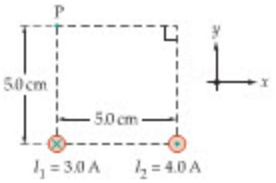
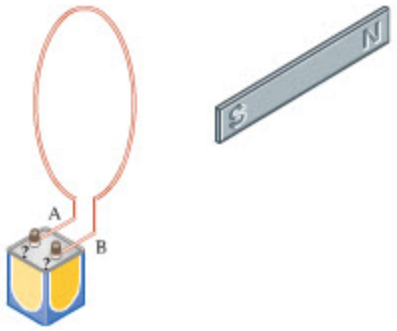


▲ **FIGURE 22–40** Problems 54 and 106

SECTION 22–7 CURRENT LOOPS AND SOLENOIDS

55. • **CE** A loop of wire is connected to the terminals of a battery, as indicated in Figure 22–41. If the loop is to attract the bar magnet, which of the terminals, A or B, should be the positive terminal of the battery? Explain.



▲ **FIGURE 22–41** Problem 55

56. • **CE Predict/Explain** The number of turns in a solenoid is doubled, and at the same time its length is doubled. Does the magnetic field within the solenoid increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. Doubling the number of turns in a solenoid doubles its magnetic field, and hence the field increases.

II. Making a solenoid longer decreases its magnetic field, and therefore the field decreases.

III. The magnetic field remains the same because the number of turns per length is unchanged.

57. • It is desired that a solenoid 38 cm long and with 430 turns produce a magnetic field within it equal to the Earth's magnetic field (5.0×10^{-5} T). What current is required?

58. • A solenoid that is 62 cm long produces a magnetic field of 1.3 T within its core when it carries a current of 8.4 A. How many turns of wire are contained in this solenoid?

59. • The maximum current in a superconducting solenoid can be as large as 3.75 kA. If the number of turns per meter in such a solenoid is 3650, what is the magnitude of the magnetic field it produces?

60. •• To construct a solenoid, you wrap insulated wire uniformly around a plastic tube 12 cm in diameter and 55 cm in length. You would like a 2.0-A current to produce a 2.5-kG magnetic field inside your solenoid. What is the total length of wire you will need to meet these specifications?

GENERAL PROBLEMS

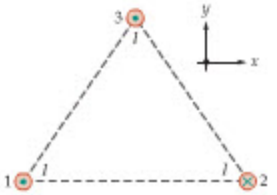
61. • **CE** At a point near the equator, the Earth's magnetic field is horizontal and points to the north. If an electron is moving vertically upward at this point, does the magnetic force acting on it point north, south, east, west, upward, or downward? Explain.

62. • **CE** A proton is to orbit the Earth at the equator using the Earth's magnetic field to supply part of the necessary centripetal force. Should the proton move eastward or westward? Explain.

63. • **CE** The accompanying photograph shows an electron beam whose initial direction of motion is horizontal, from right to left. A magnetic field deflects the beam downward. What is the direction of the magnetic field?

A horizontal electron beam is deflected downward by a magnetic field. (Problem 63)

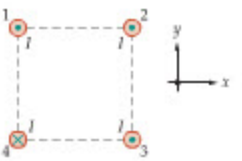
64. • **CE** The three wires shown in Figure 22–42 are long and straight, and they each carry a current of the same magnitude, I. The currents in wires 1 and 3 are out of the page; the current in wire 2 is into the page. What is the direction of the magnetic force experienced by wire 3?



▲ **FIGURE 22–42** Problems 64 and 65

65. • **CE** Each of the current-carrying wires in Figure 22–42 is long and straight, and carries the current I either into or out of the page, as shown. What is the direction of the net magnetic field produced by these three wires at the center of the triangle?

66. • **CE** The four wires shown in Figure 22–43 are long and straight, and they each carry a current of the same magnitude, I. The currents in wires 1, 2, and 3 are out of the page; the current in wire 4 is into the page. What is the direction of the magnetic force experienced by wire 2?



▲ **FIGURE 22–43** Problems 66 and 67

67. • **CE** Each of the current-carrying wires in Figure 22–43 is long and straight, and carries the current I either into or out of the page, as shown. What is the direction of the net magnetic field produced by these four wires at the center of the square?

68. • Consider a current loop immersed in a magnetic field, as in Figure 22–37 (a) (Problem 38). It is given that $B = 0.34$ T and $I = 9.5$ A. In addition, the loop is a square 0.46 m on a side. Find the magnitude of the magnetic force exerted on each side of the loop.

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69. • A stationary proton ($q = 1.60 \times 10^{-19}$ C) is located between the poles of a horseshoe magnet, where the magnetic field is 0.35 T. What is the magnitude of the magnetic force acting on the proton?

70. • **BIO Brain Function and Magnetic Fields** Experiments have shown that thought processes in the brain can be affected if the parietal lobe is exposed to a magnetic field with a strength of 1.0 T. How much current must a long, straight wire carry if it is to produce a 1.0-T magnetic field at a distance of 0.50 m? (For comparison, a typical lightning bolt carries a current of about 20,000 A, which would melt most wires.)

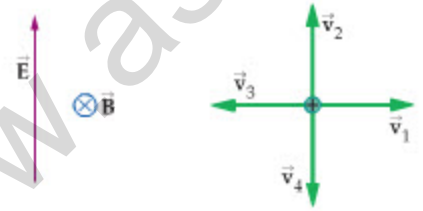
71. • A mixture of two isotopes is injected into a mass spectrometer. One isotope follows a curved path of radius $R_1 = 48.9$ cm; the other follows a curved path of radius $R_2 = 51.7$ cm. Find the mass ratio, m_1/m_2, assuming that the two isotopes have the same charge and speed.

72. • High above the surface of the Earth, charged particles (such as electrons and protons) can become trapped in the Earth's magnetic field in regions known as Van Allen belts. A typical electron in a Van Allen belt has an energy of 45 keV and travels in a roughly circular orbit with an average radius of 220 m. What is the magnitude of the Earth's magnetic field where such an electron orbits?

73. • **Credit-Card Magnetic Strips** Experiments carried out on the television show *Mythbusters* determined that a magnetic field of 1000 gauss is needed to corrupt the information on a credit card's magnetic strip. (They also busted the myth that a credit card can be demagnetized by an electric eel or an eelskin wallet.) Suppose a long, straight wire carries a current of 3.5 A. How close can a credit card be held to this wire without damaging its magnetic strip?

74. • **Superconducting Solenoid** Cryomagnetics, Inc., advertises a high-field, superconducting solenoid that produces a magnetic field of 17 T with a current of 105 A. What is the number of turns per meter in this solenoid?

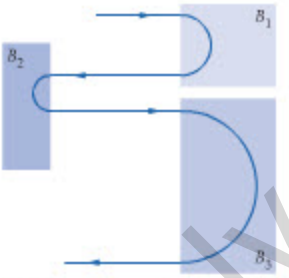
75. •• **CE** A positively charged particle moves through a region with a uniform electric field pointing toward the top of the page and a uniform magnetic field pointing into the page. The particle can have one of the four velocities shown in Figure 22–44. **(a)** Rank the four possibilities in order of increasing magnitude of the net force (F_1, F_2, F_3, and F_4) the particle experiences. Indicate ties where appropriate. **(b)** Which of the four velocities could potentially result in zero net force?



▲ **FIGURE 22–44** Problems 75 and 76

76. •• **CE** Suppose the fields in Figure 22–44 are interchanged, with the magnetic field pointing toward the top of the page and the electric field pointing into the page. **(a)** Rank the four possibilities in order of increasing magnitude of the net force (F_1, F_2, F_3, and F_4) the particle experiences. Indicate ties where appropriate. **(b)** Which of the four velocities could potentially result in zero net force?

77. •• **CE** A proton follows the path shown in Figure 22–45 as it moves through three regions with different uniform magnetic fields, B_1, B_2, and B_3. In each region the proton completes a half-circle, and the magnetic field is perpendicular to the page. **(a)** Rank the three fields in order of increasing magnitude. Indicate ties where appropriate. **(b)** Give the direction (into or out of the page) for each of the fields.



▲ **FIGURE 22–45** Problems 77, 78, and 79

78. •• **CE Predict/Explain** Suppose the initial speed of the proton in Figure 22–45 is increased. **(a)** Does the radius of each half-circular path segment increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. The radius of a circular orbit in a magnetic field is proportional to the speed of the proton; therefore, the radius of the half-circular path will increase.

II. A greater speed means the proton will experience more force from the magnetic field, resulting in a decrease of the radius.

III. The increase in speed offsets the increase in magnetic force, resulting in no change of the radius.

79. •• **CE Predict/Explain** Suppose the initial speed of the proton in Figure 22–45 is decreased. **(a)** Does the time spent in each of the field regions increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

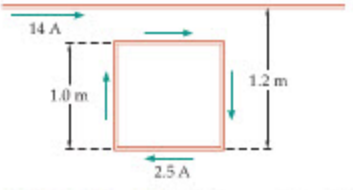
I. The proton moves more slowly, and therefore the time spent moving through each of magnetic regions will increase.

II. With a smaller speed the proton is forced out of each magnetic region more quickly, which results in a decrease in the time.

III. The time for an orbit in a magnetic field is independent of speed. Therefore, the time the proton spends in each of magnetic regions is the same no matter what its speed.

80. •• **BIO Magnetic Resonance Imaging** An MRI (magnetic resonance imaging) solenoid produces a magnetic field of 1.5 T. The solenoid is 2.5 m long, 1.0 m in diameter, and wound with insulated wires 2.2 mm in diameter. Find the current that flows in the solenoid. (Your answer should be rather large. A typical MRI solenoid uses niobium–titanium wire kept at liquid helium temperatures, where it is superconducting.)

81. •• **IP** A long, straight wire carries a current of 14 A. Next to the wire is a square loop with sides 1.0 m in length, as shown in Figure 22–46. The loop carries a current of 2.5 A in the direction indicated. **(a)** What is the direction of the net force exerted on



▲ **FIGURE 22–46** Problems 81 and 82

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the loop? Explain. **(b)** Calculate the magnitude of the net force acting on the loop.

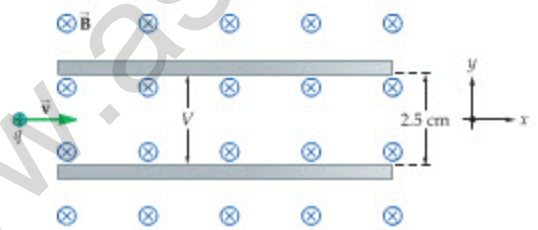
82. •• Suppose the 14-A current in the straight wire in Figure 22–46 is reversed in direction, but the current in the loop is unchanged. **(a)** Calculate the magnitude and direction of the net force acting on the loop. **(b)** If the loop is extended in the horizontal direction, so that it is 1.0 m high and 2.0 m wide, does the net force exerted on the loop increase or decrease? By what factor? Explain. **(c)** If, instead, the loop is extended in the vertical direction, so it is 2.0 m high and 1.0 m wide, does the net force exerted on the loop increase or decrease? Explain.

83. •• A charged particle moves through a region of space containing both electric and magnetic fields. The velocity of the particle is $\vec{\mathbf{V}} = (4.4 \times 10^3 \text{m/s})\hat{\mathbf{x}} + (2.7 \times 10^3 \text{ m/s})\hat{\mathbf{y}}$ and the magnetic field is $\vec{\mathbf{B}} = (0.73 \text{ T})\hat{\mathbf{z}}$. Find the electric field vector $\vec{\mathbf{E}}$ necessary to yield zero net force on the particle. (*Note:* You may wish to use cross products in this problem. They are discussed in Appendix A.)

84. •• **IP Medical X-rays** An electron in a medical X-ray machine is accelerated from rest through a voltage of 10.0 kV. **(a)** Find the maximum force a magnetic field of 0.957 T can exert on this electron. **(b)** If the voltage of the X-ray machine is increased, does the maximum force found in part (a) increase, decrease, or stay the same? Explain. **(c)** Repeat part (a) for an electron accelerated through a potential of 25.0 kV.

85. •• A particle with a charge of 34 μC moves with a speed of 73 m/s in the positive x direction. The magnetic field in this region of space has a component of 0.40 T in the positive y direction, and a component of 0.85 T in the positive z direction. What are the magnitude and direction of the magnetic force on the particle?

86. •• **IP** A beam of protons with various speeds is directed in the positive x direction. The beam enters a region with a uniform magnetic field of magnitude 0.52 T pointing in the negative z direction, as indicated in Figure 22–47. It is desired to use a uniform electric field (in addition to the magnetic field) to select from this beam only those protons with a speed of 1.42×10^5 m/s—that is, only these protons should be undeflected by the two fields. **(a)** Determine the magnitude and direction of the electric field that yields the desired result. **(b)** Suppose the electric field is to be produced by a parallel-plate capacitor with a plate separation of 2.5 cm. What potential difference is required between the plates? **(c)** Which plate in Figure 22–47 (top or bottom) should be positively charged? Explain.



▲ **FIGURE 22–47** Problem 86

87. •• **IP** A charged particle moves in a horizontal plane with a speed of 8.70×10^6 m/s. When this particle encounters a uniform magnetic field in the vertical direction it begins to move on a circular path of radius 15.9 cm. **(a)** If the magnitude of the magnetic field is 1.21 T, what is the charge-to-mass ratio (q/m) of this particle? **(b)** If the radius of the circular path were greater than 15.9 cm, would the corresponding charge-to-mass ratio be greater than, less than, or the same as that found in part (a)? Explain. (Assume that the magnetic field remains the same.)

88. •• Two parallel wires, each carrying a current of 2.2 A in the same direction, are shown in Figure 22–48. Find the direction and magnitude of the net magnetic field at points A, B, and C.

▲ **FIGURE 22–48** Problems 88 and 89

89. •• Repeat Problem 88 for the case where the current in wire 1 is reversed in direction.

90. •• **Lightning Bolts** A powerful bolt of lightning can carry a current of 225 kA. **(a)** Treating a lightning bolt as a long, thin wire, calculate the magnitude of the magnetic field produced by such a bolt of lightning at a distance of 35 m. **(b)** If two such bolts strike simultaneously at a distance of 35 m from each other, what is the magnetic force per meter exerted by one bolt on the other?

91. •• **IP** Consider the two current-carrying wires shown in Figure 22–49. The current in wire 1 is 3.7 A; the current in wire 2 is adjusted to make the net magnetic field at point A equal to zero. **(a)** Is the magnitude of the current in wire 2 greater than, less than, or the same as that in wire 1? Explain. **(b)** Find the magnitude and direction of the current in wire 2.

▲ **FIGURE 22–49** Problems 91 and 92

92. •• **IP** Consider the physical system shown in Figure 22–49, which consists of two current-carrying wires each with a length of 71 cm. **(a)** If the net magnetic field at the point A is out of the page, is the force between the wires attractive or repulsive? Explain. **(b)** Calculate the magnitude of the force exerted by each wire on the other wire, given that the magnetic field at point A is out of the page with a magnitude of 2.1×10^{-6} T.

93. •• **Magnetars** The astronomical object 4U014+61 has the distinction of creating the most powerful magnetic field ever observed. This object is referred to as a "magnetar" (a subclass of pulsars), and its magnetic field is 1.3×10^{15} times greater than the Earth's magnetic field. **(a)** Suppose a 2.5-m straight wire carrying a current of 1.1 A is placed in this magnetic field at an angle of 65° to the field lines. What force does this wire experience? **(b)** A field this strong can significantly change the behavior of an atom. To see this, consider an electron moving with a speed of 2.2×10^6 m/s. Compare the maximum magnetic force exerted on the electron to the electric force a proton exerts on an electron in a hydrogen atom. The radius of the hydrogen atom is 5.29×10^{-11} m.

94. •• Consider a system consisting of two concentric solenoids, as illustrated in Figure 22–50. The current in the outer solenoid is $I_1 = 1.25$ A, and the current in the inner solenoid is $I_2 = 2.17$ A. Given that the number of turns per centimeter is 105 for the outer solenoid and 125 for the inner solenoid, find the magnitude and direction of magnetic field **(a)** between the solenoids and **(b)** inside the inner solenoid.

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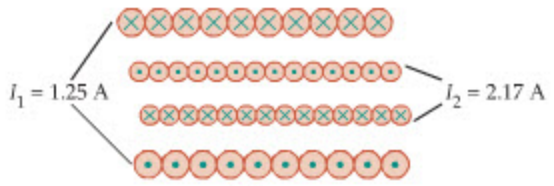


▲ **FIGURE 22–50** Problem 94

95. •• **IP** A long, straight wire on the x axis carries a current of 3.12 A in the positive x direction. The magnetic field produced by the wire combines with a uniform magnetic field of 1.45×10^{-6} T that points in the positive z direction. **(a)** Is the net magnetic field of this system equal to zero at a point on the positive y axis or at a point on the negative y axis? Explain. **(b)** Find the distance from the wire to the point where the field vanishes.

96. •• Find the angle between the plane of a loop and the magnetic field for which the magnetic torque acting on the loop is equal to x times its maximum value, where $0 \leq x \leq 1$.

97. •• Solenoids produce magnetic fields that are relatively intense for the amount of current they carry. To make a direct comparison, consider a solenoid with 55 turns per centimeter, a radius of 1.05 cm, and a current of 0.622 A. **(a)** Find the magnetic field at the center of the solenoid. **(b)** What current must a long, straight wire carry to have the same magnetic field as that found in part (a)? Let the distance from the wire be the same as the radius of the solenoid, 1.05 cm.

98. •• The current in a solenoid with 22 turns per centimeter is 0.50 A. The solenoid has a radius of 1.5 cm. A long, straight wire runs along the axis of the solenoid, carrying a current of 13 A. Find the magnitude of the net magnetic field a radial distance of 0.75 cm from the straight wire.

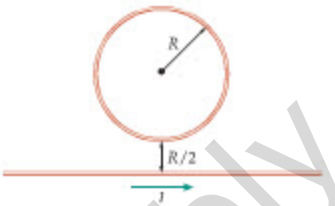
99. •• **IP** **BIO** **Transcranial Magnetic Stimulation** A recently developed method to study brain function is to produce a rapidly changing magnetic field within the brain. When this technique, known as transcranial magnetic stimulation (TMS), is applied to the prefrontal cortex, for example, it can reduce a person's ability to conjugate verbs, though other thought processes are unaffected. The rapidly varying magnetic field is produced with a circular coil of 21 turns and a radius of 6.0 cm placed directly on the head. The current in this loop increases at the rate of 1.2×10^7 A/s (by discharging a capacitor). **(a)** At what rate does the magnetic field at the center of the coil increase? **(b)** Suppose a second coil with half the area of the first coil is used instead. Would your answer to part (a) increase, decrease, or stay the same? By what factor?

100. ••• An electron with a velocity given by $\vec{\mathbf{v}} = (1.5 \times 10^5 \text{ m/s})\hat{\mathbf{x}} + (0.67 \times 10^4 \text{ m/s})\hat{\mathbf{y}}$ moves through a region of space with a magnetic field $\vec{\mathbf{B}} = (0.25 \text{ T})\hat{\mathbf{x}} - (0.11 \text{ T})\hat{\mathbf{z}}$ and an electric field $\vec{\mathbf{E}} = (220 \text{ N/C})\hat{\mathbf{x}}$. Using cross products, find the magnitude of the net force acting on the electron. (Cross products are discussed in Appendix A.)

101. ••• A thin ring of radius R and charge per length λ rotates with an angular speed ω about an axis perpendicular to its plane and passing through its center. Find the magnitude of the magnetic field at the center of the ring.

102. ••• A solenoid is made from a 25-m length of wire of resistivity $2.3 \times 10^{-8} \Omega \cdot$ m. The wire, whose radius is 2.1 mm, is wrapped uniformly onto a plastic tube 4.5 cm in diameter and 1.65 m long. Find the emf to which the ends of the wire must be connected to produce a magnetic field of 0.015 T within the solenoid.

103. ••• **IP** A single current-carrying circular loop of radius R is placed next to a long, straight wire, as shown in Figure 22–51. The current in the wire points to the right and is of magnitude I. **(a)** In which direction must current flow in the loop to produce zero magnetic field at its center? Explain. **(b)** Calculate the magnitude of the current in part (a).



▲ **FIGURE 22–51** Problem 103

104. ••• **Magnetic Fields in the Bohr Model** In the Bohr model of the hydrogen atom, the electron moves in a circular orbit of radius 5.29×10^{-11} m about the nucleus. Given that the charge on the electron is -1.60×10^{-19} C, and that its speed is 2.2×10^6 m/s, find the magnitude of the magnetic field the electron produces at the nucleus of the atom.

105. ••• A single-turn square loop carries a current of 18 A. The loop is 15 cm on a side and has a mass of 0.035 kg. Initially the loop lies flat on a horizontal tabletop. When a horizontal magnetic field is turned on, it is found that only one side of the loop experiences an upward force. Find the minimum magnetic field, B_{min}, necessary to start tipping the loop up from the table.

106. ••• Consider the physical system shown in Figure 22–40. **(a)** Find the net magnetic field (direction and magnitude) at an arbitrary point on the bottom side of the square, a distance $0 < x < 5.0$ cm to the right of wire 1. **(b)** Find the magnitude of the net magnetic field at an arbitrary point on the left side of the square, a distance $0 < y < 5.0$ cm above wire 1.

PASSAGE PROBLEMS

BIO Magnetoencephalography

To read and understand this sentence your brain must process visual input from your eyes and translate it into words and thoughts. As you do so, minute electric currents flow through the neurons in your visual cortex. These currents, like any electric current, produce magnetic fields. In fact, even your innermost thoughts and dreams produce magnetic fields that can be detected outside your head.

Magnetoencephalography (MEG) is the study of magnetic fields produced by electrical activity in the brain. Though completely noninvasive, MEG can provide detailed information on spontaneous brain function—like alpha waves and pathological epileptic spikes—as well as brain activity that is evoked by visual, auditory, and tactile stimuli.

The magnetic fields produced by brain activity are incredibly weak—roughly 100 million times smaller than the Earth's magnetic field. Even so, sensitive detectors called SQUIDS (superconducting quantum interference devices), which were invented by physicists as a research tool, can detect fields as small as 1.0×10^{-15} T. Coupled with sophisticated electronics and software, and operating at liquid helium temperatures (−269 °C), SQUIDS can localize the source of brain activity to

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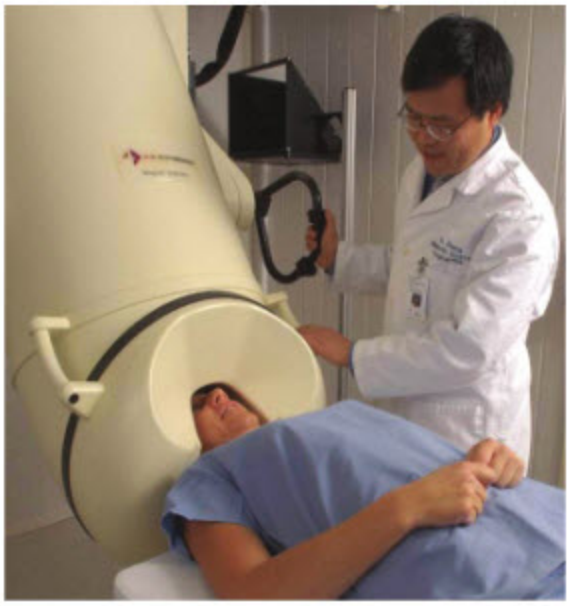
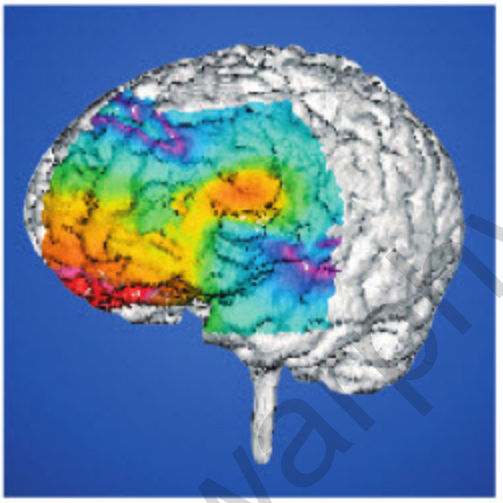
within millimeters. When the information from MEG is overlaid with the anatomical data from an MRI scan, the result is a richly detailed "map" of the electrical activity within the brain.





A magnetoencephalograph (MEG) is made by measuring the magnetic fields produced by the brain. It is a completely noninvasive process. The result is a map of the electrical activity within the brain. (Problems 107, 108, 109, and 110)

107. • Approximating a neuron by a straight wire, what electric current is needed to produce a magnetic field of 1.0×10^{-15} T at a distance of 5.0 cm?

A. 4.0×10^{-22} A **B.** 7.9×10^{-11} A

C. 2.5×10^{-10} A **D.** 1.0×10^{-7} A

108. • Suppose a neuron in the brain carries a current of 5.0×10^{-8} A. Treating the neuron as a straight wire, what is the magnetic field it produces at a distance of 7.5 cm?

A. 1.3×10^{-13} T **B.** 4.2×10^{-12} T

C. 1.1×10^{-10} T **D.** 3.3×10^{-7} T

109. • A given neuron in the brain carries a current of 3.1×10^{-8} A. If the SQUID detects a magnetic field of 2.8×10^{-14} T, how far away is the neuron? Treat the neuron as a straight wire.

A. 22 cm **B.** 70 cm

C. 140 cm **D.** 176 cm

110. •• A SQUID detects a magnetic field of 1.8×10^{-14} T at a distance of 13 cm. How many electrons flow through the neuron per second? Treat the neuron as a straight wire.

A. 1.2×10^{10} **B.** 2.3×10^{10}

C. 7.3×10^{10} **D.** 9.2×10^{10}

INTERACTIVE PROBLEMS

111. •• **IP Referring to Example 22–3** Suppose the speed of the isotopes is doubled. **(a)** Does the separation distance, d, increase, decrease, or stay the same? Explain. **(b)** Find the separation distance for this case.

112. •• **IP Referring to Example 22–3** Suppose we change the initial speed of ^{238}U, leaving everything else the same. **(a)** If we want the separation distance to be zero, should the initial speed of ^{238}U be increased or decreased? Explain. **(b)** Find the required initial speed.

113. •• **Referring to Active Example 22–2** The current I_1 is adjusted until the magnetic field halfway between the wires has a magnitude of 2.5×10^{-6} T and points *into* the page. Everything else in the system remains the same as in Active Example 22–2. Find the magnitude and direction of I_1.

114. •• **Referring to Active Example 22–2** The current I_2 is adjusted until the magnetic field 5.5 cm below wire 2 has a magnitude of 2.5×10^{-6} T and points *out* of page. Everything else in the system remains the same as in Active Example 22–2. Find the magnitude and direction of I_2.

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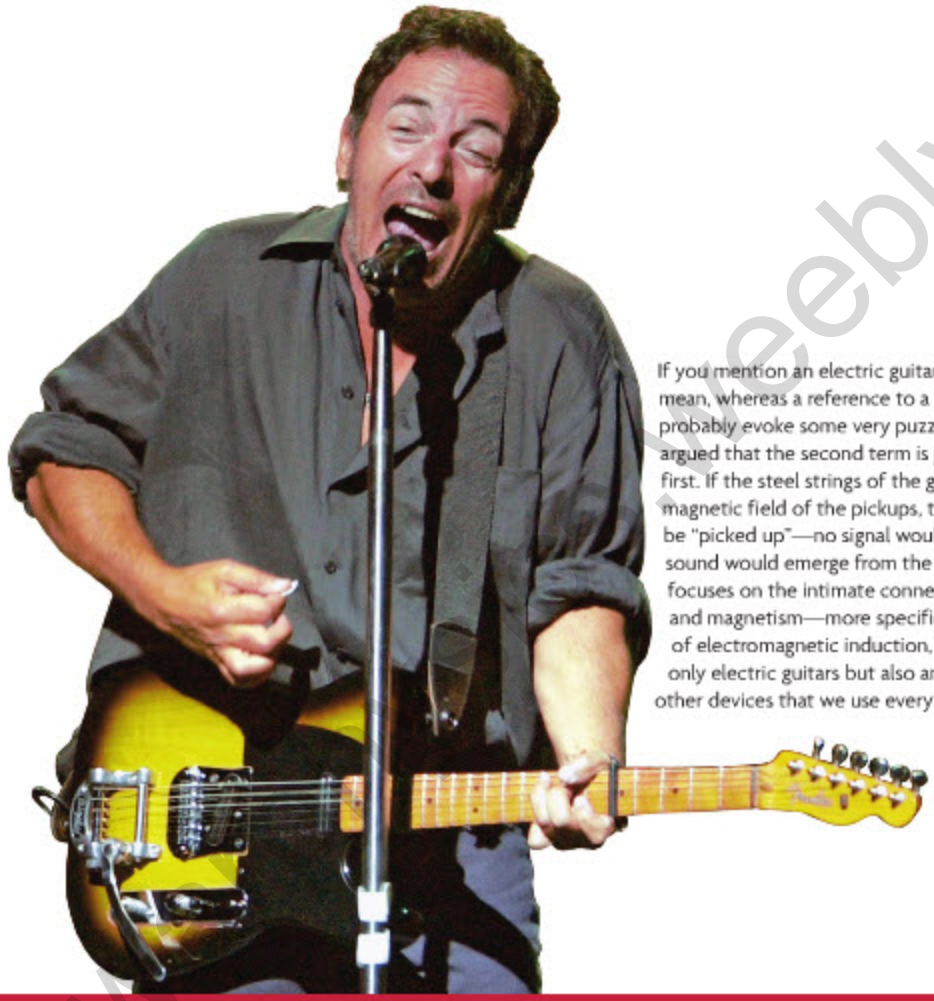
23 Magnetic Flux and Faraday's Law of Induction



If you mention an electric guitar, everyone knows what you mean, whereas a reference to a "magnetic guitar" would probably evoke some very puzzled looks. Yet it could be argued that the second term is just as appropriate as the first. If the steel strings of the guitar didn't respond to the magnetic field of the pickups, there would be nothing to be "picked up"—no signal would go to the amp, and no sound would emerge from the speaker. This chapter focuses on the intimate connection between electricity and magnetism—more specifically, on the phenomenon of electromagnetic induction, which makes possible not only electric guitars but also an enormous number of other devices that we use every day.

In the last chapter we saw a number of connections between electricity and magnetism. All the cases we considered, however, involved magnetic fields that were constant in time; that is, static fields. We now explore some of the interesting new phenomena that are produced when magnetic fields change with time.

Of particular interest is that changing magnetic fields can create electric fields, which, in turn, can cause stationary charges to move. When applied to an electric circuit, a changing magnetic field can produce an electric current, just like a battery. Effects of this nature have led to a host of important applications in everyday life, from electric motors and generators to microphones, speakers, and computer disk drives.

Finally, a recurrent theme throughout this chapter is energy—its conservation and the various forms in which it can appear. As we shall see, mechanical energy can be converted to electrical energy—with the help of a magnetic field—and the voltage and current in one electric circuit can be converted, or "transformed," into different values in another circuit. Just as in earlier chapters, we again find that energy is one of the most fundamental and versatile of all physical concepts.

23–1	Induced Electromotive Force	801
23–2	Magnetic Flux	802
23–3	Faraday's Law of Induction	804
23–4	Lenz's Law	807
23–5	Mechanical Work and Electrical Energy	810
23–6	Generators and Motors	813
23–7	Inductance	816
23–8	*RL* Circuits	819
23–9	Energy Stored in a Magnetic Field	820
23–10	Transformers	822

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23–1 Induced Electromotive Force

When Oersted observed that an electric current produces a magnetic field, it was pure serendipity. In contrast, Michael Faraday (1791–1867), an English chemist and physicist who was aware of Oersted's results, purposefully set out to determine whether a similar effect operates in the reverse direction, that is, whether a magnetic field can produce an electric field. His careful experimentation showed that such a connection does indeed exist.

In Figure 23–1 we show a simplified version of the type of experiment performed by Faraday. Two electric circuits are involved. The first, called the **primary circuit,** consists of a battery, a switch, a resistor to control the current, and a coil of several turns around an iron bar. When the switch is closed on the primary circuit, a current flows through the coil, producing a magnetic field that is particularly intense within the iron bar.

The **secondary circuit** also has a coil wrapped around the iron bar, and this coil is connected to an ammeter to detect any current in the circuit. Note, however, that there is no battery in the secondary circuit, and no direct physical contact between the two circuits. What does link the circuits, instead, is the magnetic field in the iron bar—it ensures that the field experienced by the secondary coil is approximately the same as that produced by the primary coil.

Now, let's look at the experimental results. When the switch is closed on the primary circuit, the magnetic field in the iron bar rises from zero to some finite amount, and the ammeter in the secondary coil deflects to one side briefly, and then returns to zero. As long as the current in the primary circuit is maintained at a constant value, the ammeter in the secondary circuit gives zero reading. If the switch on the primary circuit is now opened, so that the magnetic field decreases again to zero, the ammeter in the secondary circuit deflects briefly in the opposite direction, and then returns to zero. We can summarize these observations as follows:

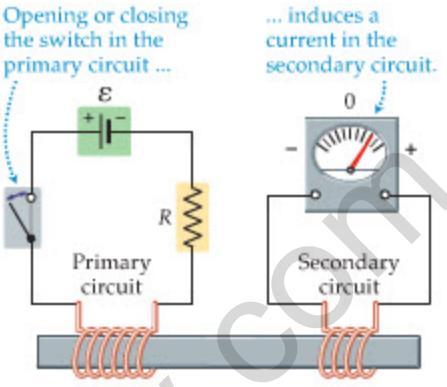
- The current in the secondary circuit is zero as long as the current in the primary circuit is constant—which means, in turn, that the magnetic field in the iron bar is constant. It does not matter whether the constant value of the magnetic field is zero or nonzero.
- When the magnetic field passing through the secondary coil increases, a current is observed to flow in one direction in the secondary circuit; when the magnetic field decreases, a current is observed in the opposite direction.

Recall that the current in the secondary circuit appears without any direct contact between the primary and secondary circuits. For this reason we refer to the secondary current as an **induced current.** Since the induced current behaves the same as a current produced by a battery with an electromotive force (emf), we say that the changing magnetic field creates an **induced emf** in the secondary circuit. This leads to our final experimental observation:

- The magnitudes of the induced current and induced emf are found to be proportional to the rate of change of the magnetic field—the more rapidly the magnetic field changes, the greater the induced emf.

In Section 23–3 we return to these observations and state them in the form of a mathematical law.

Finally, in Faraday's experiment the changing magnetic field is caused by a changing current in the primary circuit. Any means of altering the magnetic field is just as effective, however. For example, in Figure 23–2 we show a common classroom demonstration of induced emf. In this case, there is no primary circuit; instead, the magnetic field is changed by simply moving a permanent magnet toward or away from a coil connected to an ammeter. When the magnet is moved toward the coil, the meter deflects in one direction; when it is pulled away from the coil, the meter deflects in the opposite direction. There is no induced emf, however, when the magnet is held still.



▲ **FIGURE 23–1 Magnetic induction**
Basic setup of Faraday's experiment on magnetic induction. When the position of the switch on the primary circuit is changed from open to closed or from closed to open, an electromotive force (emf) is induced in the secondary circuit. The induced emf causes a current in the secondary circuit, and the current is detected by the ammeter. If the current in the primary circuit does not change, no matter how large it may be, there is no induced current in the secondary circuit.

▲ When a magnet moves toward or away from a coil, the magnetic field within the coil changes. This, in turn, induces an electric current in the coil, which is detected by the meter.

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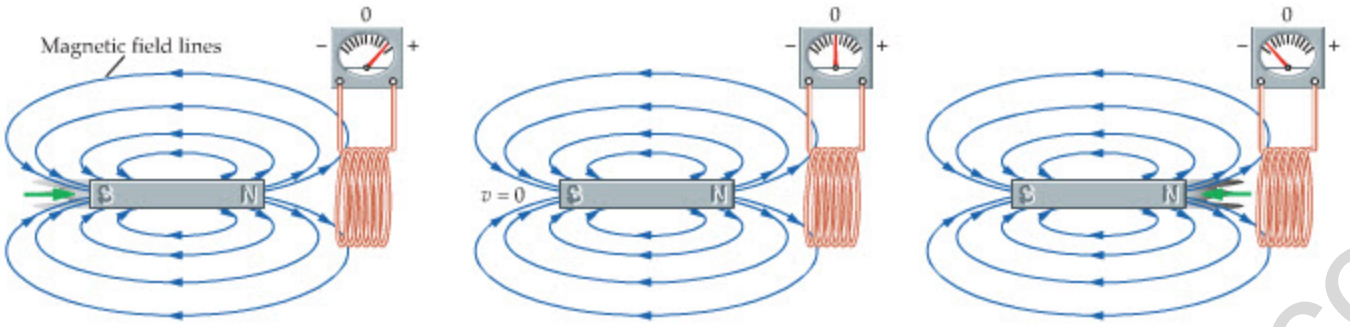


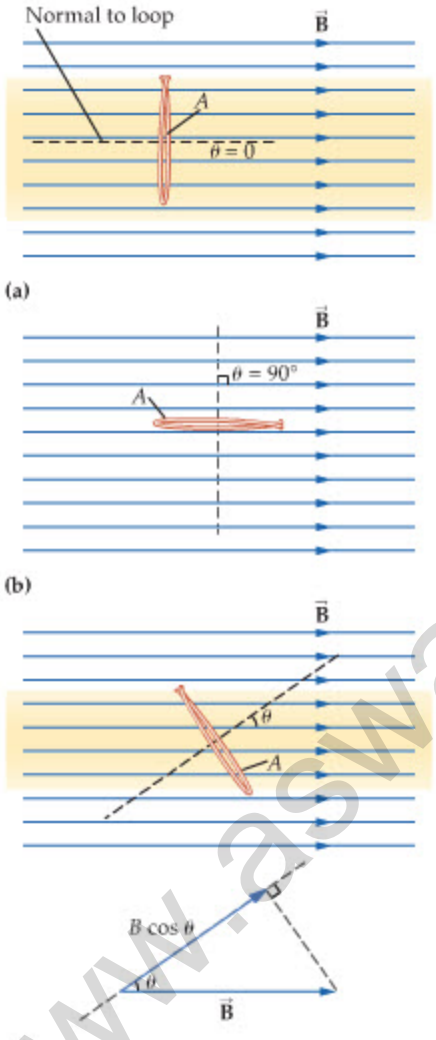
(a) Moving magnet toward coil induces current in one direction

(b) No motion, no induced current

(c) Moving magnet away from coil induces current in opposite direction

▲ **FIGURE 23–2 Induced current produced by a moving magnet**

A coil experiences an induced current when the magnetic field passing through it varies. **(a)** When the magnet moves toward the coil, the current is in one direction. **(b)** No current is induced while the magnet is held still. **(c)** When the magnet is pulled away from the coil, the current is in the other direction.



▲ **FIGURE 23–3 The magnetic flux through a loop**

The magnetic flux through a loop of area A is $\Phi = BA\cos\theta$, where θ is the angle between the normal to the loop and the magnetic field. **(a)** The loop is perpendicular to the field; hence, $\theta = 0$, and $\Phi = BA$. **(b)** The loop is parallel to the field; therefore, $\theta = 90°$ and $\Phi = 0$. **(c)** For a general angle θ, the component of the field that is perpendicular to the loop is $B\cos\theta$; hence, the flux is $\Phi = BA\cos\theta$.

23–2 Magnetic Flux

In the previous section, we saw that an emf can be induced in a coil of wire by simply changing the strength of the magnetic field that passes through the coil. This isn't the only way to induce an emf, however. More generally, we can change the direction of the magnetic field rather than its magnitude. We can even change the orientation of the coil in a constant magnetic field, or change the cross-sectional area of the coil. All of these situations can be described in terms of the change of a single physical quantity, the magnetic flux.

Basically, **magnetic flux** is a measure of the number of magnetic field lines that cross a given area, in complete analogy with the electric flux discussed in Section 19–7. Suppose, for example, that a magnetic field $\vec{\mathbf{B}}$ crosses a surface area A at right angles, as in Figure 23–3 (a). The magnetic flux, Φ, in this case is simply the magnitude of the magnetic field times the area:

$$\Phi = BA$$

If, on the other hand, the magnetic field is parallel to the surface, as in Figure 23–3 (b), we see that *no field lines cross the surface*. In a case like this the magnetic flux is zero:

$$\Phi = 0$$

In general, only the component of $\vec{\mathbf{B}}$ that is *perpendicular* to a surface contributes to the magnetic flux. The magnetic field in Figure 23–3 (c), for example, crosses the surface at an angle θ relative to the normal, and hence its perpendicular component is $B\cos\theta$. The magnetic flux, then, is simply $B\cos\theta$ times the area A:

Definition of Magnetic Flux, Φ

$$\Phi = (B\cos\theta)A = BA\cos\theta \qquad 23\text{–}1$$

SI unit: $1\ \mathrm{T\cdot m^2} = 1$ weber $= 1$ Wb

Note that $\Phi = BA\cos\theta$ gives $\Phi = BA$ when $\vec{\mathbf{B}}$ is perpendicular to the surface ($\theta = 0$), and $\Phi = 0$ when $\vec{\mathbf{B}}$ is parallel to the surface ($\theta = 90°$), as expected. Finally, the unit of magnetic flux is the **weber (Wb)**, named after the German physicist Wilhelm Weber (1804–1891). It is defined as follows:

$$1\ \mathrm{Wb} = 1\ \mathrm{T\cdot m^2} \qquad 23\text{–}2$$

As we have seen, magnetic flux depends on the magnitude of the magnetic field, B, its orientation with respect to a surface, θ, and the area of the surface, A. A change in any of these variables results in a change in the flux. For example, in the case of the permanent magnet moved toward or away from the coil in the previous section, it is the change in *magnitude* of the field, B, that results in a change in flux. In the following Example, we consider the effect of changing the *orientation* of a wire loop in a region of constant magnetic field.

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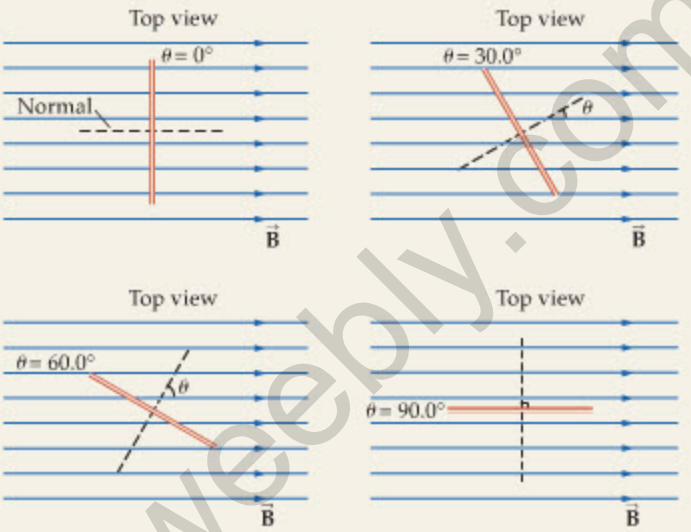
EXAMPLE 23–1 A SYSTEM IN FLUX

Consider a circular loop with a 2.50-cm radius in a constant magnetic field of 0.625 T. Find the magnetic flux through this loop when its normal makes an angle of **(a)** 0°, **(b)** 30.0°, **(c)** 60.0°, and **(d)** 90.0° with the direction of the magnetic field $\vec{\mathbf{B}}$.

PICTURE THE PROBLEM

Our sketch shows four top views of the system, looking directly down on the top edge of the circular loop. The orientation of the loop and its normal is shown in each of the four panels. Note that the specified angles between the normal and the magnetic field direction are $\theta = 0°$, 30.0°, 60.0°, and 90.0°.



STRATEGY

To find the magnetic flux, we use the following relation $\Phi = BA\cos\theta$. In this expression, B is the magnitude of the magnetic field and $A = \pi r^2$ is the area of a circular loop of radius r. The values of B, r, and θ are given in the problem statement.

SOLUTION

Part (a)

1. Substitute $\theta = 0°$ in $\Phi = BA\cos\theta$:

$$\Phi = BA\cos\theta = (0.625\ \text{T})\pi(0.0250\ \text{m})^2\cos 0° = 1.23 \times 10^{-3}\ \text{T}\cdot\text{m}^2$$

Part (b)

2. Substitute $\theta = 30.0°$ in $\Phi = BA\cos\theta$:

$$\Phi = BA\cos\theta = (0.625\ \text{T})\pi(0.0250\ \text{m})^2\cos 30.0° = 1.06 \times 10^{-3}\ \text{T}\cdot\text{m}^2$$

Part (c)

3. Substitute $\theta = 60.0°$ in $\Phi = BA\cos\theta$:

$$\Phi = BA\cos\theta = (0.625\ \text{T})\pi(0.0250\ \text{m})^2\cos 60.0° = 6.14 \times 10^{-4}\ \text{T}\cdot\text{m}^2$$

Part (d)

4. Substitute $\theta = 90.0°$ in $\Phi = BA\cos\theta$:

$$\Phi = BA\cos\theta = (0.625\ \text{T})\pi(0.0250\ \text{m})^2\cos 90.0° = 0$$

INSIGHT

Thus, even if a magnetic field is uniform in space and constant in time, the magnetic flux through a given area will change if the orientation of the area changes. This is particularly relevant to the case of a coil that rotates in a field and constantly changes its orientation. As we shall see in the next section, a changing magnetic flux can create an electric current, and Section 23–6 shows how this applies to generators and motors.

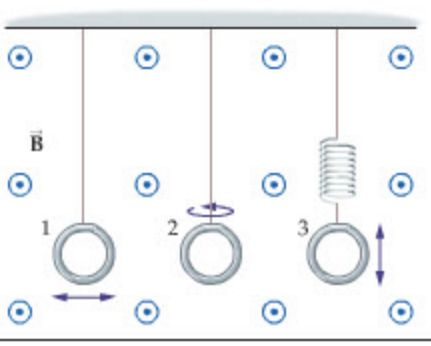
PRACTICE PROBLEM

At what angle is the flux in this system equal to $1.00 \times 10^{-4}\ \text{T}\cdot\text{m}^2$? **[Answer:** $\theta = 85.3°$**]**

Some related homework problems: Problem 1, Problem 3

CONCEPTUAL CHECKPOINT 23–1 MAGNETIC FLUX

The three loops of wire shown in the sketch are all in a region of space with a uniform, constant magnetic field. Loop 1 swings back and forth as the bob on a pendulum; loop 2 rotates about a vertical axis; and loop 3 oscillates vertically on the end of a spring. Which loop or loops have a magnetic flux that changes with time?



REASONING AND DISCUSSION

Loop 1 moves back and forth, and loop 3 moves up and down, but since the magnetic field is uniform, the flux doesn't depend on the loop's position. Loop 2, on the other hand, changes its orientation relative to the field as it rotates; hence, its flux does change with time.

ANSWER

Only loop 2 has a changing magnetic flux.

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23–3 Faraday's Law of Induction

Now that the magnetic flux is defined, it is possible to be more precise about the experimental observations described in Section 23–1. In particular, Faraday found that the secondary coil in Figure 23–1 experiences an induced emf only when the *magnetic flux* through it changes with time. Furthermore, the induced emf for a given loop is found to be proportional to the *rate* at which the flux changes with time, $\Delta\Phi/\Delta t$. If there are N loops in a coil, each with the same magnetic flux, Faraday found that the induced emf is given by the following relation:

Faraday's Law of Induction

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{\Phi_{\text{final}} - \Phi_{\text{initial}}}{t_{\text{final}} - t_{\text{initial}}} \qquad 23\text{–}3$$

In this expression, known as **Faraday's law of induction**, the minus sign in front of N indicates that the induced emf opposes the change in magnetic flux, as we show in the next section. When we are concerned only with magnitudes, as is often the case, we use the form of Faraday's law given next:

Magnitude of Induced emf

$$|\mathcal{E}| = N\left|\frac{\Delta\Phi}{\Delta t}\right| = N\left|\frac{\Phi_{\text{final}} - \Phi_{\text{initial}}}{t_{\text{final}} - t_{\text{initial}}}\right| \qquad 23\text{–}4$$

Notice that Faraday's law gives the *emf* that is induced in a circuit or a loop of wire. The current that is induced as a result of the emf depends on the characteristics of the circuit itself—for example, how much resistance it contains.

EXERCISE 23–1

The induced emf in a single loop of wire has a magnitude of 1.48 V when the magnetic flux is changed from 0.850 $\text{T}\cdot\text{m}^2$ to 0.110 $\text{T}\cdot\text{m}^2$. How much time is required for this change in flux?

SOLUTION

Solving Equation 23–4 for the time, we obtain

$$|\Delta t| = N\frac{|\Delta\Phi|}{|\mathcal{E}|} = (1)\frac{|0.110\ \text{T}\cdot\text{m}^2 - 0.850\ \text{T}\cdot\text{m}^2|}{1.48\ \text{V}} = 0.500\ \text{s}$$

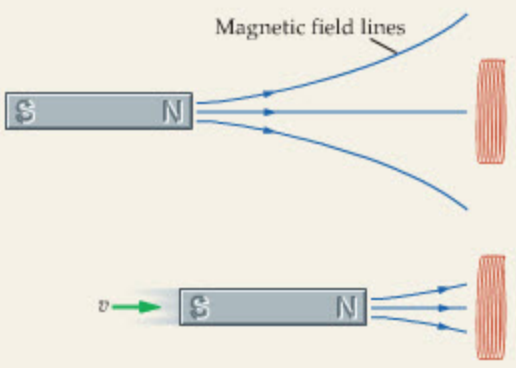
In the next Example we continue to deal with magnitudes, so we use Equation 23–4. We also consider a system in which the magnetic field varies in both magnitude and direction over the area of a coil. We can still calculate the magnetic flux in such a case if we simply replace the perpendicular component of the magnetic field ($B\cos\theta$) with its average value, B_{av}. With this substitution, the magnetic flux, $\Phi = BA\cos\theta$, becomes $\Phi = B_{\text{av}}A$.

EXAMPLE 23–2 BAR MAGNET INDUCTION

A bar magnet is moved rapidly toward a 40-turn circular coil of wire. As the magnet moves, the average value of $B\cos\theta$ over the area of the coil increases from 0.0125 T to 0.450 T in 0.250 s. If the radius of the coil is 3.05 cm, and the resistance of its wire is 3.55 Ω, find the magnitude of **(a)** the induced emf and **(b)** the induced current.

PICTURE THE PROBLEM

The motion of the bar magnet relative to the coil is shown in our sketch. As the magnet approaches the coil, the average value of the perpendicular component of the magnetic field increases. As a result, the magnetic flux through the coil increases with time.



STRATEGY

To find the induced emf and the induced current we must first calculate the rate of change of the magnetic flux. To find the flux, we use $\Phi = B_{\text{av}}A$, where $A = \pi r^2$, and B_{av} is the average value of $B\cos\theta$. Once we have found the induced emf using Faraday's law, $|\mathcal{E}| = N|\Delta\Phi/\Delta t|$, we obtain the induced current using Ohm's law, $I = V/R$, with $V = |\mathcal{E}|$.

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SOLUTION

Part (a)

1. Calculate the initial and final values of the magnetic flux through the coil:

$$\Phi_i = B_i A = (0.0125\ \text{T})\pi(0.0305\ \text{m})^2 = 3.65 \times 10^{-5}\ \text{T}\cdot\text{m}^2$$
$$\Phi_f = B_f A = (0.450\ \text{T})\pi(0.0305\ \text{m})^2 = 1.32 \times 10^{-3}\ \text{T}\cdot\text{m}^2$$

2. Use Faraday's law to find the magnitude of the induced emf:

$$|\mathcal{E}| = N\left|\frac{\Phi_{\text{final}} - \Phi_{\text{initial}}}{t_{\text{final}} - t_{\text{initial}}}\right| = (40)\left|\frac{1.32 \times 10^{-3}\ \text{T}\cdot\text{m}^2 - 3.65 \times 10^{-5}\ \text{T}\cdot\text{m}^2}{0.250\ \text{s}}\right| = 0.205\ \text{V}$$

Part (b)

3. Use Ohm's law to calculate the induced current:

$$I = \frac{V}{R} = \frac{0.205\ \text{V}}{3.55\ \Omega} = 0.0577\ \text{A}$$

INSIGHT

If the magnet is now pulled back to its original position in the same amount of time, the induced emf and current will have the same magnitudes; their directions will be reversed, however.

PRACTICE PROBLEM

How many turns of wire would be required in this coil to give an induced current of at least 0.100 A? [**Answer:** $N = 70$]

Some related homework problems: Problem 9, Problem 18, Problem 19

REAL-WORLD PHYSICS

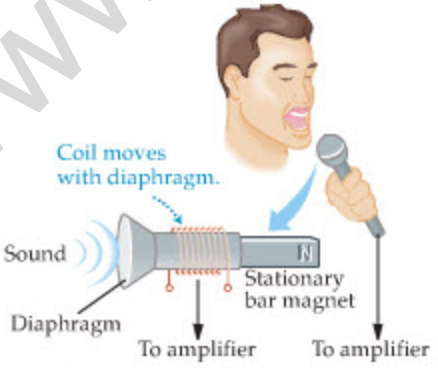
Dynamic microphones and seismographs

An everyday application of Faraday's law of induction can be found in a type of microphone known as a *dynamic microphone*. These devices employ a stationary permanent magnet and a wire coil attached to a movable diaphragm, as illustrated in Figure 23–4. When a sound wave strikes the microphone, it causes the diaphragm to oscillate, moving the coil alternately closer to and farther from the magnet. This movement changes the magnetic flux through the coil, which in turn produces an induced emf. Connecting the coil to an amplifier increases the magnitude of the induced emf enough that it can power a set of large speakers. The same principle is employed in the *seismograph*, except in this case the oscillations that produce the induced emf are generated by earthquakes and the vibrations they send through the ground.

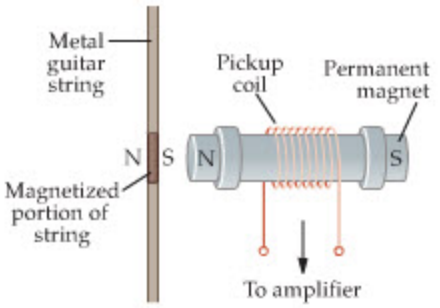
REAL-WORLD PHYSICS

Electric guitar pickups

The American pop-jazz guitarist Les Paul (1915–) applied the same basic physics to musical instruments when he made the first solid-body *electric guitar* in 1941. The pickup in an electric guitar is simply a small permanent magnet with a coil wrapped around it, as shown in Figure 23–5. This magnet produces a field that is strong enough to produce a magnetization in the steel guitar string, which is the moving part in the system. When the string is plucked, the oscillating string changes the magnetic flux in the coil, inducing an emf that can be amplified.



▲ **FIGURE 23–4** A dynamic microphone



▲ **FIGURE 23–5** The pickup on an electric guitar

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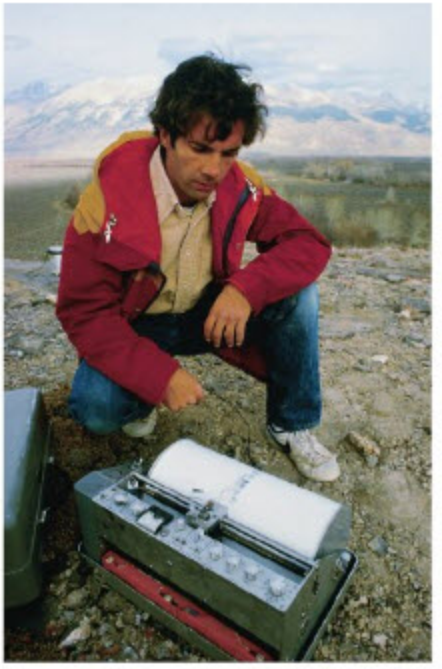
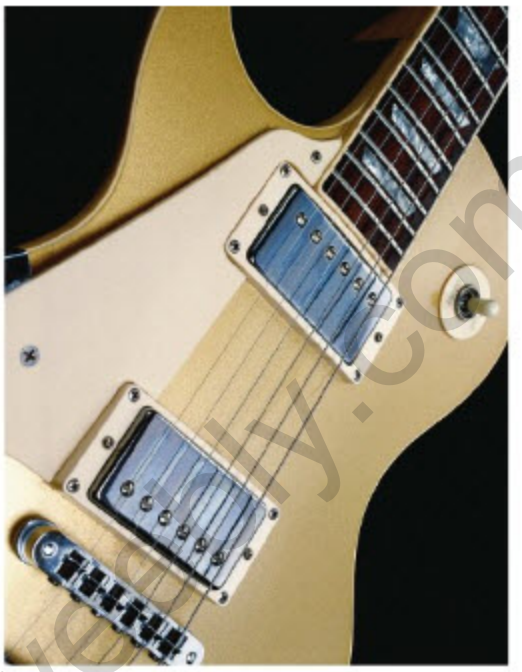
▶ Many devices take advantage of Faraday's law to translate physical oscillations into electrical impulses. In all such devices, the oscillations are used to change the relative position of a coil and a magnet, thus causing the electric current in the coil to vary. For example, the vibrations produced by an earthquake or underground explosion produce an oscillating current in a seismograph (left) that can be amplified and used to drive a plotting pen. The magnetic pickups located under the strings of an electric guitar (right) function in much the same way. In this case, plucking a string (which is magnetized by the pickup) causes it to vibrate and generate an oscillating electrical signal in a pickup. This signal is amplified and fed to speakers, which produce most of the sound we hear.



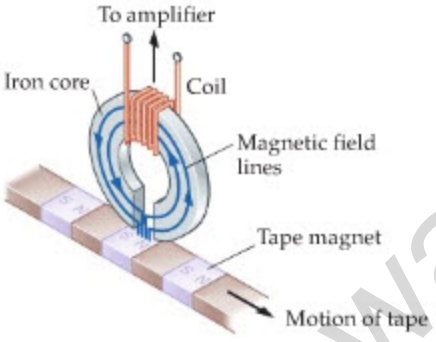


A typical electric guitar will have two or three sets of pickups, each positioned to amplify a different harmonic of the vibrating strings.

REAL-WORLD PHYSICS

Magnetic disk drives and credit card readers

Faraday's law of induction also applies to the operation of computer disk drives, credit card readers at grocery stores, audio tape players, and ATM machines. In these devices, information is recorded on a magnetic material by using an electromagnet to produce regions of opposite magnetic polarity, as indicated in Figure 23–6. When the information is to be "read out," the magnetic material is moved past the gap in an iron core that is wrapped with many turns of a wire coil. As magnetic regions of different polarity pass between the poles of the magnet, its field is alternately increased or decreased—resulting in a changing magnetic flux through the coil. The corresponding induced emf is sent to electronic circuitry to be translated into information a computer can process.



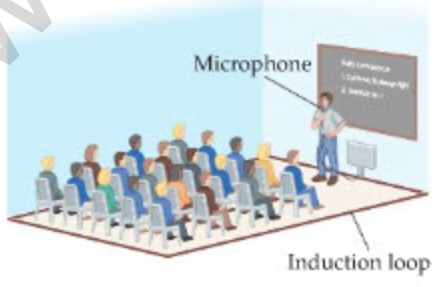
▲ **FIGURE 23–6** **Magnetic tape recording**

Finally, induced emf can be of assistance to the hearing impaired. Many hearing aids have a "T switch" that activates a coil of wire in the device known as the telecoil, or **T coil** for short. This coil is designed to pick up the varying magnetic field produced by the speaker in a telephone and to convert the corresponding induced emf into sound for the user of the hearing aid. It should be emphasized that the varying magnetic field of the telephone speaker is simply a natural byproduct of the way it operates and is not produced specifically for the benefit of persons with hearing loss. Nonetheless, a person using the T coil on a hearing aid can usually hear more clearly than if the hearing aid merely amplifies the sound that comes from the speaker.

REAL-WORLD PHYSICS

T coils and induction loops

This principle is now being used on a larger scale in churches, auditoriums, and other public places. A single loop of wire, known as an **induction loop**, is placed on the floor of a room around its perimeter, as shown in Figure 23–7. When a person gives a talk in the room, the signal from the microphone is sent not only to speakers but to the induction loop as well. The loop produces a varying magnetic field throughout the room that hearing aids with the T switch can convert directly to sound. Ironically, it is often the hearing impaired in such a case who are able to hear the speech most clearly.



◀ **FIGURE 23–7** **An induction loop for the hearing impaired**
A single loop of wire placed around the perimeter of a room can be used as an induction loop that creates a varying magnetic field. This field can be picked up by T coils in hearing aids.

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23–4 Lenz's Law

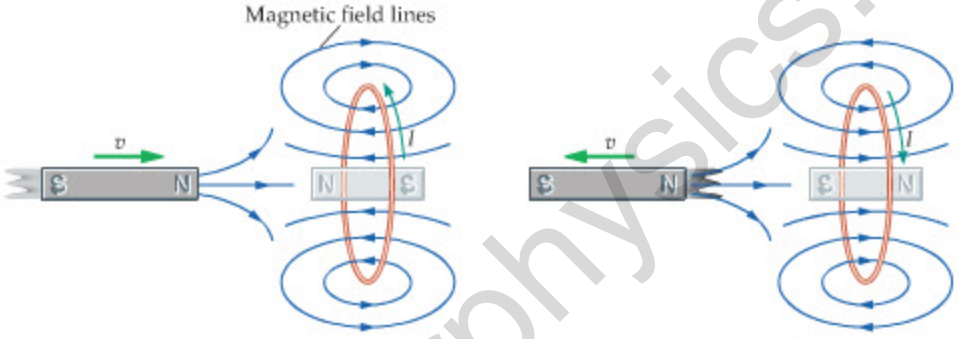
In this section we discuss **Lenz's law,** a physical way of expressing the meaning of the minus sign in Faraday's law of induction. The basic idea of Lenz's law, first stated by the Estonian physicist Heinrich Lenz (1804–1865), is the following:

> **Lenz's Law**
> An induced current always flows in a direction that *opposes* the change that caused it.

This general physical principle is closely related to energy conservation, as we shall see at several points in our discussion. The remainder of this section, however, is devoted to specific applications of Lenz's law.

As a first example, consider a bar magnet that is moved toward a conducting ring, as in Figure 23–8 (a). If the north pole of the magnet approaches the ring, a current is induced that tends to oppose the motion of the magnet. To be specific, the current in the ring creates a magnetic field that has a north pole (that is, diverging field lines) facing the north pole of the magnet, as indicated in the figure. There is now a repulsive force acting on the magnet, opposing its motion.

On the other hand, if the magnet is pulled away from the ring, as in Figure 23–8 (b), the induced current creates a south pole facing the north pole of the magnet. The resulting attractive force again opposes the magnet's motion. We can obtain the same results using Faraday's law directly, but Lenz's law gives a simpler, more physical way to approach the problem.



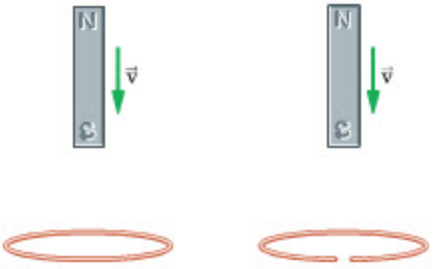
(a) Moving magnet toward coil induces a field that repels the magnet

(b) Moving magnet away from coil induces a field that attracts the magnet

FIGURE 23–8 Applying Lenz's law to a magnet moving toward and away from a current loop
(a) If the north pole of a magnet is moved toward a conducting loop, the induced current produces a north pole pointing toward the magnet's north pole. This creates a repulsive force opposing the change that caused the current. **(b)** If the north pole of a magnet is pulled away from a conducting loop, the induced current produces a south magnetic pole near the magnet's north pole. The result is an attractive force opposing the motion of the magnet.

CONCEPTUAL CHECKPOINT 23–2 FALLING MAGNETS

The magnets shown in the sketch are dropped from rest through the middle of conducting rings. Notice that the ring on the right has a small break in it, whereas the ring on the left forms a closed loop. As the magnets drop toward the rings, does the magnet on the left have an acceleration that is **(a)** more than, **(b)** less than, or **(c)** the same as that of the magnet on the right?



REASONING AND DISCUSSION

As the magnet on the left approaches the ring, it induces a circulating current. According to Lenz's law, this current produces a magnetic field that exerts a repulsive force on the magnet—to oppose its motion. In contrast, the ring on the right has a break, so it cannot have a circulating current. As a result, it exerts no force on its magnet. Therefore, the magnet on the right falls with the acceleration of gravity; the magnet on the left falls with a smaller acceleration.

ANSWER

(b) The magnet on the left has the smaller acceleration.

In the examples given so far in this section, the "change" involved the motion of a bar magnet, but Lenz's law applies no matter how the magnetic flux is changed. Suppose, for example, that a magnetic field is decreased with time, as

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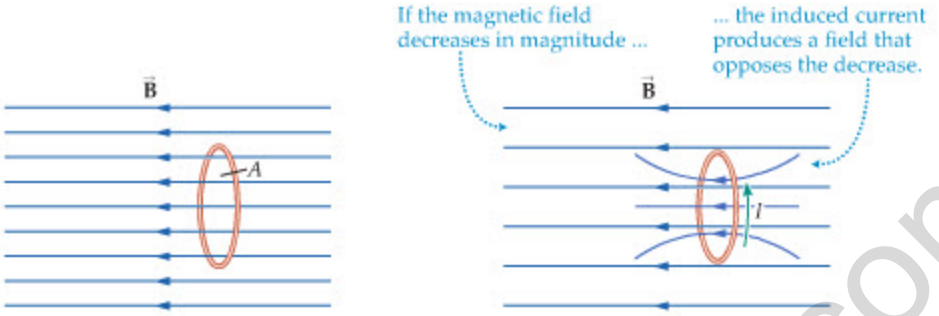


▶ **FIGURE 23–9 Lenz's law applied to a decreasing magnetic field**
As the magnetic field is decreased, the induced current produces a magnetic field that passes through the ring in the same direction as $\vec{B}$.

(a) A conducting loop in a constant magnetic field

(b) A change in the field induces a current

illustrated in Figure 23–9. In this case the change is a decrease of magnetic flux through the ring. The induced current flows in a direction that tends to oppose this change; that is, the induced current must produce a field within the ring that is in the same direction as the decreasing B field, as we see in Figure 23–9 (b).

Motional emf: Qualitative

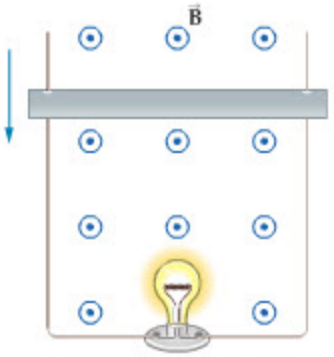
Next, we consider the physical situation shown in Figure 23–10. Here we see a metal rod that is free to move in the vertical direction, and a constant magnetic field that points out of the page. The rod is in frictionless contact with two vertical wires; hence, a current can flow in a loop through the rod, the wires, and the lightbulb. Since the motion of the rod produces an emf in this system, it is referred to as **motional emf**. The question to be answered at this point is this: What is the direction of the current when the rod is released from rest and allowed to fall?

In this system the magnetic field is constant, but the magnetic flux through the current loop, $\Phi = BA$, decreases anyway. The reason is that as the rod falls, the area A enclosed by the loop decreases. To oppose this decrease in flux, Lenz's law implies that the induced current must flow in a direction that strengthens the magnetic field $\vec{B}$ within the loop, thus increasing the flux. This direction is counterclockwise, as indicated in Figure 23–11.

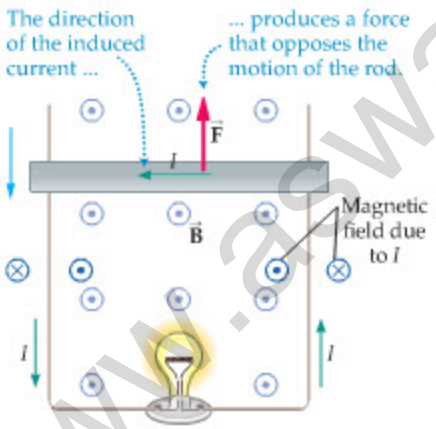
Notice that the current in the rod flows from right to left. This direction is also in agreement with Lenz's law, since it opposes the change that caused it. To be specific, the change for the rod is that it begins to fall downward. In order to oppose this change, the current through the rod must produce an upward magnetic force. As we see in Figure 23–11, using the right-hand rule, a current from right to left does just that.

When the rod is first released, there is no current and its downward acceleration is the acceleration of gravity—the rod speeds up as its gravitational potential energy is converted to kinetic energy. As the rod falls, the induced current begins to flow, however, and the rod is acted on by the upward magnetic force. This causes its acceleration to decrease. Ultimately, the magnetic force cancels the force of gravity, and the rod's acceleration vanishes. From this point on it falls with constant speed and hence constant kinetic energy. Now you may wonder what happens to the rod's gravitational potential energy. In fact, all the rod's gravitational potential energy is now being converted to heat and light in the lightbulb—none of it goes to increasing the kinetic energy of the rod.

Imagine for a moment the consequences that would follow if Lenz's law were not obeyed. If the current in the rod were from left to right, for example, the magnetic force would be downward, causing the rod to increase its downward acceleration. As the rod sped up, the induced current would become even larger and the downward force would increase as well. Thus, the rod would simply move faster and faster, without limit. Clearly, energy would not be conserved in such a case. The connections between mechanical and electrical energy are explored in greater detail in the next section.



▲ **FIGURE 23–10 Motional emf**
Motional emf is created in this system as the rod falls. The result is an induced current, which causes the light to shine.



▲ **FIGURE 23–11 Determining the direction of an induced current**
The direction of the current induced by the rod's downward motion is counterclockwise, because this direction produces a magnetic field within the loop that points out of the page—in the same direction as the original field. Notice that a current flowing in this direction through the rod interacts with the original magnetic field to give an upward force, opposing the downward motion of the rod.

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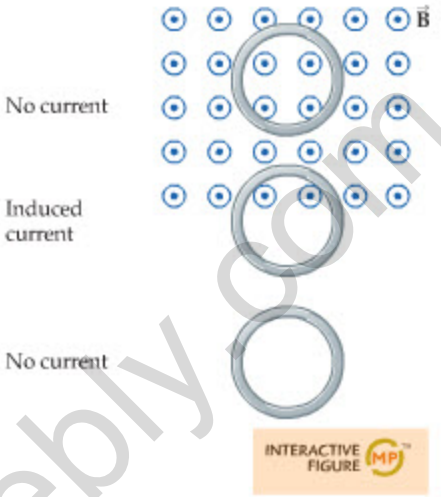
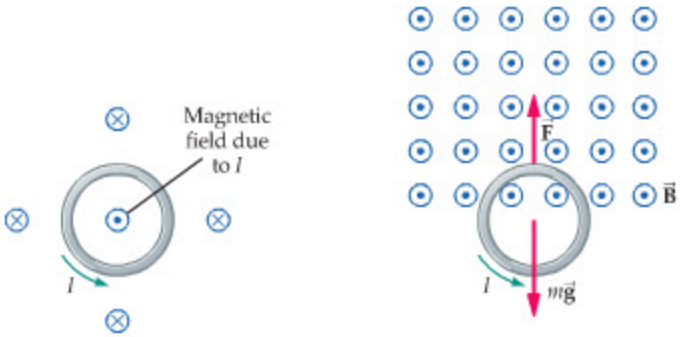
CONCEPTUAL CHECKPOINT 23–3 THE DIRECTION OF INDUCED CURRENT

Consider a system in which a metal ring is falling out of a region with a magnetic field and into a field-free region, as shown in our sketch to the right. According to Lenz's law, is the induced current in the ring **(a)** clockwise or **(b)** counterclockwise?



REASONING AND DISCUSSION

The induced current must be in a direction that opposes the change in the system. In this case, the change is that fewer magnetic field lines are piercing the area of the loop and pointing out of the page. The induced current can oppose this change by generating more field lines out of the page *within the loop*. As shown to the left in the following figure, the induced current must be counterclockwise to accomplish this.



Finally, to the right in the preceding figure, note that the induced current generates an upward magnetic force at the top of the ring, but no magnetic force on the bottom, where the magnetic field is zero. Hence, the motion of the ring is retarded as it drops out of the field.

ANSWER

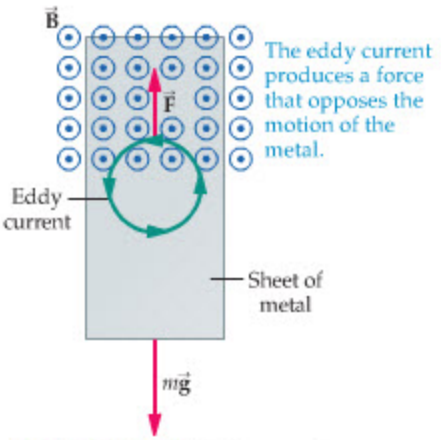
(b) The induced current is counterclockwise.

The retarding effect on a ring leaving a magnetic field, as just described in the Conceptual Checkpoint, allows us to understand the behavior of **eddy currents.** Suppose, for example, that a sheet of metal falls from a region with a magnetic field to a region with no field, as in Figure 23–12. In the portion of the sheet that is just leaving the field, a circulating current—an eddy current—is induced in the metal, just like the circulating current set up in the ring in Conceptual Checkpoint 23–3. As in the case of the ring, the current will retard the motion of the sheet of metal, somewhat like a frictional force.

The frictionlike effect of eddy currents is the basis for the phenomenon known as *magnetic braking*. An important advantage of this type of braking is that no direct physical contact is needed, thus eliminating frictional wear. In addition, the force of magnetic braking is stronger the greater the speed of the metal with respect to the magnetic field. In contrast, kinetic friction is independent of the relative speed of the surfaces. Magnetic braking is used in everything from fishing reels to exercise bicycles to roller coasters.

Perhaps the most dramatic manifestation of magnetic braking is the slowing down of rotating magnetic stars. White dwarf stars, with intense magnetic fields, produce eddy currents in the clouds of ionized material that surround them. The result is a rapid slowing of their rotational speed, almost as if they were rotating in a viscous liquid. In some stars, magnetic braking has increased the rotational period from as short as a few hours to as long as 200 years.

On a more down-to-earth scale, magnetic braking is the operating principle behind the common analog *speedometer*. In these devices, a cable is connected at one end to a gear in the car's transmission and at the other end to a small permanent magnet. When the car is under way, the magnet rotates within a metal cup. The resulting "magnetic friction" between the rotating magnet and the metal cup—which is proportional to the speed of rotation—causes the cup to rotate in the same direction. A hairspring attached to the cup stops the rotation when the force of the spring equals the force of magnetic friction. Thus, the cup—and the needle attached to it—rotate through an angle that is proportional to the speed of the car.



▲ **FIGURE 23–12 Eddy currents**
A circulating current is induced in a sheet of metal near where it leaves a region with a magnetic field. This "eddy current" is similar to the current induced in the ring described in Conceptual Checkpoint 23–3. In both cases, the induced current exerts a retarding force on the moving object, opposing its motion.

REAL-WORLD PHYSICS

Magnetic braking and speedometers

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REAL-WORLD PHYSICS
Induction stove

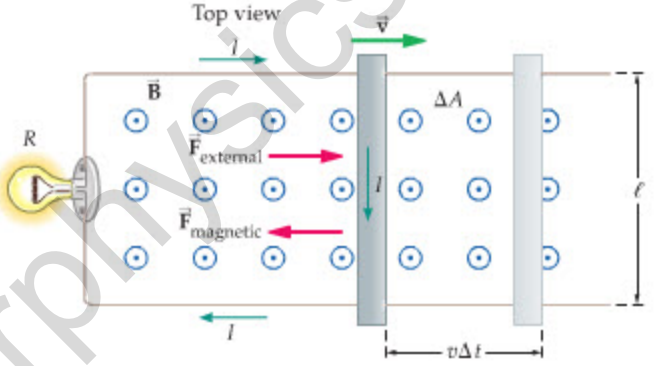
Finally, eddy currents are also used in the kitchen. In an *induction stove*, a metal coil is placed just beneath the cooking surface. When an alternating current is sent through the coil, it sets up an alternating magnetic field that, in turn, induces eddy currents in nearby metal objects. The finite resistance of a metal pan, for instance, will result in heating as the eddy current dissipates power according to the relation $P = I^2R$. Nonmetallic objects, like the surface of the stove and glassware, remain cool to the touch because they are poor electrical conductors and thus have negligible eddy currents.

23–5 Mechanical Work and Electrical Energy

To see how mechanical work can be converted to electrical energy, we turn to a detailed calculation of the forces and currents involved in a simple physical system. This analysis will form the basis for our understanding of electrical generators in the next section.

Motional emf: Quantitative

We again consider motional emf, though this time in a somewhat simpler system where gravity plays no role. As shown in Figure 23–13, our setup consists of a rod that slides horizontally without friction on a U-shaped wire connected to a lightbulb of resistance R. A constant magnetic field points out of the page, and the rod is pushed by an external agent—your hand, for instance—so that it moves to the right with a constant speed v.



▶ **FIGURE 23–13 Force and induced current**
A conducting rod slides without friction on horizontal wires in a region where the magnetic field is uniform and pointing out of the page. The motion of the rod to the right induces a clockwise current and a corresponding magnetic force to the left. An external force of magnitude $F = B^2v\ell^2/R$ is required to offset the magnetic force and to keep the rod moving with a constant speed v.

Let's begin by calculating the rate at which the magnetic flux changes with time. Since $\vec{\mathbf{B}}$ is perpendicular to the area of the loop, it follows that $\Phi = BA$. Now the area increases with time because of the motion of the rod. In fact, the rod moves a horizontal distance $v\Delta t$ in the time Δt; therefore, the area of the loop increases by the amount $\Delta A = (v\Delta t)\ell$, as indicated in Figure 23–13. As a result, the increase in magnetic flux is

$$\Delta\Phi = B\,\Delta A = Bv\ell\,\Delta t$$

With this result in hand, we can use Faraday's law to calculate the magnitude of the induced emf. We find that for this single-turn loop

$$|\mathcal{E}| = N\left|\frac{\Delta\Phi}{\Delta t}\right| = (1)\frac{Bv\ell\,\Delta t}{\Delta t} = Bv\ell \qquad 23\text{–}5$$

As one might expect, the induced emf is proportional to the strength of the magnetic field and to the speed of the rod.

Now let's calculate the electric field caused by the motion of the rod. We know that the emf from one end of the rod to the other is $\mathcal{E} = Bv\ell$, where $\mathcal{E}$ is an electric potential difference measured in volts. Recall that the difference in electric potential caused by an electric field E over a length ℓ is $V = E\ell$. Applying this to the emf in the rod, we have $E\ell = Bv\ell$, or

$$E = Bv \qquad 23\text{–}6$$

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Thus, a changing magnetic flux does indeed create an electric field, as Faraday suspected. In particular, moving through a magnetic field B at a speed v causes an electric field of magnitude $E = Bv$ in a direction that is at right angles to both the velocity and the magnetic field.

EXERCISE 23–2

As the rod in Figure 23–13 moves through a 0.445-T magnetic field, it experiences an induced electric field of 0.668 V/m. How fast is the rod moving?

SOLUTION

Solving Equation 23–6 for v, we find

$$v = \frac{E}{B} = \frac{0.668\ \text{V/m}}{0.445\ \text{T}} = 1.50\ \text{m/s}$$

If we assume that the only resistance in the system is the lightbulb, R, it follows that the current in the circuit has the following magnitude:

$$I = \frac{|\mathcal{E}|}{R} = \frac{Bv\ell}{R} \qquad 23\text{–}7$$

Using Lenz's law, we can determine that the direction of the current is clockwise, as indicated in Figure 23–13. Next, we show how both the magnitude and direction of the current play a key role in the energy aspects of this system.

Mechanical Work/Electrical Energy

We begin with a calculation of the force and power required to keep the rod moving with a constant speed v. First, note that the rod carries a current of magnitude $I = Bv\ell/R$ at right angles to a magnetic field of strength B. Since the length of the rod is ℓ, it follows that the magnetic force exerted on it has a magnitude

$$F = I\ell B = \left(\frac{Bv\ell}{R}\right)(\ell)B = \frac{B^2v\ell^2}{R} \qquad 23\text{–}8$$

The direction of the current in the rod is downward on the page in Figure 23–13; hence, the magnetic-force *RHR* shows that $\vec{\mathbf{F}}_{\text{magnetic}}$ is directed to the left. As a result, an external force, $\vec{\mathbf{F}}_{\text{external}}$, must act to the right with equal magnitude to maintain the rod's constant speed. The mechanical power delivered by the external force is simply the force times the speed:

$$P_{\text{mechanical}} = Fv = \left(\frac{B^2v\ell^2}{R}\right)v = \frac{B^2v^2\ell^2}{R} \qquad 23\text{–}9$$

Now let's compare this mechanical power with the electrical power converted to light and heat in the lightbulb. Recalling that the power dissipated in a resistor with a resistance R is $P = I^2R$, we have

$$P_{\text{electrical}} = I^2R = \left(\frac{Bv\ell}{R}\right)^2 R = \frac{B^2v^2\ell^2}{R} \qquad 23\text{–}10$$

Note that this is *exactly* the same as the mechanical power. Thus, with the aid of a magnetic field, we can convert *mechanical* power directly to *electrical* power. This simple example of motional emf illustrates the basic principle behind the generation of virtually all the world's electrical energy.

EXAMPLE 23–3 LIGHT POWER

The lightbulb in the circuit shown here has a resistance of 12 Ω and consumes 5.0 W of power; the rod is 1.25 m long and moves to the left with a constant speed of 3.1 m/s. **(a)** What is the strength of the magnetic field? **(b)** What external force is required to maintain the rod's constant speed?

CONTINUED ON NEXT PAGE

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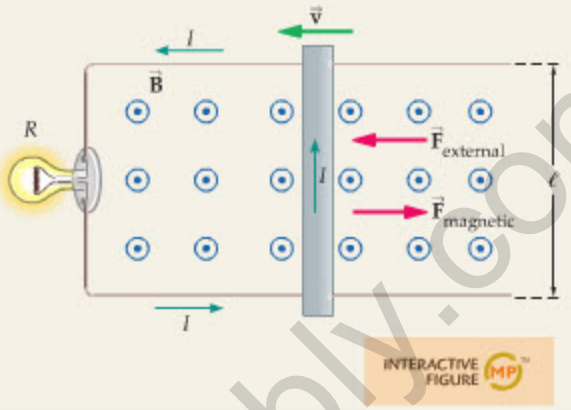
PICTURE THE PROBLEM

Our sketch shows the rod being pushed to the left by the force $\vec{F}_{\text{external}}$. The motion of the rod, in turn, generates an electric current whose direction, according to Lenz's law, produces a magnetic force, $\vec{F}_{\text{magnetic}}$, that opposes the motion of the rod. We know that the rod moves with constant speed, and therefore the applied force $\vec{F}_{\text{external}}$ exactly cancels the magnetic force $\vec{F}_{\text{magnetic}}$.



STRATEGY

a. The power consumed by the lightbulb is given by the expression $P = B^2v^2\ell^2/R$. This relation can be solved to give the strength of the magnetic field, B.

b. The magnitude of the external force is given by $F = B^2v\ell^2/R$, where B has the value obtained in part (a). Alternatively, we can solve for the force using the power relation, $P = Fv$.

SOLUTION

Part (a)

1. Solve the power expression for the magnetic field, B:

$$P = \frac{B^2v^2\ell^2}{R}$$

$$B = \frac{\sqrt{PR}}{v\ell}$$

2. Substitute numerical values:

$$B = \frac{\sqrt{PR}}{v\ell} = \frac{\sqrt{(5.0\text{ W})(12\ \Omega)}}{(3.1\text{ m/s})(1.25\text{ m})} = 2.0\text{ T}$$

Part (b)

3. Calculate the magnitude of the external force using $F = B^2v\ell^2/R$:

$$F = \frac{B^2v\ell^2}{R} = \frac{(2.0\text{ T})^2(3.1\text{ m/s})(1.25\text{ m})^2}{12\ \Omega} = 1.6\text{ N}$$

4. Find the magnitude of the external force using $P = Fv$:

$$F = \frac{P}{v} = \frac{5.0\text{ W}}{3.1\text{ m/s}} = 1.6\text{ N}$$

INSIGHT

Note that a constant external *force* is required to move the rod with constant *speed* and that the magnitude of the external force is proportional to the speed. This is similar to the behavior of a "drag" force like air resistance. Also, note that the *net* force acting on the rod is zero—as expected for an object that moves with constant speed.

PRACTICE PROBLEM

How fast will the rod move if the external force exerted on it is 1.5 N? **[Answer:** $v = 2.9$ m/s**]**

Some related homework problems: Problem 39, Problem 40, Problem 41

ACTIVE EXAMPLE 23–1 FIND THE WORK CONVERTED TO LIGHT ENERGY

Referring to Example 23–3, find **(a)** the work done by the external force in 0.58 s and **(b)** the energy consumed by the lightbulb in the same time.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Find the distance covered by the rod in 0.58 s: 1.8 m
2. Find the work done using $W = Fd$: $(1.6\text{ N})(1.8\text{ m}) = 2.9\text{ J}$

Part (b)

3. Calculate the energy used in the lightbulb by multiplying the power by the time: $(5.0\text{ W})(0.58\text{ s}) = 2.9\text{ J}$

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INSIGHT
As expected, the energy used by the lightbulb is equal to the work done by the external force as it pushes the rod against the magnetic force.

YOUR TURN
What is the current flowing in this circuit? Obtain your answer in at least two different ways.

(Answers to **Your Turn** *problems are given in the back of the book.)*

Motional emfs are generated whenever a magnet moves relative to a conductor or when a conductor moves relative to a magnetic field. For example, *magnetic antitheft devices* in libraries and stores detect the motional emf produced by a small magnetic strip placed in a book or other item. If this strip is not demagnetized, it will trigger an alarm as it moves past a sensitive set of conductors designed to pick up the induced emf.

REAL-WORLD PHYSICS
Magnetic antitheft devices

In numerous biomedical applications of the same principle, it is the conductor that moves while the magnetic field remains stationary. For example, researchers have tracked the head and body movements of several flying insects, including blowflies, hover flies, and honeybees. They attach lightweight, flexible wires to a small metal coil on the insect's head, and another on its thorax, and then allow the insect to fly in a stationary magnetic field. As the coils move through the field, they experience induced emfs that can be analyzed by computer to determine the corresponding orientation of the head and thorax.

REAL-WORLD PHYSICS: BIO
Tracking the movement of insects

The same technique is used by researchers investigating the movements of the human eye. You may not be aware of it, but your eyes are jiggling back and forth rather rapidly at this very moment. This constant motion, referred to as **saccadic motion,** prevents an image from appearing at the same position on the retina for a period of time. Without saccadic motion, our brains would tend to ignore stationary images that we are focused on in favor of moving images of little interest. To study the dynamics of saccadic motion, a small coil of wire is attached to a modified contact lens. As the eye moves the coil through a stationary magnetic field, the induced motional emf gives the eye's orientation and rotational speed.

REAL-WORLD PHYSICS: BIO
Tracking the motion of the human eye

23–6 Generators and Motors

We have just seen that a changing magnetic flux can serve as a means of converting mechanical work to electrical energy. In this section we discuss this concept in greater detail as we explore the workings of an electric generator. We also point out that the energy conversion can run in the other direction as well, with electrical energy being converted to mechanical work in an electric motor.

Electric Generators

An electric **generator** is a device designed to efficiently convert mechanical energy to electrical energy. The mechanical energy can be supplied by any of a number of sources, such as falling water in a hydroelectric dam, expanding steam in a coal-fired power plant, or the output of a small gasoline motor that powers a portable generator. In all cases the basic operating principle is the same—mechanical energy is used to move a conductor through a magnetic field to produce motional emf.

REAL-WORLD PHYSICS
Electric generators

The linear motion of a metal rod through a magnetic field, as in Figure 23–13, results in motional emf, but to continue producing an electric current in this way the rod must move through ever greater distances. A practical way to employ the same effect is to use a coil of wire that can be *rotated* in a magnetic field. By rotating the coil, which in turn changes the magnetic flux, we can continue the process indefinitely.

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▲ Hydroelectric power provides an excellent example of energy transformations. First, the gravitational potential energy of water held behind a dam is converted to kinetic energy as the water descends through pipes in the generating station (left). Next, the kinetic energy of the falling water is used to spin turbines (center). Each turbine rotates the coil of a generator in a strong magnetic field, thus converting mechanical energy into an electric current in the coil (right). Eventually the electrical energy is delivered to homes, factories, and cities where it is converted to light (in incandescent bulbs and fluorescent tubes), heat (in devices such as toasters and broilers), and work (in various machines).

Imagine, then, a coil of area A located in the magnetic field between the poles of a magnet, as illustrated in Figure 23–14. Metal rings are attached to either end of the wire that makes up the coil, and carbon brushes are in contact with the rings to allow the induced emf to be communicated to the outside world. As the coil rotates with the angular speed ω, the emf produced in it is given by Faraday's law, $\mathcal{E} = -N(\Delta\Phi/\Delta t)$. Referring to the similar case of uniform circular motion in Chapter 13, and Equation 13–6 in particular, it can be shown that the explicit expression for the emf of a rotating coil is as follows:

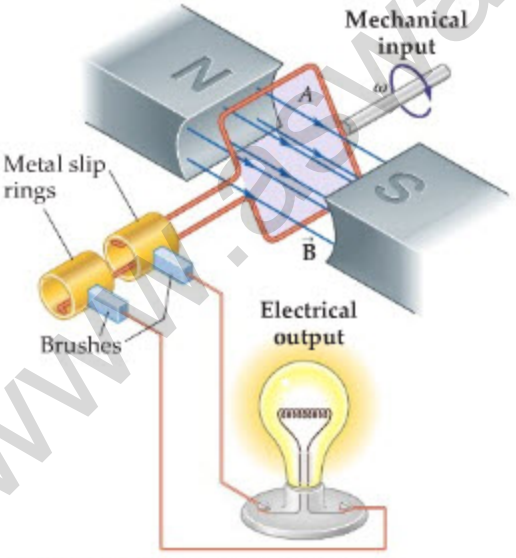
$$\mathcal{E} = NBA\omega \sin \omega t \qquad 23\text{–}11$$

This result is plotted in Figure 23–15. Notice that the induced emf in the coil alternates in sign, which means that the current in the coil alternates in direction. For this reason, this type of generator is referred to as an **alternating-current generator** or, simply, an ac generator.

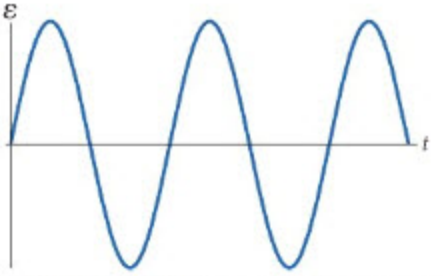
PROBLEM-SOLVING NOTE

Maximum and Minimum Values of Induced emf

Note that the induced emf of a rotating coil has a maximum value of $NBA\omega$ and a minimum value of $-NBA\omega$. This follows from Equation 23–11 and the fact that the maximum and minimum values of $\sin \omega t$ are $+1$ and -1, respectively.



▲ **FIGURE 23–14 An electric generator**
The basic operating elements of an electric generator are shown in a schematic representation. As the coil is rotated by an external source of mechanical work, it produces an emf that can be used to power an electric circuit.



▲ **FIGURE 23–15 Induced emf of a rotating coil**
Induced emf produced by an electric generator like the one shown in Figure 23–14. The emf alternates in sign, so the current changes sign as well. We say that a generator of this type produces an "alternating current."

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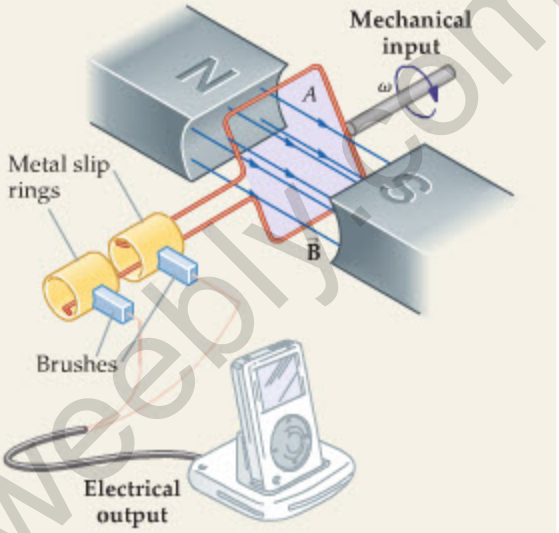
EXAMPLE 23–4 GENERATOR NEXT

The coil in an electric generator has 100 turns and an area of $2.5 \times 10^{-3}\ \text{m}^2$. It is desired that the maximum emf of the coil be 120 V when it rotates at the rate of 60.0 cycles per second. **(a)** What is the angular speed of the generator? **(b)** Find the strength of the magnetic field B that is required for this generator to operate at the desired voltage.

PICTURE THE PROBLEM

In our sketch, a square conducting coil of area A and angular speed ω rotates between the poles of a magnet, where the magnetic field has a strength B. As the coil rotates, the magnetic flux through it changes continuously. The result is an induced emf that varies sinusoidally with time.



STRATEGY

a. We need to find the angular speed, ω, since it is not given directly in the problem statement. What we are given, however, is that the frequency of the coil's rotation is $f = 60.0$ cycles per second $= 60.0$ Hz. The corresponding angular speed is $\omega = 2\pi f$. (See Chapter 13.)

b. The induced emf of the generator is given by $\mathcal{E} = NBA\omega \sin \omega t$. Therefore, the maximum emf occurs when $\sin \omega t$ has its maximum value of 1. It follows that $\mathcal{E}_{\text{max}} = NBA\omega$. This relation can be solved for the magnetic field, B.

SOLUTION

Part (a)

1. Calculate the angular speed of the coil: $\omega = 2\pi f = 2\pi(60.0\ \text{Hz}) = 377\ \text{rad/s}$

Part (b)

2. Solve for the magnetic field in terms of the maximum emf: $\mathcal{E}_{\text{max}} = NBA\omega \quad \text{or} \quad B = \dfrac{\mathcal{E}_{\text{max}}}{NA\omega}$

3. Substitute numerical values into the expression for B: $B = \dfrac{\mathcal{E}_{\text{max}}}{NA\omega} = \dfrac{120\ \text{V}}{(100)(2.5 \times 10^{-3}\ \text{m}^2)(377\ \text{rad/s})} = 1.3\ \text{T}$

INSIGHT

Note that the emf of a generator coil depends on its area A but not on its shape. In addition, each turn of the coil generates the same induced emf; therefore, the total emf of the coil is directly proportional to the number of turns. Finally, the more rapidly the coil rotates, the more rapidly the magnetic flux through it changes. As a result, the induced emf is also proportional to the angular speed of the generator.

PRACTICE PROBLEM

What is the maximum emf of this generator if its rotation frequency is reduced to 50.0 Hz? **[Answer:** 100 V]

Some related homework problems: Problem 42, Problem 43

Electric Motors

REAL-WORLD PHYSICS

Electric motors

The basic physical principle behind the operation of electric motors has already been discussed. In particular, we saw in Section 22–5 that a current-carrying loop in a magnetic field experiences a torque that tends to make it rotate. If such a loop is mounted on an axle, as in Figure 23–16, it is possible for the magnetic torque to be applied to the external world.

To see just how this works in practice, notice that the torque exerted on the loop at the moment shown in Figure 23–16 causes it to rotate clockwise *toward* the vertical position. Once it reaches this orientation, and continues past it due to its angular momentum, the alternating current from the electrical input reverses direction. This reverses the torque on the loop and causes the loop to rotate *away* from the vertical—which means it is still rotating in the clockwise sense. The next time the loop becomes vertical, the current again reverses, causing the loop to continue rotating clockwise. The result is an axle continually turning in the same direction.

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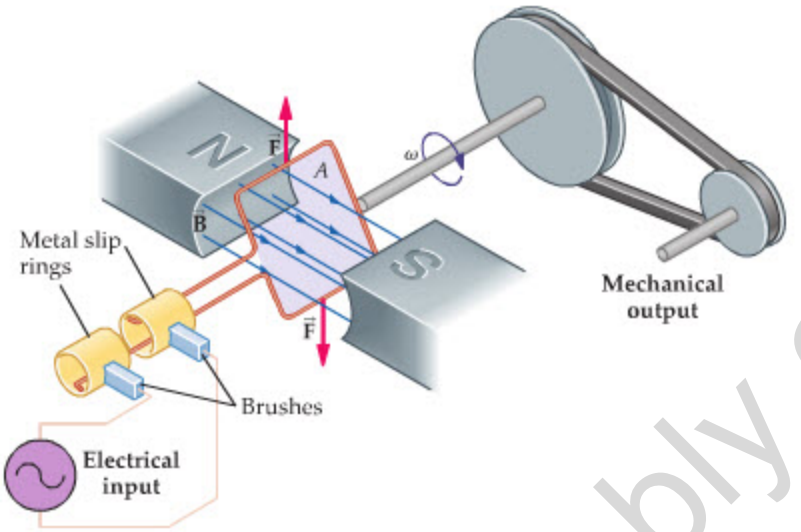
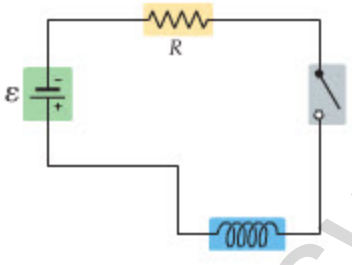


▶ **FIGURE 23–16 A simple electric motor**
An electric current causes the coil of a motor to rotate and deliver mechanical work to the outside world.

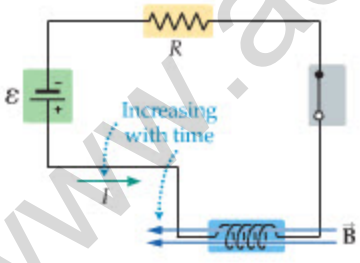
What we have just described is precisely the reverse of a generator—instead of doing work to turn a coil and produce an electric current, as in a generator, a motor uses an electric current to produce rotation in a coil, which then does work. Thus, a motor, in some sense, is a generator run in reverse. In fact, it is possible to have one electric motor turn the axle of another identical motor and thereby produce electricity from the "motor"-turned generator.

REAL-WORLD PHYSICS
Energy recovery technology in cars

Similarly, if a car is powered by an electric motor, its motor can double as a generator when it helps to slow the car during braking. This results in a more efficient mode of transportation, because some of the car's kinetic energy that is normally converted to heat during braking can instead be used to recharge the batteries and extend the range of the car. Some of the newer "hybrid" cars, such as the Honda Insight and the Toyota Prius, make use of this "energy recovery" technology.



(a) No current, no field in coil



(b) Increasing current produces increasing magnetic field in coil

▲ **FIGURE 23–17 A changing current in an inductor**
(a) A coil of wire with no current and no magnetic flux. **(b)** The current is now increasing with time, which produces a magnetic flux that also increases with time.

23–7 Inductance

We began this chapter with a description of Faraday's induction experiment, in which a changing current in one coil induces a current in another coil. This type of interaction between coils is referred to as **mutual inductance.** It is also possible, however, for a single, isolated coil to have a similar effect—that is, a coil with a changing current can induce an emf, and hence a current, in itself. This type of process is known as **self-inductance.**

To be more precise, consider a coil of wire in a region free of magnetic fields, as in Figure 23–17 (a). Initially the current in the coil is zero, so the magnetic flux through the coil is zero as well. The current in the coil is now steadily increased from zero, as indicated in Figure 23–17 (b). As a result, the magnetic flux through the coil increases with time, which, according to Faraday's law, induces an emf in the coil.

The direction of the induced emf is given by Lenz's law, which says that the induced current opposes the change that caused it. In this case, the change is the increasing current in the coil; hence, the induced emf is in a direction opposite to the current. This situation is illustrated in Figure 23–18, where we schematically show the effect of the induced emf (often referred to as a "back" emf) in the circuit. Note that in order to increase the current in the coil it is necessary to *force* the current against an opposing emf.

This is a rather interesting result. It says that even if the wires in a coil are ideal, with no resistance whatsoever, work must still be done to increase the current. Conversely, if a current already exists in a coil and it decreases with time, Lenz's law ensures that the self-induced current will oppose this change as well, in an attempt to keep the current constant. Hence, a coil tends to resist changes in its current, regardless of whether the change is an increase or a decrease.

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To make a quantitative connection between a changing current and the associated self-induced emf, we start with Faraday's law in magnitude form:

$$|\mathcal{E}| = N\left|\frac{\Delta\Phi}{\Delta t}\right|$$

Now, since the magnetic field is proportional to the current in a coil, it follows that the magnetic flux Φ is also proportional to I. As a result, the time rate of change of the magnetic flux, $\Delta\Phi/\Delta t$, must be proportional to the time rate of change of the current, $\Delta I/\Delta t$. Therefore, the induced emf can be written as

Definition of Inductance, *L*

$$|\mathcal{E}| = N\left|\frac{\Delta\Phi}{\Delta t}\right| = L\left|\frac{\Delta I}{\Delta t}\right| \qquad 23\text{–}12$$

SI unit: $1\ \text{V}\cdot\text{s/A} = 1\ \text{henry} = 1\ \text{H}$

This expression defines the constant of proportionality, L, which is referred to as the **inductance** of a coil. Rearranging Equation 23–12, we see that

$$L = N\left|\frac{\Delta\Phi}{\Delta I}\right|$$

The SI unit of inductance is the **henry (H)**, named to honor the work of Joseph Henry (1797–1878), an American physicist. As can be seen from Equation 23–12, the henry is one volt-second per amp:

$$1\ \text{H} = 1\ \text{V}\cdot\text{s/A} \qquad 23\text{–}13$$

Common inductances are generally in the millihenry (mH) range.

EXERCISE 23–3

The coil in an electromagnet has an inductance of 2.9 mH and carries a constant direct current of 5.6 A. A switch is suddenly opened, allowing the current to drop to zero over a small interval of time, Δt. If the magnitude of the average induced emf during this time is 7.3 V, what is Δt?

SOLUTION

Solving Equation 23–12 for Δt, and using only magnitudes, we find

$$\Delta t = \frac{L|\Delta I|}{|\mathcal{E}|} = \frac{(2.9\times10^{-3}\ \text{H})(5.6\ \text{A})}{7.3\ \text{V}} = 2.2\ \text{ms}$$

Of particular interest is the inductance of an ideal solenoid with N turns and length ℓ. For example, suppose the initial current in a solenoid is zero and the final current is I. It follows that the initial magnetic flux is zero. The final flux is simply $\Phi_f = BA\cos\theta$, where, according to Equation 22–12, the magnetic field in the solenoid is $B = \mu_0(N/\ell)I$. Finally, we know that the field in a solenoid is perpendicular to its cross-sectional area, A; thus $\theta = 0$ and $\Phi_f = BA$. Combining these results, we obtain

$$L = \frac{N(BA - 0)}{(I - 0)} = \frac{N[\mu_0(N/\ell)I]A}{I} = \mu_0\left(\frac{N^2}{\ell}\right)A$$

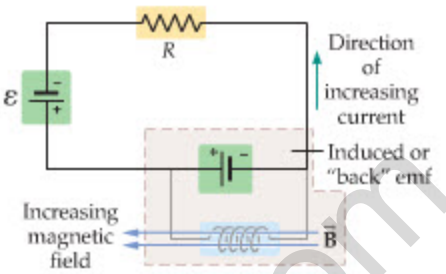
Denoting the number of turns per length as $n = N/\ell$, we can express the inductance of a solenoid in the following forms:

Inductance of a Solenoid

$$L = \mu_0\left(\frac{N^2}{\ell}\right)A = \mu_0 n^2 A\ell \qquad 23\text{–}14$$

Notice that doubling the number of turns per length quadruples the inductance. In addition, the inductance is proportional to the volume of a solenoid, $V = A\ell$.

In the following Examples we consider problems involving solenoids and their inductance.



▲ **FIGURE 23–18 The back emf of an inductor**
The effect of an increasing current in a coil is an induced emf that opposes the increase. This is indicated schematically by replacing the coil with the opposing, or "back," emf.

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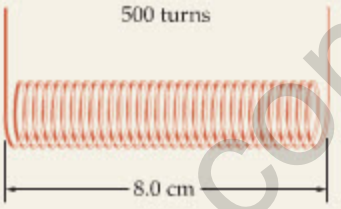
EXAMPLE 23–5 SOLENOID SELF-INDUCTION

A 500-turn solenoid is 8.0 cm long. When the current in this solenoid is increased from 0 to 2.5 A in 0.35 s, the magnitude of the induced emf is 0.012 V. Find **(a)** the inductance and **(b)** the cross-sectional area of the solenoid.

PICTURE THE PROBLEM

The solenoid described in the problem statement is shown in our sketch. Note that it is 8.0 cm in length and contains 500 turns.



STRATEGY

a. We can use the definition of inductance, $|\mathcal{E}| = L|\Delta I/\Delta t|$, to solve for the inductance.

b. Once we know L from part (a), we can find the cross-sectional area from $L = \mu_0 n^2 A\ell$, where $n = N/\ell$.

SOLUTION

Part (a)

1. Using magnitudes, solve $|\mathcal{E}| = L|\Delta I/\Delta t|$ for the inductance:
$$L = \frac{|\mathcal{E}|}{|\Delta I/\Delta t|} = \frac{0.012\ \text{V}}{(2.5\ \text{A}/0.35\ \text{s})} = 1.7\ \text{mH}$$

Part (b)

2. Solve $L = \mu_0 n^2 A\ell$ for the cross-sectional area of the solenoid:
$$A = \frac{L}{\mu_0 n^2 \ell} = \frac{L\ell}{\mu_0 N^2}$$

3. Substitute numerical values:
$$A = \frac{(1.7 \times 10^{-3}\ \text{H})(0.080\ \text{m})}{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(500)^2} = 4.3 \times 10^{-4}\ \text{m}^2$$

INSIGHT

This cross-sectional area corresponds to a radius of about 1.2 cm. Therefore, it would take about 37 m of wire to construct this inductor.

PRACTICE PROBLEM

Find the inductance and induced emf for a solenoid with a cross-sectional area of $1.3 \times 10^{-3}\ \text{m}^2$. The solenoid, which is 9.0 cm long, is made from a piece of wire that is 44 m in length. Assume that the rate of change of current remains the same. **[Answer:** $L = 2.2$ mH, $|\mathcal{E}| = 0.016$ V**]**

Some related homework problems: Problem 49, Problem 50

ACTIVE EXAMPLE 23–2 FIND THE NUMBER OF TURNS

The inductance of a solenoid 5.00 cm long is 0.157 mH. If its cross-sectional area is $1.00 \times 10^{-4}\ \text{m}^2$, how many turns does the solenoid have?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve $L = \mu_0(N^2/\ell)A$ for the number of turns, N: $N = (L\ell/\mu_0 A)^{1/2}$
2. Substitute numerical values: $N = 2.50 \times 10^2$

YOUR TURN

Which of the following changes would affect the value of the inductance more: **(a)** doubling the number of turns or **(b)** tripling the cross-sectional area? Verify your answer by giving the factor by which the inductance changes for each of these cases.

(Answers to **Your Turn** *problems are given in the back of the book.)*

A coil with a finite inductance—an **inductor** for short—resists changes in its electric current. Similarly, a particle of finite mass has the property of inertia, by which it resists changes in its velocity. These analogies, between inductance and mass on the one hand and current and velocity on the other, are useful ones, as we shall see in greater detail when we consider the behavior of ac circuits in the next chapter.

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23–8 *RL* Circuits

The circuit shown in Figure 23–19 is referred to as an ***RL* circuit;** it consists of a resistor, R, and an inductor, L, in series with a battery of emf $\mathcal{E}$. Note that the inductor is represented by a symbol reminiscent of a coil with several loops, as shown below:

We assume the inductor to be ideal, which means that the wires forming its loops have no resistance—the only resistance in the circuit is the resistor R. It follows, then, that after the switch has been closed for a long time, the current in the circuit is simply $I = \mathcal{E}/R$.

You may have noticed that we said the current would be $\mathcal{E}/R$ after the switch had been closed "for a long time." Why did we make this qualification? The point is that because of the inductor, and its tendency to resist changes in the current, it is not possible for the current to rise immediately to its final value. The inductor slows the process, causing the current to rise over a finite period of time, just as the charge on a capacitor builds up over the characteristic time $\tau = RC$ in an RC circuit.

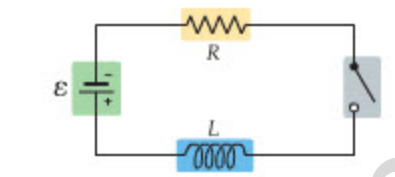
The corresponding characteristic time for an RL circuit is given by

$$\tau = \frac{L}{R} \qquad 23\text{–}15$$

As expected, the larger the inductance, the longer the time required for the current to build up. In addition, the current approaches its final value of $\mathcal{E}/R$ with an exponential time dependence, much like the case of an RC circuit. With the aid of calculus, it can be shown that the current as a function of time is given by

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) = \frac{\mathcal{E}}{R}(1 - e^{-tR/L}) \qquad 23\text{–}16$$

Note the similarity between this expression, which is plotted in Figure 23–20, and the expression given in Equation 21–18 for an RC circuit.

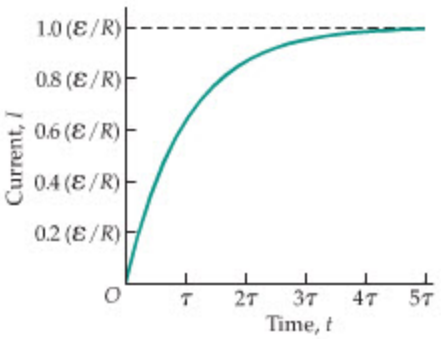


▲ **FIGURE 23–19 An *RL* circuit**
The characteristic time for an RL circuit is $\tau = L/R$. After the switch has been closed for a time much greater than τ, the current in the circuit is simply $I = \mathcal{E}/R$. That is, when the current is steady, the inductor behaves like a zero-resistance wire.

PROBLEM-SOLVING NOTE

Opening or Closing a Switch on an *RL* Circuit

When the switch is first closed in an RL circuit, the inductor has a back emf equal to that of the battery. Long after the switch is closed current flows freely through the inductor, with no resistance at all.



▲ **FIGURE 23–20 Current as a function of time in an *RL* circuit**
Notice that the current approaches $\mathcal{E}/R$ after several characteristic time intervals, $\tau = L/R$, have elapsed.

EXERCISE 23–4

Find the inductance in the RL circuit of Figure 23–19, given that the resistance is 35 Ω and the characteristic time is 7.5 ms.

SOLUTION

Solving $\tau = L/R$ for the inductance, we get

$$L = \tau R = (7.5 \times 10^{-3}\,\text{s})(35\,\Omega) = 0.26\,\text{H}$$

In the next Example we consider a circuit with one inductor and two resistors. By replacing the resistors with their equivalent resistance, however, we can still treat the circuit as a simple RL circuit.

EXAMPLE 23–6 *RL* IN PARALLEL

The circuit shown in the sketch on the next page consists of a 12-V battery, a 56-mH inductor, and two 150-Ω resistors in parallel. **(a)** Find the characteristic time for this circuit. What is the current in this circuit **(b)** one characteristic time interval after the switch is closed and **(c)** a long time after the switch is closed?

PICTURE THE PROBLEM

The circuit in question is shown in diagram (a). Noting that two resistors R connected in parallel have an equivalent resistance equal to $R/2$, we can replace the original circuit with the equivalent RL circuit shown in diagram (b).

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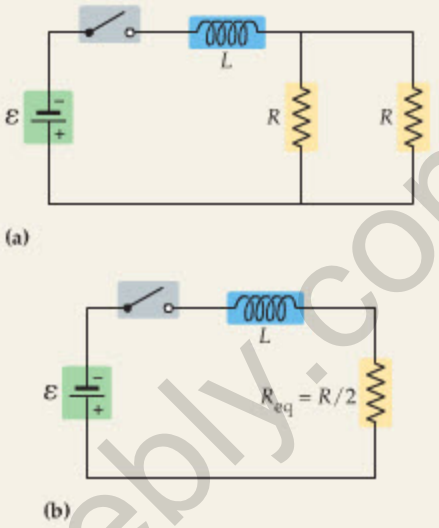
STRATEGY

a. We can find the characteristic time using $\tau = L/R_{eq}$, where $R_{eq} = R/2 = 75\ \Omega$.

b. To find the current after one characteristic time interval, we substitute $t = \tau$ in Equation 23–16.

c. A long time after the switch has been closed ($t \to \infty$), the exponential term in Equation 23–16 is essentially zero; hence, the current in this case is simply $I = \mathcal{E}/R_{eq}$.



SOLUTION

Part (a)

1. Calculate the characteristic time, $\tau = L/R_{eq}$, using 75 Ω for the resistance:

$$\tau = \frac{L}{R_{eq}} = \frac{56 \times 10^{-3}\ \text{H}}{75\ \Omega} = 7.5 \times 10^{-4}\ \text{s}$$

Part (b)

2. Substitute $t = \tau$ in Equation 23–16:

$$I = \frac{\mathcal{E}}{R_{eq}}(1 - e^{-t/\tau}) = \left(\frac{12\ \text{V}}{75\ \Omega}\right)(1 - e^{-1}) = 0.10\ \text{A}$$

Part (c)

3. Substitute $t \to \infty$ in Equation 23–16:

$$I = \frac{\mathcal{E}}{R_{eq}}(1 - e^{-t/\tau}) = \frac{\mathcal{E}}{R_{eq}}(1 - e^{-\infty}) = \frac{\mathcal{E}}{R_{eq}} = \frac{12\ \text{V}}{75\ \Omega} = 0.16\ \text{A}$$

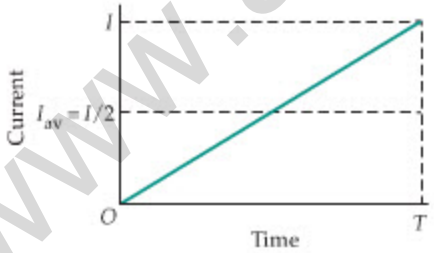
INSIGHT

Notice that the current rises to almost two-thirds of its final value after just one characteristic time interval. It takes only a few more time intervals for the current to essentially "saturate" at its final value, $\mathcal{E}/R_{eq}$.

PRACTICE PROBLEM

If L is doubled to $2(56\ \text{mH}) = 112\ \text{mH}$, will the current after one characteristic time interval be greater than, less than, or the same as that found in part (b)? [**Answer:** The current is the same when $t = \tau$ because $\mathcal{E}$ and R are unchanged. Note that the value of τ is doubled, however, so it actually takes twice as long for the current to rise to 0.10 A.]

Some related homework problems: Problem 53, Problem 55



▲ **FIGURE 23–21 Current in an *RL* circuit with $R = 0$**

In a circuit with only a battery and an inductor, the current rises linearly with time, because the characteristic time ($\tau = L/R$) with $R = 0$ is infinite. If the current after a time T is I, the average current during this time is $I_{av} = I/2$.

23–9 Energy Stored in a Magnetic Field

Work is required to establish a current in an inductor. For example, consider a circuit that consists of just two elements—a battery and an inductor. Even though there are no resistors in the circuit, the battery must still do electrical work to force charge to flow through the inductor, in opposition to its self-induced back emf. What happens to the energy expended to increase the current? It is not lost, nor is it dissipated, as there are no losses in this circuit. Instead, it is stored in the magnetic field, just as the energy in a capacitor is stored in its electric field.

To see how this energy storage works, imagine increasing the current in an inductor L from $I_i = 0$ to $I_f = I$ in the time $\Delta t = T$. The magnitude of the emf required to accomplish this increase in current is

$$\mathcal{E} = L\frac{\Delta I}{\Delta t} = L\frac{I - 0}{T} = \frac{LI}{T} \qquad 23\text{–}17$$

During this time the average current is $I/2$, as indicated in Figure 23–21. Thus, the average power supplied by the emf, $P_{av} = I_{av}V$, is

$$P_{av} = \tfrac{1}{2}I\mathcal{E} = \tfrac{1}{2}LI^2/T \qquad 23\text{–}18$$

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The total *energy* delivered by the emf is simply the average power times the total time; hence, the energy U required to increase the current in the inductor from 0 to I is

$$U = P_{av}T$$

Using the result given in Equation 23–18, we find

Energy Stored in an Inductor

$$U = \tfrac{1}{2}LI^2 \qquad 23\text{–}19$$

SI unit: joule, J

Note the similarity of Equation 23–19 to $U = \frac{1}{2}CV^2$ for the energy stored in a capacitor.

EXERCISE 23–5

An inductor carrying a current of 2.5 A stores 0.10 J of energy. What is the inductance of this inductor?

SOLUTION

Solving $U = \frac{1}{2}LI^2$ for the inductance, we find

$$L = \frac{2U}{I^2} = \frac{2(0.10\text{ J})}{(2.5\text{ A})^2} = 0.032\text{ H}$$

As mentioned previously, the energy of an inductor is stored in its magnetic field. To be specific, consider a solenoid of length ℓ with n turns per unit length. The inductance of this solenoid, as determined in Section 23–7, is

$$L = \mu_0 n^2 A\ell$$

Therefore, the energy stored in the solenoid's magnetic field when it carries a current I is

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(\mu_0 n^2 A\ell)I^2$$

Recalling that the magnetic field inside a solenoid has a magnitude given by $B = \mu_0 nI$, we can write the energy as follows:

$$U = \frac{1}{2\mu_0}B^2 A\ell$$

Finally, we note that the volume where the magnetic field exists—that is, the volume inside the solenoid—is area times length, $A\ell$. Therefore, the magnetic energy per volume is

$$u_B = \frac{\text{magnetic energy}}{\text{volume}} = \frac{B^2}{2\mu_0} \qquad 23\text{–}20$$

Although derived for the case of an ideal solenoid, this expression applies to any magnetic field no matter how it is generated.

Finally, recall that in the case of an electric field we found that the energy density was $u_E = \varepsilon_0 E^2/2$. Note, in both cases, that the energy density depends on the magnitude of the field squared. We shall return to these expressions in the next chapter and again when we study electromagnetic waves, such as radio waves and light, in Chapter 25.

PROBLEM-SOLVING NOTE

Energy Stored in an Inductor

Remember that an inductor carrying a current always stores energy in its magnetic field. Thus, energy is required to establish a current in an inductor, and energy is released when the current in an inductor decreases.

EXAMPLE 23–7 AN UNKNOWN RESISTANCE

After the switch in the circuit shown on the next page has been closed a long time, the energy stored in the inductor is 3.11×10^{-3} J. What is the value of the resistance R?

PICTURE THE PROBLEM

Our sketch shows the circuit for this system, with a 36.0-V battery, a 92.5-Ω resistor, and a 75.0-mH inductor all connected in series with an unknown resistor, R. Initially, the switch is open, but after it has been closed a long time the current settles down to a constant value, as does the amount of energy stored in the inductor.

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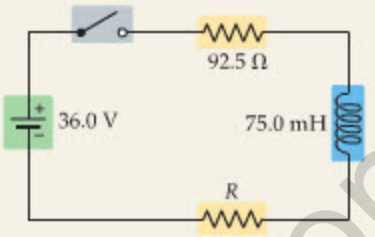
STRATEGY

First, the energy stored in the inductor, $U = \frac{1}{2}LI^2$, gives us the current in the circuit.

Second, we note that after the switch has been closed a long time the current has a constant value. Thus, there is no opposing emf produced by the inductor. As a result, the final current is simply the emf divided by the equivalent resistance of two resistors in series; $I = \mathcal{E}/(92.5\ \Omega + R)$. This relation can be used to solve for R.



SOLUTION

1. Use the energy stored in the inductor to solve for the current in the circuit:

$$U = \tfrac{1}{2}LI^2$$
$$I = \sqrt{\frac{2U}{L}}$$

2. Substitute numerical values:

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(3.11 \times 10^{-3}\ \text{J})}{75.0 \times 10^{-3}\ \text{H}}} = 0.288\ \text{A}$$

3. Use Ohm's law to solve for the resistance R:

$$I = \frac{\mathcal{E}}{92.5\ \Omega + R}$$
$$R = \frac{\mathcal{E}}{I} - 92.5\ \Omega$$

4. Substitute numerical values:

$$R = \frac{36.0\ \text{V}}{0.288\ \text{A}} - 92.5\ \Omega = 32.5\ \Omega$$

INSIGHT

Note that as long as the switch is closed, energy is dissipated in the resistors at a constant rate. On the other hand, *no* energy is dissipated in the inductor—it simply stores a constant amount of energy, like a mass moving with a constant speed.

PRACTICE PROBLEM

If R is increased, does the energy stored in the inductor increase, decrease, or stay the same? Calculate the stored energy with $R = 106\ \Omega$ as a check on your answer. [**Answer:** With a larger R, the current is less; hence U is less. With $R = 106\ \Omega$ we find $U = 1.23 \times 10^{-3}$ J.]

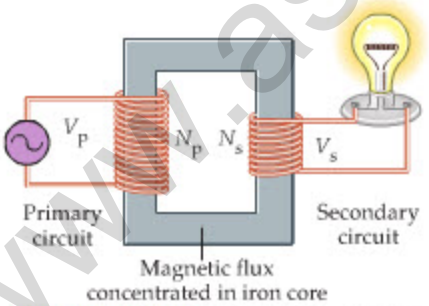
Some related homework problems: Problem 59, Problem 62

23–10 Transformers

In electrical applications it is often useful to be able to change the voltage from one value to another. For example, high-voltage power lines may operate at voltages as high as 750,000 V, but before the electrical power can be used in the home it must be "stepped down" to 120 V. Similarly, the 120 V from a wall socket may be stepped down again to 9 V or 12 V to power a portable CD player, or stepped up to give the 15,000 V needed in a television tube. The electrical device that performs these voltage conversions is called a **transformer.**

The basic idea of a transformer has already been described in connection with Faraday's induction experiment in Section 23–1. In fact, the device shown in Figure 23–1 can be thought of as a crude transformer. Basically, the magnetic flux created by a primary coil induces a voltage in a secondary coil. If the number of turns is different in the two coils, however, the voltages will be different as well.

To see how this works, consider the system shown in Figure 23–22. Here an ac generator produces an alternating current in the primary circuit of voltage V_p. The primary circuit then loops around an iron core with N_p turns. The iron core intensifies and concentrates the magnetic flux and ensures, at least to a good approximation, that the secondary coil experiences the same magnetic flux as the primary coil. The secondary coil has N_s turns around its iron core and is connected to a secondary circuit that may operate a CD player, a lightbulb, or some other device. We would now like to determine the voltage, V_s, in the secondary circuit.



▲ **FIGURE 23–22 The basic elements of a transformer**

An alternating current in the primary circuit creates an alternating magnetic flux and, hence, an alternating induced emf in the secondary circuit. The ratio of the emfs in the two circuits, V_s/V_p, is equal to the ratio of the number of turns in each circuit, N_s/N_p.

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To find this voltage, we apply Faraday's law of induction to each of the coils. For the primary coil we have

$$\mathcal{E}_p = -N_p \frac{\Delta \Phi_p}{\Delta t}$$

Similarly, the emf of the secondary coil is

$$\mathcal{E}_s = -N_s \frac{\Delta \Phi_s}{\Delta t}$$

Now, recall that the magnetic fluxes are equal, $\Delta \Phi_p = \Delta \Phi_s$, because of the iron core. As a result, we can divide these two equations (which cancels the flux) to find

$$\frac{\mathcal{E}_p}{\mathcal{E}_s} = \frac{N_p}{N_s}$$

Assuming that the resistance in the coils is negligible, so that the emf in each is essentially the same as its voltage, we obtain the following result:

Transformer Equation

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} \qquad 23\text{–}21$$

Equation 23–21 is known as the **transformer equation.**

It follows that the voltage in the secondary circuit is simply

$$V_s = V_p\left(\frac{N_s}{N_p}\right)$$

Therefore, if the number of turns in the secondary coil is less than the number of turns in the primary coil, the voltage is stepped down, and $V_s < V_p$. Conversely, if the number of turns in the secondary coil is higher, the voltage is stepped up, and $V_s > V_p$.

Now, before you begin to think that transformers give us something for nothing, we note that there is more to the story. Because energy must always be conserved, the average power in the primary circuit must be the same as the average power in the secondary circuit. Since power can be written as $P = IV$, it follows that

$$I_p V_p = I_s V_s$$

Therefore, the ratio of the current in the secondary coil to that in the primary coil is

Transformer Equation (Current and Voltage)

$$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s} \qquad 23\text{–}22$$

Thus, if the voltage is stepped up, the current is stepped down.

Suppose, for example, that the number of turns in the secondary coil is twice the number of turns in the primary coil. As a result, the transformer doubles the voltage in the secondary circuit, $V_s = 2\,V_p$, and at the same time halves the current, $I_s = I_p/2$. In general, there is always a tradeoff between voltage and current in a transformer:

> If a transformer increases the voltage by a given factor, it decreases the current by the same factor. Similarly, if it decreases the voltage, it increases the current.

This behavior is similar to that of a lever, in which there is a tradeoff between the force that can be exerted and the distance through which it is exerted. In the case of a lever, as in the case of a transformer, the tradeoff is due to energy conservation: The work done on one end of the lever, $F_1 d_1$, must be equal to the work done on the other end of the lever, $F_2 d_2$.

PROBLEM-SOLVING NOTE

Energy Conservation in Transformers

Energy conservation requires that the product of I and V be the same for each coil of a transformer. Thus, as the voltage of the secondary goes up with the number of turns, the current decreases by the same factor.

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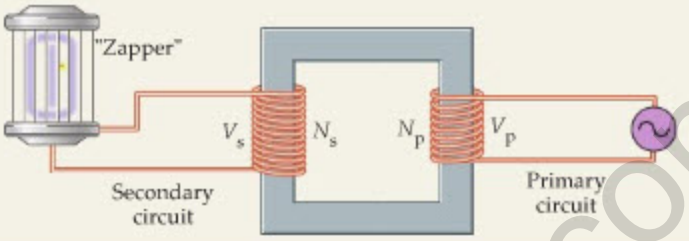
ACTIVE EXAMPLE 23–3 FIND THE NUMBER OF TURNS ON THE SECONDARY

A common summertime sound in many backyards is the *zap* heard when an unfortunate insect flies into a high-voltage "bug zapper." Typically, such devices operate with voltages of about 4000 V obtained from a transformer plugged into a standard 120-V outlet. How many turns are on the secondary coil of the transformer if the primary coil has 27 turns?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve the transformer equation for the number of turns in the secondary coil: $N_s = N_p(V_s/V_p)$
2. Substitute numerical values: $N_s = 900$

INSIGHT

Of course, any transformer with a turns ratio of $N_s/N_p = 900/27$ will produce the same secondary voltage.

YOUR TURN

Suppose the primary voltage is doubled and the number of turns in the primary circuit is quadrupled. By what factor must the number of turns in the secondary circuit be changed if the secondary voltage is to remain the same?

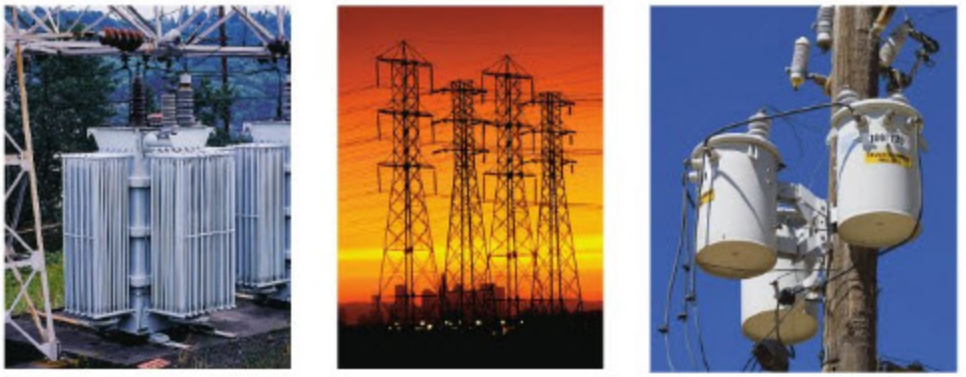
(Answers to **Your Turn** *problems are given in the back of the book.)*

Notice that the operation of a transformer depends on a *changing* magnetic flux to create an induced emf in the secondary coil. If the current is constant in direction and magnitude, there is simply no induced emf. This is an important advantage that ac circuits have over dc circuits and is one reason that most electrical power systems today operate with alternating currents.

REAL-WORLD PHYSICS

High-voltage electric power transmission

Finally, transformers play an important role in the *transmission* of electrical energy from the power plants that produce it to the communities and businesses where it is used. As was pointed out in Equation 21–3, the resistance of a wire is directly proportional to its length. Therefore, when electrical energy is transmitted over a large distance, the finite resistivity of the wires that carry the current becomes significant. If the wire carries a current I and has a resistance R, the power dissipated as waste heat is $P = I^2R$. One way to reduce this energy loss in a given wire is to reduce the current, I. A transformer that steps up the voltage of a power plant by a factor of 15 (from 10,000 V to 150,000 V, for example) at the same time reduces the current by a factor of 15 and reduces the energy dissipation by a factor of $15^2 = 225$. When the electrical energy reaches the location where it is to be used, step-down transformers can convert the voltage to levels (such as 120 V or 240 V) typically used in the home or workplace.



▲ The transmission of electric power over long distances would not be feasible without transformers. A step-up transformer near the power plant (left) boosts the plant's output voltage from 12,000 V to the 240,000 V carried by high-voltage lines that crisscross the country (center). A series of step-down transformers near the destination reduce the voltage, first to 2400 V at local substations for distribution to neighborhoods, and then to the 240 V supplied to most houses. The familiar gray cylinders commonly seen on utility poles (right) are the transformers responsible for this last voltage reduction.

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THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concept of electric flux, so useful in Gauss's law in Section 19–7, is extended to magnetic flux in this chapter, where it plays a key role in Faraday's law.

Work, energy, and power (Chapters 7 and 8) are discussed in relation to electrical energy in Section 23–5. These concepts are also important in understanding the operation of electric motors and generators in Section 23–6.

In Chapter 20 we introduced the concept of a capacitor (C), and showed how a capacitor stores energy in its electric field. Here we generalize this concept to inductors (L), which store energy in their magnetic fields. We also used our experience with RC circuits (Chapter 21) to gain a better understanding of RL circuits.

LOOKING AHEAD

Inductors are important elements in any electric circuit, but especially in alternating current (ac) circuits. This is discussed in detail in Section 24–4.

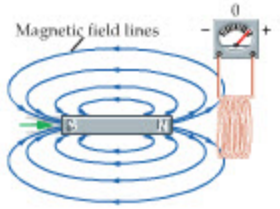
A particular type of circuit containing an inductor—the RLC circuit—plays an important role in Chapter 24. For example, we show in Section 24–5 how an RLC circuit behaves at high and low frequencies. Then, in Section 24–6, we see that an RLC circuit can show resonance, in analogy with a mass on a spring. In this analogy, the inductor plays the role of the mass.

The detailed connections between electric and magnetic fields developed in Chapters 22 and 23 are central to understanding electromagnetic waves, as described in Chapter 25. It would be hard to overstate the importance of electromagnetic waves, which cover a broad spectrum that includes radio waves, microwaves, infrared rays, visible light, ultraviolet light, X-rays, and gamma rays.

CHAPTER SUMMARY

23–1 INDUCED ELECTROMOTIVE FORCE

A changing magnetic field can induce a current in a circuit. The driving force behind the current is referred to as the induced electromotive force.

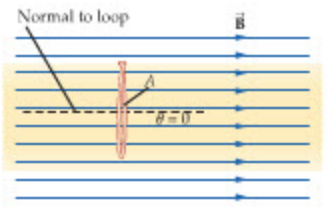


Basic Features of Magnetic Induction

An induced current occurs when there is a *change* in the magnetic field. The magnitude of the induced current depends on the *rate* of change of the magnetic field.

23–2 MAGNETIC FLUX

Magnetic flux is a measure of the number of magnetic field lines that cross a given area.



Magnetic Flux, Defined

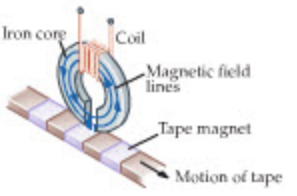
If a magnetic field of strength B crosses an area A at an angle θ relative to the area's normal, the magnetic flux is

$$\Phi = (B\cos\theta)A = BA\cos\theta \qquad 23\text{–}1$$

Units of Magnetic Flux

Magnetic flux is measured in webers (Wb) defined as follows:

$$1\ \text{Wb} = 1\ \text{T}\cdot\text{m}^2 \qquad 23\text{–}2$$



23–3 FARADAY'S LAW OF INDUCTION

Faraday's law of induction gives precise mathematical form to the basic features of magnetic induction discussed in Section 23–1. Faraday's law is a law of nature, not an approximate rule of thumb.

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Faraday's Law
If the magnetic flux in a coil of N turns changes by the amount $\Delta\Phi$ in the time Δt, the induced emf is

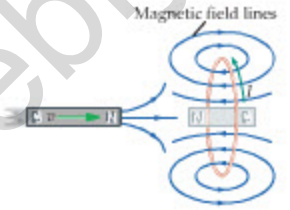
$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{\Phi_{\text{final}} - \Phi_{\text{initial}}}{t_{\text{final}} - t_{\text{initial}}} \qquad 23\text{–}3$$

Magnitude of the Induced emf
In many cases we are concerned only with the magnitude of the magnetic flux and the induced current. In such cases we use the following form of Faraday's law:

$$|\mathcal{E}| = N\left|\frac{\Delta\Phi}{\Delta t}\right| = N\left|\frac{\Phi_{\text{final}} - \Phi_{\text{initial}}}{t_{\text{final}} - t_{\text{initial}}}\right| \qquad 23\text{–}4$$

23–4 LENZ'S LAW

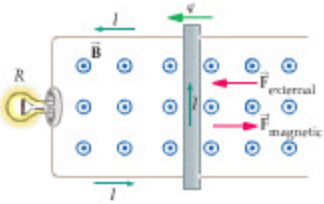
Lenz's law states that an induced current flows in the direction that opposes the change that caused the current.



Eddy Currents
Eddy currents are circulating electric currents formed in a conducting material that experiences a changing magnetic flux.

23–5 MECHANICAL WORK AND ELECTRICAL ENERGY

With the help of a magnetic field, mechanical work can be converted into electrical energy in the form of currents flowing through a circuit.



Motional emf
If a conductor of length ℓ is moved at right angles to a magnetic field of strength B with a speed v, the magnitude of the induced emf in the conductor is

$$|\mathcal{E}| = Bv\ell \qquad 23\text{–}5$$

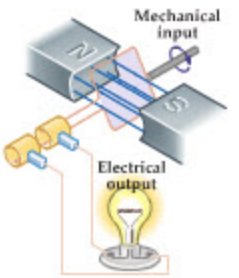
The corresponding electric field is

$$E = Bv \qquad 23\text{–}6$$

Mechanical Work/Electrical Energy
Assuming no frictional losses, the work done in moving a conductor through a magnetic field is equal to the electrical energy that can be delivered to a circuit.

23–6 GENERATORS AND MOTORS

Generators and motors are practical devices for converting between electrical and mechanical energy.



Electric Generators
Electric generators use mechanical work to produce electrical energy. The emf produced by a generator is

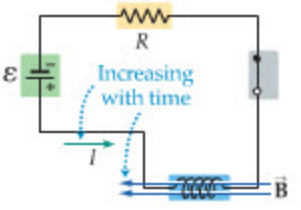
$$\mathcal{E} = NBA\omega \sin \omega t \qquad 23\text{–}11$$

where N is the number of turns in the coil, B is the magnetic field, A is the area of the coil, and ω is the angular speed of the coil.

Electric Motors
An electric motor is basically an electric generator operated in reverse; it uses electrical energy to produce mechanical work.

23–7 INDUCTANCE

Self-inductance, or inductance for short, is the effect produced when a coil with a changing current induces an emf in itself.



Inductance
Inductance, L, is defined as follows:

$$|\mathcal{E}| = L\left|\frac{\Delta I}{\Delta t}\right| \qquad 23\text{–}12$$

The SI unit of inductance is the henry (H): $1\ \text{H} = 1\ \text{V}\cdot\text{s/A}$.

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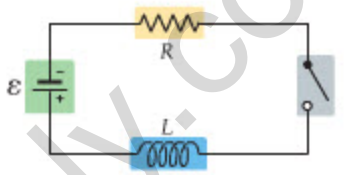
Inductance of a Solenoid
The inductance of a solenoid of length ℓ with N turns of wire and a cross-sectional area A is

$$L = \mu_0 n^2 A\ell = \mu_0\left(\frac{N^2}{\ell}\right)A \qquad 23\text{–}14$$

Note that the number of turns per length is $n = N/\ell$.

23–8 *RL* CIRCUITS

A simple RL circuit consists of a battery, a switch, a resistor R, and an inductor L connected in series.



Time Constant
The characteristic time over which the current changes in an RL circuit is

$$\tau = \frac{L}{R} \qquad 23\text{–}15$$

Increasing Current Versus Time
When the switch is closed in an RL circuit at time $t = 0$, the current increases with time as follows:

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) = \frac{\mathcal{E}}{R}(1 - e^{-tR/L}) \qquad 23\text{–}16$$

In this expression ε is the emf of the battery.

23–9 ENERGY STORED IN A MAGNETIC FIELD

Energy can be stored in a magnetic field, just as it is in an electric field.

Energy Stored in an Inductor
An inductor of inductance L carrying a current I stores the energy

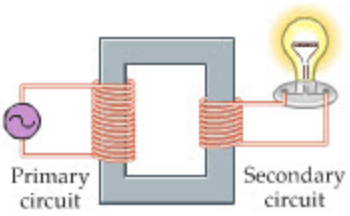
$$U = \tfrac{1}{2}LI^2 \qquad 23\text{–}19$$

Energy Density
Whenever a magnetic field of strength B exists in a given region, there is a corresponding magnetic energy density given by

$$u_B = \frac{\text{magnetic energy}}{\text{volume}} = \frac{B^2}{2\mu_0} \qquad 23\text{–}20$$

23–10 TRANSFORMERS

A transformer is an electrical device used to convert the voltage and current in one circuit to a different voltage and current in another circuit using magnetic induction.



Transformer Equation
The equation relating the voltage V, current I, and number of turns N in the primary (p) and secondary (s) circuits of a transformer is

$$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s} \qquad 23\text{–}22$$

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Calculate the magnetic flux.	The magnetic flux of a field B passing through an area A at an angle θ to the normal is $\Phi = BA\cos\theta$.	Example 23–1
Find the induced emf.	Faraday's law states that the induced emf is proportional to the rate of change of magnetic flux with time, $\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$. In the special case of a rotating coil, the induced emf is $\mathcal{E} = NBA\omega\sin\omega t$.	Examples 23–2, 23–4
Calculate the inductance of a solenoid.	A solenoid with n turns per length, area A, and length ℓ has an inductance given by $L = \mu_0 n^2 A\ell$.	Example 23–5 Active Example 23–2

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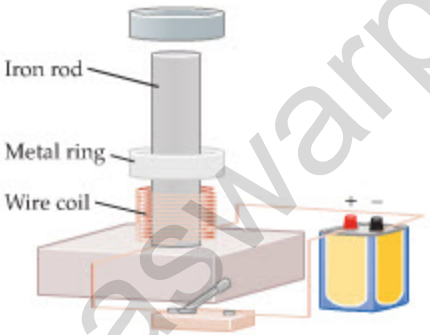
Find the current as a function of time in an *RL* circuit.	The time constant of an *RL* circuit is $\tau = L/R$. When the current increases from zero, its time dependence is $I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})$.	Example 23–6
Relate the energy stored in an inductor to its inductance and current.	The energy stored in an inductor L carrying a current I is $U = \frac{1}{2}LI^2$.	Example 23–7
Relate voltage and current to the number of turns in the primary and secondary coils of a transformer.	The voltage in the secondary coil is proportional to the number of turns in the secondary coil and inversely proportional to the number of turns in the primary coil. The current and voltage in the secondary coil are inversely proportional to each other.	Active Example 23–3

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

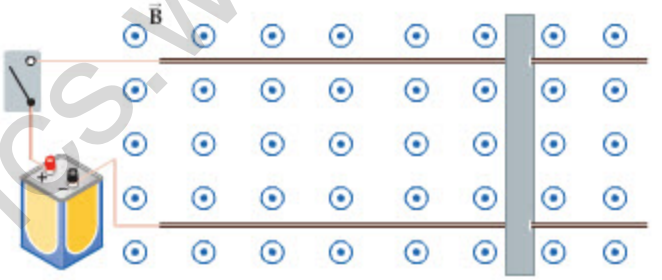
(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Explain the difference between a magnetic field and a magnetic flux.
2. A metal ring with a break in its perimeter is dropped from a field-free region of space into a region with a magnetic field. What effect does the magnetic field have on the ring?
3. In a common classroom demonstration, a magnet is dropped down a long, vertical copper tube. The magnet moves very slowly as it moves through the tube, taking several seconds to reach the bottom. Explain this behavior.
4. Many equal-arm balances have a small metal plate attached to one of the two arms. The plate passes between the poles of a magnet mounted in the base of the balance. Explain the purpose of this arrangement.
5. Figure 23–23 shows a vertical iron rod with a wire coil of many turns wrapped around its base. A metal ring slides over the rod and rests on the wire coil. Initially the switch connecting the coil to a battery is open, but when it is closed, the ring flies into the air. Explain why this happens.



▲ **FIGURE 23–23** Conceptual Questions 5 and 6

6. Referring to Conceptual Question 5, suppose the metal ring has a break in its circumference. Describe what happens when the switch is closed in this case.
7. A metal rod of resistance R can slide without friction on two zero-resistance rails, as shown in Figure 23–24. The rod and the rails are immersed in a region of constant magnetic field pointing out of the page. Describe the motion of the rod when the switch is closed. Your discussion should include the effects of motional emf.



▲ **FIGURE 23–24** Conceptual Question 7

8. A penny is placed on edge in the powerful magnetic field of an MRI solenoid. If the penny is tipped over, it takes several seconds for it to land on one of its faces. Explain.
9. Recently, NASA tested a power generation system that involves connecting a small satellite to the space shuttle with a conducting wire several miles long. Explain how such a system can generate electrical power.
10. Explain what happens when the angular speed of the coil in an electric generator is increased.
11. The inductor in an *RL* circuit determines how long it takes for the current to reach a given value, but it has no effect on the final value of the current. Explain.
12. When the switch in a circuit containing an inductor is opened, it is common for a spark to jump across the contacts of the switch. Why?

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 23–2 MAGNETIC FLUX

1. • A 0.055-T magnetic field passes through a circular ring of radius 3.1 cm at an angle of 16° with the normal. Find the magnitude of the magnetic flux through the ring.
2. • A uniform magnetic field of 0.0250 T points vertically upward. Find the magnitude of the magnetic flux through each of the five sides of the open-topped rectangular box shown in Figure 23–25, given that the dimensions of the box are $L = 32.5$ cm, $W = 12.0$ cm, and $H = 10.0$ cm.

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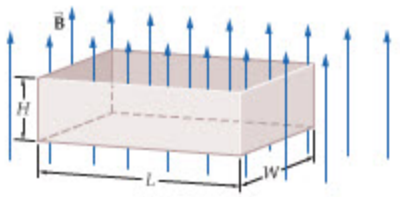


▲ **FIGURE 23–25** Problem 2

3. • A magnetic field is oriented at an angle of 47° to the normal of a rectangular area 5.1 cm by 6.8 cm. If the magnetic flux through this surface has a magnitude of $4.8 \times 10^{-5}\ \text{T} \cdot \text{m}^2$, what is the strength of the magnetic field?

4. • Find the magnitude of the magnetic flux through the floor of a house that measures 22 m by 18 m. Assume that the Earth's magnetic field at the location of the house has a horizontal component of 2.6×10^{-5} T pointing north, and a downward vertical component of 4.2×10^{-5} T.

5. • **MRI Solenoid** The magnetic field produced by an MRI solenoid 2.5 m long and 1.2 m in diameter is 1.7 T. Find the magnitude of the magnetic flux through the core of this solenoid.

6. •• At a certain location, the Earth's magnetic field has a magnitude of 5.9×10^{-5} T and points in a direction that is 72° below the horizontal. Find the magnitude of the magnetic flux through the top of a desk at this location that measures 130 cm by 82 cm.

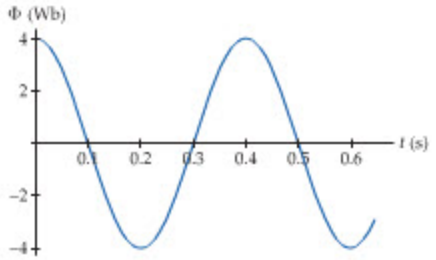
7. •• **IP** A solenoid with 385 turns per meter and a diameter of 17.0 cm has a magnetic flux through its core of magnitude $1.28 \times 10^{-4}\ \text{T} \cdot \text{m}^2$. **(a)** Find the current in this solenoid. **(b)** How would your answer to part (a) change if the diameter of the solenoid were doubled? Explain.

8. ••• A single-turn square loop of side L is centered on the axis of a long solenoid. In addition, the plane of the square loop is perpendicular to the axis of the solenoid. The solenoid has 1250 turns per meter and a diameter of 6.00 cm, and carries a current of 2.50 A. Find the magnetic flux through the loop when **(a)** $L = 3.00$ cm, **(b)** $L = 6.00$ cm, and **(c)** $L = 12.0$ cm.

SECTION 23–3 FARADAY'S LAW OF INDUCTION

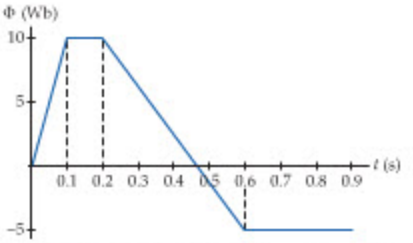
9. • A 0.45-T magnetic field is perpendicular to a circular loop of wire with 53 turns and a radius of 15 cm. If the magnetic field is reduced to zero in 0.12 s, what is the magnitude of the induced emf?

10. • Figure 23–26 shows the magnetic flux through a coil as a function of time. At what times shown in this plot do **(a)** the magnetic flux and **(b)** the induced emf have the greatest magnitude?



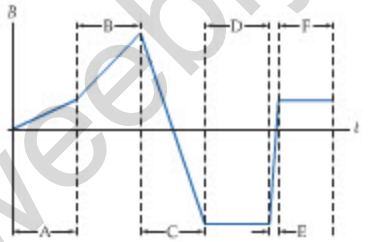
▲ **FIGURE 23–26** Problems 10 and 15

11. • Figure 23–27 shows the magnetic flux through a single-loop coil as a function of time. What is the induced emf in the coil at **(a)** $t = 0.050$ s, **(b)** $t = 0.15$ s, and **(c)** $t = 0.50$ s?



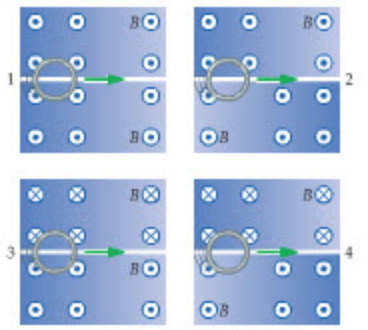
▲ **FIGURE 23–27** Problems 11 and 14

12. •• **CE** A wire loop is placed in a magnetic field that is perpendicular to its plane. The field varies with time as shown in Figure 23–28. Rank the six regions of time in order of increasing magnitude of the induced emf. Indicate ties where appropriate.



▲ **FIGURE 23–28** Problem 12

13. •• **CE** Figure 23–29 shows four different situations in which a metal ring moves to the right with constant speed through a region with a varying magnetic field. The intensity of the color indicates the intensity of the field, and in each case the field either increases or decreases at a uniform rate from the left edge of the colored region to the right edge. The direction of the field in each region is indicated. For each of the four cases, state whether the induced emf is clockwise, counterclockwise, or zero.



▲ **FIGURE 23–29** Problem 13

14. •• **IP** The magnetic flux through a single-loop coil is given by Figure 23–27. **(a)** Is the magnetic flux at $t = 0.25$ s greater than, less than, or the same as the magnetic flux at $t = 0.55$ s? Explain. **(b)** Is the induced emf at $t = 0.25$ s greater than, less than, or the same as the induced emf at $t = 0.55$ s? Explain. **(c)** Calculate the induced emf at the times $t = 0.25$ s and $t = 0.55$ s.

15. •• **IP** Consider a single-loop coil whose magnetic flux is given by Figure 23–26. **(a)** Is the magnitude of the induced emf in this coil greater near $t = 0.4$ s or near $t = 0.5$ s? Explain. **(b)** At what times in this plot do you expect the induced emf in the coil to have a maximum magnitude? Explain. **(c)** Estimate the induced emf in the coil at times near $t = 0.3$ s, $t = 0.4$ s, and $t = 0.5$ s.

16. •• A single conducting loop of wire has an area of $7.2 \times 10^{-2}\ \text{m}^2$ and a resistance of 110 Ω. Perpendicular to the plane of the loop is a magnetic field of strength 0.48 T. At what rate (in T/s)

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must this field change if the induced current in the loop is to be 0.32 A?

17. •• The area of a 120-turn coil oriented with its plane perpendicular to a 0.20-T magnetic field is 0.050 m^2. Find the average induced emf in this coil if the magnetic field reverses its direction in 0.34 s.

18. •• An emf is induced in a conducting loop of wire 1.22 m long as its shape is changed from square to circular. Find the average magnitude of the induced emf if the change in shape occurs in 4.25 s and the local 0.125-T magnetic field is perpendicular to the plane of the loop.

19. •• A magnetic field increases from 0 to 0.25 T in 1.8 s. How many turns of wire are needed in a circular coil 12 cm in diameter to produce an induced emf of 6.0 V?

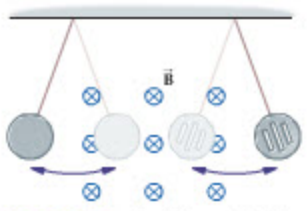
SECTION 23–4 LENZ'S LAW

20. • **CE Predict/Explain** A metal ring is dropped into a localized region of constant magnetic field, as indicated in Figure 23–30. The magnetic field is zero above and below the region where it is finite. **(a)** For each of the three indicated locations (1, 2, and 3), is the induced current clockwise, counterclockwise, or zero? **(b)** Choose the *best explanation* from among the following:
   I. Clockwise at 1 to oppose the field; zero at 2 because the field is uniform; counterclockwise at 3 to try to maintain the field.
   II. Counterclockwise at 1 to oppose the field; zero at 2 because the field is uniform; clockwise at 3 to try to maintain the field.
   III. Clockwise at 1 to oppose the field; clockwise at 2 to maintain the field; clockwise at 3 to oppose the field.

▲ **FIGURE 23–30** Problems 20 and 21

21. • **CE Predict/Explain** A metal ring is dropped into a localized region of constant magnetic field, as indicated in Figure 23–30. The magnetic field is zero above and below the region where it is finite. **(a)** For each of the three indicated locations (1, 2, and 3), is the magnetic force exerted on the ring upward, downward, or zero? **(b)** Choose the *best explanation* from among the following:
   I. Upward at 1 to oppose entering the field; zero at 2 because the field is uniform; downward at 3 to help leaving the field.
   II. Upward at 1 to oppose entering the field; upward at 2 where the field is strongest; upward at 3 to oppose leaving the field.
   III. Upward at 1 to oppose entering the field; zero at 2 because the field is uniform; upward at 3 to oppose leaving the field.

22. • **CE Predict/Explain** Figure 23–31 shows two metal disks of the same size and material oscillating in and out of a region with a magnetic field. One disk is solid; the other has a series of slots. **(a)** Is the retarding effect of eddy currents on the solid disk greater than, less than, or equal to the retarding effect on the slotted disk? **(b)** Choose the *best explanation* from among the following:
   I. The solid disk experiences a greater retarding force because eddy currents in it flow freely and are not interrupted by the slots.
   II. The slotted disk experiences the greater retarding force because the slots allow more magnetic field to penetrate the disk.
   III. The disks are the same size and made of the same material; therefore, they experience the same retarding force.



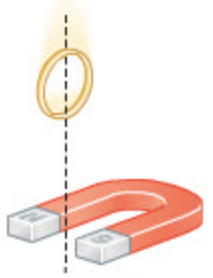
▲ **FIGURE 23–31** Problems 22, 23, and 24

23. • **CE** Consider the solid disk in Figure 23–31. When this disk has swung to the right as far as it can go, is the induced current in it a maximum or a minimum? Explain.

24. • **CE Predict/Explain** **(a)** As the solid metal disk in Figure 23–31 swings to the right, from the region with no field into the region with a finite magnetic field, is the induced current in the disk clockwise, counterclockwise, or zero? **(b)** Choose the *best explanation* from among the following:
   I. The induced current is clockwise, since this produces a field within the disk in the same direction as the magnetic field that produced the current.
   II. The induced current is counterclockwise to generate a field within the disk that points out of the page.
   III. The induced current is zero because the disk enters a region where the magnetic field is uniform.

25. • A bar magnet with its north pole pointing downward is falling toward the center of a horizontal conducting ring. As viewed from above, is the direction of the induced current in the ring clockwise or counterclockwise? Explain.

26. • **A Wire Loop and a Magnet** A loop of wire is dropped and allowed to fall between the poles of a horseshoe magnet, as shown in Figure 23–32. State whether the induced current in the loop is clockwise or counterclockwise when **(a)** the loop is above the magnet and **(b)** the loop is below the magnet.



▲ **FIGURE 23–32** Problems 26, 27, and 28

27. •• Suppose we change the situation shown in Figure 23–32 as follows: Instead of allowing the loop to fall on its own, we attach a string to it and lower it with constant speed along the path indicated by the dashed line. Is the tension in the string greater than, less than, or equal to the weight of the loop? Give specific answers for times when **(a)** the loop is above the magnet and **(b)** the loop is below the magnet. Explain in each case.

28. •• Rather than letting the loop fall downward in Figure 23–32, suppose we attach a string to it and raise it upward with constant speed along the path indicated by the dashed line. Is the tension in the string greater than, less than, or equal to the weight of the loop? Give specific answers for times when **(a)** the loop is below the magnet and **(b)** the loop is above the magnet. Explain in each case.

29. •• Figure 23–33 shows a current-carrying wire and a circuit containing a resistor R. **(a)** If the current in the wire is constant, is

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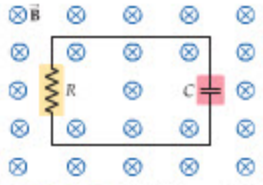
the induced current in the circuit clockwise, counterclockwise, or zero? Explain. **(b)** If the current in the wire increases, is the induced current in the circuit clockwise, counterclockwise, or zero? Explain.

▲ **FIGURE 23–33** Problems 29 and 30

30. •• Consider the physical system shown in Figure 23–33. If the current in the wire changes direction, is the induced current in the circuit clockwise, counterclockwise, or zero? Explain.

31. •• A long, straight, current-carrying wire passes through the center of a circular coil. The wire is perpendicular to the plane of the coil. **(a)** If the current in the wire is constant, is the induced emf in the coil zero or nonzero? Explain. **(b)** If the current in the wire increases, is the induced emf in the coil zero or nonzero? Explain. **(c)** Does your answer to part (b) change if the wire no longer passes through the center of the coil but is still perpendicular to its plane? Explain.

32. •• Figure 23–34 shows a circuit containing a resistor and an uncharged capacitor. Pointing into the plane of the circuit is a uniform magnetic field $\vec{\mathbf{B}}$. If the magnetic field reverses direction in a short period of time, which plate of the capacitor (top or bottom) becomes positively charged? Explain.



▲ **FIGURE 23–34** Problems 32 and 33

33. •• Referring to Problem 32, which plate of the capacitor (top or bottom) becomes positively charged if the magnetic field increases in magnitude with time? Explain.

34. •• A long, straight wire carries a current I, as indicated in Figure 23–35. Three small metal rings are placed near the current-carrying wire (A and C) or directly on top of it (B). If the current in the wire is increasing with time, indicate whether the induced emf in each of the rings is clockwise, counterclockwise, or zero. Explain your answer for each ring.

▲ **FIGURE 23–35** Problem 34

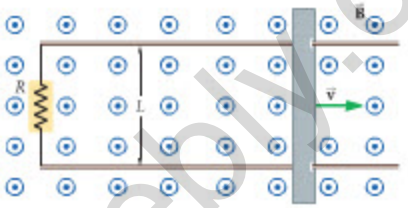
SECTION 23–5 MECHANICAL WORK AND ELECTRICAL ENERGY

35. • **CE** A conducting rod slides on two wires in a region with a magnetic field. The two wires are not connected. Is a force required to keep the rod moving with constant speed? Explain.

36. • A metal rod 0.76 m long moves with a speed of 2.0 m/s perpendicular to a magnetic field. If the induced emf between the ends of the rod is 0.45 V, what is the strength of the magnetic field?

37. •• **Airplane emf** A Boeing KC-135A airplane has a wingspan of 39.9 m and flies at constant altitude in a northerly direction with a speed of 850 km/h. If the vertical component of the Earth's magnetic field is 5.0×10^{-6} T, and its horizontal component is 1.4×10^{-6} T, what is the induced emf between the wing tips?

38. •• **IP** Figure 23–36 shows a zero-resistance rod sliding to the right on two zero-resistance rails separated by the distance $L = 0.450$ m. The rails are connected by a 12.5-Ω resistor, and the entire system is in a uniform magnetic field with a magnitude of 0.750 T. **(a)** Find the speed at which the bar must be moved to produce a current of 0.155 A in the resistor. **(b)** Would your answer to part (a) change if the bar was moving to the left instead of to the right? Explain.



▲ **FIGURE 23–36** Problems 38 and 39

39. •• Referring to part (a) of Problem 38, **(a)** find the force that must be exerted on the rod to maintain a constant current of 0.155 A in the resistor. **(b)** What is the rate of energy dissipation in the resistor? **(c)** What is the mechanical power delivered to the rod?

40. •• **(a)** Find the current that flows in the circuit shown in Example 23–3. **(b)** What speed must the rod have if the current in the circuit is to be 1.0 A?

41. •• Suppose the mechanical power delivered to the rod in Example 23–3 is 8.9 W. Find **(a)** the current in the circuit and **(b)** the speed of the rod.

SECTION 23–6 GENERATORS AND MOTORS

42. • The maximum induced emf in a generator rotating at 210 rpm is 45 V. How fast must the rotor of the generator rotate if it is to generate a maximum induced emf of 55 V?

43. • A rectangular coil 25 cm by 35 cm has 120 turns. This coil produces a maximum emf of 65 V when it rotates with an angular speed of 190 rad/s in a magnetic field of strength B. Find the value of B.

44. • A 1.6-m wire is wound into a coil with a radius of 3.2 cm. If this coil is rotated at 85 rpm in a 0.075-T magnetic field, what is its maximum emf?

45. •• **IP** A circular coil with a diameter of 22.0 cm and 155 turns rotates about a vertical axis with an angular speed of 1250 rpm. The only magnetic field in this system is that of the Earth. At the location of the coil, the horizontal component of the magnetic field is 3.80×10^{-5} T, and the vertical component is 2.85×10^{-5} T. **(a)** Which component of the magnetic field is important when calculating the induced emf in this coil? Explain. **(b)** Find the maximum emf induced in the coil.

46. •• A generator is designed to produce a maximum emf of 170 V while rotating with an angular speed of 3600 rpm. Each coil of the generator has an area of 0.016 m^2. If the magnetic field used in the generator has a magnitude of 0.050 T, how many turns of wire are needed?

SECTION 23–7 INDUCTANCE

47. • Find the induced emf when the current in a 45.0-mH inductor increases from 0 to 515 mA in 16.5 ms.

48. • How many turns should a solenoid of cross-sectional area 0.035 m^2 and length 0.22 m have if its inductance is to be 45 mH?

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49. •• The inductance of a solenoid with 450 turns and a length of 24 cm is 7.3 mH. **(a)** What is the cross-sectional area of the solenoid? **(b)** What is the induced emf in the solenoid if its current drops from 3.2 A to 0 in 55 ms?

50. •• Determine the inductance of a solenoid with 640 turns in a length of 25 cm. The circular cross section of the solenoid has a radius of 4.3 cm.

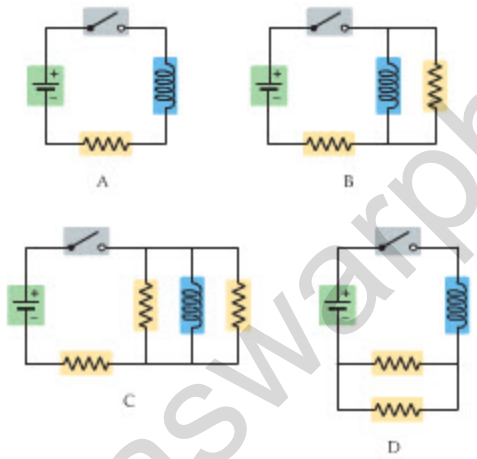
51. •• A solenoid with a cross-sectional area of $1.81 \times 10^{-3}\ \text{m}^2$ is 0.750 m long and has 455 turns per meter. Find the induced emf in this solenoid if the current in it is increased from 0 to 2.00 A in 45.5 ms.

52. ••• **IP** A solenoid has N turns of area A distributed uniformly along its length, ℓ. When the current in this solenoid increases at the rate of 2.0 A/s, an induced emf of 75 mV is observed. **(a)** What is the inductance of this solenoid? **(b)** Suppose the spacing between coils is doubled. The result is a solenoid that is twice as long but with the same area and number of turns. Will the induced emf in this new solenoid be greater than, less than, or equal to 75 mV when the current changes at the rate of 2.0 A/s? Explain. **(c)** Calculate the induced emf for part (b).

SECTION 23–8 *RL* CIRCUITS

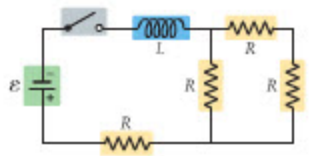
53. • How long does it take for the current in an RL circuit with $R = 130\ \Omega$ and $L = 68$ mH to reach half its final value?

54. •• **CE** The four electric circuits shown in Figure 23–37 have identical batteries, resistors, and inductors. Rank the circuits in order of increasing current supplied by the battery long after the switch is closed. Indicate ties where appropriate.



▲ **FIGURE 23–37** Problem 54

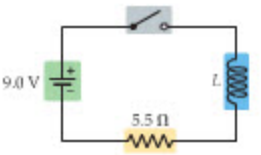
55. •• The circuit shown in Figure 23–38 consists of a 6.0-V battery, a 37-mH inductor, and four 55-Ω resistors. **(a)** Find the characteristic time for this circuit. What is the current supplied by this battery **(b)** two characteristic time intervals after closing the switch and **(c)** a long time after the switch is closed?



▲ **FIGURE 23–38** Problems 55 and 64

56. •• The current in an RL circuit increases to 95% of its final value 2.24 s after the switch is closed. **(a)** What is the time constant for this circuit? **(b)** If the inductance in the circuit is 0.275 H, what is the resistance?

57. ••• Consider the RL circuit shown in Figure 23–39. When the switch is closed, the current in the circuit is observed to increase from 0 to 0.32 A in 0.15 s. **(a)** What is the inductance L? **(b)** How long after the switch is closed does the current have the value 0.50 A? **(c)** What is the maximum current that flows in this circuit?



▲ **FIGURE 23–39** Problems 57 and 59

SECTION 23–9 ENERGY STORED IN A MAGNETIC FIELD

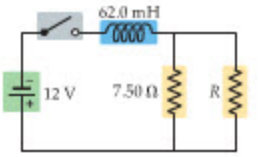
58. • **CE** The number of turns per meter in a solenoid of fixed length is doubled. At the same time, the current in the solenoid is halved. Does the energy stored in the inductor increase, decrease, or stay the same? Explain.

59. • Consider the circuit shown in Figure 23–39. Assuming the inductor in this circuit has the value $L = 6.1$ mH, how much energy is stored in the inductor after the switch has been closed a long time?

60. • A solenoid is 1.5 m long and has 470 turns per meter. What is the cross-sectional area of this solenoid if it stores 0.31 J of energy when it carries a current of 12 A?

61. •• **Alcator Fusion Experiment** In the Alcator fusion experiment at MIT, a magnetic field of 50.0 T is produced. **(a)** What is the magnetic energy density in this field? **(b)** Find the magnitude of the electric field that would have the same energy density found in part (a).

62. •• **IP** After the switch in Figure 23–40 has been closed for a long time, the energy stored in the inductor is 0.110 J. **(a)** What is the value of the resistance R? **(b)** If it is desired that more energy be stored in the inductor, should the resistance R be greater than or less than the value found in part (a)? Explain.



▲ **FIGURE 23–40** Problems 62 and 63

63. •• **IP** Suppose the resistor in Figure 23–40 has the value $R = 14\ \Omega$ and that the switch is closed at time $t = 0$. **(a)** How much energy is stored in the inductor at the time $t = \tau$? **(b)** How much energy is stored in the inductor at the time $t = 2\tau$? **(c)** If the value of R is increased, does the characteristic time, τ, increase or decrease? Explain.

64. •• **IP** Consider the circuit shown in Figure 23–38, which contains a 6.0-V battery, a 37-mH inductor, and four 55-Ω resistors. **(a)** Is more energy stored in the inductor just after the switch is closed or long after the switch is closed? Explain. **(b)** Calculate the energy stored in the inductor one characteristic time interval after the switch is closed. **(c)** Calculate the energy stored in the inductor long after the switch is closed.

65. ••• You would like to store 9.9 J of energy in the magnetic field of a solenoid. The solenoid has 580 circular turns of diameter 7.2 cm distributed uniformly along its 28-cm length. **(a)** How much current is needed? **(b)** What is the magnitude of the magnetic field inside the solenoid? **(c)** What is the energy density (energy/volume) inside the solenoid?

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SECTION 23–10 TRANSFORMERS

66. • **CE** Transformer 1 has a primary voltage V_p and a secondary voltage V_s. Transformer 2 has twice the number of turns on both its primary and secondary coils compared with transformer 1. If the primary voltage on transformer 2 is $2V_p$, what is its secondary voltage? Explain.

67. • **CE** Transformer 1 has a primary current I_p and a secondary current I_s. Transformer 2 has twice as many turns on its primary coil as transformer 1, and both transformers have the same number of turns on the secondary coil. If the primary current on transformer 2 is $3I_p$, what is its secondary current? Explain.

68. • The electric motor in a toy train requires a voltage of 3.0 V. Find the ratio of turns on the primary coil to turns on the secondary coil in a transformer that will step the 110-V household voltage down to 3.0 V.

69. • **IP** A disk drive plugged into a 120-V outlet operates on a voltage of 9.0 V. The transformer that powers the disk drive has 125 turns on its primary coil. **(a)** Should the number of turns on the secondary coil be greater than or less than 125? Explain. **(b)** Find the number of turns on the secondary coil.

70. • A transformer with a turns ratio (secondary/primary) of 1:18 is used to step down the voltage from a 120-V wall socket to be used in a battery recharging unit. What is the voltage supplied to the recharger?

71. • A neon sign that requires a voltage of 11,000 V is plugged into a 120-V wall outlet. What turns ratio (secondary/primary) must a transformer have to power the sign?

72. •• A step-down transformer produces a voltage of 6.0 V across the secondary coil when the voltage across the primary coil is 120 V. What voltage appears across the primary coil of this transformer if 120 V is applied to the secondary coil?

73. •• A step-up transformer has 25 turns on the primary coil and 750 turns on the secondary coil. If this transformer is to produce an output of 4800 V with a 12-mA current, what input current and voltage are needed?

GENERAL PROBLEMS

74. • **CE Predict/Explain** An airplane flies level to the ground toward the north pole. **(a)** Is the induced emf from wing tip to wing tip when the plane is at the equator greater than, less than, or equal to the wing-tip-to-wing-tip emf when it is at the latitude of New York? **(b)** Choose the *best explanation* from among the following:
   - **I.** The induced emf is the same because the strength of the Earth's magnetic field is the same at the equator and at New York.
   - **II.** The induced emf is greater at New York because the vertical component of the Earth's magnetic field is greater there than at the equator.
   - **III.** The induced emf is less at New York because at the equator the plane is flying parallel to the magnetic field lines.

75. • **CE** You hold a circular loop of wire at the north magnetic pole of the Earth. Consider the magnetic flux through this loop due to the Earth's magnetic field. Is the flux when the normal to the loop points horizontally greater than, less than, or equal to the flux when the normal points vertically downward? Explain.

76. • **CE** You hold a circular loop of wire at the equator. Consider the magnetic flux through this loop due to the Earth's magnetic field. Is the flux when the normal to the loop points north greater than, less than, or equal to the flux when the normal points vertically upward? Explain.

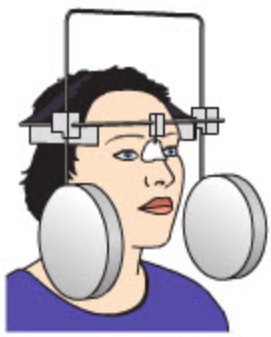
77. • **CE** The inductor shown in Figure 23–41 is connected to an electric circuit with a changing current. At the moment in question, the inductor has an induced emf with the indicated direction. Is the current in the circuit at this time increasing and to the right, increasing and to the left, decreasing and to the right, or decreasing and to the left?

▲ **FIGURE 23–41** Problem 77

78. • **Interstellar Magnetic Field** The *Voyager I* spacecraft moves through interstellar space with a speed of 8.0×10^3 m/s. The magnetic field in this region of space has a magnitude of 2.0×10^{-10} T. Assuming that the 5.0-m-long antenna on the spacecraft is at right angles to the magnetic field, find the induced emf between its ends.

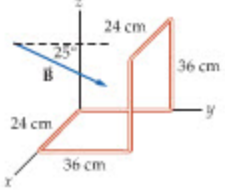
79. • **BIO Blowfly Flight** The coils used to measure the movements of a blowfly, as described in Section 23–5, have a diameter of 2.0 mm. In addition, the fly is immersed in a magnetic field of magnitude 0.15 mT. Find the maximum magnetic flux experienced by one of these coils.

80. • **Electrognathography** Computerized jaw tracking, or electrognathography (EGN), is an important tool for diagnosing and treating temporomandibular disorders (TMDs) that affect a person's ability to bite effectively. The first step in applying EGN is to attach a small permanent magnet to the patient's gum below the lower incisors. Then, as the jaw undergoes a biting motion, the resulting change in magnetic flux is picked up by wire coils placed on either side of the mouth, as shown in Figure 23–42. Suppose this person's jaw moves to her right and that the north pole of the permanent magnet also points to her right. From her point of view, is the induced current in the coil to **(a)** her right and **(b)** her left clockwise or counterclockwise? Explain.



▲ **FIGURE 23–42** Problem 80

81. •• A rectangular loop of wire 24 cm by 72 cm is bent into an L shape, as shown in Figure 23–43. The magnetic field in the vicinity of the loop has a magnitude of 0.035 T and points in a direction 25° below the y axis. The magnetic field has no x component. Find the magnitude of the magnetic flux through the loop.



▲ **FIGURE 23–43** Problem 81

82. •• **IP** A circular loop with a radius of 3.7 cm lies in the x-y plane. The magnetic field in this region of space is uniform and given by $\vec{\mathbf{B}} = (0.43\ \text{T})\hat{\mathbf{x}} + (-0.11\ \text{T})\hat{\mathbf{y}} + (0.52\ \text{T})\hat{\mathbf{z}}$. **(a)** What is

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the magnitude of the magnetic flux through this loop? **(b)** Suppose we now increase the x component of $\vec{\mathbf{B}}$, leaving the other components unchanged. Does the magnitude of the magnetic flux increase, decrease, or stay the same? Explain. **(c)** Suppose, instead, that we increase the z component of $\vec{\mathbf{B}}$, leaving the other components unchanged. Does the magnitude of the magnetic flux increase, decrease, or stay the same? Explain.

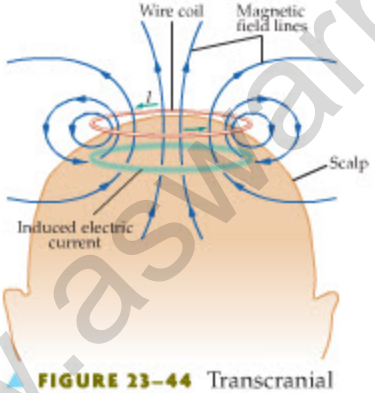
83. •• Consider a rectangular loop of wire 5.8 cm by 8.2 cm in a uniform magnetic field of magnitude 1.3 T. The loop is rotated from a position of zero magnetic flux to a position of maximum flux in 21 ms. What is the average induced emf in the loop?

84. •• **IP** A car with a vertical radio antenna 85 cm long drives due east at 25 m/s. The Earth's magnetic field at this location has a magnitude of 5.9×10^{-5} T and points northward, 72° below the horizontal. **(a)** Is the top or the bottom of the antenna at the higher potential? Explain. **(b)** Find the induced emf between the ends of the antenna.

85. •• The rectangular coils in a 325-turn generator are 11 cm by 17 cm. What is the maximum emf produced by this generator when it rotates with an angular speed of 525 rpm in a magnetic field of 0.45 T?

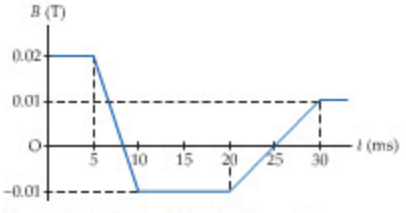
86. •• A cubical box 22 cm on a side is placed in a uniform 0.35-T magnetic field. Find the net magnetic flux through the box.

87. •• **BIO Transcranial Magnetic Stimulation** Transcranial magnetic stimulation (TMS) is a noninvasive method for studying brain function, and possibly for treatment as well. In this technique, a conducting loop is held near a person's head, as shown in Figure 23–44. When the current in the loop is changed rapidly, the magnetic field it creates can change at the rate of 3.00×10^4 T/s. This rapidly changing magnetic field induces an electric current in a restricted region of the brain that can cause a finger to twitch, bright spots to appear in the visual field (magnetophosphenes), or a feeling of complete happiness to overwhelm a person. If the magnetic field changes at the previously mentioned rate over an area of 1.13×10^{-2} m^2, what is the induced emf?



▲ **FIGURE 23–44** Transcranial magnetic stimulation (Problem 87)

88. •• A magnetic field with the time dependence shown in Figure 23–45 is at right angles to a 155-turn circular coil with a diameter of 3.75 cm. What is the induced emf in the coil at **(a)** $t = 2.50$ ms, **(b)** $t = 7.50$ ms, **(c)** $t = 15.0$ ms, and **(d)** $t = 25.0$ ms?



▲ **FIGURE 23–45** Problem 88

89. •• You would like to construct a 50.0-mH inductor by wrapping insulated copper wire (diameter = 0.0332 cm) onto a tube with a circular cross section of radius 2.67 cm. What length of wire is required if it is wrapped onto the tube in a single, close-packed layer?

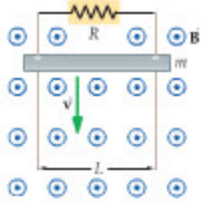
90. •• The time constant of an RL circuit with $L = 25$ mH is twice the time constant of an RC circuit with $C = 45$ μF. Both circuits have the same resistance R. Find **(a)** the value of R and **(b)** the time constant of the RL circuit.

91. •• A 6.0-V battery is connected in series with a 29-mH inductor, a 110-Ω resistor, and an open switch. **(a)** How long after the switch is closed will the current in the circuit be equal to 12 mA? **(b)** How much energy is stored in the inductor when the current reaches its maximum value?

92. •• A 9.0-V battery is connected in series with a 31-mH inductor, a 180-Ω resistor, and an open switch. **(a)** What is the current in the circuit 0.120 ms after the switch is closed? **(b)** How much energy is stored in the inductor at this time?

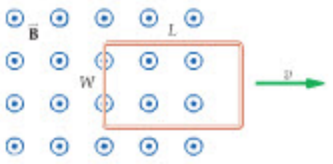
93. •• **BIO Blowfly Maneuvers** Suppose the fly described in Problem 79 turns through an angle of 90° in 37 ms. If the magnetic flux through one of the coils on the insect goes from a maximum to zero during this maneuver, and the coil has 85 turns of wire, find the magnitude of the induced emf.

94. ••• **IP** A conducting rod of mass m is in contact with two vertical conducting rails separated by a distance L, as shown in Figure 23–46. The entire system is immersed in a magnetic field of magnitude B pointing out of the page. Assuming the rod slides without friction, **(a)** describe the motion of the rod after it is released from rest. **(b)** What is the direction of the induced current (clockwise or counterclockwise) in the circuit? **(c)** Find the speed of the rod after it has fallen for a long time.



▲ **FIGURE 23–46** Problem 94

95. ••• **IP** A single-turn rectangular loop of width W and length L moves parallel to its length with a speed v. The loop moves from a region with a magnetic field $\vec{\mathbf{B}}$ perpendicular to the plane of the loop to a region where the magnetic field is zero, as shown in Figure 23–47. Find the rate of change in the magnetic flux through the loop **(a)** before it enters the region of zero field, **(b)** just after it enters the region of zero field, and **(c)** once it is fully within the region of zero field. **(d)** For each of the cases considered in parts (a), (b), and (c), state whether the induced current in the loop is clockwise, counterclockwise, or zero. Explain in each case.



▲ **FIGURE 23–47** Problem 95

96. ••• **IP** The switch in the circuit shown in Figure 23–48 is open initially. **(a)** Find the current in the circuit a long time after the switch is closed. **(b)** Describe the behavior of the lightbulb from the time the switch is closed until the current reaches the value

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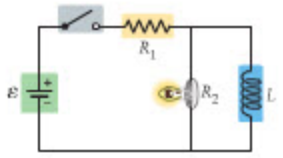
found in part (a). **(c)** Now, suppose the switch is opened after having been closed for a long time. If the inductor is large, it is observed that the light flashes brightly and then burns out. Explain this behavior. **(d)** Find the voltage across the lightbulb just before and just after the switch is opened.



▲ **FIGURE 23–48** Problem 96

97. ••• **Energy Density in *E* and *B* Fields** An electric field E and a magnetic field B have the same energy density. **(a)** Express the ratio E/B in terms of the fundamental constants ε_0 and μ_0. **(b)** Evaluate E/B numerically, and compare your result with the speed of light.

PASSAGE PROBLEMS

Loop Detectors on Roadways

"Smart" traffic lights are controlled by loops of wire embedded in the road (Figure 23–49). These "loop detectors" sense the change in magnetic field as a large metal object—such as a car or a truck—moves over the loop. Once the object is detected, electric circuits in the controller check for cross traffic, and then turn the light from red to green.

▲ **FIGURE 23–49** Problems 98, 99, 100, and 101

A typical loop detector consists of three or four loops of 14-gauge wire buried 3 in. below the pavement. You can see the marks on the road where the pavement has been cut to allow for installation of the wires. There may be more than one loop detector at a given intersection; this allows the system to recognize that an object is moving as it activates first one detector and then another over a short period of time. If the system determines that a car has entered the intersection while the light is red, it can activate one camera to take a picture of the car from the front—to see the driver's face—and then a second camera to take a picture of the car and its license plate from behind. This red-light camera system was used to good effect during an exciting chase scene through the streets of London in the movie *National Treasure: Book of Secrets*.

Motorcycles are small enough that they often fail to activate the detectors, leaving the cyclist waiting and waiting for a green light. Some companies have begun selling powerful neodymium magnets to mount on the bottom of a motorcycle to ensure that they are "seen" by the detectors.

98. • Suppose the downward vertical component of the magnetic field increases as a car drives over a loop detector. As viewed from above, is the induced current in the loop clockwise, counterclockwise, or zero?

99. • A car drives onto a loop detector and increases the downward component of the magnetic field within the loop from 1.2×10^{-5} T to 2.6×10^{-5} T in 0.38 s. What is the induced emf in the detector if it is circular, has a radius of 0.67 m, and consists of four loops of wire?

A. 0.66×10^{-4} V **B.** 1.5×10^{-4} V
C. 2.1×10^{-4} V **D.** 6.2×10^{-4} V

100. •• A truck drives onto a loop detector and increases the downward component of the magnetic field within the loop from 1.2×10^{-5} T to the larger value B in 0.38 s. The detector is circular, has a radius of 0.67 m, and consists of three loops of wire. What is B, given that the induced emf is 8.1×10^{-4} V?

A. 3.6×10^{-5} T **B.** 7.3×10^{-5} T
C. 8.5×10^{-5} T **D.** 24×10^{-5} T

101. •• Suppose a motorcycle increases the downward component of the magnetic field within a loop only from 1.2×10^{-5} T to 1.9×10^{-5} T. The detector is square, is 0.75 m on a side, and has four loops of wire. Over what period of time must the magnetic field increase if it is to induce an emf of 1.4×10^{-4} V?

A. 0.028 s **B.** 0.11 s
C. 0.35 s **D.** 0.60 s

INTERACTIVE PROBLEMS

102. •• **Referring to Conceptual Checkpoint 23–3** Suppose the ring is initially to the left of the field region, where there is no field, and is moving to the right. When the ring is partway into the field region, **(a)** is the induced current in the ring clockwise, counterclockwise, or zero, and **(b)** is the magnetic force exerted on the ring to the right, to the left, or zero? Explain.

103. •• **Referring to Conceptual Checkpoint 23–3** Suppose the ring is completely inside the field region initially and is moving to the right. **(a)** Is the induced current in the ring clockwise, counterclockwise, or zero, and **(b)** is the magnetic force on the ring to the right, to the left, or zero? Explain. The ring now begins to emerge from the field region, still moving to the right. **(c)** Is the induced current in the ring clockwise, counterclockwise, or zero, and **(d)** is the magnetic force on the ring to the right, to the left, or zero? Explain.

104. •• **Referring to Example 23–3** **(a)** What external force is required to give the rod a speed of 3.49 m/s, everything else remaining the same? **(b)** What is the current in the circuit in this case?

105. •• **IP Referring to Example 23–3** Suppose the direction of the magnetic field is reversed. Everything else in the system remains the same. **(a)** Is the magnetic force exerted on the rod to the right, to the left, or zero? Explain. **(b)** Is the direction of the induced current clockwise, counterclockwise, or zero? Explain. **(c)** Suppose we now adjust the strength of the magnetic field until the speed of the rod is 2.49 m/s, keeping the force equal to 1.60 N. What is the new magnitude of the magnetic field?

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PHYSICS IN

PERSPECTIVE

Electricity and Magnetism

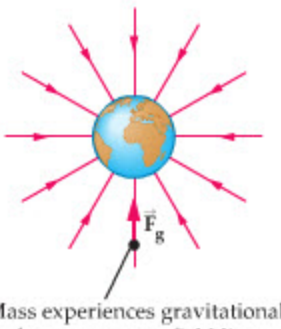
Electricity and magnetism at first seemed quite unrelated. These pages follow the detective story by which physicists discovered that they are actually facets of a single force. This discovery led to much of our current technology, including telecommunications and electronics.

1 Gravity, electricity, magnetism: Three distinct forces?

In daily life, gravity, electrostatic force, and magnetism seem distinct: socks out of the dryer cling to each other but not to refrigerators; magnets don't stick to socks; gravity pays no attention to charge or magnetism. We seem to have three forces with distinct characteristics.

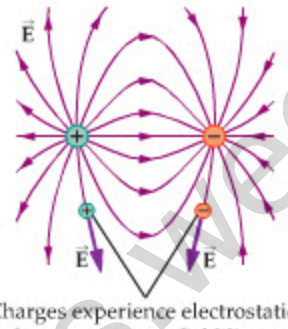
Gravitational force	Electrostatic force	Magnetic force
• Acts between masses	• Acts between electric charges	• Acts between magnetic poles
• Always attractive	• Unlike charges attract; like charges repel	• Unlike poles attract; like poles repel
• Mass comes in only one "kind"	• Positive and negative charges can be isolated	• N and S poles cannot be isolated; occur as dipoles

Gravitational field



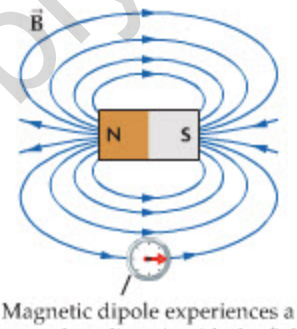
Mass experiences gravitational force tangent to field line

Electrostatic field



Charges experience electrostatic forces tangent to field lines

Magnetic field



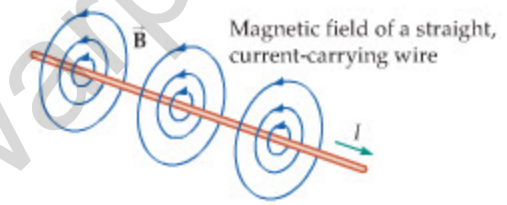
Magnetic dipole experiences a torque that aligns it with the field

But are these forces really as distinct as they seem?

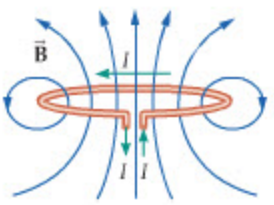
2 First clue: Moving charges generate and respond to magnetic fields

A motionless charge does not respond to magnetism. However, *moving* charges both generate a magnetic field and experience a force when moving at an angle to an external magnetic field.

Moving charges generate magnetic fields. Magnetic field lines form circles around a moving charge.

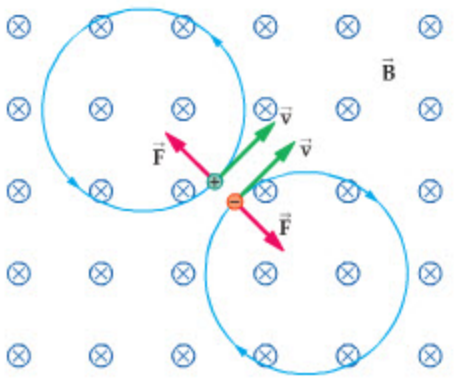


Magnetic field of a straight, current-carrying wire

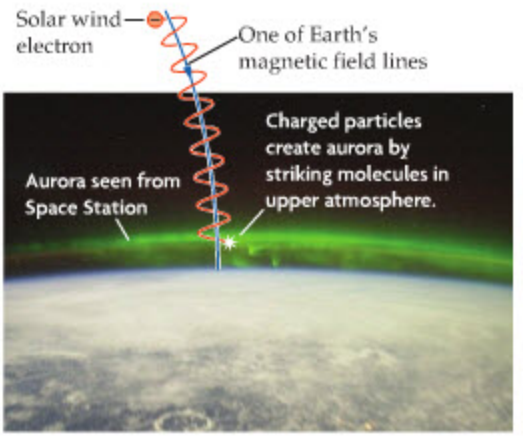


Magnetic field of a current-carrying loop—analogous to field of bar magnet

Moving charges feel a force from a magnetic field. This force causes charged particles to circle or spiral around magnetic field lines. For example, Earth's magnetic field guides charged particles from the solar wind toward Earth's magnetic poles, where they produce the auroras.



Paths of charged particles in a uniform magnetic field



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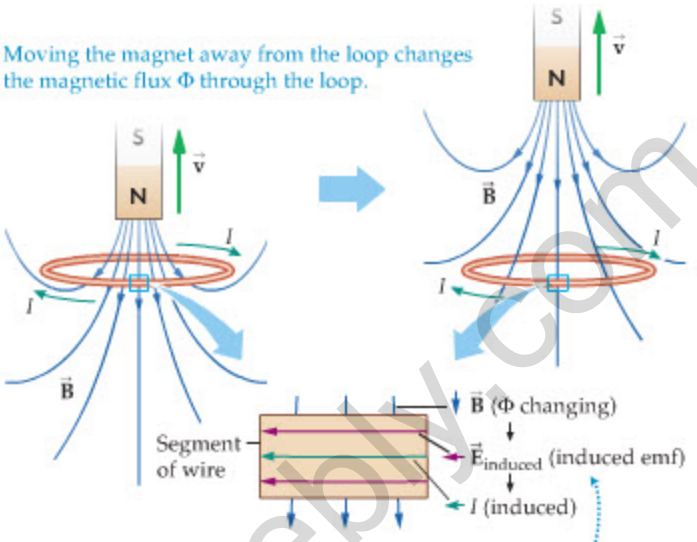
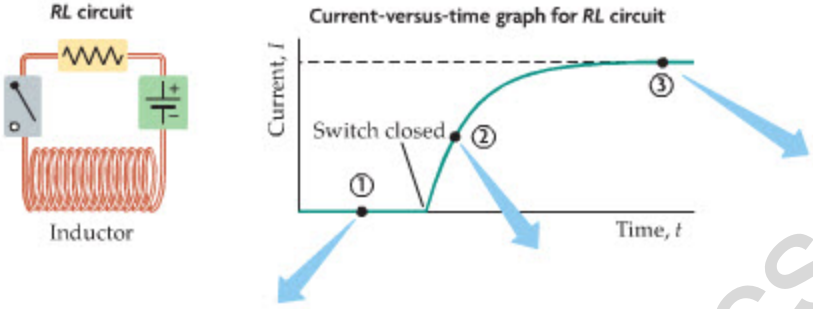
3 Second clue: A changing magnetic field generates an electric field

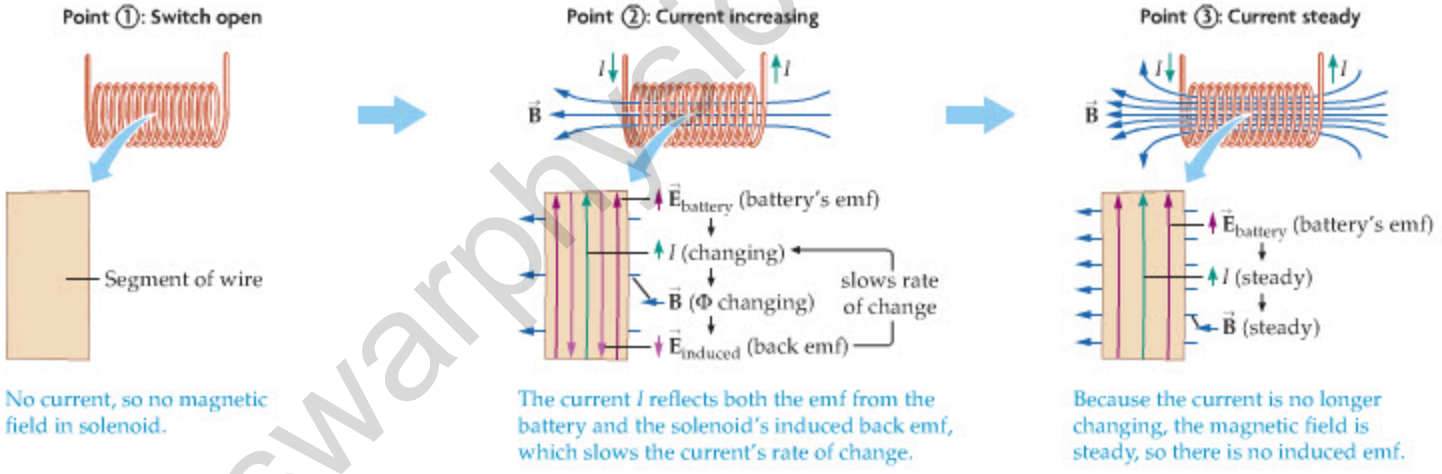
Holding a magnet next to a wire loop does not generate a current. However, if you do something that changes the magnetic flux Φ through the loop, the changing flux will induce an electric field within the wire, which will drive an induced current. The induced electric field is the emf for the induced current.

Notice that the field lines of the induced electric field do not start or end on charged particles—the electric field is generated by the magnetic field, not by charged objects.



Back emf in an *RL* circuit The following graph and diagrams explore how an induced electric field produces the back emf by which a solenoid (inductor) retards changes in current through a circuit.



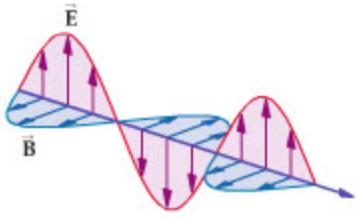


4 Final clue and conclusions

Final clue: As we'll see in Chapter 25, a changing *electric* field can generate a *magnetic* field. In fact, electromagnetic waves (including light, radio waves, and microwaves) consist of oscillating, mutually generating electric and magnetic fields.

Conclusions: Physicists realized that electric and magnetic phenomena are facets of a single *electromagnetic force*, which is one of the four fundamental forces. (The others are gravity and two nuclear forces called the weak and strong forces.)

A central goal of physics is to "unify" the fundamental forces—that is, to show that they are really all facets of a single force. So far, electromagnetism has been unified with the weak force and possibly with the strong force. Unification of these forces with gravity is the elusive goal of string theory.



Electromagnetic waves (including the microwaves put out by a cell phone) consist of mutually generating electric and magnetic fields.

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