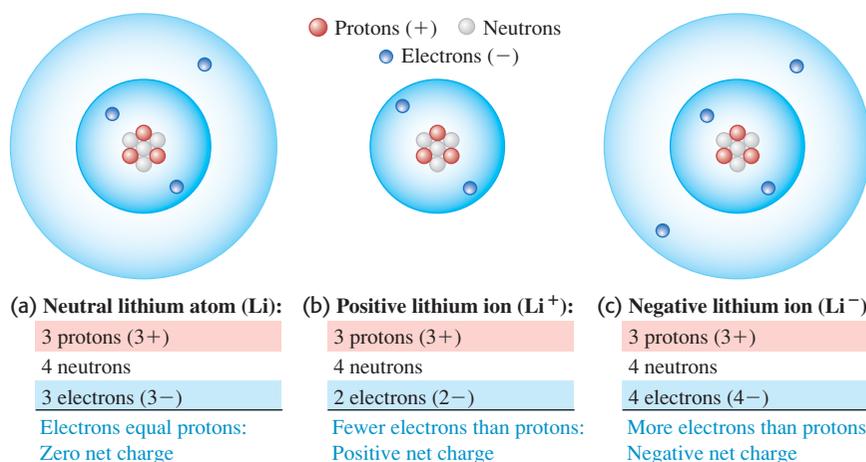
The numbers in parentheses are the uncertainties in the last two digits. Note that the masses of the proton and neutron are nearly equal and are roughly 2000 times

21.3 The structure of an atom. The particular atom depicted here is lithium (see Fig. 21.4a).



The charges of the electron and proton are equal in magnitude.

21.4 (a) A neutral atom has as many electrons as it does protons. (b) A positive ion has a deficit of electrons. (c) A negative ion has an excess of electrons. (The electron “shells” are a schematic representation of the actual electron distribution, a diffuse cloud many times larger than the nucleus.)



the mass of the electron. Over 99.9% of the mass of any atom is concentrated in its nucleus.

The negative charge of the electron has (within experimental error) *exactly* the same magnitude as the positive charge of the proton. In a neutral atom the number of electrons equals the number of protons in the nucleus, and the net electric charge (the algebraic sum of all the charges) is exactly zero (Fig. 21.4a). The number of protons or electrons in a neutral atom of an element is called the **atomic number** of the element. If one or more electrons are removed from an atom, what remains is called a **positive ion** (Fig. 21.4b). A **negative ion** is an atom that has *gained* one or more electrons (Fig. 21.4c). This gain or loss of electrons is called **ionization**.

When the total number of protons in a macroscopic body equals the total number of electrons, the total charge is zero and the body as a whole is electrically neutral. To give a body an excess negative charge, we may either *add negative* charges to a neutral body or *remove positive* charges from that body. Similarly, we can create an excess positive charge by either *adding positive* charge or *removing negative* charge. In most cases, negatively charged (and highly mobile) electrons are added or removed, and a “positively charged body” is one that has lost some of its normal complement of electrons. When we speak of the charge of a body, we always mean its *net* charge. The net charge is always a very small fraction (typically no more than 10^{-12}) of the total positive charge or negative charge in the body.

Electric Charge Is Conserved

Implicit in the foregoing discussion are two very important principles. First is the **principle of conservation of charge**:

The algebraic sum of all the electric charges in any closed system is constant.

If we rub together a plastic rod and a piece of fur, both initially uncharged, the rod acquires a negative charge (since it takes electrons from the fur) and the fur acquires a positive charge of the *same* magnitude (since it has lost as many electrons as the rod has gained). Hence the total electric charge on the two bodies together does not change. In any charging process, charge is not created or destroyed; it is merely *transferred* from one body to another.

Conservation of charge is thought to be a *universal* conservation law. No experimental evidence for any violation of this principle has ever been observed. Even in high-energy interactions in which particles are created and destroyed, such as the creation of electron–positron pairs, the total charge of any closed system is exactly constant.

The second important principle is:

The magnitude of charge of the electron or proton is a natural unit of charge.

Every observable amount of electric charge is always an integer multiple of this basic unit. We say that charge is *quantized*. A familiar example of quantization is money. When you pay cash for an item in a store, you have to do it in one-cent increments. Cash can't be divided into amounts smaller than one cent, and electric charge can't be divided into amounts smaller than the charge of one electron or proton. (The quark charges, $\pm\frac{1}{3}$ and $\pm\frac{2}{3}$ of the electron charge, are probably not observable as isolated charges.) Thus the charge on any macroscopic body is always either zero or an integer multiple (negative or positive) of the electron charge.

Understanding the electric nature of matter gives us insight into many aspects of the physical world (Fig. 21.5). The chemical bonds that hold atoms together to form molecules are due to electric interactions between the atoms. They include the strong ionic bonds that hold sodium and chlorine atoms together to make table salt and the relatively weak bonds between the strands of DNA that record your body's genetic code. The normal force exerted on you by the chair in which you're sitting arises from electric forces between charged particles in the atoms of your seat and in the atoms of your chair. The tension force in a stretched string and the adhesive force of glue are likewise due to the electric interactions of atoms.

Test Your Understanding of Section 21.1 (a) Strictly speaking, does the plastic rod in Fig. 21.1 weigh more, less, or the same after rubbing it with fur? (b) What about the glass rod after rubbing it with silk? What about (c) the fur and (d) the silk? **I**

21.2 Conductors, Insulators, and Induced Charges

Some materials permit electric charge to move easily from one region of the material to another, while others do not. For example, Fig. 21.6a shows a copper wire supported by a nylon thread. Suppose you touch one end of the wire to a charged plastic rod and attach the other end to a metal ball that is initially uncharged; you then remove the charged rod and the wire. When you bring another charged body up close to the ball (Figs. 21.6b and 21.6c), the ball is attracted or repelled, showing that the ball has become electrically charged. Electric charge has been transferred through the copper wire between the ball and the surface of the plastic rod.

The copper wire is called a **conductor** of electricity. If you repeat the experiment using a rubber band or nylon thread in place of the wire, you find that *no* charge is transferred to the ball. These materials are called **insulators**. Conductors permit the easy movement of charge through them, while insulators do not. (The supporting nylon threads shown in Fig. 21.6 are insulators, which prevents charge from leaving the metal ball and copper wire.)

As an example, carpet fibers on a dry day are good insulators. As you walk across a carpet, the rubbing of your shoes against the fibers causes charge to build up on you, and this charge remains on you because it can't flow through the insulating fibers. If you then touch a conducting object such as a doorknob, a rapid charge transfer takes place between your finger and the doorknob, and you feel a shock. One way to prevent this is to wind some of the carpet fibers around conducting cores so that any charge that builds up on you can be transferred harmlessly to the carpet. Another solution is to coat the carpet fibers with an anti-static layer that does not easily transfer electrons to or from your shoes; this prevents any charge from building up on you in the first place.

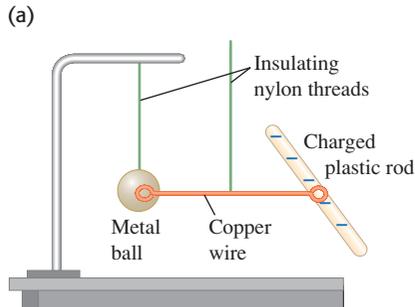
21.5 Most of the forces on this water skier are electric. Electric interactions between adjacent molecules give rise to the force of the water on the ski, the tension in the tow rope, and the resistance of the air on the skier's body. Electric interactions also hold the atoms of the skier's body together. Only one wholly nonelectric force acts on the skier: the force of gravity.



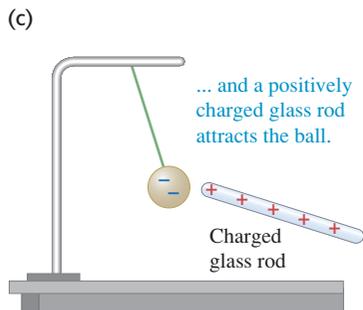
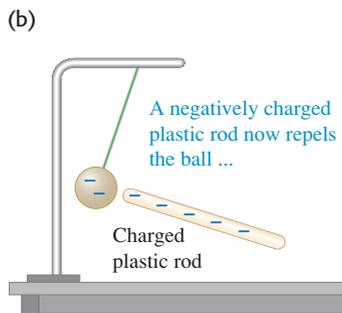
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PhET: Balloons and Static Electricity
PhET: John Travoltage

21.6 Copper is a good conductor of electricity; nylon is a good insulator. (a) The copper wire conducts charge between the metal ball and the charged plastic rod to charge the ball negatively. Afterward, the metal ball is (b) repelled by a negatively charged plastic rod and (c) attracted to a positively charged glass rod.



The wire conducts charge from the negatively charged plastic rod to the metal ball.



Most metals are good conductors, while most nonmetals are insulators. Within a solid metal such as copper, one or more outer electrons in each atom become detached and can move freely throughout the material, just as the molecules of a gas can move through the spaces between the grains in a bucket of sand. The other electrons remain bound to the positively charged nuclei, which themselves are bound in nearly fixed positions within the material. In an insulator there are no, or very few, free electrons, and electric charge cannot move freely through the material. Some materials called *semiconductors* are intermediate in their properties between good conductors and good insulators.

Charging by Induction

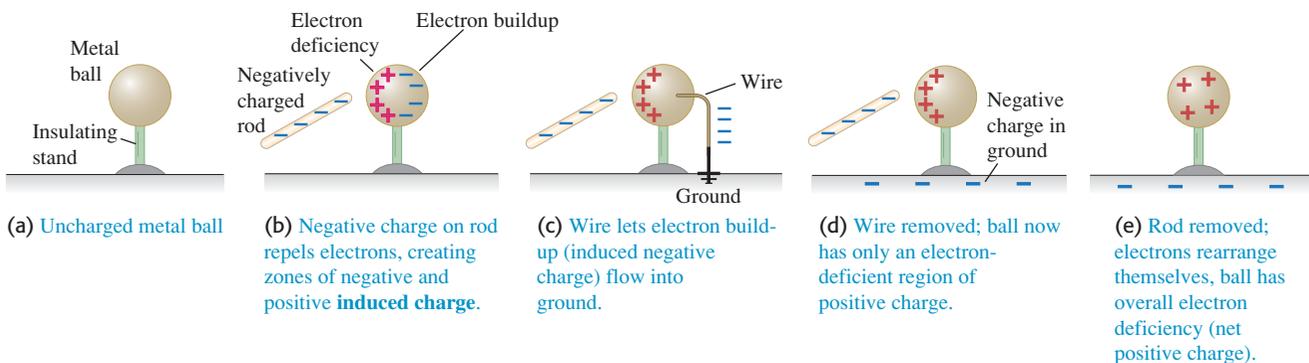
We can charge a metal ball using a copper wire and an electrically charged plastic rod, as in Fig. 21.6a. In this process, some of the excess electrons on the rod are transferred from it to the ball, leaving the rod with a smaller negative charge. But there is a different technique in which the plastic rod can give another body a charge of *opposite* sign without losing any of its own charge. This process is called charging by **induction**.

Figure 21.7 shows an example of charging by induction. An uncharged metal ball is supported on an insulating stand (Fig. 21.7a). When you bring a negatively charged rod near it, without actually touching it (Fig. 21.7b), the free electrons in the metal ball are repelled by the excess electrons on the rod, and they shift toward the right, away from the rod. They cannot escape from the ball because the supporting stand and the surrounding air are insulators. So we get excess negative charge at the right surface of the ball and a deficiency of negative charge (that is, a net positive charge) at the left surface. These excess charges are called **induced charges**.

Not all of the free electrons move to the right surface of the ball. As soon as any induced charge develops, it exerts forces toward the *left* on the other free electrons. These electrons are repelled by the negative induced charge on the right and attracted toward the positive induced charge on the left. The system reaches an equilibrium state in which the force toward the right on an electron, due to the charged rod, is just balanced by the force toward the left due to the induced charge. If we remove the charged rod, the free electrons shift back to the left, and the original neutral condition is restored.

What happens if, while the plastic rod is nearby, you touch one end of a conducting wire to the right surface of the ball and the other end to the earth (Fig. 21.7c)? The earth is a conductor, and it is so large that it can act as a practically infinite source of extra electrons or sink of unwanted electrons. Some of the negative charge flows through the wire to the earth. Now suppose you disconnect the wire (Fig. 21.7d) and then remove the rod (Fig. 21.7e); a net positive charge is left on the ball. The charge on the negatively charged rod has not changed during this process. The earth acquires a negative charge that is equal in magnitude to the induced positive charge remaining on the ball.

21.7 Charging a metal ball by induction.



(a) Uncharged metal ball

(b) Negative charge on rod repels electrons, creating zones of negative and positive **induced charge**.

(c) Wire lets electron buildup (induced negative charge) flow into ground.

(d) Wire removed; ball now has only an electron-deficient region of positive charge.

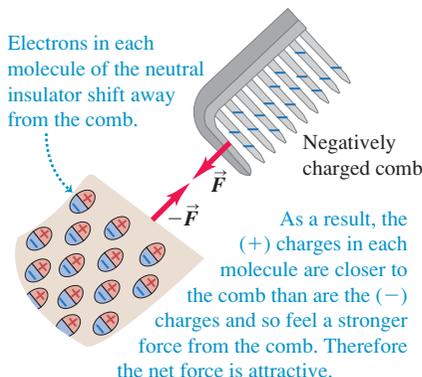
(e) Rod removed; electrons rearrange themselves, ball has overall electron deficiency (net positive charge).

21.8 The charges within the molecules of an insulating material can shift slightly. As a result, a comb with either sign of charge attracts a neutral insulator. By Newton's third law the neutral insulator exerts an equal-magnitude attractive force on the comb.

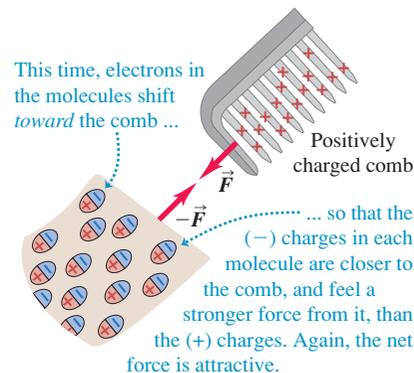
(a) A charged comb picking up uncharged pieces of plastic



(b) How a negatively charged comb attracts an insulator



(c) How a positively charged comb attracts an insulator



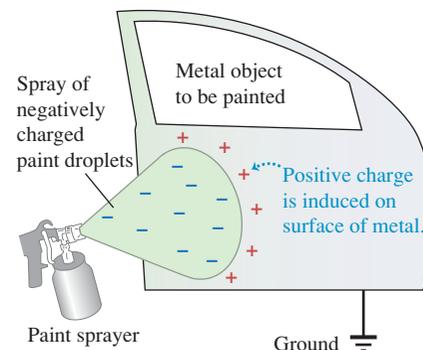
Electric Forces on Uncharged Objects

Finally, we note that a charged body can exert forces even on objects that are *not* charged themselves. If you rub a balloon on the rug and then hold the balloon against the ceiling, it sticks, even though the ceiling has no net electric charge. After you electrify a comb by running it through your hair, you can pick up uncharged bits of paper or plastic with the comb (Fig. 21.8a). How is this possible?

This interaction is an induced-charge effect. Even in an insulator, electric charges can shift back and forth a little when there is charge nearby. This is shown in Fig. 21.8b; the negatively charged plastic comb causes a slight shifting of charge within the molecules of the neutral insulator, an effect called *polarization*. The positive and negative charges in the material are present in equal amounts, but the positive charges are closer to the plastic comb and so feel an attraction that is stronger than the repulsion felt by the negative charges, giving a net attractive force. (In Section 21.3 we will study how electric forces depend on distance.) Note that a neutral insulator is also attracted to a *positively* charged comb (Fig. 21.8c). Now the charges in the insulator shift in the opposite direction; the negative charges in the insulator are closer to the comb and feel an attractive force that is stronger than the repulsion felt by the positive charges in the insulator. Hence a charged object of *either* sign exerts an attractive force on an uncharged insulator. Figure 21.9 shows an industrial application of this effect.

Test Your Understanding of Section 21.2 You have two lightweight metal spheres, each hanging from an insulating nylon thread. One of the spheres has a net negative charge, while the other sphere has no net charge. (a) If the spheres are close together but do not touch, will they (i) attract each other, (ii) repel each other, or (iii) exert no force on each other? (b) You now allow the two spheres to touch. Once they have touched, will the two spheres (i) attract each other, (ii) repel each other, or (iii) exert no force on each other?

21.9 The electrostatic painting process (compare Figs. 21.7b and 21.7c). A metal object to be painted is connected to the earth ("ground"), and the paint droplets are given an electric charge as they exit the sprayer nozzle. Induced charges of the opposite sign appear in the object as the droplets approach, just as in Fig. 21.7b, and they attract the droplets to the surface. This process minimizes overspray from clouds of stray paint particles and gives a particularly smooth finish.



21.3 Coulomb's Law

Charles Augustin de Coulomb (1736–1806) studied the interaction forces of charged particles in detail in 1784. He used a torsion balance (Fig. 21.10a) similar to the one used 13 years later by Cavendish to study the much weaker gravitational interaction, as we discussed in Section 13.1. For **point charges**, charged

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ActivPhysics 11.1: Electric Force: Coulomb's Law

ActivPhysics 11.2: Electric Force: Superposition Principle

ActivPhysics 11.3: Electric Force: Superposition (Quantitative)

Application Electric Forces, Sweat, and Cystic Fibrosis

One way to test for the genetic disease cystic fibrosis (CF) is by measuring the salt content of a person's sweat. Sweat is a mixture of water and ions, including the sodium (Na^+) and chloride (Cl^-) ions that make up ordinary salt (NaCl). When sweat is secreted by epithelial cells, some of the Cl^- ions flow from the sweat back into these cells (a process called reabsorption). The electric attraction between negative and positive charges pulls Na^+ ions along with the Cl^- . Water molecules cannot flow back into the epithelial cells, so sweat on the skin has a low salt content. However, in persons with CF the reabsorption of Cl^- ions is blocked. Hence the sweat of persons with CF is unusually salty, with up to four times the normal concentration of Cl^- and Na^+ .



bodies that are very small in comparison with the distance r between them, Coulomb found that the electric force is proportional to $1/r^2$. That is, when the distance r doubles, the force decreases to one-quarter of its initial value; when the distance is halved, the force increases to four times its initial value.

The electric force between two point charges also depends on the quantity of charge on each body, which we will denote by q or Q . To explore this dependence, Coulomb divided a charge into two equal parts by placing a small charged spherical conductor into contact with an identical but uncharged sphere; by symmetry, the charge is shared equally between the two spheres. (Note the essential role of the principle of conservation of charge in this procedure.) Thus he could obtain one-half, one-quarter, and so on, of any initial charge. He found that the forces that two point charges q_1 and q_2 exert on each other are proportional to each charge and therefore are proportional to the *product* q_1q_2 of the two charges.

Thus Coulomb established what we now call **Coulomb's law**:

The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

In mathematical terms, the magnitude F of the force that each of two point charges q_1 and q_2 a distance r apart exerts on the other can be expressed as

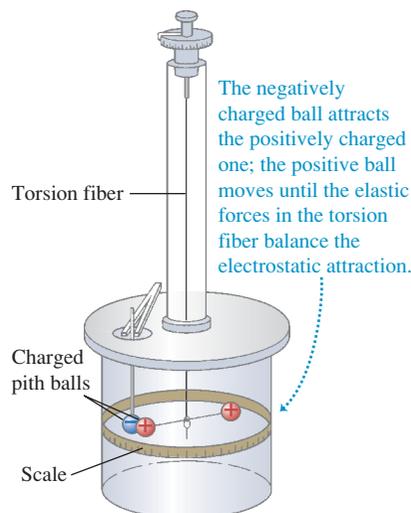
$$F = k \frac{|q_1 q_2|}{r^2} \quad (21.1)$$

where k is a proportionality constant whose numerical value depends on the system of units used. The absolute value bars are used in Eq. (21.1) because the charges q_1 and q_2 can be either positive or negative, while the force magnitude F is always positive.

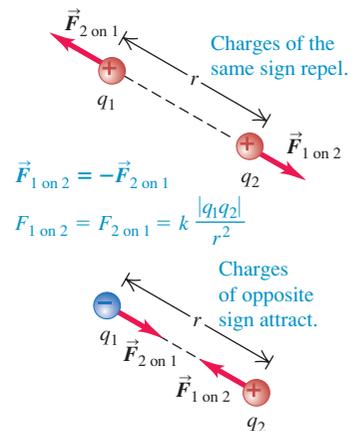
The directions of the forces the two charges exert on each other are always along the line joining them. When the charges q_1 and q_2 have the same sign, either both positive or both negative, the forces are repulsive; when the charges have opposite signs, the forces are attractive (Fig. 21.10b). The two forces obey Newton's third law; they are always equal in magnitude and opposite in direction, even when the charges are not equal in magnitude.

21.10 (a) Measuring the electric force between point charges. (b) The electric forces between point charges obey Newton's third law: $\vec{F}_{1 \text{ on } 2} = -\vec{F}_{2 \text{ on } 1}$.

(a) A torsion balance of the type used by Coulomb to measure the electric force



(b) Interactions between point charges



The proportionality of the electric force to $1/r^2$ has been verified with great precision. There is no reason to suspect that the exponent is different from precisely 2. Thus the form of Eq. (21.1) is the same as that of the law of gravitation. But electric and gravitational interactions are two distinct classes of phenomena. Electric interactions depend on electric charges and can be either attractive or repulsive, while gravitational interactions depend on mass and are always attractive (because there is no such thing as negative mass).

Fundamental Electric Constants

The value of the proportionality constant k in Coulomb's law depends on the system of units used. In our study of electricity and magnetism we will use SI units exclusively. The SI electric units include most of the familiar units such as the volt, the ampere, the ohm, and the watt. (There is *no* British system of electric units.) The SI unit of electric charge is called one **coulomb** (1 C). In SI units the constant k in Eq. (21.1) is

$$k = 8.987551787 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \cong 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

The value of k is known to such a large number of significant figures because this value is closely related to the speed of light in vacuum. (We will show this in Chapter 32 when we study electromagnetic radiation.) As we discussed in Section 1.3, this speed is *defined* to be exactly $c = 2.99792458 \times 10^8$ m/s. The numerical value of k is defined in terms of c to be precisely

$$k = (10^{-7} \text{ N} \cdot \text{s}^2/\text{C}^2)c^2$$

You should check this expression to confirm that k has the right units.

In principle we can measure the electric force F between two equal charges q at a measured distance r and use Coulomb's law to determine the charge. Thus we could regard the value of k as an operational definition of the coulomb. For reasons of experimental precision it is better to define the coulomb instead in terms of a unit of electric *current* (charge per unit time), the *ampere*, equal to 1 coulomb per second. We will return to this definition in Chapter 28.

In SI units we usually write the constant k in Eq. (21.1) as $1/4\pi\epsilon_0$, where ϵ_0 (“epsilon-nought” or “epsilon-zero”) is another constant. This appears to complicate matters, but it actually simplifies many formulas that we will encounter in later chapters. From now on, we will usually write Coulomb's law as

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1q_2|}{r^2} \quad (\text{Coulomb's law: force between two point charges}) \quad (21.2)$$

The constants in Eq. (21.2) are approximately

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \quad \text{and} \quad \frac{1}{4\pi\epsilon_0} = k = 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

In examples and problems we will often use the approximate value

$$\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

which is within about 0.1% of the correct value.

As we mentioned in Section 21.1, the most fundamental unit of charge is the magnitude of the charge of an electron or a proton, which is denoted by e . The most precise value available as of the writing of this book is

$$e = 1.602176487(40) \times 10^{-19} \text{ C}$$

One coulomb represents the negative of the total charge of about 6×10^{18} electrons. For comparison, a copper cube 1 cm on a side contains about 2.4×10^{24}

electrons. About 10^{19} electrons pass through the glowing filament of a flashlight bulb every second.

In electrostatics problems (that is, problems that involve charges at rest), it's very unusual to encounter charges as large as 1 coulomb. Two 1-C charges separated by 1 m would exert forces on each other of magnitude 9×10^9 N (about 1 million tons)! The total charge of all the electrons in a copper one-cent coin is even greater, about 1.4×10^5 C, which shows that we can't disturb electric neutrality very much without using enormous forces. More typical values of charge range from about 10^{-9} to about 10^{-6} C. The microcoulomb ($1 \mu\text{C} = 10^{-6}$ C) and the nanocoulomb ($1 \text{nC} = 10^{-9}$ C) are often used as practical units of charge.

Example 21.1 Electric force versus gravitational force

An α particle (the nucleus of a helium atom) has mass $m = 6.64 \times 10^{-27}$ kg and charge $q = +2e = 3.2 \times 10^{-19}$ C. Compare the magnitude of the electric repulsion between two α ("alpha") particles with that of the gravitational attraction between them.

SOLUTION

IDENTIFY and SET UP: This problem involves Newton's law for the gravitational force F_g between particles (see Section 13.1) and Coulomb's law for the electric force F_e between point charges. To compare these forces, we make our target variable the ratio F_e/F_g . We use Eq. (21.2) for F_e and Eq. (13.1) for F_g .

EXECUTE: Figure 21.11 shows our sketch. From Eqs. (21.2) and (13.1),

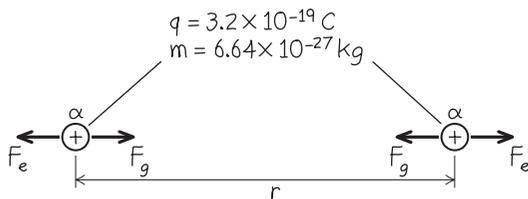
$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \quad F_g = G \frac{m^2}{r^2}$$

These are both inverse-square forces, so the r^2 factors cancel when we take the ratio:

$$\begin{aligned} \frac{F_e}{F_g} &= \frac{1}{4\pi\epsilon_0 G} \frac{q^2}{m^2} \\ &= \frac{9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} \frac{(3.2 \times 10^{-19} \text{ C})^2}{(6.64 \times 10^{-27} \text{ kg})^2} \\ &= 3.1 \times 10^{35} \end{aligned}$$

EVALUATE: This astonishingly large number shows that the gravitational force in this situation is completely negligible in comparison to the electric force. This is always true for interactions of atomic and subnuclear particles. But within objects the size of a person or a planet, the positive and negative charges are nearly equal in magnitude, and the net electric force is usually much *smaller* than the gravitational force.

21.11 Our sketch for this problem.



Superposition of Forces

Coulomb's law as we have stated it describes only the interaction of two *point* charges. Experiments show that when two charges exert forces simultaneously on a third charge, the total force acting on that charge is the *vector sum* of the forces that the two charges would exert individually. This important property, called the **principle of superposition of forces**, holds for any number of charges. By using this principle, we can apply Coulomb's law to *any* collection of charges. Two of the examples at the end of this section use the superposition principle.

Strictly speaking, Coulomb's law as we have stated it should be used only for point charges *in a vacuum*. If matter is present in the space between the charges, the net force acting on each charge is altered because charges are induced in the molecules of the intervening material. We will describe this effect later. As a practical matter, though, we can use Coulomb's law unaltered for point charges in air. At normal atmospheric pressure, the presence of air changes the electric force from its vacuum value by only about one part in 2000.

Problem-Solving Strategy 21.1 Coulomb's Law



IDENTIFY the relevant concepts: Coulomb's law describes the electric force between charged particles.

SET UP the problem using the following steps:

1. Sketch the locations of the charged particles and label each particle with its charge.
2. If the charges do not all lie on a single line, set up an xy -coordinate system.
3. The problem will ask you to find the electric force on one or more particles. Identify which these are.

EXECUTE the solution as follows:

1. For each particle that exerts an electric force on a given particle of interest, use Eq. (21.2) to calculate the magnitude of that force.
2. Using those magnitudes, sketch a free-body diagram showing the electric force vectors acting on each particle of interest. The force exerted by particle 1 on particle 2 points from particle 2 toward particle 1 if the charges have opposite signs, but points from particle 2 directly away from particle 1 if the charges have the same sign.
3. Use the principle of superposition to calculate the total electric force—a vector sum—on each particle of interest. (Review the

vector algebra in Sections 1.7 through 1.9. The method of components is often helpful.)

4. Use consistent units; SI units are completely consistent. With $1/4\pi\epsilon_0 = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$, distances must be in meters, charges in coulombs, and forces in newtons.
5. Some examples and problems in this and later chapters involve *continuous* distributions of charge along a line, over a surface, or throughout a volume. In these cases the vector sum in step 3 becomes a vector *integral*. We divide the charge distribution into infinitesimal pieces, use Coulomb's law for each piece, and integrate to find the vector sum. Sometimes this can be done without actual integration.
6. Exploit any symmetries in the charge distribution to simplify your problem solving. For example, two identical charges q exert zero net electric force on a charge Q midway between them, because the forces on Q have equal magnitude and opposite direction.

EVALUATE your answer: Check whether your numerical results are reasonable. Confirm that the direction of the net electric force agrees with the principle that charges of the same sign repel and charges of opposite sign attract.

Example 21.2 Force between two point charges

Two point charges, $q_1 = +25 \text{ nC}$ and $q_2 = -75 \text{ nC}$, are separated by a distance $r = 3.0 \text{ cm}$ (Fig. 21.12a). Find the magnitude and direction of the electric force (a) that q_1 exerts on q_2 and (b) that q_2 exerts on q_1 .

SOLUTION

IDENTIFY and SET UP: This problem asks for the electric forces that two charges exert on each other. We use Coulomb's law, Eq. (21.2), to calculate the magnitudes of the forces. The signs of the charges will determine the directions of the forces.

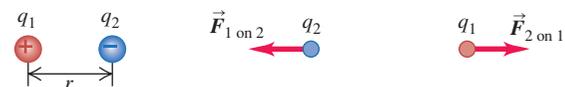
EXECUTE: (a) After converting the units of r to meters and the units of q_1 and q_2 to coulombs, Eq. (21.2) gives us

$$\begin{aligned} F_{1 \text{ on } 2} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} \\ &= (9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{|(+25 \times 10^{-9} \text{ C})(-75 \times 10^{-9} \text{ C})|}{(0.030 \text{ m})^2} \\ &= 0.019 \text{ N} \end{aligned}$$

The charges have opposite signs, so the force is attractive (to the left in Fig. 21.12b); that is, the force that acts on q_2 is directed toward q_1 along the line joining the two charges.

21.12 What force does q_1 exert on q_2 , and what force does q_2 exert on q_1 ? Gravitational forces are negligible.

- (a) The two charges (b) Free-body diagram for charge q_2 (c) Free-body diagram for charge q_1



- (b) Proceeding as in part (a), we have

$$F_{1 \text{ on } 2} = \frac{1}{4\pi\epsilon_0} \frac{|q_2 q_1|}{r^2} = F_{2 \text{ on } 1} = 0.019 \text{ N}$$

The attractive force that acts on q_1 is to the right, toward q_2 (Fig. 21.12c).

EVALUATE: Newton's third law applies to the electric force. Even though the charges have different magnitudes, the magnitude of the force that q_2 exerts on q_1 is the same as the magnitude of the force that q_1 exerts on q_2 , and these two forces are in opposite directions.

Example 21.3 Vector addition of electric forces on a line

Two point charges are located on the x -axis of a coordinate system: $q_1 = 1.0 \text{ nC}$ is at $x = +2.0 \text{ cm}$, and $q_2 = -3.0 \text{ nC}$ is at $x = +4.0 \text{ cm}$. What is the total electric force exerted by q_1 and q_2 on a charge $q_3 = 5.0 \text{ nC}$ at $x = 0$?

SOLUTION

IDENTIFY and SET UP: Figure 21.13a shows the situation. To find the total force on q_3 , our target variable, we find the vector sum of the two electric forces on it.

Continued

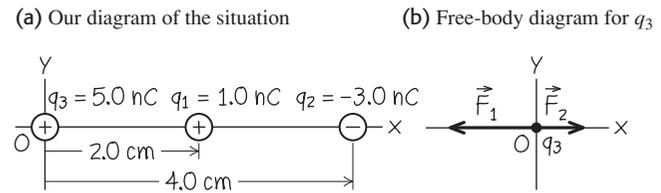
EXECUTE: Figure 21.13b is a free-body diagram for q_3 , which is repelled by q_1 (which has the same sign) and attracted to q_2 (which has the opposite sign): $\vec{F}_{1\text{ on }3}$ is in the $-x$ -direction and $\vec{F}_{2\text{ on }3}$ is in the $+x$ -direction. After unit conversions, we have from Eq. (21.2)

$$F_{1\text{ on }3} = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_3|}{r_{13}^2} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{(1.0 \times 10^{-9} \text{ C})(5.0 \times 10^{-9} \text{ C})}{(0.020 \text{ m})^2} = 1.12 \times 10^{-4} \text{ N} = 112 \mu\text{N}$$

In the same way you can show that $F_{2\text{ on }3} = 84 \mu\text{N}$. We thus have $\vec{F}_{1\text{ on }3} = (-112 \mu\text{N})\hat{i}$ and $\vec{F}_{2\text{ on }3} = (84 \mu\text{N})\hat{i}$. The net force on q_3 is

$$\vec{F}_3 = \vec{F}_{1\text{ on }3} + \vec{F}_{2\text{ on }3} = (-112 \mu\text{N})\hat{i} + (84 \mu\text{N})\hat{i} = (-28 \mu\text{N})\hat{i}$$

21.13 Our sketches for this problem.



EVALUATE: As a check, note that the magnitude of q_2 is three times that of q_1 , but q_2 is twice as far from q_3 as q_1 . Equation (21.2) then says that $F_{2\text{ on }3}$ must be $3/2^2 = 3/4 = 0.75$ as large as $F_{1\text{ on }3}$. This agrees with our calculated values: $F_{2\text{ on }3}/F_{1\text{ on }3} = (84 \mu\text{N})/(112 \mu\text{N}) = 0.75$. Because $F_{2\text{ on }3}$ is the weaker force, the direction of the net force is that of $\vec{F}_{1\text{ on }3}$ —that is, in the negative x -direction.

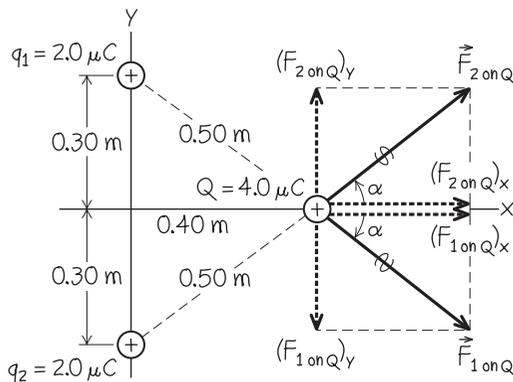
Example 21.4 Vector addition of electric forces in a plane

Two equal positive charges $q_1 = q_2 = 2.0 \mu\text{C}$ are located at $x = 0, y = 0.30 \text{ m}$ and $x = 0, y = -0.30 \text{ m}$, respectively. What are the magnitude and direction of the total electric force that q_1 and q_2 exert on a third charge $Q = 4.0 \mu\text{C}$ at $x = 0.40 \text{ m}, y = 0$?

SOLUTION

IDENTIFY and SET UP: As in Example 21.3, we must compute the force that each charge exerts on Q and then find the vector sum of those forces. Figure 21.14 shows the situation. Since the three charges do not all lie on a line, the best way to calculate the forces is to use components.

21.14 Our sketch for this problem.



EXECUTE: Figure 21.14 shows the forces $\vec{F}_{1\text{ on }Q}$ and $\vec{F}_{2\text{ on }Q}$ due to the identical charges q_1 and q_2 , which are at equal distances from Q . From Coulomb's law, *both* forces have magnitude

$$F_{1\text{ or }2\text{ on }Q} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \times \frac{(4.0 \times 10^{-6} \text{ C})(2.0 \times 10^{-6} \text{ C})}{(0.50 \text{ m})^2} = 0.29 \text{ N}$$

The x -components of the two forces are equal:

$$(F_{1\text{ or }2\text{ on }Q})_x = (F_{1\text{ or }2\text{ on }Q})\cos\alpha = (0.29 \text{ N})\frac{0.40 \text{ m}}{0.50 \text{ m}} = 0.23 \text{ N}$$

From symmetry we see that the y -components of the two forces are equal and opposite. Hence their sum is zero and the total force \vec{F} on Q has only an x -component $F_x = 0.23 \text{ N} + 0.23 \text{ N} = 0.46 \text{ N}$. The total force on Q is in the $+x$ -direction, with magnitude 0.46 N.

EVALUATE: The total force on Q points neither directly away from q_1 nor directly away from q_2 . Rather, this direction is a compromise that points away from the *system* of charges q_1 and q_2 . Can you see that the total force would *not* be in the $+x$ -direction if q_1 and q_2 were not equal or if the geometrical arrangement of the charges were not so symmetric?

Test Your Understanding of Section 21.3 Suppose that charge q_2 in Example 21.4 were $-2.0 \mu\text{C}$. In this case, the total electric force on Q would be (i) in the positive x -direction; (ii) in the negative x -direction; (iii) in the positive y -direction; (iv) in the negative y -direction; (v) zero; (vi) none of these.



21.4 Electric Field and Electric Forces

When two electrically charged particles in empty space interact, how does each one know the other is there? We can begin to answer this question, and at the same time reformulate Coulomb's law in a very useful way, by using the concept of *electric field*.

Electric Field

To introduce this concept, let's look at the mutual repulsion of two positively charged bodies A and B (Fig. 21.15a). Suppose B has charge q_0 , and let \vec{F}_0 be the electric force of A on B . One way to think about this force is as an “action-at-a-distance” force—that is, as a force that acts across empty space without needing any matter (such as a push rod or a rope) to transmit it through the intervening space. (Gravity can also be thought of as an “action-at-a-distance” force.) But a more fruitful way to visualize the repulsion between A and B is as a two-stage process. We first envision that body A , as a result of the charge that it carries, somehow *modifies the properties of the space around it*. Then body B , as a result of the charge that it carries, senses how space has been modified at its position. The response of body B is to experience the force \vec{F}_0 .

To elaborate how this two-stage process occurs, we first consider body A by itself: We remove body B and label its former position as point P (Fig. 21.15b). We say that the charged body A produces or causes an **electric field** at point P (and at all other points in the neighborhood). This electric field is present at P even if there is no charge at P ; it is a consequence of the charge on body A only. If a point charge q_0 is then placed at point P , it experiences the force \vec{F}_0 . We take the point of view that this force is exerted on q_0 by the field at P (Fig. 21.15c). Thus the electric field is the intermediary through which A communicates its presence to q_0 . Because the point charge q_0 would experience a force at *any* point in the neighborhood of A , the electric field that A produces exists at all points in the region around A .

We can likewise say that the point charge q_0 produces an electric field in the space around it and that this electric field exerts the force $-\vec{F}_0$ on body A . For each force (the force of A on q_0 and the force of q_0 on A), one charge sets up an electric field that exerts a force on the second charge. We emphasize that this is an *interaction* between *two* charged bodies. A single charge produces an electric field in the surrounding space, but this electric field cannot exert a net force on the charge that created it; as we discussed in Section 4.3, a body cannot exert a net force on itself. (If this wasn't true, you would be able to lift yourself to the ceiling by pulling up on your belt!)

The electric force on a charged body is exerted by the electric field created by other charged bodies.

To find out experimentally whether there is an electric field at a particular point, we place a small charged body, which we call a **test charge**, at the point (Fig. 21.15c). If the test charge experiences an electric force, then there is an electric field at that point. This field is produced by charges other than q_0 .

Force is a vector quantity, so electric field is also a vector quantity. (Note the use of vector signs as well as boldface letters and plus, minus, and equals signs in the following discussion.) We define the *electric field* \vec{E} at a point as the electric force \vec{F}_0 experienced by a test charge q_0 at the point, divided by the charge q_0 . That is, the electric field at a certain point is equal to the *electric force per unit charge* experienced by a charge at that point:

$$\vec{E} = \frac{\vec{F}_0}{q_0} \quad (\text{definition of electric field as electric force per unit charge}) \quad (21.3)$$

In SI units, in which the unit of force is 1 N and the unit of charge is 1 C, the unit of electric field magnitude is 1 newton per coulomb (1 N/C).

If the field \vec{E} at a certain point is known, rearranging Eq. (21.3) gives the force \vec{F}_0 experienced by a point charge q_0 placed at that point. This force is just equal to the electric field \vec{E} produced at that point by charges other than q_0 , multiplied by the charge q_0 :

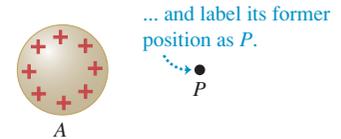
$$\vec{F}_0 = q_0 \vec{E} \quad (\text{force exerted on a point charge } q_0 \text{ by an electric field } \vec{E}) \quad (21.4)$$

21.15 A charged body creates an electric field in the space around it.

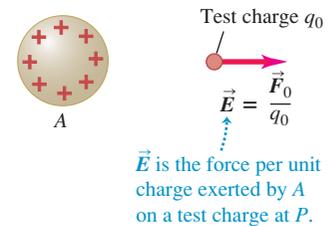
(a) A and B exert electric forces on each other.



(b) Remove body B ...



(c) Body A sets up an electric field \vec{E} at point P .

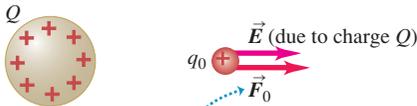


Application Sharks and the “Sixth Sense”

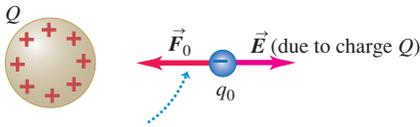
Sharks have the ability to locate prey (such as flounder and other bottom-dwelling fish) that are completely hidden beneath the sand at the bottom of the ocean. They do this by sensing the weak electric fields produced by muscle contractions in their prey. Sharks derive their sensitivity to electric fields (a “sixth sense”) from jelly-filled canals in their bodies. These canals end in pores on the shark's skin (shown in this photograph). An electric field as weak as 5×10^{-7} N/C causes charge flow within the canals and triggers a signal in the shark's nervous system. Because the shark has canals with different orientations, it can measure different components of the electric-field vector and hence determine the direction of the field.



21.16 The force $\vec{F}_0 = q_0 \vec{E}$ exerted on a point charge q_0 placed in an electric field \vec{E} .



The force on a positive test charge q_0 points in the direction of the electric field.



The force on a negative test charge q_0 points opposite to the electric field.

The charge q_0 can be either positive or negative. If q_0 is *positive*, the force \vec{F}_0 experienced by the charge is the same direction as \vec{E} ; if q_0 is *negative*, \vec{F}_0 and \vec{E} are in opposite directions (Fig. 21.16).

While the electric field concept may be new to you, the basic idea—that one body sets up a field in the space around it and a second body responds to that field—is one that you’ve actually used before. Compare Eq. (21.4) to the familiar expression for the gravitational force \vec{F}_g that the earth exerts on a mass m_0 :

$$\vec{F}_g = m_0 \vec{g} \quad (21.5)$$

In this expression, \vec{g} is the acceleration due to gravity. If we divide both sides of Eq. (21.5) by the mass m_0 , we obtain

$$\vec{g} = \frac{\vec{F}_g}{m_0}$$

Thus \vec{g} can be regarded as the gravitational force per unit mass. By analogy to Eq. (21.3), we can interpret \vec{g} as the *gravitational field*. Thus we treat the gravitational interaction between the earth and the mass m_0 as a two-stage process: The earth sets up a gravitational field \vec{g} in the space around it, and this gravitational field exerts a force given by Eq. (21.5) on the mass m_0 (which we can regard as a *test mass*). The gravitational field \vec{g} , or gravitational force per unit mass, is a useful concept because it does not depend on the mass of the body on which the gravitational force is exerted; likewise, the electric field \vec{E} , or electric force per unit charge, is useful because it does not depend on the charge of the body on which the electric force is exerted.

CAUTION $\vec{F}_0 = q_0 \vec{E}_0$ is for *point test charges only* The electric force experienced by a test charge q_0 can vary from point to point, so the electric field can also be different at different points. For this reason, Eq. (21.4) can be used only to find the electric force on a *point* charge. If a charged body is large enough in size, the electric field \vec{E} may be noticeably different in magnitude and direction at different points on the body, and calculating the net electric force on the body can become rather complicated. **I**

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ActivPhysics 11.4: Electric Field: Point Charge

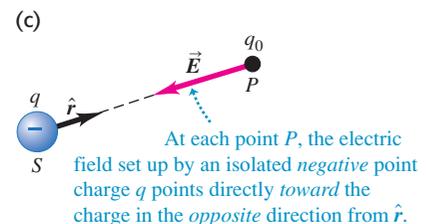
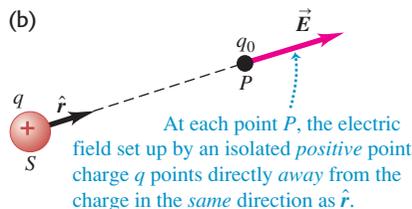
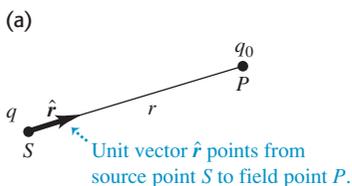
ActivPhysics 11.9: Motion of a Charge in an Electric Field: Introduction

ActivPhysics 11.10: Motion in an Electric Field: Problems

Electric Field of a Point Charge

If the source distribution is a point charge q , it is easy to find the electric field that it produces. We call the location of the charge the **source point**, and we call the point P where we are determining the field the **field point**. It is also useful to introduce a *unit vector* \hat{r} that points along the line from source point to field point (Fig. 21.17a). This unit vector is equal to the displacement vector \vec{r} from the source point to the field point, divided by the distance $r = |\vec{r}|$ between these two points; that is, $\hat{r} = \vec{r}/r$. If we place a small test charge q_0 at the field point P , at a

21.17 The electric field \vec{E} produced at point P by an isolated point charge q at S . Note that in both (b) and (c), \vec{E} is *produced* by q [see Eq. (21.7)] but *acts* on the charge q_0 at point P [see Eq. (21.4)].



distance r from the source point, the magnitude F_0 of the force is given by Coulomb's law, Eq. (21.2):

$$F_0 = \frac{1}{4\pi\epsilon_0} \frac{|qq_0|}{r^2}$$

From Eq. (21.3) the magnitude E of the electric field at P is

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} \quad (\text{magnitude of electric field of a point charge}) \quad (21.6)$$

Using the unit vector \hat{r} , we can write a *vector* equation that gives both the magnitude and direction of the electric field \vec{E} :

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \quad (\text{electric field of a point charge}) \quad (21.7)$$

By definition, the electric field of a point charge always points *away from* a positive charge (that is, in the same direction as \hat{r} ; see Fig. 21.17b) but *toward* a negative charge (that is, in the direction opposite \hat{r} ; see Fig. 21.17c).

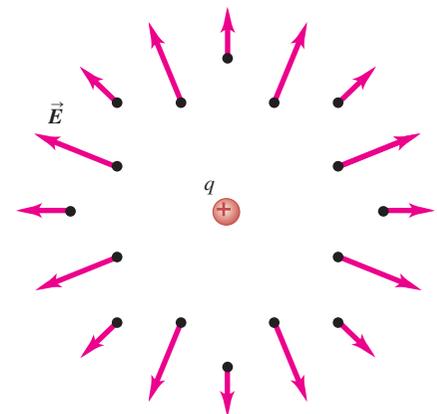
We have emphasized calculating the electric field \vec{E} at a certain point. But since \vec{E} can vary from point to point, it is not a single vector quantity but rather an *infinite* set of vector quantities, one associated with each point in space. This is an example of a **vector field**. Figure 21.18 shows a number of the field vectors produced by a positive or negative point charge. If we use a rectangular (x, y, z) coordinate system, each component of \vec{E} at any point is in general a function of the coordinates (x, y, z) of the point. We can represent the functions as $E_x(x, y, z)$, $E_y(x, y, z)$, and $E_z(x, y, z)$. Vector fields are an important part of the language of physics, not just in electricity and magnetism. One everyday example of a vector field is the velocity \vec{v} of wind currents; the magnitude and direction of \vec{v} , and hence its vector components, vary from point to point in the atmosphere.

In some situations the magnitude and direction of the field (and hence its vector components) have the same values everywhere throughout a certain region; we then say that the field is *uniform* in this region. An important example of this is the electric field inside a *conductor*. If there is an electric field within a conductor, the field exerts a force on every charge in the conductor, giving the free charges a net motion. By definition an electrostatic situation is one in which the charges have *no* net motion. We conclude that *in electrostatics the electric field at every point within the material of a conductor must be zero*. (Note that we are not saying that the field is necessarily zero in a *hole* inside a conductor.)

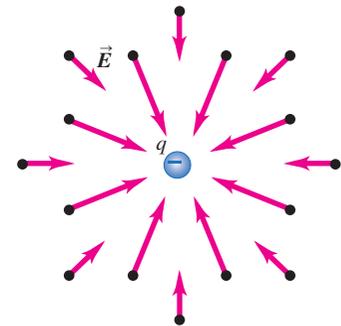
In summary, our description of electric interactions has two parts. First, a given charge distribution acts as a source of electric field. Second, the electric field exerts a force on any charge that is present in the field. Our analysis often has two corresponding steps: first, calculating the field caused by a source charge distribution; second, looking at the effect of the field in terms of force and motion. The second step often involves Newton's laws as well as the principles of electric interactions. In the next section we show how to calculate fields caused by various source distributions, but first here are three examples of calculating the field due to a point charge and of finding the force on a charge due to a given field \vec{E} .

21.18 A point charge q produces an electric field \vec{E} at *all* points in space. The field strength decreases with increasing distance.

(a) The field produced by a positive point charge points *away from* the charge.



(b) The field produced by a negative point charge points *toward* the charge.



Example 21.5 Electric-field magnitude for a point charge

What is the magnitude of the electric field \vec{E} at a field point 2.0 m from a point charge $q = 4.0$ nC?

SOLUTION

IDENTIFY and SET UP: This problem concerns the electric field due to a point charge. We are given the magnitude of the charge

Continued

and the distance from the charge to the field point, so we use Eq. (21.6) to calculate the field magnitude E .

EXECUTE: From Eq. (21.6),

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{4.0 \times 10^{-9} \text{ C}}{(2.0 \text{ m})^2} = 9.0 \text{ N/C}$$

EVALUATE: Our result $E = 9.0 \text{ N/C}$ means that if we placed a 1.0-C charge at a point 2.0 m from q , it would experience a 9.0-N force. The force on a 2.0-C charge at that point would be $(2.0 \text{ C})(9.0 \text{ N/C}) = 18 \text{ N}$, and so on.

Example 21.6 Electric-field vector for a point charge

A point charge $q = -8.0 \text{ nC}$ is located at the origin. Find the electric-field vector at the field point $x = 1.2 \text{ m}$, $y = -1.6 \text{ m}$.

SOLUTION

IDENTIFY and SET UP: We must find the electric-field vector \vec{E} due to a point charge. Figure 21.19 shows the situation. We use Eq. (21.7); to do this, we must find the distance r from the source point S (the position of the charge q , which in this example is at the ori-

gin O) to the field point P , and we must obtain an expression for the unit vector $\hat{r} = \vec{r}/r$ that points from S to P .

EXECUTE: The distance from S to P is

$$r = \sqrt{x^2 + y^2} = \sqrt{(1.2 \text{ m})^2 + (-1.6 \text{ m})^2} = 2.0 \text{ m}$$

The unit vector \hat{r} is then

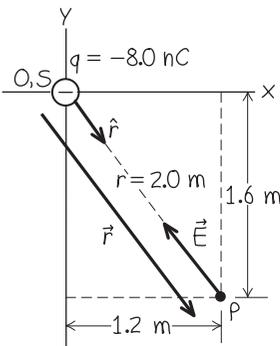
$$\begin{aligned} \hat{r} &= \frac{\vec{r}}{r} = \frac{x\hat{i} + y\hat{j}}{r} \\ &= \frac{(1.2 \text{ m})\hat{i} + (-1.6 \text{ m})\hat{j}}{2.0 \text{ m}} = 0.60\hat{i} - 0.80\hat{j} \end{aligned}$$

Then, from Eq. (21.7),

$$\begin{aligned} \vec{E} &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \\ &= (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{(-8.0 \times 10^{-9} \text{ C})}{(2.0 \text{ m})^2} (0.60\hat{i} - 0.80\hat{j}) \\ &= (-11 \text{ N/C})\hat{i} + (14 \text{ N/C})\hat{j} \end{aligned}$$

EVALUATE: Since q is negative, \vec{E} points from the field point to the charge (the source point), in the direction opposite to \hat{r} (compare Fig. 21.17c). We leave the calculation of the magnitude and direction of \vec{E} to you (see Exercise 21.36).

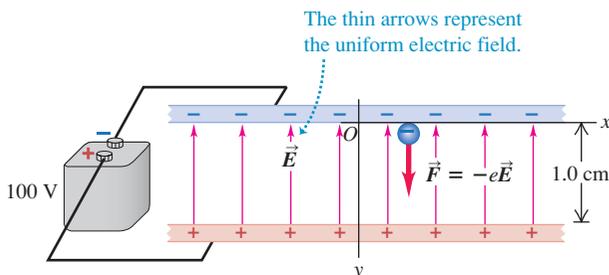
21.19 Our sketch for this problem.



Example 21.7 Electron in a uniform field

When the terminals of a battery are connected to two parallel conducting plates with a small gap between them, the resulting charges on the plates produce a nearly uniform electric field \vec{E} between the plates. (In the next section we'll see why this is.) If the plates are 1.0 cm apart and are connected to a 100-volt battery as shown in Fig. 21.20, the field is vertically upward and has magnitude

21.20 A uniform electric field between two parallel conducting plates connected to a 100-volt battery. (The separation of the plates is exaggerated in this figure relative to the dimensions of the plates.)



$E = 1.00 \times 10^4 \text{ N/C}$. (a) If an electron (charge $-e = -1.60 \times 10^{-19} \text{ C}$, mass $m = 9.11 \times 10^{-31} \text{ kg}$) is released from rest at the upper plate, what is its acceleration? (b) What speed and kinetic energy does it acquire while traveling 1.0 cm to the lower plate? (c) How long does it take to travel this distance?

SOLUTION

IDENTIFY and SET UP: This example involves the relationship between electric field and electric force. It also involves the relationship between force and acceleration, the definition of kinetic energy, and the kinematic relationships among acceleration, distance, velocity, and time. Figure 21.20 shows our coordinate system. We are given the electric field, so we use Eq. (21.4) to find the force on the electron and Newton's second law to find its acceleration. Because the field is uniform, the force is constant and we can use the constant-acceleration formulas from Chapter 2 to find the electron's velocity and travel time. We find the kinetic energy using $K = \frac{1}{2}mv^2$.

EXECUTE: (a) Although \vec{E} is upward (in the $+y$ -direction), \vec{F} is downward (because the electron's charge is negative) and so F_y is negative. Because F_y is constant, the electron's acceleration is constant:

$$a_y = \frac{F_y}{m} = \frac{-eE}{m} = \frac{(-1.60 \times 10^{-19} \text{ C})(1.00 \times 10^4 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} \\ = -1.76 \times 10^{15} \text{ m/s}^2$$

(b) The electron starts from rest, so its motion is in the y -direction only (the direction of the acceleration). We can find the electron's speed at any position y using the constant-acceleration Eq. (2.13), $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$. We have $v_{0y} = 0$ and $y_0 = 0$, so at $y = -1.0 \text{ cm} = -1.0 \times 10^{-2} \text{ m}$ we have

$$|v_y| = \sqrt{2a_y y} = \sqrt{2(-1.76 \times 10^{15} \text{ m/s}^2)(-1.0 \times 10^{-2} \text{ m})} \\ = 5.9 \times 10^6 \text{ m/s}$$

The velocity is downward, so $v_y = -5.9 \times 10^6 \text{ m/s}$. The electron's kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(5.9 \times 10^6 \text{ m/s})^2 \\ = 1.6 \times 10^{-17} \text{ J}$$

(c) From Eq. (2.8) for constant acceleration, $v_y = v_{0y} + a_y t$,

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{(-5.9 \times 10^6 \text{ m/s}) - (0 \text{ m/s})}{-1.76 \times 10^{15} \text{ m/s}^2} \\ = 3.4 \times 10^{-9} \text{ s}$$

EVALUATE: Our results show that in problems concerning subatomic particles such as electrons, many quantities—including acceleration, speed, kinetic energy, and time—will have *very* different values from those typical of everyday objects such as baseballs and automobiles.

Test Your Understanding of Section 21.4 (a) A negative point charge moves along a straight-line path directly toward a stationary positive point charge. Which aspect(s) of the electric force on the negative point charge will remain constant as it moves? (i) magnitude; (ii) direction; (iii) both magnitude and direction; (iv) neither magnitude nor direction. (b) A negative point charge moves along a circular orbit around a positive point charge. Which aspect(s) of the electric force on the negative point charge will remain constant as it moves? (i) magnitude; (ii) direction; (iii) both magnitude and direction; (iv) neither magnitude nor direction. 

21.5 Electric-Field Calculations

Equation (21.7) gives the electric field caused by a single point charge. But in most realistic situations that involve electric fields and forces, we encounter charge that is *distributed* over space. The charged plastic and glass rods in Fig. 21.1 have electric charge distributed over their surfaces, as does the imaging drum of a laser printer (Fig. 21.2). In this section we'll learn to calculate electric fields caused by various distributions of electric charge. Calculations of this kind are of tremendous importance for technological applications of electric forces. To determine the trajectories of atomic nuclei in an accelerator for cancer radiotherapy or of charged particles in a semiconductor electronic device, you have to know the detailed nature of the electric field acting on the charges.

The Superposition of Electric Fields

To find the field caused by a charge distribution, we imagine the distribution to be made up of many point charges q_1, q_2, q_3, \dots . (This is actually quite a realistic description, since we have seen that charge is carried by electrons and protons that are so small as to be almost pointlike.) At any given point P , each point charge produces its own electric field $\vec{E}_1, \vec{E}_2, \vec{E}_3, \dots$, so a test charge q_0 placed at P experiences a force $\vec{F}_1 = q_0\vec{E}_1$ from charge q_1 , a force $\vec{F}_2 = q_0\vec{E}_2$ from charge q_2 , and so on. From the principle of superposition of forces discussed in Section 21.3, the *total* force \vec{F}_0 that the charge distribution exerts on q_0 is the vector sum of these individual forces:

$$\vec{F}_0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots = q_0\vec{E}_1 + q_0\vec{E}_2 + q_0\vec{E}_3 + \dots$$

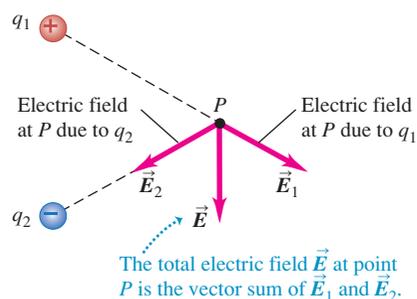
The combined effect of all the charges in the distribution is described by the *total* electric field \vec{E} at point P . From the definition of electric field, Eq. (21.3), this is

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$

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ActivPhysics 11.5: Electric Field Due to a Dipole
ActivPhysics 11.6: Electric Field: Problems

21.21 Illustrating the principle of superposition of electric fields.



The total electric field at P is the vector sum of the fields at P due to each point charge in the charge distribution (Fig. 21.21). This is the **principle of superposition of electric fields**.

When charge is distributed along a line, over a surface, or through a volume, a few additional terms are useful. For a line charge distribution (such as a long, thin, charged plastic rod), we use λ (the Greek letter lambda) to represent the **linear charge density** (charge per unit length, measured in C/m). When charge is distributed over a surface (such as the surface of the imaging drum of a laser printer), we use σ (sigma) to represent the **surface charge density** (charge per unit area, measured in C/m²). And when charge is distributed through a volume, we use ρ (rho) to represent the **volume charge density** (charge per unit volume, C/m³).

Some of the calculations in the following examples may look fairly intricate. After you've worked through the examples one step at a time, the process will seem less formidable. We will use many of the calculational techniques in these examples in Chapter 28 to calculate the *magnetic* fields caused by charges in motion.

Problem-Solving Strategy 21.2 Electric-Field Calculations



IDENTIFY the relevant concepts: Use the principle of superposition to calculate the electric field due to a discrete or continuous charge distribution.

SET UP the problem using the following steps:

1. Make a drawing showing the locations of the charges and your choice of coordinate axes.
2. On your drawing, indicate the position of the *field point* P (the point at which you want to calculate the electric field \vec{E}).

EXECUTE the solution as follows:

1. Use consistent units. Distances must be in meters and charge must be in coulombs. If you are given centimeters or nanocoulombs, don't forget to convert.
2. Distinguish between the source point S and the field point P . The field produced by a point charge always points from S to P if the charge is positive, and from P to S if the charge is negative.
3. Use *vector* addition when applying the principle of superposition; review the treatment of vector addition in Chapter 1 if necessary.

4. Simplify your calculations by exploiting any symmetries in the charge distribution.
5. If the charge distribution is continuous, define a small element of charge that can be considered as a point, find its electric field at P , and find a way to add the fields of all the charge elements by doing an integral. Usually it is easiest to do this for each component of \vec{E} separately, so you may need to evaluate more than one integral. Ensure that the limits on your integrals are correct; especially when the situation has symmetry, don't count a charge twice.

EVALUATE your answer: Check that the direction of \vec{E} is reasonable. If your result for the electric-field magnitude E is a function of position (say, the coordinate x), check your result in any limits for which you know what the magnitude should be. When possible, check your answer by calculating it in a different way.

Example 21.8 Field of an electric dipole

Point charges $q_1 = +12$ nC and $q_2 = -12$ nC are 0.100 m apart (Fig. 21.22). (Such pairs of point charges with equal magnitude and opposite sign are called *electric dipoles*.) Compute the electric field caused by q_1 , the field caused by q_2 , and the total field (a) at point a ; (b) at point b ; and (c) at point c .

SOLUTION

IDENTIFY and SET UP: We must find the total electric field at various points due to two point charges. We use the principle of superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2$. Figure 21.22 shows the coordinate system and the locations of the field points a , b , and c .

EXECUTE: At each field point, \vec{E} depends on \vec{E}_1 and \vec{E}_2 there; we first calculate the magnitudes E_1 and E_2 at each field point. At a the magnitude of the field \vec{E}_{1a} caused by q_1 is

$$\begin{aligned} E_{1a} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r^2} = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{12 \times 10^{-9} \text{ C}}{(0.060 \text{ m})^2} \\ &= 3.0 \times 10^4 \text{ N/C} \end{aligned}$$

We calculate the other field magnitudes in a similar way. The results are

$$E_{1a} = 3.0 \times 10^4 \text{ N/C} \quad E_{1b} = 6.8 \times 10^4 \text{ N/C}$$

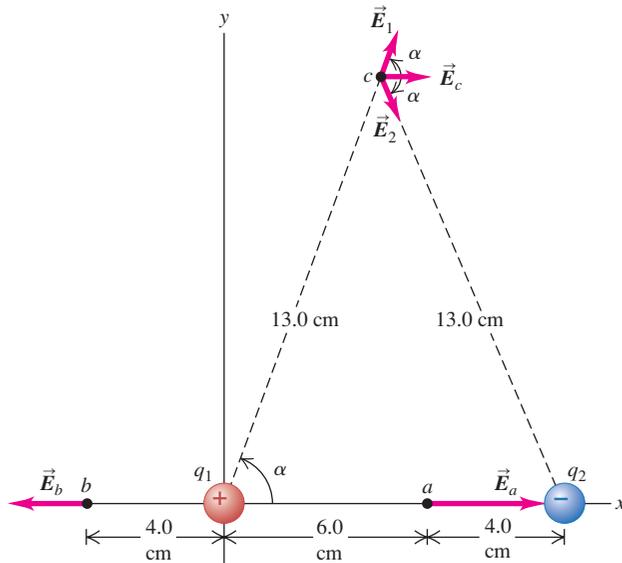
$$E_{1c} = 6.39 \times 10^3 \text{ N/C}$$

$$E_{2a} = 6.8 \times 10^4 \text{ N/C} \quad E_{2b} = 0.55 \times 10^4 \text{ N/C}$$

$$E_{2c} = E_{1c} = 6.39 \times 10^3 \text{ N/C}$$

The *directions* of the corresponding fields are in all cases *away* from the positive charge q_1 and *toward* the negative charge q_2 .

21.22 Electric field at three points, a , b , and c , set up by charges q_1 and q_2 , which form an electric dipole.



(a) At a , \vec{E}_{1a} and \vec{E}_{2a} are both directed to the right, so

$$\vec{E}_a = E_{1a}\hat{i} + E_{2a}\hat{i} = (9.8 \times 10^4 \text{ N/C})\hat{i}$$

(b) At b , \vec{E}_{1b} is directed to the left and \vec{E}_{2b} is directed to the right, so

$$\vec{E}_b = -E_{1b}\hat{i} + E_{2b}\hat{i} = (-6.2 \times 10^4 \text{ N/C})\hat{i}$$

(c) Figure 21.22 shows the directions of \vec{E}_1 and \vec{E}_2 at c . Both vectors have the same x -component:

$$\begin{aligned} E_{1cx} = E_{2cx} &= E_{1c}\cos\alpha = (6.39 \times 10^3 \text{ N/C})\left(\frac{5}{13}\right) \\ &= 2.46 \times 10^3 \text{ N/C} \end{aligned}$$

From symmetry, E_{1y} and E_{2y} are equal and opposite, so their sum is zero. Hence

$$\vec{E}_c = 2(2.46 \times 10^3 \text{ N/C})\hat{i} = (4.9 \times 10^3 \text{ N/C})\hat{i}$$

EVALUATE: We can also find \vec{E}_c using Eq. (21.7) for the field of a point charge. The displacement vector \vec{r}_1 from q_1 to point c is $\vec{r}_1 = r\cos\alpha\hat{i} + r\sin\alpha\hat{j}$. Hence the unit vector that points from q_1 to point c is $\hat{r}_1 = \vec{r}_1/r = \cos\alpha\hat{i} + \sin\alpha\hat{j}$. By symmetry, the unit vector that points from q_2 to point c has the opposite x -component but the same y -component: $\hat{r}_2 = -\cos\alpha\hat{i} + \sin\alpha\hat{j}$. We can now use Eq. (21.7) to write the fields \vec{E}_{1c} and \vec{E}_{2c} at c in vector form, then find their sum. Since $q_2 = -q_1$ and the distance r to c is the same for both charges,

$$\begin{aligned} \vec{E}_c &= \vec{E}_{1c} + \vec{E}_{2c} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \hat{r}_1 + \frac{1}{4\pi\epsilon_0} \frac{q_2}{r^2} \hat{r}_2 \\ &= \frac{1}{4\pi\epsilon_0 r^2} (q_1 \hat{r}_1 + q_2 \hat{r}_2) = \frac{q_1}{4\pi\epsilon_0 r^2} (\hat{r}_1 - \hat{r}_2) \\ &= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} (2\cos\alpha\hat{i}) \\ &= 2(9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{12 \times 10^{-9} \text{ C}}{(0.13 \text{ m})^2} \left(\frac{5}{13}\right)\hat{i} \\ &= (4.9 \times 10^3 \text{ N/C})\hat{i} \end{aligned}$$

This is the same as we calculated in part (c).

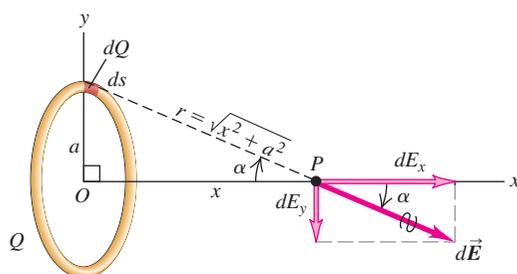
Example 21.9 Field of a ring of charge

Charge Q is uniformly distributed around a conducting ring of radius a (Fig. 21.23). Find the electric field at a point P on the ring axis at a distance x from its center.

SOLUTION

IDENTIFY and SET UP: This is a problem in the superposition of electric fields. Each bit of charge around the ring produces an electric field at an arbitrary point on the x -axis; our target variable is the total field at this point due to all such bits of charge.

21.23 Calculating the electric field on the axis of a ring of charge. In this figure, the charge is assumed to be positive.



EXECUTE: We divide the ring into infinitesimal segments ds as shown in Fig. 21.23. In terms of the linear charge density $\lambda = Q/2\pi a$, the charge in a segment of length ds is $dQ = \lambda ds$. Consider two identical segments, one as shown in the figure at $y = a$ and another halfway around the ring at $y = -a$. From Example 21.4, we see that the net force $d\vec{F}$ they exert on a point test charge at P , and thus their net field $d\vec{E}$, are directed along the x -axis. The same is true for any such pair of segments around the ring, so the *net* field at P is along the x -axis: $\vec{E} = E_x\hat{i}$.

To calculate E_x , note that the square of the distance r from a single ring segment to the point P is $r^2 = x^2 + a^2$. Hence the magnitude of this segment's contribution $d\vec{E}$ to the electric field at P is

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dQ}{x^2 + a^2}$$

The x -component of this field is $dE_x = dE\cos\alpha$. We know $dQ = \lambda ds$ and Fig. 21.23 shows that $\cos\alpha = x/r = x/(x^2 + a^2)^{1/2}$, so

$$\begin{aligned} dE_x &= dE\cos\alpha = \frac{1}{4\pi\epsilon_0} \frac{dQ}{x^2 + a^2} \frac{x}{\sqrt{x^2 + a^2}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{(x^2 + a^2)^{3/2}} ds \end{aligned}$$

Continued

To find E_x we integrate this expression over the entire ring—that is, for s from 0 to $2\pi a$ (the circumference of the ring). The integrand has the same value for all points on the ring, so it can be taken outside the integral. Hence we get

$$\begin{aligned} E_x &= \int dE_x = \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{(x^2 + a^2)^{3/2}} \int_0^{2\pi a} ds \\ &= \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{(x^2 + a^2)^{3/2}} (2\pi a) \\ \vec{E} &= E_x \hat{i} = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}} \hat{i} \end{aligned} \quad (21.8)$$

EVALUATE: Equation (21.8) shows that $\vec{E} = \mathbf{0}$ at the center of the ring ($x = 0$). This makes sense; charges on opposite sides of the ring push in opposite directions on a test charge at the center, and the vector sum of each such pair of forces is zero. When the field point P is much farther from the ring than the ring's radius, we have $x \gg a$ and the denominator in Eq. (21.8) becomes approximately equal to x^3 . In this limit the electric field at P is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{x^2} \hat{i}$$

That is, when the ring is so far away that its radius is negligible in comparison to the distance x , its field is the same as that of a point charge.

Example 21.10 Field of a charged line segment

Positive charge Q is distributed uniformly along the y -axis between $y = -a$ and $y = +a$. Find the electric field at point P on the x -axis at a distance x from the origin.

SOLUTION

IDENTIFY and SET UP: Figure 21.24 shows the situation. As in Example 21.9, we must find the electric field due to a continuous distribution of charge. Our target variable is an expression for the electric field at P as a function of x . The x -axis is a perpendicular bisector of the segment, so we can use a symmetry argument.

EXECUTE: We divide the line charge of length $2a$ into infinitesimal segments of length dy . The linear charge density is $\lambda = Q/2a$, and the charge in a segment is $dQ = \lambda dy = (Q/2a)dy$. The distance r from a segment at height y to the field point P is $r = (x^2 + y^2)^{1/2}$, so the magnitude of the field at P due to the segment at height y is

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} \frac{dy}{(x^2 + y^2)}$$

Figure 21.24 shows that the x - and y -components of this field are $dE_x = dE \cos \alpha$ and $dE_y = -dE \sin \alpha$, where $\cos \alpha = x/r$ and $\sin \alpha = y/r$. Hence

$$\begin{aligned} dE_x &= \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} \frac{xy}{(x^2 + y^2)^{3/2}} \\ dE_y &= \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} \frac{ydy}{(x^2 + y^2)^{3/2}} \end{aligned}$$

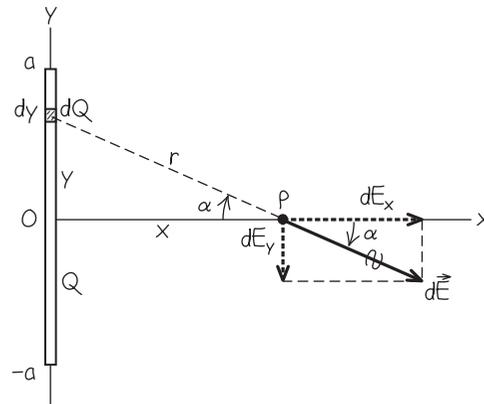
To find the total field at P , we must sum the fields from all segments along the line—that is, we must integrate from $y = -a$ to $y = +a$. You should work out the details of the integration (a table of integrals will help). The results are

$$\begin{aligned} E_x &= \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} \int_{-a}^{+a} \frac{xy}{(x^2 + y^2)^{3/2}} dy = \frac{Q}{4\pi\epsilon_0} \frac{1}{x\sqrt{x^2 + a^2}} \\ E_y &= \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} \int_{-a}^{+a} \frac{ydy}{(x^2 + y^2)^{3/2}} = 0 \end{aligned}$$

or, in vector form,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2 + a^2}} \hat{i} \quad (21.9)$$

21.24 Our sketch for this problem.



\vec{E} points away from the line of charge if λ is positive and toward the line of charge if λ is negative.

EVALUATE: Using a symmetry argument as in Example 21.9, we could have guessed that E_y would be zero; if we place a positive test charge at P , the upper half of the line of charge pushes downward on it, and the lower half pushes up with equal magnitude. Symmetry also tells us that the upper and lower halves of the segment contribute equally to the total field at P .

If the segment is very *short* (or the field point is very far from the segment) so that $x \gg a$, we can neglect a in the denominator of Eq. (21.9). Then the field becomes that of a point charge, just as in Example 21.9:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{x^2} \hat{i}$$

To see what happens if the segment is very *long* (or the field point is very close to it) so that $a \gg x$, we first rewrite Eq. (21.9) slightly:

$$\vec{E} = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{x\sqrt{(x^2/a^2) + 1}} \hat{i} \quad (21.10)$$

In the limit $a \gg x$ we can neglect x^2/a^2 in the denominator of Eq. (21.10), so

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 x} \hat{i}$$

This is the field of an *infinitely long* line of charge. At any point P at a perpendicular distance r from the line in *any* direction, \vec{E} has magnitude

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad (\text{infinite line of charge})$$

Note that this field is proportional to $1/r$ rather than to $1/r^2$ as for a point charge.

Example 21.11 Field of a uniformly charged disk

A nonconducting disk of radius R has a uniform positive surface charge density σ . Find the electric field at a point along the axis of the disk a distance x from its center. Assume that x is positive.

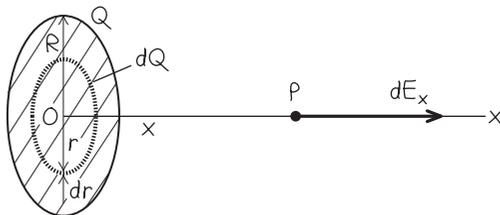
SOLUTION

IDENTIFY and SET UP: Figure 21.25 shows the situation. We represent the charge distribution as a collection of concentric rings of charge dQ . In Example 21.9 we obtained Eq. (21.8) for the field on the axis of a single uniformly charged ring, so all we need do here is integrate the contributions of our rings.

EXECUTE: A typical ring has charge dQ , inner radius r , and outer radius $r + dr$. Its area is approximately equal to its width dr times its circumference $2\pi r$, or $dA = 2\pi r dr$. The charge per unit area is $\sigma = dQ/dA$, so the charge of the ring is $dQ = \sigma dA = 2\pi\sigma r dr$. We use dQ in place of Q in Eq. (21.8), the expression for the field due to a ring that we found in Example 21.9, and replace the ring radius a with r . Then the field component dE_x at point P due to this ring is

$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{2\pi\sigma r x dr}{(x^2 + r^2)^{3/2}}$$

21.25 Our sketch for this problem.



There's really no such thing in nature as an infinite line of charge. But when the field point is close enough to the line, there's very little difference between the result for an infinite line and the real-life finite case. For example, if the distance r of the field point from the center of the line is 1% of the length of the line, the value of E differs from the infinite-length value by less than 0.02%.

To find the total field due to all the rings, we integrate dE_x over r from $r = 0$ to $r = R$ (not from $-R$ to R):

$$E_x = \int_0^R \frac{1}{4\pi\epsilon_0} \frac{(2\pi\sigma r dr)x}{(x^2 + r^2)^{3/2}} = \frac{\sigma x}{4\epsilon_0} \int_0^R \frac{2r dr}{(x^2 + r^2)^{3/2}}$$

You can evaluate this integral by making the substitution $t = x^2 + r^2$ (which yields $dt = 2r dr$); you can work out the details. The result is

$$\begin{aligned} E_x &= \frac{\sigma x}{2\epsilon_0} \left[-\frac{1}{\sqrt{x^2 + R^2}} + \frac{1}{x} \right] \\ &= \frac{\sigma}{2\epsilon_0} \left[1 - \frac{1}{\sqrt{(R^2/x^2) + 1}} \right] \end{aligned} \quad (21.11)$$

EVALUATE: If the disk is very large (or if we are very close to it), so that $R \gg x$, the term $1/\sqrt{(R^2/x^2) + 1}$ in Eq. (21.11) is very much less than 1. Then Eq. (21.11) becomes

$$E = \frac{\sigma}{2\epsilon_0} \quad (21.12)$$

Our final result does not contain the distance x from the plane. Hence the electric field produced by an *infinite* plane sheet of charge is *independent of the distance from the sheet*. The field direction is everywhere perpendicular to the sheet, away from it. There is no such thing as an infinite sheet of charge, but if the dimensions of the sheet are much larger than the distance x of the field point P from the sheet, the field is very nearly given by Eq. (21.12).

If P is to the *left* of the plane ($x < 0$), the result is the same except that the direction of \vec{E} is to the left instead of the right. If the surface charge density is negative, the directions of the fields on both sides of the plane are toward it rather than away from it.

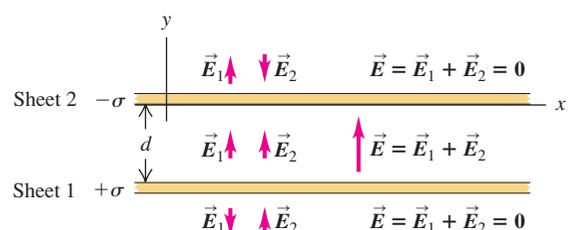
Example 21.12 Field of two oppositely charged infinite sheets

Two infinite plane sheets with uniform surface charge densities $+\sigma$ and $-\sigma$ are placed parallel to each other with separation d (Fig. 21.26). Find the electric field between the sheets, above the upper sheet, and below the lower sheet.

SOLUTION

IDENTIFY and SET UP: Equation (21.12) gives the electric field due to a single infinite plane sheet of charge. To find the field due to *two* such sheets, we combine the fields using the principle of superposition (Fig. 21.26).

21.26 Finding the electric field due to two oppositely charged infinite sheets. The sheets are seen edge-on; only a portion of the infinite sheets can be shown!



Continued

EXECUTE: From Eq. (21.12), both \vec{E}_1 and \vec{E}_2 have the same magnitude at all points, independent of distance from either sheet:

$$E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$$

From Example 21.11, \vec{E}_1 is everywhere directed away from sheet 1, and \vec{E}_2 is everywhere directed toward sheet 2.

Between the sheets, \vec{E}_1 and \vec{E}_2 reinforce each other; above the upper sheet and below the lower sheet, they cancel each other. Thus the total field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} \mathbf{0} & \text{above the upper sheet} \\ \frac{\sigma}{\epsilon_0} \hat{j} & \text{between the sheets} \\ \mathbf{0} & \text{below the lower sheet} \end{cases}$$

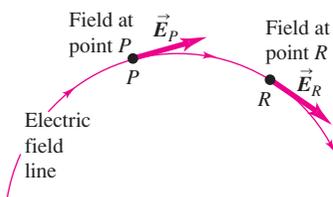
EVALUATE: Because we considered the sheets to be infinite, our result does not depend on the separation d . Our result shows that the field between oppositely charged plates is essentially uniform if the plate separation is much smaller than the dimensions of the plates. We actually used this result in Example 21.7 (Section 21.4).

CAUTION Electric fields are not “flows” You may have thought that the field \vec{E}_1 of sheet 1 would be unable to “penetrate” sheet 2, and that field \vec{E}_2 caused by sheet 2 would be unable to “penetrate” sheet 1. You might conclude this if you think of the electric field as some kind of physical substance that “flows” into or out of charges. But in fact there is no such substance, and the electric fields \vec{E}_1 and \vec{E}_2 depend only on the individual charge distributions that create them. The *total* field at every point is just the vector sum of \vec{E}_1 and \vec{E}_2 . **|**

Test Your Understanding of Section 21.5 Suppose that the line of charge in Fig. 21.25 (Example 21.11) had charge $+Q$ distributed uniformly between $y = 0$ and $y = +a$ and had charge $-Q$ distributed uniformly between $y = 0$ and $y = -a$. In this situation, the electric field at P would be (i) in the positive x -direction; (ii) in the negative x -direction; (iii) in the positive y -direction; (iv) in the negative y -direction; (v) zero; (vi) none of these. **|**



21.27 The direction of the electric field at any point is tangent to the field line through that point.



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21.6 Electric Field Lines

The concept of an electric field can be a little elusive because you can't see an electric field directly. Electric field *lines* can be a big help for visualizing electric fields and making them seem more real. An **electric field line** is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric-field vector at that point. Figure 21.27 shows the basic idea. (We used a similar concept in our discussion of fluid flow in Section 12.5. A *streamline* is a line or curve whose tangent at any point is in the direction of the velocity of the fluid at that point. However, the similarity between electric field lines and fluid streamlines is a mathematical one only; there is nothing “flowing” in an electric field.) The English scientist Michael Faraday (1791–1867) first introduced the concept of field lines. He called them “lines of force,” but the term “field lines” is preferable.

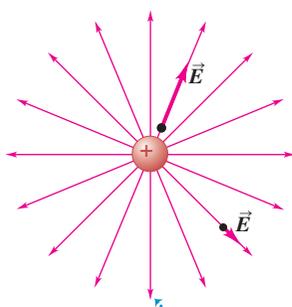
Electric field lines show the direction of \vec{E} at each point, and their spacing gives a general idea of the *magnitude* of \vec{E} at each point. Where \vec{E} is strong, we draw lines bunched closely together; where \vec{E} is weaker, they are farther apart. At any particular point, the electric field has a unique direction, so only one field line can pass through each point of the field. In other words, *field lines never intersect*.

Figure 21.28 shows some of the electric field lines in a plane containing (a) a single positive charge; (b) two equal-magnitude charges, one positive and one negative (a dipole); and (c) two equal positive charges. Diagrams such as these are sometimes called *field maps*; they are cross sections of the actual three-dimensional patterns. The direction of the total electric field at every point in each diagram is along the tangent to the electric field line passing through the point. Arrowheads indicate the direction of the \vec{E} -field vector along each field line. The actual field vectors have been drawn at several points in each pattern. Notice that in general, the magnitude of the electric field is different at different points on a given field line; a field line is *not* a curve of constant electric-field magnitude!

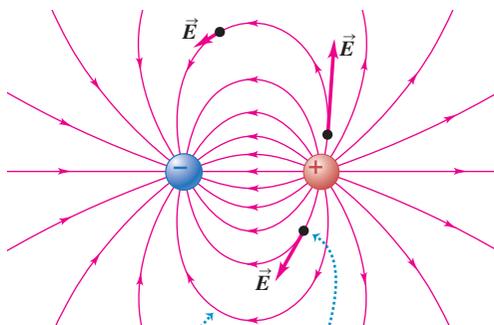
Figure 21.28 shows that field lines are directed *away* from positive charges (since close to a positive point charge, \vec{E} points away from the charge) and

21.28 Electric field lines for three different charge distributions. In general, the magnitude of \vec{E} is different at different points along a given field line.

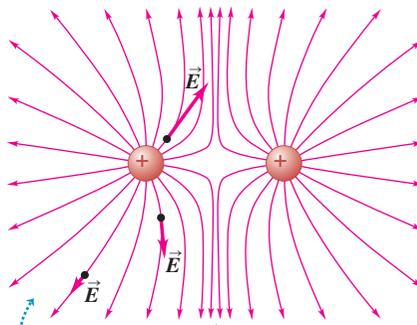
(a) A single positive charge



(b) Two equal and opposite charges (a dipole)



(c) Two equal positive charges



Field lines always point away from (+) charges and toward (-) charges.

At each point in space, the electric field vector is tangent to the field line passing through that point.

Field lines are close together where the field is strong, farther apart where it is weaker.

toward negative charges (since close to a negative point charge, \vec{E} points toward the charge). In regions where the field magnitude is large, such as between the positive and negative charges in Fig. 21.28b, the field lines are drawn close together. In regions where the field magnitude is small, such as between the two positive charges in Fig. 21.28c, the lines are widely separated. In a *uniform* field, the field lines are straight, parallel, and uniformly spaced, as in Fig. 21.20.

Figure 21.29 is a view from above of a demonstration setup for visualizing electric field lines. In the arrangement shown here, the tips of two positively charged wires are inserted in a container of insulating liquid, and some grass seeds are floated on the liquid. The grass seeds are electrically neutral insulators, but the electric field of the two charged wires causes *polarization* of the grass seeds; there is a slight shifting of the positive and negative charges within the molecules of each seed, like that shown in Fig. 21.8. The positively charged end of each grass seed is pulled in the direction of \vec{E} and the negatively charged end is pulled opposite \vec{E} . Hence the long axis of each grass seed tends to orient parallel to the electric field, in the direction of the field line that passes through the position of the seed (Fig. 21.29b).

CAUTION **Electric field lines are not the same as trajectories** It's a common misconception that if a charged particle of charge q is in motion where there is an electric field, the particle must move along an electric field line. Because \vec{E} at any point is tangent to the field line that passes through that point, it is indeed true that the force $\vec{F} = q\vec{E}$ on the particle, and hence the particle's acceleration, are tangent to the field line. But we learned in Chapter 3 that when a particle moves on a curved path, its acceleration *cannot* be tangent to the path. So in general, the trajectory of a charged particle is *not* the same as a field line. **|**

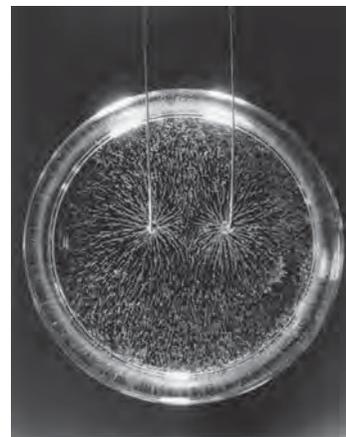
Test Your Understanding of Section 21.6 Suppose the electric field lines in a region of space are straight lines. If a charged particle is released from rest in that region, will the trajectory of the particle be along a field line? **|**

21.7 Electric Dipoles

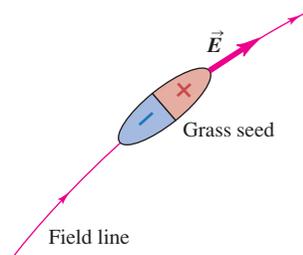
An **electric dipole** is a pair of point charges with equal magnitude and opposite sign (a positive charge q and a negative charge $-q$) separated by a distance d . We introduced electric dipoles in Example 21.8 (Section 21.5); the concept is worth exploring further because many physical systems, from molecules to TV antennas, can be described as electric dipoles. We will also use this concept extensively in our discussion of dielectrics in Chapter 24.

21.29 (a) Electric field lines produced by two equal point charges. The pattern is formed by grass seeds floating on a liquid above two charged wires. Compare this pattern with Fig. 21.28c. (b) The electric field causes polarization of the grass seeds, which in turn causes the seeds to align with the field.

(a)

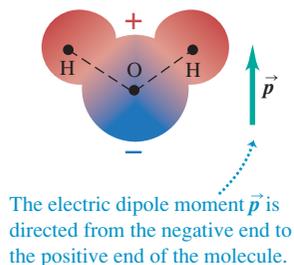


(b)



21.30 (a) A water molecule is an example of an electric dipole. (b) Each test tube contains a solution of a different substance in water. The large electric dipole moment of water makes it an excellent solvent.

(a) A water molecule, showing positive charge as red and negative charge as blue



(b) Various substances dissolved in water



21.31 The net force on this electric dipole is zero, but there is a torque directed into the page that tends to rotate the dipole clockwise.

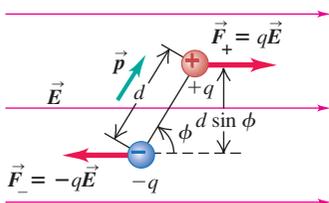


Figure 21.30 a shows a molecule of water (H_2O), which in many ways behaves like an electric dipole. The water molecule as a whole is electrically neutral, but the chemical bonds within the molecule cause a displacement of charge; the result is a net negative charge on the oxygen end of the molecule and a net positive charge on the hydrogen end, forming an electric dipole. The effect is equivalent to shifting one electron only about 4×10^{-11} m (about the radius of a hydrogen atom), but the consequences of this shift are profound. Water is an excellent solvent for ionic substances such as table salt (sodium chloride, NaCl) precisely because the water molecule is an electric dipole (Fig. 21.30b). When dissolved in water, salt dissociates into a positive sodium ion (Na^+) and a negative chlorine ion (Cl^-), which tend to be attracted to the negative and positive ends, respectively, of water molecules; this holds the ions in solution. If water molecules were not electric dipoles, water would be a poor solvent, and almost all of the chemistry that occurs in aqueous solutions would be impossible. This includes all of the biochemical reactions that occur in all of the life on earth. In a very real sense, your existence as a living being depends on electric dipoles!

We examine two questions about electric dipoles. First, what forces and torques does an electric dipole experience when placed in an external electric field (that is, a field set up by charges outside the dipole)? Second, what electric field does an electric dipole itself produce?

Force and Torque on an Electric Dipole

To start with the first question, let's place an electric dipole in a *uniform* external electric field \vec{E} , as shown in Fig. 21.31. The forces \vec{F}_+ and \vec{F}_- on the two charges both have magnitude qE , but their directions are opposite, and they add to zero. *The net force on an electric dipole in a uniform external electric field is zero.*

However, the two forces don't act along the same line, so their *torques* don't add to zero. We calculate torques with respect to the center of the dipole. Let the angle between the electric field \vec{E} and the dipole axis be ϕ ; then the lever arm for both \vec{F}_+ and \vec{F}_- is $(d/2) \sin \phi$. The torque of \vec{F}_+ and the torque of \vec{F}_- both have the same magnitude of $(qE)(d/2) \sin \phi$, and both torques tend to rotate the dipole clockwise (that is, $\vec{\tau}$ is directed into the page in Fig. 21.31). Hence the magnitude of the net torque is twice the magnitude of either individual torque:

$$\tau = (qE)(d \sin \phi) \quad (21.13)$$

where $d \sin \phi$ is the perpendicular distance between the lines of action of the two forces.

The product of the charge q and the separation d is the magnitude of a quantity called the **electric dipole moment**, denoted by p :

$$p = qd \quad (\text{magnitude of electric dipole moment}) \quad (21.14)$$

The units of p are charge times distance ($\text{C} \cdot \text{m}$). For example, the magnitude of the electric dipole moment of a water molecule is $p = 6.13 \times 10^{-30} \text{C} \cdot \text{m}$.

CAUTION The symbol p has multiple meanings Be careful not to confuse dipole moment with momentum or pressure. There aren't as many letters in the alphabet as there are physical quantities, so some letters are used several times. The context usually makes it clear what we mean, but be careful. **|**

We further define the electric dipole moment to be a *vector* quantity \vec{p} . The magnitude of \vec{p} is given by Eq. (21.14), and its direction is along the dipole axis from the negative charge to the positive charge as shown in Fig. 21.31.

In terms of p , Eq. (21.13) for the magnitude τ of the torque exerted by the field becomes

$$\tau = pE \sin \phi \quad (\text{magnitude of the torque on an electric dipole}) \quad (21.15)$$

Since the angle ϕ in Fig. 21.31 is the angle between the directions of the vectors \vec{p} and \vec{E} , this is reminiscent of the expression for the magnitude of the *vector product* discussed in Section 1.10. (You may want to review that discussion.) Hence we can write the torque on the dipole in vector form as

$$\vec{\tau} = \vec{p} \times \vec{E} \quad (\text{torque on an electric dipole, in vector form}) \quad (21.16)$$

You can use the right-hand rule for the vector product to verify that in the situation shown in Fig. 21.31, $\vec{\tau}$ is directed into the page. The torque is greatest when \vec{p} and \vec{E} are perpendicular and is zero when they are parallel or antiparallel. The torque always tends to turn \vec{p} to line it up with \vec{E} . The position $\phi = 0$, with \vec{p} parallel to \vec{E} , is a position of stable equilibrium, and the position $\phi = \pi$, with \vec{p} and \vec{E} antiparallel, is a position of unstable equilibrium. The polarization of a grass seed in the apparatus of Fig. 21.29b gives it an electric dipole moment; the torque exerted by \vec{E} then causes the seed to align with \vec{E} and hence with the field lines.

Potential Energy of an Electric Dipole

When a dipole changes direction in an electric field, the electric-field torque does *work* on it, with a corresponding change in potential energy. The work dW done by a torque τ during an infinitesimal displacement $d\phi$ is given by Eq. (10.19): $dW = \tau d\phi$. Because the torque is in the direction of decreasing ϕ , we must write the torque as $\tau = -pE \sin \phi$, and

$$dW = \tau d\phi = -pE \sin \phi d\phi$$

In a finite displacement from ϕ_1 to ϕ_2 the total work done on the dipole is

$$\begin{aligned} W &= \int_{\phi_1}^{\phi_2} (-pE \sin \phi) d\phi \\ &= pE \cos \phi_2 - pE \cos \phi_1 \end{aligned}$$

The work is the negative of the change of potential energy, just as in Chapter 7: $W = U_1 - U_2$. So a suitable definition of potential energy U for this system is

$$U(\phi) = -pE \cos \phi \quad (21.17)$$

In this expression we recognize the *scalar product* $\vec{p} \cdot \vec{E} = pE \cos \phi$, so we can also write

$$U = -\vec{p} \cdot \vec{E} \quad (\text{potential energy for a dipole in an electric field}) \quad (21.18)$$

The potential energy has its minimum (most negative) value $U = -pE$ at the stable equilibrium position, where $\phi = 0$ and \vec{p} is parallel to \vec{E} . The potential energy is maximum when $\phi = \pi$ and \vec{p} is antiparallel to \vec{E} ; then $U = +pE$. At $\phi = \pi/2$, where \vec{p} is perpendicular to \vec{E} , U is zero. We could define U differently so that it is zero at some other orientation of \vec{p} , but our definition is simplest.

Equation (21.18) gives us another way to look at the effect shown in Fig. 21.29. The electric field \vec{E} gives each grass seed an electric dipole moment, and the grass seed then aligns itself with \vec{E} to minimize the potential energy.

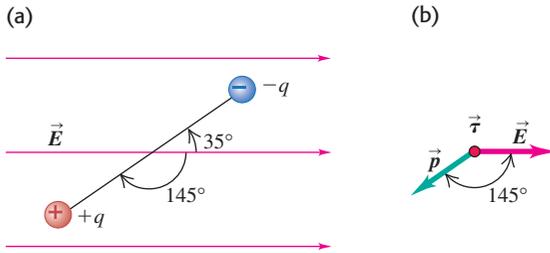
Example 21.13 Force and torque on an electric dipole

Figure 21.32a shows an electric dipole in a uniform electric field of magnitude $5.0 \times 10^5 \text{ N/C}$ that is directed parallel to the plane of the figure. The charges are $\pm 1.6 \times 10^{-19} \text{ C}$; both lie in the plane

and are separated by $0.125 \text{ nm} = 0.125 \times 10^{-9} \text{ m}$. Find (a) the net force exerted by the field on the dipole; (b) the magnitude and

Continued

21.32 (a) An electric dipole. (b) Directions of the electric dipole moment, electric field, and torque ($\vec{\tau}$ points out of the page).



direction of the electric dipole moment; (c) the magnitude and direction of the torque; (d) the potential energy of the system in the position shown.

SOLUTION

IDENTIFY and SET UP: This problem uses the ideas of this section about an electric dipole placed in an electric field. We use the relationship $\vec{F} = q\vec{E}$ for each point charge to find the force on the dipole as a whole. Equation (21.14) gives the dipole moment, Eq. (21.16) gives the torque on the dipole, and Eq. (21.18) gives the potential energy of the system.

EXECUTE: (a) The field is uniform, so the forces on the two charges are equal and opposite. Hence the total force on the dipole is zero.

(b) The magnitude p of the electric dipole moment \vec{p} is

$$\begin{aligned} p &= qd = (1.6 \times 10^{-19} \text{ C})(0.125 \times 10^{-9} \text{ m}) \\ &= 2.0 \times 10^{-29} \text{ C} \cdot \text{m} \end{aligned}$$

The direction of \vec{p} is from the negative to the positive charge, 145° clockwise from the electric-field direction (Fig. 21.32b).

(c) The magnitude of the torque is

$$\begin{aligned} \tau &= pE \sin \phi = (2.0 \times 10^{-29} \text{ C} \cdot \text{m})(5.0 \times 10^5 \text{ N/C})(\sin 145^\circ) \\ &= 5.7 \times 10^{-24} \text{ N} \cdot \text{m} \end{aligned}$$

From the right-hand rule for vector products (see Section 1.10), the direction of the torque $\vec{\tau} = \vec{p} \times \vec{E}$ is out of the page. This corresponds to a counterclockwise torque that tends to align \vec{p} with \vec{E} .

(d) The potential energy

$$\begin{aligned} U &= -pE \cos \phi \\ &= -(2.0 \times 10^{-29} \text{ C} \cdot \text{m})(5.0 \times 10^5 \text{ N/C})(\cos 145^\circ) \\ &= 8.2 \times 10^{-24} \text{ J} \end{aligned}$$

EVALUATE: The charge magnitude, the distance between the charges, the dipole moment, and the potential energy are all very small, but are all typical of molecules.

In this discussion we have assumed that \vec{E} is uniform, so there is no net force on the dipole. If \vec{E} is not uniform, the forces at the ends may not cancel completely, and the net force may not be zero. Thus a body with zero net charge but an electric dipole moment can experience a net force in a nonuniform electric field. As we mentioned in Section 21.1, an uncharged body can be polarized by an electric field, giving rise to a separation of charge and an electric dipole moment. This is how uncharged bodies can experience electrostatic forces (see Fig. 21.8).

Field of an Electric Dipole

Now let's think of an electric dipole as a *source* of electric field. What does the field look like? The general shape of things is shown by the field map of Fig. 21.28b. At each point in the pattern the total \vec{E} field is the vector sum of the fields from the two individual charges, as in Example 21.8 (Section 21.5). Try drawing diagrams showing this vector sum for several points.

To get quantitative information about the field of an electric dipole, we have to do some calculating, as illustrated in the next example. Notice the use of the principle of superposition of electric fields to add up the contributions to the field of the individual charges. Also notice that we need to use approximation techniques even for the relatively simple case of a field due to two charges. Field calculations often become very complicated, and computer analysis is typically used to determine the field due to an arbitrary charge distribution.

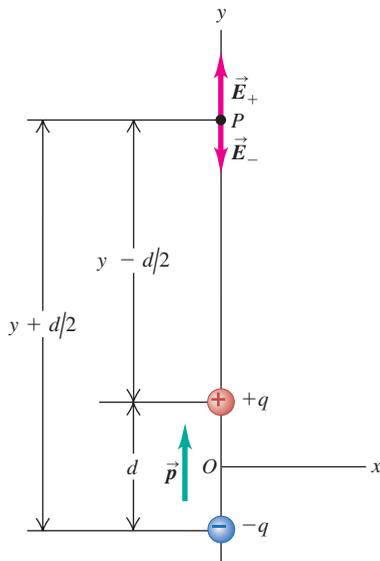
Example 21.14 Field of an electric dipole, revisited

An electric dipole is centered at the origin, with \vec{p} in the direction of the $+y$ -axis (Fig. 21.33). Derive an approximate expression for the electric field at a point P on the y -axis for which y is much larger than d . To do this, use the binomial expansion $(1+x)^n \cong 1+nx+n(n-1)x^2/2+\dots$ (valid for the case $|x| < 1$).

SOLUTION

IDENTIFY and SET UP: We use the principle of superposition: The total electric field is the vector sum of the field produced by the positive charge and the field produced by the negative charge. At the field point P shown in Fig. 21.33, the field \vec{E}_+ of the positive charge has a positive (upward) y -component and the field \vec{E}_- of

21.33 Finding the electric field of an electric dipole at a point on its axis.



the negative charge has a negative (downward) y -component. We add these components to find the total field and then apply the approximation that y is much greater than d .

EXECUTE: The total y -component E_y of electric field from the two charges is

$$\begin{aligned} E_y &= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(y - d/2)^2} - \frac{1}{(y + d/2)^2} \right] \\ &= \frac{q}{4\pi\epsilon_0 y^2} \left[\left(1 - \frac{d}{2y}\right)^{-2} - \left(1 + \frac{d}{2y}\right)^{-2} \right] \end{aligned}$$

We used this same approach in Example 21.8 (Section 21.5). Now the approximation: When we are far from the dipole compared to its size, so $y \gg d$, we have $d/2y \ll 1$. With $n = -2$ and with $d/2y$ replacing x in the binomial expansion, we keep only the first two terms (the terms we discard are much smaller). We then have

$$\left(1 - \frac{d}{2y}\right)^{-2} \cong 1 + \frac{d}{y} \quad \text{and} \quad \left(1 + \frac{d}{2y}\right)^{-2} \cong 1 - \frac{d}{y}$$

Hence E_y is given approximately by

$$E_y \cong \frac{q}{4\pi\epsilon_0 y^2} \left[1 + \frac{d}{y} - \left(1 - \frac{d}{y}\right) \right] = \frac{qd}{2\pi\epsilon_0 y^3} = \frac{p}{2\pi\epsilon_0 y^3}$$

EVALUATE: An alternative route to this result is to put the fractions in the first expression for E_y over a common denominator, add, and then approximate the denominator $(y - d/2)^2(y + d/2)^2$ as y^4 . We leave the details to you (see Exercise 21.60).

For points P off the coordinate axes, the expressions are more complicated, but at *all* points far away from the dipole (in any direction) the field drops off as $1/r^3$. We can compare this with the $1/r^2$ behavior of a point charge, the $1/r$ behavior of a long line charge, and the independence of r for a large sheet of charge. There are charge distributions for which the field drops off even more quickly. At large distances, the field of an *electric quadrupole*, which consists of two equal dipoles with opposite orientation, separated by a small distance, drops off as $1/r^4$.

Test Your Understanding of Section 21.7 An electric dipole is placed in a region of uniform electric field \vec{E} , with the electric dipole moment \vec{p} , pointing in the direction opposite to \vec{E} . Is the dipole (i) in stable equilibrium, (ii) in unstable equilibrium, or (iii) neither? (*Hint:* You may want to review Section 7.5.)

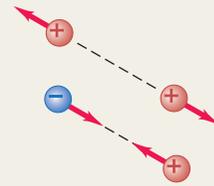


I

Electric charge, conductors, and insulators: The fundamental quantity in electrostatics is electric charge. There are two kinds of charge, positive and negative. Charges of the same sign repel each other; charges of opposite sign attract. Charge is conserved; the total charge in an isolated system is constant.

All ordinary matter is made of protons, neutrons, and electrons. The positive protons and electrically neutral neutrons in the nucleus of an atom are bound together by the nuclear force; the negative electrons surround the nucleus at distances much greater than the nuclear size. Electric interactions are chiefly responsible for the structure of atoms, molecules, and solids.

Conductors are materials in which charge moves easily; in insulators, charge does not move easily. Most metals are good conductors; most nonmetals are insulators.

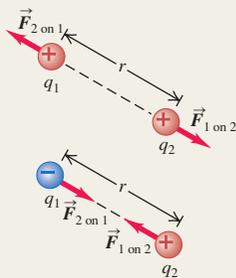


Coulomb's law: For charges q_1 and q_2 separated by a distance r , the magnitude of the electric force on either charge is proportional to the product q_1q_2 and inversely proportional to r^2 . The force on each charge is along the line joining the two charges—repulsive if q_1 and q_2 have the same sign, attractive if they have opposite signs. In SI units the unit of electric charge is the coulomb, abbreviated C. (See Examples 21.1 and 21.2.)

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1q_2|}{r^2} \quad (21.2)$$

$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

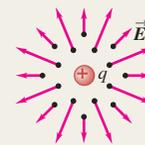
When two or more charges each exert a force on a charge, the total force on that charge is the vector sum of the forces exerted by the individual charges. (See Examples 21.3 and 21.4.)



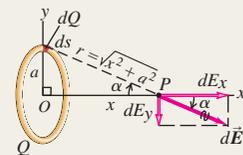
Electric field: Electric field \vec{E} , a vector quantity, is the force per unit charge exerted on a test charge at any point. The electric field produced by a point charge is directed radially away from or toward the charge. (See Examples 21.5–21.7.)

$$\vec{E} = \frac{\vec{F}_0}{q_0} \quad (21.3)$$

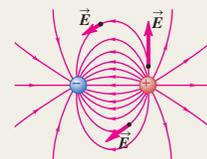
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \quad (21.7)$$



Superposition of electric fields: The electric field \vec{E} of any combination of charges is the vector sum of the fields caused by the individual charges. To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements, calculate the field caused by each element, and then carry out the vector sum, usually by integrating. Charge distributions are described by linear charge density λ , surface charge density σ , and volume charge density ρ . (See Examples 21.8–21.12.)



Electric field lines: Field lines provide a graphical representation of electric fields. At any point on a field line, the tangent to the line is in the direction of \vec{E} at that point. The number of lines per unit area (perpendicular to their direction) is proportional to the magnitude of \vec{E} at the point.

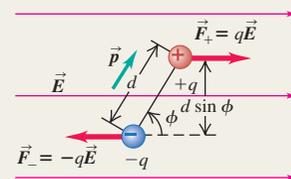


Electric dipoles: An electric dipole is a pair of electric charges of equal magnitude q but opposite sign, separated by a distance d . The electric dipole moment \vec{p} has magnitude $p = qd$. The direction of \vec{p} is from negative toward positive charge. An electric dipole in an electric field \vec{E} experiences a torque $\vec{\tau}$ equal to the vector product of \vec{p} and \vec{E} . The magnitude of the torque depends on the angle ϕ between \vec{p} and \vec{E} . The potential energy U for an electric dipole in an electric field also depends on the relative orientation of \vec{p} and \vec{E} . (See Examples 21.13 and 21.14.)

$$\tau = pE \sin \phi \quad (21.15)$$

$$\vec{\tau} = \vec{p} \times \vec{E} \quad (21.16)$$

$$U = -\vec{p} \cdot \vec{E} \quad (21.18)$$



BRIDGING PROBLEM

Calculating Electric Field: Half a Ring of Charge

Positive charge Q is uniformly distributed around a semicircle of radius a as shown in Fig. 21.34. Find the magnitude and direction of the resulting electric field at point P , the center of curvature of the semicircle.

SOLUTION GUIDE

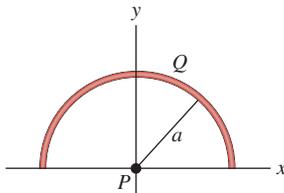
See MasteringPhysics® study area for a Video Tutor solution.



IDENTIFY and SET UP

1. The target variables are the components of the electric field at P .
2. Divide the semicircle into infinitesimal segments, each of which is a short circular arc of radius a and angle $d\theta$. What is the length of such a segment? How much charge is on a segment?

21.34



3. Consider an infinitesimal segment located at an angular position θ on the semicircle, measured from the lower right corner of the semicircle at $x = a, y = 0$. (Thus $\theta = \pi/2$ at $x = 0, y = a$ and $\theta = \pi$ at $x = -a, y = 0$.) What are the x - and y -components of the electric field at P (dE_x and dE_y) produced by just this segment?

EXECUTE

4. Integrate your expressions for dE_x and dE_y from $\theta = 0$ to $\theta = \pi$. The results will be the x -component and y -component of the electric field at P .
5. Use your results from step 4 to find the magnitude and direction of the field at P .

EVALUATE

6. Does your result for the electric-field magnitude have the correct units?
7. Explain how you could have found the x -component of the electric field using a symmetry argument.
8. What would be the electric field at P if the semicircle were extended to a full circle centered at P ?

Problems

For instructor-assigned homework, go to www.masteringphysics.com



•, ••, •••: Problems of increasing difficulty. CP: Cumulative problems incorporating material from earlier chapters. CALC: Problems requiring calculus. BIO: Biosciences problems.

DISCUSSION QUESTIONS

Q21.1 If you peel two strips of transparent tape off the same roll and immediately let them hang near each other, they will repel each other. If you then stick the sticky side of one to the shiny side of the other and rip them apart, they will attract each other. Give a plausible explanation, involving transfer of electrons between the strips of tape, for this sequence of events.

Q21.2 Two metal spheres are hanging from nylon threads. When you bring the spheres close to each other, they tend to attract. Based on this information alone, discuss all the possible ways that the spheres could be charged. Is it possible that after the spheres touch, they will cling together? Explain.

Q21.3 The electric force between two charged particles becomes weaker with increasing distance. Suppose instead that the electric force were *independent* of distance. In this case, would a charged comb still cause a neutral insulator to become polarized as in Fig. 21.8? Why or why not? Would the neutral insulator still be attracted to the comb? Again, why or why not?

Q21.4 Your clothing tends to cling together after going through the dryer. Why? Would you expect more or less clinging if all your clothing were made of the same material (say, cotton) than if you dried different kinds of clothing together? Again, why? (You may want to experiment with your next load of laundry.)

Q21.5 An uncharged metal sphere hangs from a nylon thread. When a positively charged glass rod is brought close to the metal

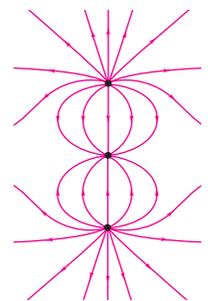
sphere, the sphere is drawn toward the rod. But if the sphere touches the rod, it suddenly flies away from the rod. Explain why the sphere is first attracted and then repelled.

Q21.6 The free electrons in a metal are gravitationally attracted toward the earth. Why, then, don't they all settle to the bottom of the conductor, like sediment settling to the bottom of a river?

Q21.7 • Figure Q21.7 shows some of the electric field lines due to three point charges arranged along the vertical axis. All three charges have the same magnitude. (a) What are the signs of the three charges? Explain your reasoning. (b) At what point(s) is the magnitude of the electric field the smallest? Explain your reasoning. Explain how the fields produced by each individual point charge combine to give a small net field at this point or points.

Q21.8 Good electrical conductors, such as metals, are typically good conductors of heat; electrical insulators, such as wood, are typically poor conductors of heat. Explain why there should be a relationship between electrical conduction and heat conduction in these materials.

Figure Q21.7



Q21.9 • Suppose the charge shown in Fig. 21.28a is fixed in position. A small, positively charged particle is then placed at some point in the figure and released. Will the trajectory of the particle follow an electric field line? Why or why not? Suppose instead that the particle is placed at some point in Fig. 21.28b and released (the positive and negative charges shown in the figure are fixed in position). Will its trajectory follow an electric field line? Again, why or why not? Explain any differences between your answers for the two different situations.

Q21.10 Two identical metal objects are mounted on insulating stands. Describe how you could place charges of opposite sign but exactly equal magnitude on the two objects.

Q21.11 You can use plastic food wrap to cover a container by stretching the material across the top and pressing the overhanging material against the sides. What makes it stick? (*Hint:* The answer involves the electric force.) Does the food wrap stick to itself with equal tenacity? Why or why not? Does it work with metallic containers? Again, why or why not?

Q21.12 If you walk across a nylon rug and then touch a large metal object such as a doorknob, you may get a spark and a shock. Why does this tend to happen more on dry days than on humid days? (*Hint:* See Fig. 21.30.) Why are you less likely to get a shock if you touch a *small* metal object, such as a paper clip?

Q21.13 You have a negatively charged object. How can you use it to place a net negative charge on an insulated metal sphere? To place a net positive charge on the sphere?

Q21.14 When two point charges of equal mass and charge are released on a frictionless table, each has an initial acceleration a_0 . If instead you keep one fixed and release the other one, what will be its initial acceleration: a_0 , $2a_0$, or $a_0/2$? Explain.

Q21.15 A point charge of mass m and charge Q and another point charge of mass m but charge $2Q$ are released on a frictionless table. If the charge Q has an initial acceleration a_0 , what will be the acceleration of $2Q$: a_0 , $2a_0$, $4a_0$, $a_0/2$, or $a_0/4$? Explain.

Q21.16 A proton is placed in a uniform electric field and then released. Then an electron is placed at this same point and released. Do these two particles experience the same force? The same acceleration? Do they move in the same direction when released?

Q21.17 In Example 21.1 (Section 21.3) we saw that the electric force between two α particles is of the order of 10^{35} times as strong as the gravitational force. So why do we readily feel the gravity of the earth but no electrical force from it?

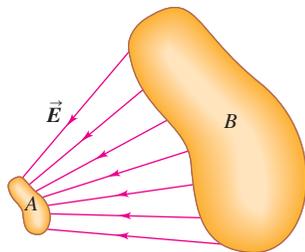
Q21.18 What similarities do electrical forces have with gravitational forces? What are the most significant differences?

Q21.19 Two irregular objects A and B carry charges of opposite sign. Figure Q21.19 shows the electric field lines near each of these objects. (a) Which object is positive, A or B ? How do you know? (b) Where is the electric field stronger, close to A or close to B ? How do you know?

Q21.20 Atomic nuclei are made of protons and neutrons. This shows that there must be another kind of interaction in addition to gravitational and electric forces. Explain.

Q21.21 Sufficiently strong electric fields can cause atoms to become positively ionized—that is, to lose one or more electrons. Explain how this can happen. What determines how strong the field must be to make this happen?

Figure Q21.19

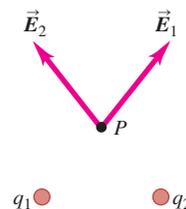


Q21.22 The electric fields at point P due to the positive charges q_1 and q_2 are shown in Fig. Q21.22. Does the fact that they cross each other violate the statement in Section 21.6 that electric field lines never cross? Explain.

Q21.23 The air temperature and the velocity of the air have different values at different places in the earth's atmosphere. Is the air velocity a vector field? Why or why not?

Is the air temperature a vector field? Again, why or why not?

Figure Q21.22



EXERCISES

Section 21.3 Coulomb's Law

21.1 •• Excess electrons are placed on a small lead sphere with mass 8.00 g so that its net charge is -3.20×10^{-9} C. (a) Find the number of excess electrons on the sphere. (b) How many excess electrons are there per lead atom? The atomic number of lead is 82, and its atomic mass is 207 g/mol.

21.2 • Lightning occurs when there is a flow of electric charge (principally electrons) between the ground and a thundercloud. The maximum rate of charge flow in a lightning bolt is about 20,000 C/s; this lasts for 100 μ s or less. How much charge flows between the ground and the cloud in this time? How many electrons flow during this time?

21.3 •• **BIO** Estimate how many electrons there are in your body. Make any assumptions you feel are necessary, but clearly state what they are. (*Hint:* Most of the atoms in your body have equal numbers of electrons, protons, and neutrons.) What is the combined charge of all these electrons?

21.4 • **Particles in a Gold Ring.** You have a pure (24-karat) gold ring with mass 17.7 g. Gold has an atomic mass of 197 g/mol and an atomic number of 79. (a) How many protons are in the ring, and what is their total positive charge? (b) If the ring carries no net charge, how many electrons are in it?

21.5 • **BIO Signal Propagation in Neurons.** *Neurons* are components of the nervous system of the body that transmit signals as electrical impulses travel along their length. These impulses propagate when charge suddenly rushes into and then out of a part of the neuron called an *axon*. Measurements have shown that, during the inflow part of this cycle, approximately 5.6×10^{11} Na^+ (sodium ions) per meter, each with charge $+e$, enter the axon. How many coulombs of charge enter a 1.5-cm length of the axon during this process?

21.6 • Two small spheres spaced 20.0 cm apart have equal charge. How many excess electrons must be present on each sphere if the magnitude of the force of repulsion between them is 4.57×10^{-21} N?

21.7 •• An average human weighs about 650 N. If two such generic humans each carried 1.0 coulomb of excess charge, one positive and one negative, how far apart would they have to be for the electric attraction between them to equal their 650-N weight?

21.8 •• Two small aluminum spheres, each having mass 0.0250 kg, are separated by 80.0 cm. (a) How many electrons does each sphere contain? (The atomic mass of aluminum is 26.982 g/mol, and its atomic number is 13.) (b) How many electrons would have to be removed from one sphere and added to the other to cause an attractive force between the spheres of magnitude 1.00×10^4 N (roughly 1 ton)? Assume that the spheres may be treated as point charges. (c) What fraction of all the electrons in each sphere does this represent?

21.9 •• Two small plastic spheres are given positive electrical charges. When they are 15.0 cm apart, the repulsive force between them has magnitude 0.220 N. What is the charge on each sphere (a) if the two charges are equal and (b) if one sphere has four times the charge of the other?

21.10 •• What If We Were Not Neutral? A 75-kg person holds out his arms so that his hands are 1.7 m apart. Typically, a person's hand makes up about 1.0% of his or her body weight. For round numbers, we shall assume that all the weight of each hand is due to the calcium in the bones, and we shall treat the hands as point charges. One mole of Ca contains 40.18 g, and each atom has 20 protons and 20 electrons. Suppose that only 1.0% of the positive charges in each hand were unbalanced by negative charge. (a) How many Ca atoms does each hand contain? (b) How many coulombs of unbalanced charge does each hand contain? (c) What force would the person's arms have to exert on his hands to prevent them from flying off? Does it seem likely that his arms are capable of exerting such a force?

21.11 •• Two very small 8.55-g spheres, 15.0 cm apart from center to center, are charged by adding equal numbers of electrons to each of them. Disregarding all other forces, how many electrons would you have to add to each sphere so that the two spheres will accelerate at 25.0g when released? Which way will they accelerate?

21.12 •• Just How Strong Is the Electric Force? Suppose you had two small boxes, each containing 1.0 g of protons. (a) If one were placed on the moon by an astronaut and the other were left on the earth, and if they were connected by a very light (and very long!) string, what would be the tension in the string? Express your answer in newtons and in pounds. Do you need to take into account the gravitational forces of the earth and moon on the protons? Why? (b) What gravitational force would each box of protons exert on the other box?

21.13 • In an experiment in space, one proton is held fixed and another proton is released from rest a distance of 2.50 mm away. (a) What is the initial acceleration of the proton after it is released? (b) Sketch qualitative (no numbers!) acceleration–time and velocity–time graphs of the released proton's motion.

21.14 • A negative charge of $-0.550 \mu\text{C}$ exerts an upward 0.200-N force on an unknown charge 0.300 m directly below it. (a) What is the unknown charge (magnitude and sign)? (b) What are the magnitude and direction of the force that the unknown charge exerts on the $-0.550\text{-}\mu\text{C}$ charge?

21.15 •• Three point charges are arranged on a line. Charge $q_3 = +5.00 \text{ nC}$ and is at the origin. Charge $q_2 = -3.00 \text{ nC}$ and is at $x = +4.00 \text{ cm}$. Charge q_1 is at $x = +2.00 \text{ cm}$. What is q_1 (magnitude and sign) if the net force on q_3 is zero?

21.16 •• In Example 21.4, suppose the point charge on the y -axis at $y = -0.30 \text{ m}$ has negative charge $-2.0 \mu\text{C}$, and the other charges remain the same. Find the magnitude and direction of the net force on Q . How does your answer differ from that in Example 21.4? Explain the differences.

21.17 •• In Example 21.3, calculate the net force on charge q_1 .

21.18 •• In Example 21.4, what is the net force (magnitude and direction) on charge q_1 exerted by the other two charges?

21.19 •• Three point charges are arranged along the x -axis. Charge $q_1 = +3.00 \mu\text{C}$ is at the origin, and charge $q_2 = -5.00 \mu\text{C}$ is at $x = 0.200 \text{ m}$. Charge $q_3 = -8.00 \mu\text{C}$. Where is q_3 located if the net force on q_1 is 7.00 N in the $-x$ -direction?

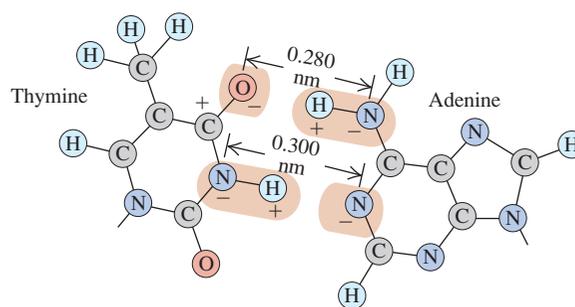
21.20 •• Repeat Exercise 21.19 for $q_3 = +8.00 \mu\text{C}$.

21.21 •• Two point charges are located on the y -axis as follows: charge $q_1 = -1.50 \text{ nC}$ at $y = -0.600 \text{ m}$, and charge $q_2 = +3.20 \text{ nC}$ at the origin ($y = 0$). What is the total force (magnitude and direction) exerted by these two charges on a third charge $q_3 = +5.00 \text{ nC}$ located at $y = -0.400 \text{ m}$?

21.22 •• Two point charges are placed on the x -axis as follows: Charge $q_1 = +4.00 \text{ nC}$ is located at $x = 0.200 \text{ m}$, and charge $q_2 = +5.00 \text{ nC}$ is at $x = -0.300 \text{ m}$. What are the magnitude and direction of the total force exerted by these two charges on a negative point charge $q_3 = -6.00 \text{ nC}$ that is placed at the origin?

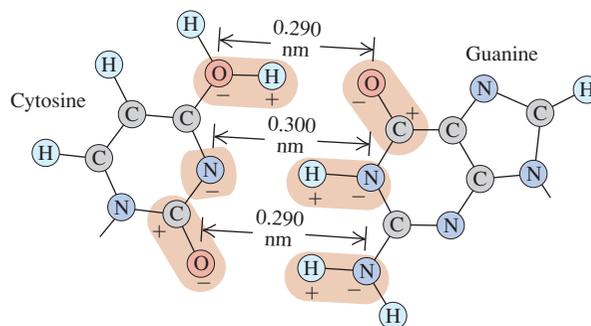
21.23 •• BIO Base Pairing in DNA, I. The two sides of the DNA double helix are connected by pairs of bases (adenine, thymine, cytosine, and guanine). Because of the geometric shape of these molecules, adenine bonds with thymine and cytosine bonds with guanine. Figure E21.23 shows the thymine–adenine bond. Each charge shown is $\pm e$, and the H–N distance is 0.110 nm. (a) Calculate the net force that thymine exerts on adenine. Is it attractive or repulsive? To keep the calculations fairly simple, yet reasonable, consider only the forces due to the O–H–N and the N–H–N combinations, assuming that these two combinations are parallel to each other. Remember, however, that in the O–H–N set, the O^- exerts a force on both the H^+ and the N^- , and likewise along the N–H–N set. (b) Calculate the force on the electron in the hydrogen atom, which is 0.0529 nm from the proton. Then compare the strength of the bonding force of the electron in hydrogen with the bonding force of the adenine–thymine molecules.

Figure E21.23



21.24 •• BIO Base Pairing in DNA, II. Refer to Exercise 21.23. Figure E21.24 shows the bonding of the cytosine and guanine molecules. The O–H and H–N distances are each 0.110 nm. In this case, assume that the bonding is due only to the forces along the O–H–O, N–H–N, and O–H–N combinations, and assume also that these three combinations are parallel to each other. Calculate the net force that cytosine exerts on guanine due to the preceding three combinations. Is this force attractive or repulsive?

Figure E21.24



Section 21.4 Electric Field and Electric Forces

21.25 • CP A proton is placed in a uniform electric field of $2.75 \times 10^3 \text{ N/C}$. Calculate: (a) the magnitude of the electric force felt by the proton; (b) the proton's acceleration; (c) the proton's speed after 1.00 μs in the field, assuming it starts from rest.

21.26 • A particle has charge -3.00 nC. (a) Find the magnitude and direction of the electric field due to this particle at a point 0.250 m directly above it. (b) At what distance from this particle does its electric field have a magnitude of 12.0 N/C?

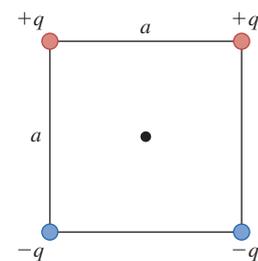
21.27 • CP A proton is traveling horizontally to the right at 4.50×10^6 m/s. (a) Find the magnitude and direction of the weakest electric field that can bring the proton uniformly to rest over a distance of 3.20 cm. (b) How much time does it take the proton to stop after entering the field? (c) What minimum field (magnitude and direction) would be needed to stop an electron under the conditions of part (a)?

21.28 • CP An electron is released from rest in a uniform electric field. The electron accelerates vertically upward, traveling 4.50 m in the first 3.00 μ s after it is released. (a) What are the magnitude and direction of the electric field? (b) Are we justified in ignoring the effects of gravity? Justify your answer quantitatively.

21.29 •• (a) What must the charge (sign and magnitude) of a 1.45 -g particle be for it to remain stationary when placed in a downward-directed electric field of magnitude 650 N/C? (b) What is the magnitude of an electric field in which the electric force on a proton is equal in magnitude to its weight?

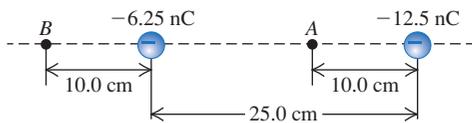
21.30 •• A point charge is placed at each corner of a square with side length a . The charges all have the same magnitude q . Two of the charges are positive and two are negative, as shown in Fig. E21.30. What is the direction of the net electric field at the center of the square due to the four charges, and what is its magnitude in terms of q and a ?

Figure E21.30



21.31 • Two point charges are separated by 25.0 cm (Fig. E21.31). Find the net electric field these charges produce at (a) point A and (b) point B . (c) What would be the magnitude and direction of the electric force this combination of charges would produce on a proton at A ?

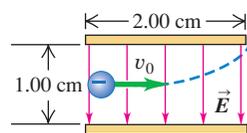
Figure E21.31



21.32 •• Electric Field of the Earth. The earth has a net electric charge that causes a field at points near its surface equal to 150 N/C and directed in toward the center of the earth. (a) What magnitude and sign of charge would a 60 -kg human have to acquire to overcome his or her weight by the force exerted by the earth's electric field? (b) What would be the force of repulsion between two people each with the charge calculated in part (a) and separated by a distance of 100 m? Is use of the earth's electric field a feasible means of flight? Why or why not?

21.33 •• CP An electron is projected with an initial speed $v_0 = 1.60 \times 10^6$ m/s into the uniform field between the parallel plates in Fig. E21.33. Assume that the field between the plates is uniform and directed vertically downward, and that the field outside the plates is zero. The electron enters the field at a point

Figure E21.33



midway between the plates. (a) If the electron just misses the upper plate as it emerges from the field, find the magnitude of the electric field. (b) Suppose that in Fig. E21.33 the electron is replaced by a proton with the same initial speed v_0 . Would the proton hit one of the plates? If the proton would not hit one of the plates, what would be the magnitude and direction of its vertical displacement as it exits the region between the plates? (c) Compare the paths traveled by the electron and the proton and explain the differences. (d) Discuss whether it is reasonable to ignore the effects of gravity for each particle.

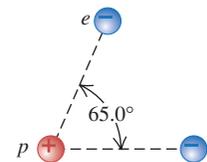
21.34 •• Point charge $q_1 = -5.00$ nC is at the origin and point charge $q_2 = +3.00$ nC is on the x -axis at $x = 3.00$ cm. Point P is on the y -axis at $y = 4.00$ cm. (a) Calculate the electric fields \vec{E}_1 and \vec{E}_2 at point P due to the charges q_1 and q_2 . Express your results in terms of unit vectors (see Example 21.6). (b) Use the results of part (a) to obtain the resultant field at P , expressed in unit vector form.

21.35 •• CP In Exercise 21.33, what is the speed of the electron as it emerges from the field?

21.36 • (a) Calculate the magnitude and direction (relative to the $+x$ -axis) of the electric field in Example 21.6. (b) A -2.5 -nC point charge is placed at point P in Fig. 21.19. Find the magnitude and direction of (i) the force that the -8.0 -nC charge at the origin exerts on this charge and (ii) the force that this charge exerts on the -8.0 -nC charge at the origin.

21.37 •• If two electrons are each 1.50×10^{-10} m from a proton, as shown in Fig. E21.37, find the magnitude and direction of the net electric force they will exert on the proton.

Figure E21.37

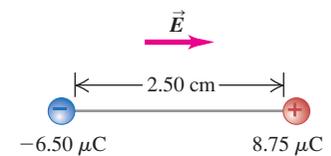


21.38 •• CP A uniform electric field exists in the region between two oppositely charged plane parallel plates. A proton is released from rest at the surface of the positively charged plate and strikes the surface of the opposite plate, 1.60 cm distant from the first, in a time interval of 1.50×10^{-6} s. (a) Find the magnitude of the electric field. (b) Find the speed of the proton when it strikes the negatively charged plate.

21.39 • A point charge is at the origin. With this point charge as the source point, what is the unit vector \hat{r} in the direction of (a) the field point at $x = 0$, $y = -1.35$ m; (b) the field point at $x = 12.0$ cm, $y = 12.0$ cm; (c) the field point at $x = -1.10$ m, $y = 2.60$ m? Express your results in terms of the unit vectors \hat{i} and \hat{j} .

21.40 •• A $+8.75$ - μ C point charge is glued down on a horizontal frictionless table. It is tied to a -6.50 - μ C point charge by a light, nonconducting 2.50 -cm wire. A uniform electric field of magnitude 1.85×10^8 N/C is directed parallel to the wire, as shown in Fig. E21.40. (a) Find the tension in the wire. (b) What would the tension be if both charges were negative?

Figure E21.40



21.41 •• (a) An electron is moving east in a uniform electric field of 1.50 N/C directed to the west. At point A , the velocity of the electron is 4.50×10^5 m/s toward the east. What is the speed of the electron when it reaches point B , 0.375 m east of point A ? (b) A proton is moving in the uniform electric field of part (a). At point A , the velocity of the proton is 1.90×10^4 m/s, east. What is the speed of the proton at point B ?

Section 21.5 Electric-Field Calculations

21.42 • Two point charges Q and $+q$ (where q is positive) produce the net electric field shown at point P in Fig. E21.42. The field points parallel to the line connecting the two charges. (a) What can you conclude about the sign and magnitude of Q ? Explain your reasoning. (b) If the lower charge were negative instead, would it be possible for the field to have the direction shown in the figure? Explain your reasoning.

21.43 •• Two positive point charges q are placed on the x -axis, one at $x = a$ and one at $x = -a$. (a) Find the magnitude and direction of the electric field at $x = 0$. (b) Derive an expression for the electric field at points on the x -axis. Use your result to graph the x -component of the electric field as a function of x , for values of x between $-4a$ and $+4a$.

21.44 • The two charges q_1 and q_2 shown in Fig. E21.44 have equal magnitudes. What is the direction of the net electric field due to these two charges at points A (midway between the charges), B , and C if (a) both charges are negative, (b) both charges are positive, (c) q_1 is positive and q_2 is negative.

21.45 • A $+2.00$ -nC point charge is at the origin, and a second -5.00 -nC point charge is on the x -axis at $x = 0.800$ m. (a) Find the electric field (magnitude and direction) at each of the following points on the x -axis: (i) $x = 0.200$ m; (ii) $x = 1.20$ m; (iii) $x = -0.200$ m. (b) Find the net electric force that the two charges would exert on an electron placed at each point in part (a).

21.46 •• Repeat Exercise 21.44, but now let $q_1 = -4.00$ nC.

21.47 • Three negative point charges lie along a line as shown in Fig. E21.47. Find the magnitude and direction of the electric field this combination of charges produces at point P , which lies 6.00 cm from the -2.00 - μC charge measured perpendicular to the line connecting the three charges.

21.48 •• **BIO Electric Field of Axons.** A nerve signal is transmitted through a neuron when an excess of Na^+ ions suddenly enters the axon, a long cylindrical part of the neuron. Axons are approximately 10.0 μm in diameter, and measurements show that about 5.6×10^{11} Na^+ ions per meter (each of charge $+e$) enter during this process. Although the axon is a long cylinder, the charge does not all enter everywhere at the same time. A plausible model would be a series of point charges moving along the axon. Let us look at a 0.10 -mm length of the axon and model it as a point charge. (a) If the charge that enters each meter of the axon gets distributed uniformly along it, how many coulombs of charge enter a 0.10 -mm length of the axon? (b) What electric field (magnitude and direction) does the sudden influx of charge produce at the surface of the body if the axon is 5.00 cm below the skin? (c) Certain sharks can respond to electric fields as weak as 1.0 $\mu\text{N}/\text{C}$.

Figure E21.42

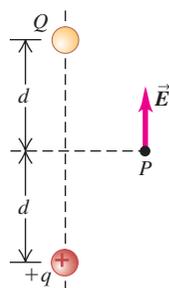


Figure E21.44

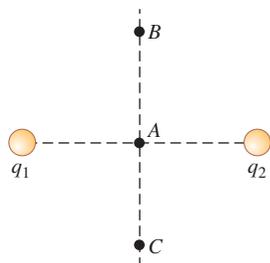
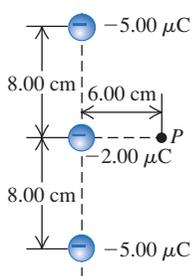


Figure E21.47



How far from this segment of axon could a shark be and still detect its electric field?

21.49 • In a rectangular coordinate system a positive point charge $q = 6.00 \times 10^{-9}$ C is placed at the point $x = +0.150$ m, $y = 0$, and an identical point charge is placed at $x = -0.150$ m, $y = 0$. Find the x - and y -components, the magnitude, and the direction of the electric field at the following points: (a) the origin; (b) $x = 0.300$ m, $y = 0$; (c) $x = 0.150$ m, $y = -0.400$ m; (d) $x = 0$, $y = 0.200$ m.

21.50 •• A point charge $q_1 = -4.00$ nC is at the point $x = 0.600$ m, $y = 0.800$ m, and a second point charge $q_2 = +6.00$ nC is at the point $x = 0.600$ m, $y = 0$. Calculate the magnitude and direction of the net electric field at the origin due to these two point charges.

21.51 •• Repeat Exercise 21.49 for the case where the point charge at $x = +0.150$ m, $y = 0$ is positive and the other is negative, each with magnitude 6.00×10^{-9} C.

21.52 •• A very long, straight wire has charge per unit length 1.50×10^{-10} C/m. At what distance from the wire is the electric-field magnitude equal to 2.50 N/C?

21.53 • A ring-shaped conductor with radius $a = 2.50$ cm has a total positive charge $Q = +0.125$ nC uniformly distributed around it, as shown in Fig. 21.23. The center of the ring is at the origin of coordinates O . (a) What is the electric field (magnitude and direction) at point P , which is on the x -axis at $x = 40.0$ cm? (b) A point charge $q = -2.50$ μC is placed at the point P described in part (a). What are the magnitude and direction of the force exerted by the charge q on the ring?

21.54 •• A straight, nonconducting plastic wire 8.50 cm long carries a charge density of $+175$ nC/m distributed uniformly along its length. It is lying on a horizontal tabletop. (a) Find the magnitude and direction of the electric field this wire produces at a point 6.00 cm directly above its midpoint. (b) If the wire is now bent into a circle lying flat on the table, find the magnitude and direction of the electric field it produces at a point 6.00 cm directly above its center.

21.55 •• A charge of -6.50 nC is spread uniformly over the surface of one face of a nonconducting disk of radius 1.25 cm. (a) Find the magnitude and direction of the electric field this disk produces at a point P on the axis of the disk a distance of 2.00 cm from its center. (b) Suppose that the charge were all pushed away from the center and distributed uniformly on the outer rim of the disk. Find the magnitude and direction of the electric field at point P . (c) If the charge is all brought to the center of the disk, find the magnitude and direction of the electric field at point P . (d) Why is the field in part (a) stronger than the field in part (b)? Why is the field in part (c) the strongest of the three fields?

Section 21.7 Electric Dipoles

21.56 • The ammonia molecule (NH_3) has a dipole moment of 5.0×10^{-30} C \cdot m. Ammonia molecules in the gas phase are placed in a uniform electric field \vec{E} with magnitude 1.6×10^6 N/C. (a) What is the change in electric potential energy when the dipole moment of a molecule changes its orientation with respect to \vec{E} from parallel to perpendicular? (b) At what absolute temperature T is the average translational kinetic energy $\frac{3}{2}kT$ of a molecule equal to the change in potential energy calculated in part (a)? (Note: Above this temperature, thermal agitation prevents the dipoles from aligning with the electric field.)

21.57 • Point charges $q_1 = -4.5$ nC and $q_2 = +4.5$ nC are separated by 3.1 mm, forming an electric dipole. (a) Find the electric dipole moment (magnitude and direction). (b) The charges are in a uniform electric field whose direction makes an angle of 36.9° with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has magnitude 7.2×10^{-9} N·m?

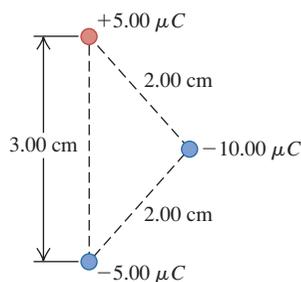
21.58 • The dipole moment of the water molecule (H_2O) is 6.17×10^{-30} C·m. Consider a water molecule located at the origin whose dipole moment \vec{p} points in the $+x$ -direction. A chlorine ion (Cl^-), of charge -1.60×10^{-19} C, is located at $x = 3.00 \times 10^{-9}$ m. Find the magnitude and direction of the electric force that the water molecule exerts on the chlorine ion. Is this force attractive or repulsive? Assume that x is much larger than the separation d between the charges in the dipole, so that the approximate expression for the electric field along the dipole axis derived in Example 21.14 can be used.

21.59 • **Torque on a Dipole.** An electric dipole with dipole moment \vec{p} is in a uniform electric field \vec{E} . (a) Find the orientations of the dipole for which the torque on the dipole is zero. (b) Which of the orientations in part (a) is stable, and which is unstable? (*Hint:* Consider a small displacement away from the equilibrium position and see what happens.) (c) Show that for the stable orientation in part (b), the dipole's own electric field tends to oppose the external field.

21.60 • Consider the electric dipole of Example 21.14. (a) Derive an expression for the magnitude of the electric field produced by the dipole at a point on the x -axis in Fig. 21.33. What is the direction of this electric field? (b) How does the electric field at points on the x -axis depend on x when x is very large?

21.61 • Three charges are at the corners of an isosceles triangle as shown in Fig. E21.61. The ± 5.00 - μC charges form a dipole. (a) Find the force (magnitude and direction) the -10.00 - μC charge exerts on the dipole. (b) For an axis perpendicular to the line connecting the ± 5.00 - μC charges at the midpoint of this line, find the torque (magnitude and direction) exerted on the dipole by the -10.00 - μC charge.

Figure E21.61



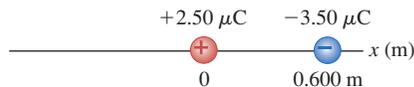
21.62 • A dipole consisting of charges $\pm e$, 220 nm apart, is placed between two very large (essentially infinite) sheets carrying equal but opposite charge densities of 125 $\mu\text{C}/\text{m}^2$. (a) What is the maximum potential energy this dipole can have due to the sheets, and how should it be oriented relative to the sheets to attain this value? (b) What is the maximum torque the sheets can exert on the dipole, and how should it be oriented relative to the sheets to attain this value? (c) What net force do the two sheets exert on the dipole?

PROBLEMS

21.63 • Four identical charges Q are placed at the corners of a square of side L . (a) In a free-body diagram, show all of the forces that act on one of the charges. (b) Find the magnitude and direction of the total force exerted on one charge by the other three charges.

21.64 • Two charges, one of 2.50 μC and the other of -3.50 μC , are placed on the x -axis, one at the origin and the other at $x = 0.600$ m, as shown in Fig. P21.64. Find the position on the x -axis where the net force on a small charge $+q$ would be zero.

Figure P21.64



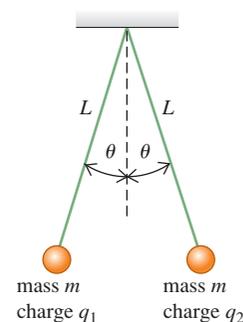
21.65 • Three point charges are arranged along the x -axis. Charge $q_1 = -4.50$ nC is located at $x = 0.200$ m, and charge $q_2 = +2.50$ nC is at $x = -0.300$ m. A positive point charge q_3 is located at the origin. (a) What must the value of q_3 be for the net force on this point charge to have magnitude 4.00 μN ? (b) What is the direction of the net force on q_3 ? (c) Where along the x -axis can q_3 be placed and the net force on it be zero, other than the trivial answers of $x = +\infty$ and $x = -\infty$?

21.66 • A charge $q_1 = +5.00$ nC is placed at the origin of an xy -coordinate system, and a charge $q_2 = -2.00$ nC is placed on the positive x -axis at $x = 4.00$ cm. (a) If a third charge $q_3 = +6.00$ nC is now placed at the point $x = 4.00$ cm, $y = 3.00$ cm, find the x - and y -components of the total force exerted on this charge by the other two. (b) Find the magnitude and direction of this force.

21.67 • **CP** Two positive point charges Q are held fixed on the x -axis at $x = a$ and $x = -a$. A third positive point charge q , with mass m , is placed on the x -axis away from the origin at a coordinate x such that $|x| \ll a$. The charge q , which is free to move along the x -axis, is then released. (a) Find the frequency of oscillation of the charge q . (*Hint:* Review the definition of simple harmonic motion in Section 14.2. Use the binomial expansion $(1+z)^n = 1 + nz + n(n-1)z^2/2 + \dots$, valid for the case $|z| < 1$.) (b) Suppose instead that the charge q were placed on the y -axis at a coordinate y such that $|y| \ll a$, and then released. If this charge is free to move anywhere in the xy -plane, what will happen to it? Explain your answer.

21.68 • **CP** Two identical spheres with mass m are hung from silk threads of length L , as shown in Fig. P21.68. Each sphere has the same charge, so $q_1 = q_2 = q$. The radius of each sphere is very small compared to the distance between the spheres, so they may be treated as point charges. Show that if the angle θ is small, the equilibrium separation d between the spheres is $d = (q^2 L / 2\pi\epsilon_0 mg)^{1/3}$. (*Hint:* If θ is small, then $\tan \theta \cong \sin \theta$.)

Figure P21.68



21.69 • **CP** Two small spheres with mass $m = 15.0$ g are hung by silk threads of length $L = 1.20$ m from a common point (Fig. P21.68). When the spheres are given equal quantities of negative charge, so that $q_1 = q_2 = q$, each thread hangs at $\theta = 25.0^\circ$ from the vertical. (a) Draw a diagram showing the forces on each sphere. Treat the spheres as point charges. (b) Find the magnitude of q . (c) Both threads are now shortened to length $L = 0.600$ m, while the charges q_1 and q_2 remain unchanged. What new angle will each thread make with the vertical? (*Hint:* This part of the problem can be solved numerically)

by using trial values for θ and adjusting the values of θ until a self-consistent answer is obtained.)

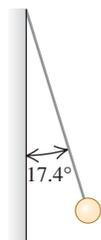
21.70 •• CP Two identical spheres are each attached to silk threads of length $L = 0.500$ m and hung from a common point (Fig. P21.68). Each sphere has mass $m = 8.00$ g. The radius of each sphere is very small compared to the distance between the spheres, so they may be treated as point charges. One sphere is given positive charge q_1 , and the other a different positive charge q_2 ; this causes the spheres to separate so that when the spheres are in equilibrium, each thread makes an angle $\theta = 20.0^\circ$ with the vertical. (a) Draw a free-body diagram for each sphere when in equilibrium, and label all the forces that act on each sphere. (b) Determine the magnitude of the electrostatic force that acts on each sphere, and determine the tension in each thread. (c) Based on the information you have been given, what can you say about the magnitudes of q_1 and q_2 ? Explain your answers. (d) A small wire is now connected between the spheres, allowing charge to be transferred from one sphere to the other until the two spheres have equal charges; the wire is then removed. Each thread now makes an angle of 30.0° with the vertical. Determine the original charges. (*Hint:* The total charge on the pair of spheres is conserved.)

21.71 •• Sodium chloride (NaCl, ordinary table salt) is made up of positive sodium ions (Na^+) and negative chloride ions (Cl^-). (a) If a point charge with the same charge and mass as all the Na^+ ions in 0.100 mol of NaCl is 2.00 cm from a point charge with the same charge and mass as all the Cl^- ions, what is the magnitude of the attractive force between these two point charges? (b) If the positive point charge in part (a) is held in place and the negative point charge is released from rest, what is its initial acceleration? (See Appendix D for atomic masses.) (c) Does it seem reasonable that the ions in NaCl could be separated in this way? Why or why not? (In fact, when sodium chloride dissolves in water, it breaks up into Na^+ and Cl^- ions. However, in this situation there are additional electric forces exerted by the water molecules on the ions.)

21.72 •• A -5.00 -nC point charge is on the x -axis at $x = 1.20$ m. A second point charge Q is on the x -axis at -0.600 m. What must be the sign and magnitude of Q for the resultant electric field at the origin to be (a) 45.0 N/C in the $+x$ -direction, (b) 45.0 N/C in the $-x$ -direction?

21.73 •• CP A small 12.3-g plastic ball is tied to a very light 28.6-cm string that is attached to the vertical wall of a room (Fig. P21.73). A uniform horizontal electric field exists in this room. When the ball has been given an excess charge of -1.11 μC , you observe that it remains suspended, with the string making an angle of 17.4° with the wall. Find the magnitude and direction of the electric field in the room.

Figure P21.73

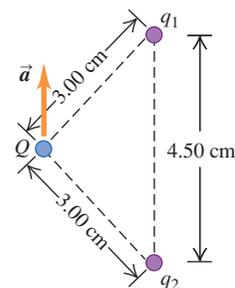


21.74 •• CP At $t = 0$ a very small object with mass 0.400 mg and charge $+9.00$ μC is traveling at 125 m/s in the $-x$ -direction. The charge is moving in a uniform electric field that is in the $+y$ -direction and that has magnitude $E = 895$ N/C. The gravitational force on the particle can be neglected. How far is the particle from the origin at $t = 7.00$ ms?

21.75 •• Two particles having charges $q_1 = 0.500$ nC and $q_2 = 8.00$ nC are separated by a distance of 1.20 m. At what point along the line connecting the two charges is the total electric field due to the two charges equal to zero?

21.76 ••• Two point charges q_1 and q_2 are held in place 4.50 cm apart. Another point charge $Q = -1.75$ μC of mass 5.00 g is initially located 3.00 cm from each of these charges (Fig. P21.76) and released from rest. You observe that the initial acceleration of Q is 324 m/s² upward, parallel to the line connecting the two point charges. Find q_1 and q_2 .

Figure P21.76



21.77 • Three identical point charges q are placed at each of three corners of a square of side L . Find the magnitude and direction of the net force on a point charge $-3q$ placed (a) at the center of the square and (b) at the vacant corner of the square. In each case, draw a free-body diagram showing the forces exerted on the $-3q$ charge by each of the other three charges.

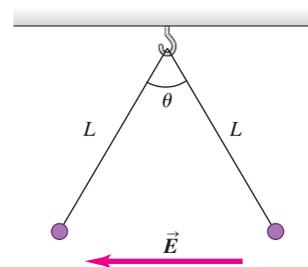
21.78 ••• Three point charges are placed on the y -axis: a charge q at $y = a$, a charge $-2q$ at the origin, and a charge q at $y = -a$. Such an arrangement is called an electric quadrupole. (a) Find the magnitude and direction of the electric field at points on the positive x -axis. (b) Use the binomial expansion to find an approximate expression for the electric field valid for $x \gg a$. Contrast this behavior to that of the electric field of a point charge and that of the electric field of a dipole.

21.79 •• CP Strength of the Electric Force. Imagine two 1.0-g bags of protons, one at the earth's north pole and the other at the south pole. (a) How many protons are in each bag? (b) Calculate the gravitational attraction and the electrical repulsion that each bag exerts on the other. (c) Are the forces in part (b) large enough for you to feel if you were holding one of the bags?

21.80 • Electric Force Within the Nucleus. Typical dimensions of atomic nuclei are of the order of 10^{-15} m (1 fm). (a) If two protons in a nucleus are 2.0 fm apart, find the magnitude of the electric force each one exerts on the other. Express the answer in newtons and in pounds. Would this force be large enough for a person to feel? (b) Since the protons repel each other so strongly, why don't they shoot out of the nucleus?

21.81 •• If Atoms Were Not Neutral . . . Because the charges on the electron and proton have the same absolute value, atoms are electrically neutral. Suppose this were not precisely true, and the absolute value of the charge of the electron were less than the charge of the proton by 0.00100%. (a) Estimate what the net charge of this textbook would be under these circumstances. Make any assumptions you feel are justified, but state clearly what they are. (*Hint:* Most of the atoms in this textbook have equal numbers of electrons, protons, and neutrons.) (b) What would be the magnitude of the electric force between two textbooks placed 5.0 m apart? Would this force be attractive or repulsive? Estimate what the acceleration of each book would be if the books were 5.0 m apart and there were no non-electric forces on them. (c) Discuss how the fact that ordinary matter is stable shows that the absolute values of the charges on the electron and proton must be identical to a very high level of accuracy.

Figure P21.82



21.82 ••• CP Two tiny spheres of mass 6.80 mg carry charges of equal magnitude,

72.0 nC, but opposite sign. They are tied to the same ceiling hook by light strings of length 0.530 m. When a horizontal uniform electric field E that is directed to the left is turned on, the spheres hang at rest with the angle θ between the strings equal to 50.0° (Fig. P21.82). (a) Which ball (the one on the right or the one on the left) has positive charge? (b) What is the magnitude E of the field?

21.83 •• CP Consider a model of a hydrogen atom in which an electron is in a circular orbit of radius $r = 5.29 \times 10^{-11}$ m around a stationary proton. What is the speed of the electron in its orbit?

21.84 •• CP A small sphere with mass $9.00 \mu\text{g}$ and charge $-4.30 \mu\text{C}$ is moving in a circular orbit around a stationary sphere that has charge $+7.50 \mu\text{C}$. If the speed of the small sphere is 5.90×10^3 m/s, what is the radius of its orbit? Treat the spheres as point charges and ignore gravity.

21.85 •• Two small copper spheres each have radius 1.00 mm. (a) How many atoms does each sphere contain? (b) Assume that each copper atom contains 29 protons and 29 electrons. We know that electrons and protons have charges of exactly the same magnitude, but let's explore the effect of small differences (see also Problem 21.81). If the charge of a proton is $+e$ and the magnitude of the charge of an electron is 0.100% smaller, what is the net charge of each sphere and what force would one sphere exert on the other if they were separated by 1.00 m?

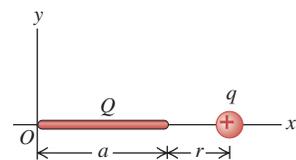
21.86 ••• CP Operation of an Inkjet Printer. In an inkjet printer, letters are built up by squirting drops of ink at the paper from a rapidly moving nozzle. The ink drops, which have a mass of 1.4×10^{-8} g each, leave the nozzle and travel toward the paper at 20 m/s, passing through a charging unit that gives each drop a positive charge q by removing some electrons from it. The drops then pass between parallel deflecting plates 2.0 cm long where there is a uniform vertical electric field with magnitude 8.0×10^4 N/C. If a drop is to be deflected 0.30 mm by the time it reaches the end of the deflection plates, what magnitude of charge must be given to the drop?

21.87 •• CP A proton is projected into a uniform electric field that points vertically upward and has magnitude E . The initial velocity of the proton has a magnitude v_0 and is directed at an angle α below the horizontal. (a) Find the maximum distance h_{max} that the proton descends vertically below its initial elevation. You can ignore gravitational forces. (b) After what horizontal distance d does the proton return to its original elevation? (c) Sketch the trajectory of the proton. (d) Find the numerical values of h_{max} and d if $E = 500$ N/C, $v_0 = 4.00 \times 10^5$ m/s, and $\alpha = 30.0^\circ$.

21.88 • A negative point charge $q_1 = -4.00$ nC is on the x -axis at $x = 0.60$ m. A second point charge q_2 is on the x -axis at $x = -1.20$ m. What must the sign and magnitude of q_2 be for the net electric field at the origin to be (a) 50.0 N/C in the $+x$ -direction and (b) 50.0 N/C in the $-x$ -direction?

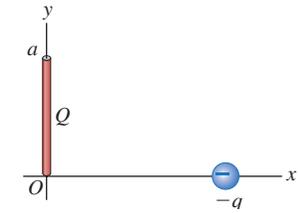
21.89 •• CALC Positive charge Q is distributed uniformly along the x -axis from $x = 0$ to $x = a$. A positive point charge q is located on the positive x -axis at $x = a + r$, a distance r to the right of the end of Q (Fig. P21.89). (a) Calculate the x - and y -components of the electric field produced by the charge distribution Q at points on the positive x -axis where $x > a$. (b) Calculate the force (magnitude and direction) that the charge distribution Q exerts on q . (c) Show that if $r \gg a$, the magnitude of the force in part (b) is approximately $Qq/4\pi\epsilon_0 r^2$. Explain why this result is obtained.

Figure P21.89



21.90 •• CALC Positive charge Q is distributed uniformly along the positive y -axis between $y = 0$ and $y = a$. A negative point charge $-q$ lies on the positive x -axis, a distance x from the origin (Fig. P21.90). (a) Calculate the x - and y -components of the electric field produced by the charge distribution Q at points on the positive x -axis. (b) Calculate the x - and y -components of the force that the charge distribution Q exerts on q . (c) Show that if $x \gg a$, $F_x \cong -Qq/4\pi\epsilon_0 x^2$ and $F_y \cong +Qqa/8\pi\epsilon_0 x^3$. Explain why this result is obtained.

Figure P21.90



21.91 •• A charged line like that shown in Fig. 21.24 extends from $y = 2.50$ cm to $y = -2.50$ cm. The total charge distributed uniformly along the line is -7.00 nC. (a) Find the electric field (magnitude and direction) on the x -axis at $x = 10.0$ cm. (b) Is the magnitude of the electric field you calculated in part (a) larger or smaller than the electric field 10.0 cm from a point charge that has the same total charge as this finite line of charge? In terms of the approximation used to derive $E = Q/4\pi\epsilon_0 x^2$ for a point charge from Eq. (21.9), explain why this is so. (c) At what distance x does the result for the finite line of charge differ by 1.0% from that for the point charge?

21.92 • CP A Parallel Universe. Imagine a parallel universe in which the electric force has the same properties as in our universe but there is no gravity. In this parallel universe, the sun carries charge Q , the earth carries charge $-Q$, and the electric attraction between them keeps the earth in orbit. The earth in the parallel universe has the same mass, the same orbital radius, and the same orbital period as in our universe. Calculate the value of Q . (Consult Appendix F as needed.)

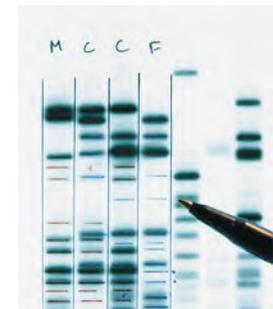
21.93 ••• A uniformly charged disk like the disk in Fig. 21.25 has radius 2.50 cm and carries a total charge of 7.0×10^{-12} C. (a) Find the electric field (magnitude and direction) on the x -axis at $x = 20.0$ cm. (b) Show that for $x \gg R$, Eq. (21.11) becomes $E = Q/4\pi\epsilon_0 x^2$, where Q is the total charge on the disk. (c) Is the magnitude of the electric field you calculated in part (a) larger or smaller than the electric field 20.0 cm from a point charge that has the same total charge as this disk? In terms of the approximation used in part (b) to derive $E = Q/4\pi\epsilon_0 x^2$ for a point charge from Eq. (21.11), explain why this is so. (d) What is the percent difference between the electric fields produced by the finite disk and by a point charge with the same charge at $x = 20.0$ cm and at $x = 10.0$ cm?

21.94 •• BIO Electrophoresis.

Electrophoresis is a process used by biologists to separate different biological molecules (such as proteins) from each other according to their ratio of charge to size. The materials to be separated are in a viscous solution that produces a drag force F_D proportional to the size and speed of the molecule.

We can express this relationship as $F_D = KRv$, where R is the radius of the molecule (modeled as being spherical), v is its speed, and K is a constant that depends on the viscosity of the

Figure P21.94



solution. The solution is placed in an external electric field E so that the electric force on a particle of charge q is $F = qE$. (a) Show that when the electric field is adjusted so that the two forces (electric and viscous drag) just balance, the ratio of q to R is Kv/E . (b) Show that if we leave the electric field on for a time T , the distance x that the molecule moves during that time is $x = (ET/k)(q/R)$. (c) Suppose you have a sample containing three different biological molecules for which the molecular ratio q/R for material 2 is twice that of material 1 and the ratio for material 3 is three times that of material 1. Show that the distances migrated by these molecules after the same amount of time are $x_2 = 2x_1$ and $x_3 = 3x_1$. In other words, material 2 travels twice as far as material 1, and material 3 travels three times as far as material 1. Therefore, we have separated these molecules according to their ratio of charge to size. In practice, this process can be carried out in a special gel or paper, along which the biological molecules migrate. (Fig. P21.94). The process can be rather slow, requiring several hours for separations of just a centimeter or so.

21.95 • CALC Positive charge $+Q$ is distributed uniformly along the $+x$ -axis from $x = 0$ to $x = a$. Negative charge $-Q$ is distributed uniformly along the $-x$ -axis from $x = 0$ to $x = -a$. (a) A positive point charge q lies on the positive y -axis, a distance y from the origin. Find the force (magnitude and direction) that the positive and negative charge distributions together exert on q . Show that this force is proportional to y^{-3} for $y \gg a$. (b) Suppose instead that the positive point charge q lies on the positive x -axis, a distance $x > a$ from the origin. Find the force (magnitude and direction) that the charge distribution exerts on q . Show that this force is proportional to x^{-3} for $x \gg a$.

21.96 •• CP A small sphere with mass m carries a positive charge q and is attached to one end of a silk fiber of length L . The other end of the fiber is attached to a large vertical insulating sheet that has a positive surface charge density σ . Show that when the sphere is in equilibrium, the fiber makes an angle equal to $\arctan(q\sigma/2mg\epsilon_0)$ with the vertical sheet.

21.97 •• CALC Negative charge $-Q$ is distributed uniformly around a quarter-circle of radius a that lies in the first quadrant, with the center of curvature at the origin. Find the x - and y -components of the net electric field at the origin.

21.98 •• CALC A semicircle of radius a is in the first and second quadrants, with the center of curvature at the origin. Positive charge $+Q$ is distributed uniformly around the left half of the semicircle, and negative charge $-Q$ is distributed uniformly around the right half of the semicircle (Fig. P21.98). What are the magnitude and direction of the net electric field at the origin produced by this distribution of charge?

21.99 •• Two 1.20-m nonconducting wires meet at a right angle. One segment carries $+2.50 \mu\text{C}$ of charge distributed uniformly along its length, and the other carries $-2.50 \mu\text{C}$ distributed uniformly along it, as shown in Fig. P21.99.

Figure P21.98

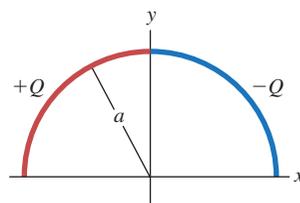
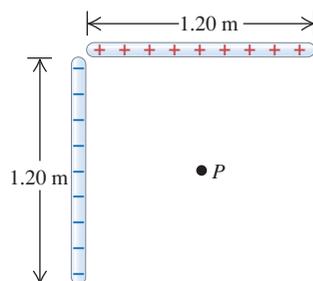


Figure P21.99



(a) Find the magnitude and direction of the electric field these wires produce at point P , which is 60.0 cm from each wire. (b) If an electron is released at P , what are the magnitude and direction of the net force that these wires exert on it?

21.100 • Two very large parallel sheets are 5.00 cm apart. Sheet A carries a uniform surface charge density of $-9.50 \mu\text{C}/\text{m}^2$, and sheet B , which is to the right of A , carries a uniform charge density of $-11.6 \mu\text{C}/\text{m}^2$. Assume the sheets are large enough to be treated as infinite. Find the magnitude and direction of the net electric field these sheets produce at a point (a) 4.00 cm to the right of sheet A ; (b) 4.00 cm to the left of sheet A ; (c) 4.00 cm to the right of sheet B .

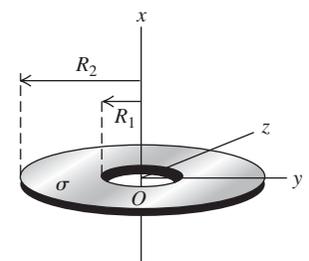
21.101 • Repeat Problem 21.100 for the case where sheet B is positive.

21.102 • Two very large horizontal sheets are 4.25 cm apart and carry equal but opposite uniform surface charge densities of magnitude σ . You want to use these sheets to hold stationary in the region between them an oil droplet of mass $324 \mu\text{g}$ that carries an excess of five electrons. Assuming that the drop is in vacuum, (a) which way should the electric field between the plates point, and (b) what should σ be?

21.103 •• An infinite sheet with positive charge per unit area σ lies in the xy -plane. A second infinite sheet with negative charge per unit area $-\sigma$ lies in the yz -plane. Find the net electric field at all points that do not lie in either of these planes. Express your answer in terms of the unit vectors \hat{i} , \hat{j} , and \hat{k} .

21.104 •• CP A thin disk with a circular hole at its center, called an *annulus*, has inner radius R_1 and outer radius R_2 (Fig. P21.104). The disk has a uniform positive surface charge density σ on its surface. (a) Determine the total electric charge on the annulus. (b) The annulus lies in the yz -plane, with its center at the origin. For an arbitrary point on the x -axis (the axis of the annulus),

Figure P21.104

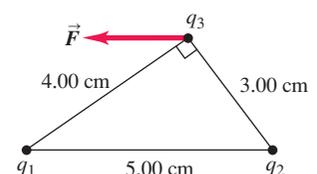


find the magnitude and direction of the electric field \vec{E} . Consider points both above and below the annulus in Fig. P21.104. (c) Show that at points on the x -axis that are sufficiently close to the origin, the magnitude of the electric field is approximately proportional to the distance between the center of the annulus and the point. How close is “sufficiently close”? (d) A point particle with mass m and negative charge $-q$ is free to move along the x -axis (but cannot move off the axis). The particle is originally placed at rest at $x = 0.01R_1$ and released. Find the frequency of oscillation of the particle. (*Hint*: Review Section 14.2. The annulus is held stationary.)

CHALLENGE PROBLEMS

21.105 ••• Three charges are placed as shown in Fig. P21.105. The magnitude of q_1 is $2.00 \mu\text{C}$, but its sign and the value of the charge q_2 are not known. Charge q_3 is $+4.00 \mu\text{C}$, and the net force \vec{F} on q_3 is entirely in the negative x -direction. (a) Considering the different possible signs of q_1 , there are four possible force diagrams representing the forces \vec{F}_1 and \vec{F}_2 that q_1 and q_2 exert on q_3 . Sketch these four possible force configurations.

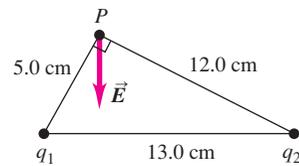
Figure P21.105



(b) Using the sketches from part (a) and the direction of \vec{F} , deduce the signs of the charges q_1 and q_2 . (c) Calculate the magnitude of q_2 . (d) Determine F , the magnitude of the net force on q_3 .

21.106 •• Two charges are placed as shown in Fig. P21.106. The magnitude of q_1 is $3.00 \mu\text{C}$, but its sign and the value of the charge q_2 are not known. The direction of the net electric field \vec{E} at point P is entirely in the negative y -direction. (a) Considering the different possible signs of q_1 and q_2 , there are four possible diagrams that could represent the electric fields \vec{E}_1 and \vec{E}_2 produced by q_1 and q_2 . Sketch the four possible electric-field configurations. (b) Using the sketches from part (a) and the direction of \vec{E} , deduce the signs of q_1 and q_2 . (c) Determine the magnitude of \vec{E} .

Figure P21.106



21.107 ••• **CALC** Two thin rods of length L lie along the x -axis, one between $x = a/2$ and $x = a/2 + L$ and the other between $x = -a/2$ and $x = -a/2 - L$. Each rod has positive charge Q distributed uniformly along its length. (a) Calculate the electric field produced by the second rod at points along the positive x -axis. (b) Show that the magnitude of the force that one rod exerts on the other is

$$F = \frac{Q^2}{4\pi\epsilon_0 L^2} \ln \left[\frac{(a+L)^2}{a(a+2L)} \right]$$

(c) Show that if $a \gg L$, the magnitude of this force reduces to $F = Q^2/4\pi\epsilon_0 a^2$. (Hint: Use the expansion $\ln(1+z) = z - z^2/2 + z^3/3 - \dots$, valid for $|z| \ll 1$. Carry all expansions to at least order L^2/a^2 .) Interpret this result.

Answers

Chapter Opening Question ?

Water molecules have a permanent electric dipole moment: One end of the molecule has a positive charge and the other end has a negative charge. These ends attract negative and positive ions, respectively, holding the ions apart in solution. Water is less effective as a solvent for materials whose molecules do not ionize (called *nonionic* substances), such as oils.

Test Your Understanding Questions

21.1 Answers: (a) the plastic rod weighs more, (b) the glass rod weighs less, (c) the fur weighs less, (d) the silk weighs more The plastic rod gets a negative charge by taking electrons from the fur, so the rod weighs a little more and the fur weighs a little less after the rubbing. By contrast, the glass rod gets a positive charge by giving electrons to the silk. Hence, after they are rubbed together, the glass rod weighs a little less and the silk weighs a little more. The weight change is *very* small: The number of electrons transferred is a small fraction of a mole, and a mole of electrons has a mass of only $(6.02 \times 10^{23} \text{ electrons})(9.11 \times 10^{-31} \text{ kg/electron}) = 5.48 \times 10^{-7} \text{ kg} = 0.548 \text{ milligram}$!

21.2 Answers: (a) (i), (b) (ii) Before the two spheres touch, the negatively charged sphere exerts a repulsive force on the electrons in the other sphere, causing zones of positive and negative induced charge (see Fig. 21.7b). The positive zone is closer to the negatively charged sphere than the negative zone, so there is a net force of attraction that pulls the spheres together, like the comb and insulator in Fig. 21.8b. Once the two metal spheres touch, some of the excess electrons on the negatively charged sphere will flow onto the other sphere (because metals are conductors). Then both spheres will have a net negative charge and will repel each other.

21.3 Answer: (iv) The force exerted by q_1 on Q is still as in Example 21.4. The magnitude of the force exerted by q_2 on Q is still equal to $F_{1 \text{ on } Q}$, but the direction of the force is now *toward* q_2 at an angle α below the x -axis. Hence the x -components of the two forces cancel while the (negative) y -components add together, and the total electric force is in the negative y -direction.

21.4 Answers: (a) (ii), (b) (i) The electric field \vec{E} produced by a positive point charge points directly away from the charge (see Fig. 21.18a) and has a magnitude that depends on the distance r from the charge to the field point. Hence a second, negative point charge $q < 0$ will feel a force $\vec{F} = q\vec{E}$ that points directly toward the positive charge and has a magnitude that depends on the distance r between the two charges. If the negative charge moves directly toward the positive charge, the direction of the force remains the same but the force magnitude increases as the distance r decreases. If the negative charge moves in a circle around the positive charge, the force magnitude stays the same (because the distance r is constant) but the force direction changes.

21.5 Answer: (iv) Think of a pair of segments of length dy , one at coordinate $y > 0$ and the other at coordinate $-y < 0$. The upper segment has a positive charge and produces an electric field $d\vec{E}$ at P that points away from the segment, so this $d\vec{E}$ has a positive x -component and a negative y -component, like the vector $d\vec{E}$ in Fig. 21.24. The lower segment has the same amount of negative charge. It produces a $d\vec{E}$ that has the same magnitude but points *toward* the lower segment, so it has a negative x -component and a negative y -component. By symmetry, the two x -components are equal but opposite, so they cancel. Thus the total electric field has only a negative y -component.

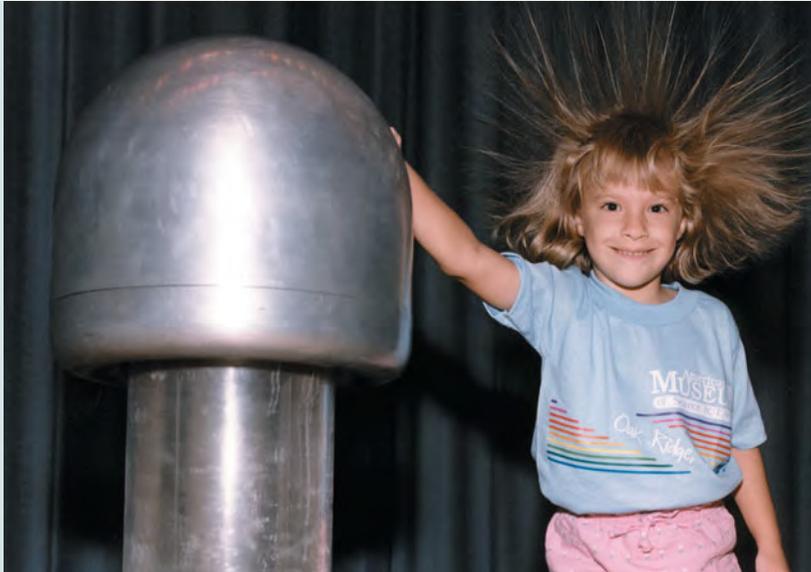
21.6 Answer: yes If the field lines are straight, \vec{E} must point in the same direction throughout the region. Hence the force $\vec{F} = q\vec{E}$ on a particle of charge q is always in the same direction. A particle released from rest accelerates in a straight line in the direction of \vec{F} , and so its trajectory is a straight line along a field line.

21.7 Answer: (ii) Equations (21.17) and (21.18) tell us that the potential energy for a dipole in an electric field is $U = -\vec{p} \cdot \vec{E} = -pE \cos \phi$, where ϕ is the angle between the directions of \vec{p} and \vec{E} . If \vec{p} and \vec{E} point in opposite directions, so that $\phi = 180^\circ$, we have $\cos \phi = -1$ and $U = +pE$. This is the maximum value that U can have. From our discussion of energy diagrams in Section 7.5, it follows that this is a situation of unstable equilibrium.

Bridging Problem

Answer: $E = 2kQ/\pi a^2$ in the $-y$ -direction

GAUSS'S LAW



? This child acquires an electric charge by touching the charged metal sphere. The charged hairs on the child's head repel and stand out. If the child stands *inside* a large, charged metal sphere, will her hair stand on end?

Often, there are both an easy way and a hard way to do a job; the easy way may involve nothing more than using the right tools. In physics, an important tool for simplifying problems is the *symmetry properties* of systems. Many physical systems have symmetry; for example, a cylindrical body doesn't look any different after you've rotated it around its axis, and a charged metal sphere looks just the same after you've turned it about any axis through its center.

Gauss's law is part of the key to using symmetry considerations to simplify electric-field calculations. For example, the field of a straight-line or plane-sheet charge distribution, which we derived in Section 21.5 using some fairly strenuous integrations, can be obtained in a few lines with the help of Gauss's law. But Gauss's law is more than just a way to make certain calculations easier. Indeed, it is a fundamental statement about the relationship between electric charges and electric fields. Among other things, Gauss's law can help us understand how electric charge distributes itself over conducting bodies.

Here's what Gauss's law is all about. Given any general distribution of charge, we surround it with an imaginary surface that encloses the charge. Then we look at the electric field at various points on this imaginary surface. Gauss's law is a relationship between the field at *all* the points on the surface and the total charge enclosed within the surface. This may sound like a rather indirect way of expressing things, but it turns out to be a tremendously useful relationship. Above and beyond its use as a calculational tool, Gauss's law can help us gain deeper insights into electric fields. We will make use of these insights repeatedly in the next several chapters as we pursue our study of electromagnetism.

22.1 Charge and Electric Flux

In Chapter 21 we asked the question, "Given a charge distribution, what is the electric field produced by that distribution at a point P ?" We saw that the answer could be found by representing the distribution as an assembly of point charges,

LEARNING GOALS

By studying this chapter, you will learn:

- How you can determine the amount of charge within a closed surface by examining the electric field on the surface.
- What is meant by electric flux, and how to calculate it.
- How Gauss's law relates the electric flux through a closed surface to the charge enclosed by the surface.
- How to use Gauss's law to calculate the electric field due to a symmetric charge distribution.
- Where the charge is located on a charged conductor.

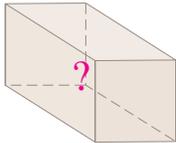
The discussion of Gauss's law in this section is based on and inspired by the innovative ideas of Ruth W. Chabay and Bruce A. Sherwood in *Electric and Magnetic Interactions* (John Wiley & Sons, 1994).

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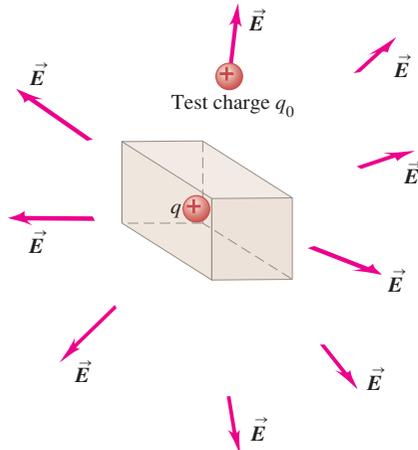
ActivPhysics 11.7: Electric Flux

22.1 How can you measure the charge inside a box without opening it?

(a) A box containing an unknown amount of charge



(b) Using a test charge outside the box to probe the amount of charge inside the box



each of which produces an electric field \vec{E} given by Eq. (21.7). The total field at P is then the vector sum of the fields due to all the point charges.

But there is an alternative relationship between charge distributions and electric fields. To discover this relationship, let's stand the question of Chapter 21 on its head and ask, "If the electric field pattern is known in a given region, what can we determine about the charge distribution in that region?"

Here's an example. Consider the box shown in Fig. 22.1a, which may or may not contain electric charge. We'll imagine that the box is made of a material that has no effect on any electric fields; it's of the same breed as the massless rope and the frictionless incline. Better still, let the box represent an *imaginary* surface that may or may not enclose some charge. We'll refer to the box as a **closed surface** because it completely encloses a volume. How can you determine how much (if any) electric charge lies within the box?

Knowing that a charge distribution produces an electric field and that an electric field exerts a force on a test charge, you move a test charge q_0 around the vicinity of the box. By measuring the force \vec{F} experienced by the test charge at different positions, you make a three-dimensional map of the electric field $\vec{E} = \vec{F}/q_0$ outside the box. In the case shown in Fig. 22.1b, the map turns out to be the same as that of the electric field produced by a positive point charge (Fig. 21.28a). From the details of the map, you can find the exact value of the point charge inside the box.

To determine the contents of the box, we actually need to measure \vec{E} only on the *surface* of the box. In Fig. 22.2a there is a single *positive* point charge inside the box, and in Fig. 22.2b there are two such charges. The field patterns on the surfaces of the boxes are different in detail, but in each case the electric field points *out* of the box. Figures 22.2c and 22.2d show cases with one and two *negative* point charges, respectively, inside the box. Again, the details of \vec{E} are different for the two cases, but the electric field points *into* each box.

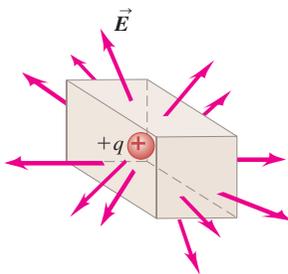
Electric Flux and Enclosed Charge

In Section 21.4 we mentioned the analogy between electric-field vectors and the velocity vectors of a fluid in motion. This analogy can be helpful, even though an electric field does not actually "flow." Using this analogy, in Figs. 22.2a and 22.2b, in which the electric field vectors point out of the surface, we say that there is an *outward electric flux*. (The word "flux" comes from a Latin word meaning "flow.") In Figs. 22.2c and 22.2d the \vec{E} vectors point into the surface, and the electric flux is *inward*.

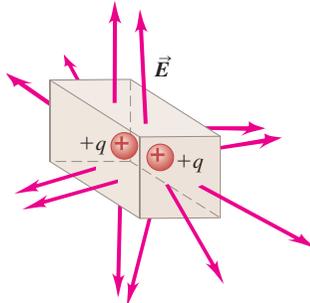
Figure 22.2 suggests a simple relationship: Positive charge inside the box goes with an outward electric flux through the box's surface, and negative charge inside goes with an inward electric flux. What happens if there is *zero* charge

22.2 The electric field on the surface of boxes containing (a) a single positive point charge, (b) two positive point charges, (c) a single negative point charge, or (d) two negative point charges.

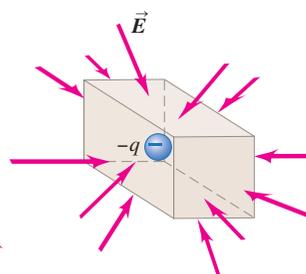
(a) Positive charge inside box, outward flux



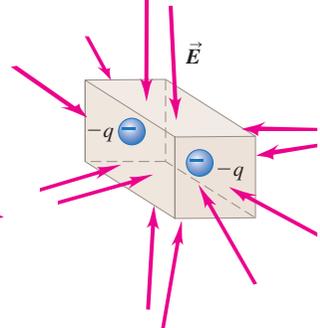
(b) Positive charges inside box, outward flux



(c) Negative charge inside box, inward flux



(d) Negative charges inside box, inward flux



inside the box? In Fig. 22.3a the box is empty and $\vec{E} = \mathbf{0}$ everywhere, so there is no electric flux into or out of the box. In Fig. 22.3b, one positive and one negative point charge of equal magnitude are enclosed within the box, so the *net* charge inside the box is zero. There is an electric field, but it “flows into” the box on half of its surface and “flows out of” the box on the other half. Hence there is no *net* electric flux into or out of the box.

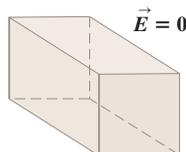
The box is again empty in Fig. 22.3c. However, there is charge present *outside* the box; the box has been placed with one end parallel to a uniformly charged infinite sheet, which produces a uniform electric field perpendicular to the sheet (as we learned in Example 21.11 of Section 21.5). On one end of the box, \vec{E} points into the box; on the opposite end, \vec{E} points out of the box; and on the sides, \vec{E} is parallel to the surface and so points neither into nor out of the box. As in Fig. 22.3b, the inward electric flux on one part of the box exactly compensates for the outward electric flux on the other part. So in all of the cases shown in Fig. 22.3, there is no *net* electric flux through the surface of the box, and no *net* charge is enclosed in the box.

Figures 22.2 and 22.3 demonstrate a connection between the *sign* (positive, negative, or zero) of the *net* charge enclosed by a closed surface and the sense (outward, inward, or none) of the net electric flux through the surface. There is also a connection between the *magnitude* of the net charge inside the closed surface and the *strength* of the net “flow” of \vec{E} over the surface. In both Figs. 22.4a and 22.4b there is a single point charge inside the box, but in Fig. 22.4b the magnitude of the charge is twice as great, and so \vec{E} is everywhere twice as great in magnitude as in Fig. 22.4a. If we keep in mind the fluid-flow analogy, this means that the net outward electric flux is also twice as great in Fig. 22.4b as in Fig. 22.4a. This suggests that the net electric flux through the surface of the box is *directly proportional* to the magnitude of the net charge enclosed by the box.

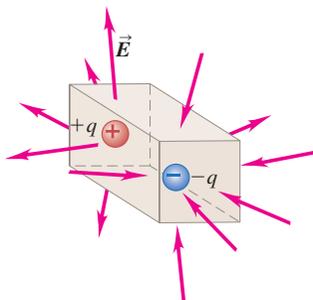
This conclusion is independent of the size of the box. In Fig. 22.4c the point charge $+q$ is enclosed by a box with twice the linear dimensions of the box in Fig. 22.4a. The magnitude of the electric field of a point charge decreases with distance according to $1/r^2$, so the average magnitude of \vec{E} on each face of the large box in Fig. 22.4c is just $\frac{1}{4}$ of the average magnitude of \vec{E} on the corresponding face in Fig. 22.4a. But each face of the large box has exactly four times the area of the corresponding face of the small box. Hence the outward electric flux is the *same* for the two boxes if we *define* electric flux as follows: For each face of the box, take the product of the average perpendicular component of \vec{E} and the area of that face; then add up the results from all faces of the box. With this definition the net electric flux due to a single point charge inside the box is independent of the size of the box and depends only on the net charge inside the box.

22.3 Three cases in which there is zero *net* charge inside a box and no net electric flux through the surface of the box. (a) An empty box with $\vec{E} = \mathbf{0}$. (b) A box containing one positive and one equal-magnitude negative point charge. (c) An empty box immersed in a uniform electric field.

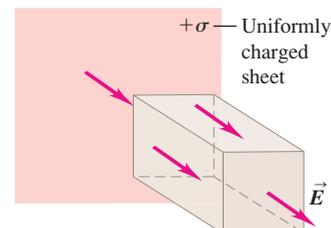
(a) No charge inside box,
zero flux



(b) Zero *net* charge inside box,
inward flux cancels outward flux.

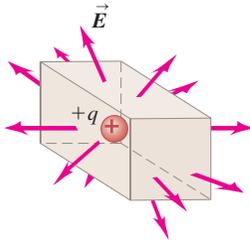


(c) No charge inside box,
inward flux cancels outward flux.

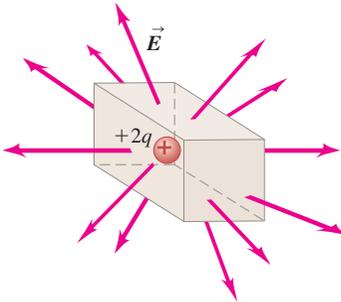


22.4 (a) A box enclosing a positive point charge $+q$. (b) Doubling the charge causes the magnitude of \vec{E} to double, and it doubles the electric flux through the surface. (c) If the charge stays the same but the dimensions of the box are doubled, the flux stays the same. The magnitude of \vec{E} on the surface decreases by a factor of $\frac{1}{4}$, but the area through which \vec{E} “flows” increases by a factor of 4.

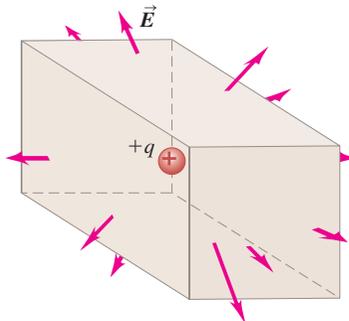
(a) A box containing a charge



(b) Doubling the enclosed charge doubles the flux.



(c) Doubling the box dimensions does not change the flux.



To summarize, for the special cases of a closed surface in the shape of a rectangular box and charge distributions made up of point charges or infinite charged sheets, we have found:

1. Whether there is a net outward or inward electric flux through a closed surface depends on the sign of the enclosed charge.
2. Charges *outside* the surface do not give a net electric flux through the surface.
3. The net electric flux is directly proportional to the net amount of charge enclosed within the surface but is otherwise independent of the size of the closed surface.

These observations are a qualitative statement of *Gauss's law*.

Do these observations hold true for other kinds of charge distributions and for closed surfaces of arbitrary shape? The answer to these questions will prove to be yes. But to explain why this is so, we need a precise mathematical statement of what we mean by electric flux. We develop this in the next section.

Test Your Understanding of Section 22.1 If all of the dimensions of the box in Fig. 22.2a are increased by a factor of 3, what effect will this change have on the electric flux through the box? (i) The flux will be $3^2 = 9$ times greater; (ii) the flux will be 3 times greater; (iii) the flux will be unchanged; (iv) the flux will be $\frac{1}{3}$ as great; (v) the flux will be $(\frac{1}{3})^2 = \frac{1}{9}$ as great; (vi) not enough information is given to decide.



22.2 Calculating Electric Flux

In the preceding section we introduced the concept of *electric flux*. We used this to give a rough qualitative statement of Gauss's law: The net electric flux through a closed surface is directly proportional to the net charge inside that surface. To be able to make full use of this law, we need to know how to *calculate* electric flux. To do this, let's again make use of the analogy between an electric field \vec{E} and the field of velocity vectors \vec{v} in a flowing fluid. (Again, keep in mind that this is only an analogy; an electric field is *not* a flow.)

Flux: Fluid-Flow Analogy

Figure 22.5 shows a fluid flowing steadily from left to right. Let's examine the volume flow rate dV/dt (in, say, cubic meters per second) through the wire rectangle with area A . When the area is perpendicular to the flow velocity \vec{v} (Fig. 22.5a) and the flow velocity is the same at all points in the fluid, the volume flow rate dV/dt is the area A multiplied by the flow speed v :

$$\frac{dV}{dt} = vA$$

When the rectangle is tilted at an angle ϕ (Fig. 22.5b) so that its face is not perpendicular to \vec{v} , the area that counts is the silhouette area that we see when we look in the direction of \vec{v} . This area, which is outlined in red and labeled A_{\perp} in Fig. 22.5b, is the *projection* of the area A onto a surface perpendicular to \vec{v} . Two sides of the projected rectangle have the same length as the original one, but the other two are foreshortened by a factor of $\cos \phi$, so the projected area A_{\perp} is equal to $A \cos \phi$. Then the volume flow rate through A is

$$\frac{dV}{dt} = vA \cos \phi$$

If $\phi = 90^\circ$, $dV/dt = 0$; the wire rectangle is edge-on to the flow, and no fluid passes through the rectangle.

Also, $v \cos \phi$ is the component of the vector \vec{v} perpendicular to the plane of the area A . Calling this component v_{\perp} , we can rewrite the volume flow rate as

$$\frac{dV}{dt} = v_{\perp} A$$

We can express the volume flow rate more compactly by using the concept of *vector area* \vec{A} , a vector quantity with magnitude A and a direction perpendicular to the plane of the area we are describing. The vector area \vec{A} describes both the size of an area and its orientation in space. In terms of \vec{A} , we can write the volume flow rate of fluid through the rectangle in Fig. 22.5b as a scalar (dot) product:

$$\frac{dV}{dt} = \vec{v} \cdot \vec{A}$$

Flux of a Uniform Electric Field

Using the analogy between electric field and fluid flow, we now define electric flux in the same way as we have just defined the volume flow rate of a fluid; we simply replace the fluid velocity \vec{v} by the electric field \vec{E} . The symbol that we use for electric flux is Φ_E (the capital Greek letter phi; the subscript E is a reminder that this is *electric* flux). Consider first a flat area A perpendicular to a uniform electric field \vec{E} (Fig. 22.6a). We define the electric flux through this area to be the product of the field magnitude E and the area A :

$$\Phi_E = EA$$

Roughly speaking, we can picture Φ_E in terms of the field lines passing through A . Increasing the area means that more lines of \vec{E} pass through the area, increasing the flux; a stronger field means more closely spaced lines of \vec{E} and therefore more lines per unit area, so again the flux increases.

If the area A is flat but not perpendicular to the field \vec{E} , then fewer field lines pass through it. In this case the area that counts is the silhouette area that we see when looking in the direction of \vec{E} . This is the area A_{\perp} in Fig. 22.6b and is equal to $A \cos \phi$ (compare to Fig. 22.5b). We generalize our definition of electric flux for a uniform electric field to

$$\Phi_E = EA \cos \phi \quad (\text{electric flux for uniform } \vec{E}, \text{ flat surface}) \quad (22.1)$$

Since $E \cos \phi$ is the component of \vec{E} perpendicular to the area, we can rewrite Eq. (22.1) as

$$\Phi_E = E_{\perp} A \quad (\text{electric flux for uniform } \vec{E}, \text{ flat surface}) \quad (22.2)$$

In terms of the vector area \vec{A} perpendicular to the area, we can write the electric flux as the scalar product of \vec{E} and \vec{A} :

$$\Phi_E = \vec{E} \cdot \vec{A} \quad (\text{electric flux for uniform } \vec{E}, \text{ flat surface}) \quad (22.3)$$

Equations (22.1), (22.2), and (22.3) express the electric flux for a *flat* surface and a *uniform* electric field in different but equivalent ways. The SI unit for electric flux is $1 \text{ N} \cdot \text{m}^2/\text{C}$. Note that if the area is edge-on to the field, \vec{E} and \vec{A} are perpendicular and the flux is zero (Fig. 22.6c).

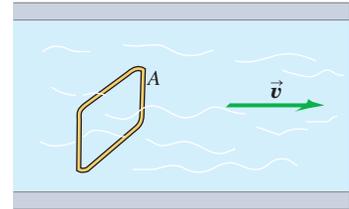
We can represent the direction of a vector area \vec{A} by using a *unit vector* \hat{n} perpendicular to the area; \hat{n} stands for “normal.” Then

$$\vec{A} = A\hat{n} \quad (22.4)$$

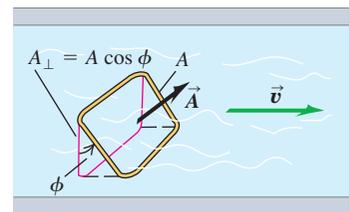
A surface has two sides, so there are two possible directions for \hat{n} and \vec{A} . We must always specify which direction we choose. In Section 22.1 we related the charge inside a *closed* surface to the electric flux through the surface. With a closed surface we will always choose the direction of \hat{n} to be *outward*, and we

22.5 The volume flow rate of fluid through the wire rectangle (a) is vA when the area of the rectangle is perpendicular to \vec{v} and (b) is $vA \cos \phi$ when the rectangle is tilted at an angle ϕ .

(a) A wire rectangle in a fluid



(b) The wire rectangle tilted by an angle ϕ



Application Flux Through a Basking Shark's Mouth

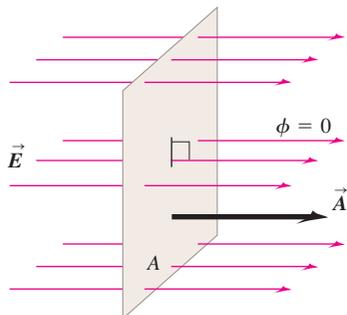
Unlike aggressive carnivorous sharks such as great whites, a basking shark feeds passively on plankton in the water that passes through the shark's gills as it swims. To survive on these tiny organisms requires a huge flux of water through a basking shark's immense mouth, which can be up to a meter across. The water flux—the product of the shark's speed through the water and the area of its mouth—can be up to $0.5 \text{ m}^3/\text{s}$ (500 liters per second, or almost 5×10^5 gallons per hour). In a similar way, the flux of electric field through a surface depends on the magnitude of the field and the area of the surface (as well as the relative orientation of the field and surface).



22.6 A flat surface in a uniform electric field. The electric flux Φ_E through the surface equals the scalar product of the electric field \vec{E} and the area vector \vec{A} .

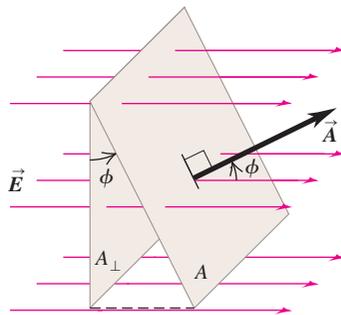
(a) Surface is face-on to electric field:

- \vec{E} and \vec{A} are parallel (the angle between \vec{E} and \vec{A} is $\phi = 0$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA$.



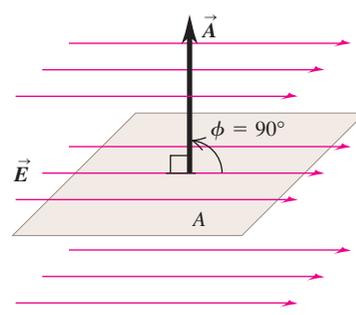
(b) Surface is tilted from a face-on orientation by an angle ϕ :

- The angle between \vec{E} and \vec{A} is ϕ .
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \phi$.



(c) Surface is edge-on to electric field:

- \vec{E} and \vec{A} are perpendicular (the angle between \vec{E} and \vec{A} is $\phi = 90^\circ$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos 90^\circ = 0$.



will speak of the flux *out of* a closed surface. Thus what we called “outward electric flux” in Section 22.1 corresponds to a *positive* value of Φ_E , and what we called “inward electric flux” corresponds to a *negative* value of Φ_E .

Flux of a Nonuniform Electric Field

What happens if the electric field \vec{E} isn't uniform but varies from point to point over the area A ? Or what if A is part of a curved surface? Then we divide A into many small elements dA , each of which has a unit vector \hat{n} perpendicular to it and a vector area $d\vec{A} = \hat{n} dA$. We calculate the electric flux through each element and integrate the results to obtain the total flux:

$$\Phi_E = \int E \cos \phi \, dA = \int E_{\perp} \, dA = \int \vec{E} \cdot d\vec{A} \quad (\text{general definition of electric flux}) \quad (22.5)$$

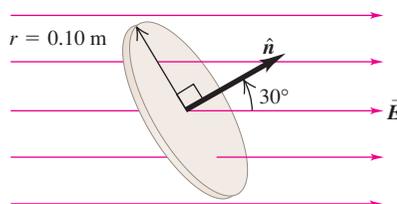
We call this integral the **surface integral** of the component E_{\perp} over the area, or the surface integral of $\vec{E} \cdot d\vec{A}$. In specific problems, one form of the integral is sometimes more convenient than another. Example 22.3 at the end of this section illustrates the use of Eq. (22.5).

In Eq. (22.5) the electric flux $\int E_{\perp} \, dA$ is equal to the *average* value of the perpendicular component of the electric field, multiplied by the area of the surface. This is the same definition of electric flux that we were led to in Section 22.1, now expressed more mathematically. In the next section we will see the connection between the total electric flux through *any* closed surface, no matter what its shape, and the amount of charge enclosed within that surface.

Example 22.1 Electric flux through a disk

A disk of radius 0.10 m is oriented with its normal unit vector \hat{n} at 30° to a uniform electric field \vec{E} of magnitude $2.0 \times 10^3 \text{ N/C}$ (Fig. 22.7). (Since this isn't a closed surface, it has no “inside” or “outside.” That's why we have to specify the direction of \hat{n} in the figure.) (a) What is the electric flux through the disk? (b) What is the flux through the disk if it is turned so that \hat{n} is perpendicular to \vec{E} ? (c) What is the flux through the disk if \hat{n} is parallel to \vec{E} ?

22.7 The electric flux Φ_E through a disk depends on the angle between its normal \hat{n} and the electric field \vec{E} .



SOLUTION

IDENTIFY and SET UP: This problem is about a flat surface in a uniform electric field, so we can apply the ideas of this section. We calculate the electric flux using Eq. (22.1).

EXECUTE: (a) The area is $A = \pi(0.10 \text{ m})^2 = 0.0314 \text{ m}^2$ and the angle between \vec{E} and $\vec{A} = A\hat{n}$ is $\phi = 30^\circ$, so from Eq. (22.1),

$$\begin{aligned}\Phi_E &= EA \cos \phi = (2.0 \times 10^3 \text{ N/C})(0.0314 \text{ m}^2)(\cos 30^\circ) \\ &= 54 \text{ N} \cdot \text{m}^2/\text{C}\end{aligned}$$

(b) The normal to the disk is now perpendicular to \vec{E} , so $\phi = 90^\circ$, $\cos \phi = 0$, and $\Phi_E = 0$.

(c) The normal to the disk is parallel to \vec{E} , so $\phi = 0$ and $\cos \phi = 1$:

$$\begin{aligned}\Phi_E &= EA \cos \phi = (2.0 \times 10^3 \text{ N/C})(0.0314 \text{ m}^2)(1) \\ &= 63 \text{ N} \cdot \text{m}^2/\text{C}\end{aligned}$$

EVALUATE: As a check on our results, note that our answer to part (b) is smaller than that to part (a), which is in turn smaller than that to part (c). Is all this as it should be?

Example 22.2 Electric flux through a cube

An imaginary cubical surface of side L is in a region of uniform electric field \vec{E} . Find the electric flux through each face of the cube and the total flux through the cube when (a) it is oriented with two of its faces perpendicular to \vec{E} (Fig. 22.8a) and (b) the cube is turned by an angle θ about a vertical axis (Fig. 22.8b).

SOLUTION

IDENTIFY and SET UP: Since \vec{E} is uniform and each of the six faces of the cube is flat, we find the flux Φ_{Ei} through each face using Eqs. (22.3) and (22.4). The total flux through the cube is the sum of the six individual fluxes.

EXECUTE: (a) Figure 22.8a shows the unit vectors \hat{n}_1 through \hat{n}_6 for each face; each unit vector points *outward* from the cube's closed surface. The angle between \vec{E} and \hat{n}_1 is 180° , the angle between \vec{E}

and \hat{n}_2 is 0° , and the angle between \vec{E} and each of the other four unit vectors is 90° . Each face of the cube has area L^2 , so the fluxes through the faces are

$$\begin{aligned}\Phi_{E1} &= \vec{E} \cdot \hat{n}_1 A = EL^2 \cos 180^\circ = -EL^2 \\ \Phi_{E2} &= \vec{E} \cdot \hat{n}_2 A = EL^2 \cos 0^\circ = +EL^2 \\ \Phi_{E3} &= \Phi_{E4} = \Phi_{E5} = \Phi_{E6} = EL^2 \cos 90^\circ = 0\end{aligned}$$

The flux is negative on face 1, where \vec{E} is directed into the cube, and positive on face 2, where \vec{E} is directed out of the cube. The total flux through the cube is

$$\begin{aligned}\Phi_E &= \Phi_{E1} + \Phi_{E2} + \Phi_{E3} + \Phi_{E4} + \Phi_{E5} + \Phi_{E6} \\ &= -EL^2 + EL^2 + 0 + 0 + 0 + 0 = 0\end{aligned}$$

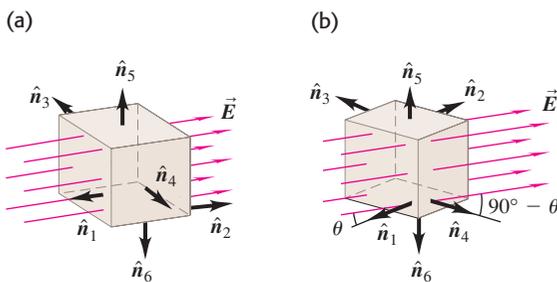
(b) The field \vec{E} is directed into faces 1 and 3, so the fluxes through them are negative; \vec{E} is directed out of faces 2 and 4, so the fluxes through them are positive. We find

$$\begin{aligned}\Phi_{E1} &= \vec{E} \cdot \hat{n}_1 A = EL^2 \cos(180^\circ - \theta) = -EL^2 \cos \theta \\ \Phi_{E2} &= \vec{E} \cdot \hat{n}_2 A = +EL^2 \cos \theta \\ \Phi_{E3} &= \vec{E} \cdot \hat{n}_3 A = EL^2 \cos(90^\circ + \theta) = -EL^2 \sin \theta \\ \Phi_{E4} &= \vec{E} \cdot \hat{n}_4 A = EL^2 \cos(90^\circ - \theta) = +EL^2 \sin \theta \\ \Phi_{E5} &= \Phi_{E6} = EL^2 \cos 90^\circ = 0\end{aligned}$$

The total flux $\Phi_E = \Phi_{E1} + \Phi_{E2} + \Phi_{E3} + \Phi_{E4} + \Phi_{E5} + \Phi_{E6}$ through the surface of the cube is again zero.

EVALUATE: We came to the same conclusion in our discussion of Fig. 22.3c: There is zero net flux of a uniform electric field through a closed surface that contains no electric charge.

22.8 Electric flux of a uniform field \vec{E} through a cubical box of side L in two orientations.

**Example 22.3 Electric flux through a sphere**

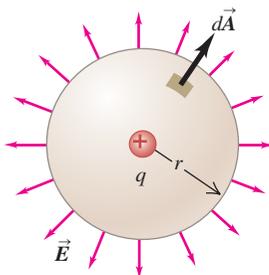
A point charge $q = +3.0 \mu\text{C}$ is surrounded by an imaginary sphere of radius $r = 0.20 \text{ m}$ centered on the charge (Fig. 22.9). Find the resulting electric flux through the sphere.

SOLUTION

IDENTIFY and SET UP: The surface is not flat and the electric field is not uniform, so to calculate the electric flux (our target variable)

we must use the general definition, Eq. (22.5). We use Eq. (22.5) to calculate the electric flux (our target variable). Because the sphere is centered on the point charge, at any point on the spherical surface, \vec{E} is directed out of the sphere perpendicular to the surface. The positive direction for both \hat{n} and E_\perp is outward, so $E_\perp = E$ and the flux through a surface element dA is $\vec{E} \cdot d\vec{A} = E dA$. This greatly simplifies the integral in Eq. (22.5).

Continued

22.9 Electric flux through a sphere centered on a point charge.

EXECUTE: We must evaluate the integral of Eq. (22.5), $\Phi_E = \int E \, dA$. At any point on the sphere of radius r the electric field has the same magnitude $E = q/4\pi\epsilon_0 r^2$. Hence E can be taken outside the integral, which becomes $\Phi_E = E \int dA = EA$, where A is the

area of the spherical surface: $A = 4\pi r^2$. Hence the total flux through the sphere is

$$\begin{aligned}\Phi_E &= EA = \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0} \\ &= \frac{3.0 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = 3.4 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}\end{aligned}$$

EVALUATE: The radius r of the sphere cancels out of the result for Φ_E . We would have obtained the same flux with a sphere of radius 2.0 m or 200 m. We came to essentially the same conclusion in our discussion of Fig. 22.4 in Section 22.1, where we considered rectangular closed surfaces of two different sizes enclosing a point charge. There we found that the flux of \vec{E} was independent of the size of the surface; the same result holds true for a spherical surface. Indeed, the flux through *any* surface enclosing a single point charge is independent of the shape or size of the surface, as we'll soon see.

Test Your Understanding of Section 22.2 Rank the following surfaces in order from most positive to most negative electric flux. (i) a flat rectangular surface with vector area $\vec{A} = (6.0 \text{ m}^2)\hat{i}$ in a uniform electric field $\vec{E} = (4.0 \text{ N/C})\hat{j}$; (ii) a flat circular surface with vector area $\vec{A} = (3.0 \text{ m}^2)\hat{j}$ in a uniform electric field $\vec{E} = (4.0 \text{ N/C})\hat{i} + (2.0 \text{ N/C})\hat{j}$; (iii) a flat square surface with vector area $\vec{A} = (3.0 \text{ m}^2)\hat{i} + (7.0 \text{ m}^2)\hat{j}$ in a uniform electric field $\vec{E} = (4.0 \text{ N/C})\hat{i} - (2.0 \text{ N/C})\hat{j}$; (iv) a flat oval surface with vector area $\vec{A} = (3.0 \text{ m}^2)\hat{i} - (7.0 \text{ m}^2)\hat{j}$ in a uniform electric field $\vec{E} = (4.0 \text{ N/C})\hat{i} - (2.0 \text{ N/C})\hat{j}$.



22.10 Carl Friedrich Gauss helped develop several branches of mathematics, including differential geometry, real analysis, and number theory. The “bell curve” of statistics is one of his inventions. Gauss also made state-of-the-art investigations of the earth’s magnetism and calculated the orbit of the first asteroid to be discovered.



22.3 Gauss's Law

Gauss's law is an alternative to Coulomb's law. While completely equivalent to Coulomb's law, Gauss's law provides a different way to express the relationship between electric charge and electric field. It was formulated by Carl Friedrich Gauss (1777–1855), one of the greatest mathematicians of all time (Fig. 22.10).

Point Charge Inside a Spherical Surface

Gauss's law states that the total electric flux through any closed surface (a surface enclosing a definite volume) is proportional to the total (net) electric charge inside the surface. In Section 22.1 we observed this relationship qualitatively for certain special cases; now we'll develop it more rigorously. We'll start with the field of a single positive point charge q . The field lines radiate out equally in all directions. We place this charge at the center of an imaginary spherical surface with radius R . The magnitude E of the electric field at every point on the surface is given by

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$$

At each point on the surface, \vec{E} is perpendicular to the surface, and its magnitude is the same at every point, just as in Example 22.3 (Section 22.2). The total electric flux is the product of the field magnitude E and the total area $A = 4\pi R^2$ of the sphere:

$$\Phi_E = EA = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} (4\pi R^2) = \frac{q}{\epsilon_0} \quad (22.6)$$

The flux is independent of the radius R of the sphere. It depends only on the charge q enclosed by the sphere.

We can also interpret this result in terms of field lines. Figure 22.11 shows two spheres with radii R and $2R$ centered on the point charge q . Every field line that passes through the smaller sphere also passes through the larger sphere, so the total flux through each sphere is the same.

What is true of the entire sphere is also true of any portion of its surface. In Fig. 22.11 an area dA is outlined on the sphere of radius R and then projected onto the sphere of radius $2R$ by drawing lines from the center through points on the boundary of dA . The area projected on the larger sphere is clearly $4dA$. But since the electric field due to a point charge is inversely proportional to r^2 , the field magnitude is $\frac{1}{4}$ as great on the sphere of radius $2R$ as on the sphere of radius R . Hence the electric flux is the same for both areas and is independent of the radius of the sphere.

Point Charge Inside a Nonspherical Surface

This projection technique shows us how to extend this discussion to nonspherical surfaces. Instead of a second sphere, let us surround the sphere of radius R by a surface of irregular shape, as in Fig. 22.12a. Consider a small element of area dA on the irregular surface; we note that this area is *larger* than the corresponding element on a spherical surface at the same distance from q . If a normal to dA makes an angle ϕ with a radial line from q , two sides of the area projected onto the spherical surface are foreshortened by a factor $\cos \phi$ (Fig. 22.12b). The other two sides are unchanged. Thus the electric flux through the spherical surface element is equal to the flux $E dA \cos \phi$ through the corresponding irregular surface element.

We can divide the entire irregular surface into elements dA , compute the electric flux $E dA \cos \phi$ for each, and sum the results by integrating, as in Eq. (22.5). Each of the area elements projects onto a corresponding spherical surface element. Thus the *total* electric flux through the irregular surface, given by any of the forms of Eq. (22.5), must be the same as the total flux through a sphere, which Eq. (22.6) shows is equal to q/ϵ_0 . Thus, for the irregular surface,

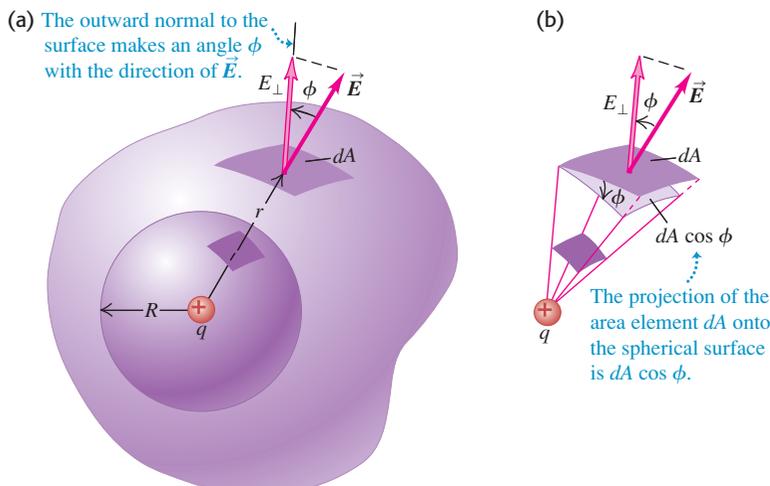
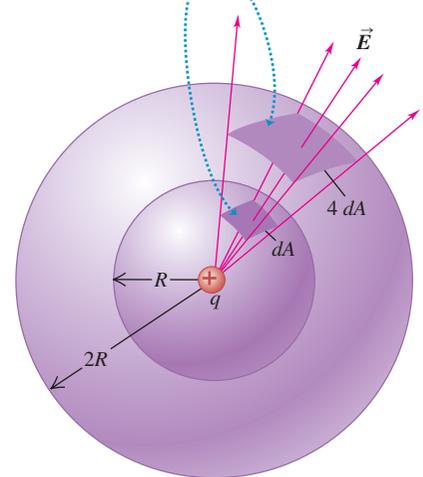
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} \quad (22.7)$$

Equation (22.7) holds for a surface of *any* shape or size, provided only that it is a *closed* surface enclosing the charge q . The circle on the integral sign reminds us that the integral is always taken over a *closed* surface.

The area elements $d\vec{A}$ and the corresponding unit vectors \hat{n} always point *out* of the volume enclosed by the surface. The electric flux is then positive in areas

22.11 Projection of an element of area dA of a sphere of radius R onto a concentric sphere of radius $2R$. The projection multiplies each linear dimension by 2, so the area element on the larger sphere is $4dA$.

The same number of field lines and the same flux pass through both of these area elements.



22.12 Calculating the electric flux through a nonspherical surface.

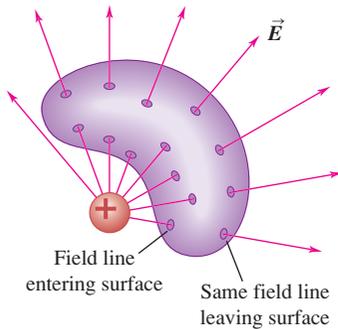
where the electric field points out of the surface and negative where it points inward. Also, E_{\perp} is positive at points where \vec{E} points out of the surface and negative at points where \vec{E} points into the surface.

If the point charge in Fig. 22.12 is negative, the \vec{E} field is directed radially *inward*; the angle ϕ is then greater than 90° , its cosine is negative, and the integral in Eq. (22.7) is negative. But since q is also negative, Eq. (22.7) still holds.

For a closed surface enclosing *no* charge,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = 0$$

22.13 A point charge *outside* a closed surface that encloses no charge. If an electric field line from the external charge enters the surface at one point, it must leave at another.



This is the mathematical statement that when a region contains no charge, any field lines caused by charges *outside* the region that enter on one side must leave again on the other side. (In Section 22.1 we came to the same conclusion by considering the special case of a rectangular box in a uniform field.) Figure 22.13 illustrates this point. *Electric field lines can begin or end inside a region of space only when there is charge in that region.*

General Form of Gauss's Law

Now comes the final step in obtaining the general form of Gauss's law. Suppose the surface encloses not just one point charge q but several charges q_1, q_2, q_3, \dots . The total (resultant) electric field \vec{E} at any point is the vector sum of the \vec{E} fields of the individual charges. Let Q_{encl} be the *total* charge enclosed by the surface: $Q_{\text{encl}} = q_1 + q_2 + q_3 + \dots$. Also let \vec{E} be the *total* field at the position of the surface area element $d\vec{A}$, and let E_{\perp} be its component perpendicular to the plane of that element (that is, parallel to $d\vec{A}$). Then we can write an equation like Eq. (22.7) for each charge and its corresponding field and add the results. When we do, we obtain the general statement of Gauss's law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \quad (\text{Gauss's law}) \quad (22.8)$$

The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ϵ_0 .

CAUTION **Gaussian surfaces are imaginary** Remember that the closed surface in Gauss's law is *imaginary*; there need not be any material object at the position of the surface. We often refer to a closed surface used in Gauss's law as a **Gaussian surface**. **I**

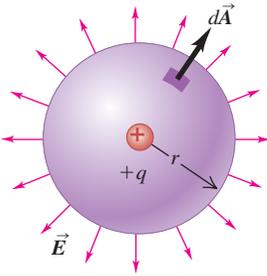
Using the definition of Q_{encl} and the various ways to express electric flux given in Eq. (22.5), we can express Gauss's law in the following equivalent forms:

$$\Phi_E = \oint E \cos \phi \, dA = \oint E_{\perp} \, dA = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \quad (\text{various forms of Gauss's law}) \quad (22.9)$$

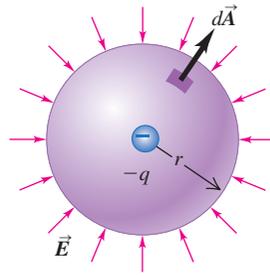
As in Eq. (22.5), the various forms of the integral all express the same thing, the total electric flux through the Gaussian surface, in different terms. One form is sometimes more convenient than another.

As an example, Fig. 22.14a shows a spherical Gaussian surface of radius r around a positive point charge $+q$. The electric field points out of the Gaussian surface, so at every point on the surface \vec{E} is in the same direction as $d\vec{A}$, $\phi = 0$, and E_{\perp} is equal to the field magnitude $E = q/4\pi\epsilon_0 r^2$. Since E is the same at all points

(a) Gaussian surface around positive charge:
positive (outward) flux



(b) Gaussian surface around negative charge:
negative (inward) flux



22.14 Spherical Gaussian surfaces around (a) a positive point charge and (b) a negative point charge.

on the surface, we can take it outside the integral in Eq. (22.9). Then the remaining integral is $\int dA = A = 4\pi r^2$, the area of the sphere. Hence Eq. (22.9) becomes

$$\Phi_E = \oint E_{\perp} dA = \oint \left(\frac{q}{4\pi\epsilon_0 r^2} \right) dA = \frac{q}{4\pi\epsilon_0 r^2} \oint dA = \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0}$$

The enclosed charge Q_{encl} is just the charge $+q$, so this agrees with Gauss's law. If the Gaussian surface encloses a *negative* point charge as in Fig. 22.14b, then \vec{E} points *into* the surface at each point in the direction opposite $d\vec{A}$. Then $\phi = 180^\circ$ and E_{\perp} is equal to the negative of the field magnitude: $E_{\perp} = -E = -| -q | / 4\pi\epsilon_0 r^2 = -q / 4\pi\epsilon_0 r^2$. Equation (22.9) then becomes

$$\Phi_E = \oint E_{\perp} dA = \oint \left(\frac{-q}{4\pi\epsilon_0 r^2} \right) dA = \frac{-q}{4\pi\epsilon_0 r^2} \oint dA = \frac{-q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{-q}{\epsilon_0}$$

This again agrees with Gauss's law because the enclosed charge in Fig. 22.14b is $Q_{\text{encl}} = -q$.

In Eqs. (22.8) and (22.9), Q_{encl} is always the algebraic sum of all the positive and negative charges enclosed by the Gaussian surface, and \vec{E} is the *total* field at each point on the surface. Also note that in general, this field is caused partly by charges inside the surface and partly by charges outside. But as Fig. 22.13 shows, the outside charges do *not* contribute to the total (net) flux through the surface. So Eqs. (22.8) and (22.9) are correct even when there are charges outside the surface that contribute to the electric field at the surface. When $Q_{\text{encl}} = 0$, the total flux through the Gaussian surface must be zero, even though some areas may have positive flux and others may have negative flux (see Fig. 22.3b).

Gauss's law is the definitive answer to the question we posed at the beginning of Section 22.1: "If the electric field pattern is known in a given region, what can we determine about the charge distribution in that region?" It provides a relationship between the electric field on a closed surface and the charge distribution within that surface. But in some cases we can use Gauss's law to answer the reverse question: "If the charge distribution is known, what can we determine about the electric field that the charge distribution produces?" Gauss's law may seem like an unappealing way to address this question, since it may look as though evaluating the integral in Eq. (22.8) is a hopeless task. Sometimes it is, but other times it is surprisingly easy. Here's an example in which *no* integration is involved at all; we'll work out several more examples in the next section.

Conceptual Example 22.4 Electric flux and enclosed charge

Figure 22.15 shows the field produced by two point charges $+q$ and $-q$ (an electric dipole). Find the electric flux through each of the closed surfaces A, B, C, and D.

SOLUTION

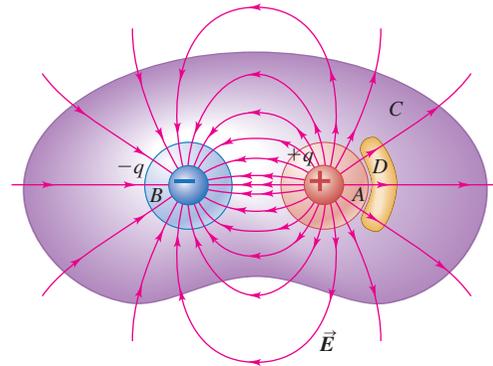
Gauss's law, Eq. (22.8), says that the total electric flux through a closed surface is equal to the total enclosed charge divided by ϵ_0 . In

Continued

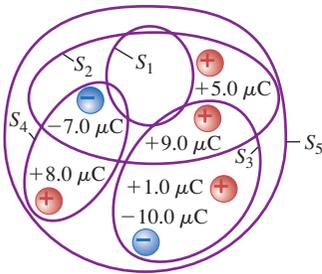
Fig. 22.15, surface A (shown in red) encloses the positive charge, so $Q_{\text{encl}} = +q$; surface B (in blue) encloses the negative charge, so $Q_{\text{encl}} = -q$; surface C (in purple) encloses *both* charges, so $Q_{\text{encl}} = +q + (-q) = 0$; and surface D (in yellow) encloses no charges, so $Q_{\text{encl}} = 0$. Hence, without having to do any integration, we have $\Phi_{EA} = +q/\epsilon_0$, $\Phi_{EB} = -q/\epsilon_0$, and $\Phi_{EC} = \Phi_{ED} = 0$. These results depend only on the charges enclosed within each Gaussian surface, not on the precise shapes of the surfaces.

We can draw similar conclusions by examining the electric field lines. All the field lines that cross surface A are directed out of the surface, so the flux through A must be positive. Similarly, the flux through B must be negative since all of the field lines that cross that surface point inward. For both surface C and surface D , there are as many field lines pointing into the surface as there are field lines pointing outward, so the flux through each of these surfaces is zero.

22.15 The net number of field lines leaving a closed surface is proportional to the total charge enclosed by that surface.



22.16 Five Gaussian surfaces and six point charges.



Test Your Understanding of Section 22.3 Figure 22.16 shows six point charges that all lie in the same plane. Five Gaussian surfaces— S_1 , S_2 , S_3 , S_4 , and S_5 —each enclose part of this plane, and Fig. 22.16 shows the intersection of each surface with the plane. Rank these five surfaces in order of the electric flux through them, from most positive to most negative.



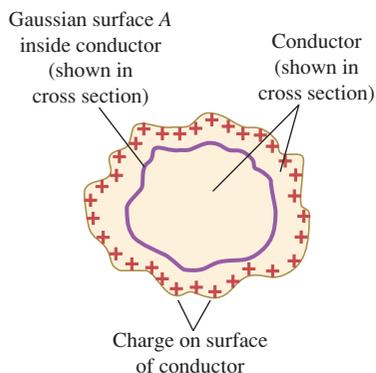
22.4 Applications of Gauss's Law

Gauss's law is valid for *any* distribution of charges and for *any* closed surface. Gauss's law can be used in two ways. If we know the charge distribution, and if it has enough symmetry to let us evaluate the integral in Gauss's law, we can find the field. Or if we know the field, we can use Gauss's law to find the charge distribution, such as charges on conducting surfaces.

In this section we present examples of both kinds of applications. As you study them, watch for the role played by the symmetry properties of each system. We will use Gauss's law to calculate the electric fields caused by several simple charge distributions; the results are collected in a table in the chapter summary.

In practical problems we often encounter situations in which we want to know the electric field caused by a charge distribution on a conductor. These calculations are aided by the following remarkable fact: *When excess charge is placed on a solid conductor and is at rest, it resides entirely on the surface, not in the interior of the material.* (By *excess* we mean charges other than the ions and free electrons that make up the neutral conductor.) Here's the proof. We know from Section 21.4 that in an electrostatic situation (with all charges at rest) the electric field \vec{E} at every point in the interior of a conducting material is zero. If \vec{E} were *not* zero, the excess charges would move. Suppose we construct a Gaussian surface inside the conductor, such as surface A in Fig. 22.17. Because $\vec{E} = \mathbf{0}$ everywhere on this surface, Gauss's law requires that the net charge inside the surface is zero. Now imagine shrinking the surface like a collapsing balloon until it encloses a region so small that we may consider it as a point P ; then the charge at that point must be zero. We can do this anywhere inside the conductor, so *there can be no excess charge at any point within a solid conductor; any excess charge must reside on the conductor's surface.* (This result is for a *solid* conductor. In the next section we'll discuss what can happen if the conductor has cavities in its interior.) We will make use of this fact frequently in the examples that follow.

22.17 Under electrostatic conditions (charges not in motion), any excess charge on a solid conductor resides entirely on the conductor's surface.



Problem-Solving Strategy 22.1 Gauss's Law



IDENTIFY the relevant concepts: Gauss's law is most useful when the charge distribution has spherical, cylindrical, or planar symmetry. In these cases the symmetry determines the direction of \vec{E} . Then Gauss's law yields the magnitude of \vec{E} if we are given the charge distribution, and vice versa. In either case, begin the analysis by asking the question: What is the symmetry?

SET UP the problem using the following steps:

1. List the known and unknown quantities and identify the target variable.
2. Select the appropriate closed, imaginary Gaussian surface. For spherical symmetry, use a concentric spherical surface. For cylindrical symmetry, use a coaxial cylindrical surface with flat ends perpendicular to the axis of symmetry (like a soup can). For planar symmetry, use a cylindrical surface (like a tuna can) with its flat ends parallel to the plane.

EXECUTE the solution as follows:

1. Determine the appropriate size and placement of your Gaussian surface. To evaluate the field magnitude at a particular point, the surface must include that point. It may help to place one end of a can-shaped surface within a conductor, where \vec{E} and therefore Φ are zero, or to place its ends equidistant from a charged plane.
2. Evaluate the integral $\oint E_{\perp} dA$ in Eq. (22.9). In this equation E_{\perp} is the perpendicular component of the total electric field at each point on the Gaussian surface. A well-chosen Gaussian surface should make integration trivial or unnecessary. If the surface comprises several separate surfaces, such as the sides and ends

of a cylinder, the integral $\oint E_{\perp} dA$ over the entire closed surface is the sum of the integrals $\int E_{\perp} dA$ over the separate surfaces. Consider points 3–6 as you work.

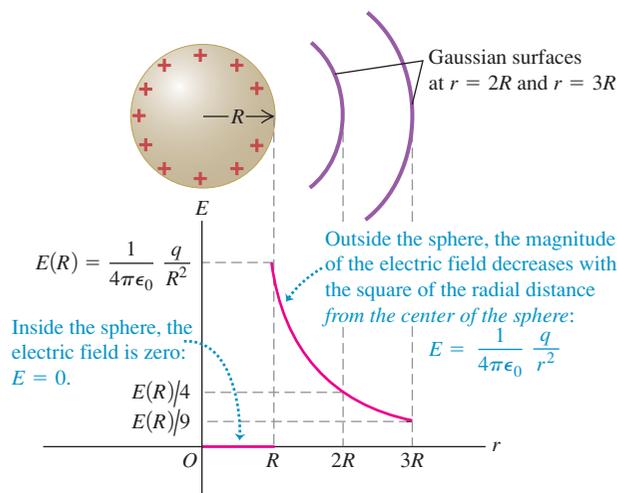
3. If \vec{E} is perpendicular (normal) at every point to a surface with area A , if it points outward from the interior of the surface, and if it has the same magnitude at every point on the surface, then $E_{\perp} = E = \text{constant}$, and $\int E_{\perp} dA$ over that surface is equal to EA . (If \vec{E} is inward, then $E_{\perp} = -E$ and $\int E_{\perp} dA = -EA$.) This should be the case for part or all of your Gaussian surface. If \vec{E} is tangent to a surface at every point, then $E_{\perp} = 0$ and the integral over that surface is zero. This may be the case for parts of a cylindrical Gaussian surface. If $\vec{E} = \mathbf{0}$ at every point on a surface, the integral is zero.
4. Even when there is no charge within a Gaussian surface, the field at any given point on the surface is not necessarily zero. In that case, however, the total electric flux through the surface is always zero.
5. The flux integral $\oint E_{\perp} dA$ can be approximated as the difference between the numbers of electric lines of force leaving and entering the Gaussian surface. In this sense the flux gives the sign of the enclosed charge, but is only proportional to it; zero flux corresponds to zero enclosed charge.
6. Once you have evaluated $\oint E_{\perp} dA$, use Eq. (22.9) to solve for your target variable.

EVALUATE your answer: If your result is a function that describes how the magnitude of the electric field varies with position, ensure that it makes sense.

Example 22.5 Field of a charged conducting sphere

We place a total positive charge q on a solid conducting sphere with radius R (Fig. 22.18). Find \vec{E} at any point inside or outside the sphere.

22.18 Calculating the electric field of a conducting sphere with positive charge q . Outside the sphere, the field is the same as if all of the charge were concentrated at the center of the sphere.



SOLUTION

IDENTIFY and SET UP: As we discussed earlier in this section, all of the charge must be on the surface of the sphere. The charge is free to move on the conductor, and there is no preferred position on the surface; the charge is therefore distributed uniformly over the surface, and the system is spherically symmetric. To exploit this symmetry, we take as our Gaussian surface a sphere of radius r centered on the conductor. We can calculate the field inside or outside the conductor by taking $r < R$ or $r > R$, respectively. In either case, the point at which we want to calculate \vec{E} lies on the Gaussian surface.

EXECUTE: The spherical symmetry means that the direction of the electric field must be radial; that's because there is no preferred direction parallel to the surface, so \vec{E} can have no component parallel to the surface. There is also no preferred orientation of the sphere, so the field magnitude E can depend only on the distance r from the center and must have the same value at all points on the Gaussian surface.

For $r > R$ the entire conductor is within the Gaussian surface, so the enclosed charge is q . The area of the Gaussian surface is $4\pi r^2$, and \vec{E} is uniform over the surface and perpendicular to it at each point. The flux integral $\oint E_{\perp} dA$ is then just $E(4\pi r^2)$, and Eq. (22.8) gives

Continued

$$E(4\pi r^2) = \frac{q}{\epsilon_0} \quad \text{and}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad (\text{outside a charged conducting sphere})$$

This expression is the same as that for a point charge; outside the charged sphere, its field is the same as though the entire charge were concentrated at its center. Just outside the surface of the sphere, where $r = R$,

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \quad (\text{at the surface of a charged conducting sphere})$$

CAUTION Flux can be positive or negative Remember that we have chosen the charge q to be *positive*. If the charge is negative, the electric field is radially *inward* instead of radially outward, and the electric flux through the Gaussian surface is negative. The electric-field magnitudes outside and at the surface of the sphere are given by the same expressions as above, except that q denotes the *magnitude* (absolute value) of the charge. \blacksquare

For $r < R$ we again have $E(4\pi r^2) = Q_{\text{encl}}/\epsilon_0$. But now our Gaussian surface (which lies entirely within the conductor)

encloses *no* charge, so $Q_{\text{encl}} = 0$. The electric field inside the conductor is therefore zero.

EVALUATE: We already knew that $\vec{E} = \mathbf{0}$ inside a solid conductor (whether spherical or not) when the charges are at rest. Figure 22.18 shows E as a function of the distance r from the center of the sphere. Note that in the limit as $R \rightarrow 0$, the sphere becomes a point charge; there is then only an “outside,” and the field is everywhere given by $E = q/4\pi\epsilon_0 r^2$. Thus we have deduced Coulomb’s law from Gauss’s law. (In Section 22.3 we deduced Gauss’s law from Coulomb’s law; the two laws are equivalent.)

We can also use this method for a conducting spherical *shell* (a spherical conductor with a concentric spherical hole inside) if there is no charge inside the hole. We use a spherical Gaussian surface with radius r less than the radius of the hole. If there *were* a field inside the hole, it would have to be radial and spherically symmetric as before, so $E = Q_{\text{encl}}/4\pi\epsilon_0 r^2$. But now there is no enclosed charge, so $Q_{\text{encl}} = 0$ and $E = 0$ inside the hole.

Can you use this same technique to find the electric field in the region between a charged sphere and a concentric hollow conducting sphere that surrounds it?

Example 22.6 Field of a uniform line charge

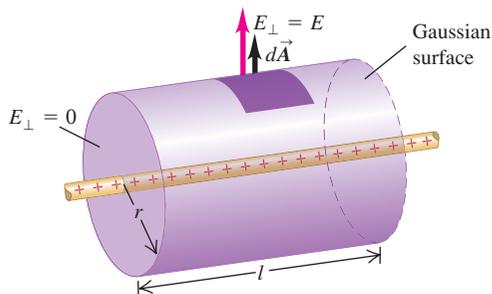
Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is λ (assumed positive). Find the electric field using Gauss’s law.

SOLUTION

IDENTIFY and SET UP: We found in Example 21.10 (Section 21.5) that the field \vec{E} of a uniformly charged, infinite wire is radially outward if λ is positive and radially inward if λ is negative, and that the field magnitude E depends only on the radial distance from the wire. This suggests that we use a *cylindrical* Gaussian surface, of radius r and arbitrary length l , coaxial with the wire and with its ends perpendicular to the wire (Fig. 22.19).

EXECUTE: The flux through the flat ends of our Gaussian surface is zero because the radial electric field is parallel to these ends, and so $\vec{E} \cdot \hat{n} = 0$. On the cylindrical part of our surface we have $\vec{E} \cdot \hat{n} = E_{\perp} = E$ everywhere. (If λ were negative, we would have

22.19 A coaxial cylindrical Gaussian surface is used to find the electric field outside an infinitely long, charged wire.



$\vec{E} \cdot \hat{n} = E_{\perp} = -E$ everywhere.) The area of the cylindrical surface is $2\pi rl$, so the flux through it—and hence the total flux Φ_E through the Gaussian surface—is $EA = 2\pi rlE$. The total enclosed charge is $Q_{\text{encl}} = \lambda l$, and so from Gauss’s law, Eq. (22.8),

$$\Phi_E = 2\pi rlE = \frac{\lambda l}{\epsilon_0} \quad \text{and}$$

$$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} \quad (\text{field of an infinite line of charge})$$

We found this same result in Example 21.10 with *much* more effort.

If λ is *negative*, \vec{E} is directed radially inward, and in the above expression for E we must interpret λ as the absolute value of the charge per unit length.

EVALUATE: We saw in Example 21.10 that the *entire* charge on the wire contributes to the field at any point, and yet we consider only that part of the charge $Q_{\text{encl}} = \lambda l$ within the Gaussian surface when we apply Gauss’s law. There’s nothing inconsistent here; it takes the entire charge to give the field the properties that allow us to calculate Φ_E so easily, and Gauss’s law always applies to the enclosed charge only. If the wire is short, the symmetry of the infinite wire is lost, and E is not uniform over a coaxial, cylindrical Gaussian surface. Gauss’s law then *cannot* be used to find Φ_E ; we must solve the problem the hard way, as in Example 21.10.

We can use the Gaussian surface in Fig. 22.19 to show that the field outside a long, uniformly charged cylinder is the same as though all the charge were concentrated on a line along its axis (see Problem 22.42). We can also calculate the electric field in the space between a charged cylinder and a coaxial hollow conducting cylinder surrounding it (see Problem 22.39).

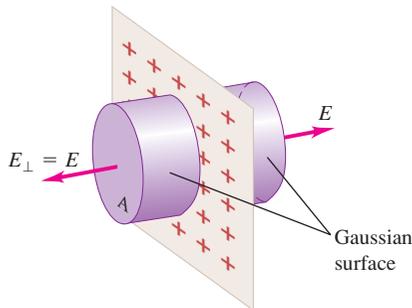
Example 22.7 Field of an infinite plane sheet of charge

Use Gauss's law to find the electric field caused by a thin, flat, infinite sheet with a uniform positive surface charge density σ .

SOLUTION

IDENTIFY and SET UP: In Example 21.11 (Section 21.5) we found that the field \vec{E} of a uniformly charged infinite sheet is normal to the sheet, and that its magnitude is independent of the distance from the sheet. To take advantage of these symmetry properties, we use a cylindrical Gaussian surface with ends of area A and with its axis perpendicular to the sheet of charge (Fig. 22.20).

22.20 A cylindrical Gaussian surface is used to find the field of an infinite plane sheet of charge.



EXECUTE: The flux through the cylindrical part of our Gaussian surface is zero because $\vec{E} \cdot \hat{n} = 0$ everywhere. The flux through each flat end of the surface is $+EA$ because $\vec{E} \cdot \hat{n} = E_{\perp} = E$ everywhere, so the total flux through both ends—and hence the total flux Φ_E through the Gaussian surface—is $+2EA$. The total enclosed charge is $Q_{\text{encl}} = \sigma A$, and so from Gauss's law,

$$2EA = \frac{\sigma A}{\epsilon_0} \quad \text{and}$$

$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{field of an infinite sheet of charge})$$

In Example 21.11 we found this same result using a much more complex calculation.

If σ is negative, \vec{E} is directed *toward* the sheet, the flux through the Gaussian surface in Fig. 22.20 is negative, and σ in the expression $E = \sigma/2\epsilon_0$ denotes the magnitude (absolute value) of the charge density.

EVALUATE: Again we see that, given favorable symmetry, we can deduce electric fields using Gauss's law much more easily than using Coulomb's law.

Example 22.8 Field between oppositely charged parallel conducting plates

Two large plane parallel conducting plates are given charges of equal magnitude and opposite sign; the surface charge densities are $+\sigma$ and $-\sigma$. Find the electric field in the region between the plates.

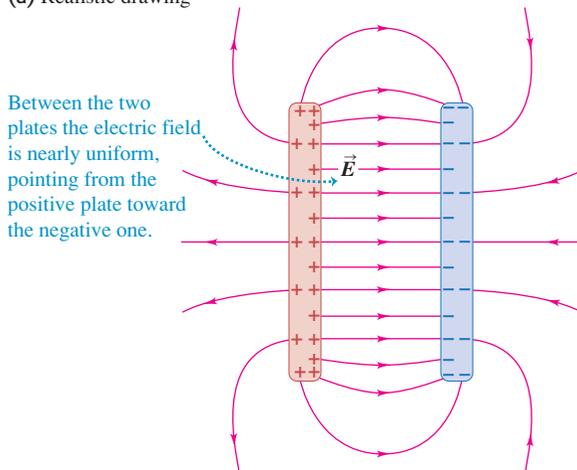
SOLUTION

IDENTIFY and SET UP: Figure 22.21a shows the field. Because opposite charges attract, most of the charge accumulates at the opposing faces of the plates. A small amount of charge resides on the *outer* surfaces of the plates, and there is some spreading or “fringing” of

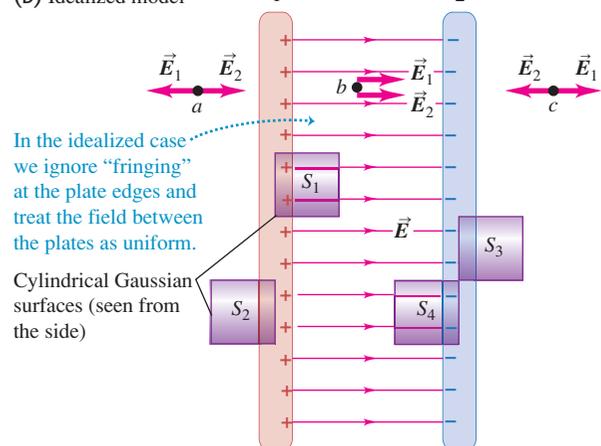
the field at the edges. But if the plates are very large in comparison to the distance between them, the amount of charge on the outer surfaces is negligibly small, and the fringing can be neglected except near the edges. In this case we can assume that the field is uniform in the interior region between the plates, as in Fig. 22.21b, and that the charges are distributed uniformly over the opposing surfaces. To exploit this symmetry, we can use the shaded Gaussian surfaces S_1 , S_2 , S_3 , and S_4 . These surfaces are cylinders with flat ends of area A ; one end of each surface lies *within* a plate.

22.21 Electric field between oppositely charged parallel plates.

(a) Realistic drawing



(b) Idealized model



Continued

EXECUTE: The left-hand end of surface S_1 is within the positive plate 1. Since the field is zero within the volume of any solid conductor under electrostatic conditions, there is no electric flux through this end. The electric field between the plates is perpendicular to the right-hand end, so on that end, E_{\perp} is equal to E and the flux is EA ; this is positive, since \vec{E} is directed out of the Gaussian surface. There is no flux through the side walls of the cylinder, since these walls are parallel to \vec{E} . So the total flux integral in Gauss's law is EA . The net charge enclosed by the cylinder is σA , so Eq. (22.8) yields $EA = \sigma A/\epsilon_0$; we then have

$$E = \frac{\sigma}{\epsilon_0} \quad (\text{field between oppositely charged conducting plates})$$

Example 22.9 Field of a uniformly charged sphere

Positive electric charge Q is distributed uniformly throughout the volume of an insulating sphere with radius R . Find the magnitude of the electric field at a point P a distance r from the center of the sphere.

SOLUTION

IDENTIFY and SET UP: As in Example 22.5, the system is spherically symmetric. Hence we can use the conclusions of that example about the direction and magnitude of \vec{E} . To make use of the spherical symmetry, we choose as our Gaussian surface a sphere with radius r , concentric with the charge distribution.

EXECUTE: From symmetry, the direction of \vec{E} is radial at every point on the Gaussian surface, so $E_{\perp} = E$ and the field magnitude E is the same at every point on the surface. Hence the total electric flux through the Gaussian surface is the product of E and the total area of the surface $A = 4\pi r^2$ —that is, $\Phi_E = 4\pi r^2 E$.

The amount of charge enclosed within the Gaussian surface depends on r . To find E inside the sphere, we choose $r < R$. The volume charge density ρ is the charge Q divided by the volume of the entire charged sphere of radius R :

$$\rho = \frac{Q}{4\pi R^3/3}$$

The volume V_{encl} enclosed by the Gaussian surface is $\frac{4}{3}\pi r^3$, so the total charge Q_{encl} enclosed by that surface is

$$Q_{\text{encl}} = \rho V_{\text{encl}} = \left(\frac{Q}{4\pi R^3/3}\right)\left(\frac{4}{3}\pi r^3\right) = Q \frac{r^3}{R^3}$$

Then Gauss's law, Eq. (22.8), becomes

$$4\pi r^2 E = \frac{Q}{\epsilon_0} \frac{r^3}{R^3} \quad \text{or}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Qr}{R^3} \quad (\text{field inside a uniformly charged sphere})$$

The field magnitude is proportional to the distance r of the field point from the center of the sphere (see the graph of E versus r in Fig. 22.22).

To find E outside the sphere, we take $r > R$. This surface encloses the entire charged sphere, so $Q_{\text{encl}} = Q$, and Gauss's law gives

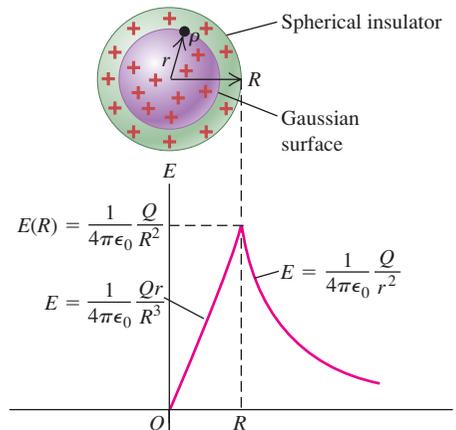
$$4\pi r^2 E = \frac{Q}{\epsilon_0} \quad \text{or}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \quad (\text{field outside a uniformly charged sphere})$$

The field is uniform and perpendicular to the plates, and its magnitude is independent of the distance from either plate. The Gaussian surface S_4 yields the same result. Surfaces S_2 and S_3 yield $E = 0$ to the left of plate 1 and to the right of plate 2, respectively. We leave these calculations to you (see Exercise 22.29).

EVALUATE: We obtained the same results in Example 21.11 by using the principle of superposition of electric fields. The fields due to the two sheets of charge (one on each plate) are \vec{E}_1 and \vec{E}_2 ; from Example 22.7, both of these have magnitude $\sigma/2\epsilon_0$. The total electric field at any point is the vector sum $\vec{E} = \vec{E}_1 + \vec{E}_2$. At points a and c in Fig. 22.21b, \vec{E}_1 and \vec{E}_2 point in opposite directions, and their sum is zero. At point b , \vec{E}_1 and \vec{E}_2 are in the same direction; their sum has magnitude $E = \sigma/\epsilon_0$, just as we found above using Gauss's law.

22.22 The magnitude of the electric field of a uniformly charged insulating sphere. Compare this with the field for a conducting sphere (see Fig. 22.18).



The field outside any spherically symmetric charged body varies as $1/r^2$, as though the entire charge were concentrated at the center. This is graphed in Fig. 22.22.

If the charge is negative, \vec{E} is radially inward and in the expressions for E we interpret Q as the absolute value of the charge.

EVALUATE: Notice that if we set $r = R$ in either expression for E , we get the same result $E = Q/4\pi\epsilon_0 R^2$ for the magnitude of the field at the surface of the sphere. This is because the magnitude E is a continuous function of r . By contrast, for the charged conducting sphere of Example 22.5 the electric-field magnitude is discontinuous at $r = R$ (it jumps from $E = 0$ just inside the sphere to $E = Q/4\pi\epsilon_0 R^2$ just outside the sphere). In general, the electric field \vec{E} is discontinuous in magnitude, direction, or both wherever there is a sheet of charge, such as at the surface of a charged conducting sphere (Example 22.5), at the surface of an infinite charged sheet (Example 22.7), or at the surface of a charged conducting plate (Example 22.8).

The approach used here can be applied to any spherically symmetric distribution of charge, even if it is not radially uniform, as it was here. Such charge distributions occur within many atoms and atomic nuclei, so Gauss's law is useful in atomic and nuclear physics.