

# Chapter 11

# Energy and Its Conservation

## What You'll Learn

- You will learn that energy is a property of an object that can change the object's position, motion, or its environment.
- You will learn that energy changes from one form to another, and that the total amount of energy in a closed system remains constant.

## Why It's Important

Energy turns the wheels of our world. People buy and sell energy to operate electric appliances, automobiles, and factories.

**Skiing** The height of the ski jump determines the energy the skier has at the bottom of the ramp before jumping into the air and flying many meters down the slope. The distance that the ski jumper travels depends on his or her use of physical principles such as air resistance, balance, and energy.

## Think About This ►

How does the height of the ski ramp affect the distance that the skier can jump?

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## How can you analyze the energy of a bouncing basketball?

### Question

What is the relationship between the height a basketball is dropped from and the height it reaches when it bounces back?

### Procedure



1. Place a meterstick against a wall. Choose an initial height from which to drop a basketball. Record the height in the data table.
2. Drop the ball and record how high the ball bounced.
3. Repeat steps 1 and 2 by dropping the basketball from three other heights.
4. **Make and Use Graphs** Construct a graph of bounce height ( $y$ ) versus drop height ( $x$ ). Find the best-fit line.

### Analysis

Use the graph to find how high a basketball would bounce if it were dropped from a height of 10.0 m.

When the ball is lifted and ready to drop, it possesses energy. What are the factors that influence this energy?

**Critical Thinking** Why doesn't the ball bounce back to the height from which it was dropped?



## 11.1 The Many Forms of Energy

The word *energy* is used in many different ways in everyday speech. Some fruit-and-cereal bars are advertised as energy sources. Athletes use energy in sports. Companies that supply your home with electricity, natural gas, or heating fuel are called energy companies.

Scientists and engineers use the term *energy* much more precisely. As you learned in the last chapter, work causes a change in the energy of a system. That is, work transfers energy between a system and the external world.

In this chapter, you will explore how objects can have energy in a variety of ways. Energy is like ice cream—it comes in different varieties. You can have vanilla, chocolate, or peach ice cream. They are different varieties, but they are all ice cream and serve the same purpose. However, unlike ice cream, energy can be changed from one variety to another. In this chapter, you will learn how energy is transformed from one variety (or form) to another and how to keep track of the changes.

### ► Objectives

- **Use** a model to relate work and energy.
- **Calculate** kinetic energy.
- **Determine** the gravitational potential energy of a system.
- **Identify** how elastic potential energy is stored.

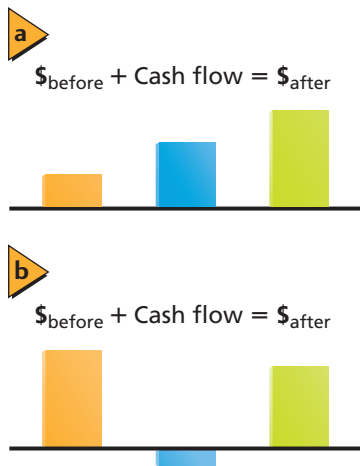
### ► Vocabulary

rotational kinetic energy  
gravitational potential energy  
reference level  
elastic potential energy





■ **Figure 11-1** When you earn money, the amount of cash that you have increases **(a)**. When you spend money, the amount of cash that you have decreases **(b)**.



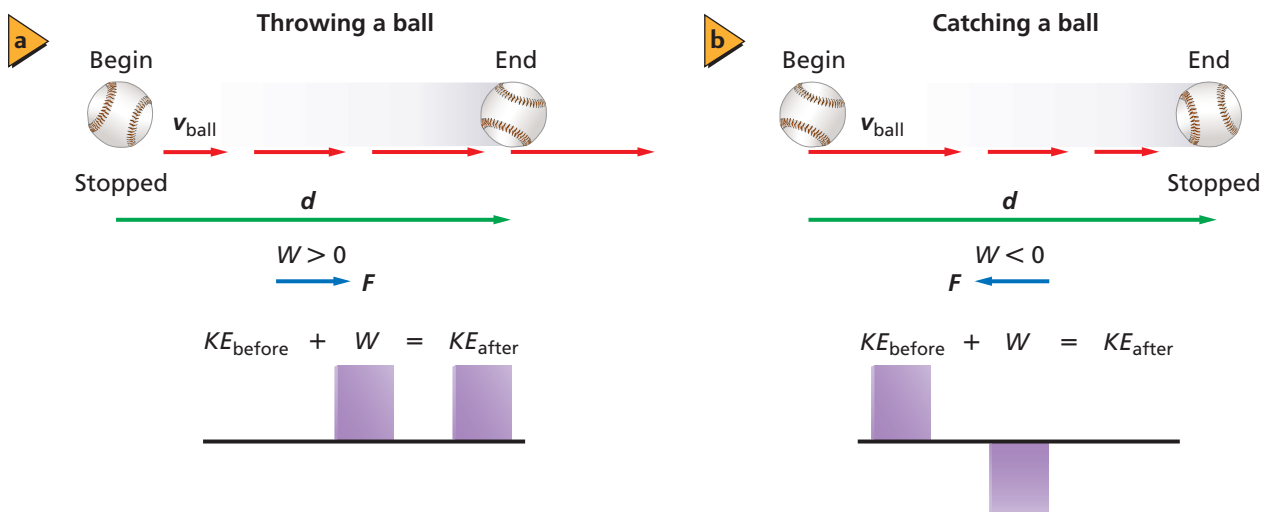
## A Model of the Work-Energy Theorem

In the last chapter, you were introduced to the work-energy theorem. You learned that when work is done on a system, the energy of that system increases. On the other hand, if the system does work, then the energy of the system decreases. These are abstract ideas, but keeping track of energy is much like keeping track of your spending money.

If you have a job, the amount of money that you have increases every time you are paid. This process can be represented with a bar graph, as shown in **Figure 11-1a**. The orange bar represents how much money you had to start with, and the blue bar represents the amount that you were paid. The green bar is the total amount that you possess after the payment. An accountant would say that your cash flow was positive. What happens when you spend the money that you have? The total amount of money that you have decreases. As shown in **Figure 11-1b**, the bar that represents the amount of money that you had before you bought that new CD is higher than the bar that represents the amount of money remaining after your shopping trip. The difference is the cost of the CD. Cash flow is shown as a bar below the axis because it represents money going out, which can be shown as a negative number. Energy is similar to your spending money. The amount of money that you have changes only when you earn more or spend it. Similarly, energy can be stored, and when energy is spent, it affects the motion of a system.

**Throwing a ball** Gaining and losing energy also can be illustrated by throwing and catching a ball. In Chapter 10, you learned that when you exert a constant force,  $F$ , on an object through a distance,  $d$ , in the direction of the force, you do an amount of work, represented by  $W = Fd$ . The work is positive because the force and motion are in the same direction, and the energy of the object increases by an amount equal to  $W$ . Suppose the object is a ball, and you exert a force to throw the ball. As a result of the force you apply, the ball gains kinetic energy. This process is shown in **Figure 11-2a**. You can again use a bar graph to represent the process. This time, the height of the bar represents the amount of work, or energy, measured in joules. The kinetic energy after the work is done is equal to the sum of the initial kinetic energy plus the work done on the ball.

■ **Figure 11-2** The kinetic energy after throwing or catching a ball is equal to the kinetic energy before plus the input work.

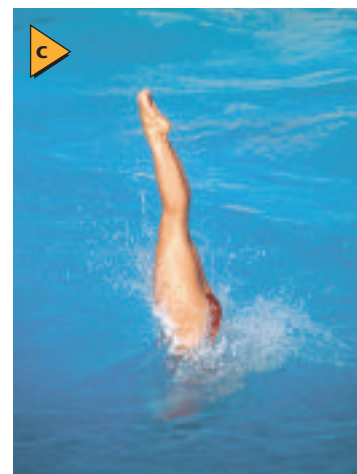
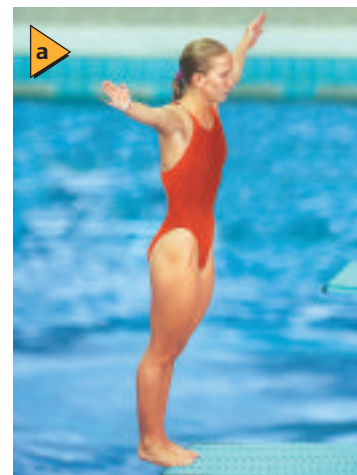


**Catching a ball** What happens when you catch a ball? Before hitting your hands or glove, the ball is moving, so it has kinetic energy. In catching it, you exert a force on the ball in the direction opposite to its motion. Therefore, you do negative work on it, causing it to stop. Now that the ball is not moving, it has no kinetic energy. This process and the bar graph that represents it are shown in **Figure 11-2b**. Kinetic energy is always positive, so the initial kinetic energy of the ball is positive. The work done on the ball is negative and the final kinetic energy is zero. Again, the kinetic energy after the ball has stopped is equal to the sum of the initial kinetic energy plus the work done on the ball.

## Kinetic Energy

Recall that kinetic energy,  $KE = \frac{1}{2}mv^2$ , where  $m$  is the mass of the object and  $v$  is the magnitude of its velocity. The kinetic energy is proportional to the object's mass. A 7.26-kg shot put thrown through the air has much more kinetic energy than a 0.148-kg baseball with the same velocity, because the shot put has a greater mass. The kinetic energy of an object is also proportional to the square of the object's velocity. A car speeding at 20 m/s has four times the kinetic energy of the same car moving at 10 m/s. Kinetic energy also can be due to rotational motion. If you spin a toy top in one spot, does it have kinetic energy? You might say that it does not because the top is not moving anywhere. However, to make the top rotate, someone had to do work on it. Therefore, the top has **rotational kinetic energy**. This is one of the several varieties of energy. Rotational kinetic energy can be calculated using  $KE_{\text{rot}} = \frac{1}{2}I\omega^2$ , where  $I$  is the object's moment of inertia and  $\omega$  is the object's angular velocity.

The diver, shown in **Figure 11-3a**, does work as she pushes off of the diving board. This work produces both linear and rotational kinetic energies. When the diver's center of mass moves as she leaps, linear kinetic energy is produced. When she rotates about her center of mass, as shown in **Figure 11-3b**, rotational kinetic energy is produced. Because she is moving toward the water and rotating at the same time while in the tuck position, she has both linear and rotational kinetic energy. When she slices into the water, as shown in **Figure 11-3c**, she has linear kinetic energy.



■ **Figure 11-3** The diver does work as she pushes off of the diving board **(a)**. This work produces rotational kinetic energy as she rotates about her center of mass **(b)** and she has linear kinetic energy when she slices into the water **(c)**.

### ▶ PRACTICE Problems

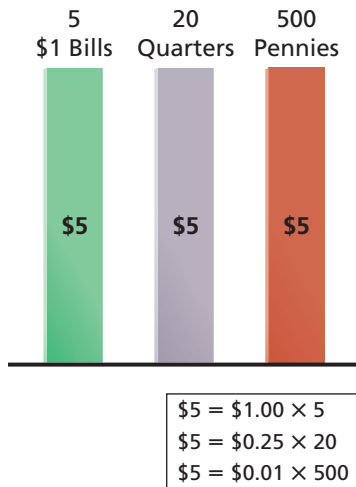
### Additional Problems, Appendix B

1. A skater with a mass of 52.0 kg moving at 2.5 m/s glides to a stop over a distance of 24.0 m. How much work did the friction of the ice do to bring the skater to a stop? How much work would the skater have to do to speed up to 2.5 m/s again?
2. An 875.0-kg compact car speeds up from 22.0 m/s to 44.0 m/s while passing another car. What are its initial and final energies, and how much work is done on the car to increase its speed?
3. A comet with a mass of  $7.85 \times 10^{11}$  kg strikes Earth at a speed of 25.0 km/s. Find the kinetic energy of the comet in joules, and compare the work that is done by Earth in stopping the comet to the  $4.2 \times 10^{15}$  J of energy that was released by the largest nuclear weapon ever built.





■ **Figure 11-4** Money in the form of bills, quarters, and pennies are different forms of the same thing.



■ **Figure 11-5** Kinetic and potential energy are constantly being exchanged when juggling.



## Stored Energy

Imagine a group of boulders high on a hill. These boulders have been lifted up by geological processes against the force of gravity; thus, they have stored energy. In a rock slide, the boulders are shaken loose. They fall and pick up speed as their stored energy is converted to kinetic energy.

In the same way, a small, spring-loaded toy, such as a jack-in-the-box, has stored energy, but the energy is stored in a compressed spring. While both of these examples represent energy stored by mechanical means, there are many other means of storing energy. Automobiles, for example, carry their energy stored in the form of chemical energy in the gasoline tank. Energy is made useful or causes motion when it changes from one form to another.

How does the money model that was discussed earlier illustrate the transformation of energy from one form to another? Money, too, can come in different forms. You can have one five-dollar bill, 20 quarters, or 500 pennies. In all of these cases, you still have five dollars. The height of the bar graph in **Figure 11-4** represents the amount of money in each form. In the same way, you can use a bar graph to represent the amount of energy in various forms that a system has.

## Gravitational Potential Energy

Look at the oranges being juggled in **Figure 11-5**. If you consider the system to be only one orange, then it has several external forces acting on it. The force of the juggler's hand does work, giving the orange its original kinetic energy. After the orange leaves the juggler's hand, only the force of gravity acts on it. How much work does gravity do on the orange as its height changes?

**Work done by gravity** Let  $h$  represent the orange's height measured from the juggler's hand. On the way up, its displacement is upward, but the force on the orange,  $F_g$ , is downward, so the work done by gravity is negative:  $W_g = -mgh$ . On the way back down, the force and displacement are in the same direction, so the work done by gravity is positive:  $W_g = mgh$ . Thus, while the orange is moving upward, gravity does negative work, slowing the orange to a stop. On the way back down, gravity does positive work, increasing the orange's speed and thereby increasing its kinetic energy. The orange recovers all of the kinetic energy it originally had when it returns to the height at which it left the juggler's hand. It is as if the orange's kinetic energy is stored in another form as the ball rises and is transformed back to kinetic energy as the ball falls.

Consider a system that consists of an object plus Earth. The gravitational attraction between the object and Earth is a force that always does work on the object as it moves. If the object moves away from Earth, energy is stored in the system as a result of the gravitational force between the object and Earth. This stored energy is called **gravitational potential energy** and is represented by the symbol  $PE$ . The height to which the object has risen is determined by using a **reference level**, the position where  $PE$  is defined to be zero. For an object with mass,  $m$ , that has risen to a height,  $h$ , above the reference level, gravitational potential energy is represented by the following equation.

**Gravitational Potential Energy**  $PE = mgh$   
 The gravitational potential energy of an object is equal to the product of its mass, the acceleration due to gravity, and the distance from the reference level.

In the equation for gravitational potential energy,  $g$  is the acceleration due to gravity. Gravitational potential energy, like kinetic energy, is measured in joules.

**Kinetic energy and potential energy of a system** Consider the energy of a system consisting of an orange used by the juggler plus Earth. The energy in the system exists in two forms: kinetic energy and gravitational potential energy. At the beginning of the orange's flight, all the energy is in the form of kinetic energy, as shown in **Figure 11-6a**. On the way up, as the orange slows down, energy changes from kinetic energy to potential energy. At the highest point of the orange's flight, the velocity is zero. Thus, all the energy is in the form of gravitational potential energy. On the way back down, potential energy changes back into kinetic energy. The sum of kinetic energy and potential energy is constant at all times because no work is done on the system by any external forces.

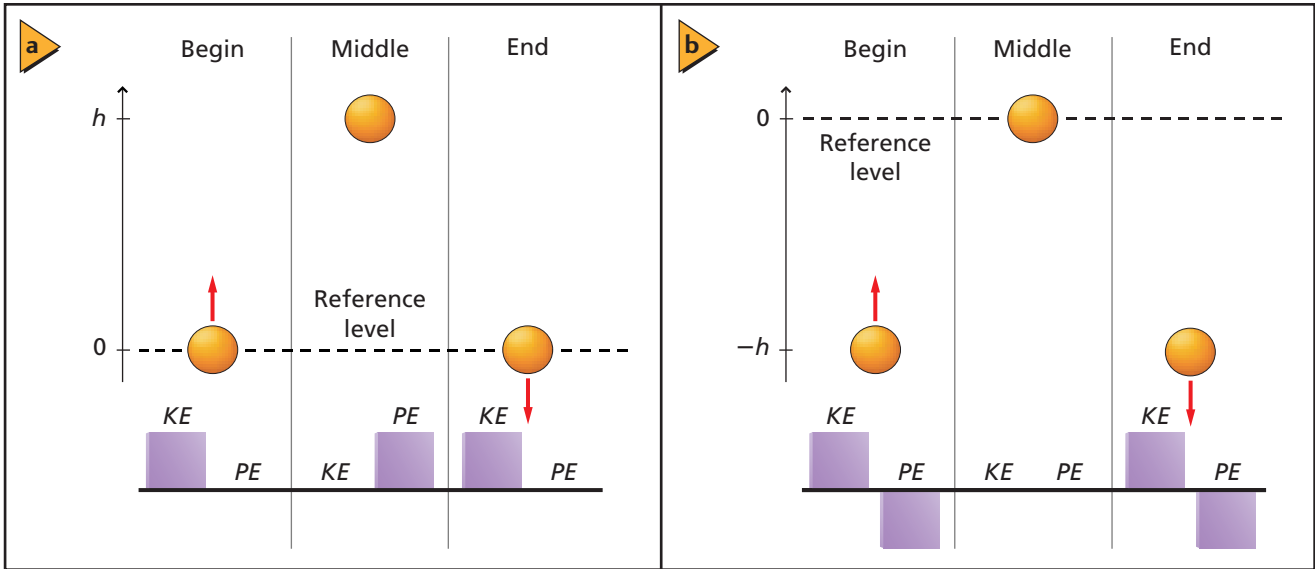
**Reference levels** In Figure 11-6a, the reference level is the juggler's hand. That is, the height of the orange is measured from the juggler's hand. Thus, at the juggler's hand,  $h = 0$  m and  $PE = 0$  J. You can set the reference level at any height that is convenient for solving a given problem.

Suppose the reference level is set at the highest point of the orange's flight. Then,  $h = 0$  m and the system's  $PE = 0$  J at that point, as illustrated in **Figure 11-6b**. The potential energy of the system is negative at the beginning of the orange's flight, zero at the highest point, and negative at the end of the orange's flight. If you were to calculate the total energy of the system represented in Figure 11-6a, it would be different from the total energy of the system represented in Figure 11-6b. This is because the reference levels are different in each case. However, the total energy of the system in each situation would be constant at all times during the flight of the orange. Only changes in energy determine the motion of a system.

► **Potential Energy of an Atom**

It is interesting to consider the relative sizes of potential energy per atom. For instance, a carbon atom has a mass of about  $2 \times 10^{-26}$  kg. If you lift it 1 m above the ground, its gravitational potential energy is about  $2 \times 10^{-25}$  J. The electrostatic energy that holds the electrons on the atom has a value of about  $10^{-19}$  J, and the nuclear potential energy that holds the nucleus together is greater than  $10^{-12}$  J. The nuclear potential energy is at least a million million times greater than the gravitational potential energy. ◀

■ **Figure 11-6** The energy of an orange is converted from one form to another in various stages of its flight **(a)**. Note that the choice of a reference level is arbitrary, but that the total energy remains constant **(b)**.

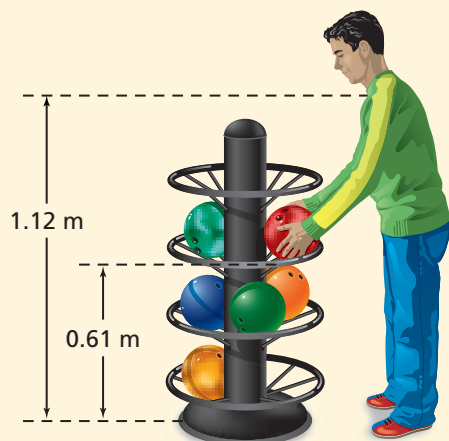




## ▶ EXAMPLE Problem 1

**Gravitational Potential Energy** You lift a 7.30-kg bowling ball from the storage rack and hold it up to your shoulder. The storage rack is 0.610 m above the floor and your shoulder is 1.12 m above the floor.

- When the bowling ball is at your shoulder, what is the bowling ball's gravitational potential energy relative to the floor?
- When the bowling ball is at your shoulder, what is its gravitational potential energy relative to the storage rack?
- How much work was done by gravity as you lifted the ball from the rack to shoulder level?



### 1 Analyze and Sketch the Problem

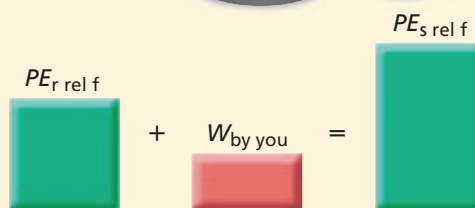
- Sketch the situation.
- Choose a reference level.
- Draw a bar graph showing the gravitational potential energy with the floor as the reference level.

#### Known:

$$\begin{aligned} m &= 7.30 \text{ kg} \\ h_r &= 0.610 \text{ m (relative to the floor)} \\ h_s &= 1.12 \text{ m (relative to the floor)} \\ g &= 9.80 \text{ m/s}^2 \end{aligned}$$

#### Unknown:

$$\begin{aligned} PE_{s \text{ rel } f} &= ? \\ PE_{s \text{ rel } r} &= ? \end{aligned}$$



### 2 Solve for the Unknown

- Set the reference level to be at the floor.

Solve for the potential energy of the ball at shoulder level.

$$\begin{aligned} PE_{s \text{ rel } f} &= mgh_s \\ &= (7.30 \text{ kg})(9.80 \text{ m/s}^2)(1.12 \text{ m}) \\ &= 80.1 \text{ J} \end{aligned}$$

Substitute  $m = 7.30 \text{ kg}$ ,  $g = 9.80 \text{ m/s}^2$ ,  $h_{\text{shoulder}} = 1.12 \text{ m}$

- Set the reference level to be at the rack height.

Solve for the height of your shoulder relative to the rack.

$$h = h_s - h_r$$

Solve for the potential energy of the ball.

$$\begin{aligned} PE_{s \text{ rel } r} &= mgh \\ &= mg(h_s - h_r) \\ &= (7.30 \text{ kg})(9.80 \text{ m/s}^2)(1.12 \text{ m} - 0.610 \text{ m}) \\ &= 36.5 \text{ J} \end{aligned}$$

Substitute  $h = h_s - h_r$

Substitute  $m = 7.3 \text{ kg}$ ,  $g = 9.80 \text{ m/s}^2$ ,

$h_s = 1.12 \text{ m}$ ,  $h_r = 0.610 \text{ m}$

This also is equal to the work done by you.

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Order of Operations  
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- The work done by gravity is the weight of the ball times the distance the ball was lifted.

$$\begin{aligned} W &= Fd \\ &= -(mg)h \quad \text{Because the weight opposes the motion of lifting, the work is negative.} \\ &= -(mg)(h_s - h_r) \\ &= -(7.30 \text{ kg})(9.80 \text{ m/s}^2)(1.12 \text{ m} - 0.610 \text{ m}) \\ &= -36.5 \text{ J} \end{aligned}$$

Substitute  $m = 7.30 \text{ kg}$ ,  $g = 9.80 \text{ m/s}^2$ ,

$h_s = 1.12 \text{ m}$ ,  $h_r = 0.610 \text{ m}$

### 3 Evaluate the Answer

- Are the units correct?** The potential energy and work are both measured in joules.
- Is the magnitude realistic?** The ball should have a greater potential energy relative to the floor than relative to the rack, because the ball's distance above the reference level is greater.



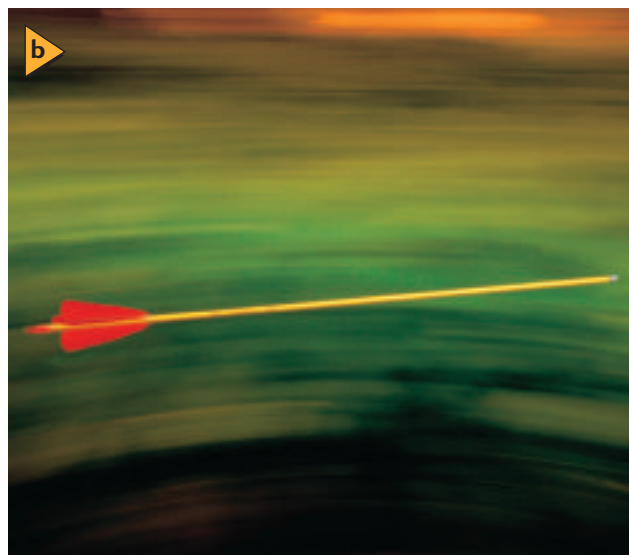
4. In Example Problem 1, what is the potential energy of the bowling ball relative to the rack when it is on the floor?
5. If you slowly lower a 20.0-kg bag of sand 1.20 m from the trunk of a car to the driveway, how much work do you do?
6. A boy lifts a 2.2-kg book from his desk, which is 0.80 m high, to a bookshelf that is 2.10 m high. What is the potential energy of the book relative to the desk?
7. If a 1.8-kg brick falls to the ground from a chimney that is 6.7 m high, what is the change in its potential energy?
8. A warehouse worker picks up a 10.1-kg box from the floor and sets it on a long, 1.1-m-high table. He slides the box 5.0 m along the table and then lowers it back to the floor. What were the changes in the energy of the box, and how did the total energy of the box change? (Ignore friction.)

## Elastic Potential Energy

When the string on the bow shown in **Figure 11-7** is pulled, work is done on the bow, storing energy in it. Thus, the energy of the system increases. Identify the system as the bow, the arrow, and Earth. When the string and arrow are released, energy is changed into kinetic energy. The stored energy in the pulled string is called **elastic potential energy**, which is often stored in rubber balls, rubber bands, slingshots, and trampolines.

Energy also can be stored in the bending of an object. When stiff metal or bamboo poles were used in pole-vaulting, the poles did not bend easily. Little work was done on the poles, and consequently, the poles did not store much potential energy. Since flexible fiberglass poles were introduced, however, record pole-vaulting heights have soared.

■ **Figure 11-7** Elastic potential energy is stored in the string of this bow. Before the string is released, the energy is all potential **(a)**. As the string is released, the energy is transferred to the arrow as kinetic energy **(b)**.







■ **Figure 11-8** When a pole-vaulter jumps, elastic potential energy is changed into kinetic energy and gravitational potential energy.

A pole-vaulter runs with a flexible pole and plants its end into the socket in the ground. When the pole-vaulter bends the pole, as shown in **Figure 11-8**, some of the pole-vaulter's kinetic energy is converted to elastic potential energy. When the pole straightens, the elastic potential energy is converted to gravitational potential energy and kinetic energy as the pole-vaulter is lifted as high as 6 m above the ground. Unlike stiff metal poles or bamboo poles, fiberglass poles have an increased capacity for storing elastic potential energy. Thus, pole-vaulters are able to clear bars that are set very high.

**Mass** Albert Einstein recognized yet another form of potential energy: mass itself. He said that mass, by its very nature, is energy. This energy,  $E_0$ , is called rest energy and is represented by the following famous formula.

$$\text{Rest Energy } E_0 = mc^2$$

The rest energy of an object is equal to the object's mass times the speed of light squared.

According to this formula, stretching a spring or bending a vaulting pole causes the spring or pole to gain mass. In these cases, the change in mass is too small to be detected. When forces within the nucleus of an atom are involved, however, the energy released into other forms, such as kinetic energy, by changes in mass can be quite large.

## 11.1 Section Review

9. **Elastic Potential Energy** You get a spring-loaded toy pistol ready to fire by compressing the spring. The elastic potential energy of the spring pushes the rubber dart out of the pistol. You use the toy pistol to shoot the dart straight up. Draw bar graphs that describe the forms of energy present in the following instances.
  - a. The dart is pushed into the gun barrel, thereby compressing the spring.
  - b. The spring expands and the dart leaves the gun barrel after the trigger is pulled.
  - c. The dart reaches the top of its flight.
10. **Potential Energy** A 25.0-kg shell is shot from a cannon at Earth's surface. The reference level is Earth's surface. What is the gravitational potential energy of the system when the shell is at 425 m? What is the change in potential energy when the shell falls to a height of 225 m?
11. **Rotational Kinetic Energy** Suppose some children push a merry-go-round so that it turns twice as fast as it did before they pushed it. What are the relative changes in angular momentum and rotational kinetic energy?
12. **Work-Energy Theorem** How can you apply the work-energy theorem to lifting a bowling ball from a storage rack to your shoulder?
13. **Potential Energy** A 90.0-kg rock climber first climbs 45.0 m up to the top of a quarry, then descends 85.0 m from the top to the bottom of the quarry. If the initial height is the reference level, find the potential energy of the system (the climber and Earth) at the top and at the bottom. Draw bar graphs for both situations.
14. **Critical Thinking** Karl uses an air hose to exert a constant horizontal force on a puck, which is on a frictionless air table. He keeps the hose aimed at the puck, thereby creating a constant force as the puck moves a fixed distance.
  - a. Explain what happens in terms of work and energy. Draw bar graphs.
  - b. Suppose Karl uses a different puck with half the mass of the first one. All other conditions remain the same. How will the kinetic energy and work differ from those in the first situation?
  - c. Explain what happened in parts a and b in terms of impulse and momentum.



## 11.2 Conservation of Energy

Consider a ball near the surface of Earth. The sum of gravitational potential energy and kinetic energy in that system is constant. As the height of the ball changes, energy is converted from kinetic energy to potential energy, but the total amount of energy stays the same.

### Conservation of Energy

In our everyday world, it may not seem as if energy is conserved. A hockey puck eventually loses its kinetic energy and stops moving, even on smooth ice. A pendulum stops swinging after some time. The money model can again be used to illustrate what is happening in these cases.

Suppose you have a total of \$50 in cash. One day, you count your money and discover that you are \$3 short. Would you assume that the money just disappeared? You probably would try to remember whether you spent it, and you might even search for it. In other words, rather than giving up on the conservation of money, you would try to think of different places where it might have gone.

**Law of conservation of energy** Scientists do the same thing as you would if you could not account for a sum of money. Whenever they observe energy leaving a system, they look for new forms into which the energy could have been transferred. This is because the total amount of energy in a system remains constant as long as the system is closed and isolated from external forces. The **law of conservation of energy** states that in a closed, isolated system, energy can neither be created nor destroyed; rather, energy is conserved. Under these conditions, energy changes from one form to another while the total energy of the system remains constant.

**Conservation of mechanical energy** The sum of the kinetic energy and gravitational potential energy of a system is called **mechanical energy**. In any given system, if no other forms of energy are present, mechanical energy is represented by the following equation.

$$\text{Mechanical Energy of a System } E = KE + PE$$

The mechanical energy of a system is equal to the sum of the kinetic energy and potential energy if no other forms of energy are present.

Imagine a system consisting of a 10.0-N ball and Earth, as shown in **Figure 11-9**. Suppose the ball is released from 2.00 m above the ground, which you set to be the reference level. Because the ball is not yet moving, it has no kinetic energy. Its potential energy is represented by the following equation:

$$PE = mgh = (10.0 \text{ N})(2.00 \text{ m}) = 20.0 \text{ J}$$

The ball's total mechanical energy, therefore, is 20.0 J. As the ball falls, it loses potential energy and gains kinetic energy. When the ball is 1.00 m above Earth's surface:  $PE = mgh = (10.0 \text{ N})(1.00 \text{ m}) = 10.0 \text{ J}$ .

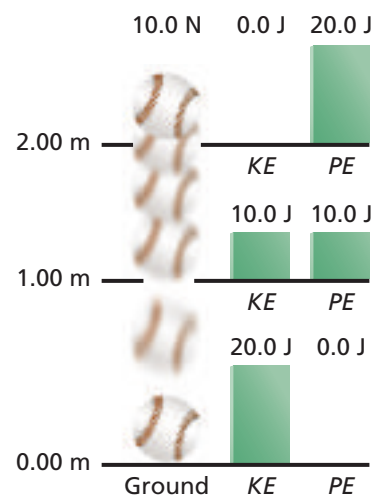
#### Objectives

- **Solve** problems using the law of conservation of energy.
- **Analyze** collisions to find the change in kinetic energy.

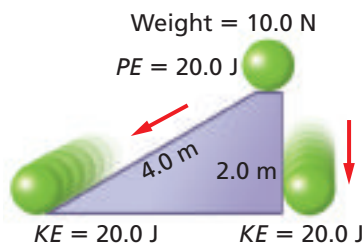
#### Vocabulary

- law of conservation of energy
- mechanical energy
- thermal energy
- elastic collision
- inelastic collision

**Figure 11-9** A decrease in potential energy is equal to the increase in kinetic energy.







■ **Figure 11-10** The path that an object follows in reaching the ground does not affect the final kinetic energy of the object.

What is the ball's kinetic energy when it is at a height of 1.00 m? The system consisting of the ball and Earth is closed and isolated because no external forces are acting upon it. Hence, the total energy of the system,  $E$ , remains constant at 20.0 J.

$$E = KE + PE, \text{ so } KE = E - PE$$

$$KE = 20.0 \text{ J} - 10.0 \text{ J} = 10.0 \text{ J}$$

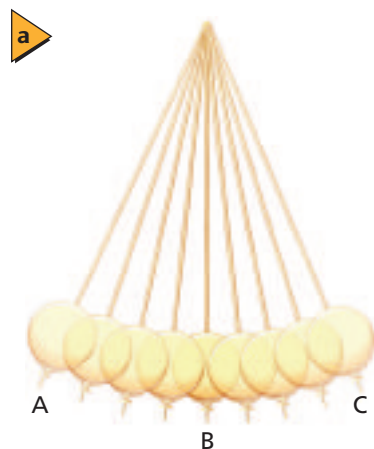
When the ball reaches ground level, its potential energy is zero, and its kinetic energy is 20.0 J. The equation that describes conservation of mechanical energy can be written as follows.

### Conservation of Mechanical Energy

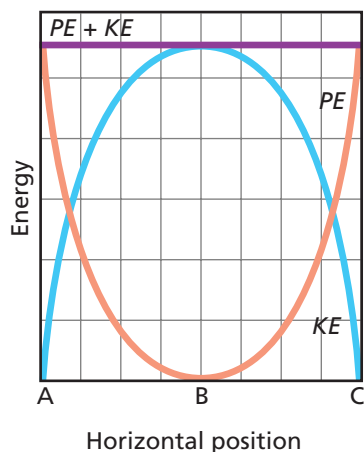
$$KE_{\text{before}} + PE_{\text{before}} = KE_{\text{after}} + PE_{\text{after}}$$

When mechanical energy is conserved, the sum of the kinetic energy and potential energy present in the system before the event is equal to the sum of the kinetic energy and potential energy in the system after the event.

■ **Figure 11-11** For the simple harmonic motion of a pendulum bob (a), the mechanical energy—the sum of the potential and kinetic energies—is a constant (b).



b Energy v. Position



What happens if the ball does not fall down, but rolls down a ramp, as shown in **Figure 11-10**? If there is no friction, there are no external forces acting on the system. Thus, the system remains closed and isolated. The ball still moves down a vertical distance of 2.00 m, so its loss of potential energy is 20.0 J. Therefore, it gains 20.0 J of kinetic energy. As long as there is no friction, the path that the ball takes does not matter.

**Roller coasters** In the case of a roller coaster that is nearly at rest at the top of the first hill, the total mechanical energy in the system is the coaster's gravitational potential energy at that point. Suppose some other hill along the track were higher than the first one. The roller coaster would not be able to climb the higher hill because the energy required to do so would be greater than the total mechanical energy of the system.

**Skiing** Suppose you ski down a steep slope. When you begin from rest at the top of the slope, your total mechanical energy is simply your gravitational potential energy. Once you start skiing downhill, your gravitational potential energy is converted to kinetic energy. As you ski down the slope, your speed increases as more of your potential energy is converted to kinetic energy. In ski jumping, the height of the ramp determines the amount of energy that the jumper has to convert into kinetic energy at the beginning of his or her flight.

**Pendulums** The simple oscillation of a pendulum also demonstrates conservation of energy. The system is the pendulum bob and Earth. Usually, the reference level is chosen to be the height of the bob at the lowest point, when it is at rest. If an external force pulls the bob to one side, the force does work that gives the system mechanical energy. At the instant the bob is released, all the energy is in the form of potential energy, but as the bob swings downward, the energy is converted to kinetic energy. **Figure 11-11** shows a graph of the changing potential and kinetic energies of a pendulum. When the bob is at the lowest point, its gravitational potential energy is zero, and its kinetic energy is equal to the total mechanical



energy in the system. Note that the total mechanical energy of the system is constant if we assume that there is no friction. You will learn more about pendulums in Chapter 14.

**Loss of mechanical energy** The oscillations of a pendulum eventually come to a stop, a bouncing ball comes to rest, and the heights of roller-coaster hills get lower and lower. Where does the mechanical energy in such systems go? Any object moving through the air experiences the forces of air resistance. In a roller coaster, there are frictional forces between the wheels and the tracks.

When a ball bounces off of a surface, all of the elastic potential energy that is stored in the deformed ball is not converted back into kinetic energy after the bounce. Some of the energy is converted into thermal energy and sound energy. As in the cases of the pendulum and the roller coaster, some of the original mechanical energy in the system is converted into another form of energy within members of the system or transmitted to energy outside the system, as in air resistance. Usually, this new energy causes the temperature of objects to rise slightly. You will learn more about this form of energy, called **thermal energy**, in Chapter 12. The following strategies will be helpful to you when solving problems related to conservation of energy.

### ▶ PROBLEM-SOLVING Strategies

#### Conservation of Energy

When solving problems related to the conservation of energy, use the following strategies.

1. Carefully identify the system. Make sure it is closed. In a closed system, no objects enter or leave the system.
2. Identify the forms of energy in the system.
3. Identify the initial and final states of the system.
4. Is the system isolated?
  - a. If there are no external forces acting on the system, then the system is isolated and the total energy of the system is constant.

$$E_{\text{before}} = E_{\text{after}}$$

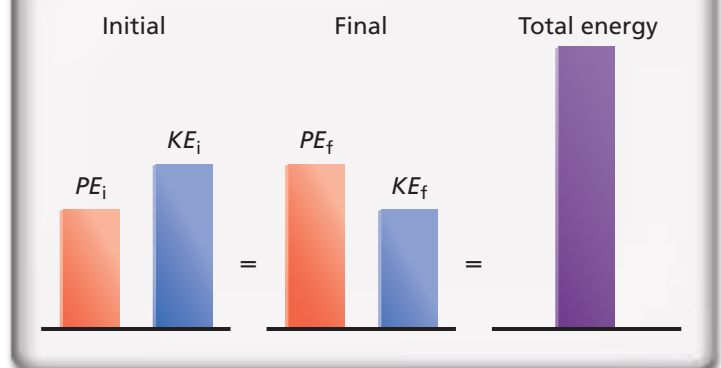
- b. If there are external forces, then the following is true.

$$E_{\text{before}} + W = E_{\text{after}}$$

5. If mechanical energy is conserved, decide on the reference level for potential energy. Draw bar graphs showing initial and final energy like the bar graphs shown to the right.

#### ▶ Connecting Math to Physics

#### Energy Bar Graphs



## ▶ EXAMPLE Problem 2

**Conservation of Mechanical Energy** During a hurricane, a large tree limb, with a mass of 22.0 kg and a height of 13.3 m above the ground, falls on a roof that is 6.0 m above the ground.

- Ignoring air resistance, find the kinetic energy of the limb when it reaches the roof.
- What is the speed of the limb when it reaches the roof?

### 1 Analyze and Sketch the Problem

- Sketch the initial and final conditions.
- Choose a reference level.
- Draw a bar graph.

**Known:**

$$\begin{aligned} m &= 22.0 \text{ kg} & g &= 9.80 \text{ m/s}^2 \\ h_{\text{limb}} &= 13.3 \text{ m} & v_i &= 0.0 \text{ m/s} \\ h_{\text{roof}} &= 6.0 \text{ m} & KE_i &= 0.0 \text{ J} \end{aligned}$$

**Unknown:**

$$\begin{aligned} PE_i &= ? & KE_f &= ? \\ PE_f &= ? & v_f &= ? \end{aligned}$$

### 2 Solve for the Unknown

- Set the reference level as the height of the roof. Solve for the initial height of the limb relative to the roof.

$$\begin{aligned} h &= h_{\text{limb}} - h_{\text{roof}} \\ &= 13.3 \text{ m} - 6.0 \text{ m} && \text{Substitute } h_{\text{limb}} = 13.3 \text{ m}, h_{\text{roof}} = 6.0 \text{ m} \\ &= 7.3 \text{ m} \end{aligned}$$

Solve for the initial potential energy of the limb.

$$\begin{aligned} PE_i &= mgh \\ &= (22.0 \text{ kg})(9.80 \text{ m/s}^2)(7.3 \text{ m}) && \text{Substitute } m = 22.0 \text{ kg}, g = 9.80 \text{ m/s}^2, h = 7.3 \text{ m} \\ &= 1.6 \times 10^3 \text{ J} \end{aligned}$$

Identify the initial kinetic energy of the limb.

$$KE_i = 0.0 \text{ J} \quad \text{The tree limb is initially at rest.}$$

The kinetic energy of the limb when it reaches the roof is equal to its initial potential energy because energy is conserved.

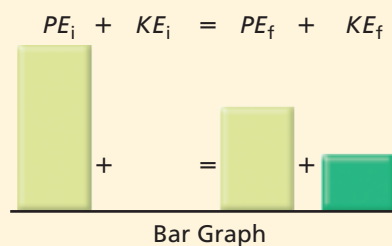
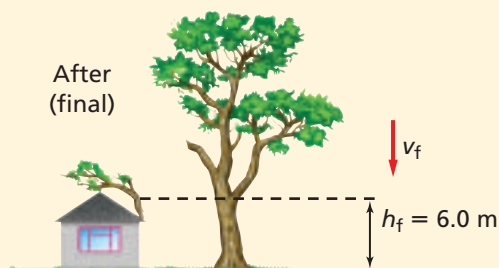
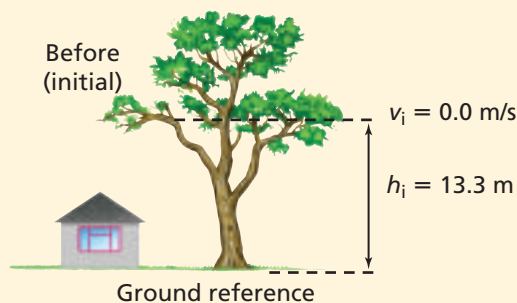
$$\begin{aligned} KE_f &= PE_i && PE_f = 0.0 \text{ J because } h = 0.0 \text{ m at the reference level.} \\ &= 1.6 \times 10^3 \text{ J} \end{aligned}$$

- Solve for the speed of the limb.

$$\begin{aligned} KE_f &= \frac{1}{2}mv_f^2 \\ v_f &= \sqrt{\frac{2KE_f}{m}} \\ &= \sqrt{\frac{2(1.6 \times 10^3 \text{ J})}{22.0 \text{ kg}}} && \text{Substitute } KE_f = 1.6 \times 10^3 \text{ J}, m = 22.0 \text{ kg} \\ &= 12 \text{ m/s} \end{aligned}$$

### 3 Evaluate the Answer

- Are the units correct?** Velocity is measured in m/s and energy is measured in  $\text{kg} \cdot \text{m}^2/\text{s}^2 = \text{J}$ .
- Do the signs make sense?**  $KE$  and the magnitude of velocity are always positive.



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15. A bike rider approaches a hill at a speed of 8.5 m/s. The combined mass of the bike and the rider is 85.0 kg. Choose a suitable system. Find the initial kinetic energy of the system. The rider coasts up the hill. Assuming there is no friction, at what height will the bike come to rest?
16. Suppose that the bike rider in problem 15 pedaled up the hill and never came to a stop. In what system is energy conserved? From what form of energy did the bike gain mechanical energy?
17. A skier starts from rest at the top of a 45.0-m-high hill, skis down a 30° incline into a valley, and continues up a 40.0-m-high hill. The heights of both hills are measured from the valley floor. Assume that you can neglect friction and the effect of the ski poles. How fast is the skier moving at the bottom of the valley? What is the skier's speed at the top of the next hill? Do the angles of the hills affect your answers?
18. In a belly-flop diving contest, the winner is the diver who makes the biggest splash upon hitting the water. The size of the splash depends not only on the diver's style, but also on the amount of kinetic energy that the diver has. Consider a contest in which each diver jumps from a 3.00-m platform. One diver has a mass of 136 kg and simply steps off the platform. Another diver has a mass of 102 kg and leaps upward from the platform. How high would the second diver have to leap to make a competitive splash?

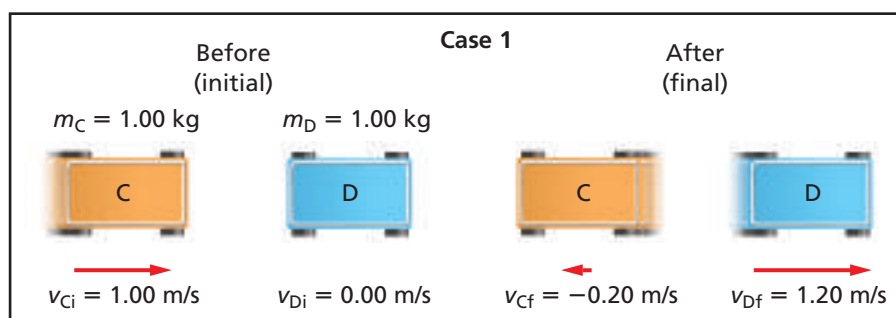
## Analyzing Collisions

A collision between two objects, whether the objects are automobiles, hockey players, or subatomic particles, is one of the most common situations analyzed in physics. Because the details of a collision can be very complex during the collision itself, the strategy is to find the motion of the objects just before and just after the collision. What conservation laws can be used to analyze such a system? If the system is isolated, then momentum and energy are conserved. However, the potential energy or thermal energy in the system may decrease, remain the same, or increase. Therefore, you cannot predict whether or not kinetic energy is conserved. **Figure 11-12** and **Figure 11-13** on the next page show three different kinds of collisions. In case 1, the momentum of the system before and after the collision is represented by the following:

$$p_i = p_{Ci} + p_{Di} = (1.00 \text{ kg})(1.00 \text{ m/s}) + (1.00 \text{ kg})(0.00 \text{ m/s}) = 1.00 \text{ kg}\cdot\text{m/s}$$

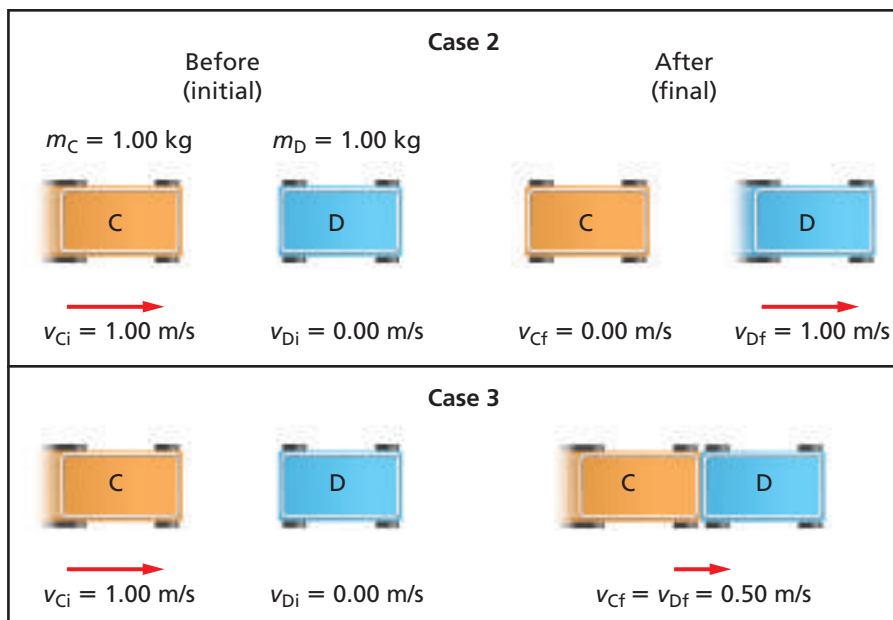
$$p_f = p_{Cf} + p_{Df} = (1.00 \text{ kg})(-0.20 \text{ m/s}) + (1.00 \text{ kg})(1.20 \text{ m/s}) = 1.00 \text{ kg}\cdot\text{m/s}$$

Thus, in case 1, the momentum is conserved. Look again at Figure 11-13 and verify for yourself that momentum is conserved in cases 2 and 3.



■ **Figure 11-12** Two moving objects can have different types of collisions. Case 1: the two objects move apart in opposite directions.

■ **Figure 11-13** Case 2: the moving object comes to rest and the stationary object begins to move. Case 3: the two objects are stuck together and move as one.



Next, consider the kinetic energy of the system in each of these cases. For case 1 the kinetic energy of the system before and after the collision is represented by the following equations:

$$KE_{Ci} + KE_{Di} = \frac{1}{2}(1.00 \text{ kg})(1.00 \text{ m/s})^2 + \frac{1}{2}(1.00 \text{ kg})(0.00 \text{ m/s})^2 = 0.50 \text{ J}$$

$$KE_{Cf} + KE_{Df} = \frac{1}{2}(1.00 \text{ kg})(-0.20 \text{ m/s})^2 + \frac{1}{2}(1.00 \text{ kg})(1.20 \text{ m/s})^2 = 0.74 \text{ J}$$

In case 1, the kinetic energy of the system increased. If energy in the system is conserved, then one or more of the other forms of energy must have decreased. Perhaps when the two carts collided, a compressed spring was released, adding kinetic energy to the system. This kind of collision is sometimes called a superelastic or explosive collision.

After the collision in case 2, the kinetic energy is equal to:

$$KE_{Cf} + KE_{Df} = (1.0 \text{ kg})(0.00 \text{ m/s})^2 + \frac{1}{2}(1.0 \text{ kg})(1.0 \text{ m/s})^2 = 0.50 \text{ J}$$

Kinetic energy remained the same after the collision. This type of collision, in which the kinetic energy does not change, is called an **elastic collision**. Collisions between hard, elastic objects, such as those made of steel, glass, or hard plastic, often are called nearly elastic collisions.

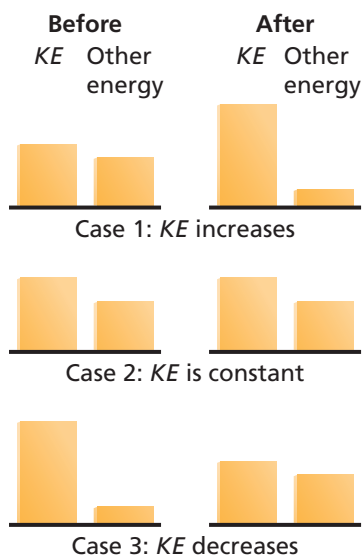
After the collision in case 3, the kinetic energy is equal to:

$$KE_{Cf} + KE_{Df} = \frac{1}{2}(1.00 \text{ kg})(0.50 \text{ m/s})^2 + \frac{1}{2}(1.00 \text{ kg})(0.50 \text{ m/s})^2 = 0.25 \text{ J}$$

Kinetic energy decreased and some of it was converted to thermal energy. This kind of collision, in which kinetic energy decreases, is called an **inelastic collision**. Objects made of soft, sticky material, such as clay, act in this way.

The three kinds of collisions can be represented using bar graphs, such as those shown in **Figure 11-14**. Although the kinetic energy before and after the collisions can be calculated, only the change in other forms of energy can be found. In automobile collisions, kinetic energy is transferred into other forms of energy, such as heat and sound.

■ **Figure 11-14** Bar graphs can be drawn to represent the three kinds of collisions.



### ▶ EXAMPLE Problem 3

**Kinetic Energy** In an accident on a slippery road, a compact car with a mass of 575 kg moving at 15.0 m/s smashes into the rear end of a car with mass 1575 kg moving at 5.00 m/s in the same direction.

- What is the final velocity if the wrecked cars lock together?
- How much kinetic energy was lost in the collision?
- What fraction of the original kinetic energy was lost?

#### 1 Analyze and Sketch the Problem

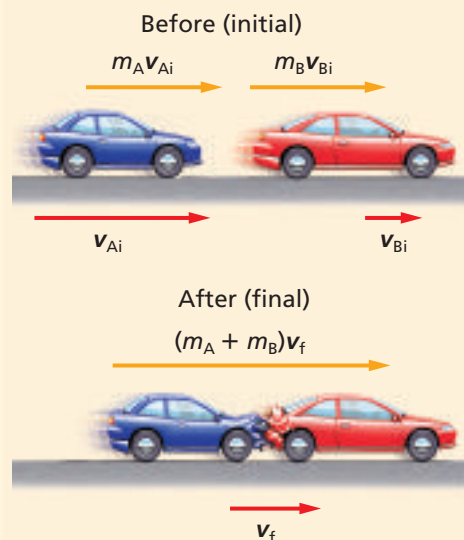
- Sketch the initial and final conditions.
- Sketch the momentum diagram.

**Known:**

$$\begin{aligned} m_A &= 575 \text{ kg} & m_B &= 1575 \text{ kg} \\ v_{Ai} &= 15.0 \text{ m/s} & v_{Bi} &= 5.00 \text{ m/s} \\ & & v_{Af} &= v_{Bf} = v_f \end{aligned}$$

**Unknown:**

$$\begin{aligned} v_f &= ? & \Delta KE &= KE_f - KE_i = ? \\ \text{Fraction of } KE_i \text{ lost, } \Delta KE/KE_i &= ? \end{aligned}$$



#### 2 Solve for the Unknown

- Use the conservation of momentum equation to find the final velocity.

$$\begin{aligned} p_{Ai} + p_{Bi} &= p_{Af} + p_{Bf} \\ m_A v_{Ai} + m_B v_{Bi} &= (m_A + m_B) v_f \\ v_f &= \frac{(m_A v_{Ai} + m_B v_{Bi})}{(m_A + m_B)} \\ &= \frac{(575 \text{ kg})(15.0 \text{ m/s}) + (1575 \text{ kg})(5.00 \text{ m/s})}{(575 \text{ kg} + 1575 \text{ kg})} \\ &= 7.67 \text{ m/s, in the direction of the motion before the collision} \end{aligned}$$

**Substitute**  $m_A = 575 \text{ kg}$ ,  $v_{Ai} = 15.0 \text{ m/s}$ ,  
 $m_B = 1575 \text{ kg}$ ,  $v_{Bi} = 5.00 \text{ m/s}$

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- To determine the change in kinetic energy of the system,  $KE_f$  and  $KE_i$  are needed.

$$\begin{aligned} KE_f &= \frac{1}{2} m v^2 \\ &= \frac{1}{2} (m_A + m_B) v_f^2 && \text{Substitute } m = m_A + m_B \\ &= \frac{1}{2} (575 \text{ kg} + 1575 \text{ kg}) (7.67 \text{ m/s})^2 && \text{Substitute } m_A = 575 \text{ kg}, m_B = 1575 \text{ kg}, v_f = 7.67 \text{ m/s} \\ &= 6.32 \times 10^4 \text{ J} \end{aligned}$$

$$\begin{aligned} KE_i &= KE_{Ai} + KE_{Bi} \\ &= \frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 && \text{Substitute } KE_{Ai} = \frac{1}{2} m_A v_{Ai}^2, KE_{Bi} = \frac{1}{2} m_B v_{Bi}^2 \\ &= \frac{1}{2} (575 \text{ kg}) (15.0 \text{ m/s})^2 + \frac{1}{2} (1575 \text{ kg}) (5.00 \text{ m/s})^2 && \text{Substitute } m_A = 575 \text{ kg}, m_B = 1575 \text{ kg}, \\ &= 8.44 \times 10^4 \text{ J} && v_{Ai} = 15.0 \text{ m/s}, v_{Bi} = 5.00 \text{ m/s} \end{aligned}$$

Solve for the change in kinetic energy of the system.

$$\begin{aligned} \Delta KE &= KE_f - KE_i \\ &= 6.32 \times 10^4 \text{ J} - 8.44 \times 10^4 \text{ J} && \text{Substitute } KE_f = 6.32 \times 10^4 \text{ J}, KE_i = 8.44 \times 10^4 \text{ J} \\ &= -2.12 \times 10^4 \text{ J} \end{aligned}$$

- Calculate the fraction of the original kinetic energy that is lost.

$$\begin{aligned} \frac{\Delta KE}{KE_i} &= \frac{-2.12 \times 10^4 \text{ J}}{8.44 \times 10^4 \text{ J}} && \text{Substitute } \Delta KE = -2.12 \times 10^4 \text{ J}, KE_i = 8.44 \times 10^4 \text{ J} \\ &= -0.251 = 25.1\% \text{ of the original kinetic energy in the system was lost.} \end{aligned}$$

#### 3 Evaluate the Answer

- Are the units correct?** Velocity is measured in m/s; energy is measured in J.
- Does the sign make sense?** Velocity is positive, consistent with the original velocities.





19. An 8.00-g bullet is fired horizontally into a 9.00-kg block of wood on an air table and is embedded in it. After the collision, the block and bullet slide along the frictionless surface together with a speed of 10.0 cm/s. What was the initial speed of the bullet?
20. A 0.73-kg magnetic target is suspended on a string. A 0.025-kg magnetic dart, shot horizontally, strikes the target head-on. The dart and the target together, acting like a pendulum, swing 12.0 cm above the initial level before instantaneously coming to rest.
  - a. Sketch the situation and choose a system.
  - b. Decide what is conserved in each part and explain your decision.
  - c. What was the initial velocity of the dart?
21. A 91.0-kg hockey player is skating on ice at 5.50 m/s. Another hockey player of equal mass, moving at 8.1 m/s in the same direction, hits him from behind. They slide off together.
  - a. What are the total energy and momentum in the system before the collision?
  - b. What is the velocity of the two hockey players after the collision?
  - c. How much energy was lost in the collision?

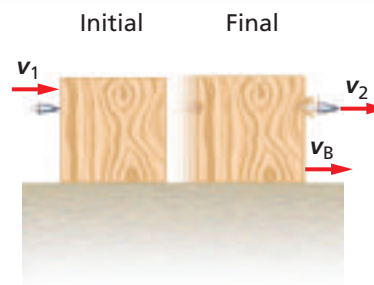
In collisions, you can see how momentum and energy are really very different. Momentum is almost always conserved in a collision. Energy is conserved only in elastic collisions. Momentum is what makes objects stop. A 10.0-kg object moving at 5.00 m/s will stop a 20.0-kg object moving at 2.50 m/s if they have a head-on collision. However, in this case, the smaller object has much more kinetic energy. The kinetic energy of the smaller object is  $KE = \frac{1}{2}(10.0 \text{ kg})(5.00 \text{ m/s})^2 = 125 \text{ J}$ . The kinetic energy of the larger object is  $KE = \frac{1}{2}(20.0 \text{ kg})(2.50 \text{ m/s})^2 = 62.5 \text{ J}$ . Based on the work-energy theorem, you can conclude that it takes more work to make the 10.0-kg object move at 5.00 m/s than it does to move the 20.0-kg object at 2.50 m/s. It sometimes is said that in automobile collisions, the momentum stops the cars but it is the energy in the collision that causes the damage.

It also is possible to have a collision in which nothing collides. If two lab carts sit motionless on a table, connected by a compressed spring, their total momentum is zero. If the spring is released, the carts will be forced to move away from each other. The potential energy of the spring will be transformed into the kinetic energy of the carts. The carts will still move away from each other so that their total momentum is zero.

● CHALLENGE PROBLEM

A bullet of mass  $m$ , moving at speed  $v_1$ , goes through a motionless wooden block and exits with speed  $v_2$ . After the collision, the block, which has mass  $m_B$ , is moving.

1. What is the final speed,  $v_B$ , of the block?
2. How much energy was lost to the bullet?
3. How much energy was lost to friction inside the block?



**Energy Exchange**  

1. Select different-sized steel balls and determine their masses.
  2. Stand a spring-loaded laboratory cart on end with the spring mechanism pointing upward.
  3. Place a ball on top of the spring mechanism and press down until the ball is touching the cart.
  4. Quickly release the ball so that the spring shoots it upward.
- CAUTION: Stay clear of the ball when launching.**
5. Repeat the process several times, and measure the average height.
  6. **Estimate** how high the other sizes of steel balls will rise.

**Analyze and Conclude**

7. **Classify** the balls by height attained. What can you conclude?

It is useful to remember two simple examples of collisions. One is the elastic collision between two objects of equal mass, such as when a cue ball with velocity,  $v$ , hits a motionless billiard ball head-on. In this case, after the collision, the cue ball is motionless and the other ball rolls off at velocity,  $v$ . It is easy to prove that both momentum and energy are conserved in this collision.

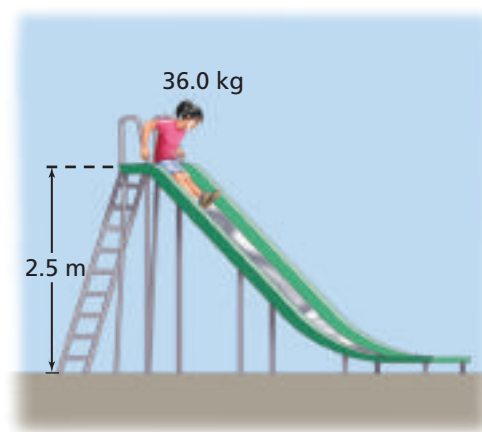
The other simple example is to consider a skater of mass  $m$ , with velocity  $v$ , running into another skater of equal mass who happens to be standing motionless on the ice. If they hold on to each other after the collision, they will slide off at a velocity of  $\frac{1}{2}v$  because of the conservation of momentum. The final kinetic energy of the pair would be equal to  $KE = \frac{1}{2}(2m)(\frac{1}{2}v)^2 = \frac{1}{4}mv^2$ , which is half the initial kinetic energy. This is because the collision was inelastic.

You have investigated examples in which the conservation of energy, and sometimes the conservation of momentum, can be used to calculate the motions of a system of objects. These systems would be too complicated to comprehend using only Newton's second law of motion. The understanding of the forms of energy and how energy flows from one form to another is one of the most useful concepts in science. The term *energy conservation* appears in everything from scientific papers to electric appliance commercials. Scientists use the concept of energy to explore topics much more complicated than colliding billiard balls.

## 11.2 Section Review

22. **Closed Systems** Is Earth a closed, isolated system? Support your answer.
23. **Energy** A child jumps on a trampoline. Draw bar graphs to show the forms of energy present in the following situations.
  - a. The child is at the highest point.
  - b. The child is at the lowest point.
24. **Kinetic Energy** Suppose a glob of chewing gum and a small, rubber ball collide head-on in midair and then rebound apart. Would you expect kinetic energy to be conserved? If not, what happens to the energy?
25. **Kinetic Energy** In table tennis, a very light but hard ball is hit with a hard rubber or wooden paddle. In tennis, a much softer ball is hit with a racket. Why are the two sets of equipment designed in this way? Can you think of other ball-paddle pairs in sports? How are they designed?
26. **Potential Energy** A rubber ball is dropped from a height of 8.0 m onto a hard concrete floor. It hits the floor and bounces repeatedly. Each time it hits the floor, it loses  $\frac{1}{5}$  of its total energy. How many times will it bounce before it bounces back up to a height of only about 4 m?

27. **Energy** As shown in **Figure 11-15**, a 36.0-kg child slides down a playground slide that is 2.5 m high. At the bottom of the slide, she is moving at 3.0 m/s. How much energy was lost as she slid down the slide?



■ Figure 11-15

28. **Critical Thinking** A ball drops 20 m. When it has fallen half the distance, or 10 m, half of its energy is potential and half is kinetic. When the ball has fallen for half the amount of time it takes to fall, will more, less, or exactly half of its energy be potential energy?

## Conservation of Energy

Alternate CBL instructions can be found on the Web site.  
[physicspp.com](http://physicspp.com)

There are many examples of situations where energy is conserved. One such example is a rock falling from a given height. If the rock starts at rest, at the moment the rock is dropped, it only has potential energy. As it falls, its potential energy decreases as its height decreases, but its kinetic energy increases. The sum of potential energy and kinetic energy remains constant if friction is neglected. When the rock is about to hit the ground, all of its potential energy has been converted to kinetic energy. In this experiment, you will model a falling object and calculate its speed as it hits the ground.

### QUESTION

**How does the transfer of an object's potential energy to kinetic energy demonstrate conservation of energy?**

#### Objectives

- **Calculate** the speed of a falling object as it hits the ground by using a model.
- **Interpret data** to find the relationship between potential energy and kinetic energy of a falling object.

#### Safety Precautions



#### Materials

- |                              |                     |
|------------------------------|---------------------|
| grooved track (two sections) | electronic balance  |
| marble or steel ball         | metric ruler        |
| stopwatch                    | graphing calculator |
| block of wood                |                     |

#### Procedure

1. Place the two sections of grooved track together, as shown in **Figure 1**. Raise one end of the track and place the block under it, about 5 cm from the raised end. Make sure the ball can roll smoothly across the junction of the two tracks.
2. Record the length of the level portion of the track in the data table. Place a ball on the track directly above the point supported by the block. Release the ball. Start the stopwatch when the ball reaches the level section of track. Stop timing when the ball reaches the end of the level portion of the track. Record the time required for the ball to travel that distance in the data table.
3. Move the support block so that it is under the midsection of the inclined track, as shown in **Figure 2**. Place the ball on the track just above the point supported by the block. Release the ball and measure the time needed for the ball to roll the length of the level portion of the track and record it in the data table. Notice that even though the incline is steeper, the ball is released from the same height as in step 2.
4. Calculate the speed of the ball on the level portion of the track in steps 2 and 3. Move the support block to a point about three-quarters down the length of the inclined track, as shown in **Figure 3**.



Figure 1



Figure 2



Figure 3



Data Table			
Release Height (m)	Distance (m)	Time (s)	Speed (m/s)
0.05			
0.05			
0.05			
0.01			
0.02			
0.03			

- Predict the amount of time the ball will take to travel the length of the level portion of the track. Record your prediction. Test your prediction.
- Place the support block at the midpoint of the inclined track (Figure 2). Measure a point on the inclined portion of the track that is 1.0 cm above the level portion of the track. Be sure to measure 1.0 cm above the level portion, and not 1.0 cm above the table.
- Release the ball from this point and measure the time required for the ball to travel on the level portion of the track and record it in the data table.
- Use a ruler to measure a point that is 2.0 cm above the level track. Release the ball from this point and measure the time required for the ball to travel on the level portion of the track. Record the time in the data table.
- Repeat step 8 for 3.0 cm, 4.0 cm, 5.0 cm, 6.0 cm, 7.0 cm, and 8.0 cm. Record the times.

### Analyze

- Infer** What effect did changing the slope of the inclined plane in steps 2–6 have on the speed of the ball on the level portion of the track?
- Analyze** Perform a power law regression for this graph using your graphing calculator. Record the equation of this function. Graph this by inputting the equation into  $Y=$ . Draw a sketch of the graph.
- Using the data from step 9 for the release point of 8.0 cm, find the potential energy of the ball before it was released. Use an electronic balance to find the mass of the ball. Note that height must be in m, and mass in kg.
- Using the speed data from step 9 for the release point of 8.0 cm calculate the kinetic energy of the ball on the level portion of the track. Remember, speed must be in m/s and mass in kg.

### Conclude and Apply

- Solve for speed,  $y$ , in terms of height,  $x$ . Begin by setting  $PE_i = KE_f$ .
- How does the equation found in the previous question relate to the power law regression calculated earlier?
- Suppose you want the ball to roll twice as fast on the level part of the track as it did when you released it from the 2-cm mark. Using the power law regression performed earlier, calculate the height from which you should release the ball.
- Explain how this experiment only models dropping a ball and finding its kinetic energy just as it hits the ground.
- Compare and Contrast** Compare the potential energy of the ball before it is released (step 8) to the kinetic energy of the ball on the level track (step 9). Explain why they are the same or why they are different.
- Draw Conclusions** Does this experiment demonstrate conservation of energy? Explain.

### Going Further

What are potential sources of error in this experiment, and how can they be reduced?

### Real-World Physics

How does your favorite roller coaster demonstrate the conservation of energy by the transfer of potential energy to kinetic energy?

**Physics**  **online**

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# Technology and Society

## Running Smarter

**The Physics of Running Shoes** Today's running shoes are high-tech marvels. They enhance performance and protect your body by acting as shock absorbers. How do running shoes help you win a race? They reduce your energy consumption, as well as allow you to use energy more efficiently. Good running shoes must be flexible enough to bend with your feet as you run, support your feet, and hold them in place. They must be lightweight and provide traction to prevent slipping.

**Running Shoes as Shock Absorbers** Today, much of the focus of running shoe technology centers on the cushioned midsole that plays a key role as a shock absorber and performance enhancer. Each time a runner's foot hits the ground, the ground exerts an equal and opposite force on the runner's foot. This force can be nearly four times a runner's weight, causing aches and pains, shin splints, and damage to knees and ankles over long distances.

Cushioning is used in running shoes to decrease the force absorbed by the runner. As a runner's foot hits the ground and comes to a stop, its momentum changes. The change in momentum is  $\Delta p = F\Delta t$ , where  $F$  is the force on that object and  $\Delta t$  is the time during which the force acts. The cushioning causes the change of momentum to occur over an extended time and reduces the force of the foot on the ground. The decreased force reduces the damage to the runner's body.

### Running Shoes Boost Performance

A shoe's cushioning system also affects energy consumption. The bones, muscles, ligaments,

and tendons of the foot and leg are a natural cushioning system. But operating this system requires the body to use stored energy to contract muscles. So if a shoe can be worn that assists a runner's natural cushioning system, the runner does not expend as much of his or her own stored energy. The energy the runner saved can be spent to run farther or faster.

The cushioned midsole uses the law of conservation of energy to return as much of the energy to the runner as possible. The runner's kinetic energy transforms to elastic potential energy, plus heat, when the runner's foot hits the running surface. If the runner can reduce the amount of energy that is lost as heat, the runner's elastic potential energy can be converted back to useful kinetic energy.

Bouncy, springy, elastic materials that resist crushing over time commonly are used to create the cushioned midsole. Options now range from silicon gel pads to complex fluid-filled systems and even springs to give a runner extra energy efficiency.



### Going Further

- 1. Use Scientific Explanations** Use physics to explain why manufacturers put cushioned midsoles in running shoes.
- 2. Analyze** Which surface would provide more cushioning when running: a grassy field or a concrete sidewalk? Explain why that surface provides better cushioning.
- 3. Research** Some people prefer to run barefoot, even in marathon races. Why might this be so?

### 11.1 The Many Forms of Energy

#### Vocabulary

- rotational kinetic energy (p. 287)
- gravitational potential energy (p. 288)
- reference level (p. 288)
- elastic potential energy (p. 291)

#### Key Concepts

- The kinetic energy of an object is proportional to its mass and the square of its velocity.
- The rotational kinetic energy of an object is proportional to the object's moment of inertia and the square of its angular velocity.
- When Earth is included in a system, the work done by gravity is replaced by gravitational potential energy.
- The gravitational potential energy of an object depends on the object's weight and its distance from Earth's surface.

$$PE = mgh$$

- The reference level is the position where the gravitational potential energy is defined to be zero.
- Elastic potential energy may be stored in an object as a result of its change in shape.
- Albert Einstein recognized that mass itself has potential energy. This energy is called rest energy.

$$E_0 = mc^2$$

### 11.2 Conservation of Energy

#### Vocabulary

- law of conservation of energy (p. 293)
- mechanical energy (p. 293)
- thermal energy (p. 295)
- elastic collision (p. 298)
- inelastic collision (p. 298)

#### Key Concepts

- The sum of kinetic and potential energy is called mechanical energy.

$$E = KE + PE$$

- If no objects enter or leave a system, the system is considered to be a closed system.
- If there are no external forces acting on a system, the system is considered to be an isolated system.
- The total energy of a closed, isolated system is constant. Within the system, energy can change form, but the total amount of energy does not change. Thus, energy is conserved.

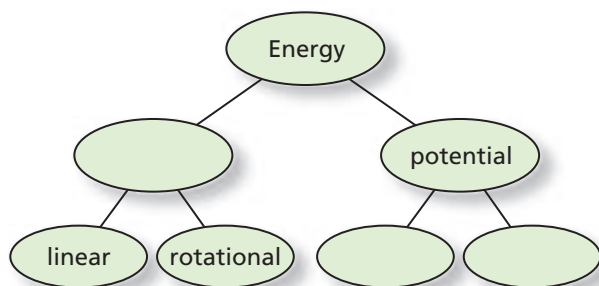
$$KE_{\text{before}} + PE_{\text{before}} = KE_{\text{after}} + PE_{\text{after}}$$

- The type of collision in which the kinetic energy after the collision is less than the kinetic energy before the collision is called an inelastic collision.
- The type of collision in which the kinetic energy before and after the collision is the same is called an elastic collision.
- Momentum is conserved in collisions if the external force is zero. The mechanical energy may be unchanged or decreased by the collision, depending on whether the collision is elastic or inelastic.



## Concept Mapping

29. Complete the concept map using the following terms: *gravitational potential energy, elastic potential energy, kinetic energy*.



## Mastering Concepts

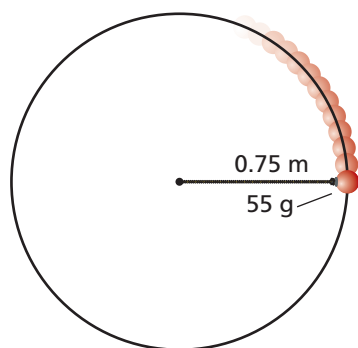
Unless otherwise directed, assume that air resistance is negligible.

30. Explain how work and a change in energy are related. (11.1)
31. What form of energy does a wound-up watch spring have? What form of energy does a functioning mechanical watch have? When a watch runs down, what has happened to the energy? (11.1)
32. Explain how energy change and force are related. (11.1)
33. A ball is dropped from the top of a building. You choose the top of the building to be the reference level, while your friend chooses the bottom. Explain whether the energy calculated using these two reference levels is the same or different for the following situations. (11.1)
- the ball's potential energy at any point
  - the change in the ball's potential energy as a result of the fall
  - the kinetic energy of the ball at any point
34. Can the kinetic energy of a baseball ever be negative? (11.1)
35. Can the gravitational potential energy of a baseball ever be negative? Explain without using a formula. (11.1)
36. If a sprinter's velocity increases to three times the original velocity, by what factor does the kinetic energy increase? (11.1)
37. What energy transformations take place when an athlete is pole-vaulting? (11.2)
38. The sport of pole-vaulting was drastically changed when the stiff, wooden poles were replaced by flexible, fiberglass poles. Explain why. (11.2)
39. You throw a clay ball at a hockey puck on ice. The smashed clay ball and the hockey puck stick together and move slowly. (11.2)
- Is momentum conserved in the collision? Explain.
  - Is kinetic energy conserved? Explain.
40. Draw energy bar graphs for the following processes. (11.2)
- An ice cube, initially at rest, slides down a frictionless slope.
  - An ice cube, initially moving, slides up a frictionless slope and instantaneously comes to rest.
41. Describe the transformations from kinetic energy to potential energy and vice versa for a roller-coaster ride. (11.2)
42. Describe how the kinetic energy and elastic potential energy are lost in a bouncing rubber ball. Describe what happens to the motion of the ball. (11.2)

## Applying Concepts

43. The driver of a speeding car applies the brakes and the car comes to a stop. The system includes the car but not the road. Apply the work-energy theorem to the following situations.
- The car's wheels do not skid.
  - The brakes lock and the car's wheels skid.
44. A compact car and a trailer truck are both traveling at the same velocity. Did the car engine or the truck engine do more work in accelerating its vehicle?
45. **Catapults** Medieval warriors used catapults to assault castles. Some catapults worked by using a tightly wound rope to turn the catapult arm. What forms of energy are involved in catapulting a rock to the castle wall?
46. Two cars collide and come to a complete stop. Where did all of their energy go?
47. During a process, positive work is done on a system, and the potential energy decreases. Can you determine anything about the change in kinetic energy of the system? Explain.
48. During a process, positive work is done on a system, and the potential energy increases. Can you tell whether the kinetic energy increased, decreased, or remained the same? Explain.
49. **Skating** Two skaters of unequal mass have the same speed and are moving in the same direction. If the ice exerts the same frictional force on each skater, how will the stopping distances of their bodies compare?

50. You swing a 55-g mass on the end of a 0.75-m string around your head in a nearly horizontal circle at constant speed, as shown in **Figure 11-16**.
- How much work is done on the mass by the tension of the string in one revolution?
  - Is your answer to part **a** in agreement with the work-energy theorem? Explain.



■ **Figure 11-16**

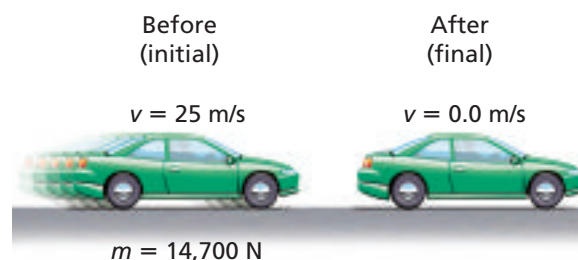
51. Give specific examples that illustrate the following processes.
- Work is done on a system, thereby increasing kinetic energy with no change in potential energy.
  - Potential energy is changed to kinetic energy with no work done on the system.
  - Work is done on a system, increasing potential energy with no change in kinetic energy.
  - Kinetic energy is reduced, but potential energy is unchanged. Work is done by the system.
52. **Roller Coaster** You have been hired to make a roller coaster more exciting. The owners want the speed at the bottom of the first hill doubled. How much higher must the first hill be built?
53. Two identical balls are thrown from the top of a cliff, each with the same speed. One is thrown straight up, the other straight down. How do the kinetic energies and speeds of the balls compare as they strike the ground?

## Mastering Problems

Unless otherwise directed, assume that air resistance is negligible.

### 11.1 The Many Forms of Energy

54. A 1600-kg car travels at a speed of 12.5 m/s. What is its kinetic energy?
55. A racing car has a mass of 1525 kg. What is its kinetic energy if it has a speed of 108 km/h?
56. Shawn and his bike have a combined mass of 45.0 kg. Shawn rides his bike 1.80 km in 10.0 min at a constant velocity. What is Shawn's kinetic energy?
57. Tony has a mass of 45 kg and is moving with a speed of 10.0 m/s.
- Find Tony's kinetic energy.
  - Tony's speed changes to 5.0 m/s. Now what is his kinetic energy?
  - What is the ratio of the kinetic energies in parts **a** and **b**? Explain.
58. Katia and Angela each have a mass of 45 kg, and they are moving together with a speed of 10.0 m/s.
- What is their combined kinetic energy?
  - What is the ratio of their combined mass to Katia's mass?
  - What is the ratio of their combined kinetic energy to Katia's kinetic energy? Explain.
59. **Train** In the 1950s, an experimental train, which had a mass of  $2.50 \times 10^4$  kg, was powered across a level track by a jet engine that produced a thrust of  $5.00 \times 10^5$  N for a distance of 509 m.
- Find the work done on the train.
  - Find the change in kinetic energy.
  - Find the final kinetic energy of the train if it started from rest.
  - Find the final speed of the train if there had been no friction.
60. **Car Brakes** A 14,700-N car is traveling at 25 m/s. The brakes are applied suddenly, and the car slides to a stop, as shown in **Figure 11-17**. The average braking force between the tires and the road is 7100 N. How far will the car slide once the brakes are applied?



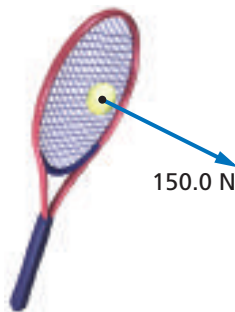
■ **Figure 11-17**

61. A 15.0-kg cart is moving with a velocity of 7.50 m/s down a level hallway. A constant force of 10.0 N acts on the cart, and its velocity becomes 3.20 m/s.
- What is the change in kinetic energy of the cart?
  - How much work was done on the cart?
  - How far did the cart move while the force acted?
62. How much potential energy does DeAnna with a mass of 60.0 kg, gain when she climbs a gymnasium rope a distance of 3.5 m?
63. **Bowling** A 6.4-kg bowling ball is lifted 2.1 m into a storage rack. Calculate the increase in the ball's potential energy.



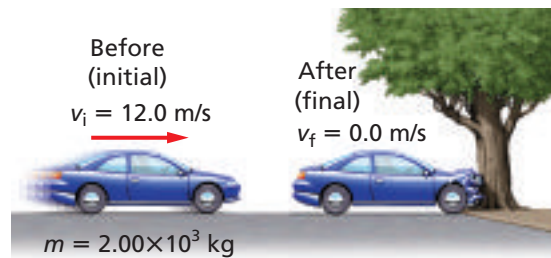
## Chapter 11 Assessment

64. Mary weighs 505 N. She walks down a flight of stairs to a level 5.50 m below her starting point. What is the change in Mary's potential energy?
65. **Weightlifting** A weightlifter raises a 180-kg barbell to a height of 1.95 m. What is the increase in the potential energy of the barbell?
66. A 10.0-kg test rocket is fired vertically from Cape Canaveral. Its fuel gives it a kinetic energy of 1960 J by the time the rocket engine burns all of the fuel. What additional height will the rocket rise?
67. Antwan raised a 12.0-N physics book from a table 75 cm above the floor to a shelf 2.15 m above the floor. What was the change in the potential energy of the system?
68. A hallway display of energy is constructed in which several people pull on a rope that lifts a block 1.00 m. The display indicates that 1.00 J of work is done. What is the mass of the block?
69. **Tennis** It is not uncommon during the serve of a professional tennis player for the racket to exert an average force of 150.0 N on the ball. If the ball has a mass of 0.060 kg and is in contact with the strings of the racket, as shown in **Figure 11-18**, for 0.030 s, what is the kinetic energy of the ball as it leaves the racket? Assume that the ball starts from rest.



■ Figure 11-18

70. Pam, wearing a rocket pack, stands on frictionless ice. She has a mass of 45 kg. The rocket supplies a constant force for 22.0 m, and Pam acquires a speed of 62.0 m/s.
- What is the magnitude of the force?
  - What is Pam's final kinetic energy?
71. **Collision** A  $2.00 \times 10^3$ -kg car has a speed of 12.0 m/s. The car then hits a tree. The tree doesn't move, and the car comes to rest, as shown in **Figure 11-19**.
- Find the change in kinetic energy of the car.
  - Find the amount of work done as the front of the car crashes into the tree.
  - Find the size of the force that pushed in the front of the car by 50.0 cm.

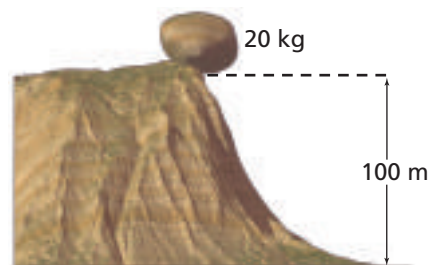


■ Figure 11-19

72. A constant net force of 410 N is applied upward to a stone that weighs 32 N. The upward force is applied through a distance of 2.0 m, and the stone is then released. To what height, from the point of release, will the stone rise?

### 11.2 Conservation of Energy

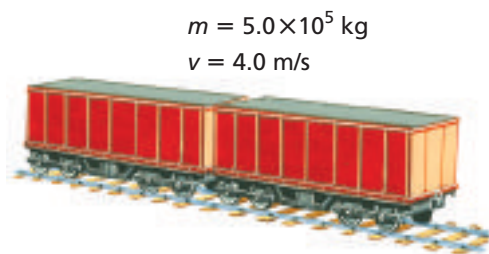
73. A 98.0-N sack of grain is hoisted to a storage room 50.0 m above the ground floor of a grain elevator.
- How much work was done?
  - What is the increase in potential energy of the sack of grain at this height?
  - The rope being used to lift the sack of grain breaks just as the sack reaches the storage room. What kinetic energy does the sack have just before it strikes the ground floor?
74. A 20-kg rock is on the edge of a 100-m cliff, as shown in **Figure 11-20**.
- What potential energy does the rock possess relative to the base of the cliff?
  - The rock falls from the cliff. What is its kinetic energy just before it strikes the ground?
  - What speed does the rock have as it strikes the ground?



■ Figure 11-20

75. **Archery** An archer puts a 0.30-kg arrow to the bowstring. An average force of 201 N is exerted to draw the string back 1.3 m.
- Assuming that all the energy goes into the arrow, with what speed does the arrow leave the bow?
  - If the arrow is shot straight up, how high does it rise?

76. A 2.0-kg rock that is initially at rest loses 407 J of potential energy while falling to the ground. Calculate the kinetic energy that the rock gains while falling. What is the rock's speed just before it strikes the ground?
77. A physics book of unknown mass is dropped 4.50 m. What speed does the book have just before it hits the ground?
78. **Railroad Car** A railroad car with a mass of  $5.0 \times 10^5$  kg collides with a stationary railroad car of equal mass. After the collision, the two cars lock together and move off at 4.0 m/s, as shown in **Figure 11-21**.
- Before the collision, the first railroad car was moving at 8.0 m/s. What was its momentum?
  - What was the total momentum of the two cars after the collision?
  - What were the kinetic energies of the two cars before and after the collision?
  - Account for the loss of kinetic energy.



■ **Figure 11-21**

79. From what height would a compact car have to be dropped to have the same kinetic energy that it has when being driven at  $1.00 \times 10^2$  km/h?
80. Kelli weighs 420 N, and she is sitting on a playground swing that hangs 0.40 m above the ground. Her mom pulls the swing back and releases it when the seat is 1.00 m above the ground.
- How fast is Kelli moving when the swing passes through its lowest position?
  - If Kelli moves through the lowest point at 2.0 m/s, how much work was done on the swing by friction?
81. Hakeem throws a 10.0-g ball straight down from a height of 2.0 m. The ball strikes the floor at a speed of 7.5 m/s. What was the initial speed of the ball?
82. **Slide** Lorena's mass is 28 kg. She climbs the 4.8-m ladder of a slide and reaches a velocity of 3.2 m/s at the bottom of the slide. How much work was done by friction on Lorena?
83. A person weighing 635 N climbs up a ladder to a height of 5.0 m. Use the person and Earth as the system.
- Draw energy bar graphs of the system before the person starts to climb the ladder and after the person stops at the top. Has the mechanical energy changed? If so, by how much?
  - Where did this energy come from?

## Mixed Review

84. Suppose a chimpanzee swings through the jungle on vines. If it swings from a tree on a 13-m-long vine that starts at an angle of  $45^\circ$ , what is the chimp's velocity when it reaches the ground?
85. An 0.80-kg cart rolls down a frictionless hill of height 0.32 m. At the bottom of the hill, the cart rolls on a flat surface, which exerts a frictional force of 2.0 N on the cart. How far does the cart roll on the flat surface before it comes to a stop?
86. **High Jump** The world record for the men's high jump is about 2.45 m. To reach that height, what is the minimum amount of work that a 73.0-kg jumper must exert in pushing off the ground?
87. A stuntwoman finds that she can safely break her fall from a one-story building by landing in a box filled to a 1-m depth with foam peanuts. In her next movie, the script calls for her to jump from a five-story building. How deep a box of foam peanuts should she prepare?
88. **Football** A 110-kg football linebacker has a head-on collision with a 150-kg defensive end. After they collide, they come to a complete stop. Before the collision, which player had the greater momentum and which player had the greater kinetic energy?
89. A 2.0-kg lab cart and a 1.0-kg lab cart are held together by a compressed spring. The lab carts move at 2.1 m/s in one direction. The spring suddenly becomes uncompressed and pushes the two lab carts apart. The 2-kg lab cart comes to a stop, and the 1.0-kg lab cart moves ahead. How much energy did the spring add to the lab carts?
90. A 55.0-kg scientist roping through the top of a tree in the jungle sees a lion about to attack a tiny antelope. She quickly swings down from her 12.0-m-high perch and grabs the antelope (21.0 kg) as she swings. They barely swing back up to a tree limb out of reach of the lion. How high is this tree limb?





## Chapter 11 Assessment

91. An 0.80-kg cart rolls down a  $30.0^\circ$  hill from a vertical height of 0.50 m as shown in **Figure 11-22**. The distance that the cart must roll to the bottom of the hill is  $0.50 \text{ m}/\sin 30.0^\circ = 1.0 \text{ m}$ . The surface of the hill exerts a frictional force of 5.0 N on the cart. Does the cart roll to the bottom of the hill?

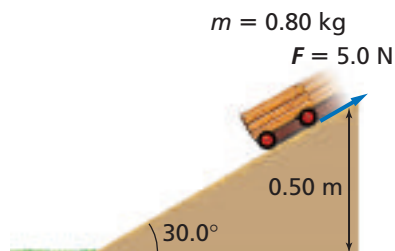


Figure 11-22

92. Object A, sliding on a frictionless surface at 3.2 m/s, hits a 2.0-kg object, B, which is motionless. The collision of A and B is completely elastic. After the collision, A and B move away from each other at equal and opposite speeds. What is the mass of object A?
93. **Hockey** A 90.0-kg hockey player moving at 5.0 m/s collides head-on with a 110-kg hockey player moving at 3.0 m/s in the opposite direction. After the collision, they move off together at 1.0 m/s. How much energy was lost in the collision?

### Thinking Critically

94. **Apply Concepts** A golf ball with a mass of 0.046 kg rests on a tee. It is struck by a golf club with an effective mass of 0.220 kg and a speed of 44 m/s. Assuming that the collision is elastic, find the speed of the ball when it leaves the tee.
95. **Apply Concepts** A fly hitting the windshield of a moving pickup truck is an example of a collision in which the mass of one of the objects is many times larger than the other. On the other hand, the collision of two billiard balls is one in which the masses of both objects are the same. How is energy transferred in these collisions? Consider an elastic collision in which billiard ball  $m_1$  has velocity  $v_1$  and ball  $m_2$  is motionless.
- If  $m_1 = m_2$ , what fraction of the initial energy is transferred to  $m_2$ ?
  - If  $m_1 \gg m_2$ , what fraction of the initial energy is transferred to  $m_2$ ?
  - In a nuclear reactor, neutrons must be slowed down by causing them to collide with atoms. (A neutron is about as massive as a proton.) Would hydrogen, carbon, or iron atoms be more desirable to use for this purpose?

96. **Analyze and Conclude** In a perfectly elastic collision, both momentum and mechanical energy are conserved. Two balls, with masses  $m_A$  and  $m_B$ , are moving toward each other with speeds  $v_A$  and  $v_B$ , respectively. Solve the appropriate equations to find the speeds of the two balls after the collision.
97. **Analyze and Conclude** A 25-g ball is fired with an initial speed of  $v_1$  toward a 125-g ball that is hanging motionless from a 1.25-m string. The balls have a perfectly elastic collision. As a result, the 125-g ball swings out until the string makes an angle of  $37.0^\circ$  with the vertical. What is  $v_1$ ?

### Writing in Physics

98. All energy comes from the Sun. In what forms has this solar energy come to us to allow us to live and to operate our society? Research the ways that the Sun's energy is turned into a form that we can use. After we use the Sun's energy, where does it go? Explain.
99. All forms of energy can be classified as either kinetic or potential energy. How would you describe nuclear, electric, chemical, biological, solar, and light energy, and why? For each of these types of energy, research what objects are moving and how energy is stored in those objects.

### Cumulative Review

100. A satellite is placed in a circular orbit with a radius of  $1.0 \times 10^7 \text{ m}$  and a period of  $9.9 \times 10^3 \text{ s}$ . Calculate the mass of Earth. *Hint: Gravity is the net force on such a satellite. Scientists have actually measured the mass of Earth this way.* (Chapter 7)
101. A 5.00-g bullet is fired with a velocity of 100.0 m/s toward a 10.00-kg stationary solid block resting on a frictionless surface. (Chapter 9)
- What is the change in momentum of the bullet if it is embedded in the block?
  - What is the change in momentum of the bullet if it ricochets in the opposite direction with a speed of 99 m/s?
  - In which case does the block end up with a greater speed?
102. An automobile jack must exert a lifting force of at least 15 kN. (Chapter 10)
- If you want to limit the effort force to 0.10 kN, what mechanical advantage is needed?
  - If the jack is 75% efficient, over what distance must the effort force be exerted in order to raise the auto 33 cm?



# Standardized Test Practice

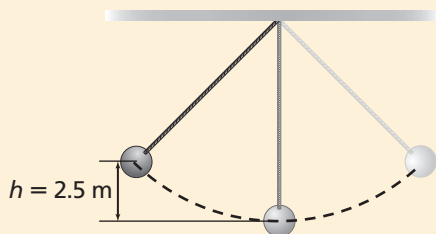
## Multiple Choice

1. A bicyclist increases her speed from 4.0 m/s to 6.0 m/s. The combined mass of the bicyclist and the bicycle is 55 kg. How much work did the bicyclist do in increasing her speed?

(A) 11 J                      (C) 55 J  
(B) 28 J                      (D) 550 J

2. The illustration below shows a ball swinging freely in a plane. The mass of the ball is 4.0 kg. Ignoring friction, what is the maximum kinetic energy of the ball as it swings back and forth?

(A) 0.14 m/s                (C) 7.0 m/s  
(B) 21 m/s                 (D) 49 m/s



3. You lift a 4.5-kg box from the floor and place it on a shelf that is 1.5 m above the ground. How much energy did you use in lifting the box?

(A) 9.0 J                      (C) 11 J  
(B) 49 J                      (D) 66 J

4. You drop a  $6.0 \times 10^{-2}$ -kg ball from a height of 1.0 m above a hard, flat surface. The ball strikes the surface and loses 0.14 J of its energy. It then bounces back upward. How much kinetic energy does the ball have just after it bounces off the flat surface?

(A) 0.20 J                      (C) 0.45 J  
(B) 0.59 J                      (D) 0.73 J

5. You move a 2.5-kg book from a shelf that is 1.2 m above the ground to a shelf that is 2.6 m above the ground. What is the change in the book's potential energy?

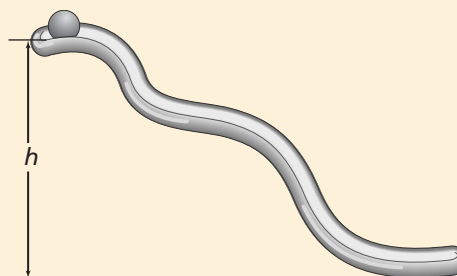
(A) 1.4 J                      (C) 3.5 J  
(B) 25 J                      (D) 34 J

6. A ball of mass  $m$  rolls along a flat surface with a speed of  $v_1$ . It strikes a padded wall and bounces back in the opposite direction. The energy of the ball after striking the wall is half its initial energy. Ignoring friction, which of the following expressions gives the ball's new speed as a function of its initial speed?

(A)  $\frac{1}{2}v_1$                       (C)  $\sqrt{2}(v_1)$   
(B)  $\frac{\sqrt{2}}{2}(v_1)$                 (D)  $2v_1$

7. The illustration below shows a ball on a curved track. The ball starts with zero velocity at the top of the track. It then rolls from the top of the track to the horizontal part at the ground. Ignoring friction, its velocity just at the moment it reaches the ground is 14 m/s. What is the height,  $h$ , from the ground to the top of the track?

(A) 7 m                        (C) 10 m  
(B) 14 m                      (D) 20 m



## Extended Answer

8. A box sits on a platform supported by a compressed spring. The box has a mass of 1.0 kg. When the spring is released, it gives 4.9 J of energy to the box, and the box flies upward. What will be the maximum height above the platform reached by the box before it begins to fall?

### ✓ Test-Taking TIP

#### Use the Process of Elimination

On any multiple-choice test, there are two ways to find the correct answer to each question. Either you can choose the right answer immediately or you can eliminate the answers that you know are wrong.





# Thermal Energy

## What You'll Learn

- You will learn how temperature relates to the potential and kinetic energies of atoms and molecules.
- You will distinguish heat from work.
- You will calculate heat transfer and the absorption of thermal energy.

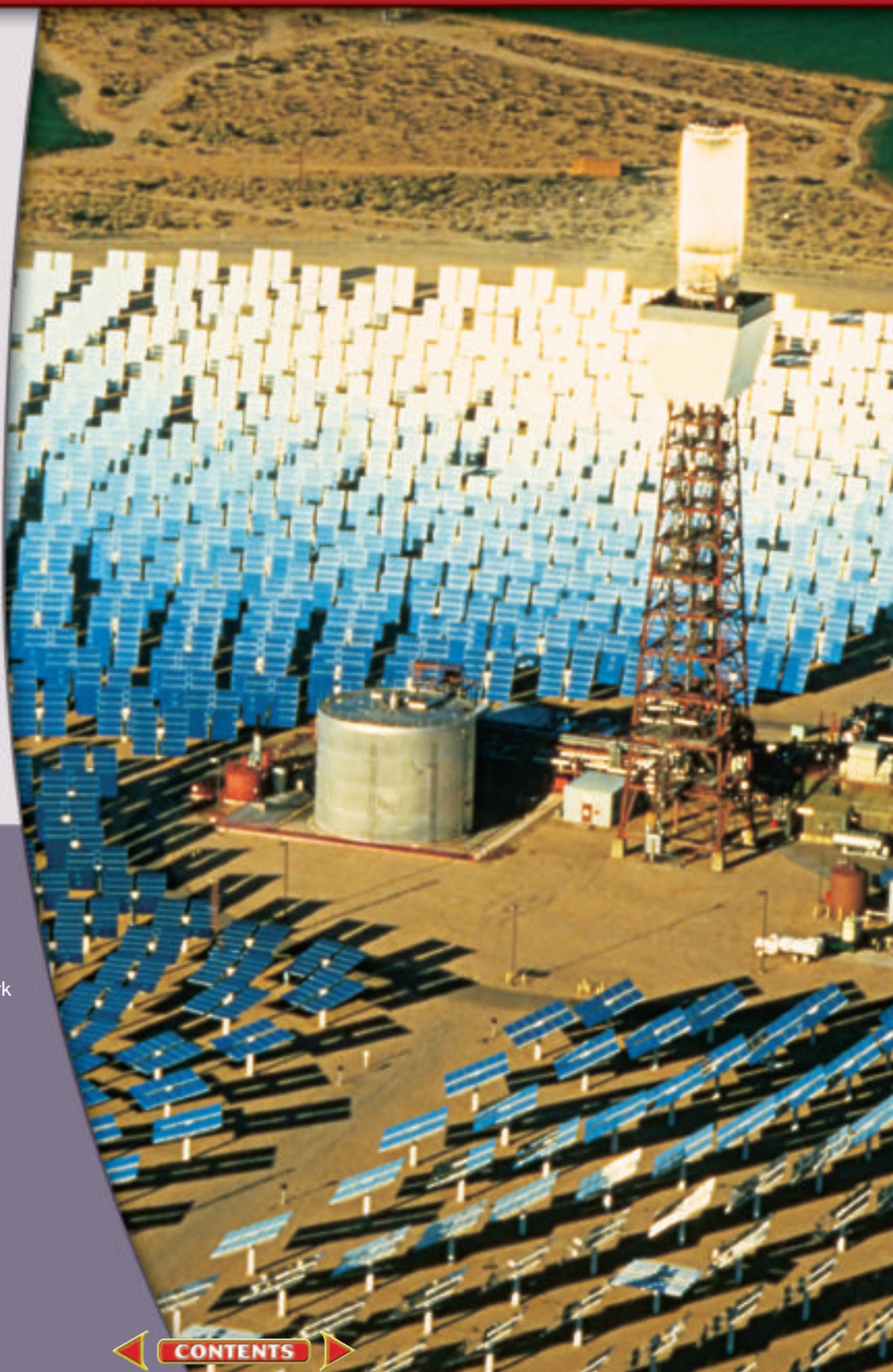
## Why It's Important

Thermal energy is vital for living creatures, chemical reactions, and the working of engines.

**Solar Energy** A strategy used to produce electric power from sunlight concentrates the light with many mirrors onto one collector that becomes very hot. The energy collected at a high temperature is then used to drive an engine, which turns an electric generator.

## Think About This ►

What forms of energy does light from the Sun take in the process of converting solar energy into useful work through an engine?





## What happens when you provide thermal energy by holding a glass of water?

### Question

What happens to the temperature of water when you hold a glass of water in your hand?

### Procedure



1. You will need to use a 250-mL beaker and 150 mL of water.
2. Fill the beaker with the 150 mL of water.
3. Record the initial temperature of the water by holding a thermometer in the water in the beaker. Note that the bulb end of the thermometer must not touch the bottom or sides of the beaker, nor should it touch a table or your hands.
4. Remove the thermometer and hold the beaker of water for 2 min by cupping it with both hands, as shown in the figure.
5. Have your lab partner record the final temperature of the water by placing the thermometer in the beaker. Be sure that the bulb end of the thermometer is not touching the bottom or sides of the beaker.

### Analysis

Calculate the change in temperature of the water. If you had more water in the beaker, would it affect the change in temperature?

**Critical Thinking** Explain what caused the water temperature to change.



## 12.1 Temperature and Thermal Energy

The study of heat transformations into other forms of energy, called thermodynamics, began with the eighteenth-century engineers who built the first steam engines. These steam engines were used to power trains, factories, and water pumps for coal mines, and thus they contributed greatly to the Industrial Revolution in Europe and in the United States. In learning to design more efficient engines, the engineers developed new concepts about how heat is related to useful work. Although the study of thermodynamics began in the eighteenth century, it was not until around 1900 that the concepts of thermodynamics were linked to the motions of atoms and molecules in solids, liquids, and gases.

Today, the concepts of thermodynamics are widely used in various applications that involve heat and temperature. Engineers use the laws of thermodynamics to continually develop higher performance refrigerators, automobile engines, aircraft engines, and numerous other machines.

### ▶ Objectives

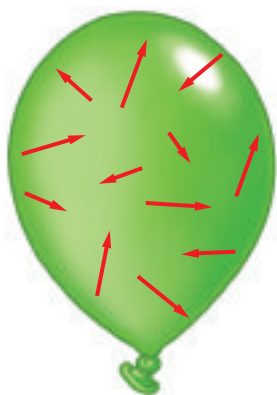
- **Describe** thermal energy and compare it to potential and kinetic energies.
- **Distinguish** temperature from thermal energy.
- **Define** specific heat and **calculate** heat transfer.

### ▶ Vocabulary

conduction  
thermal equilibrium  
heat  
convection  
radiation  
specific heat







Helium balloon

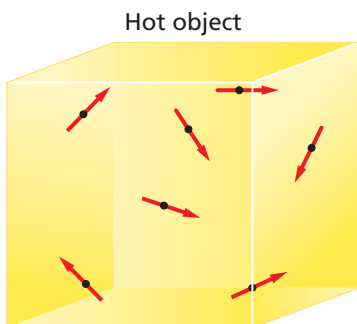
■ **Figure 12-1** Helium atoms in a balloon collide with the rubber wall and cause the balloon to expand.

## Thermal Energy

You already have studied how objects collide and trade kinetic energies. For example, the many molecules present in a gas have linear and rotational kinetic energies. The molecules also may have potential energy in their vibrations and bending. The gas molecules collide with each other and with the walls of their container, transferring energy among each other in the process. There are numerous molecules moving freely in a gas, resulting in many collisions. Therefore, it is convenient to discuss the total energy of the molecules and the average energy per molecule. The total energy of the molecules is called thermal energy, and the average energy per molecule is related to the temperature of the gas.

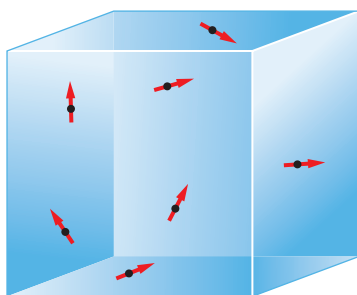
**Hot objects** What makes an object hot? When you fill up a balloon with helium, the rubber in the balloon is stretched by the repeated pounding from helium atoms. Each of the billions of helium atoms in the balloon collides with the rubber wall, bounces back, and hits the other side of the balloon, as shown in **Figure 12-1**. If you put a balloon in sunlight, you might notice that the balloon gets slightly larger. The energy from the Sun makes each of the gas atoms move faster and bounce off the rubber walls of the balloon more often. Each atomic collision with the balloon wall puts a greater force on the balloon and stretches the rubber. Thus, the balloon expands.

On the other hand, if you refrigerate a balloon, you will find that it shrinks slightly. Lowering the temperature slows the movement of the helium atoms. Hence, their collisions do not transfer enough momentum to stretch the balloon quite as much. Even though the balloon contains the same number of atoms, the balloon shrinks.



Hot object

$$KE_{\text{hot}} > KE_{\text{cold}}$$



Cold object

■ **Figure 12-2** Particles in a hot object have greater kinetic and potential energies than particles in a cold object do.

**Solids** The atoms in solids also have kinetic energy, but they are unable to move freely as gas atoms do. One way to illustrate the molecular structure of a solid is to picture a number of atoms that are connected to each other by springs. Because of the springs, the atoms bounce back and forth, with some bouncing more than others. Each atom has some kinetic energy and some potential energy from the springs that are attached to it. If a solid has  $N$  number of atoms, then the total thermal energy in the solid is equal to the average kinetic and potential energy per atom times  $N$ .

## Thermal Energy and Temperature

According to the previous discussion of gases and solids, a hot object has more thermal energy than a similar cold object, as shown in **Figure 12-2**. This means that, as a whole, the particles in a hot object have greater thermal energy than the particles in a cold object. This does not mean that all the particles in an object have exactly the same amount of energy; they have a wide range of energies. However, the average energy of the particles in a hot object is higher than the average energy of the particles in a cold object. To understand this, consider the heights of students in a twelfth-grade class. Although the students' heights vary, you can calculate the average height of the students in the class. This average is likely to be greater than the average height of students in a ninth-grade class, even though some ninth-grade students may be taller than some twelfth-grade students.



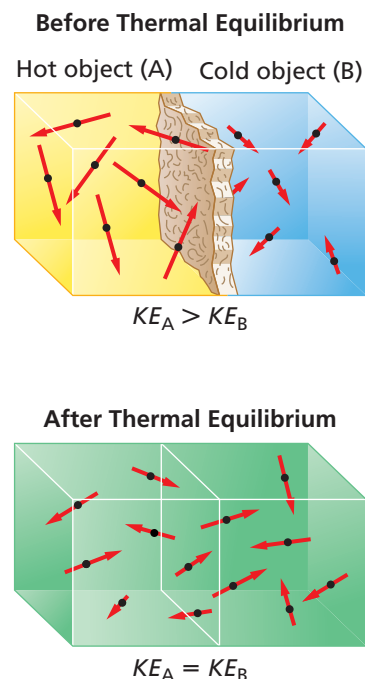
**Temperature** Temperature depends only on the average kinetic energy of the particles in the object. Because temperature depends on average kinetic energy, it does not depend on the number of atoms in an object. To understand this, consider two blocks of steel. The first block has a mass of 1 kg, and the second block has a mass of 2 kg. If the 1-kg block is at the same temperature as the 2-kg block, the average kinetic energy of the particles in each block is the same. However, the 2-kg block has twice the mass of the 1-kg block. Hence, the 2-kg block has twice the amount of particles as the 1-kg block. Thus, the total amount of kinetic energy of the particles in the 2-kg block is twice that of the 1-kg mass. Total kinetic energy is divided by the total number of particles in an object to calculate its average kinetic energy. Therefore, the thermal energy in an object is proportional to the number of particles in it. Temperature, however, is not dependent on the number of particles in an object.

## Equilibrium and Thermometry

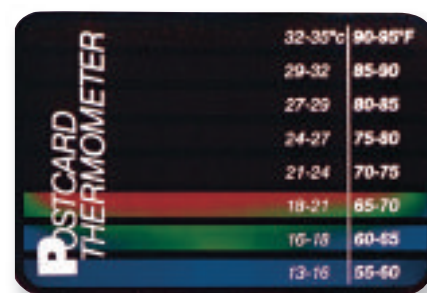
How do you measure your body temperature? For example, if you suspect that you have a fever, you might place a thermometer in your mouth and wait for a few minutes before checking the thermometer for your temperature reading. The microscopic process involved in measuring temperature involves collisions and energy transfers between the thermometer and your body. Your body is hot compared to the thermometer, which means that the particles in your body have greater thermal energy and are moving faster than the particles in the thermometer. When the cold glass tube of the thermometer touches your skin, which is warmer than the glass, the faster-moving particles in your skin collide with the slower-moving particles in the glass. Energy is then transferred from your skin to the glass particles by the process of **conduction**, which is the transfer of kinetic energy when particles collide. The thermal energy of the particles that make up the thermometer increases, while at the same time, the thermal energy of the particles in your skin decreases.

**Thermal equilibrium** As the particles in the glass gain more energy, they begin to give some of their energy back to the particles in your body. At some point, the rate of transfer of energy between the glass and your body becomes equal, and your body and the thermometer are then at the same temperature. At this point, your body and the thermometer are said to have reached **thermal equilibrium**, the state in which the rate of energy flow between two objects is equal and the objects are at the same temperature, as shown in **Figure 12-3**.

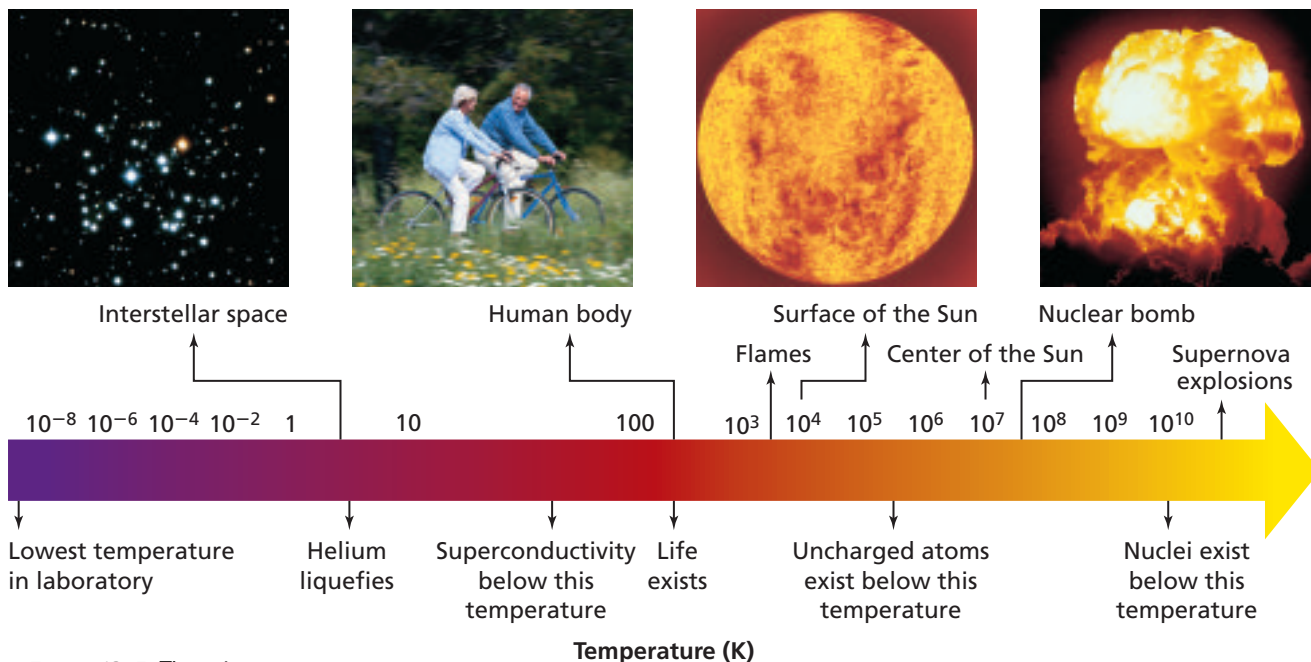
The operation of a thermometer depends on some property, such as volume, which changes with temperature. Many household thermometers contain colored alcohol that expands when heated and rises in a narrow tube. The hotter the thermometer, the more the alcohol expands and the higher it rises in the tube. In liquid-crystal thermometers, such as the one shown in **Figure 12-4**, a set of different kinds of liquid crystals is used. Each crystal's molecules rearrange at a specific temperature, which causes the color of the crystal to change and indicates the temperature by color. Medical thermometers and the thermometers that monitor automobile engines use very small, temperature-sensitive electronic circuits to take rapid measurements.



■ **Figure 12-3** Thermal energy is transferred from a hot object to a cold object. When thermal equilibrium is reached, the transfer of energy between objects is equal.



■ **Figure 12-4** Thermometers use a change in physical properties to measure temperature. A liquid-crystal thermometer changes color with a temperature change.



■ **Figure 12-5** There is an extremely wide range of temperatures throughout the universe. Note that the scale has been expanded in areas of particular interest.

## Temperature Scales: Celsius and Kelvin

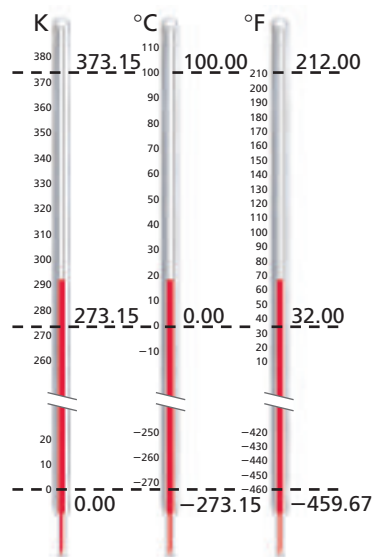
Over the years, scientists developed temperature scales so that they could compare their measurements with those of other scientists. A scale based on the properties of water was devised in 1741 by Swedish astronomer and physicist Anders Celsius. On this scale, now called the Celsius scale, the freezing point of pure water is defined to be  $0^{\circ}\text{C}$ . The boiling point of pure water at sea level is defined to be  $100^{\circ}\text{C}$ .

**Temperature limits** The wide range of temperatures present in the universe is shown in **Figure 12-5**. Temperatures do not appear to have an upper limit. The interior of the Sun is at least  $1.5 \times 10^7^{\circ}\text{C}$ . Temperatures do, however, have a lower limit. Generally, materials contract as they cool. If an ideal gas, such as the helium in a balloon is cooled, it contracts in such a way that it occupies a volume that is only the size of the helium atoms at  $-273.15^{\circ}\text{C}$ . At this temperature, all the thermal energy that can be removed has been removed from the gas. It is impossible to reduce the temperature any further. Therefore, there can be no temperature lower than  $-273.15^{\circ}\text{C}$ , which is called absolute zero.

The Celsius scale is useful for day-to-day measurements of temperature. It is not conducive for working on science and engineering problems, however, because it has negative temperatures. Negative temperatures suggest a molecule could have negative kinetic energy, which is not possible because kinetic energy is always positive. The solution to this issue is to use a temperature scale based on absolute zero.

The zero point of the Kelvin scale is defined to be absolute zero. On the Kelvin scale, the freezing point of water ( $0^{\circ}\text{C}$ ) is about 273 K and the boiling point of water is about 373 K. Each interval on this scale, called a kelvin, is equal to  $1^{\circ}\text{C}$ . Thus,  $T_{\text{C}} + 273 = T_{\text{K}}$ . **Figure 12-6** shows representative temperatures on the three most-common scales: Fahrenheit, Celsius, and Kelvin.

Very cold temperatures are reached by liquefying gases. Helium liquefies at 4.2 K, or  $-269^{\circ}\text{C}$ . Even colder temperatures can be reached by making use of special properties of solids, helium isotopes, and atoms and lasers.



■ **Figure 12-6** The three most-common temperature scales are Kelvin, Celsius, and Fahrenheit.

- Convert the following Kelvin temperatures to Celsius temperatures.
  - 115 K
  - 172 K
  - 125 K
  - 402 K
  - 425 K
  - 212 K
- Find the Celsius and Kelvin temperatures for the following.
  - room temperature
  - a typical refrigerator
  - a hot summer day in North Carolina
  - a winter night in Minnesota

## Heat and the Flow of Thermal Energy

When two objects come in contact with each other, they transfer energy. This energy that is transferred between the objects is called **heat**. Heat is described as the energy that always flows from the hotter object to the cooler object. Left to itself heat never flows from a colder object to a hotter object. The symbol  $Q$  is used to represent an amount of heat, which, like other forms of energy, is measured in joules. If  $Q$  has a negative value, heat has left the object; if  $Q$  has a positive value, heat has been absorbed by the object.

**Conduction** If you place one end of a metal rod in a flame, the hot gas particles in the flame conduct heat to the rod. The other end of the rod also becomes warm within a short period of time. Heat is conducted because the particles in the rod are in direct contact with each other.

**Convection** Thermal energy transfer can occur even if the particles in an object are not in direct contact with each other. Have you ever looked into a pot of water just about to boil? The water at the bottom of the pot is heated by conduction and rises to the top, while the colder water at the top sinks to the bottom. Heat flows between the rising hot water and the descending cold water. This motion of fluid in a liquid or gas caused by temperature differences is called **convection**. Atmospheric turbulence is caused by convection of gases in the atmosphere. Thunderstorms are excellent examples of large-scale atmospheric convection. Ocean currents that cause changes in weather patterns also result from convection.

**Radiation** The third method of thermal transfer, unlike the first two, does not depend on the presence of matter. The Sun warms Earth from over 150 million km away via **radiation**, which is the transfer of energy by electromagnetic waves. These waves carry the energy from the hot Sun through the vacuum of space to the much cooler Earth.

## Specific Heat

Some objects are easier to heat than others. On a bright summer day, the Sun warms the sand on a beach and the ocean water. However, the sand on the beach gets quite hot, while the ocean water stays relatively cool. When heat flows into an object, its thermal energy and temperature increase. The amount of the increase in temperature depends on the size of the object and on the material from which the object is made.

## APPLYING PHYSICS

▶ **Steam Heating** In a steam heating system of a building, water is turned into steam in a boiler located in a maintenance area or the basement. The steam then flows through insulated pipes to each room in the building. In the radiator, the steam is condensed as liquid water and then flows back through pipes to the boiler to be revaporized. The hot steam physically carries the heat from the boiler, and that energy is released when the steam condenses in the radiator. Some disadvantages of steam heating are that it requires expensive boilers and pipes must contain steam under pressure. ◀

## Meteorology Connection



**Table 12-1**

**Specific Heat of Common Substances**

Material	Specific Heat (J/kg·K)	Material	Specific Heat (J/kg·K)
Aluminum	897	Lead	130
Brass	376	Methanol	2450
Carbon	710	Silver	235
Copper	385	Steam	2020
Glass	840	Water	4180
Ice	2060	Zinc	388
Iron	450		

The **specific heat** of a material is the amount of energy that must be added to the material to raise the temperature of a unit mass by one temperature unit. In SI units, specific heat, represented by  $C$ , is measured in  $\text{J}/\text{kg}\cdot\text{K}$ . **Table 12-1** provides values of specific heat for some common substances. For example, 897 J must be added to 1 kg of aluminum to raise its temperature by 1 K. The specific heat of aluminum is therefore 897  $\text{J}/\text{kg}\cdot\text{K}$ .

The heat gained or lost by an object as its temperature changes depends on the mass, the change in temperature, and the specific heat of the substance. By using the following equation, you can calculate the amount of heat,  $Q$ , that must be transferred to change the temperature of an object.

**Heat Transfer**  $Q = mC\Delta T = mC(T_f - T_i)$

Heat transfer is equal to the mass of an object times the specific heat of the object times the difference between the final and initial temperatures.

Liquid water has a high specific heat compared to the other substance in Table 12-1. When the temperature of 10.0 kg of water is increased by 5.0 K, the heat absorbed is  $Q = (10.0 \text{ kg})(4180 \text{ J}/\text{kg}\cdot\text{K})(5.0 \text{ K}) = 2.1 \times 10^5 \text{ J}$ . Remember that the temperature interval for kelvins is the same as that for Celsius degrees. For this reason, you can calculate  $\Delta T$  in kelvins or in degrees Celsius.

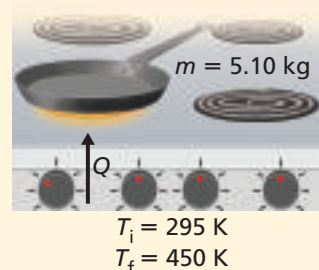
**EXAMPLE Problem 1**

**Heat Transfer** A 5.10-kg cast-iron skillet is heated on the stove from 295 K to 450 K. How much heat had to be transferred to the iron?

**1 Analyze and Sketch the Problem**

- Sketch the flow of heat into the skillet from the stove top.

**Known:**  $m = 5.10 \text{ kg}$      $C = 450 \text{ J}/\text{kg}\cdot\text{K}$     **Unknown:**  $Q = ?$   
 $T_i = 295 \text{ K}$      $T_f = 450 \text{ K}$



**2 Solve for the Unknown**

$$Q = mC(T_f - T_i)$$

$$= (5.10 \text{ kg})(450 \text{ J}/\text{kg}\cdot\text{K})(450 \text{ K} - 295 \text{ K}) \quad \text{Substitute } m = 5.10 \text{ kg}, C = 450 \text{ J}/\text{kg}\cdot\text{K}, T_f = 450 \text{ K}, T_i = 295 \text{ K}$$

$$= 3.6 \times 10^5 \text{ J}$$

**3 Evaluate the Answer**

- **Are the units correct?** Heat is measured in J.
- **Does the sign make sense?** Temperature increased, so  $Q$  is positive.

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3. When you turn on the hot water to wash dishes, the water pipes have to heat up. How much heat is absorbed by a copper water pipe with a mass of 2.3 kg when its temperature is raised from 20.0°C to 80.0°C?
4. The cooling system of a car engine contains 20.0 L of water (1 L of water has a mass of 1 kg).
  - a. What is the change in the temperature of the water if the engine operates until 836.0 kJ of heat is added?
  - b. Suppose that it is winter, and the car's cooling system is filled with methanol. The density of methanol is 0.80 g/cm<sup>3</sup>. What would be the increase in temperature of the methanol if it absorbed 836.0 kJ of heat?
  - c. Which is the better coolant, water or methanol? Explain.
5. Electric power companies sell electricity by the kWh, where 1 kWh = 3.6 × 10<sup>6</sup> J. Suppose that it costs \$0.15 per kWh to run an electric water heater in your neighborhood. How much does it cost to heat 75 kg of water from 15°C to 43°C to fill a bathtub?

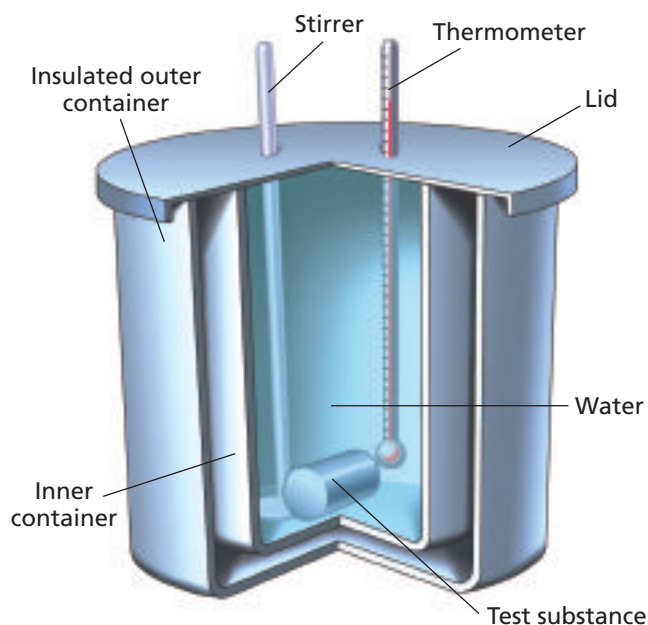
## Calorimetry: Measuring Specific Heat

A simple calorimeter, such as the one shown in **Figure 12-7**, is a device used to measure changes in thermal energy. A calorimeter is carefully insulated so that heat transfer to the external world is kept to a minimum. A measured mass of a substance that has been heated to a high temperature is placed in the calorimeter. The calorimeter also contains a known mass of cold water at a measured temperature. The heat released by the substance is transferred to the cooler water. The change in thermal energy of the substance is calculated using the resulting increase in the water temperature. More sophisticated types of calorimeters are used to measure chemical reactions and the energy content of various foods.

The operation of a calorimeter depends on the conservation of energy in an isolated, closed system. Energy can neither enter nor leave this system. As a result, if the energy of one part of the system increases, the energy of another part of the system must decrease by the same amount. Consider a system composed of two blocks of metal, block A and block B, shown in **Figure 12-8a** on the next page. The total energy of the system is constant, as represented by the following equation.

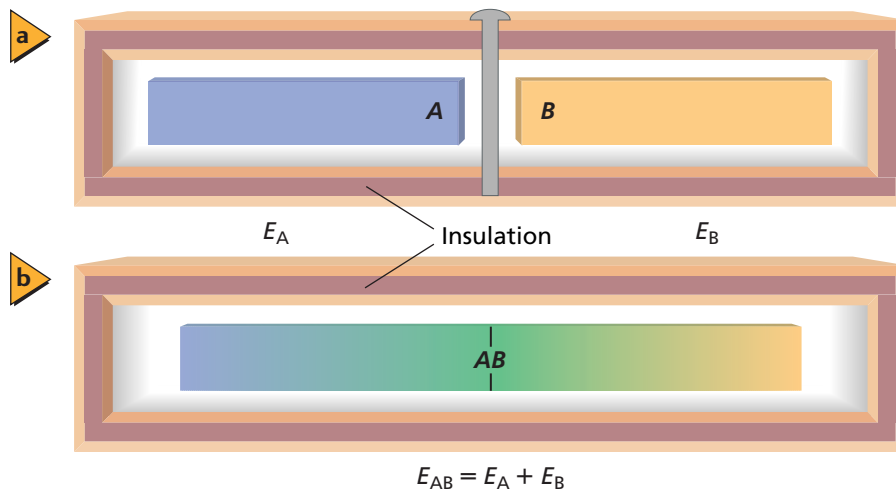
**Conservation of Energy**  $E_A + E_B = \text{constant}$

In an isolated, closed system, the thermal energy of object A plus the thermal energy of object B is constant.



■ **Figure 12-7** A calorimeter provides an isolated, closed system in which to measure energy transfer.

■ **Figure 12-8** A system is composed of two model blocks at different temperatures that initially are separated **(a)**. When the blocks are brought together, heat flows from the hotter block to the colder block **(b)**. Total energy remains constant.



Suppose that the two blocks initially are separated but can be placed in contact with each other. If the thermal energy of block A changes by an amount  $\Delta E_A$ , then the change in thermal energy of block B,  $\Delta E_B$ , must be related by the equation,  $\Delta E_A + \Delta E_B = 0$ . Thus,  $\Delta E_A = -\Delta E_B$ . The change in energy of one block is positive, while the change in energy of the other block is negative. For the block whose thermal energy change is positive, the temperature of the block rises. For the block whose thermal energy change is negative, the temperature falls.

Assume that the initial temperatures of the two blocks are different. When the blocks are brought together, heat flows from the hotter block to the colder block, as shown in **Figure 12-8b**. The heat flow continues until the blocks are in thermal equilibrium, which is when the blocks have the same temperature.

In an isolated, closed system, the change in thermal energy is equal to the heat transferred because no work is done. Therefore, the change in energy for each block can be expressed by the following equation:

$$\Delta E = Q = mC\Delta T$$

The increase in thermal energy of block A is equal to the decrease in thermal energy of block B. Thus, the following relationship is true:

$$m_A C_A \Delta T_A + m_B C_B \Delta T_B = 0$$

The change in temperature is the difference between the final and initial temperatures; that is,  $\Delta T = T_f - T_i$ . If the temperature of a block increases,  $T_f > T_i$ , and  $\Delta T$  is positive. If the temperature of the block decreases,  $T_f < T_i$ , and  $\Delta T$  is negative. The final temperatures of the two blocks are equal. The following is the equation for the transfer of energy:

$$m_A C_A (T_f - T_A) + m_B C_B (T_f - T_B) = 0$$

To solve for  $T_f$  expand the equation.

$$m_A C_A T_f - m_A C_A T_A + m_B C_B T_f - m_B C_B T_B = 0$$

$$T_f (m_A C_A + m_B C_B) = m_A C_A T_A + m_B C_B T_B$$

$$T_f = \frac{m_A C_A T_A + m_B C_B T_B}{m_A C_A + m_B C_B}$$



## ▶ EXAMPLE Problem 2

**Transferring Heat in a Calorimeter** A calorimeter contains 0.50 kg of water at 15°C. A 0.040-kg block of zinc at 115°C is placed in the water. What is the final temperature of the system?

### 1 Analyze and Sketch the Problem

- Let zinc be sample A and water be sample B.
- Sketch the transfer of heat from the hotter zinc to the cooler water.

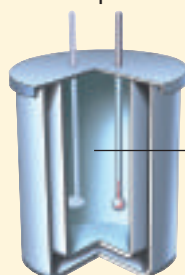
**Known:**

$$\begin{aligned}m_A &= 0.040 \text{ kg} \\C_A &= 388 \text{ J/kg}\cdot^\circ\text{C} \\T_A &= 115^\circ\text{C} \\m_B &= 0.50 \text{ kg} \\C_B &= 4180 \text{ J/kg}\cdot^\circ\text{C} \\T_B &= 15.0^\circ\text{C}\end{aligned}$$

**Unknown:**

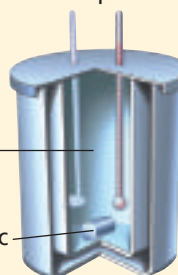
$$T_f = ?$$

Before block of zinc is placed



$$\begin{aligned}m_B &= 0.50 \text{ kg} \\T_B &= 15^\circ\text{C}\end{aligned}$$

After block of zinc is placed



$$\begin{aligned}m_A &= 0.040 \text{ kg} \\T_A &= 115^\circ\text{C} \\T_f &= ?\end{aligned}$$

Water

Zinc

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### 2 Solve for the Unknown

Determine the final temperature using the following equation.

$$\begin{aligned}T_f &= \frac{m_A C_A T_A + m_B C_B T_B}{m_A C_A + m_B C_B} \\&= \frac{(0.040 \text{ kg})(388 \text{ J/kg}\cdot^\circ\text{C})(115^\circ\text{C}) + (0.50 \text{ kg})(4180 \text{ J/kg}\cdot^\circ\text{C})(15.0^\circ\text{C})}{(0.040 \text{ kg})(388 \text{ J/kg}\cdot^\circ\text{C}) + (0.50 \text{ kg})(4180 \text{ J/kg}\cdot^\circ\text{C})} \\&= 16^\circ\text{C}\end{aligned}$$

Substitute  $m_A = 0.040 \text{ kg}$ ,  
 $C_A = 388 \text{ J/kg}\cdot^\circ\text{C}$ ,  $T_A = 115^\circ\text{C}$ ,  
 $m_B = 0.50 \text{ kg}$ ,  $C_B = 4180 \text{ J/kg}\cdot^\circ\text{C}$ ,  
 $T_B = 15^\circ\text{C}$

### 3 Evaluate the Answer

- **Are the units correct?** Temperature is measured in Celsius.
- **Is the magnitude realistic?** The answer is between the initial temperatures of the two samples, as is expected when using a calorimeter.

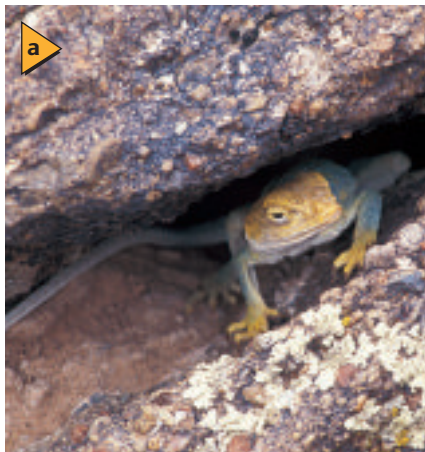
## ▶ PRACTICE Problems

### Additional Problems, Appendix B

6. A  $2.00 \times 10^2$ -g sample of water at  $80.0^\circ\text{C}$  is mixed with  $2.00 \times 10^2$  g of water at  $10.0^\circ\text{C}$ . Assume that there is no heat loss to the surroundings. What is the final temperature of the mixture?
7. A  $4.00 \times 10^2$ -g sample of methanol at  $16.0^\circ\text{C}$  is mixed with  $4.00 \times 10^2$  g of water at  $85.0^\circ\text{C}$ . Assume that there is no heat loss to the surroundings. What is the final temperature of the mixture?
8. Three lead fishing weights, each with a mass of  $1.00 \times 10^2$  g and at a temperature of  $100.0^\circ\text{C}$ , are placed in  $1.00 \times 10^2$  g of water at  $35.0^\circ\text{C}$ . The final temperature of the mixture is  $45.0^\circ\text{C}$ . What is the specific heat of the lead in the weights?
9. A  $1.00 \times 10^2$ -g aluminum block at  $100.0^\circ\text{C}$  is placed in  $1.00 \times 10^2$  g of water at  $10.0^\circ\text{C}$ . The final temperature of the mixture is  $25.0^\circ\text{C}$ . What is the specific heat of the aluminum?



■ **Figure 12-9** A lizard regulates its body temperature by hiding under a rock when the atmosphere is hot **(a)** and sunbathing when the atmosphere gets cold **(b)**.



### Biology Connection

Animals can be divided into two groups based on their body temperatures. Most are cold-blooded animals whose body temperatures depend on the environment. The others are warm-blooded animals whose body temperatures are controlled internally. That is, a warm-blooded animal's body temperature remains stable regardless of the temperature of the environment. In contrast, when the temperature of the environment is high, the body temperature of a cold-blooded animal also becomes high. A cold-blooded animal, such as the lizard shown in **Figure 12-9**, regulates this heat flow by hiding under a rock or crevice, thereby reducing its body temperature. Humans are warm-blooded and have a body temperature of about  $37^{\circ}\text{C}$ . To regulate its body temperature, a warm-blooded animal increases or decreases the level of its metabolic processes. Thus, a warm-blooded animal may hibernate in winter and reduce its body temperature to approach the freezing point of water.

## 12.1 Section Review

10. **Temperature** Make the following conversions.
  - a.  $5^{\circ}\text{C}$  to kelvins
  - b. 34 K to degrees Celsius
  - c.  $212^{\circ}\text{C}$  to kelvins
  - d. 316 K to degrees Celsius
11. **Conversions** Convert the following Celsius temperatures to Kelvin temperatures.
  - a.  $28^{\circ}\text{C}$
  - b.  $154^{\circ}\text{C}$
  - c.  $568^{\circ}\text{C}$
  - d.  $-55^{\circ}\text{C}$
  - e.  $-184^{\circ}\text{C}$
12. **Thermal Energy** Could the thermal energy of a bowl of hot water equal that of a bowl of cold water? Explain your answer.
13. **Heat Flow** On a dinner plate, a baked potato always stays hot longer than any other food. Why?
14. **Heat** The hard tile floor of a bathroom always feels cold to bare feet even though the rest of the room is warm. Is the floor colder than the rest of the room?
15. **Specific Heat** If you take a plastic spoon out of a cup of hot cocoa and put it in your mouth, you are not likely to burn your tongue. However, you could very easily burn your tongue if you put the hot cocoa in your mouth. Why?
16. **Heat** Chefs often use cooking pans made of thick aluminum. Why is thick aluminum better than thin aluminum for cooking?
17. **Heat and Food** It takes much longer to bake a whole potato than to cook french fries. Why?
18. **Critical Thinking** As water heats in a pot on a stove, the water might produce some mist above its surface right before the water begins to roll. What is happening, and where is the coolest part of the water in the pot?



## 12.2 Changes of State and the Laws of Thermodynamics

**E**ighteenth-century steam-engine builders used heat to turn liquid water into steam. The steam pushed a piston to turn the engine, and then the steam was cooled and condensed into a liquid again. Adding heat to the liquid water changed not only its temperature, but also its structure. You will learn that changing state means changing form as well as changing the way in which atoms store thermal energy.

### Changes of State

The three most common states of matter are solids, liquids, and gases. As the temperature of a solid is raised, it usually changes to a liquid. At even higher temperatures, it becomes a gas. How can these changes be explained? Consider a material in a solid state. When the thermal energy of the solid is increased, the motion of the particles also increases, as does the temperature.

**Figure 12-10** diagrams the changes of state as thermal energy is added to 1.0 g of water starting at 243 K (ice) and continuing until it reaches 473 K (steam). Between points A and B, the ice is warmed to 273 K. At some point, the added thermal energy causes the particles to move rapidly enough that their motion overcomes the forces holding the particles together in a fixed location. The particles are still touching each other, but they have more freedom of movement. Eventually, the particles become free enough to slide past each other.

**Melting point** At this point, the substance has changed from a solid to a liquid. The temperature at which this change occurs is the melting point of the substance. When a substance is melting, all of the added thermal energy goes to overcome the forces holding the particles together in the solid state. None of the added thermal energy increases the kinetic energy of the particles. This can be observed between points B and C in Figure 12-10, where the added thermal energy melts the ice at a constant 273 K. Because the kinetic energy of the particles does not increase, the temperature does not increase between points B and C.

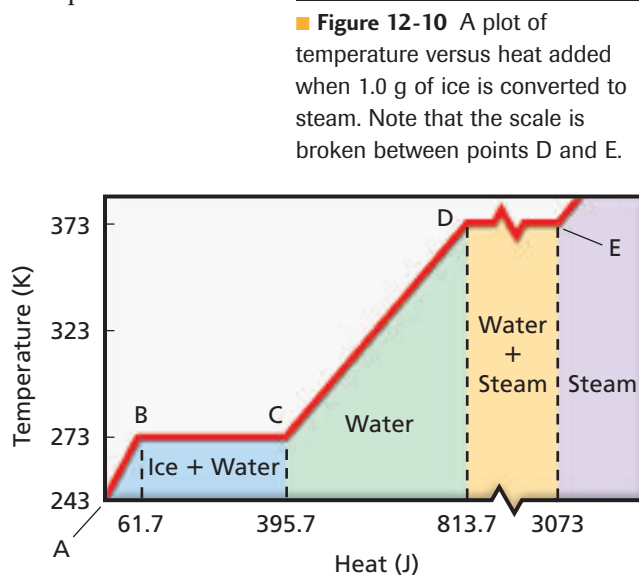
**Boiling point** Once a solid is completely melted, there are no more forces holding the particles in the solid state. Adding more thermal energy again increases the motion of the particles, and the temperature of the liquid rises. In the diagram, this process occurs between points C and D. As the temperature increases further, some particles in the liquid acquire enough energy to break free from the other particles. At a specific temperature, known as the boiling point, further addition of energy causes the substance to undergo another change of state. All the added thermal energy converts the substance from the liquid state to the gaseous state.

#### ► Objectives

- **Define** heats of fusion and vaporization.
- **State** the first and second laws of thermodynamics.
- **Distinguish** between heat and work.
- **Define** entropy.

#### ► Vocabulary

heat of fusion  
heat of vaporization  
first law of thermodynamics  
heat engine  
entropy  
second law of thermodynamics



**Table 12-2**

Heats of Fusion and Vaporization of Common Substances		
Material	Heat of Fusion $H_f$ (J/kg)	Heat of Vaporization $H_v$ (J/kg)
Copper	$2.05 \times 10^5$	$5.07 \times 10^6$
Mercury	$1.15 \times 10^4$	$2.72 \times 10^5$
Gold	$6.30 \times 10^4$	$1.64 \times 10^6$
Methanol	$1.09 \times 10^5$	$8.78 \times 10^5$
Iron	$2.66 \times 10^5$	$6.29 \times 10^6$
Silver	$1.04 \times 10^5$	$2.36 \times 10^6$
Lead	$2.04 \times 10^4$	$8.64 \times 10^5$
Water (ice)	$3.34 \times 10^5$	$2.26 \times 10^6$

As in melting, the temperature does not rise while a liquid boils. In Figure 12-10, this transition is represented between points D and E. After the material is entirely converted to gas, any added thermal energy again increases the motion of the particles, and the temperature rises. Above point E, steam is heated to temperatures greater than 373 K.

**Heat of fusion** The amount of energy needed to melt 1 kg of a substance is called the **heat of fusion** of that substance. For example, the heat of fusion of ice is  $3.34 \times 10^5$  J/kg. If 1 kg

of ice at its melting point, 273 K, absorbs  $3.34 \times 10^5$  J, the ice becomes 1 kg of water at the same temperature, 273 K. The added energy causes a change in state but not in temperature. The horizontal distance in Figure 12-10 from point B to point C represents the heat of fusion.

**Heat of vaporization** At normal atmospheric pressure, water boils at 373 K. The thermal energy needed to vaporize 1 kg of a liquid is called the **heat of vaporization**. For water, the heat of vaporization is  $2.26 \times 10^6$  J/kg. The distance from point D to point E in Figure 12-10 represents the heat of vaporization. Every material has a characteristic heat of vaporization.

Between points A and B, there is a definite slope to the line as the temperature is raised. This slope represents the specific heat of the ice. The slope between points C and D represents the specific heat of water, and the slope above point E represents the specific heat of steam. Note that the slope for water is less than those of both ice and steam. This is because water has a greater specific heat than does ice or steam. The heat,  $Q$ , required to melt a solid of mass  $m$  is given by the following equation.

$$\text{Heat Required to Melt a Solid } Q = mH_f$$

The heat required to melt a solid is equal to the mass of the solid times the heat of fusion of the solid.

Similarly, the heat,  $Q$ , required to vaporize a mass,  $m$ , of liquid is given by the following equation.

$$\text{Heat Required to Vaporize a Liquid } Q = mH_v$$

The heat required to vaporize a liquid is equal to the mass of the liquid times the heat of vaporization of the liquid.

When a liquid freezes, an amount of heat,  $Q = -mH_f$ , must be removed from the liquid to turn it into a solid. The negative sign indicates that the heat is transferred from the sample to the external world. In the same way, when a vapor condenses to a liquid, an amount of heat,  $Q = -mH_v$ , must be removed from the vapor. The values of some heats of fusion,  $H_f$ , and heats of vaporization,  $H_v$ , are shown in **Table 12-2**.

## MINI LAB

### Melting

1. Label two foam cups *A* and *B*.
2. Measure and pour 75 mL of room-temperature water into each cup. Wipe up any spilled liquid.
3. Add an ice cube to cup *A*, and add ice water to cup *B* until the water levels are equal.
4. Measure the temperature of the water in each cup at 1-min intervals until the ice has melted.
5. Record the temperatures in a data table and plot a graph.

### Analyze and Conclude

6. Do the samples reach the same final temperature? Why?

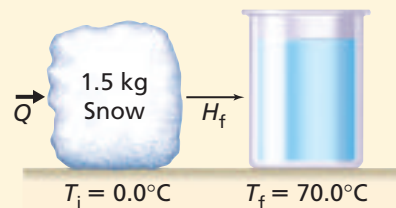


### ▶ EXAMPLE Problem 3

**Heat** Suppose that you are camping in the mountains. You need to melt 1.50 kg of snow at 0.0°C and heat it to 70.0°C to make hot cocoa. How much heat will be needed?

#### 1 Analyze and Sketch the Problem

- Sketch the relationship between heat and water in its solid and liquid states.
- Sketch the transfer of heat as the temperature of the water increases.



**Known:**

$$\begin{aligned} m &= 1.50 \text{ kg} & H_f &= 3.34 \times 10^5 \text{ J/kg} \\ T_i &= 0.0^\circ\text{C} & T_f &= 70.0^\circ\text{C} \\ C &= 4180 \text{ J/kg}\cdot^\circ\text{C} \end{aligned}$$

**Unknown:**

$$\begin{aligned} Q_{\text{melt ice}} &= ? \\ Q_{\text{heat liquid}} &= ? \\ Q_{\text{total}} &= ? \end{aligned}$$

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#### 2 Solve for the Unknown

Calculate the heat needed to melt ice.

$$\begin{aligned} Q_{\text{melt ice}} &= mH_f \\ &= (1.50 \text{ kg})(3.34 \times 10^5 \text{ J/kg}) && \text{Substitute } m = 1.50 \text{ kg, } H_f = 3.34 \times 10^5 \text{ J/kg} \\ &= 5.01 \times 10^5 \text{ J} \\ &= 5.01 \times 10^2 \text{ kJ} \end{aligned}$$

Calculate the temperature change.

$$\begin{aligned} \Delta T &= T_f - T_i \\ &= 70.0^\circ\text{C} - 0.0^\circ\text{C} && \text{Substitute } T_f = 70.0^\circ\text{C, } T_i = 0.0^\circ\text{C} \\ &= 70.0^\circ\text{C} \end{aligned}$$

Calculate the heat needed to raise the water temperature.

$$\begin{aligned} Q_{\text{heat liquid}} &= mC\Delta T \\ &= (1.50 \text{ kg})(4180 \text{ J/kg}\cdot^\circ\text{C})(70.0^\circ\text{C}) && \text{Substitute } m = 1.50 \text{ kg, } C = 4180 \text{ J/kg}\cdot^\circ\text{C, } \Delta T = 70.0^\circ\text{C} \\ &= 4.39 \times 10^5 \text{ J} \\ &= 4.39 \times 10^2 \text{ kJ} \end{aligned}$$

Calculate the total amount of heat needed.

$$\begin{aligned} Q_{\text{total}} &= Q_{\text{melt ice}} + Q_{\text{heat liquid}} \\ &= 5.01 \times 10^2 \text{ kJ} + 4.39 \times 10^2 \text{ kJ} && \text{Substitute } Q_{\text{melt ice}} = 5.01 \times 10^2 \text{ kJ, } Q_{\text{heat liquid}} = 4.39 \times 10^2 \text{ kJ} \\ &= 9.40 \times 10^2 \text{ kJ} \end{aligned}$$

#### 3 Evaluate the Answer

- **Are the units correct?** Energy units are in joules.
- **Does the sign make sense?**  $Q$  is positive when heat is absorbed.
- **Is the magnitude realistic?** The amount of heat needed to melt the ice is greater than the amount of heat needed to increase the water temperature by 70.0°C. It takes more energy to overcome the forces holding the particles in the solid state than to raise the temperature of water.

### ▶ PRACTICE Problems

Additional Problems, Appendix B

19. How much heat is absorbed by  $1.00 \times 10^2$  g of ice at  $-20.0^\circ\text{C}$  to become water at  $0.0^\circ\text{C}$ ?
20. A  $2.00 \times 10^2$ -g sample of water at  $60.0^\circ\text{C}$  is heated to steam at  $140.0^\circ\text{C}$ . How much heat is absorbed?
21. How much heat is needed to change  $3.00 \times 10^2$  g of ice at  $-30.0^\circ\text{C}$  to steam at  $130.0^\circ\text{C}$ ?





## The First Law of Thermodynamics

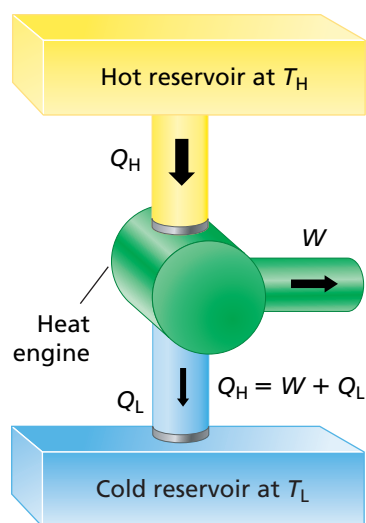
Before thermal energy was linked to the motion of atoms, the study of heat and temperature was considered to be a separate science. The first law developed for this science was a statement about what thermal energy is and where it can go. As you know, you can heat a nail by holding it over a flame or by pounding it with a hammer. That is, you can increase the nail's thermal energy by adding heat or by doing work on it. We do not normally think that the nail does work on the hammer. However, the work done by the nail on the hammer is equal to the negative of the work done by the hammer on the nail. The **first law of thermodynamics** states that the change in thermal energy,  $\Delta U$ , of an object is equal to the heat,  $Q$ , that is added to the object minus the work,  $W$ , done by the object. Note that  $\Delta U$ ,  $Q$ , and  $W$  are all measured in joules, the unit of energy.

### The First Law of Thermodynamics $\Delta U = Q - W$

The change in thermal energy of an object is equal to the heat added to the object minus the work done by the object.

Thermodynamics also involves the study of the changes in thermal properties of matter. The first law of thermodynamics is merely a restatement of the law of conservation of energy, which states that energy is neither created nor destroyed, but can be changed into other forms.

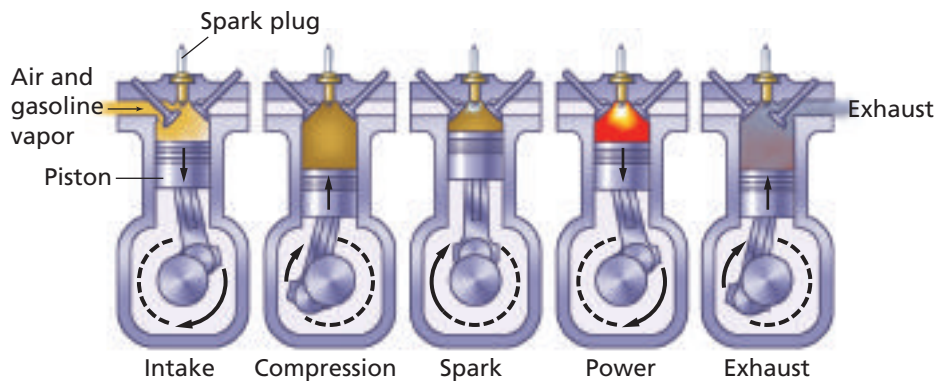
Another example of changing the amount of thermal energy in a system is a hand pump used to inflate a bicycle tire. As a person pumps, the air and the hand pump become warm. The mechanical energy in the moving piston is converted into thermal energy of the gas. Similarly, other forms of energy, such as light, sound, and electric energy, can be changed into thermal energy. For example, a toaster converts electric energy into heat when it toasts bread, and the Sun warms Earth with light from a distance of over 150 million km away.



■ **Figure 12-11** A heat engine transforms heat at high temperature into mechanical energy and low-temperature waste heat.

**Heat engines** The warmth that you experience when you rub your hands together is a result of the conversion of mechanical energy into thermal energy. The conversion of mechanical energy into thermal energy occurs easily. However, the reverse process, the conversion of thermal energy into mechanical energy, is more difficult. A device that is able to continuously convert thermal energy to mechanical energy is called a **heat engine**.

A heat engine requires a high-temperature source from which thermal energy can be removed; a low-temperature receptacle, called a sink, into which thermal energy can be delivered; and a way to convert the thermal energy into work. A diagram of a heat engine is shown in **Figure 12-11**. An automobile internal-combustion engine, such as the one shown in **Figure 12-12**, is one example of a heat engine. In the engine, a mixture of air and gasoline vapor is ignited and produces a high-temperature flame. Input heat,  $Q_H$ , flows from the flame to the air in the cylinder. The hot air expands and pushes on a piston, thereby changing thermal energy into mechanical energy. To obtain continuous mechanical energy, the engine must be returned to its starting condition. The heated air is expelled and replaced by new air, and the piston is returned to the top of the cylinder.



■ **Figure 12-12** The heat produced by burning gasoline causes the gases that are produced to expand and to exert force and do work on the piston.

The entire cycle is repeated many times each minute. The thermal energy from the burning of gasoline is converted into mechanical energy, which eventually results in the movement of the car.

Not all of the thermal energy from the high-temperature flame in an automobile engine is converted into mechanical energy. When the automobile engine is functioning, the exhaust gases and the engine parts become hot. As the exhaust comes in contact with outside air and transfers heat to it, the temperature of the outside air is raised. In addition, heat from the engine is transferred to a radiator. Outside air passes through the radiator and the air temperature is raised.

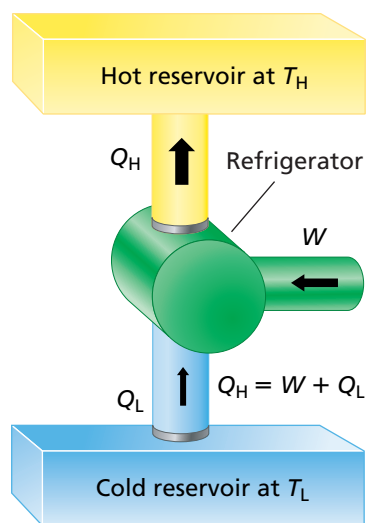
All of this energy,  $Q_L$ , transferred out of the automobile engine is called waste heat, that is, heat that has not been converted into work. When the engine is working continuously, the internal energy of the engine does not change, or  $\Delta U = 0 = Q - W$ . The net heat going into the engine is  $Q = Q_H - Q_L$ . Thus, the work done by the engine is  $W = Q_H - Q_L$ . In an automobile engine, the thermal energy in the flame produces the mechanical energy and the waste heat that is expelled. All heat engines generate waste heat, and therefore no engine can ever convert all of the energy into useful motion or work.

**Efficiency** Engineers and car salespeople often talk about the fuel efficiency of automobile engines. They are referring to the amount of the input heat,  $Q_H$ , that is turned into useful work,  $W$ . The actual efficiency of an engine is given by the ratio  $W/Q_H$ . The efficiency could equal 100 percent only if all of the input heat were turned into work by the engine. Because there is always waste heat, even the most efficient engines fall short of 100-percent efficiency.

In solar collectors, heat is collected at high temperatures and used to drive engines. The Sun's energy is transmitted as electromagnetic waves and increases the internal energy of the solar collectors. This energy is then transmitted as heat to the engine, where it is turned into useful work and waste heat.

**Refrigerators** Heat flows spontaneously from a warm object to a cold object. However, it is possible to remove thermal energy from a colder object and add it to a warmer object if work is done. A refrigerator is a common example of a device that accomplishes this transfer with the use of mechanical work. Electric energy runs a motor that does work on a gas and compresses it.





■ **Figure 12-13** A refrigerator absorbs heat,  $Q_L$ , from the cold reservoir and gives off heat,  $Q_H$ , to the hot reservoir. Work,  $W$ , is done on the refrigerator.

The gas draws heat from the interior of the refrigerator, passes from the compressor through the condenser coils on the outside of the refrigerator, and cools into a liquid. Thermal energy is transferred into the air in the room. The liquid reenters the interior, vaporizes, and absorbs thermal energy from its surroundings. The gas returns to the compressor and the process is repeated. The overall change in the thermal energy of the gas is zero. Thus, according to the first law of thermodynamics, the sum of the heat removed from the refrigerator's contents and the work done by the motor is equal to the heat expelled, as shown in **Figure 12-13**.

**Heat pumps** A heat pump is a refrigerator that can be run in two directions. In the summer, the pump removes heat from a house and thus cools the house. In the winter, heat is removed from the cold outside air and transferred into the warmer house. In both cases, mechanical energy is required to transfer heat from a cold object to a warmer one.

### ▶ PRACTICE Problems Additional Problems, Appendix B

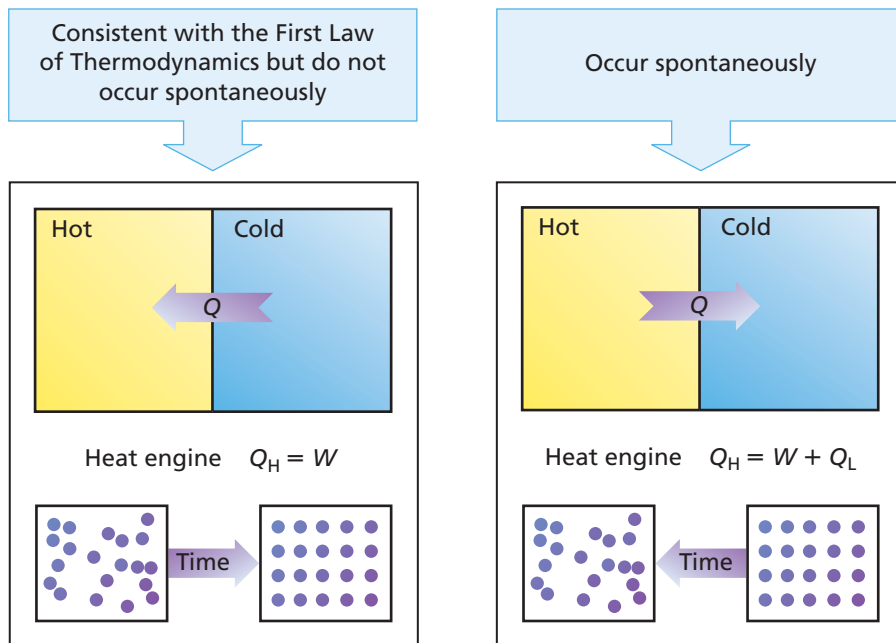
22. A gas balloon absorbs 75 J of heat. The balloon expands but stays at the same temperature. How much work did the balloon do in expanding?
23. A drill bores a small hole in a 0.40-kg block of aluminum and heats the aluminum by  $5.0^\circ\text{C}$ . How much work did the drill do in boring the hole?
24. How many times would you have to drop a 0.50-kg bag of lead shot from a height of 1.5 m to heat the shot by  $1.0^\circ\text{C}$ ?
25. When you stir a cup of tea, you do about 0.050 J of work each time you circle the spoon in the cup. How many times would you have to stir the spoon to heat a 0.15-kg cup of tea by  $2.0^\circ\text{C}$ ?
26. How can the first law of thermodynamics be used to explain how to reduce the temperature of an object?

## The Second Law of Thermodynamics

Many processes that are consistent with the first law of thermodynamics have never been observed to occur spontaneously. Three such processes are presented in **Figure 12-14**. For example, the first law of thermodynamics does not prohibit heat flowing from a cold object to a hot object. However, when hot objects have been placed in contact with cold objects, the hot objects have never been observed to become hotter. Similarly, the cold objects have never been observed to become colder.

**Entropy** If heat engines completely converted thermal energy into mechanical energy with no waste heat, then the first law of thermodynamics would be obeyed. However, waste heat is always generated, and randomly distributed particles of a gas are not observed to spontaneously arrange themselves in specific ordered patterns. In the nineteenth century, French engineer Sadi Carnot studied the ability of engines to convert thermal energy into mechanical energy. He developed a logical proof that even an ideal engine would generate some waste heat. Carnot's result is best described in terms of a quantity called **entropy**, which is a measure of the disorder in a system.





■ **Figure 12-14** Many processes that do not violate the first law of thermodynamics do not occur spontaneously. The spontaneous processes obey both the first and second law of thermodynamics.

When a baseball is dropped and falls due to gravity, it possesses potential and kinetic energies that can be recovered to do work. However, when the baseball falls through the air, it collides with many air molecules that absorb some of its energy. This causes air molecules to move in random directions and at random speeds. The energy absorbed from the baseball causes more disorder among the molecules. The greater the range of speeds exhibited by the molecules, the greater the disorder, which in turn increases the entropy. It is highly unlikely that the molecules that have been dispersed in all directions will come back together, give their energies back to the baseball, and cause it to rise.

Entropy, like thermal energy, is contained in an object. If heat is added to an object, entropy is increased. If heat is removed from an object, entropy is decreased. If an object does work with no change in temperature, the entropy does not change, as long as friction is ignored. The change in entropy,  $\Delta S$ , is expressed by the following equation, in which entropy has units of J/K and the temperature is measured in kelvins.

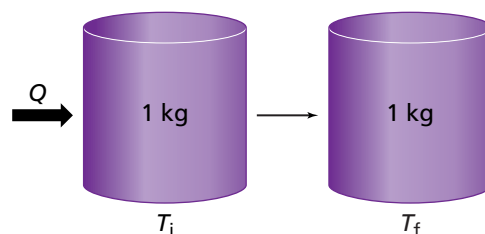
**Change in Entropy**  $\Delta S = \frac{Q}{T}$

The change in entropy of an object is equal to the heat added to the object divided by the temperature of the object in kelvins.

### CHALLENGE PROBLEM

Entropy has some interesting properties. Compare the following situations. Explain how and why these changes in entropy are different.

1. Heating 1.0 kg of water from 273 K to 274 K.
2. Heating 1.0 kg of water from 353 K to 354 K.
3. Completely melting 1.0 kg of ice at 273 K.
4. Heating 1.0 kg of lead from 273 K to 274 K.





■ **Figure 12-15** The spontaneous mixing of the food coloring and water is an example of the second law of thermodynamics.



The **second law of thermodynamics** states that natural processes go in a direction that maintains or increases the total entropy of the universe. That is, all things will become more and more disordered unless some action is taken to keep them ordered. The increase in entropy and the second law of thermodynamics can be thought of as statements of the probability of events happening. **Figure 12-15** illustrates an increase in entropy as food-coloring molecules, originally separate from the clear water, are thoroughly mixed with the water molecules over time. **Figure 12-16** shows an example of the second law of thermodynamics that might be familiar to many teenagers.

The second law of thermodynamics predicts that heat flows spontaneously only from a hot object to a cold object. Consider a hot iron bar and a cold cup of water. On the average, the particles in the iron will be moving very fast, whereas the particles in the water will be moving slowly. When the bar is plunged into the water and thermal equilibrium is eventually reached, the average kinetic energy of the particles in the iron and the water will be the same. More particles now have an increased random motion than was true for the initial state. This final state is less ordered than the initial state. The fast particles are no longer confined solely to the iron, and the slower particles are no longer confined only to the water; all speeds are evenly distributed. The entropy of the final state is greater than that of the initial state.

**Violations of the second law** We take for granted many daily events that occur spontaneously, or naturally, in one direction. We would be shocked, however, if the reverse of the same events occurred spontaneously. You are not surprised when a metal spoon, heated at one end, soon becomes uniformly hot. Consider your reaction, however, if a spoon lying on a table suddenly, on its own, became red hot at one end and icy cold at the other. If you dive into a swimming pool, you take for granted that you push the water molecules away as you enter the water. However, you would be amazed if you were swimming in the pool and all the water molecules spontaneously threw you up onto the diving board. Neither of these imagined reverse processes would violate the first law of thermodynamics. They are simply examples of the countless events that do not occur because their processes would violate the second law of thermodynamics.



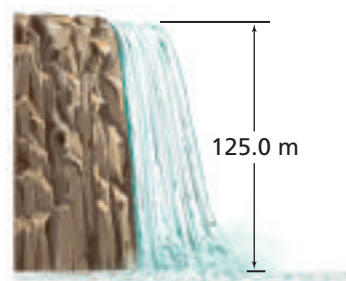
■ **Figure 12-16** If no work is done on a system, entropy spontaneously reaches a maximum.

The second law of thermodynamics and the increase in entropy also give new meaning to what has been commonly called the *energy crisis*. The energy crisis refers to the continued use of limited resources of fossil fuels, such as natural gas and petroleum. When you use a resource, such as natural gas to heat your home, you do not use up the energy in the gas. As the gas ignites, the internal chemical energy contained in the molecules of the gas is converted into thermal energy of the flame. The thermal energy of the flame is then transferred to thermal energy in the air of your home. Even if this warm air leaks to the outside, the energy is not lost. Energy has not been used up. The entropy, however, has increased.

The chemical structure of natural gas is very ordered. As you have learned, when a substance becomes warmer, the average kinetic energy of the particles in the substance increases. In contrast, the random motion of warmed air is very disordered. While it is mathematically possible for the original chemical order to be reestablished, the probability of this occurring is essentially zero. For this reason, entropy often is used as a measure of the unavailability of useful energy. The energy in the warmed air in a home is not as available to do mechanical work or to transfer heat to other objects as the original gas molecules were. The lack of usable energy is actually a surplus of entropy.

## 12.2 Section Review

- 27. Heat of Vaporization** Old-fashioned heating systems sent steam into radiators in each room of a house. In the radiators, the steam condensed back to water. Analyze this process and explain how it heated a room.
- 28. Heat of Vaporization** How much heat is needed to change 50.0 g of water at 80.0°C to steam at 110.0°C?
- 29. Heat of Vaporization** The specific heat of mercury is 140 J/kg·°C. Its heat of vaporization is  $3.06 \times 10^5$  J/kg. How much energy is needed to heat 1.0 kg of mercury metal from 10.0°C to its boiling point and vaporize it completely? The boiling point of mercury is 357°C.
- 30. Mechanical Energy and Thermal Energy** James Joule carefully measured the difference in temperature of water at the top and bottom of a waterfall. Why did he expect a difference?
- 31. Mechanical Energy and Thermal Energy** A man uses a 320-kg hammer moving at 5.0 m/s to smash a 3.0-kg block of lead against a 450-kg rock. When he measured the temperature he found that it had increased by 5.0°C. Explain how this happened.
- 32. Mechanical Energy and Thermal Energy** Water flows over a fall that is 125.0 m high, as shown in **Figure 12-17**. If the potential energy of the water is all converted to thermal energy, calculate the temperature difference between the water at the top and the bottom of the fall.



■ Figure 12-17

- 33. Entropy** Evaluate why heating a home with natural gas results in an increased amount of disorder.
- 34. Critical Thinking** A new deck of cards has all the suits (clubs, diamonds, hearts, and spades) in order, and the cards are ordered by number within the suits. If you shuffle the cards many times, are you likely to return the cards to their original order? Explain. Of what physical law is this an example?



## Heating and Cooling

Alternate CBL instructions can be found on the Web site.  
[physicspp.com](http://physicspp.com)

When a beaker of water is set on a hot plate and the hot plate is turned on, heat is transferred. It first is transferred to the beaker and then to the water at the bottom of the beaker by conduction. The water then transfers heat from the bottom to the top by moving hot water to the top through convection. Once the heat source is removed or shut off, the water radiates thermal energy until it reaches room temperature. How quickly the water heats up is a function of the amount of heat added, the mass of the water, and the specific heat of water.

### QUESTION

How does the constant supply of thermal energy affect the temperature of water?

#### Objectives

- **Measure**, in SI, temperature and mass.
- **Make and use graphs** to help describe the change in temperature of water as it heats up and cools down.
- **Explain** any similarities and differences in these two changes.

#### Safety Precautions



- **Be careful when using a hot plate. It can burn the skin.**

#### Materials

hot plate (or Bunsen burner)  
250-mL ovenproof glass beaker  
50–200 g of water  
two thermometers (non-mercury)  
stopwatch (or timer)



#### Procedure

1. Set the hot plate to the highest setting, or as recommended by your teacher. Allow a few minutes for the plate to heat up.
2. Measure the mass of the empty beaker.
3. Pour 150 mL of water into the beaker and measure the combined mass of the water and the beaker.
4. Calculate and record the mass of the water in the beaker.
5. Create a data and observations table.
6. Record the initial temperature of the water and the air in the classroom. Note that the bulb end of the thermometers must not touch the bottom or sides of the beaker, nor should it touch a table or your hands.
7. Place the beaker on the hot plate and record the temperature every minute for 5 min.
8. Carefully remove the beaker from the hot plate and record the temperature every minute for the next 10 min.
9. At the end of 10 min, record the temperature of the air.
10. Turn off the hot plate.
11. When finished, allow the equipment to cool and dispose of the water as instructed by your teacher.

Data Table		
Mass of water		
Initial air temperature		
Final air temperature		
Change in air temperature		
Time (min)	Temperature (°C)	Heating or Cooling

### Analyze

1. Calculate the change in air temperature to determine if air temperature may be an extraneous variable.
2. Make a scatter-plot graph of temperature (vertical axis) versus time (horizontal axis). Use a computer or a calculator to construct the graph, if possible.
3. **Calculate** What was the change in water temperature as the water heated up?
4. **Calculate** What was the drop in water temperature when the heat source was removed?
5. Calculate the average slope for the temperature increase by dividing change in temperature by the amount of time the water was heating up.
6. Calculate the average slope for the temperature decrease by dividing change in temperature by the amount of time the heat source was removed.

### Conclude and Apply

1. **Summarize** What was the change in water temperature when a heat source was applied?
2. **Summarize** What was the change in water temperature once the heat source was removed?
3. What would happen to the water temperature after the next 10 min? Would it continue cooling down forever?
4. Did the water appear to heat up or cool down quicker? Why do you think this is so? *Hint: Examine the slopes you calculated.*
5. **Hypothesize** Where did the thermal energy in the water go once the water began to cool down? Support your hypothesis.

### Going Further

1. Does placing your thermometer at the top of the water in your beaker result in different readings than if it is placed at the bottom of the beaker? Explain.
2. Hypothesize what the temperature changes might look like if you had the following amounts of water in the beaker: 50 mL, 250 mL.
3. Suppose you insulated the beaker you were using. How would the beaker's ability to heat up and cool down be affected?

### Real-World Physics

1. Suppose you were to use vegetable oil in the beaker instead of water. Hypothesize what the temperature changes might look like if you were to follow the same steps and perform the experiment.
2. If you were to take soup at room temperature and cook it in a microwave oven for 3 min, would the soup return to room temperature in 3 min? Explain your answer.



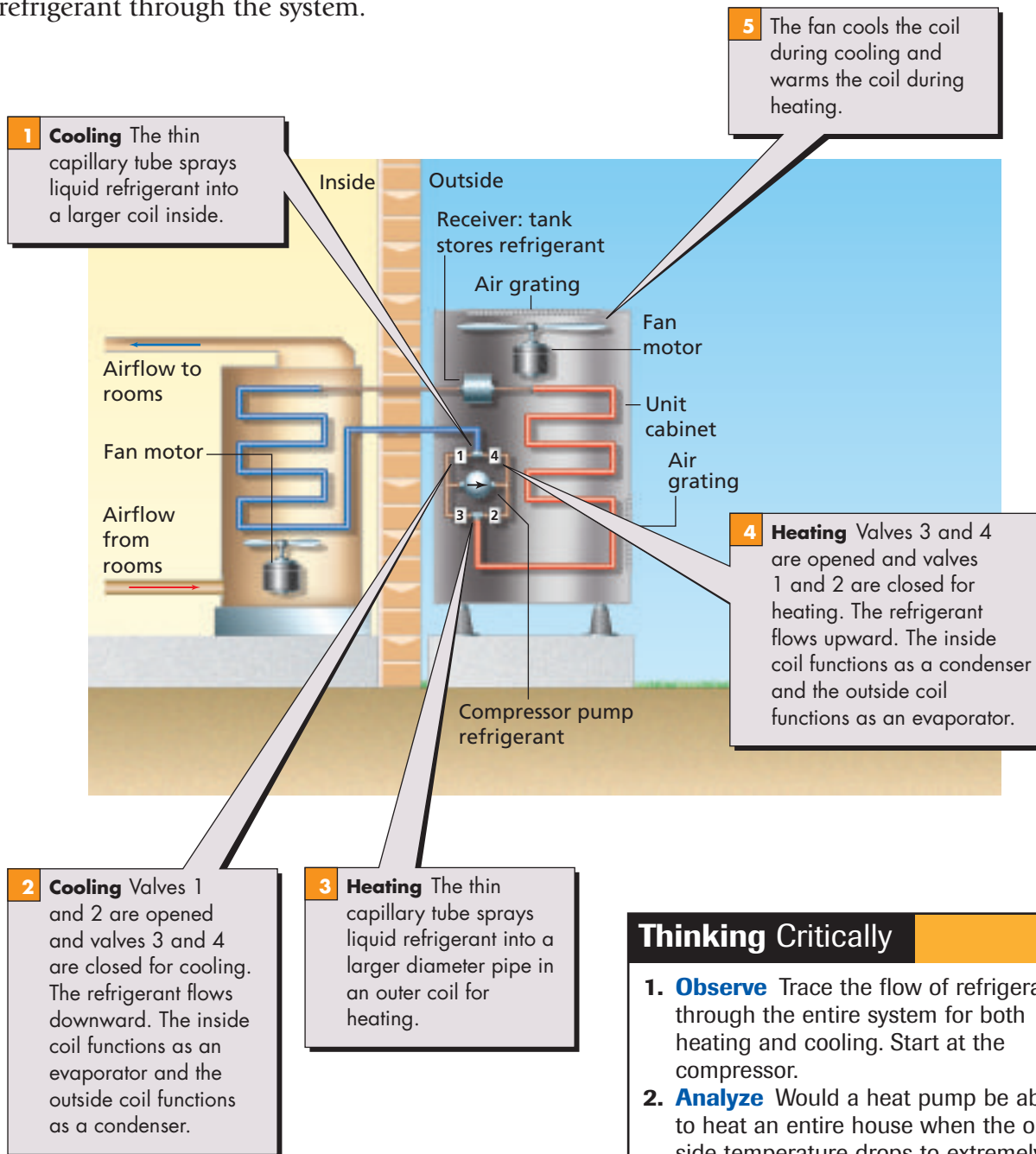
To find out more about thermal energy, visit the Web site: [physicspp.com](http://physicspp.com)



# How it Works

## The Heat Pump

Heat pumps, also called reversible air conditioners, were invented in the 1940s. They are used to heat and cool homes and hotel rooms. Heat pumps change from heaters to air conditioners by reversing the flow of refrigerant through the system.



### Thinking Critically

- 1. Observe** Trace the flow of refrigerant through the entire system for both heating and cooling. Start at the compressor.
- 2. Analyze** Would a heat pump be able to heat an entire house when the outside temperature drops to extremely cold levels?

## 12.1 Temperature and Thermal Energy

**Vocabulary**

- conduction (p. 315)
- thermal equilibrium (p. 315)
- heat (p. 317)
- convection (p. 317)
- radiation (p. 317)
- specific heat (p. 318)

**Key Concepts**

- The temperature of a gas is proportional to the average kinetic energy of its particles.
- Thermal energy is a measure of the internal motion of an object's particles.
- A thermometer reaches thermal equilibrium with the object that it comes in contact with, and then a temperature-dependent property of the thermometer indicates the temperature.
- The Celsius and Kelvin temperature scales are used in scientific work. The magnitude of 1 K is equal to the magnitude of 1°C.
- At absolute zero, no more thermal energy can be removed from a substance.
- Heat is energy transferred because of a difference in temperature.

$$Q = mC\Delta T = mC(T_f - T_i)$$

- Specific heat is the quantity of heat required to raise the temperature of 1 kg of a substance by 1 K.
- In a closed, isolated system, heat may flow and change the thermal energy of parts of the system, but the total energy of the system is constant.

$$E_A + E_B = \text{constant}$$

## 12.2 Changes of State and the Laws of Thermodynamics

**Vocabulary**

- heat of fusion (p. 324)
- heat of vaporization (p. 324)
- first law of thermodynamics (p. 326)
- heat engine (p. 326)
- entropy (p. 328)
- second law of thermodynamics (p. 330)

**Key Concepts**

- The heat of fusion is the quantity of heat needed to change 1 kg of a substance from its solid to liquid state at its melting point.

$$Q = mH_f$$

- The heat of vaporization is the quantity of heat needed to change 1 kg of a substance from its liquid to gaseous state at its boiling point.

$$Q = mH_v$$

- Heat transferred during a change of state does not change the temperature of a substance.
- The change in energy of an object is the sum of the heat added to it minus the work done by the object.

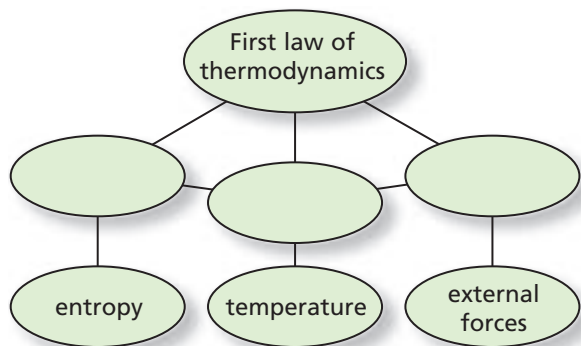
$$\Delta U = Q - W$$

- A heat engine continuously converts thermal energy to mechanical energy.
- A heat pump and a refrigerator use mechanical energy to transfer heat from a region of lower temperature to one of higher temperature.
- Entropy is a measure of the disorder of a system.
- The change in entropy of an object is defined to be the heat added to the object divided by the temperature of the object.

$$\Delta S = \frac{Q}{T}$$

## Concept Mapping

35. Complete the following concept map using the following terms: *heat, work, internal energy*.



## Mastering Concepts

36. Explain the differences among the mechanical energy of a ball, its thermal energy, and its temperature. (12.1)
37. Can temperature be assigned to a vacuum? Explain. (12.1)
38. Do all of the molecules or atoms in a liquid have the same speed? (12.1)
39. Is your body a good judge of temperature? On a cold winter day, a metal doorknob feels much colder to your hand than a wooden door does. Explain why this is true. (12.1)
40. When heat flows from a warmer object in contact with a colder object, do the two have the same temperature changes? (12.1)
41. Can you add thermal energy to an object without increasing its temperature? Explain. (12.2)
42. When wax freezes, does it absorb or release energy? (12.2)
43. Explain why water in a canteen that is surrounded by dry air stays cooler if it has a canvas cover that is kept wet. (12.2)
44. Which process occurs at the coils of a running air conditioner inside a house, vaporization or condensation? Explain. (12.2)

## Applying Concepts

45. **Cooking** Sally is cooking pasta in a pot of boiling water. Will the pasta cook faster if the water is boiling vigorously or if it is boiling gently?
46. Which liquid would an ice cube cool faster, water or methanol? Explain.

47. Equal masses of aluminum and lead are heated to the same temperature. The pieces of metal are placed on a block of ice. Which metal melts more ice? Explain.
48. Why do easily vaporized liquids, such as acetone and methanol, feel cool to the skin?
49. Explain why fruit growers spray their trees with water when frost is expected to protect the fruit from freezing.
50. Two blocks of lead have the same temperature. Block A has twice the mass of block B. They are dropped into identical cups of water of equal temperatures. Will the two cups of water have equal temperatures after equilibrium is achieved? Explain.
51. **Windows** Often, architects design most of the windows of a house on the north side. How does putting windows on the south side affect the heating and cooling of the house?

## Mastering Problems

### 12.1 Temperature and Thermal Energy

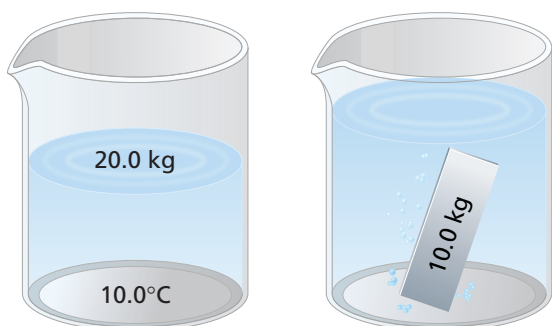
52. How much heat is needed to raise the temperature of 50.0 g of water from 4.5°C to 83.0°C?
53. A  $5.00 \times 10^2$ -g block of metal absorbs 5016 J of heat when its temperature changes from 20.0°C to 30.0°C. Calculate the specific heat of the metal.
54. **Coffee Cup** A  $4.00 \times 10^2$ -g glass coffee cup is 20.0°C at room temperature. It is then plunged into hot dishwasher water at a temperature of 80.0°C, as shown in **Figure 12-18**. If the temperature of the cup reaches that of the dishwasher, how much heat does the cup absorb? Assume that the mass of the dishwasher water is large enough so that its temperature does not change appreciably.



■ Figure 12-18

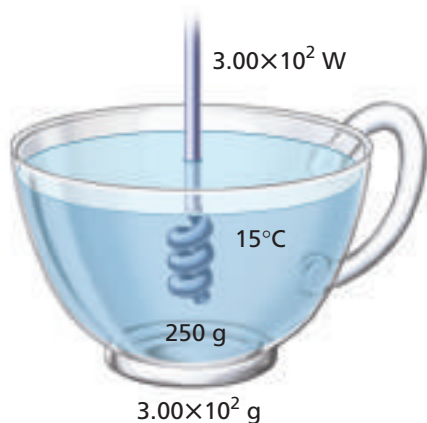
55. A  $1.00 \times 10^2$ -g mass of tungsten at 100.0°C is placed in  $2.00 \times 10^2$  g of water at 20.0°C. The mixture reaches equilibrium at 21.6°C. Calculate the specific heat of tungsten.

56. A  $6.0 \times 10^2$ -g sample of water at  $90.0^\circ\text{C}$  is mixed with  $4.00 \times 10^2$  g of water at  $22.0^\circ\text{C}$ . Assume that there is no heat loss to the surroundings. What is the final temperature of the mixture?
57. A 10.0-kg piece of zinc at  $71.0^\circ\text{C}$  is placed in a container of water, as shown in **Figure 12-19**. The water has a mass of 20.0 kg and a temperature of  $10.0^\circ\text{C}$  before the zinc is added. What is the final temperature of the water and the zinc?



■ **Figure 12-19**

58. The kinetic energy of a compact car moving at 100 km/h is  $2.9 \times 10^5$  J. To get a feeling for the amount of energy needed to heat water, what volume of water (in liters) would  $2.9 \times 10^5$  J of energy warm from room temperature ( $20.0^\circ\text{C}$ ) to boiling ( $100.0^\circ\text{C}$ )?
59. **Water Heater** A  $3.0 \times 10^2$ -W electric immersion heater is used to heat a cup of water, as shown in **Figure 12-20**. The cup is made of glass, and its mass is  $3.00 \times 10^2$  g. It contains 250 g of water at  $15^\circ\text{C}$ . How much time is needed to bring the water to the boiling point? Assume that the temperature of the cup is the same as the temperature of the water at all times and that no heat is lost to the air.



■ **Figure 12-20**

60. **Car Engine** A  $2.50 \times 10^2$ -kg cast-iron car engine contains water as a coolant. Suppose that the engine's temperature is  $35.0^\circ\text{C}$  when it is shut off, and the air temperature is  $10.0^\circ\text{C}$ . The heat given off by the engine and water in it as they cool to air temperature is  $4.40 \times 10^6$  J. What mass of water is used to cool the engine?

## 12.2 Changes of State and the Laws of Thermodynamics

61. Years ago, a block of ice with a mass of about 20.0 kg was used daily in a home icebox. The temperature of the ice was  $0.0^\circ\text{C}$  when it was delivered. As it melted, how much heat did the block of ice absorb?
62. A 40.0-g sample of chloroform is condensed from a vapor at  $61.6^\circ\text{C}$  to a liquid at  $61.6^\circ\text{C}$ . It liberates 9870 J of heat. What is the heat of vaporization of chloroform?
63. A 750-kg car moving at 23 m/s brakes to a stop. The brakes contain about 15 kg of iron, which absorbs the energy. What is the increase in temperature of the brakes?
64. How much heat is added to 10.0 g of ice at  $-20.0^\circ\text{C}$  to convert it to steam at  $120.0^\circ\text{C}$ ?
65. A 4.2-g lead bullet moving at 275 m/s strikes a steel plate and comes to a stop. If all its kinetic energy is converted to thermal energy and none leaves the bullet, what is its temperature change?
66. **Soft Drink** A soft drink from Australia is labeled "Low-Joule Cola." The label says "100 mL yields 1.7 kJ." The can contains 375 mL of cola. Chandra drinks the cola and then wants to offset this input of food energy by climbing stairs. How high would Chandra have to climb if she has a mass of 65.0 kg?

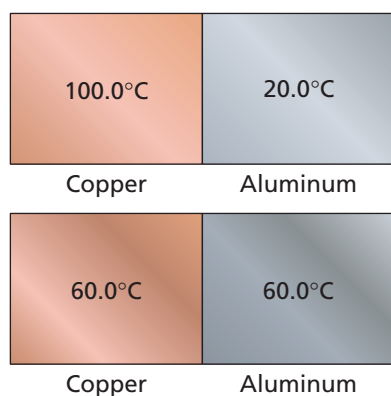
## Mixed Review

67. What is the efficiency of an engine that produces 2200 J/s while burning enough gasoline to produce 5300 J/s? How much waste heat does the engine produce per second?
68. **Stamping Press** A metal stamping machine in a factory does 2100 J of work each time it stamps out a piece of metal. Each stamped piece is then dipped in a 32.0-kg vat of water for cooling. By how many degrees does the vat heat up each time a piece of stamped metal is dipped into it?
69. A 1500-kg automobile comes to a stop from 25 m/s. All of the energy of the automobile is deposited in the brakes. Assuming that the brakes are about 45 kg of aluminum, what would be the change in temperature of the brakes?



## Chapter 12 Assessment

70. **Iced Tea** To make iced tea, you start by brewing the tea with hot water. Then you add ice. If you start with 1.0 L of  $90.0^{\circ}\text{C}$  tea, what is the minimum amount of ice needed to cool it to  $0^{\circ}\text{C}$ ? Would it be better to let the tea cool to room temperature before adding the ice?
71. A block of copper at  $100.0^{\circ}\text{C}$  comes in contact with a block of aluminum at  $20.0^{\circ}\text{C}$ , as shown in **Figure 12-21**. The final temperature of the blocks is  $60.0^{\circ}\text{C}$ . What are the relative masses of the blocks?



■ **Figure 12-21**

72. A 0.35-kg block of copper sliding on the floor hits an identical block moving at the same speed from the opposite direction. The two blocks come to a stop together after the collision. Their temperatures increase by  $0.20^{\circ}\text{C}$  as a result of the collision. What was their velocity before the collision?
73. A 2.2-kg block of ice slides across a rough floor. Its initial velocity is 2.5 m/s and its final velocity is 0.50 m/s. How much of the ice block melted as a result of the work done by friction?

### Thinking Critically

74. **Analyze and Conclude** A certain heat engine removes 50.0 J of thermal energy from a hot reservoir at temperature  $T_H = 545\text{ K}$  and expels 40.0 J of heat to a colder reservoir at temperature  $T_L = 325\text{ K}$ . In the process, it also transfers entropy from one reservoir to the other.
- How does the operation of the engine change the total entropy of the reservoirs?
  - What would be the total entropy change in the reservoirs if  $T_L = 205\text{ K}$ ?
75. **Analyze and Conclude** During a game, the metabolism of basketball players often increases by as much as 30.0 W. How much perspiration must a player vaporize per hour to dissipate this extra thermal energy?

76. **Analyze and Conclude** Chemists use calorimeters to measure the heat produced by chemical reactions. For instance, a chemist dissolves  $1.0 \times 10^{22}$  molecules of a powdered substance into a calorimeter containing 0.50 kg of water. The molecules break up and release their binding energy to the water. The water temperature increases by  $2.3^{\circ}\text{C}$ . What is the binding energy per molecule for this substance?
77. **Apply Concepts** All of the energy on Earth comes from the Sun. The surface temperature of the Sun is approximately  $10^4\text{ K}$ . What would be the effect on our world if the Sun's surface temperature were  $10^3\text{ K}$ ?

### Writing in Physics

78. Our understanding of the relationship between heat and energy was influenced by a soldier named Benjamin Thompson, Count Rumford; and a brewer named James Prescott Joule. Both relied on experimental results to develop their ideas. Investigate what experiments they did and evaluate whether or not it is fair that the unit of energy is called the Joule and not the Thompson.
79. Water has an unusually large specific heat and large heats of fusion and vaporization. Our weather and ecosystems depend upon water in all three states. How would our world be different if water's thermodynamic properties were like other materials, such as methanol?

### Cumulative Review

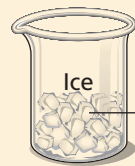
80. A rope is wound around a drum with a radius of 0.250 m and a moment of inertia of  $2.25\text{ kg m}^2$ . The rope is connected to a 4.00-kg block. (**Chapter 8**)
- Find the linear acceleration of the block.
  - Find the angular acceleration of the drum.
  - Find the tension,  $F_T$ , in the rope.
  - Find the angular velocity of the drum after the block has fallen 5.00 m.
81. A weight lifter raises a 180-kg barbell to a height of 1.95 m. How much work is done by the weight lifter in lifting the barbell? (**Chapter 10**)
82. In a Greek myth, the man Sisyphus is condemned by the gods to forever roll an enormous rock up a hill. Each time he reaches the top, the rock rolls back down to the bottom. If the rock has a mass of 215 kg, the hill is 33 m in height, and Sisyphus can produce an average power of 0.2 kW, how many times in 1 h can he roll the rock up the hill? (**Chapter 11**)



# Standardized Test Practice

## Multiple Choice

- Which of the following temperature conversions is incorrect?  
 (A)  $-273^{\circ}\text{C} = 0\text{ K}$        (C)  $298\text{ K} = 571^{\circ}\text{C}$   
 (B)  $273^{\circ}\text{C} = 546\text{ K}$        (D)  $88\text{ K} = -185^{\circ}\text{C}$
- What are the units of entropy?  
 (A) J/K       (C) J  
 (B) K/J       (D) kJ
- Which of the following statements about thermal equilibrium is false?  
 (A) When two objects are at equilibrium, heat radiation between the objects continues to occur.  
 (B) Thermal equilibrium is used to create energy in a heat engine.  
 (C) The principle of thermal equilibrium is used for calorimetry calculations.  
 (D) When two objects are not at equilibrium, heat will flow from the hotter object to the cooler object.
- How much heat is required to heat 87 g of methanol ice at 14 K to vapor at 340 K? (melting point =  $-97.6^{\circ}\text{C}$ , boiling point =  $64.6^{\circ}\text{C}$ )  
 (A) 17 kJ       (C)  $1.4 \times 10^2$  kJ  
 (B) 69 kJ       (D)  $1.5 \times 10^2$  kJ
- Which statement is true about energy, entropy, and changes of state?  
 (A) Freezing ice increases in energy as it gains molecular order as a solid.  
 (B) The higher the specific heat capacity of a substance, the higher its melting point will be.  
 (C) States of matter with increased kinetic energy have higher entropy.  
 (D) Energy and entropy cannot increase at the same time.
- How much heat is needed to warm 363 mL of water in a baby bottle from  $24^{\circ}\text{C}$  to  $38^{\circ}\text{C}$ ?  
 (A) 21 kJ       (C) 121 kJ  
 (B) 36 kJ       (D) 820 kJ
- Why is there always some waste heat in a heat engine?  
 (A) Heat cannot flow from a cold object to a hot object.  
 (B) Friction slows the engine down.  
 (C) The entropy increases at each stage.  
 (D) The heat pump uses energy.
- How much heat is absorbed from the surroundings when 81 g of  $0.0^{\circ}\text{C}$  ice in a beaker melts and warms to  $10^{\circ}\text{C}$ ?  
 (A) 0.34 kJ       (C) 30 kJ  
 (B) 27 kJ       (D) 190 kJ



- You do 0.050 J of work on the coffee in your cup each time you stir it. What would be the increase in entropy in 125 mL of coffee at  $65^{\circ}\text{C}$  when you stir it 85 times?  
 (A) 0.013 J/K       (C) 0.095 J/K  
 (B) 0.050 J       (D) 4.2 J

## Extended Answer

- What is the difference in heat required to melt 454 g of ice at  $0.00^{\circ}\text{C}$ , and to turn 454 g of water at  $100.0^{\circ}\text{C}$  into steam? Is the amount of this difference greater or less than the amount of energy required to heat the 454 g of water from  $0.00^{\circ}\text{C}$  to  $100.0^{\circ}\text{C}$ ?

### ✓ Test-Taking TIP

#### Your Mistakes Can Teach You

The mistakes you make before the test are helpful because they show you areas in which you need more work. When calculating the heat needed to melt and warm a substance, remember to calculate the heat needed for melting as well as the heat needed for raising the temperature of the substance.

# Chapter 13

# States of Matter

## What You'll Learn

- You will explain the expansion and contraction of matter caused by changes in temperature.
- You will apply Pascal's, Archimedes', and Bernoulli's principles in everyday situations.

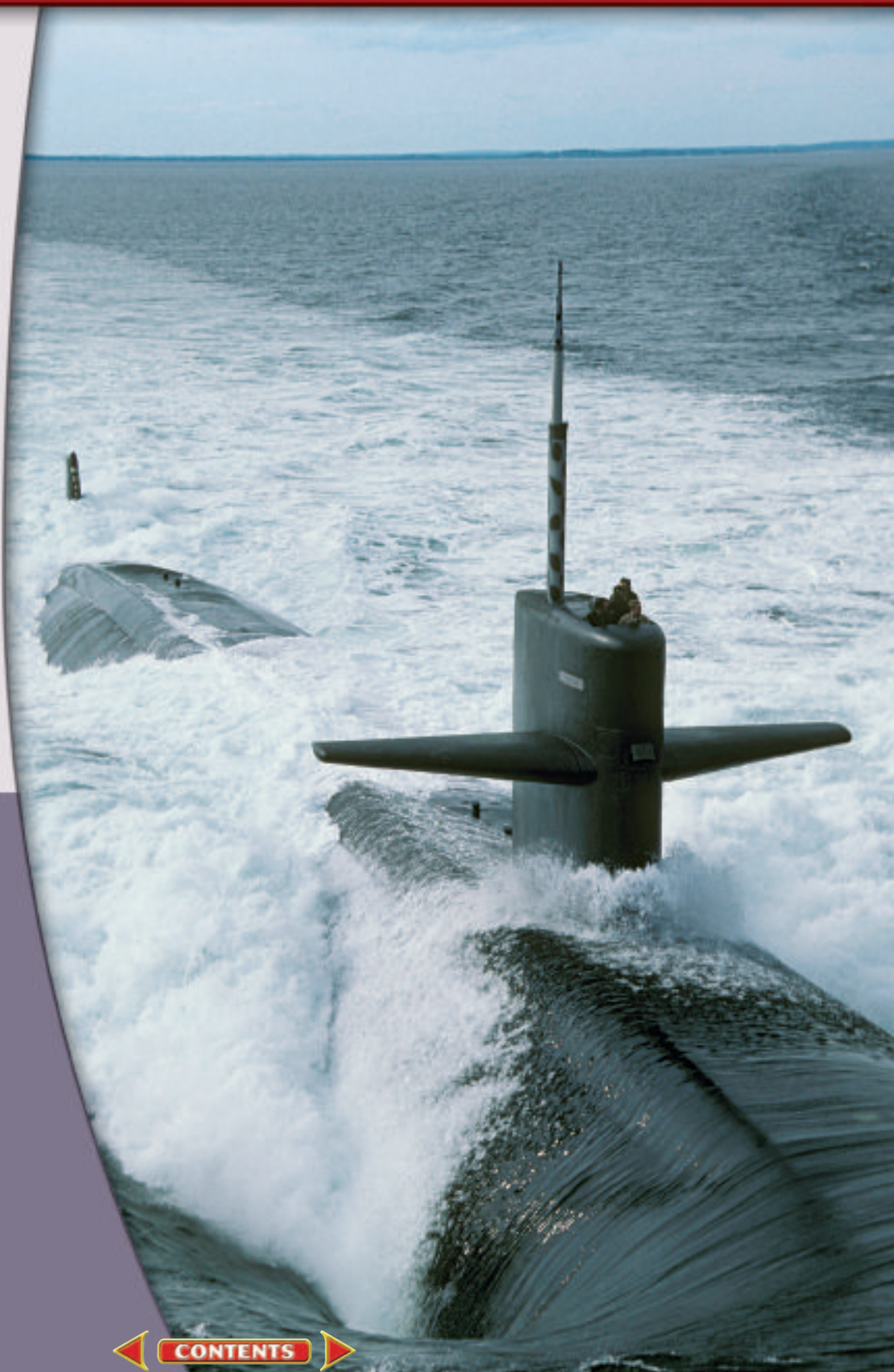
## Why It's Important

Fluids and the forces that they exert enable us to swim and dive, balloons to float, and planes to fly. Thermal expansion affects the designs of buildings, roads, bridges, and machines.

**Submarines** A nuclear submarine is designed to maneuver at all levels in the ocean. It must withstand great differences in pressure and temperature as it moves deeper under water.

## Think About This ►

How is the submarine able to float at the surface of the ocean and to dive far beneath it?



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## Does it float or sink?

### Question

How can you measure the buoyancy of objects?

### Procedure

1. Obtain a small vial (with a cap or a seal) and a 500-mL graduated cylinder. Attach a rubber band to the vial in order to suspend it from a spring scale.
2. Use the spring scale to find the weight of the vial. Then, use the graduated cylinder to find the volume of water displaced by the sealed vial when it is floating. Record both of these figures. Immediately wipe up any spilled liquid.
3. Place one nickel into the vial and close the lid. Repeat the procedures in step 2, recording the weight of the vial and the nickel, as well as the volume of water displaced. Also, record whether the vial floats or sinks.
4. Repeat steps 2 and 3, each time adding one nickel until the vial no longer floats. When the vial sinks, use the spring scale to find the apparent weight of the vial. Be sure that the vial is not touching the graduated cylinder when it is suspended under water.

### Analysis

Use the information you recorded to calculate the density of the vial-and-nickel system in each of your trials. Also, calculate the mass of the water displaced by the system in each trial.

How does density appear to be related to floating?

### Critical Thinking

How does the mass of the vial-nickel system appear to be related to the mass of the water displaced by the system? Does this relationship hold regardless of whether the system is floating?



## 13.1 Properties of Fluids

**W**ater and air are probably two of the most common substances in the everyday lives of people. We feel their effects when we drink, when we bathe, and literally with every breath we take. In your everyday experience, it might not seem that water and air have a great deal in common. If you think further about them, however, you will recognize that they have common properties. Both water and air flow, and unlike solids, neither one of them has a definite shape. Gases and liquids are two states of matter in which atoms and molecules have great freedom to move.

In this chapter, you will explore states of matter. Beginning with gases and liquids, you will learn about the principles that explain how matter responds to changes in temperature and pressure, how hydraulic systems can multiply forces, and how huge metallic ships can float on water. You also will investigate the properties of solids, discovering how they expand and contract, why some solids are elastic, and why some solids seem to straddle the line between solid and liquid.

### ► Objectives

- **Describe** how fluids create pressure.
- **Calculate** the pressure, volume, and number of moles of a gas.
- **Compare** gases and plasma.

### ► Vocabulary

fluids  
pressure  
pascal  
combined gas law  
ideal gas law  
thermal expansion  
plasma





■ **Figure 13-1** The ice cubes, which are solids, have definite shapes. However, the liquid water, a fluid, takes the shape of its container. What fluid is filling the space above the water?

■ **Figure 13-2** The astronaut and the landing module both exert pressure on the lunar surface. If the lunar module had a mass of approximately 7300 kg and rested on four pads that were each 91 cm in diameter, what pressure did it exert on the Moon's surface? How could you estimate the pressure exerted by the astronaut?



## Pressure

Suppose that you put an ice cube in an empty glass. The ice cube has a certain mass and shape, and neither of these quantities depends on the size or shape of the glass. What happens, however, when the ice melts? Its mass remains the same, but its shape changes. The water flows to take the shape of its container and forms a definite, flat, upper surface, as in **Figure 13-1**. If you boiled the water, it would change into a gas in the form of water vapor, and it also would flow and expand to fill the room. However, the water vapor would not have any definite surface. Both liquids and gases are **fluids**, which are materials that flow and have no definite shape of their own. For now, you can assume that you are dealing with ideal fluids, whose particles take up no space and have no intermolecular attractive forces.

**Pressure in fluids** You have applied the law of conservation of energy to solid objects. Can this law also be applied to fluids? Work and energy can be defined if we introduce the concept of **pressure**, which is the force on a surface, divided by the area of the surface. Since pressure is force exerted over a surface, anything that exerts pressure is capable of producing change and doing work.

$$\text{Pressure } P = \frac{F}{A}$$

Pressure equals force divided by surface area.

Pressure ( $P$ ) is a scalar quantity. In the SI system, the unit of pressure is the **pascal** (Pa), which is  $1 \text{ N/m}^2$ . Because the pascal is a small unit, the kilopascal (kPa), equal to 1000 Pa, is more commonly used. The force,  $F$ , on a surface is assumed to be perpendicular to the surface area,  $A$ . **Figure 13-2** illustrates the relationships between force, area, and pressure. **Table 13-1** shows how pressures vary in different situations.

**Solids, liquids, and pressure** Imagine that you are standing on the surface of a frozen lake. The forces that your feet exert on the ice are spread over the area of your shoes, resulting in pressure on the ice. Ice is a solid that is made up of vibrating water molecules, and the forces that hold the water molecules in place cause the ice to exert upward forces on your feet that equal your weight. If the ice melted, most of the bonds between the water molecules would be weakened. Although the molecules would continue to vibrate and remain close to each other, they also would slide past one another, and you would break through the surface. The moving water molecules would continue to exert forces on your body.

**Gas particles and pressure** The pressure exerted by a gas can be understood by applying the kinetic-molecular theory of gases. The kinetic-molecular theory explains the properties of an ideal gas. In reality, the particles of a gas take up space and have intermolecular attractive forces, but an ideal gas is an accurate model of a real gas under most conditions. According to the kinetic-molecular theory, the particles in a gas are in random motion at high speeds and undergoing elastic collisions with each other. When a gas particle hits a container's surface, it rebounds, which changes its momentum. The impulses exerted by many of these collisions result in gas pressure on the surface.



**Atmospheric pressure** On every square centimeter of Earth's surface at sea level, the atmospheric gas exerts a force of approximately 10 N, about the weight of a 1-kg object. The pressure of Earth's atmosphere on your body is so well balanced by your body's outward forces that you seldom notice it. You probably become aware of this pressure only when your ears pop as the result of pressure changes, as when you ride an elevator in a tall building or fly in an airplane. Atmospheric pressure is about 10 N per 1 cm<sup>2</sup> (10<sup>-4</sup> m<sup>2</sup>), which is about 1.0×10<sup>5</sup> N/m<sup>2</sup>, or 100 kPa. Other planets in our solar system also have atmospheres. The pressure exerted by these atmospheres, however, varies widely. For example, the pressure at the surface of Venus is about 92 times the pressure at the surface of Earth, while the pressure at the surface of Mars is less than 1 percent of Earth's.

Table 13-1	
Some Typical Pressures	
Location	Pressure (Pa)
The center of the Sun	3×10 <sup>16</sup>
The center of Earth	4×10 <sup>11</sup>
The deepest ocean trench	1.1×10 <sup>8</sup>
Standard atmosphere	1.01325×10 <sup>5</sup>
Blood pressure	1.6×10 <sup>4</sup>
Air pressure on top of Mt. Everest	3×10 <sup>4</sup>
The best vacuum	1×10 <sup>-13</sup>

### ▶ EXAMPLE Problem 1

**Calculating Pressure** A child weighs 364 N and sits on a three-legged stool, which weighs 41 N. The bottoms of the stool's legs touch the ground over a total area of 19.3 cm<sup>2</sup>.

- What is the average pressure that the child and stool exert on the ground?
- How does the pressure change when the child leans over so that only two legs of the stool touch the floor?

#### 1 Analyze and Sketch the Problem

- Sketch the child and the stool, labeling the total force that they exert on the ground.
- List the variables, including the force that the child and stool exert on the ground and the areas for parts **a** and **b**.

	<b>Known:</b>	<b>Unknown:</b>
$F_{g \text{ child}} = 364 \text{ N}$	$A_A = 19.3 \text{ cm}^2$	$P_A = ?$
$F_{g \text{ stool}} = 41 \text{ N}$	$A_B = \frac{2}{3} \times 19.3 \text{ cm}^2$	$P_B = ?$
$F_{g \text{ total}} = F_{g \text{ child}} + F_{g \text{ stool}}$	$= 12.9 \text{ cm}^2$	
$= 364 \text{ N} + 41 \text{ N}$		
$= 405 \text{ N}$		



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#### 2 Solve for the Unknown

Find each pressure.

$$P = F/A$$

a.  $P_A = \left( \frac{405 \text{ N}}{19.3 \text{ cm}^2} \right) \left( \frac{(100 \text{ cm})^2}{(1 \text{ m})^2} \right)$       **Substitute  $F = F_{g \text{ total}} = 405 \text{ N}$ ,  $A = A_A = 19.3 \text{ cm}^2$**   
 $= 2.10 \times 10^2 \text{ kPa}$

b.  $P_B = \left( \frac{405 \text{ N}}{12.9 \text{ cm}^2} \right) \left( \frac{(100 \text{ cm})^2}{(1 \text{ m})^2} \right)$       **Substitute  $F = F_{g \text{ total}} = 405 \text{ N}$ ,  $A = A_B = 12.9 \text{ cm}^2$**   
 $= 3.14 \times 10^2 \text{ kPa}$

#### 3 Evaluate the Answer

- Are the units correct?** The units for pressure should be Pa, and 1 N/m<sup>2</sup> = 1 Pa.

1. The atmospheric pressure at sea level is about  $1.0 \times 10^5$  Pa. What is the force at sea level that air exerts on the top of a desk that is 152 cm long and 76 cm wide?
2. A car tire makes contact with the ground on a rectangular area of 12 cm by 18 cm. If the car's mass is 925 kg, what pressure does the car exert on the ground as it rests on all four tires?
3. A lead brick, 5.0 cm  $\times$  10.0 cm  $\times$  20.0 cm, rests on the ground on its smallest face. Lead has a density of 11.8 g/cm<sup>3</sup>. What pressure does the brick exert on the ground?
4. In a tornado, the pressure can be 15 percent below normal atmospheric pressure. Suppose that a tornado occurred outside a door that is 195 cm high and 91 cm wide. What net force would be exerted on the door by a sudden 15 percent drop in normal atmospheric pressure? In what direction would the force be exerted?
5. In industrial buildings, large pieces of equipment must be placed on wide steel plates that spread the weight of the equipment over larger areas. If an engineer plans to install a 454-kg device on a floor that is rated to withstand additional pressure of  $5.0 \times 10^4$  Pa, how large should the steel support plate be?

## The Gas Laws

As scientists first studied gases and pressure, they began to notice some interesting relationships. The first relationship to emerge was named Boyle's law, after seventeenth-century chemist and physicist Robert Boyle. Boyle's law states that for a fixed sample of gas at constant temperature, the volume of the gas varies inversely with the pressure. Because the product of inversely related variables is a constant, Boyle's law can be written  $PV = \text{constant}$ , or  $P_1V_1 = P_2V_2$ . The subscripts that you see in the gas laws will help you keep track of different variables, such as pressure and volume, as they change throughout a problem. These variables can be rearranged to solve for an unknown pressure or volume. As shown in **Figure 13-3**, the relationship between the pressure and the volume of a gas is critical to the sport of scuba diving.

A second relationship was discovered about 100 years after Boyle's work by Jacques Charles. When Charles cooled a gas, the volume shrank by  $\frac{1}{273}$  of its original volume for every degree cooled, which is a linear relationship. At the time, Charles could not cool gases to the extremely low temperatures achieved in modern laboratories. In order to see what lower limits might be possible, he extended, or extrapolated, the graph of his data to these temperatures. This extrapolation suggested that if the temperature were reduced to  $-273^\circ\text{C}$ , a gas would have zero volume. The temperature at which a gas would have zero volume is now called absolute zero, which is represented by the zero of the Kelvin temperature scale.

These experiments indicated that under constant pressure, the volume of a sample of gas varies directly with its Kelvin temperature, a result that is now called Charles's law. Charles's law can be written  $V/T = \text{constant}$ , or  $V_1/T_1 = V_2/T_2$ .

■ **Figure 13-3** The gas in the tank on the diver's back is at high pressure. This pressure is reduced by the regulator so that the pressure of the gas the diver breathes is equal to the water pressure. In the photo, you can see bubbles coming from the regulator.



Combining Boyle's law and Charles's law relates the pressure, temperature, and volume of a fixed amount of ideal gas, which leads to the equation called the **combined gas law**.

**Combined Gas Law**  $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \text{constant}$

For a fixed amount of an ideal gas, the pressure times the volume, divided by the Kelvin temperature equals a constant.

As shown in **Figure 13-4**, the combined gas law reduces to Boyle's law under conditions of constant temperature and to Charles's law under conditions of constant pressure.

**The ideal gas law** You can use the kinetic-molecular theory to discover how the constant in the combined gas law depends on the number of particles,  $N$ . Suppose that the volume and temperature of an ideal gas are held constant. If the number of particles increases, the number of collisions that the particles make with the container will increase, thereby increasing the pressure. Removing particles decreases the number of collisions, and thus, decreases the pressure. You can conclude that the constant in the combined gas law equation is proportional to  $N$ .

$$\frac{PV}{T} = kN$$

The constant,  $k$ , is called Boltzmann's constant, and its value is  $1.38 \times 10^{-23} \text{ Pa} \cdot \text{m}^3/\text{K}$ . Of course,  $N$ , the number of particles, is a very large number. Instead of using  $N$ , scientists often use a unit called a mole. One mole (abbreviated mol and represented in equations by  $n$ ) is similar to one dozen, except that instead of representing 12 items, one mole represents  $6.022 \times 10^{23}$  particles. This number is called Avogadro's number, after Italian scientist Amedeo Avogadro.

Avogadro's number is numerically equal to the number of particles in a sample of matter whose mass equals the molar mass of the substance. You can use this relationship to convert between mass and  $n$ , the number of moles present. Using moles instead of the number of particles changes Boltzmann's constant. This new constant is abbreviated  $R$ , and it has the value  $8.31 \text{ Pa} \cdot \text{m}^3/\text{mol} \cdot \text{K}$ . Rearranging, you can write the **ideal gas law** in its most familiar form.

**Ideal Gas Law**  $PV = nRT$

For an ideal gas, the pressure times the volume is equal to the number of moles multiplied by the constant  $R$  and the Kelvin temperature.

Note that with the given value of  $R$ , volume must be expressed in  $\text{m}^3$ , temperature in K, and pressure in Pa. In practice, the ideal gas law predicts the behavior of gases remarkably well, except under conditions of high pressures or low temperatures.

## MINI LAB

### Pressure

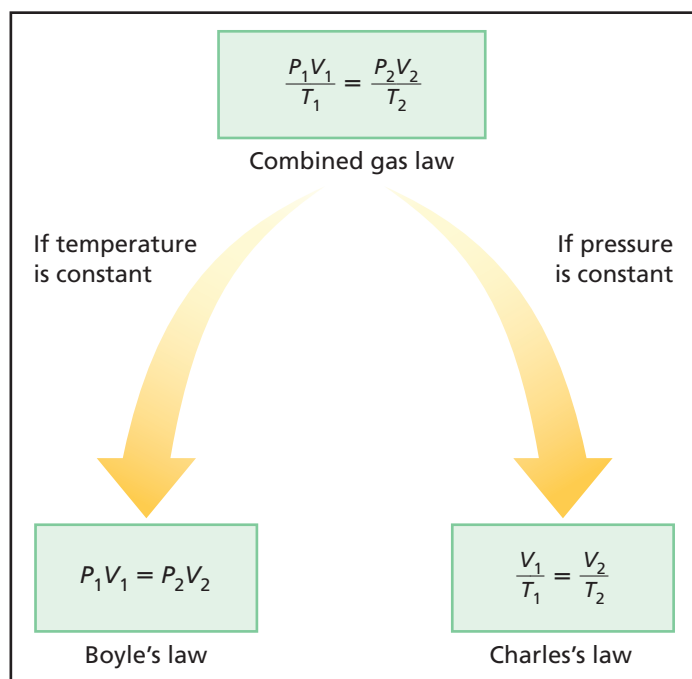
How much pressure do you exert when standing on one foot? Have a partner trace your foot, and then use the outline to estimate its area.

- Determine** your weight in newtons and the area of the outline in  $\text{m}^2$ .
- Calculate** the pressure.
- Compare and contrast** the pressure you exert on the ground with the pressure exerted by various objects. For example, you could weigh a brick and determine the pressure it exerts when resting on different faces.

### Analyze and Conclude

- How do shoes with high heels affect the pressure that a person exerts on the ground?

**Figure 13-4** You can use the combined gas law to derive both Boyle's and Charles's laws. What happens if you hold volume constant?





## ▶ EXAMPLE Problem 2

**Gas Laws** A 20.0-L sample of argon gas at 273 K is at atmospheric pressure, 101.3 kPa. The temperature is lowered to 120 K, and the pressure is increased to 145 kPa.

- What is the new volume of the argon sample?
- Find the number of moles of argon atoms in the argon sample.
- Find the mass of the argon sample. The molar mass,  $M$ , of argon is 39.9 g/mol.

### 1 Analyze and Sketch the Problem

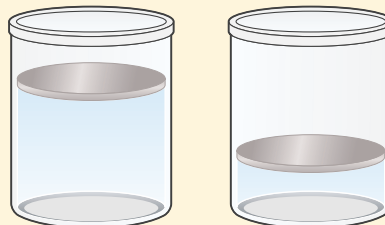
- Sketch the situation. Indicate the conditions in the container of argon before and after the change in temperature and pressure.
- List the known and unknown variables.

**Known:**

$$\begin{aligned}V_1 &= 20.0 \text{ L} \\P_1 &= 101.3 \text{ kPa} \\T_1 &= 273 \text{ K} \\P_2 &= 145 \text{ kPa} \\T_2 &= 120 \text{ K} \\R &= 8.31 \text{ Pa}\cdot\text{m}^3/\text{mol}\cdot\text{K} \\M_{\text{argon}} &= 39.9 \text{ g/mol}\end{aligned}$$

**Unknown:**

$$\begin{aligned}V_2 &= ? \\ \text{moles of argon} &= ? \\ \text{mass of argon sample} &= ?\end{aligned}$$



$$\begin{aligned}T_1 &= 273 \text{ K} \\P_1 &= 101.3 \text{ kPa} \\V_1 &= 20.0 \text{ L}\end{aligned}$$

$$\begin{aligned}T_2 &= 120 \text{ K} \\P_2 &= 145 \text{ kPa} \\V_2 &= ?\end{aligned}$$

### 2 Solve for the Unknown

- Use the combined gas law and solve for  $V_2$ .

$$\begin{aligned}\frac{P_1 V_1}{T_1} &= \frac{P_2 V_2}{T_2} \\V_2 &= \frac{P_1 V_1 T_2}{P_2 T_1} \\ &= \frac{(101.3 \text{ kPa})(20.0 \text{ L})(120 \text{ K})}{(145 \text{ kPa})(273 \text{ K})} \\ &= 6.1 \text{ L}\end{aligned}$$

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Substitute  $P_1 = 101.3 \text{ kPa}$ ,  $P_2 = 145 \text{ kPa}$ ,  
 $V_1 = 20.0 \text{ L}$ ,  $T_1 = 273 \text{ K}$ ,  $T_2 = 120 \text{ K}$

- Use the ideal gas law and solve for  $n$ .

$$\begin{aligned}PV &= nRT \\n &= \frac{PV}{RT} \\ &= \frac{(101.3 \times 10^3 \text{ Pa})(0.0200 \text{ m}^3)}{(8.31 \text{ Pa}\cdot\text{m}^3/\text{mol}\cdot\text{K})(273 \text{ K})} \\ &= 0.893 \text{ mol}\end{aligned}$$

Substitute  $P = 101.3 \times 10^3 \text{ Pa}$ ,  $V = 0.0200 \text{ m}^3$ ,  
 $R = 8.31 \text{ m}^3/\text{mol}\cdot\text{K}$ ,  $T = 273 \text{ K}$

- Use the molar mass to convert from moles of argon in the sample to mass of the sample.

$$\begin{aligned}m &= Mn \\m_{\text{argon sample}} &= (39.9 \text{ g/mol})(0.893 \text{ mol}) \\ &= 35.6 \text{ g}\end{aligned}$$

Substitute  $M = 39.9 \text{ g/mol}$ ,  $n = 0.893 \text{ mol}$

### 3 Evaluate the Answer

- Are the units correct?** The volume,  $V_2$ , is in liters, and the mass of the sample is in grams.
- Is the magnitude realistic?** The change in volume is consistent with an increase in pressure and decrease in temperature. The calculated mass of the argon sample is reasonable.

6. A tank of helium gas used to inflate toy balloons is at a pressure of  $15.5 \times 10^6$  Pa and a temperature of 293 K. The tank's volume is  $0.020 \text{ m}^3$ . How large a balloon would it fill at 1.00 atmosphere and 323 K?
7. What is the mass of the helium gas in the previous problem? The molar mass of helium gas is 4.00 g/mol.
8. A tank containing 200.0 L of hydrogen gas at  $0.0^\circ\text{C}$  is kept at 156 kPa. The temperature is raised to  $95^\circ\text{C}$ , and the volume is decreased to 175 L. What is the new pressure of the gas?
9. The average molar mass of the components of air (mainly diatomic oxygen gas and diatomic nitrogen gas) is about 29 g/mol. What is the volume of 1.0 kg of air at atmospheric pressure and  $20.0^\circ\text{C}$ ?

## Thermal Expansion

As you applied the combined gas law, you discovered how gases expand as their temperatures increase. When heated, all forms of matter—solids, liquids, and gases—generally become less dense and expand to fill more space. This property, known as **thermal expansion**, has many useful applications, such as circulating air in a room. When the air near the floor of a room is warmed, gravity pulls the denser, colder air near the ceiling down, which pushes the warmer air upward. This circulation of air within a room is called a convection current. **Figure 13-5** shows convection currents starting as the hot air above the flames rises. You also can see convection currents in a pot of hot, but not boiling, water on a stove. When the pot is heated from the bottom, the colder and denser water sinks to the bottom where it is warmed and then pushed up by the continuous flow of cooler water from the top.

This thermal expansion occurs in most liquids. A good model for all liquids does not exist, but it is useful to think of a liquid as a finely ground solid. Groups of two, three, or more particles move together as if they were tiny pieces of a solid. When a liquid is heated, particle motion causes these groups to expand in the same way that particles in a solid are pushed apart. The spaces between groups increase. As a result, the whole liquid expands. With an equal change in temperature, liquids expand considerably more than solids, but not as much as gases.

**Why ice floats** Because matter expands as it is heated, you might predict that ice would be more dense than water, and therefore, it should sink. However, when water is heated from  $0^\circ\text{C}$  to  $4^\circ\text{C}$ , instead of expanding, it contracts as the forces between particles increase and the ice crystals collapse. These forces between water molecules are strong, and the crystals that make up ice have a very open structure. Even when ice melts, tiny crystals remain. These remaining crystals are melting, and the volume of the water decreases until the temperature reaches  $4^\circ\text{C}$ . However, once the temperature of water moves above  $4^\circ\text{C}$ , its volume increases because of greater molecular motion. The practical result is that water is most dense at  $4^\circ\text{C}$  and ice floats. This fact is very important to our lives and environment. If ice sank, lakes would freeze from the bottom each winter and many would never melt completely in the summer.

■ **Figure 13-5** This image was made by a special technique that enables you to see different densities in the air. Convection currents are set up as warmer, less dense air rises and cooler, denser air sinks.



■ **Figure 13-6** The colorful lighting effects in neon signs are caused by luminous plasmas formed in the glass tubing.



## Plasma

If you heat a solid, it melts to form a liquid. Further heating results in a gas. What happens if you increase the temperature still further? Collisions between the particles become violent enough to tear the electrons off the atoms, thereby producing positively charged ions. The gaslike state of negatively charged electrons and positively charged ions is called **plasma**. Plasma is considered to be another fluid state of matter.

The plasma state may seem to be uncommon; however, most of the matter in the universe is plasma. Stars consist mostly of plasma at extremely high temperatures. Much of the matter between stars and galaxies consists of energetic hydrogen that has no electrons. This hydrogen is in the plasma state. The primary difference between gas and plasma is that plasma can conduct electricity, whereas gas cannot. Lightning bolts are in the plasma state. Neon signs, such as the one shown in **Figure 13-6**, fluorescent bulbs, and sodium vapor lamps all contain glowing plasma.

## 13.1 Section Review

- 10. Pressure and Force** Suppose that you have two boxes. One is  $20\text{ cm} \times 20\text{ cm} \times 20\text{ cm}$ . The other is  $20\text{ cm} \times 20\text{ cm} \times 40\text{ cm}$ .
  - a. How does the pressure of the air on the outside of the two boxes compare?
  - b. How does the magnitude of the total force of the air on the two boxes compare?
- 11. Meteorology** A weather balloon used by meteorologists is made of a flexible bag that allows the gas inside to freely expand. If a weather balloon containing  $25.0\text{ m}^3$  of helium gas is released from sea level, what is the volume of gas when the balloon reaches a height of  $2100\text{ m}$ , where the pressure is  $0.82 \times 10^5\text{ Pa}$ ? Assume that the temperature is unchanged.
- 12. Gas Compression** In a certain internal-combustion engine,  $0.0021\text{ m}^3$  of air at atmospheric pressure and  $303\text{ K}$  is rapidly compressed to a pressure of  $20.1 \times 10^5\text{ Pa}$  and a volume of  $0.0003\text{ m}^3$ . What is the final temperature of the compressed gas?
- 13. Density and Temperature** Starting at  $0^\circ\text{C}$ , how will the density of water change if it is heated to  $4^\circ\text{C}$ ? To  $8^\circ\text{C}$ ?
- 14. The Standard Molar Volume** What is the volume of  $1\text{ mol}$  of a gas at atmospheric pressure and a temperature of  $273\text{ K}$ ?
- 15. The Air in a Refrigerator** How many moles of air are in a refrigerator with a volume of  $0.635\text{ m}^3$  at a temperature of  $2.00^\circ\text{C}$ ? If the average molar mass of air is  $29\text{ g/mol}$ , what is the mass of the air in the refrigerator?
- 16. Critical Thinking** Compared to the particles that make up carbon dioxide gas, the particles that make up helium gas are very small. What can you conclude about the number of particles in a  $2.0\text{-L}$  sample of carbon dioxide gas compared to the number of particles in a  $2.0\text{-L}$  sample of helium gas if both samples are at the same temperature and pressure?



## 13.2 Forces Within Liquids

The liquids considered thus far have been ideal liquids, in which the particles are totally free to slide past one another. The unexpected behavior of water between 0°C and 4°C, however, illustrates that in real fluids, particles exert electromagnetic forces of attraction, called **cohesive forces**, on each other. These and other forces affect the behavior of fluids.

### Cohesive Forces

Have you ever noticed that dewdrops on spiderwebs and falling drops of oil are nearly spherical? What happens when rain falls on a freshly washed and waxed car? The water drops bead up into rounded shapes, as shown in the spiderweb in **Figure 13-7**. All of these phenomena are examples of surface tension, which is the tendency of the surface of a liquid to contract to the smallest possible area. Surface tension is a result of the cohesive forces among the particles of a liquid.

Notice that beneath the surface of the liquid shown in **Figure 13-8a** on the next page, each particle of the liquid is attracted equally in all directions by neighboring particles, and even to the particles of the wall of the container. As a result, no net force acts on any of the particles beneath the surface. At the surface, however, the particles are attracted downward and to the sides, but not upward. There is a net downward force, which acts on the top layers and causes the surface layer to be slightly compressed. The surface layer acts like a tightly stretched rubber sheet or a film that is strong enough to support the weight of very light objects, such as the water strider in **Figure 13-8b** on the next page. The surface tension of water also can support a steel paper clip, even though the density of steel is nine times greater than that of water. Try it!

Why does surface tension produce spherical drops? The force pulling the surface particles into a liquid causes the surface to become as small as possible, and the shape that has the least surface for a given volume is a sphere. The higher the surface tension of the liquid, the more resistant the liquid is to having its surface broken. For example, liquid mercury has much stronger cohesive forces than water does. Thus, liquid mercury forms spherical drops, even when it is placed on a smooth surface. On the other hand, liquids such as alcohol and ether have weaker cohesive forces. A drop of either of these liquids flattens out on a smooth surface.

**Viscosity** In nonideal fluids, the cohesive forces and collisions between fluid molecules cause internal friction that slows the fluid flow and dissipates mechanical energy. The measure of this internal friction is called the viscosity of the liquid. Water is not very viscous, but motor oil is very viscous. As a result of its viscosity, motor oil flows slowly over the parts of an engine to coat the metal and reduce rubbing. Lava, molten rock that flows from a volcano or vent in Earth's surface, is one of the most viscous fluids. There are several types of lava, and the viscosity of each type varies with composition and temperature.

#### ► Objectives

- **Explain** how cohesive forces cause surface tension.
- **Explain** how adhesive forces cause capillary action.
- **Discuss** evaporative cooling and the role of condensation in cloud formation.

#### ► Vocabulary

cohesive forces  
adhesive forces

■ **Figure 13-7** Rainwater beads up on a spider's web because water drops have surface tension.

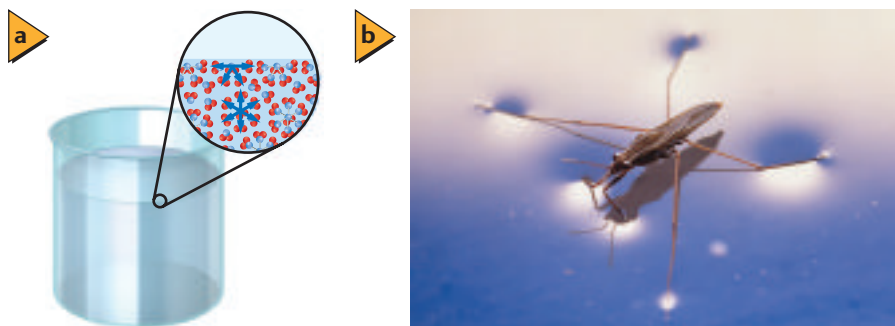


#### Geology Connection





■ **Figure 13-8** Molecules in the interior of a liquid are attracted in all directions **(a)**. A water strider can walk on water because molecules at the surface have a net inward attraction that results in surface tension **(b)**.



## Adhesive Forces

Similar to cohesive forces, **adhesive forces** are electromagnetic attractive forces that act between particles of different substances. If a glass tube with a small inner diameter is placed in water, the water rises inside the tube. The water rises because the adhesive forces between glass and water molecules are stronger than the cohesive forces between water molecules. This phenomenon is called capillary action. The water continues to rise until the weight of the water that is lifted balances the total adhesive force between the glass and water molecules. If the radius of the tube increases, the volume and the weight of the water will increase proportionally faster than the surface area of the tube. Thus, water is lifted higher in a narrow tube than in a wider one. Capillary action causes molten wax to rise in a candle's wick and water to move up through the soil and into the roots of plants.

When a glass tube is placed in a beaker of water, the surface of the water climbs the outside of the tube, as shown in **Figure 13-9a**. The adhesive forces between the glass molecules and water molecules are greater than the cohesive forces between the water molecules. In contrast, the cohesive forces between mercury molecules are greater than the adhesive forces between the mercury and glass molecules, so the liquid does not climb the tube. These forces also cause the center of the mercury's surface to depress, as shown in **Figure 13-9b**.

## Evaporation and Condensation

Why does a puddle of water disappear on a hot, dry day? As you learned in Chapter 12, the particles in a liquid are moving at random speeds. If a fast-moving particle can break through the surface layer, it will escape from the liquid. Because there is a net downward cohesive force at the surface, however, only the most energetic particles escape. This escape of particles is called evaporation.



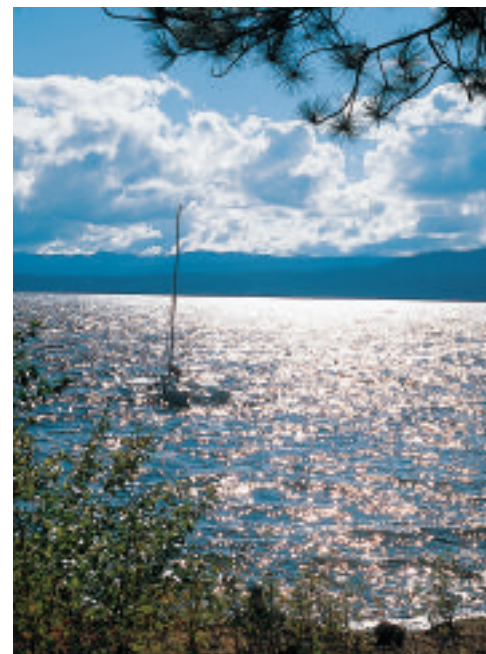
■ **Figure 13-9** Water climbs the outside wall of this glass tube **(a)**, while the mercury is depressed by the rod **(b)**. The forces of attraction between mercury atoms are stronger than any adhesive forces between the mercury and the glass.

**Evaporative cooling** Evaporation has a cooling effect. On a hot day, your body perspires, and the evaporation of your sweat cools you down. In a puddle of water, evaporation causes the remaining liquid to cool down. Each time a particle with higher-than-average kinetic energy escapes from the water, the average kinetic energy of the remaining particles decreases. As you learned in Chapter 12, a decrease in average kinetic energy is a decrease in temperature. You can test this cooling effect by pouring a small amount of rubbing alcohol in the palm of your hand. Alcohol molecules evaporate easily because they have weak cohesive forces. As the molecules evaporate, the cooling effect is quite noticeable. A liquid that evaporates quickly is called a volatile liquid.

Have you ever wondered why humid days feel warmer than dry days at the same temperature? On a day that is humid, the water vapor content of the air is high. Because there are already many water molecules in the air, the water molecules in perspiration are less likely to evaporate from the skin. Evaporation is the body's primary cooling mechanism, so the body is not able to cool itself as effectively on a humid day.

Particles of liquid that have evaporated into the air can also return to the liquid phase if the kinetic energy or temperature decreases, a process called condensation. What happens if you bring a cold glass into a hot, humid area? The outside of the glass soon becomes coated with condensed water. Water molecules moving randomly in the air surrounding the glass strike the cold surface, and if they lose enough energy, the cohesive forces become strong enough to prevent their escape.

The air above any body of water, as shown in **Figure 13-10**, contains evaporated water vapor, which is water in the form of gas. If the temperature is reduced, the water vapor condenses around tiny dust particles in the air and produces droplets only 0.01 mm in diameter. A cloud of these droplets is called fog. Fog often forms when moist air is chilled by the cold ground. Fog also can form in your home. When a carbonated drink is opened, the sudden decrease in pressure causes the temperature of the gas in the container to drop, which condenses the water vapor dissolved in that gas.



■ **Figure 13-10** Warm, moist, surface air rises until it reaches a height where the temperature is at the point at which water vapor condenses and forms clouds.

## 13.2 Section Review

- 17. Evaporation and Cooling** In the past, when a baby had a high fever, the doctor might have suggested gently sponging off the baby with rubbing alcohol. Why would this help?
- 18. Surface Tension** A paper clip, which has a density greater than that of water, can be made to stay on the surface of water. What procedures must you follow for this to happen? Explain.
- 19. Language and Physics** The English language includes the terms *adhesive tape* and *working as a cohesive group*. In these terms, are *adhesive* and *cohesive* being used in the same context as their meanings in physics?
- 20. Adhesion and Cohesion** In terms of adhesion and cohesion, explain why alcohol clings to the surface of a glass rod but mercury does not.
- 21. Floating** How can you tell that the paper clip in problem 18 was not floating?
- 22. Critical Thinking** On a hot, humid day, Beth sat on the patio with a glass of cold water. The outside of the glass was coated with water. Her younger sister, Jo, suggested that the water had leaked through the glass from the inside to the outside. Suggest an experiment that Beth could do to show Jo where the water came from.



## 13.3 Fluids at Rest and in Motion

### Objectives

- **Relate** Pascal's principle to simple machines and occurrences.
- **Apply** Archimedes' principle to buoyancy.
- **Apply** Bernoulli's principle to airflow.

### Vocabulary

Pascal's principle  
buoyant force  
Archimedes' principle  
Bernoulli's principle  
streamlines

You have learned how fluids exert pressure, the force per unit area. You also know that the pressure exerted by fluids changes; for example, atmospheric pressure drops as you climb a mountain. In this section, you will learn about the forces exerted by resting and moving fluids.

### Fluids at Rest

If you have ever dived deep into a swimming pool or lake, you know that your body, especially your ears, is sensitive to changes in pressure. You may have noticed that the pressure you felt on your ears did not depend on whether your head was upright or tilted, but that if you swam deeper, the pressure increased.

**Pascal's principle** Blaise Pascal, a French physician, noted that the pressure in a fluid depends upon the depth of the fluid and has nothing to do with the shape of the fluid's container. He also discovered that any change in pressure applied at any point on a confined fluid is transmitted undiminished throughout the fluid, a fact that is now known as **Pascal's principle**. Every time you squeeze a tube of toothpaste, you demonstrate Pascal's principle. The pressure that your fingers exert at the bottom of the tube is transmitted through the toothpaste and forces the paste out at the top. Likewise, if you squeeze one end of a helium balloon, the other end of the balloon expands.

When fluids are used in machines to multiply forces, Pascal's principle is being applied. In a common hydraulic system, a fluid is confined to two connecting chambers, as shown in **Figure 13-11**. Each chamber has a piston that is free to move, and the pistons have different surface areas. If a force,  $F_1$ , is exerted on the first piston with a surface area of  $A_1$ , the pressure,  $P_1$ , exerted on the fluid can be determined by using the following equation.

$$P_1 = \frac{F_1}{A_1}$$

This equation is simply the definition of pressure: pressure equals the force per unit area. The pressure exerted by the fluid on the second piston, with a surface area  $A_2$ , can also be determined.

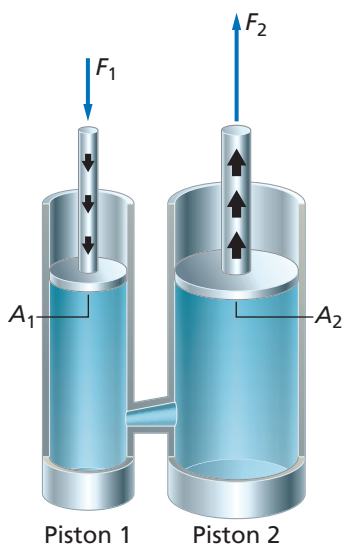
$$P_2 = \frac{F_2}{A_2}$$

According to Pascal's principle, pressure is transmitted without change throughout a fluid, so pressure  $P_2$  is equal in value to  $P_1$ . You can determine the force exerted by the second piston by using  $F_1/A_1 = F_2/A_2$  and solving for  $F_2$ . This force is shown by the following equation.

$$\text{Force Exerted by a Hydraulic Lift } F_2 = \frac{F_1 A_2}{A_1}$$

The force exerted by the second piston is equal to the force exerted by the first piston multiplied by the ratio of the area of the second piston to the area of the first piston.

■ **Figure 13-11** The pressure exerted by the force of the small piston is transmitted throughout the fluid and results in a multiplied force on the larger piston.



23. Dentists' chairs are examples of hydraulic-lift systems. If a chair weighs 1600 N and rests on a piston with a cross-sectional area of 1440 cm<sup>2</sup>, what force must be applied to the smaller piston, with a cross-sectional area of 72 cm<sup>2</sup>, to lift the chair?
24. A mechanic exerts a force of 55 N on a 0.015 m<sup>2</sup> hydraulic piston to lift a small automobile. The piston that the automobile sits on has an area of 2.4 m<sup>2</sup>. What is the weight of the automobile?
25. By multiplying a force, a hydraulic system serves the same purpose as a lever or seesaw. If a 400-N child standing on one piston is balanced by a 1100-N adult standing on another piston, what is the ratio of the areas of their pistons?
26. In a machine shop, a hydraulic lift is used to raise heavy equipment for repairs. The system has a small piston with a cross-sectional area of  $7.0 \times 10^{-2}$  m<sup>2</sup> and a large piston with a cross-sectional area of  $2.1 \times 10^{-1}$  m<sup>2</sup>. An engine weighing  $2.7 \times 10^3$  N rests on the large piston.
  - a. What force must be applied to the small piston to lift the engine?
  - b. If the engine rises 0.20 m, how far does the smaller piston move?

## Swimming Under Pressure

When you are swimming, you feel the pressure of the water increase as you dive deeper. This pressure is actually a result of gravity; it is related to the weight of the water above you. The deeper you go, the more water there is above you, and the greater the pressure. The pressure of the water is equal to the weight,  $F_g$ , of the column of water above you divided by the column's cross-sectional area,  $A$ . Even though gravity pulls only in the downward direction, the fluid transmits the pressure in all directions: up, down, and to the sides. You can find the pressure of the water by applying the following equation.

$$P = \frac{F_g}{A}$$

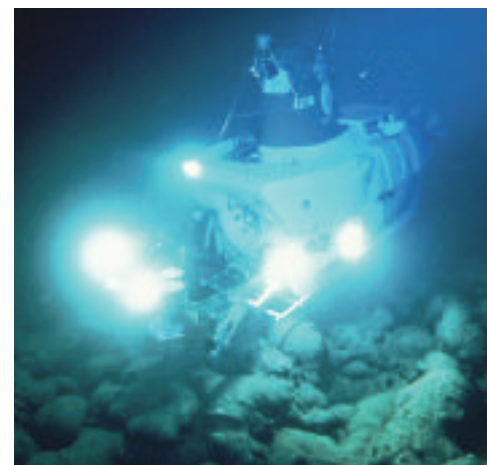
The weight of the column of water is  $F_g = mg$ , and the mass is equal to the density,  $\rho$ , of the water times its volume,  $m = \rho V$ . You also know that the volume of the water is the area of the base of the column times its height,  $V = Ah$ . Therefore,  $F_g = \rho Ahg$ . Substituting  $\rho Ahg$  for  $F_g$  in the equation for water pressure gives  $P = F_g/A = \rho Ahg/A$ . Divide  $A$  from the numerator and denominator to arrive at the simplest form of the equation for the pressure exerted by a column of water on a submerged body.

### Pressure of Water on a Body $P = \rho hg$

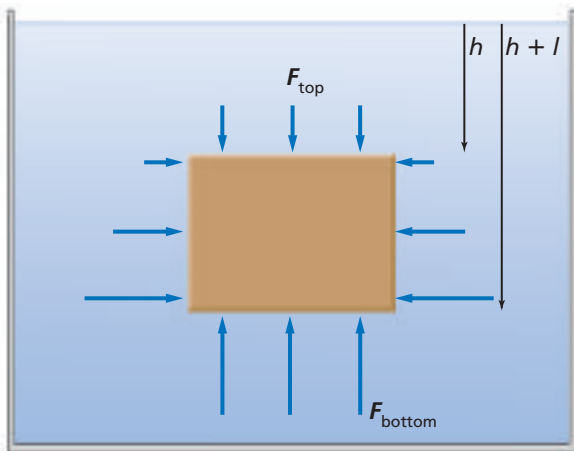
The pressure that a column of water exerts on a body is equal to the density of water times the height of the column times the acceleration due to gravity.

This formula works for all fluids, not just water. The pressure of a fluid on a body depends on the density of the fluid, its depth, and  $g$ . If there were water on the Moon, the pressure of the water at any depth would be one-sixth as great as on Earth. As illustrated in **Figure 13-12**, submersibles, both crewed and robotic, have explored the deepest ocean trenches and encountered pressures in excess of 1000 times standard air pressure.

■ **Figure 13-12** In 1960, the *Trieste*, a crewed submersible, descended to the bottom of the Marianas Trench, a depth of over 10,500 m. The crewed submersible *Alvin*, shown below, can safely dive to a depth of 4500 m.







■ **Figure 13-13** A fluid exerts a greater upward force on the bottom of an immersed object than the downward force on the top of the object. The net upward force is called the buoyant force.

**Buoyancy** What produces the upward force that allows you to swim? The increase in pressure with increasing depth creates an upward force called the **buoyant force**. By comparing the buoyant force on an object with its weight, you can predict whether the object will sink or float.

Suppose that a box is immersed in water. It has a height of  $l$  and its top and bottom each have a surface area of  $A$ . Its volume, then, is  $V = lA$ . Water pressure exerts forces on all sides, as shown in **Figure 13-13**. Will the box sink or float? As you know, the pressure on the box depends on its depth,  $h$ . To find out whether the box will float in water, you will need to determine how the pressure on the top of the box compares with the pressure from below the box. Compare these two equations:

$$F_{\text{top}} = P_{\text{top}} A = \rho h g A$$

$$F_{\text{bottom}} = P_{\text{bottom}} A = \rho(l + h)gA$$

On the four vertical sides, the forces are equal in all directions, so there is no net horizontal force. The upward force on the bottom is larger than the downward force on the top, so there is a net upward force. The buoyant force can now be determined.

$$\begin{aligned} F_{\text{buoyant}} &= F_{\text{bottom}} - F_{\text{top}} \\ &= \rho(l + h)gA - \rho h g A \\ &= \rho l g A = \rho V g \end{aligned}$$

These calculations show the net upward force to be proportional to the volume of the box. This volume equals the volume of the fluid displaced, or pushed out of the way, by the box. Therefore, the magnitude of the buoyant force,  $\rho V g$ , equals the weight of the fluid displaced by the object.

**Buoyant Force**  $F_{\text{buoyant}} = \rho_{\text{fluid}} V g$

The buoyant force on an object is equal to the weight of the fluid displaced by the object, which is equal to the density of the fluid in which the object is immersed multiplied by the object's volume and the acceleration due to gravity.

This relationship was discovered in the third century B.C. by Greek scientist Archimedes. **Archimedes' principle** states that an object immersed in a fluid has an upward force on it that is equal to the weight of the fluid displaced by the object. The force does not depend on the weight of the object, only on the weight of the displaced fluid.

**Sink or float?** If you want to know whether an object sinks or floats, you have to take into account all of the forces acting on the object. The buoyant force pushes up, but the weight of the object pulls it down. The difference between the buoyant force and the object's weight determines whether an object sinks or floats.

Suppose that you submerge three objects in a tank filled with water ( $\rho_{\text{water}} = 1.00 \times 10^3 \text{ kg/m}^3$ ). Each of the objects has a volume of  $100 \text{ cm}^3$ , or  $1.00 \times 10^{-4} \text{ m}^3$ . The first object is a steel block with a mass of  $0.90 \text{ kg}$ .



The second is an aluminum soda can with a mass of 0.10 kg. The third is an ice cube with a mass of 0.090 kg. How will each item move when it is immersed in water? The upward force on all three objects, as shown in **Figure 13-14**, is the same, because all displace the same weight of water. This buoyant force can be calculated as follows.

$$\begin{aligned} F_{\text{buoyant}} &= \rho_{\text{water}} Vg \\ &= (1.00 \times 10^3 \text{ kg/m}^3)(1.00 \times 10^{-4} \text{ m}^3)(9.80 \text{ m/s}^2) \\ &= 0.980 \text{ N} \end{aligned}$$

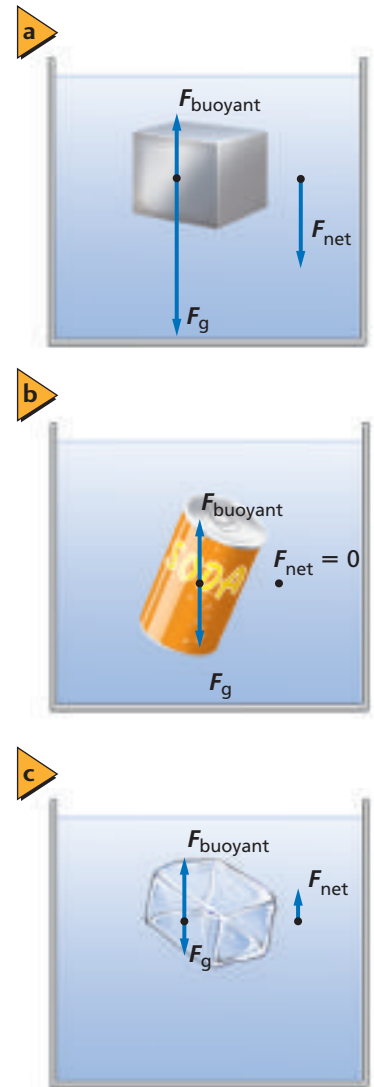
The weight of the block of steel is 8.8 N, much greater than the buoyant force. There is a net downward force, so the block will sink to the bottom of the tank. The net downward force, its apparent weight, is less than its real weight. All objects in a liquid, even those that sink, have an apparent weight that is less than when the object is in air. The apparent weight can be expressed by the equation  $F_{\text{apparent}} = F_g - F_{\text{buoyant}}$ . For the block of steel, the apparent weight is  $8.8 \text{ N} - 0.98 \text{ N}$ , or  $7.8 \text{ N}$ .

The weight of the soda can is 0.98 N, the same as the weight of the water displaced. There is, therefore, no net force, and the can will remain wherever it is placed in the water. It has neutral buoyancy. Objects with neutral buoyancy are described as being weightless; their apparent weight is zero. This property is similar to that experienced by astronauts in orbit, which is why astronaut training sometimes takes place in swimming pools.

The weight of the ice cube is 0.88 N, less than the buoyant force, so there is a net upward force, and the ice cube will rise. At the surface, the net upward force will lift part of the ice cube out of the water. As a result, less water will be displaced, and the upward force will be reduced. The ice cube will float with enough volume in the water so that the weight of water displaced equals the weight of the ice cube. An object will float if its density is less than the density of the fluid in which it is immersed.

**Ships** Archimedes' principle explains why ships can be made of steel and still float; if the hull is hollow and large enough so that the average density of the ship is less than the density of water, the ship will float. You may have noticed that a ship loaded with cargo rides lower in the water than a ship with an empty cargo hold. You can demonstrate this effect by fashioning a small boat out of folded aluminum foil. The boat should float easily, and it will ride lower in the water if you add a cargo of paper clips. If the foil is crumpled into a tight ball, the boat will sink because of its increased density. Similarly, the continents of Earth float upon a denser material below the surface. The drifting motion of these continental plates is responsible for the present shapes and locations of the continents.

Other examples of Archimedes' principle in action include submarines and fishes. Submarines take advantage of Archimedes' principle as water is pumped into or out of a number of different chambers to change the submarine's average density, causing it to rise or sink. Fishes that have swim bladders also use Archimedes' principle to control their depths. Such a fish can expand or contract its swim bladder, just like you can puff up your cheeks. To move upward in the water, the fish expands its swim bladder to displace more water and increase the buoyant force. The fish moves downward by contracting the volume of its swim bladder.



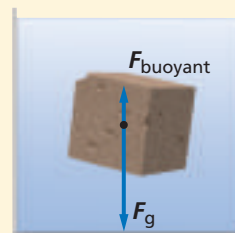
**Figure 13-14** A block of steel **(a)**, an aluminum can of soda **(b)**, and an ice cube **(c)** all have the same volume, displace the same amount of water, and experience the same buoyant force. However, because their weights are different, the net forces on the three objects are also different.

### Geology Connection

## ▶ EXAMPLE Problem 3

**Archimedes' Principle** A cubic decimeter,  $1.00 \times 10^{-3} \text{ m}^3$ , of a granite building block is submerged in water. The density of granite is  $2.70 \times 10^3 \text{ kg/m}^3$ .

- What is the magnitude of the buoyant force acting on the block?
- What is the apparent weight of the block?



### 1 Analyze and Sketch the Problem

- Sketch the cubic decimeter of granite immersed in water.
- Show the upward buoyant force and the downward force due to gravity acting on the granite.

**Known:**

$$V = 1.00 \times 10^{-3} \text{ m}^3$$

$$\rho_{\text{granite}} = 2.70 \times 10^3 \text{ kg/m}^3$$

$$\rho_{\text{water}} = 1.00 \times 10^3 \text{ kg/m}^3$$

**Unknown:**

$$F_{\text{buoyant}} = ?$$

$$F_{\text{apparent}} = ?$$

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### 2 Solve for the Unknown

- Calculate the buoyant force on the granite block.

$$\begin{aligned} F_{\text{buoyant}} &= \rho_{\text{water}} Vg \\ &= (1.00 \times 10^3 \text{ kg/m}^3)(1.00 \times 10^{-3} \text{ m}^3)(9.80 \text{ m/s}^2) \\ &= 9.80 \text{ N} \end{aligned}$$

Substitute  $\rho_{\text{water}} = 1.00 \times 10^3 \text{ kg/m}^3$ ,  
 $V = 1.00 \times 10^{-3} \text{ m}^3$ ,  $g = 9.80 \text{ m/s}^2$

- Calculate the granite's weight and then find its apparent weight.

$$\begin{aligned} F_g &= mg = \rho_{\text{granite}} Vg \\ &= (2.70 \times 10^3 \text{ kg/m}^3)(1.00 \times 10^{-3} \text{ m}^3)(9.80 \text{ m/s}^2) \\ &= 26.5 \text{ N} \end{aligned}$$

Substitute  $\rho_{\text{granite}} = 2.70 \times 10^3 \text{ kg/m}^3$ ,  
 $V = 1.00 \times 10^{-3} \text{ m}^3$ ,  $g = 9.80 \text{ m/s}^2$

$$\begin{aligned} F_{\text{apparent}} &= F_g - F_{\text{buoyant}} \\ &= 26.5 \text{ N} - 9.80 \text{ N} = 16.7 \text{ N} \end{aligned}$$

Substitute  $F_g = 26.5 \text{ N}$ ,  $F_{\text{buoyant}} = 9.80 \text{ N}$

### 3 Evaluate the Answer

- Are the units correct?** The forces and apparent weight are in newtons, as expected.
- Is the magnitude realistic?** The buoyant force is about one-third the weight of the granite, a sensible answer because the density of water is about one-third that of granite.

## ▶ PRACTICE Problems

### Additional Problems, Appendix B

- Common brick is about 1.8 times denser than water. What is the apparent weight of a  $0.20 \text{ m}^3$  block of bricks under water?
- A girl is floating in a freshwater lake with her head just above the water. If she weighs 610 N, what is the volume of the submerged part of her body?
- What is the tension in a wire supporting a 1250-N camera submerged in water? The volume of the camera is  $16.5 \times 10^{-3} \text{ m}^3$ .
- Plastic foam is about 0.10 times as dense as water. What weight of bricks could you stack on a  $1.0 \text{ m} \times 1.0 \text{ m} \times 0.10 \text{ m}$  slab of foam so that the slab of foam floats in water and is barely submerged, leaving the bricks dry?
- Canoes often have plastic foam blocks mounted under the seats for flotation in case the canoe fills with water. What is the approximate minimum volume of foam needed for flotation for a 480-N canoe?

## Fluids in Motion: Bernoulli's Principle

Try the experiment shown in **Figure 13-15**. Hold a strip of notebook paper just under your lower lip. Then blow hard across the top surface. Why does the strip of paper rise? The blowing of the air has decreased the air pressure above the paper. Because this pressure decreases, the pressure in the still air below the paper pushes the paper upward. The relationship between the velocity and pressure exerted by a moving fluid is named for Swiss scientist Daniel Bernoulli. **Bernoulli's principle** states that as the velocity of a fluid increases, the pressure exerted by that fluid decreases. This principle is a statement of work and energy conservation as applied to fluids.

One instance in which fluid velocity can increase is when it flows through a constriction. The nozzles on some garden hoses can be opened or narrowed so that the velocity of the water spray can be changed. You may have seen the water in a stream speed up as it passed through narrowed sections of the stream bed. As the nozzle of the hose and the stream channel become wider or narrower, the velocity of the fluid changes to maintain the overall flow of water. In addition to streams and hoses, the pressure of blood in our circulatory systems depends partly on Bernoulli's principle. Treatments of heart disease involve removing obstructions in the arteries and veins and preventing clots in the blood.

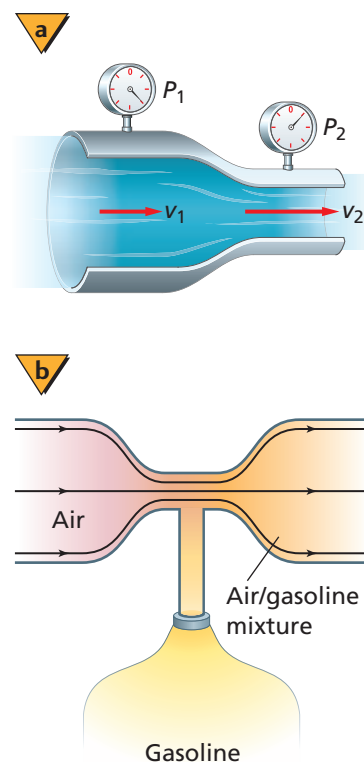
Consider a horizontal pipe completely filled with a smoothly flowing ideal fluid. If a certain mass of the fluid enters one end of the pipe, then an equal mass must come out the other end. Now consider a section of pipe with a cross section that becomes narrower, as shown in **Figure 13-16a**. To keep the same mass of fluid moving through the narrow section in a fixed amount of time, the velocity of the fluid must increase. As the fluid's velocity increases, so does its kinetic energy. This means that net work has been done on the swifter fluid. This net work comes from the difference between the work that was done to move the mass of fluid into the pipe and the work that was done by the fluid pushing the same mass out of the pipe. The work is proportional to the force on the fluid, which, in turn, depends on the pressure. If the net work is positive, the pressure at the input end of the section, where the velocity is lower, must be larger than the pressure at the output end, where the velocity is higher.

**Applications of Bernoulli's principle** There are many common applications of Bernoulli's principle, such as paint sprayers and perfume bottles. The simple atomizer on a perfume bottle works by blowing air across the top of a tube sunk into perfume, which creates lower pressure at the top of the tube than in the bottle. As a result, perfume is forced into the air flow. A gasoline engine's carburetor, which is where air and gas are mixed, is another common application of Bernoulli's principle. Part of the carburetor is a tube with a constriction, as shown in the diagram in **Figure 13-16b**. The pressure on the gasoline in the fuel supply is the same as the pressure in the thicker part of the tube. Air flowing through the narrow section of the tube, which is attached to the fuel supply, is at a lower pressure, so fuel is forced into the air flow. By regulating the flow of air in the tube, the amount of fuel mixed into the air can be varied. Newer cars tend to have fuel injectors rather than carburetors, but carburetors are common in older cars and the motors of small gasoline-powered machines, such as lawn mowers.



■ **Figure 13-15** Blowing across the surface of a sheet of paper demonstrates Bernoulli's principle.

■ **Figure 13-16** Pressure  $P_1$  is greater than  $P_2$  because  $v_1$  is less than  $v_2$  (a). In a carburetor, low pressure in the narrow part of the tube draws fuel into the air flow (b).





■ **Figure 13-17** The smooth streamlines show the air flowing above a car that is being tested in a wind tunnel.



**Streamlines** Automobile and aircraft manufacturers spend a great deal of time and money testing new designs in wind tunnels to ensure the greatest efficiency of movement through air. The flow of fluids around objects is represented by **streamlines**, as shown in **Figure 13-17**. Objects require less energy to move through a smooth streamlined flow.

Streamlines can best be illustrated by a simple demonstration. Imagine carefully squeezing tiny drops of food coloring into a smoothly flowing fluid. If the colored lines that form stay thin and well defined, the flow is said to be streamlined. Notice that if the flow narrows, the streamlines move closer together. Closely spaced streamlines indicate greater velocity and therefore reduced pressure. If streamlines swirl and become diffused, the flow of the fluid is said to be turbulent. Bernoulli's principle does not apply to turbulent flow.

## 13.3 Section Review

- 32. Floating and Sinking** Does a full soda pop can float or sink in water? Try it. Does it matter whether or not the drink is diet? All soda pop cans contain the same volume of liquid, 354 mL, and displace the same volume of water. What is the difference between a can that sinks and one that floats?
- 33. Floating and Density** A fishing bobber made of cork floats with one-tenth of its volume below the water's surface. What is the density of cork?
- 34. Floating in Air** A helium balloon rises because of the buoyant force of the air lifting it. The density of helium is  $0.18 \text{ kg/m}^3$ , and the density of air is  $1.3 \text{ kg/m}^3$ . How large a volume would a helium balloon need to lift a 10-N lead brick?
- 35. Transmission of Pressure** A toy rocket launcher is designed so that a child stomps on a rubber cylinder, which increases the air pressure in a launching tube and pushes a foam rocket into the sky. If the child stomps with a force of 150 N on a  $2.5 \times 10^{-3} \text{ m}^2$  area piston, what is the additional force transmitted to the  $4.0 \times 10^{-4} \text{ m}^2$  launch tube?
- 36. Pressure and Force** An automobile weighing  $2.3 \times 10^4 \text{ N}$  is lifted by a hydraulic cylinder with an area of  $0.15 \text{ m}^2$ .
- What is the pressure in the hydraulic cylinder?
  - The pressure in the lifting cylinder is produced by pushing on a  $0.0082 \text{ m}^2$  cylinder. What force must be exerted on this small cylinder to lift the automobile?
- 37. Displacement** Which of the following displaces more water when it is placed in an aquarium?
- A 1.0-kg block of aluminum or a 1.0-kg block of lead?
  - A  $10\text{-cm}^3$  block of aluminum or a  $10\text{-cm}^3$  block of lead?
- 38. Critical Thinking** As you discovered in Practice Problem 4, a tornado passing over a house sometimes makes the house explode from the inside out. How might Bernoulli's principle explain this phenomenon? What could be done to reduce the danger of a door or window exploding outward?



## 13.4 Solids

**H**ow do solids and liquids differ? Solids are stiff, they can be cut in pieces, and they retain their shapes. You can push on solids. Liquids flow and if you push your finger on water, your finger will move through it. However, if you have ever watched butter warm and begin to lose its shape, you may have wondered if the line between solids and liquids is always distinct.

### Solid Bodies

Under certain conditions, solids and liquids are not easily distinguished. As bottle glass is heated through the molten state, the change from solid to liquid is so gradual that it is difficult to tell which is which. Some solids, such as crystalline quartz, are made of particles that are lined up in orderly patterns. Other solids, such as glass, are made of jumbled arrangements of particles, just like a liquid. As shown in **Figure 13-18**, quartz and fused quartz (also called quartz glass) are the same chemically, but their physical properties are quite different.

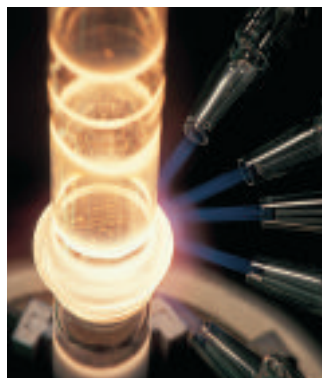
When the temperature of a liquid is lowered, the average kinetic energy of the particles decreases. As the particles slow down, the cohesive forces have more effect, and for many solids, the particles become frozen into a fixed pattern called a **crystal lattice**, shown in **Figure 13-19** on the next page. Although the cohesive forces hold the particles in place, the particles in a crystalline solid do not stop moving completely. Rather, they vibrate around their fixed positions. In other materials, such as butter and glass, the particles do not form a fixed crystalline pattern. Such a substance, which has no regular crystal structure but does have a definite volume and shape, is called an **amorphous solid**. Amorphous solids also are classified as viscous, or slowly flowing, liquids.

**Pressure and freezing** As a liquid becomes a solid, its particles usually fit more closely together than in the liquid state, making solids more dense than liquids. As you have learned, however, water is an exception because it is most dense at 4°C. Water is also an exception to another general rule. For most liquids, an increase in the pressure on the surface of the liquid increases its freezing point. Because water expands as it freezes, an increase in pressure forces the molecules closer together and opposes the freezing. Therefore, higher pressure lowers the freezing point of water very slightly.

a



b



#### ► Objectives

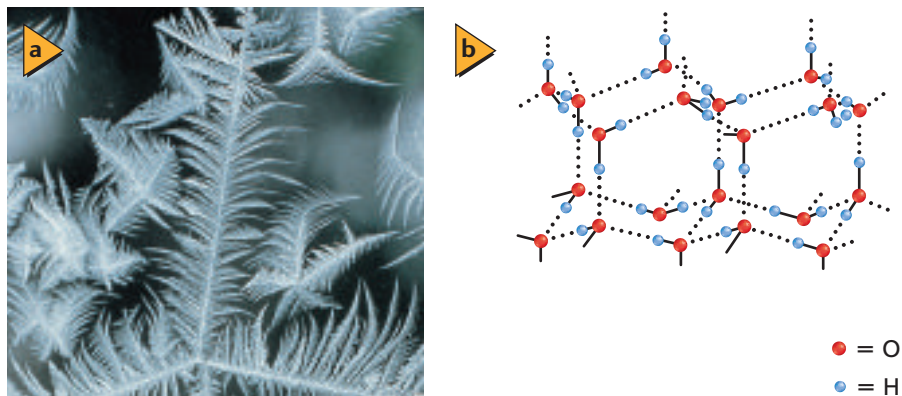
- **Relate** the properties of solids to their structures.
- **Explain** why solids expand and contract when the temperature changes.
- **Calculate** the expansion of solids.
- **Explain** the importance of thermal expansion.

#### ► Vocabulary

crystal lattice  
amorphous solid  
coefficient of linear expansion  
coefficient of volume expansion

■ **Figure 13-18** In crystalline quartz, the particles are arranged in an orderly pattern **(a)**. A crystalline solid melts at a specific temperature. Fused quartz is the same chemical as crystalline quartz, but the particles are jumbled randomly in the solid. When fused quartz melts, its properties change slowly over a range of temperatures, allowing it to be worked in a fashion similar to everyday glass **(b)**.

■ **Figure 13-19** Ice, the solid form of water, has a larger volume than an equal mass of its liquid form **(a)**. The crystalline structure of ice is in the form of a lattice **(b)**.



It has been hypothesized that the drop in water's freezing point caused by the pressure of an ice-skater's blades produces a thin film of liquid between the ice and the blades. Calculations of the pressure caused by even the sharpest blade show that the ice is still too cold to melt. More recent measurements, however, have shown that the friction between the blade and the ice generates enough thermal energy to melt the ice and create a thin layer of water. This explanation is supported by measurements of the spray of ice particles, which are considerably warmer than the ice itself. The same process of melting occurs during snow skiing.

**Elasticity of solids** External forces applied to a solid object may twist or bend it out of shape. The ability of a solid object to return to its original form when the external forces are removed is called the elasticity of the solid. If too much deformation occurs, the object will not return to its original shape because its elastic limit has been exceeded. Elasticity depends on the electromagnetic forces that hold the particles of a substance together. Malleability and ductility are two properties that depend on the structure and elasticity of a substance. Because gold can be flattened and shaped into thin sheets, it is said to be malleable. Copper is a ductile metal because it can be pulled into thin strands of wire.

■ **Figure 13-20** The extreme temperatures of a summer day caused these railroad tracks to buckle.



## Thermal Expansion of Solids

It is standard practice for engineers to design small gaps, called expansion joints, into concrete-and-steel highway bridges to allow for the expansion of parts in the heat of summer. Objects expand only a small amount when they are heated, but that small amount could be several centimeters in a 100-m-long bridge. If expansion gaps were not present, the bridge could buckle or parts of it could break. High temperatures can also damage railroad tracks that are laid without expansion joints, as shown in **Figure 13-20**. Some materials, such as the ovenproof glass that is used for cooking and laboratory experiments, are designed to have the least possible thermal expansion. Large telescope mirrors are made of a ceramic material that is designed to undergo essentially no thermal expansion.

To understand the expansion of heated solids, picture a solid as a collection of particles connected by springs that represent the attractive forces between the particles. When the particles get too close, the springs push them apart. When a solid is heated, the kinetic energy of the particles increases, and they vibrate rapidly and move farther apart, weakening the

attractive forces between the particles. As a result, when the particles vibrate more violently with increased temperature, their average separation increases and the solid expands.

The change in length of a solid is proportional to the change in temperature, as shown in **Figure 13-21**. A solid will expand in length twice as much when its temperature is increased by 20°C than when it is increased by 10°C. The expansion also is proportional to its length. A 2-m bar will expand twice as much as a 1-m bar with the same change in temperature. The length,  $L_2$ , of a solid at temperature  $T_2$  can be found with the following equation, where  $L_1$  is the length at temperature  $T_1$  and alpha,  $\alpha$ , is the **coefficient of linear expansion**.

$$L_2 = L_1 + \alpha L_1(T_2 - T_1)$$

Using simple algebra, you can solve for  $\alpha$ .

$$L_2 - L_1 = \alpha L_1(T_2 - T_1)$$

$$\Delta L = \alpha L_1 \Delta T$$

**Coefficient of Linear Expansion**  $\alpha = \frac{\Delta L}{L_1 \Delta T}$

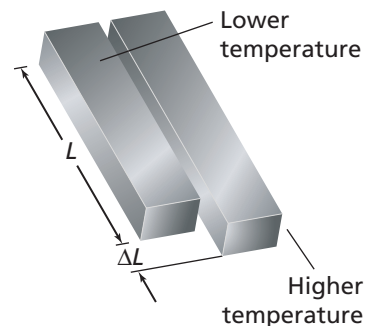
The coefficient of linear expansion is equal to the change in length, divided by the original length and the change in temperature.

The unit for the coefficient of linear expansion is  $1/^\circ\text{C}$ , or  $(^\circ\text{C})^{-1}$ . Since solids expand in three directions, the **coefficient of volume expansion**,  $\beta$ , is about three times the coefficient of linear expansion.

**Coefficient of Volume Expansion**  $\beta = \frac{\Delta V}{V_1 \Delta T}$

The coefficient of volume expansion is equal to the change in volume divided by the original volume and the change in temperature.

Again, the unit for  $\beta$  is  $1/^\circ\text{C}$ , or  $(^\circ\text{C})^{-1}$ . The two coefficients of thermal expansion for a variety of materials are given in **Table 13-2**.



**Figure 13-21** The change in length of a material is proportional to the original length and the change in temperature.

Table 13-2		
Coefficients of Thermal Expansion at 20°C		
Material	Coefficient of Linear Expansion, $\alpha(^\circ\text{C})^{-1}$	Coefficient of Volume Expansion, $\beta(^\circ\text{C})^{-1}$
<b>Solids</b>		
Aluminum	$25 \times 10^{-6}$	$75 \times 10^{-6}$
Glass (soft)	$9 \times 10^{-6}$	$27 \times 10^{-6}$
Glass (ovenproof)	$3 \times 10^{-6}$	$9 \times 10^{-6}$
Concrete	$12 \times 10^{-6}$	$36 \times 10^{-6}$
Copper	$16 \times 10^{-6}$	$48 \times 10^{-6}$
<b>Liquids</b>		
Methanol		$1200 \times 10^{-6}$
Gasoline		$950 \times 10^{-6}$
Water		$210 \times 10^{-6}$



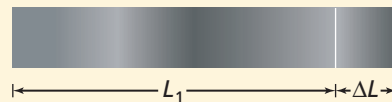


## ▶ EXAMPLE Problem 4

**Linear Expansion** A metal bar is 1.60 m long at room temperature, 21°C. The bar is put into an oven and heated to a temperature of 84°C. It is then measured and found to be 1.7 mm longer. What is the coefficient of linear expansion of this material?

### 1 Analyze and Sketch the Problem

- Sketch the bar, which is 1.7 mm longer at 84°C than at 21°C.
- Identify the initial length of the bar,  $L_1$ , and the change in length,  $\Delta L$ .



**Known:**

$$\begin{aligned}L_1 &= 1.60 \text{ m} \\ \Delta L &= 1.7 \times 10^{-3} \text{ m} \\ T_1 &= 21^\circ\text{C} \\ T_2 &= 84^\circ\text{C}\end{aligned}$$

**Unknown:**

$$\alpha = ?$$

### 2 Solve for the Unknown

Calculate the coefficient of linear expansion using the known length, change in length, and change in temperature.

$$\begin{aligned}\alpha &= \frac{\Delta L}{L_1 \Delta T} \\ &= \frac{1.7 \times 10^{-3} \text{ m}}{(1.60 \text{ m})(84^\circ\text{C} - 21^\circ\text{C})} && \text{Substitute } \Delta L = 1.7 \times 10^{-3} \text{ m, } L_1 = 1.60 \text{ m,} \\ & && \Delta T = (T_2 - T_1) = 84^\circ\text{C} - 21^\circ\text{C} \\ &= 1.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1}\end{aligned}$$

#### Math Handbook

Operations with Significant Digits pages 835–836

### 3 Evaluate the Answer

- **Are the units correct?** The units are correctly expressed in  $^\circ\text{C}^{-1}$ .
- **Is the magnitude realistic?** The magnitude of the coefficient is close to the accepted value for copper.

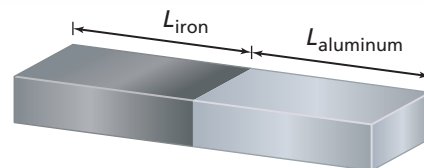
## ▶ PRACTICE Problems

Additional Problems, Appendix B

39. A piece of aluminum house siding is 3.66 m long on a cold winter day of  $-28^\circ\text{C}$ . How much longer is it on a very hot summer day at  $39^\circ\text{C}$ ?
40. A piece of steel is 11.5 cm long at  $22^\circ\text{C}$ . It is heated to  $1221^\circ\text{C}$ , close to its melting temperature. How long is it?
41. A 400-mL glass beaker at room temperature is filled to the brim with cold water at  $4.4^\circ\text{C}$ . When the water warms up to  $30.0^\circ\text{C}$ , how much water will spill from the beaker?
42. A tank truck takes on a load of 45,725 L of gasoline in Houston, where the temperature is  $28.0^\circ\text{C}$ . The truck delivers its load in Minneapolis, where the temperature is  $-12.0^\circ\text{C}$ .
  - a. How many liters of gasoline does the truck deliver?
  - b. What happened to the gasoline?
43. A hole with a diameter of 0.85 cm is drilled into a steel plate. At  $30.0^\circ\text{C}$ , the hole exactly accommodates an aluminum rod of the same diameter. What is the spacing between the plate and the rod when they are cooled to  $0.0^\circ\text{C}$ ?
44. A steel ruler is marked in millimeters so that the ruler is absolutely correct at  $30.0^\circ\text{C}$ . By what percentage would the ruler be incorrect at  $-30.0^\circ\text{C}$ ?

## CHALLENGE PROBLEM

You need to make a 1.00-m-long bar that expands with temperature in the same way as a 1.00-m-long bar of copper would. As shown in the figure at the right, your bar must be made from a bar of iron and a bar of aluminum attached end to end. How long should each of them be?



**Applications of thermal expansion** Different materials expand at different rates, as indicated by the different coefficients of expansion given in Table 13-2. Engineers must consider these different expansion rates when designing structures. Steel bars are often used to reinforce concrete, and therefore the steel and concrete must have the same expansion coefficient. Otherwise, the structure could crack on a hot day. Similarly, a dentist must use filling materials that expand and contract at the same rate as tooth enamel.

Different rates of expansion have useful applications. For example, engineers have taken advantage of these differences to construct a useful device called a bimetallic strip, which is used in thermostats. A bimetallic strip consists of two strips of different metals welded or riveted together. Usually, one strip is brass and the other is iron. When heated, brass expands more than iron does. Thus, when the bimetallic strip of brass and iron is heated, the brass part of the strip becomes longer than the iron part. The bimetallic strip bends with the brass on the outside of the curve. If the bimetallic strip is cooled, it bends in the opposite direction. The brass is then on the inside of the curve.

In a home thermostat, shown in **Figure 13-22**, the bimetallic strip is installed so that it bends toward an electric contact as the room cools. When the room cools below the setting on the thermostat, the bimetallic strip bends enough to make electric contact with the switch, which turns on the heater. As the room warms, the bimetallic strip bends in the other direction. When the room's temperature reaches the setting on the thermostat, the electric circuit is broken and the heater switches off.



■ **Figure 13-22** In this thermostat, a coiled bimetallic strip controls the flow of mercury for opening and closing electrical switches.

## 13.4 Section Review

45. **Relative Thermal Contraction** On a hot day, you are installing an aluminum screen door in a concrete door frame. You want the door to fit well on a cold winter day. Should you make the door fit tightly in the frame or leave extra room?
46. **States of Matter** Why could candle wax be considered a solid? Why might it also be considered a viscous liquid?
47. **Thermal Expansion** Can you heat a piece of copper enough to double its length?
48. **States of Matter** Does Table 13-2 provide a way to distinguish between solids and liquids?
49. **Solids and Liquids** A solid can be defined as a material that can be bent and will resist bending. Explain how these properties relate to the binding of atoms in a solid, but do not apply to a liquid.
50. **Critical Thinking** The iron ring in **Figure 13-23** was made by cutting a small piece from a solid ring. If the ring in the figure is heated, will the gap become wider or narrower? Explain your answer.



■ **Figure 13-23**

## Evaporative Cooling

Alternate CBL instructions can be found on the Web site.  
[physicspp.com](http://physicspp.com)

If you have ever spilled a small amount of rubbing alcohol on your skin, you probably noticed how cool it felt. You have learned that this coolness is caused by evaporation. In this experiment, you will test the rates at which different types of alcohol evaporate. An alcohol is a substance that has a hydroxyl functional group ( $-OH$ ) attached to a carbon or a chain of carbons. From your observations of evaporative cooling, you will infer the relative strength of the cohesive forces in the tested alcohols.

### QUESTION

How do the rates of evaporation compare for different alcohols?

#### Objectives

- **Collect and organize data** for the evaporation of alcohols.
- **Compare and contrast** the rates of evaporation for various alcohols.
- **Analyze** why some alcohols evaporate faster than others.
- **Infer** the relationship between cohesive forces and the rate of evaporation.

#### Safety Precautions



- **The chemicals used in this experiment are flammable and poisonous. Do not inhale the fumes from these chemicals. Do not have any open flame near these chemicals. Use in a well-ventilated room or fume hood.**
- **Avoid contact with the chemicals on your skin or clothing. Notify your teacher immediately if an accident or spill occurs.**
- **Wash your hands after the lab is over.**

#### Materials

methanol (methyl alcohol)  
 ethanol (ethyl alcohol)  
 2-propanol (isopropyl alcohol)  
 masking tape (two pieces)  
 thermometer (non-mercury)  
 filter paper (three pieces,  $2.5\text{ cm} \times 2.5\text{ cm}$ )  
 small rubber bands

#### Procedure

1. Wrap the thermometer with a square piece of filter paper fastened by a small rubber band. To do this, first slip the rubber band onto the thermometer. Then, wrap the paper around the thermometer and roll the rubber band over the wrapped paper. The paper should fit snugly over the thermometer's end.
2. Obtain a small beaker of methanol. Place the paper-covered end of the thermometer in the container of methanol. Do not let the container fall over. Keep the thermometer in the container for 1 min.



Data Table			
Liquid	$T_2$ (°C)	$T_1$ (°C)	$\Delta T$ (°C)
Methyl alcohol			
Ethyl alcohol			
Isopropyl alcohol			

- After 1 min has elapsed, record the temperature reading on the thermometer in the data table under  $T_1$ . This is the initial temperature of the methanol.
- Remove the thermometer from the methanol. Place the thermometer over the edge of a table top so that the thermometer's tip extends about 5 cm beyond the edge of the table. Use the masking tape to anchor the thermometer in place.
- Observe the temperature during the experiment. After 4 min have elapsed, observe and record the temperature in the data table in the column marked  $T_2$ .
- Roll the rubber band up the thermometer and dispose of the filter paper as directed by your teacher.
- Repeat steps 1–6, but use ethanol as the liquid. Record your results in the data table.
- Repeat steps 1–6, but use isopropyl alcohol as the liquid. Record your results in the data table.

### Analyze

- Interpret Data** Did the thermometer show a temperature increase or decrease for your trials? Why?
- Calculate  $\Delta T$  for each of your liquids by finding the difference between the ending temperatures and the initial temperatures of the liquids ( $T_2 - T_1$ ).
- Using the chemical formulas for methanol ( $\text{CH}_3\text{OH}$ ), ethanol ( $\text{C}_2\text{H}_5\text{OH}$ ), and isopropyl alcohol ( $\text{C}_3\text{H}_7\text{OH}$ ), determine the molar mass of each of the liquids you tested. You will need to refer to the periodic table to determine the molar masses.
- Infer** What can the  $\Delta T$  for each trial tell you about the rates of evaporation of the alcohols?
- Think Critically** Why was paper used on the thermometer instead of using only the thermometer?

### Conclude and Apply

- Using the rates of evaporation of the alcohols you studied, how can you determine which alcohol had the strongest cohesive forces?
- Which alcohol had the weakest cohesive forces?
- What general trend did you find between the change in temperature ( $\Delta T$ ) and the molar mass of an alcohol?
- Hypothesize** Would a fan blowing in the lab room change the room's air temperature? Would it change the  $\Delta T$  that you observed? Explain.

### Going Further

Predict the size of  $\Delta T$  for 1-butanol, which has the formula  $\text{C}_4\text{H}_9\text{OH}$ , relative to the alcohols that you tested.

### Real-World Physics

The National Weather Service began using a new windchill index in 2001. The old chart was based on data derived from water-freezing experiments done in Antarctica in the 1940s. Explain how windchill relates to evaporative cooling, why this phenomenon is important in cold weather, and how the new chart improves upon the old chart.



To find out more about states of matter, visit the Web site: [physicspp.com](http://physicspp.com)



## A Strange Matter

You now are familiar with the four most common states of matter: solid, liquid, gas, and plasma. But did you know that there is a fifth state of matter? Meet the Bose-Einstein Condensate (BEC).

### What is a Bose-Einstein Condensate?

The origins of the BEC are in the 1920s in Satyendra Nath Bose's studies of the quantum rules governing photon energies. Einstein applied Bose's equations to atoms. The equations showed that if the temperature of certain atoms were low enough, most of the atoms would be in the same quantum level. In other words, at extremely low temperatures, atoms that had occupied different energy states suddenly fall into the lowest possible energy state. At these temperatures, which are not found in nature, but are created in the lab through some ingenious technology, the atoms of a BEC cannot be distinguished. Even their positions are identical.

### How is a BEC created?

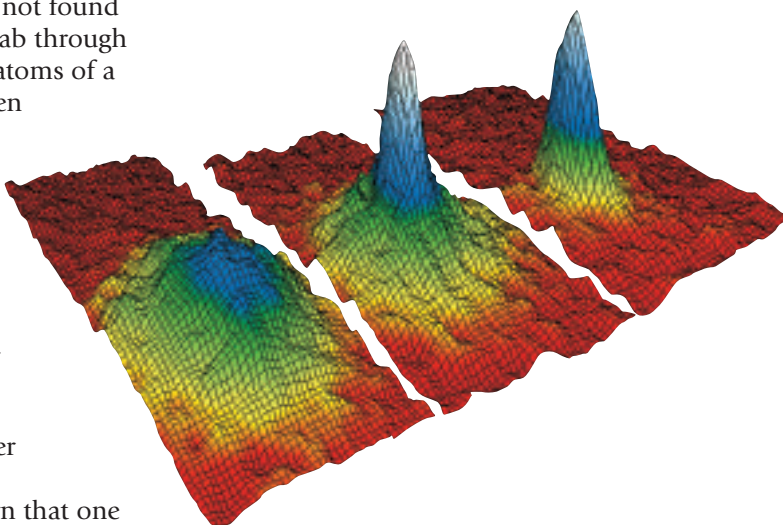
The first BEC was created in 1995 by Eric Cornell and Carl Wieman of Boulder, Colorado. To make their BEC, Cornell and Wieman used rubidium atoms. They had to figure out how to cool these atoms to a lower temperature than had ever been achieved.

You might be surprised to learn that one important step in reaching the necessary temperatures was to use lasers to cool the rubidium atoms. Lasers can burn through metal, but they also can cool a sample of atoms if they are tuned so that their photons bounce off the atoms. In this case, the photons will carry off some of the atoms' energy, lowering the temperature of the sample. But the laser will not cool the sample unless it is precisely tuned.

When the laser is tuned to the proper frequency, the result is a sample of very cold atoms known as "optical molasses." How can the optical molasses be contained? Thermal contact between the optical molasses and a

material container surely would warm the chilled atoms because any container would have a higher temperature than the atoms. So the container of choice for the BEC is a nonmaterial one—a container formed by combining lasers with a magnetic field.

While optical molasses is cold (about  $1/10,000$  K), it is not cold enough to form a BEC. Scientists use evaporative cooling to make the final step to the required temperatures. In evaporative cooling, the optical molasses is contained in a stronger magnetic container that allows the highest energy atoms to escape. The atoms with the lowest possible energy are left behind. These are the atoms that suddenly condense to form a BEC. The images below show the final cooling of a sample of atoms to form a BEC (the central lump) at fifty-billionths of a kelvin.



In these three images, the central peak forms as atoms condense to a BEC.

## Going Further

1. **Evaluate** What sorts of difficulties might scientists have to overcome in finding applications for the BEC?
2. **Compare and Contrast** Is the evaporative cooling process that creates the BEC the same process that helps keep you cool on a hot day? Explain.

## 13.1 Properties of Fluids

**Vocabulary**

- fluids (p. 342)
- pressure (p. 342)
- pascal (p. 342)
- combined gas law (p. 345)
- ideal gas law (p. 345)
- thermal expansion (p. 347)
- plasma (p. 348)

**Key Concepts**

- Matter in the fluid state flows and has no definite shape of its own.
- Pressure is the force divided by the area on which it is exerted.

$$P = \frac{F}{A}$$

- The combined gas law can be used to calculate changes in the volume, temperature, and pressure of an ideal gas.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

- The ideal gas law can be written as follows.

$$PV = nRT$$

## 13.2 Forces Within Liquids

**Vocabulary**

- cohesive forces (p. 349)
- adhesive forces (p. 350)

**Key Concepts**

- Cohesive forces are the attractive forces that like particles exert on one another. Surface tension and viscosity both result from cohesive forces.
- Adhesive forces are the attractive forces that particles of different substances exert on one another. Capillary action results from adhesive forces.

## 13.3 Fluids at Rest and in Motion

**Vocabulary**

- Pascal's principle (p. 352)
- buoyant force (p. 354)
- Archimedes' principle (p. 354)
- Bernoulli's principle (p. 357)
- streamlines (p. 358)

**Key Concepts**

- According to Pascal's principle, an applied pressure change is transmitted undiminished throughout a fluid.

$$F_2 = \frac{F_1 A_2}{A_1}$$

- Pressure at any depth is proportional to the fluid's weight above that depth.

$$P = \rho hg$$

- According to Archimedes' principle, the buoyant force equals the weight of the fluid displaced by an object.

$$F_{\text{buoyant}} = \rho_{\text{fluid}} V g$$

- Bernoulli's principle states that the pressure exerted by a fluid decreases as its velocity increases.

## 13.4 Solids

**Vocabulary**

- crystal lattice (p. 359)
- amorphous solid (p. 359)
- coefficient of linear expansion (p. 361)
- coefficient of volume expansion (p. 361)

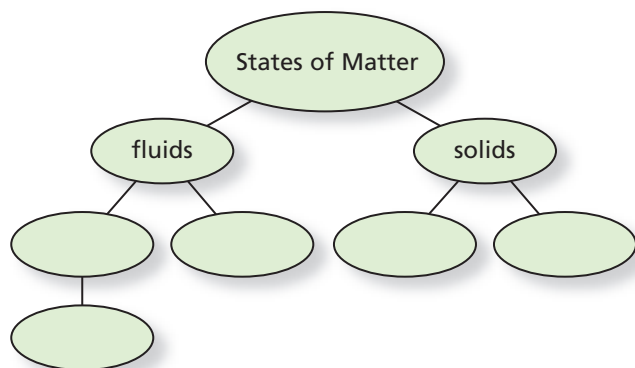
**Key Concepts**

- A crystalline solid has a regular pattern of particles, and an amorphous solid has an irregular pattern of particles.
- The thermal expansion of a solid is proportional to the temperature change and original size, and it depends on the material.

$$\alpha = \frac{\Delta L}{L_1 \Delta T} \quad \beta = \frac{\Delta V}{V_1 \Delta T}$$

## Concept Mapping

51. Complete the concept map below using the following terms: *density*, *viscosity*, *elasticity*, *pressure*. A term may be used more than once.



## Mastering Concepts

52. How are force and pressure different? (13.1)
53. A gas is placed in a sealed container, and some liquid is placed in a container of the same size. The gas and liquid both have definite volume. How do they differ? (13.1)
54. In what way are gases and plasmas similar? In what way are they different? (13.1)
55. The Sun is made of plasma. How is this plasma different from the plasmas on Earth? (13.1)
56. **Lakes** A frozen lake melts in the spring. What effect does this have on the temperature of the air above the lake? (13.2)
57. **Hiking** Canteens used by hikers often are covered with canvas bags. If you wet the canvas bag covering a canteen, the water in the canteen will be cooled. Explain. (13.2)
58. What do the equilibrium tubes in **Figure 13-24** tell you about the pressure exerted by a liquid? (13.3)
59. According to Pascal's principle, what happens to the pressure at the top of a container if the pressure at the bottom is increased? (13.3)
60. How does the water pressure 1 m below the surface of a small pond compare with the water pressure the same distance below the surface of a lake? (13.3)
61. Does Archimedes' principle apply to an object inside a flask that is inside a spaceship in orbit? (13.3)
62. A stream of water goes through a garden hose into a nozzle. As the water speeds up, what happens to the water pressure? (13.3)
63. How does the arrangement of atoms in a crystalline substance differ from that in an amorphous substance? (13.4)
64. Does the coefficient of linear expansion depend on the unit of length used? Explain. (13.4)



■ Figure 13-24

## Applying Concepts

65. A rectangular box with its largest surface resting on a table is rotated so that its smallest surface is now on the table. Has the pressure on the table increased, decreased, or remained the same?
66. Show that a pascal is equivalent to a  $\text{kg}/\text{m}\cdot\text{s}^2$ .
67. **Shipping Cargo** Compared to an identical empty ship, would a ship filled with table-tennis balls sink deeper into the water or rise in the water? Explain.
68. Drops of mercury, water, ethanol, and acetone are placed on a smooth, flat surface, as shown in **Figure 13-25**. From this figure, what can you conclude about the cohesive forces in these liquids?

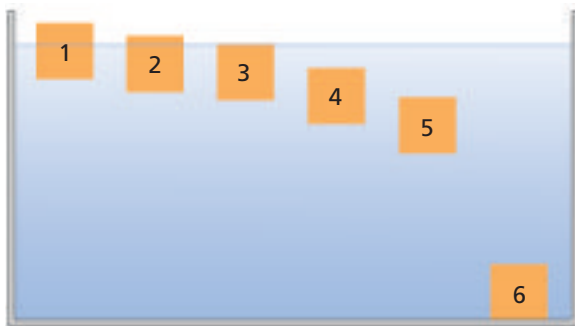


■ Figure 13-25

69. How deep would a water container have to be to have the same pressure at the bottom as that found at the bottom of a 10.0-cm deep beaker of mercury, which is 13.55 times as dense as water?

70. Alcohol evaporates more quickly than water does at the same temperature. What does this observation allow you to conclude about the properties of the particles in the two liquids?
71. Suppose you use a hole punch to make a circular hole in aluminum foil. If you heat the foil, will the size of the hole decrease or increase? Explain.
72. Equal volumes of water are heated in two narrow tubes that are identical, except that tube A is made of soft glass and tube B is made of ovenproof glass. As the temperature increases, the water level rises higher in tube B than in tube A. Give a possible explanation.
73. A platinum wire easily can be sealed in a glass tube, but a copper wire does not form a tight seal with the glass. Explain.
74. Five objects with the following densities are put into a tank of water.  
 a.  $0.85 \text{ g/cm}^3$                       d.  $1.15 \text{ g/cm}^3$   
 b.  $0.95 \text{ g/cm}^3$                       e.  $1.25 \text{ g/cm}^3$   
 c.  $1.05 \text{ g/cm}^3$

The density of water is  $1.00 \text{ g/cm}^3$ . The diagram in **Figure 13-26** shows six possible positions of these objects. Select a position, from 1 to 6, for each of the five objects. Not all positions need to be selected.



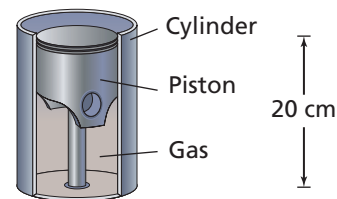
■ Figure 13-26

## Mastering Problems

### 13.1 Properties of Fluids

75. **Textbooks** A 0.85-kg physics book with dimensions of  $24.0 \text{ cm} \times 20.0 \text{ cm}$  is at rest on a table.  
 a. What force does the book apply to the table?  
 b. What pressure does the book apply?
76. A 75-kg solid cylinder that is 2.5 m long and has an end radius of 7.0 cm stands on one end. How much pressure does it exert?
77. What is the total downward force of the atmosphere on the top of your head right now? Assume that the top of your head has an area of about  $0.025 \text{ m}^2$ .

78. **Soft Drinks** Sodas are made fizzy by the carbon dioxide ( $\text{CO}_2$ ) dissolved in the liquid. An amount of carbon dioxide equal to about 8.0 L of carbon dioxide gas at atmospheric pressure and 300.0 K can be dissolved in a 2-L bottle of soda. The molar mass of  $\text{CO}_2$  is 44 g/mol.  
 a. How many moles of carbon dioxide are in the 2-L bottle? ( $1 \text{ L} = 0.001 \text{ m}^3$ )  
 b. What is the mass of the carbon dioxide in the 2-L bottle of soda?
79. As shown in **Figure 13-27**, a constant-pressure thermometer is made with a cylinder containing a piston that can move freely inside the cylinder. The pressure and the amount of gas enclosed in the cylinder are kept constant. As the temperature increases or decreases, the piston moves up or down in the cylinder. At  $0^\circ\text{C}$ , the height of the piston is 20 cm. What is the height of the piston at  $100^\circ\text{C}$ ?

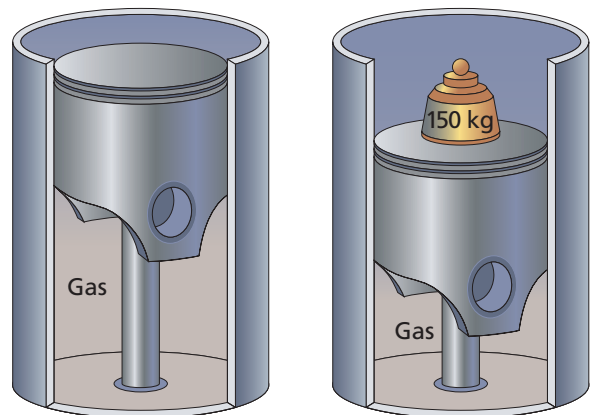


■ Figure 13-27

80. A piston with an area of  $0.015 \text{ m}^2$  encloses a constant amount of gas in a cylinder with a volume of  $0.23 \text{ m}^3$ . The initial pressure of the gas is  $1.5 \times 10^5 \text{ Pa}$ . A 150-kg mass is then placed on the piston, and the piston moves downward to a new position, as shown in **Figure 13-28**. If the temperature is constant, what is the new volume of the gas in the cylinder?

Volume =  $0.23 \text{ m}^3$   
 Piston area =  $0.015 \text{ m}^2$

Volume = ?



■ Figure 13-28



## Chapter 13 Assessment

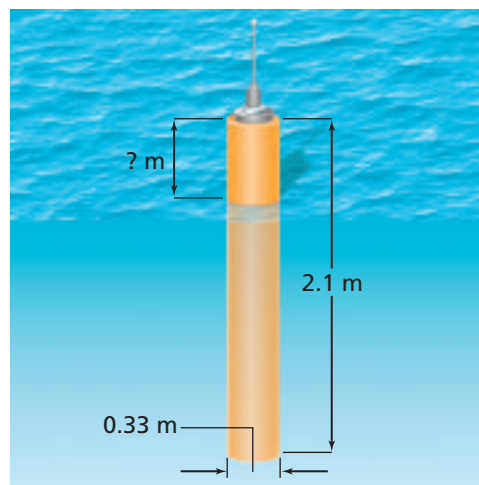
- 81. Automobiles** A certain automobile tire is specified to be used at a gauge pressure of 30.0 psi, or 30.0 pounds per square inch. (One pound per square inch equals  $6.90 \times 10^3$  Pa.) The term *gauge pressure* means the pressure above atmospheric pressure. Thus, the actual pressure in the tire is  $1.01 \times 10^5$  Pa + (30.0 psi)( $6.90 \times 10^3$  Pa/psi) =  $3.08 \times 10^5$  Pa. As the car is driven, the tire's temperature increases, and the volume and pressure increase. Suppose you filled a car's tire to a volume of  $0.55 \text{ m}^3$  at a temperature of 280 K. The initial pressure was 30.0 psi, but during the drive, the tire's temperature increased to 310 K and the tire's volume increased to  $0.58 \text{ m}^3$ .
- What is the new pressure in the tire?
  - What is the new gauge pressure?

### 13.3 Fluids at Rest and in Motion

- 82. Reservoirs** A reservoir behind a dam is 17-m deep. What is the pressure of the water at the following locations?
- the base of the dam
  - 4.0 m from the top of the dam
- 83.** A test tube standing vertically in a test-tube rack contains 2.5 cm of oil ( $\rho = 0.81 \text{ g/cm}^3$ ) and 6.5 cm of water. What is the pressure exerted by the two liquids on the bottom of the test tube?
- 84. Antiques** An antique yellow metal statuette of a bird is suspended from a spring scale. The scale reads 11.81 N when the statuette is suspended in air, and it reads 11.19 N when the statuette is completely submerged in water.
- Find the volume of the statuette.
  - Is the bird made of gold ( $\rho = 19.3 \times 10^3 \text{ kg/m}^3$ ) or gold-plated aluminum ( $\rho = 2.7 \times 10^3 \text{ kg/m}^3$ )?

- 85.** During an ecology experiment, an aquarium half-filled with water is placed on a scale. The scale shows a weight of 195 N.
- A rock weighing 8 N is added to the aquarium. If the rock sinks to the bottom of the aquarium, what will the scale read?
  - The rock is removed from the aquarium, and the amount of water is adjusted until the scale again reads 195 N. A fish weighing 2 N is added to the aquarium. What is the scale reading with the fish in the aquarium?
- 86.** What is the size of the buoyant force on a 26.0-N ball that is floating in fresh water?
- 87.** What is the apparent weight of a rock submerged in water if the rock weighs 45 N in air and has a volume of  $2.1 \times 10^{-3} \text{ m}^3$ ?

- 88.** What is the maximum weight that a balloon filled with  $1.00 \text{ m}^3$  of helium can lift in air? Assume that the density of air is  $1.20 \text{ kg/m}^3$  and that of helium is  $0.177 \text{ kg/m}^3$ . Neglect the mass of the balloon.
- 89.** If a rock weighs 54 N in air and has an apparent weight of 46 N when submerged in a liquid with a density twice that of water, what will be its apparent weight when it is submerged in water?
- 90. Oceanography** As shown in **Figure 13-29**, a large buoy used to support an oceanographic research instrument is made of a cylindrical, hollow iron tank. The tank is 2.1 m in height and 0.33 m in diameter. The total mass of the buoy and the research instrument is about 120 kg. The buoy must float so that one end is above the water to support a radio transmitter. Assuming that the mass of the buoy is evenly distributed, how much of the buoy will be above the waterline when it is floating?



■ Figure 13-29

### 13.4 Solids

- 91.** A bar of an unknown metal has a length of 0.975 m at  $45^\circ\text{C}$  and a length of 0.972 m at  $23^\circ\text{C}$ . What is its coefficient of linear expansion?
- 92.** An inventor constructs a thermometer from an aluminum bar that is 0.500 m in length at 273 K. He measures the temperature by measuring the length of the aluminum bar. If the inventor wants to measure a 1.0-K change in temperature, how precisely must he measure the length of the bar?
- 93. Bridges** How much longer will a 300-m steel bridge be on a  $30^\circ\text{C}$  day in August than on a  $-10^\circ\text{C}$  night in January?
- 94.** What is the change in length of a 2.00-m copper pipe if its temperature is raised from  $23^\circ\text{C}$  to  $978^\circ\text{C}$ ?



95. What is the change in volume of a  $1.0\text{-m}^3$  concrete block if its temperature is raised  $45^\circ\text{C}$ ?
96. **Bridges** Bridge builders often use rivets that are larger than the rivet hole to make the joint tighter. The rivet is cooled before it is put into the hole. Suppose that a builder drills a hole  $1.2230\text{ cm}$  in diameter for a steel rivet  $1.2250\text{ cm}$  in diameter. To what temperature must the rivet be cooled if it is to fit into the rivet hole, which is at  $20^\circ\text{C}$ ?
97. A steel tank filled with methanol is  $2.000\text{ m}$  in diameter and  $5.000\text{ m}$  in height. It is completely filled at  $10.0^\circ\text{C}$ . If the temperature rises to  $40.0^\circ\text{C}$ , how much methanol (in liters) will flow out of the tank, given that both the tank and the methanol will expand?
98. An aluminum sphere is heated from  $11^\circ\text{C}$  to  $580^\circ\text{C}$ . If the volume of the sphere is  $1.78\text{ cm}^3$  at  $11^\circ\text{C}$ , what is the increase in volume of the sphere at  $580^\circ\text{C}$ ?
99. The volume of a copper sphere is  $2.56\text{ cm}^3$  after being heated from  $12^\circ\text{C}$  to  $984^\circ\text{C}$ . What was the volume of the copper sphere at  $12^\circ\text{C}$ ?
100. A square of iron plate that is  $0.3300\text{ m}$  on each side is heated from  $0^\circ\text{C}$  to  $95^\circ\text{C}$ .
- What is the change in the length of the sides of the square?
  - What is the relative change in area of the square?
101. An aluminum cube with a volume of  $0.350\text{ m}^3$  at  $350.0\text{ K}$  is cooled to  $270.0\text{ K}$ .
- What is its volume at  $270.0\text{ K}$ ?
  - What is the length of a side of the cube at  $270.0\text{ K}$ ?
102. **Industry** A machinist builds a rectangular mechanical part for a special refrigerator system from two rectangular pieces of steel and two rectangular pieces of aluminum. At  $293\text{ K}$ , the part is a perfect square, but at  $170\text{ K}$ , the part becomes warped, as shown in **Figure 13-30**. Which parts were made of steel and which were made of aluminum?

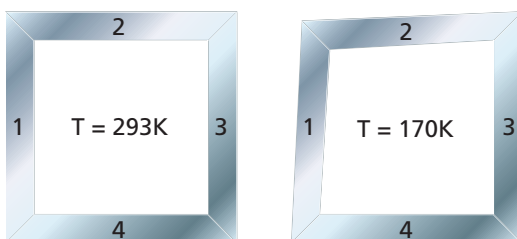


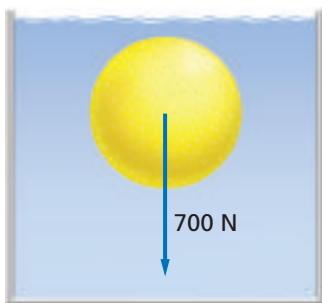
Figure 13-30

## Mixed Review

103. What is the pressure on the hull of a submarine at a depth of  $65\text{ m}$ ?
104. **Scuba Diving** A scuba diver swimming at a depth of  $5.0\text{ m}$  under water exhales a  $4.2 \times 10^{-6}\text{ m}^3$  bubble of air. What is the volume of that bubble just before it reaches the surface of the water?
105. An  $18\text{-N}$  bowling ball floats with about half of the ball submerged.
- What is the diameter of the bowling ball?
  - What would be the approximate apparent weight of a  $36\text{-N}$  bowling ball?
106. An aluminum bar is floating in a bowl of mercury. When the temperature is increased, does the aluminum float higher or sink deeper into the mercury?
107. There is  $100.0\text{ mL}$  of water in an  $800.0\text{-mL}$  soft-glass beaker at  $15.0^\circ\text{C}$ . How much will the water level have dropped or risen when the bottle and water are heated to  $50.0^\circ\text{C}$ ?
108. **Auto Maintenance** A hydraulic jack used to lift cars for repairs is called a three-ton jack. The large piston is  $22\text{ mm}$  in diameter, and the small one is  $6.3\text{ mm}$  in diameter. Assume that a force of  $3\text{ tons}$  is  $3.0 \times 10^4\text{ N}$ .
- What force must be exerted on the small piston to lift a  $3\text{-ton}$  weight?
  - Most jacks use a lever to reduce the force needed on the small piston. If the resistance arm is  $3.0\text{ cm}$ , how long must the effort arm of an ideal lever be to reduce the force to  $100.0\text{ N}$ ?
109. **Ballooning** A hot-air balloon contains a fixed volume of gas. When the gas is heated, it expands and pushes some gas out at the lower, open end. As a result, the mass of the gas in the balloon is reduced. Why would the air in a balloon have to be hotter to lift the same number of people above Vail, Colorado, which has an altitude of  $2400\text{ m}$ , than above the tidewater flats of Virginia, which have an altitude of  $6\text{ m}$ ?
110. **The Living World** Some plants and animals are able to live in conditions of extreme pressure.
- What is the pressure exerted by the water on the skin of a fish or worm that lives near the bottom of the Puerto Rico Trench,  $8600\text{ m}$  below the surface of the Atlantic Ocean? Use  $1030\text{ kg/m}^3$  for the density of seawater.
  - What would be the density of air at that pressure, relative to its density above the surface of the ocean?

## Thinking Critically

- 111. Apply Concepts** You are washing dishes in the sink. A serving bowl has been floating in the sink. You fill the bowl with water from the sink, and it sinks to the bottom. Did the water level in the sink go up or down when the bowl was submerged?
- 112. Apply Concepts** Persons confined to bed are less likely to develop bedsores if they use a waterbed rather than an ordinary mattress. Explain.
- 113. Analyze and Conclude** One method of measuring the percentage of body fat is based on the fact that fatty tissue is less dense than muscle tissue. How can a person's average density be assessed with a scale and a swimming pool? What measurements does a physician need to record to find a person's average percentage of body fat?
- 114. Analyze and Conclude** A downward force of 700 N is required to fully submerge a plastic foam sphere, as shown in **Figure 13-31**. The density of the foam is  $95 \text{ kg/m}^3$ .
- What percentage of the sphere would be submerged if the sphere were released to float freely?
  - What is the weight of the sphere in air?
  - What is the volume of the sphere?



■ **Figure 13-31**

- 115. Apply Concepts** Tropical fish for aquariums are often transported home from pet shops in transparent plastic bags filled mostly with water. If you placed a fish in its unopened transport bag in a home aquarium, which of the cases in **Figure 13-32** best represents what would happen? Explain your reasoning.



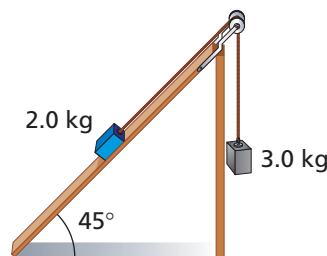
■ **Figure 13-32**

## Writing in Physics

- 116.** Some solid materials expand when they are cooled. Water between  $4^\circ$  and  $0^\circ\text{C}$  is the most common example, but rubber bands also expand in length when cooled. Research what causes this expansion.
- 117.** Research Joseph Louis Gay-Lussac and his contributions to the gas laws. How did Gay-Lussac's work contribute to the discovery of the formula for water?

## Cumulative Review

- 118.** Two blocks are connected by a string over a frictionless, massless pulley such that one is resting on an inclined plane and the other is hanging over the top edge of the plane, as shown in **Figure 13-33**. The hanging block has a mass of 3.0 kg and the block on the plane has a mass of 2.0 kg. The coefficient of kinetic friction between the block and the inclined plane is 0.19. Answer the following questions assuming the blocks are released from rest. (**Chapter 5**)
- What is the acceleration of the blocks?
  - What is the tension in the string connecting the blocks?



■ **Figure 13-33**

- 119.** A compact car with a mass of 875 kg, moving south at 15 m/s, is struck by a full-sized car with a mass of 1584 kg, moving east at 12 m/s. The two cars stick together, and momentum is conserved. (**Chapter 9**)
- Sketch the situation, assigning coordinate axes and identifying "before" and "after."
  - Find the direction and speed of the wreck immediately after the collision, remembering that momentum is a vector quantity.
  - The wreck skids along the ground and comes to a stop. The coefficient of kinetic friction while the wreck is skidding is 0.55. Assume that the acceleration is constant. How far does the wreck skid after impact?
- 120.** A 188-W motor will lift a load at the rate (speed) of 6.50 cm/s. How great a load can the motor lift at this rate? (**Chapter 10**)

# Standardized Test Practice

## Multiple Choice

1. Gas with a volume of 10.0 L is trapped in an expandable cylinder. If the pressure is tripled and the temperature is increased by 80.0 percent (as measured on the Kelvin scale), what will be the new volume of the gas?

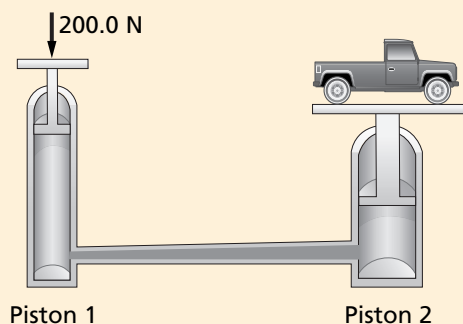
(A) 2.70 L                      (C) 16.7 L  
(B) 6.00 L                      (D) 54.0 L

2. Nitrogen gas at standard atmospheric pressure, 101.3 kPa, has a volume of 0.080 m<sup>3</sup>. If there are 3.6 mol of the gas, what is the temperature?

(A) 0.27 K                      (C) 0.27°C  
(B) 270 K                      (D) 270°C

3. As diagrammed below, an operator applies a force of 200.0 N to the first piston of a hydraulic lift, which has an area of 5.4 cm<sup>2</sup>. What is the pressure applied to the hydraulic fluid?

(A)  $3.7 \times 10^1$  Pa              (C)  $3.7 \times 10^3$  Pa  
(B)  $2.0 \times 10^3$  Pa              (D)  $3.7 \times 10^5$  Pa



4. If the second piston in the lift diagrammed above exerts a force of 41,000 N, what is the area of the second piston?

(A) 0.0049 m<sup>2</sup>              (C) 0.11 m<sup>2</sup>  
(B) 0.026 m<sup>2</sup>              (D) 11 m<sup>2</sup>

5. The density of cocobolo wood from Costa Rica is 1.10 g/cm<sup>3</sup>. What is the apparent weight of a figurine that displaces 786 mL when submerged in a freshwater lake?

(A) 0.770 N                      (C) 7.70 N  
(B) 0.865 N                      (D) 8.47 N

6. What is the buoyant force on a 17-kg object that displaces 85 L of water?

(A)  $1.7 \times 10^2$  N              (C)  $1.7 \times 10^5$  N  
(B)  $8.3 \times 10^2$  N              (D)  $8.3 \times 10^5$  N

7. Which one of the following items does not contain matter in the plasma state?

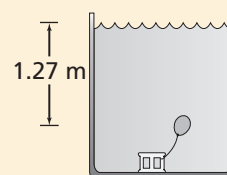
(A) neon lighting  
(B) stars  
(C) lightning  
(D) incandescent lighting

8. What is the mass of 365 mL of carbon dioxide gas at 3.0 atm pressure (1 atm = 101.3 kPa) and 24°C? The molar mass for carbon dioxide is 44.0 g/mol.

(A) 0.045 g                      (C) 45 g  
(B) 2.0 g                      (D) 2.0 kg

## Extended Answer

9. A balloon has a volume of 125 mL of air at standard atmospheric pressure, 101.3 kPa. If the balloon is anchored 1.27 m under the surface of a swimming pool, as illustrated in the diagram below, what is the new volume of the balloon?



## ✓ Test-Taking TIP

### Work Weak Muscles; Maintain Strong Ones

If you're preparing for a standardized test that covers many topics, it's sometimes difficult to focus on all the topics that require your attention. Focus most of your energy on your weaker areas and review your stronger topics frequently.



# Chapter 14

# Vibrations and Waves

## What You'll Learn

- You will examine vibrational motion and learn how it relates to waves.
- You will determine how waves transfer energy.
- You will describe wave behavior and discuss its practical significance.

## Why It's Important

Knowledge of the behavior of vibrations and waves is essential to the understanding of resonance and how safe buildings and bridges are built, as well as how communications through radio and television are achieved.

### “Galloping Gertie”

Shortly after it was opened to traffic, the Tacoma Narrows Bridge near Tacoma, Washington, began to vibrate whenever the wind blew (see inset). One day, the oscillations became so large that the bridge broke apart and collapsed into the water below.



## Think About This ►

How could a light wind cause the bridge in the inset photo to vibrate with such large waves that it eventually collapsed?



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## How do waves behave in a coiled spring?

### Question

How do pulses that are sent down a coiled spring behave when the other end of the spring is stationary?

### Procedure



1. Stretch out a coiled spiral spring, but do not overstretch it. One person should hold one end still, while the other person generates a sideways pulse in the spring. Observe the pulse while it travels along the spring and when it hits the held end. Record your observations.
2. Repeat step 1 with a larger pulse. Record your observations.
3. Generate a different pulse by compressing the spring at one end and letting go. Record your observations.
4. Generate a third type of pulse by twisting one end of the spring and then releasing it. Record your observations.

### Analysis

What happens to the pulses as they travel through the spring? What happens as they hit the end of the spring? How did the pulse in step 1 compare to that generated in step 2?

**Critical Thinking** What are some properties that seem to control how a pulse moves through the spring?



## 14.1 Periodic Motion

You've probably seen a clock pendulum swing back and forth. You would have noticed that every swing followed the same path, and each trip back and forth took the same amount of time. This action is an example of vibrational motion. Other examples include a metal block bobbing up and down on a spring and a vibrating guitar string. These motions, which all repeat in a regular cycle, are examples of **periodic motion**.

In each example, the object has one position at which the net force on it is zero. At that position, the object is in equilibrium. Whenever the object is pulled away from its equilibrium position, the net force on the system becomes nonzero and pulls the object back toward equilibrium. If the force that restores the object to its equilibrium position is directly proportional to the displacement of the object, the motion that results is called **simple harmonic motion**.

Two quantities describe simple harmonic motion. The **period**,  $T$ , is the time needed for an object to repeat one complete cycle of the motion, and the **amplitude** of the motion is the maximum distance that the object moves from equilibrium.

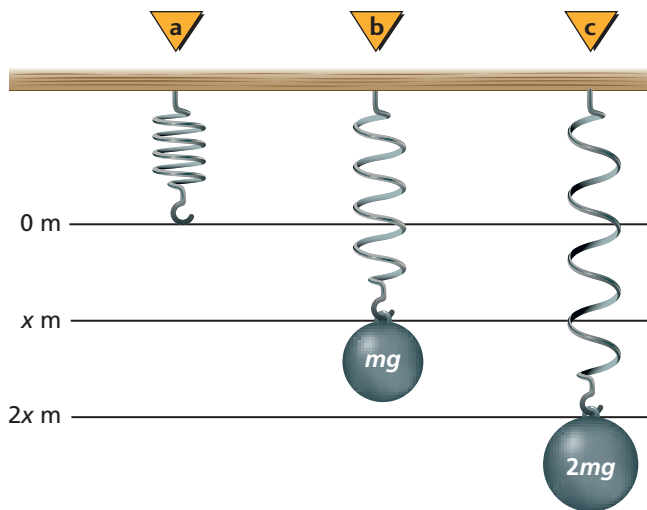
### Objectives

- **Describe** the force in an elastic spring.
- **Determine** the energy stored in an elastic spring.
- **Compare** simple harmonic motion and the motion of a pendulum.

### Vocabulary

periodic motion  
simple harmonic motion  
period  
amplitude  
Hooke's law  
pendulum  
resonance





■ **Figure 14-1** The force exerted by a spring is directly proportional to the distance the spring is stretched.

## The Mass on a Spring

How does a spring react to a force that is applied to it? **Figure 14-1a** shows a spring hanging from a support with nothing attached to it. The spring does not stretch because no external force is exerted on it. **Figure 14-1b** shows the same spring with an object of weight  $mg$  hanging from it. The spring has stretched by distance  $x$  so that the upward force it exerts balances the downward force of gravity acting on the object. **Figure 14-1c** shows the same spring stretched twice as far,  $2x$ , to support twice the weight,  $2mg$ , hanging from it. **Hooke's law** states that the force exerted by a spring is directly proportional to the amount that the spring

is stretched. A spring that acts in this way is said to obey Hooke's law, which can be expressed by the following equation.

$$\text{Hooke's Law } F = -kx$$

The force exerted by a spring is equal to the spring constant times the distance the spring is compressed or stretched from its equilibrium position.

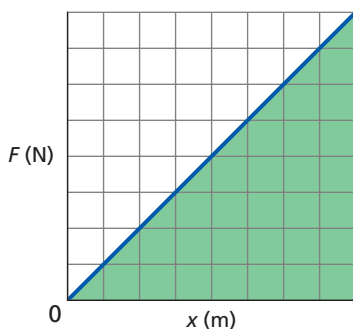
In this equation,  $k$  is the spring constant, which depends on the stiffness and other properties of the spring, and  $x$  is the distance that the spring is stretched from its equilibrium position. Not all springs obey Hooke's law, but many do. Those that do are called elastic.

**Potential energy** When a force is applied to stretch a spring, such as by hanging an object on its end, there is a direct linear relationship between the exerted force and the displacement, as shown by the graph in **Figure 14-2**. The slope of the graph is equal to the spring constant, given in units of newtons per meter. The area under the curve represents the work done to stretch the spring, and therefore equals the elastic potential energy that is stored in the spring as a result of that work. The base of the triangle is  $x$ , and the height is the force, which, according to the equation for Hooke's law, is equal to  $kx$ , so the potential energy in the spring is given by the following equation.

$$\text{Potential Energy in a Spring } PE_{\text{sp}} = \frac{1}{2}kx^2$$

The potential energy in a spring is equal to one-half times the product of the spring constant and the square of the displacement.

■ **Figure 14-2** The spring constant of a spring can be determined from the graph of force versus displacement of the spring.

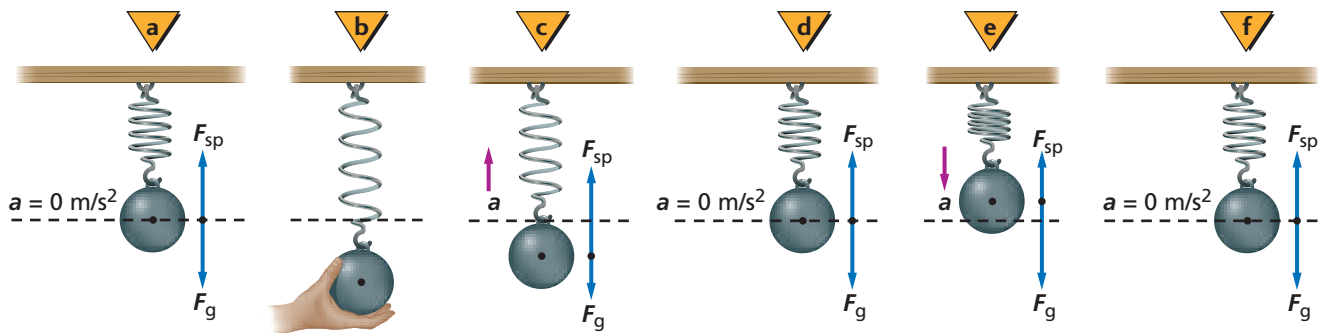


The units of the area, and thus, of the potential energy, are newton · meters, or joules.

How does the net force depend upon position? When an object hangs on a spring, the spring stretches until its upward force,  $F_{\text{sp}}$ , balances the object's weight,  $F_g$ , as shown in **Figure 14-3a**. The block is then in its equilibrium position. If you pull the object down, as in **Figure 14-3b**, the spring force increases, producing a net force upward. When you let go of the object, it accelerates in the upward direction, as in **Figure 14-3c**. However, as the stretch of the spring is reduced, the upward force decreases.







■ **Figure 14-3** Simple harmonic motion is demonstrated by the vibrations of an object hanging on a spring.

In **Figure 14-3d**, the upward force of the spring and the object's weight are equal—there is no acceleration. Because there is no net force, the object continues its upward velocity, moving above the equilibrium position. In **Figure 14-3e**, the net force is in the direction opposite the displacement of the object and is directly proportional to the displacement, so the object moves with a simple harmonic motion. The object returns to the equilibrium position, as in **Figure 14-3f**.

### ▶ EXAMPLE Problem 1

**The Spring Constant and the Energy in a Spring** A spring stretches by 18 cm when a bag of potatoes weighing 56 N is suspended from its end.

- Determine the spring constant.
- How much elastic potential energy is stored in the spring when it is stretched this far?

#### 1 Analyze and Sketch the Problem

- Sketch the situation.
- Show and label the distance that the spring has stretched and its equilibrium position.

**Known:**  $x = 18 \text{ cm}$   
**Unknown:**  $F = 56 \text{ N}$

$k = ?$   
 $PE_{\text{sp}} = ?$

#### 2 Solve for the Unknown

- Use  $F = -kx$  and solve for  $k$ .

$$k = \frac{F}{x}$$

The minus sign can be dropped because it just means that the force is restoring.

$$= \frac{56 \text{ N}}{0.18 \text{ m}}$$

Substitute  $F = 56 \text{ N}$ ,  $x = 0.18 \text{ m}$

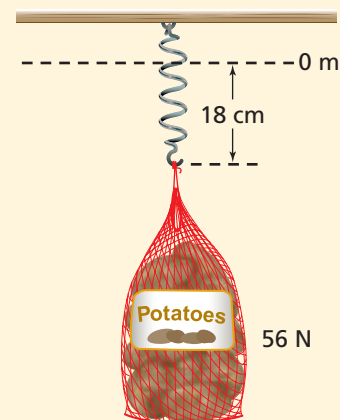
$$= 310 \text{ N/m}$$

- $PE_{\text{sp}} = \frac{1}{2}kx^2$

$$= \frac{1}{2}(310 \text{ N/m})(0.18 \text{ m})^2$$

Substitute  $k = 310 \text{ N/m}$ ,  $x = 0.18 \text{ m}$

$$= 5.0 \text{ J}$$



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#### 3 Evaluate the Answer

- Are the units correct?** N/m are the correct units for the spring constant.  $(\text{N/m})(\text{m}^2) = \text{N}\cdot\text{m} = \text{J}$ , which is the correct unit for energy.
- Is the magnitude realistic?** The spring constant is consistent with a scale used, for example, to weigh groceries. The energy of 5.0 J is equal to the value obtained from  $W = Fx = mgh$ , when the average force of 28 N is applied.



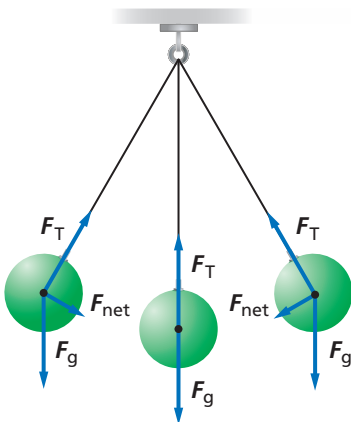


1. How much force is necessary to stretch a spring 0.25 m when the spring constant is 95 N/m?
2. A spring has a spring constant of 56 N/m. How far will it stretch when a block weighing 18 N is hung from its end?
3. What is the spring constant of a spring that stretches 12 cm when an object weighing 24 N is hung from it?
4. A spring with a spring constant of 144 N/m is compressed by a distance of 16.5 cm. How much elastic potential energy is stored in the spring?
5. A spring has a spring constant of 256 N/m. How far must it be stretched to give it an elastic potential energy of 48 J?

When the external force holding the object is released, as in Figure 14-3c, the net force and the acceleration are at their maximum, and the velocity is zero. As the object passes through the equilibrium point, Figure 14-3d, the net force is zero, and so is the acceleration. Does the object stop? No, it would take a net downward force to slow the object, and that will not exist until the object rises above the equilibrium position. When the object comes to the highest position in its oscillation, the net force and the acceleration are again at their maximum, and the velocity is zero. The object moves down through the equilibrium position to its starting point and continues to move in this vibratory manner. The period of oscillation,  $T$ , depends upon the mass of the object and the strength of the spring.

**Automobiles** Elastic potential energy is an important part of the design and building of today's automobiles. Every year, new models of cars are tested to see how well they withstand damage when they crash into barricades at low speeds. A car's ability to retain its integrity depends upon how much of the kinetic energy it had before the crash can be converted into the elastic potential energy of the frame after the crash. Many bumpers are modified springs that store energy as a car hits a barrier in a slow-speed collision. After the car stops and the spring is compressed, the spring returns to its equilibrium position, and the car recoils from the barrier.

■ **Figure 14-4**  $F_{\text{net}}$ , the vector sum of  $F_{\text{T}}$  and  $F_{\text{g}}$ , is the restoring force for the pendulum.



## Pendulums

Simple harmonic motion also can be demonstrated by the swing of a pendulum. A simple **pendulum** consists of a massive object, called the bob, suspended by a string or light rod of length  $l$ . After the bob is pulled to one side and released, it swings back and forth, as shown in **Figure 14-4**. The string or rod exerts a tension force,  $F_{\text{T}}$ , and gravity exerts a force,  $F_{\text{g}}$ , on the bob. The vector sum of the two forces produces the net force, shown at three positions in Figure 14-4. At the left and right positions shown in Figure 14-4, the net force and acceleration are maximum, and the velocity is zero. At the middle position in Figure 14-4, the net force and acceleration are zero, and the velocity is maximum. You can see that the net force is a restoring force; that is, it is opposite the direction of the displacement of the bob and is trying to restore the bob to its equilibrium position.

For small angles (less than about 15°) the restoring force is proportional to the displacement, so the movement is simple harmonic motion. The period of a pendulum is given by the following equation.

**Period of a Pendulum**  $T = 2\pi\sqrt{\frac{l}{g}}$

The period of a pendulum is equal to two pi times the square root of the length of the pendulum divided by the acceleration due to gravity.

Notice that the period depends only upon the length of the pendulum and the acceleration due to gravity, not on the mass of the bob or the amplitude of oscillation. One application of the pendulum is to measure  $g$ , which can vary slightly at different locations on Earth.

## ▶ EXAMPLE Problem 2

**Finding  $g$  Using a Pendulum** A pendulum with a length of 36.9 cm has a period of 1.22 s. What is the acceleration due to gravity at the pendulum's location?

### 1 Analyze and Sketch the Problem

- Sketch the situation.
- Label the length of the pendulum.

**Known:** **Unknown:**

$l = 36.9 \text{ cm}$       $g = ?$

$T = 1.22 \text{ s}$

### 2 Solve for the Unknown

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Solve for  $g$ .

$$g = \frac{(2\pi)^2 l}{T^2}$$

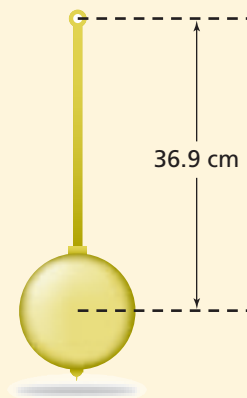
$$= \frac{4\pi^2(0.369 \text{ m})}{(1.22 \text{ s})^2}$$

Substitute  $l = 0.369 \text{ m}$ ,  $T = 1.22 \text{ s}$

$$= 9.78 \text{ m/s}^2$$

### 3 Evaluate the Answer

- **Are the units correct?**  $\text{m/s}^2$  are the correct units for acceleration.
- **Is the magnitude realistic?** The calculated value of  $g$  is quite close to the standard value of  $g$ ,  $9.80 \text{ m/s}^2$ . This pendulum could be at a high elevation above sea level.



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## ▶ PRACTICE Problems

Additional Problems, Appendix B

6. What is the period on Earth of a pendulum with a length of 1.0 m?
7. How long must a pendulum be on the Moon, where  $g = 1.6 \text{ m/s}^2$ , to have a period of 2.0 s?
8. On a planet with an unknown value of  $g$ , the period of a 0.75-m-long pendulum is 1.8 s. What is  $g$  for this planet?

## ● CHALLENGE PROBLEM

A car of mass  $m$  rests at the top of a hill of height  $h$  before rolling without friction into a crash barrier located at the bottom of the hill. The crash barrier contains a spring with a spring constant,  $k$ , which is designed to bring the car to rest with minimum damage.

1. Determine, in terms of  $m$ ,  $h$ ,  $k$ , and  $g$ , the maximum distance,  $x$ , that the spring will be compressed when the car hits it.
2. If the car rolls down a hill that is twice as high, how much farther will the spring be compressed?
3. What will happen after the car has been brought to rest?

## APPLYING PHYSICS

► **Foucault Pendulum** A Foucault pendulum has a long wire, about 16 m in length, with a heavy weight of about 109 kg attached to one end. According to Newton's first law of motion, a swinging pendulum will keep swinging in the same direction unless it is pushed or pulled in another direction. However, because Earth rotates every 24 h underneath the pendulum, to an observer it would seem as though the pendulum's direction of swing has changed. To demonstrate this, pegs are arranged in a circle on the floor beneath so that the swinging pendulum will knock them down as the floor rotates. At the north pole, this apparent rotation would be  $15^\circ/\text{h}$ . ◀

## Resonance

To get a playground swing going, you “pump” it by leaning back and pulling the chains at the same point in each swing, or a friend gives you repeated pushes at just the right times. **Resonance** occurs when small forces are applied at regular intervals to a vibrating or oscillating object and the amplitude of the vibration increases. The time interval between applications of the force is equal to the period of oscillation. Other familiar examples of resonance include rocking a car to free it from a snowbank and jumping rhythmically on a trampoline or a diving board. The large-amplitude oscillations caused by resonance can create stresses. Audiences in theater balconies, for example, sometimes damage the structures by jumping up and down with a period equal to the natural oscillation period of the balcony.

Resonance is a special form of simple harmonic motion in which the additions of small amounts of force at specific times in the motion of an object cause a larger and larger displacement. Resonance from wind, combined with the design of the bridge supports, may have caused the original Tacoma Narrows Bridge to collapse.

## 14.1 Section Review

9. **Hooke's Law** Two springs look alike but have different spring constants. How could you determine which one has the greater spring constant?
10. **Hooke's Law** Objects of various weights are hung from a rubber band that is suspended from a hook. The weights of the objects are plotted on a graph against the stretch of the rubber band. How can you tell from the graph whether or not the rubber band obeys Hooke's law?
11. **Pendulum** How must the length of a pendulum be changed to double its period? How must the length be changed to halve the period?
12. **Energy of a Spring** What is the difference between the energy stored in a spring that is stretched 0.40 m and the energy stored in the same spring when it is stretched 0.20 m?
13. **Resonance** If a car's wheel is out of balance, the car will shake strongly at a specific speed, but not when it is moving faster or slower than that speed. Explain.
14. **Critical Thinking** How is uniform circular motion similar to simple harmonic motion? How are they different?



## 14.2 Wave Properties

**B**oth particles and waves carry energy, but there is an important difference in how they do this. Think of a ball as a particle. If you toss the ball to a friend, the ball moves from you to your friend and carries energy. However, if you and your friend hold the ends of a rope and you give your end a quick shake, the rope remains in your hand. Even though no matter is transferred, the rope still carries energy through the wave that you created. A **wave** is a disturbance that carries energy through matter or space.

You have learned how Newton's laws of motion and principles of conservation of energy govern the behavior of particles. These laws and principles also govern the motion of waves. There are many kinds of waves that transmit energy, including the waves you cannot see.

### Mechanical Waves

Water waves, sound waves, and the waves that travel down a rope or spring are types of mechanical waves. Mechanical waves require a medium, such as water, air, ropes, or a spring. Because many other waves cannot be directly observed, mechanical waves can serve as models.

**Transverse waves** The two disturbances shown in **Figure 14-5a** are called wave pulses. A **wave pulse** is a single bump or disturbance that travels through a medium. If the wave moves up and down at the same rate, a **periodic wave** is generated. Notice in Figure 14-5a that the rope is disturbed in the vertical direction, but the pulse travels horizontally. A wave with this type of motion is called a transverse wave. A **transverse wave** is one that vibrates perpendicular to the direction of the wave's motion.

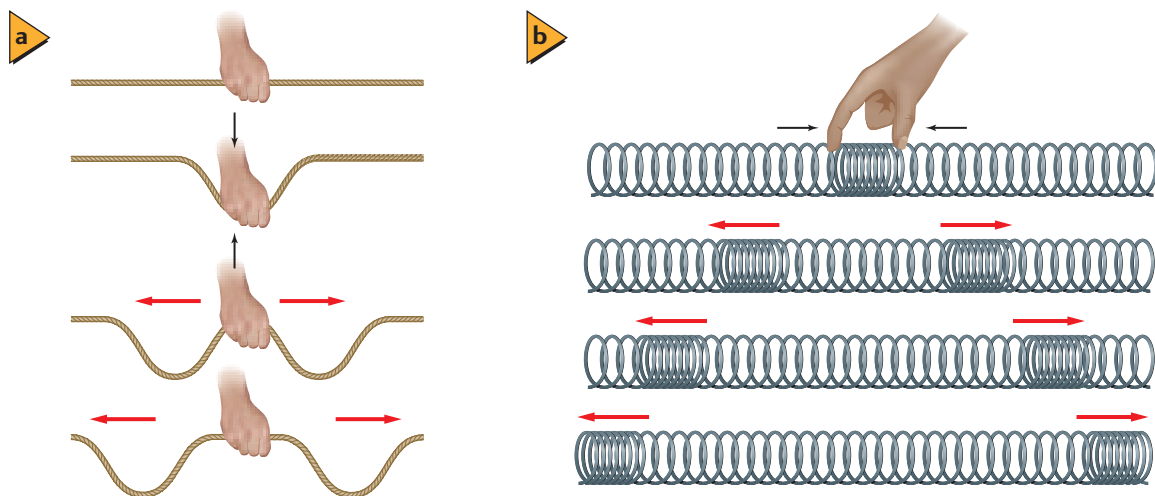
**Longitudinal waves** In a coiled-spring toy, you can create a wave pulse in a different way. If you squeeze together several turns of the coiled-spring toy and then suddenly release them, pulses of closely-spaced turns will move away in both directions, as shown in **Figure 14-5b**. This is called a **longitudinal wave**. The disturbance is in the same direction as, or parallel to, the direction of the wave's motion. Sound waves are longitudinal waves. Fluids usually transmit only longitudinal waves.

#### ► Objectives

- **Identify** how waves transfer energy without transferring matter.
- **Contrast** transverse and longitudinal waves.
- **Relate** wave speed, wavelength, and frequency.

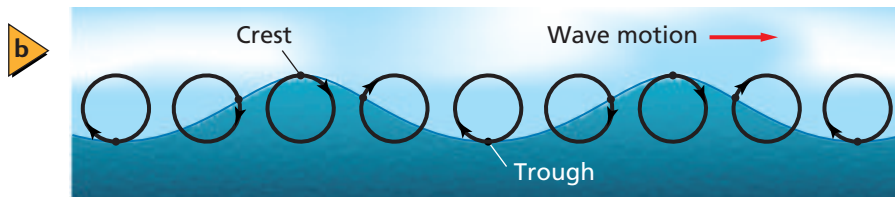
#### ► Vocabulary

wave  
wave pulse  
periodic wave  
transverse wave  
longitudinal wave  
surface wave  
trough  
crest  
wavelength  
frequency



■ **Figure 14-5** A quick shake of a rope sends out transverse wave pulses in both directions **(a)**. The squeeze and release of a coiled-spring toy sends out longitudinal wave pulses in both directions **(b)**.





■ **Figure 14-6** Surface waves have properties of both transverse and longitudinal waves **(a)**. The paths of the individual particles are circular **(b)**.

**Surface waves** Waves that are deep in a lake or ocean are longitudinal; at the surface of the water, however, the particles move in a direction that is both parallel and perpendicular to the direction of wave motion, as shown in **Figure 14-6**. Each of the waves is a **surface wave**, which has characteristics of both transverse and longitudinal waves. The energy of water waves usually comes from distant storms, whose energy initially came from the heating of Earth by solar energy. This energy, in turn, was carried to Earth by transverse electromagnetic waves from the Sun.

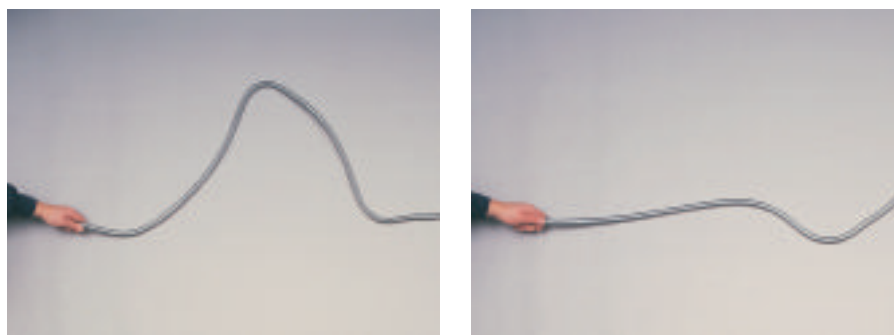
## Measuring a Wave

There are many ways to describe or measure a wave. Some characteristics depend on how the wave is produced, whereas others depend on the medium through which the wave travels.

**Speed** How fast does a wave move? The speed of the pulse shown in **Figure 14-7** can be found in the same way as the speed of a moving car is determined. First, measure the displacement of the wave peak,  $\Delta d$ , then divide this by the time interval,  $\Delta t$ , to find the speed, given by  $v = \Delta d / \Delta t$ . The speed of a periodic wave can be found in the same way. For most mechanical waves, both transverse and longitudinal, the speed depends only on the medium through which the waves move.

**Amplitude** How does the pulse generated by gently shaking a rope differ from the pulse produced by a violent shake? The difference is similar to the difference between a ripple in a pond and an ocean breaker: they have different amplitudes. You have learned that the amplitude of a wave is the maximum displacement of the wave from its position of rest, or equilibrium. Two similar waves having different amplitudes are shown in **Figure 14-8**.

A wave's amplitude depends on how it is generated, but not on its speed. More work must be done to generate a wave with a greater amplitude. For example, strong winds produce larger water waves than those formed by gentle breezes. Waves with greater amplitudes transfer more energy.



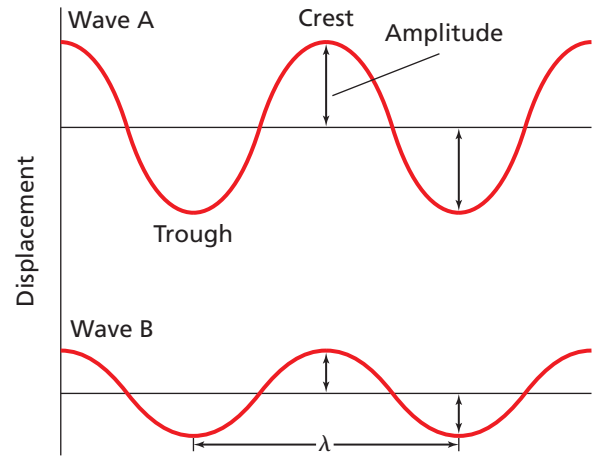
■ **Figure 14-7** These two photographs were taken 0.20 s apart. During that time, the crest moved 0.80 m. The velocity of the wave is 4.0 m/s.

Whereas a small wave might move sand on a beach a few centimeters, a giant wave can uproot and move a tree. For waves that move at the same speed, the rate at which energy is transferred is proportional to the square of the amplitude. Thus, doubling the amplitude of a wave increases the amount of energy it transfers each second by a factor of 4.

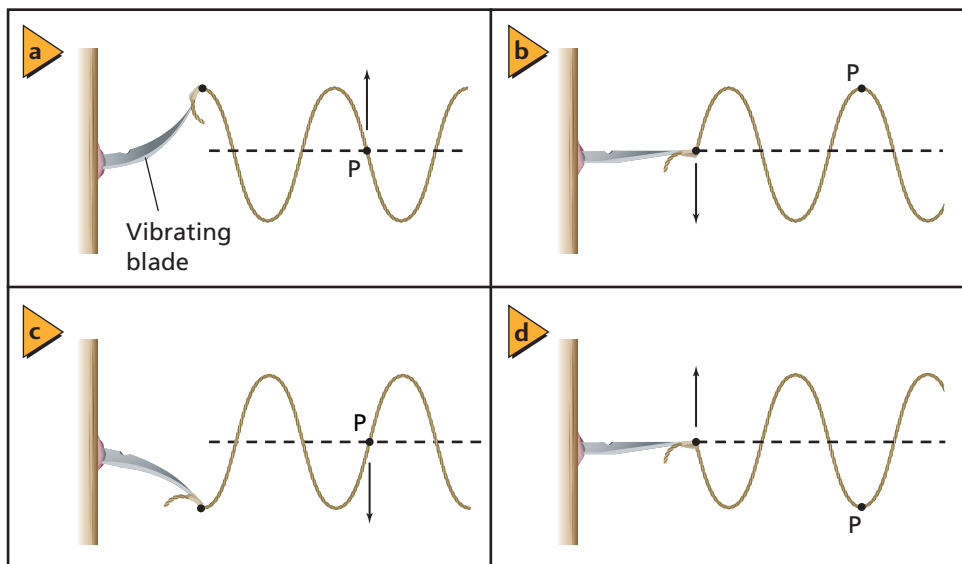
**Wavelength** Rather than focusing on one point on a wave, imagine taking a snapshot of the wave so that you can see the whole wave at one instant in time. Figure 14-8 shows each low point, called a **trough**, and each high point, called a **crest**, of a wave. The shortest distance between points where the wave pattern repeats itself is called the **wavelength**. Crests are spaced by one wavelength. Each trough also is one wavelength from the next. The Greek letter lambda,  $\lambda$ , represents wavelength.

**Phase** Any two points on a wave that are one or more whole wavelengths apart are in phase. Particles in the medium are said to be in phase with one another when they have the same displacement from equilibrium and the same velocity. Particles in the medium with opposite displacements and velocities are  $180^\circ$  out of phase. A crest and a trough, for example, are  $180^\circ$  out of phase with each other. Two particles in a wave can be anywhere from  $0^\circ$  to  $180^\circ$  out of phase with one another.

**Period and frequency** Although wave speed and amplitude can describe both pulses and periodic waves, period,  $T$ , and frequency,  $f$ , apply only to periodic waves. You have learned that the period of a simple harmonic oscillator, such as a pendulum, is the time it takes for the motion of the oscillator to complete one cycle. Such an oscillator is usually the source, or cause, of a periodic wave. The period of a wave is equal to the period of the source. In **Figures 14-9a** through **14-9d**, the period,  $T$ , equals 0.04 s, which is the time it takes the source to complete one cycle. The same time is taken by P, a point on the rope, to return to its initial phase.



■ **Figure 14-8** The amplitude of wave A is larger than that of wave B.



■ **Figure 14-9** One end of a string, with a piece of tape at point P, is attached to a blade vibrating 25 times per second. Note the change in position of point P over time.

The **frequency** of a wave,  $f$ , is the number of complete oscillations it makes each second. Frequency is measured in hertz. One hertz (Hz) is one oscillation per second. The frequency and period of a wave are related by the following equation.

$$\text{Frequency of a Wave } f = \frac{1}{T}$$

The frequency of a wave is equal to the reciprocal of the period.

Both the period and the frequency of a wave depend only on its source. They do not depend on the wave's speed or the medium.

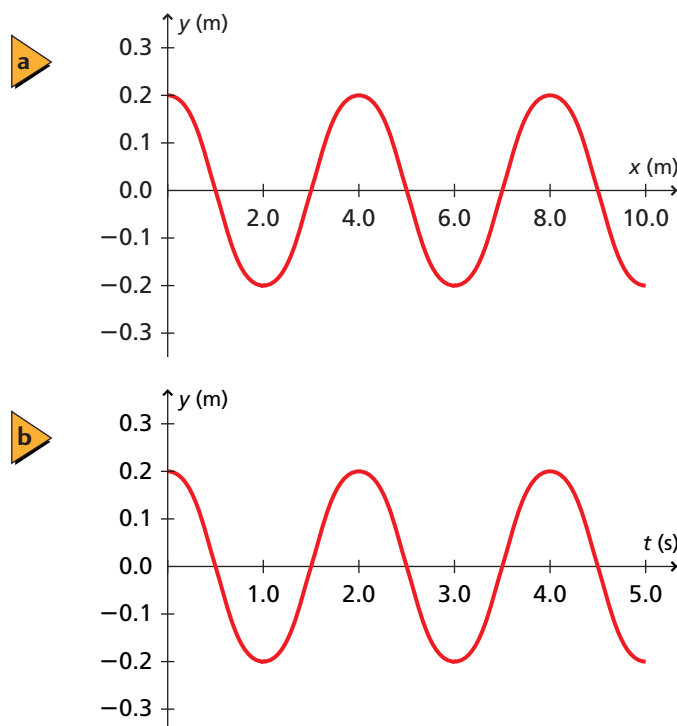
Although you can directly measure a wavelength, the wavelength depends on both the frequency of the oscillator and the speed of the wave. In the time interval of one period, a wave moves one wavelength. Therefore, the wavelength of a wave is the speed multiplied by the period,  $\lambda = vT$ . Because the frequency is usually more easily found than the period, this equation is most often written in the following way.

$$\text{Wavelength } \lambda = \frac{v}{f}$$

The wavelength of a wave is equal to the velocity divided by the frequency.

**Picturing waves** If you took a snapshot of a transverse wave on a spring, it might look like one of the waves shown in Figure 14-8. This snapshot could be placed on a graph grid to show more information about the wave, as in **Figure 14-10a**. Similarly, if you record the motion of a single particle, such as point P in Figure 14-9, that motion can be plotted on a displacement-versus-time graph, as in **Figure 14-10b**. The period is found using the time axis of the graph. Longitudinal waves can also be depicted by graphs, where the  $y$ -axis could represent pressure, for example.

■ **Figure 14-10** Waves can be represented by graphs. The wavelength of this wave is 4.0 m **(a)**. The period is 2.0 s **(b)**. The amplitude, or displacement, is 0.2 m in both graphs. If these graphs represent the same wave, what is its speed?



### ▶ EXAMPLE Problem 3

**Characteristics of a Wave** A sound wave has a frequency of 192 Hz and travels the length of a football field, 91.4 m, in 0.271 s.

- What is the speed of the wave?
- What is the wavelength of the wave?
- What is the period of the wave?
- If the frequency was changed to 442 Hz, what would be the new wavelength and period?

#### 1 Analyze and Sketch the Problem

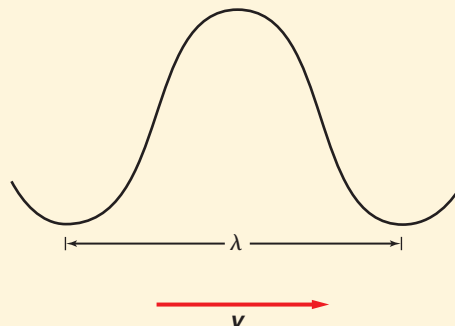
- Draw a model of the football field.
- Diagram a velocity vector.

**Known:**                      **Unknown:**

$$f = 192 \text{ Hz} \quad v = ?$$

$$d = 91.4 \text{ m} \quad \lambda = ?$$

$$t = 0.271 \text{ s} \quad T = ?$$



#### 2 Solve for the Unknown

- a. Solve for  $v$ .

$$\begin{aligned} v &= \frac{d}{t} \\ &= \frac{91.4 \text{ m}}{0.271 \text{ s}} \\ &= 337 \text{ m/s} \end{aligned}$$

Substitute  $d = 91.4 \text{ m}$ ,  $t = 0.271 \text{ s}$

- b. Solve for  $\lambda$ .

$$\begin{aligned} \lambda &= \frac{v}{f} \\ &= \frac{337 \text{ m/s}}{192 \text{ Hz}} \\ &= 1.76 \text{ m} \end{aligned}$$

Substitute  $v = 337 \text{ m/s}$ ,  $f = 192 \text{ Hz}$

- c. Solve for  $T$ .

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{192 \text{ Hz}} \\ &= 0.00521 \text{ s} \end{aligned}$$

Substitute  $f = 192 \text{ Hz}$

- d.  $\lambda = \frac{v}{f}$

$$\begin{aligned} &= \frac{337 \text{ m/s}}{442 \text{ Hz}} \\ &= 0.762 \text{ m} \end{aligned}$$

Substitute  $v = 337 \text{ m/s}$ ,  $f = 442 \text{ Hz}$

$$T = \frac{1}{f}$$

$$\begin{aligned} &= \frac{1}{442 \text{ Hz}} \\ &= 0.00226 \text{ s} \end{aligned}$$

Substitute  $f = 442 \text{ Hz}$

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#### 3 Evaluate the Answer

- Are the units correct?** Hz has the units  $\text{s}^{-1}$ , so  $(\text{m/s})/\text{Hz} = (\text{m/s}) \cdot \text{s} = \text{m}$ , which is correct.
- Are the magnitudes realistic?** A typical sound wave travels approximately 343 m/s, so 337 m/s is reasonable. The frequencies and periods are reasonable for sound waves. 442 Hz is close to a 440-Hz A above middle-C on a piano.



15. A sound wave produced by a clock chime is heard 515 m away 1.50 s later.
  - a. What is the speed of sound of the clock's chime in air?
  - b. The sound wave has a frequency of 436 Hz. What is the period of the wave?
  - c. What is the wave's wavelength?
16. A hiker shouts toward a vertical cliff 465 m away. The echo is heard 2.75 s later.
  - a. What is the speed of sound of the hiker's voice in air?
  - b. The wavelength of the sound is 0.750 m. What is its frequency?
  - c. What is the period of the wave?
17. If you want to increase the wavelength of waves in a rope, should you shake it at a higher or lower frequency?
18. What is the speed of a periodic wave disturbance that has a frequency of 3.50 Hz and a wavelength of 0.700 m?
19. The speed of a transverse wave in a string is 15.0 m/s. If a source produces a disturbance that has a frequency of 6.00 Hz, what is its wavelength?
20. Five pulses are generated every 0.100 s in a tank of water. What is the speed of propagation of the wave if the wavelength of the surface wave is 1.20 cm?
21. A periodic longitudinal wave that has a frequency of 20.0 Hz travels along a coil spring. If the distance between successive compressions is 0.600 m, what is the speed of the wave?

You probably have been intuitively aware that waves carry energy that can do work. You may have seen the massive damage done by the huge storm surge of a hurricane or the slower erosion of cliffs and beaches done by small, everyday waves. It is important to remember that while the amplitude of a mechanical wave determines the amount of energy it carries, only the medium determines the wave's speed.

## 14.2 Section Review

22. **Speed in Different Media** If you pull on one end of a coiled-spring toy, does the pulse reach the other end instantaneously? What happens if you pull on a rope? What happens if you hit the end of a metal rod? Compare and contrast the pulses traveling through these three materials.
23. **Wave Characteristics** You are creating transverse waves in a rope by shaking your hand from side to side. Without changing the distance that your hand moves, you begin to shake it faster and faster. What happens to the amplitude, wavelength, frequency, period, and velocity of the wave?
24. **Waves Moving Energy** Suppose that you and your lab partner are asked to demonstrate that a transverse wave transports energy without transferring matter. How could you do it?
25. **Longitudinal Waves** Describe longitudinal waves. What types of media transmit longitudinal waves?
26. **Critical Thinking** If a raindrop falls into a pool, it creates waves with small amplitudes. If a swimmer jumps into a pool, waves with large amplitudes are produced. Why doesn't the heavy rain in a thunderstorm produce large waves?

## 14.3 Wave Behavior

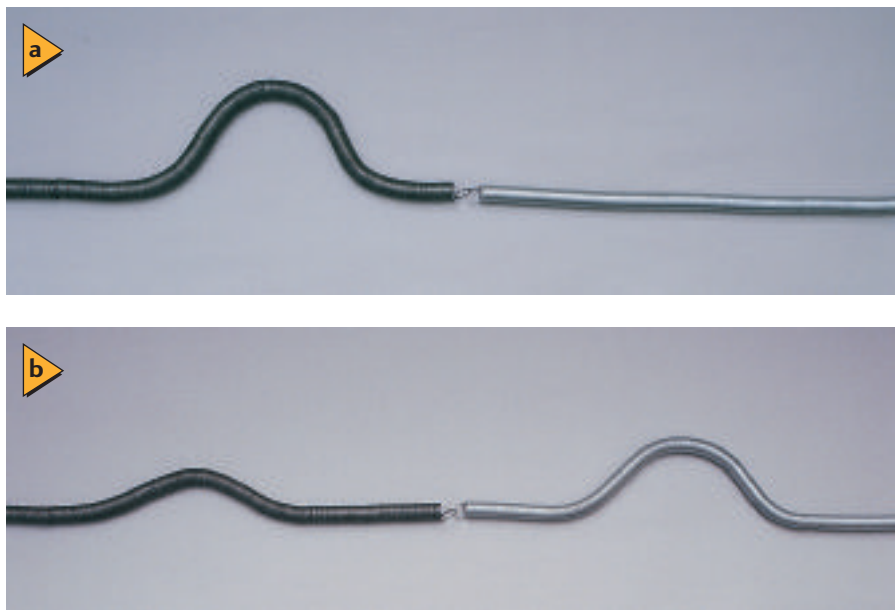
When a wave encounters the boundary of the medium in which it is traveling, it often reflects back into the medium. In other instances, some or all of the wave passes through the boundary into another medium, often changing direction at the boundary. In addition, many properties of wave behavior result from the fact that two or more waves can exist in the same medium at the same time—quite unlike particles.

### Waves at Boundaries

Recall from Section 14.2 that the speed of a mechanical wave depends only on the properties of the medium it passes through, not on the wave's amplitude or frequency. For water waves, the depth of the water affects wave speed. For sound waves in air, the temperature affects wave speed. For waves on a spring, the speed depends upon the spring's tension and mass per unit length.

Examine what happens when a wave moves across a boundary from one medium into another, as in two springs of different thicknesses joined end-to-end. **Figure 14-11** shows a wave pulse moving from a large spring into a smaller one. The wave that strikes the boundary is called the **incident wave**. One pulse from the larger spring continues in the smaller spring, but at the specific speed of waves traveling through the smaller spring. Note that this transmitted wave pulse remains upward.

Some of the energy of the incident wave's pulse is reflected backward into the larger spring. This returning wave is called the **reflected wave**. Whether or not the reflected wave is upright or inverted depends on the characteristics of the two springs. For example, if the waves in the smaller spring have a higher speed because the spring is heavier or stiffer, then the reflected wave will be inverted.



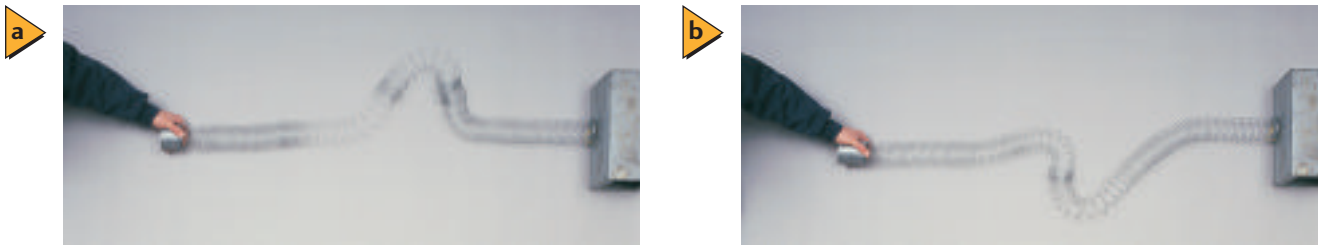
#### ► Objectives

- **Relate** a wave's speed to the medium in which the wave travels.
- **Describe** how waves are reflected and refracted at boundaries between media.
- **Apply** the principle of superposition to the phenomenon of interference.

#### ► Vocabulary

incident wave  
reflected wave  
principle of superposition  
interference  
node  
antinode  
standing wave  
wave front  
ray  
normal  
law of reflection  
refraction

■ **Figure 14-11** The junction of the two springs is a boundary between two media. A pulse reaching the boundary (**a**) is partially reflected and partially transmitted (**b**).



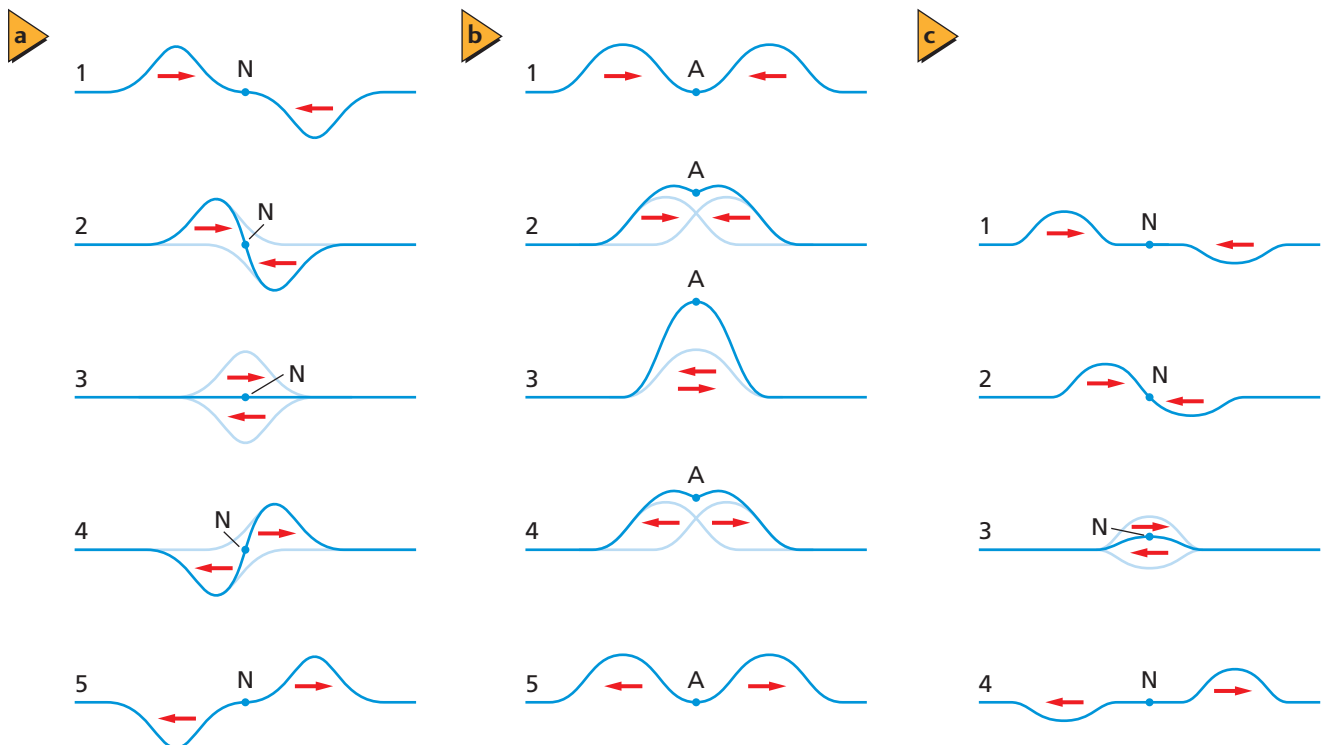
■ **Figure 14-12** A pulse approaches a rigid wall **(a)** and is reflected back **(b)**. Note that the amplitude of the reflected pulse is nearly equal to the amplitude of the incident pulse, but it is inverted.

What happens if the boundary is a wall rather than another spring? When a wave pulse is sent down a spring connected to a rigid wall, the energy transmitted is reflected back from the wall, as shown in **Figure 14-12**. The wall is the boundary of a new medium through which the wave attempts to pass. Instead of passing through, the pulse is reflected from the wall with almost exactly the same amplitude as the pulse of the incident wave. Thus, almost all the wave's energy is reflected back. Very little energy is transmitted into the wall. Also note that the pulse is inverted. If the spring were attached to a loose ring around a pole, a free-moving boundary, the wave would not be inverted.

### Superposition of Waves

Suppose a pulse traveling down a spring meets a reflected pulse coming back. In this case, two waves exist in the same place in the medium at the same time. Each wave affects the medium independently. The **principle of superposition** states that the displacement of a medium caused by two or more waves is the algebraic sum of the displacements caused by the individual waves. In other words, two or more waves can combine to form a new wave. If the waves move in opposite directions, they can cancel or form a new wave of lesser or greater amplitude. The result of the superposition of two or more waves is called **interference**.

■ **Figure 14-13** When two equal pulses meet, there is a point, called the node (N), where the medium remains undisturbed **(a)**. Constructive interference results in maximum interference at the antinode (A) **(b)**. If the opposite pulses have unequal amplitudes, cancellation is incomplete **(c)**.



**Wave Interaction**



With a coiled-spring toy, you can create pressure waves, as well as transverse waves of various amplitudes, speeds, and orientations.

**1. Design an experiment** to test what happens when waves from different directions meet.

**2.** Perform your experiment and record your observations.

**Analyze and Conclude**

**3.** Does the speed of either wave change?

**4.** Do the waves bounce off each other, or do they pass through each other?

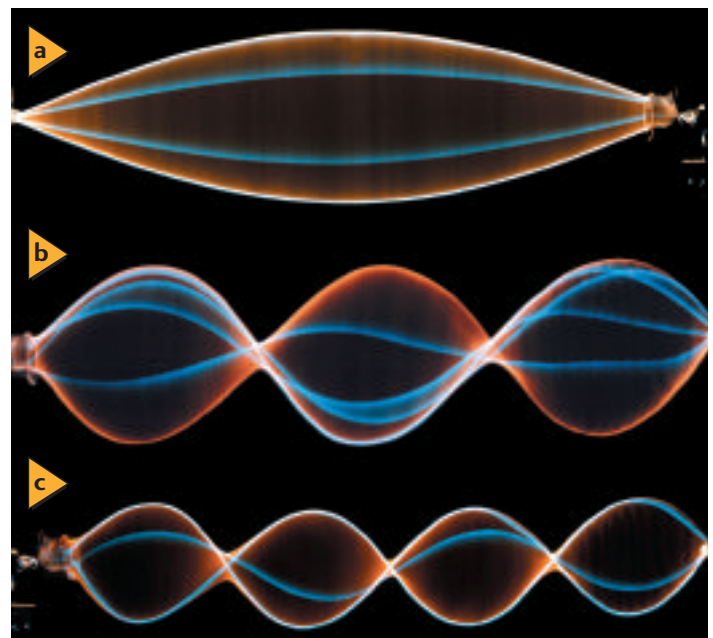
**Wave interference** Wave interference can be either constructive or destructive. When two pulses with equal but opposite amplitudes meet, the displacement of the medium at each point in the overlap region is reduced. The superposition of waves with equal but opposite amplitudes causes destructive interference, as shown in **Figure 14-13a**. When the pulses meet and are in the same location, the displacement is zero. Point N, which does not move at all, is called a **node**. The pulses continue to move and eventually resume their original form.

Constructive interference occurs when wave displacements are in the same direction. The result is a wave that has an amplitude greater than those of any of the individual waves. **Figure 14-13b** shows the constructive interference of two equal pulses. A larger pulse appears at point A when the two waves meet. Point A has the largest displacement and is called the **antinode**. The two pulses pass through each other without changing their shapes or sizes. If the pulses have unequal amplitudes, the resultant pulse at the overlap is the algebraic sum of the two pulses, as shown in **Figure 14-13c**.

**Standing waves** You can apply the concept of superimposed waves to the control of the formation of large amplitude waves. If you attach one end of a rope or coiled spring to a fixed point, such as a doorknob, and then start to vibrate the other end, the wave leaves your hand, is reflected at the fixed end, is inverted, and returns to your hand. When it reaches your hand, the reflected wave is inverted and travels back down the rope. Thus, when the wave leaves your hand the second time, its displacement is in the same direction as it was when it left your hand the first time.

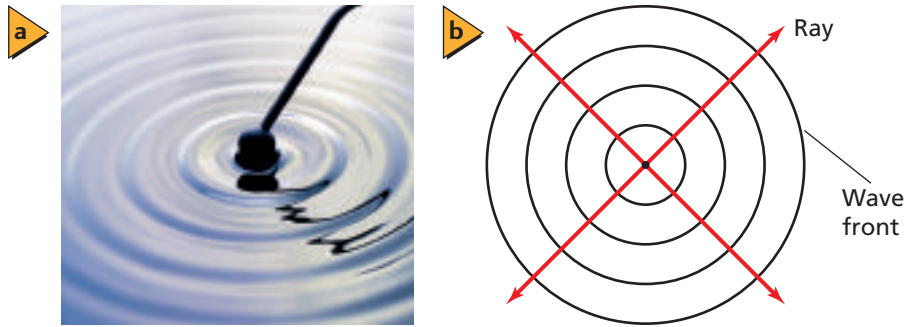
What if you want to increase the amplitude of the wave that you create? Suppose you adjust the motion of your hand so that the period of vibration equals the time needed for the wave to make one round-trip from your hand to the door and back. Then, the displacement given by your hand to the rope each time will add to the displacement of the reflected wave. As a result, the oscillation of the rope in one segment will be much greater than the motion of your hand. You would expect this based on your knowledge of constructive interference. This large-amplitude oscillation is an example of mechanical resonance. The nodes are at the ends of the rope and an antinode is in the middle, as shown in **Figure 14-14a**. Thus, the wave appears to be standing still and is called a **standing wave**. You should note, however, that the standing wave is the interference of the two traveling waves moving in opposite directions. If you double the frequency of vibration, you can produce one more node and one more antinode in the rope. Then it appears to vibrate in two segments. Further increases in frequency produce even more nodes and antinodes, as shown in **Figures 14-14b** and **c**.

**Figure 14-14** Interference produces standing waves in a rope. As the frequency is increased, as shown from top to bottom, the number of nodes and antinodes increases.





■ **Figure 14-15** Circular waves spread outward from their source **(a)**. The wave can be represented by circles drawn at their crests **(b)**. Notice that the rays are perpendicular to the wave fronts.



## Waves in Two Dimensions

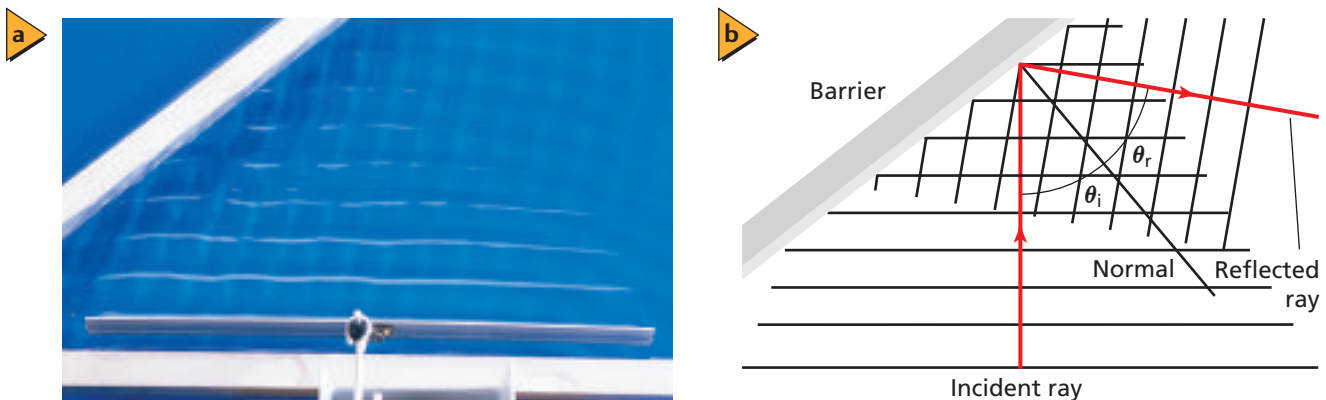
You have studied waves on a rope and on a spring reflecting from rigid supports, where the amplitude of the waves is forced to be zero by destructive interference. These mechanical waves move in only one dimension. However, waves on the surface of water move in two dimensions, and sound waves and electromagnetic waves will later be shown to move in three dimensions. How can two-dimensional waves be demonstrated?

**Picturing waves in two dimensions** When you throw a small stone into a calm pool of water, you see the circular crests and troughs of the resulting waves spreading out in all directions. You can sketch those waves by drawing circles to represent the wave crests. If you dip your finger into water with a constant frequency, the resulting sketch would be a series of concentric circles, called wave fronts, centered on your finger. A **wave front** is a line that represents the crest of a wave in two dimensions, and it can be used to show waves of any shape, including circular waves and straight waves. **Figure 14-15a** shows circular waves in water, and **Figure 14-15b** shows the wave fronts that represent those water waves. Wave fronts drawn to scale show the wavelengths of the waves, but not their amplitudes.

Whatever their shape, two-dimensional waves always travel in a direction that is perpendicular to their wave fronts. That direction can be represented by a **ray**, which is a line drawn at a right angle to the crest of the wave. When all you want to show is the direction in which a wave is traveling, it is convenient to draw rays instead of wave fronts.

**Reflection of waves in two dimensions** A ripple tank can be used to show the properties of two-dimensional waves. A ripple tank contains a thin layer of water. Vibrating boards produce wave pulses, or, in the case of

■ **Figure 14-16** A wave pulse in a ripple tank is reflected by a barrier **(a)**. The ray diagram models the wave in time sequence as it approaches the barrier and is then reflected to the right **(b)**.



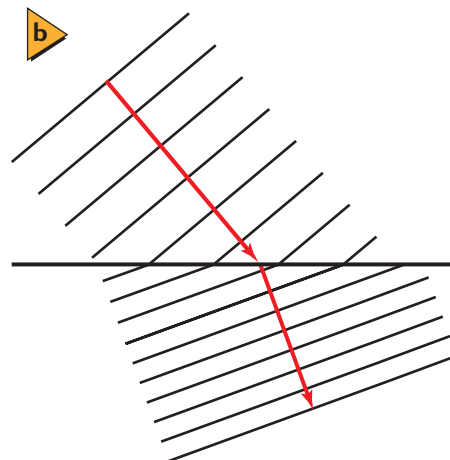
**Figure 14-16a**, traveling waves of water with constant frequency. A lamp above the tank produces shadows below the tank that show the locations of the crests of the waves. The wave pulse travels toward a rigid barrier that reflects the wave: the incident wave moves upward, and the reflected wave moves to the right.

The direction of wave motion can be modeled by a ray diagram. **Figure 14-16b** shows the ray diagram for the waves in the ripple tank. The ray representing the incident wave is the arrow pointing upward. The ray representing the reflected wave points to the right.

The direction of the barrier also is shown by a line, which is drawn at a right angle, or perpendicular, to the barrier, called the **normal**. The angle between the incident ray and the normal is called the angle of incidence. The angle between the normal and the reflected ray is called the angle of reflection. The **law of reflection** states that the angle of incidence is equal to the angle of reflection.

**Refraction of waves in two dimensions** A ripple tank also can be used to model the behavior of waves as they move from one medium into another. **Figure 14-17a** shows a glass plate placed in a ripple tank. The water above the plate is shallower than the water in the rest of the tank and acts like a different medium. As the waves move from deep to shallow water, their speed decreases, and the direction of the waves changes. Because the waves in the shallow water are generated by the waves in the deep water, their frequency is not changed. Based on the equation  $\lambda = v/f$ , the decrease in the speed of the waves means that the wavelength is shorter in the shallower water. The change in the direction of waves at the boundary between two different media is known as **refraction**. **Figure 14-17b** shows a wave front and ray model of refraction. When you study the reflection and refraction of light in Chapter 17, you will learn the law of refraction, called Snell's law.

You may not be aware that echoes are caused by the reflection of sound off hard surfaces, such as the walls of a large warehouse or a distant cliff face. Refraction is partly responsible for rainbows. When white light passes through a raindrop, refraction separates the light into its individual colors.



■ **Figure 14-17** As the water waves move over a shallower region of the ripple tank where a glass plate is placed, they slow down and their wavelength decreases **(a)**. Refraction can be represented by a diagram of wave fronts and rays **(b)**.

## 14.3 Section Review

27. **Waves at Boundaries** Which of the following wave characteristics remain unchanged when a wave crosses a boundary into a different medium: frequency, amplitude, wavelength, velocity, and/or direction?
28. **Refraction of Waves** Notice in Figure 14-17a how the wave changes direction as it passes from one medium to another. Can two-dimensional waves cross a boundary between two media without changing direction? Explain.
29. **Standing Waves** In a standing wave on a string fixed at both ends, how is the number of nodes related to the number of antinodes?
30. **Critical Thinking** As another way to understand wave reflection, cover the right-hand side of each drawing in Figure 14-13a with a piece of paper. The edge of the paper should be at point N, the node. Now, concentrate on the resultant wave, shown in darker blue. Note that it acts like a wave reflected from a boundary. Is the boundary a rigid wall, or is it open-ended? Repeat this exercise for Figure 14-13b.

## Pendulum Vibrations

A pendulum can provide a simple model for the investigation of wave properties. In this experiment, you will design a procedure to use the pendulum to examine amplitude, period, and frequency of a wave. You also will determine the acceleration due to gravity from the period and length of the pendulum.

### QUESTION

How can a pendulum demonstrate the properties of waves?

#### Objectives

- **Determine** what variables affect a pendulum's period.
- **Investigate** the frequency and period amplitude of a pendulum.
- **Measure**  $g$ , the acceleration due to gravity, using the period and length of a pendulum.

#### Safety Precautions



#### Possible Materials

string (1.5 m)  
 three sinkers  
 paper clip  
 ring stand with ring  
 stopwatch

#### Procedure

1. Design a pendulum using a ring stand, a string with a paper clip, and a sinker attached to the paper clip. Be sure to check with your teacher and have your design approved before you proceed with the lab.
2. For this investigation, the length of the pendulum is the length of the string plus half the length of the bob. The amplitude is how far the bob is pulled from its equilibrium point. The frequency is the cycles/s of the bob. The period is the time for the bob to travel back and forth (one cycle). When collecting data for the period, find the time it takes to make ten cycles, and then calculate the period in s/cycles. When finding frequency, count how many cycles occur in 10 s, and then convert your value to cycles/s.
3. Design a procedure that keeps the mass of the bob and the amplitude constant, but varies the length. Determine the frequency and period of the pendulum. Record your results in the data table. Use several trials at several lengths to collect your data.
4. Design a procedure that keeps length and amplitude constant, but varies the mass of the bob. Determine the frequency and period of the pendulum. Record your results in the data table. Use several trials to collect your data.
5. Design a procedure that keeps length and mass of the bob constant, but varies the amplitude of the pendulum. Determine the frequency and period of the pendulum. Record your results in the data table. Use several trials to collect your data.
6. Design a procedure using the pendulum to calculate  $g$ , the acceleration due to gravity, using the equation  $T = 2\pi\sqrt{\ell/g}$ .  $T$  is the period, and  $\ell$  is the length of the pendulum string. Remember to use several trials to collect your data.



### Data Table 1

This data table format can be used for steps 2–5.

	Trial 1	Trial 2	Trial 3	Average	Period (s/cycle)	Frequency (cycles/s)
Length 1					_____	
Length 2						_____
Length 3						
Mass 1					_____	
Mass 2						_____
Mass 3						
Amplitude 1					_____	
Amplitude 2						_____
Amplitude 3						

### Data Table 2

This data table format can be used for step 6, finding  $g$ .

	Trial 1	Trial 2	Trial 3	Average	Period (s/cycle)	Length of String (m)
Length 1						
Length 2						
Length 3						

#### Analyze

- Summarize** What is the relationship between the pendulum's amplitude and its period?
- Summarize** What is the relationship between the pendulum's bob mass and its period?
- Compare and Contrast** How are the period and length of a pendulum related?
- Analyze** Calculate  $g$  from your data in step 6.
- Error Analysis** What is the percent error of your experimental  $g$  value? What are some possible reasons for the difference between your experimental value of  $g$  and the accepted value of  $g$ ?
- Analyze and Conclude** When does the pendulum bob have the greatest kinetic energy?
- Analyze and Conclude** When does the pendulum bob have the greatest potential energy?

#### Going Further

Suppose you had a very long pendulum. What other observations could be made, over the period of a day, of this pendulum's motion?

#### Real-World Physics

Pendulums are used to drive some types of clocks. Using the observations from your experiments, what design problems are there in using your pendulum as a time-keeping instrument?

#### Conclude and Apply

- Infer** What variable(s) affects a pendulum's period?
- Analyze** Why is it better to run three or more trials to obtain the frequency and period of each pendulum?
- Compare** How is the motion of a pendulum like that of a wave?



To find out more about the behavior of waves, visit the Web site: [physicspp.com](http://physicspp.com)

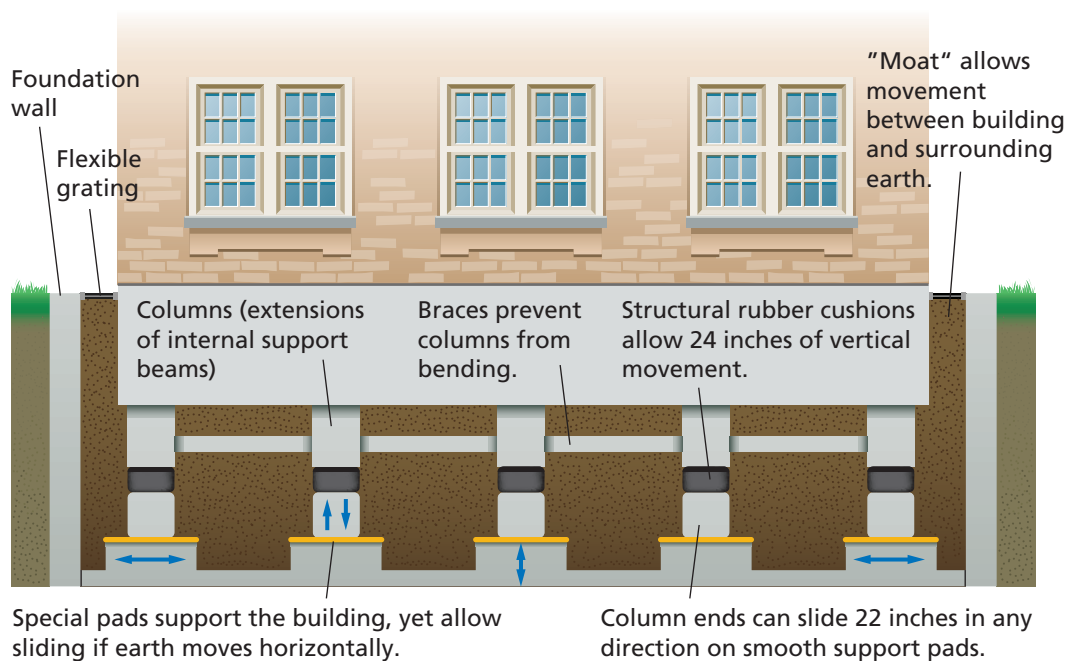


# Technology and Society

## Earthquake Protection

An **earthquake** is the equivalent of a violent explosion somewhere beneath the surface of Earth. The mechanical waves that radiate from an earthquake are both transverse and longitudinal. Transverse waves shake a structure horizontally, while longitudinal waves cause vertical shaking. Earthquakes cannot be predicted or prevented, so we must construct our buildings to withstand them.

and its foundation. To minimize vertical shaking of a building, springs are inserted into the vertical members of the framework. These springs are made of a strong rubber compound compressed within heavy structural steel cylinders. Sideways shaking is diminished by placing sliding supports beneath the building columns. These allow the structure to remain stationary if the ground beneath it moves sideways.



### New building designs reduce damage by earthquakes.

As our knowledge of earthquakes increases, existing buildings must be retrofitted to withstand newly discovered types of earthquake-related failures.

**Reducing Damage** Most bridges and parking ramps were built by stacking steel-reinforced concrete sections atop one another. Gravity keeps them in place. These structures are immensely strong under normal conditions, but they can be shaken apart by a strong earthquake. New construction codes dictate that their parts must be bonded together by heavy steel straps.

Earthquake damage to buildings also can be reduced by allowing a small amount of controlled movement between the building frame

and its foundation. Long structures, like tunnels and bridges, must be constructed to survive vertical or horizontal shearing fractures of the earth beneath. The Bay Area Rapid Transit tunnel that runs beneath San Francisco Bay has flexible couplings for stability should the bay floor buckle.

### Going Further

1. **Research** What is the framework of your school made of and how were the foundations built?
2. **Observe** Find a brick building that has a crack in one of its walls. See if you can tell why the crack formed and why it took the path that it did. What might this have to do with earthquakes?

### 14.1 Periodic Motion

#### Vocabulary

- periodic motion (p. 375)
- simple harmonic motion (p. 375)
- period (p. 375)
- amplitude (p. 375)
- Hooke's law (p. 376)
- pendulum (p. 378)
- resonance (p. 380)

#### Key Concepts

- Periodic motion is any motion that repeats in a regular cycle.
- Simple harmonic motion results when the restoring force on an object is directly proportional to the object's displacement from equilibrium. Such a force obeys Hooke's law.

$$F = -kx$$

- The elastic potential energy stored in a spring that obeys Hooke's law is expressed by the following equation.

$$PE_{\text{sp}} = \frac{1}{2}kx^2$$

- The period of a pendulum can be found with the following equation.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

### 14.2 Wave Properties

#### Vocabulary

- wave (p. 381)
- wave pulse (p. 381)
- periodic wave (p. 381)
- transverse wave (p. 381)
- longitudinal wave (p. 381)
- surface wave (p. 382)
- trough (p. 383)
- crest (p. 383)
- wavelength (p. 383)
- frequency (p. 384)

#### Key Concepts

- Waves transfer energy without transferring matter.
- In transverse waves, the displacement of the medium is perpendicular to the direction of wave motion. In longitudinal waves, the displacement is parallel to the direction of wave motion.
- Frequency is the number of cycles per second and is related to period by:

$$f = \frac{1}{T}$$

- The wavelength of a continuous wave can be found by using the following equation.

$$\lambda = \frac{v}{f}$$

### 14.3 Wave Behavior

#### Vocabulary

- incident wave (p. 387)
- reflected wave (p. 387)
- principle of superposition (p. 388)
- interference (p. 388)
- node (p. 389)
- antinode (p. 389)
- standing wave (p. 389)
- wave front (p. 390)
- ray (p. 390)
- normal (p. 391)
- law of reflection (p. 391)
- refraction (p. 391)

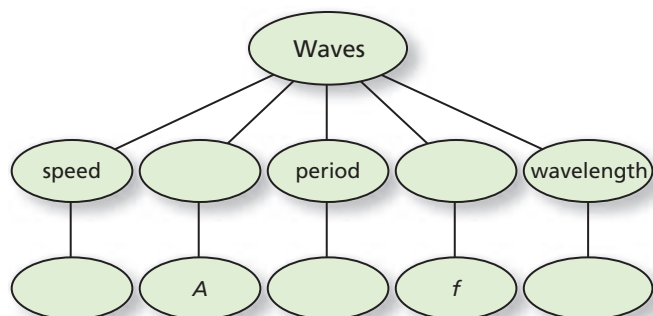
#### Key Concepts

- When a wave crosses a boundary between two media, it is partially transmitted and partially reflected.
- The principle of superposition states that the displacement of a medium resulting from two or more waves is the algebraic sum of the displacements of the individual waves.
- Interference occurs when two or more waves move through a medium at the same time.
- When two-dimensional waves are reflected from boundaries, the angles of incidence and reflection are equal.
- The change in direction of waves at the boundary between two different media is called refraction.



## Concept Mapping

31. Complete the concept map using the following terms and symbols: *amplitude*, *frequency*,  $v$ ,  $\lambda$ ,  $T$ .



## Mastering Concepts

32. What is periodic motion? Give three examples of periodic motion. (14.1)
33. What is the difference between frequency and period? How are they related? (14.1)
34. What is simple harmonic motion? Give an example of simple harmonic motion. (14.1)
35. If a spring obeys Hooke's law, how does it behave? (14.1)
36. How can the spring constant of a spring be determined from a graph of force versus displacement? (14.1)
37. How can the potential energy in a spring be determined from the graph of force versus displacement? (14.1)
38. Does the period of a pendulum depend on the mass of the bob? The length of the string? Upon what else does the period depend? (14.1)
39. What conditions are necessary for resonance to occur? (14.1)
40. How many general methods of energy transfer are there? Give two examples of each. (14.2)
41. What is the primary difference between a mechanical wave and an electromagnetic wave? (14.2)
42. What are the differences among transverse, longitudinal, and surface waves? (14.2)
43. Waves are sent along a spring of fixed length. (14.2)
- Can the speed of the waves in the spring be changed? Explain.
  - Can the frequency of a wave in the spring be changed? Explain.
44. What is the wavelength of a wave? (14.2)
45. Suppose you send a pulse along a rope. How does the position of a point on the rope before the pulse arrives compare to the point's position after the pulse has passed? (14.2)
46. What is the difference between a wave pulse and a periodic wave? (14.2)
47. Describe the difference between wave frequency and wave velocity. (14.2)
48. Suppose you produce a transverse wave by shaking one end of a spring from side to side. How does the frequency of your hand compare with the frequency of the wave? (14.2)
49. When are points on a wave in phase with each other? When are they out of phase? Give an example of each. (14.2)
50. What is the amplitude of a wave and what does it represent? (14.2)
51. Describe the relationship between the amplitude of a wave and the energy it carries. (14.2)
52. When a wave reaches the boundary of a new medium, what happens to it? (14.3)
53. When a wave crosses a boundary between a thin and a thick rope, as shown in **Figure 14-18**, its wavelength and speed change, but its frequency does not. Explain why the frequency is constant. (14.3)

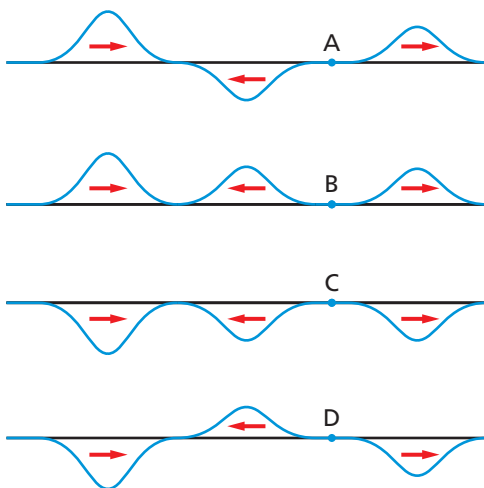


■ **Figure 14-18**

54. How does a spring pulse reflected from a rigid wall differ from the incident pulse? (14.3)
55. Describe interference. Is interference a property of only some types of waves or all types of waves? (14.3)
56. What happens to a spring at the nodes of a standing wave? (14.3)
57. **Violins** A metal plate is held fixed in the center and sprinkled with sugar. With a violin bow, the plate is stroked along one edge and made to vibrate. The sugar begins to collect in certain areas and move away from others. Describe these regions in terms of standing waves. (14.3)
58. If a string is vibrating in four parts, there are points where it can be touched without disturbing its motion. Explain. How many of these points exist? (14.3)
59. Wave fronts pass at an angle from one medium into a second medium, where they travel with a different speed. Describe two changes in the wave fronts. What does not change? (14.3)

### Applying Concepts

60. A ball bounces up and down on the end of a spring. Describe the energy changes that take place during one complete cycle. Does the total mechanical energy change?
61. Can a pendulum clock be used in the orbiting *International Space Station*? Explain.
62. Suppose you hold a 1-m metal bar in your hand and hit its end with a hammer, first, in a direction parallel to its length, and second, in a direction at right angles to its length. Describe the waves produced in the two cases.
63. Suppose you repeatedly dip your finger into a sink full of water to make circular waves. What happens to the wavelength as you move your finger faster?
64. What happens to the period of a wave as the frequency increases?
65. What happens to the wavelength of a wave as the frequency increases?
66. Suppose you make a single pulse on a stretched spring. How much energy is required to make a pulse with twice the amplitude?
67. You can make water slosh back and forth in a shallow pan only if you shake the pan with the correct frequency. Explain.
68. In each of the four waves in **Figure 14-19**, the pulse on the left is the original pulse moving toward the right. The center pulse is a reflected pulse; the pulse on the right is a transmitted pulse. Describe the rigidity of the boundaries at A, B, C, and D.

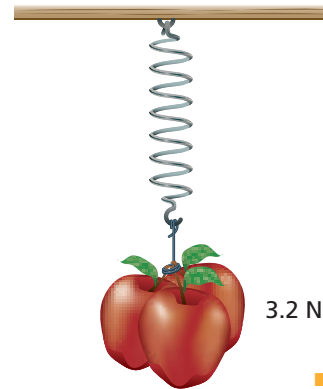


■ Figure 14-19

### Mastering Problems

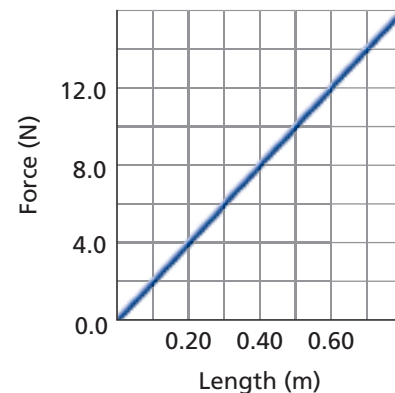
#### 14.1 Periodic Motion

69. A spring stretches by 0.12 m when some apples weighing 3.2 N are suspended from it, as shown in **Figure 14-20**. What is the spring constant of the spring?



■ Figure 14-20

70. **Car Shocks** Each of the coil springs of a car has a spring constant of 25,000 N/m. How much is each spring compressed if it supports one-fourth of the car's 12,000-N weight?
71. How much potential energy is stored in a spring with a spring constant of 27 N/m if it is stretched by 16 cm?
72. **Rocket Launcher** A toy rocket-launcher contains a spring with a spring constant of 35 N/m. How far must the spring be compressed to store 1.5 J of energy?
73. Force-versus-length data for a spring are plotted on the graph in **Figure 14-21**.
  - a. What is the spring constant of the spring?
  - b. What is the energy stored in the spring when it is stretched to a length of 0.50 m?



■ Figure 14-21





## Chapter 14 Assessment

74. How long must a pendulum be to have a period of 2.3 s on the Moon, where  $g = 1.6 \text{ m/s}^2$ ?

### 14.2 Wave Properties

75. **Building Motion** The Sears Tower in Chicago, shown in **Figure 14-22**, sways back and forth in the wind with a frequency of about 0.12 Hz. What is its period of vibration?



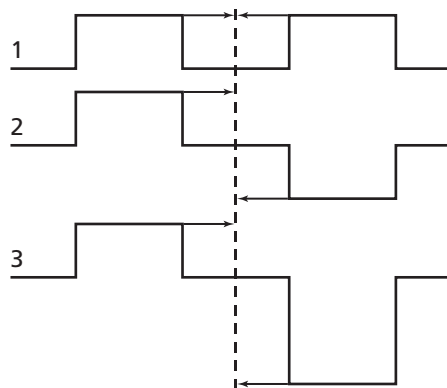
■ Figure 14-22

76. **Ocean Waves** An ocean wave has a length of 12.0 m. A wave passes a fixed location every 3.0 s. What is the speed of the wave?
77. Water waves in a shallow dish are 6.0-cm long. At one point, the water moves up and down at a rate of 4.8 oscillations/s.
- What is the speed of the water waves?
  - What is the period of the water waves?
78. Water waves in a lake travel 3.4 m in 1.8 s. The period of oscillation is 1.1 s.
- What is the speed of the water waves?
  - What is their wavelength?
79. **Sonar** A sonar signal of frequency  $1.00 \times 10^6$  Hz has a wavelength of 1.50 mm in water.
- What is the speed of the signal in water?
  - What is its period in water?
  - What is its period in air?
80. A sound wave of wavelength 0.60 m and a velocity of 330 m/s is produced for 0.50 s.
- What is the frequency of the wave?
  - How many complete waves are emitted in this time interval?
  - After 0.50 s, how far is the front of the wave from the source of the sound?

81. The speed of sound in water is 1498 m/s. A sonar signal is sent straight down from a ship at a point just below the water surface, and 1.80 s later, the reflected signal is detected. How deep is the water?
82. Pepe and Alfredo are resting on an offshore raft after a swim. They estimate that 3.0 m separates a trough and an adjacent crest of each surface wave on the lake. They count 12 crests that pass by the raft in 20.0 s. Calculate how fast the waves are moving.
83. **Earthquakes** The velocity of the transverse waves produced by an earthquake is 8.9 km/s, and that of the longitudinal waves is 5.1 km/s. A seismograph records the arrival of the transverse waves 68 s before the arrival of the longitudinal waves. How far away is the earthquake?

### 14.3 Wave Behavior

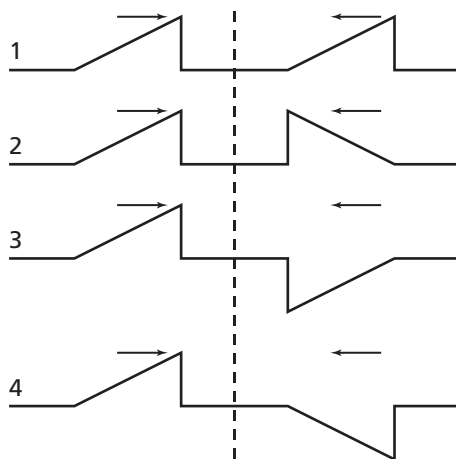
84. Sketch the result for each of the three cases shown in **Figure 14-23**, when the centers of the two approaching wave pulses lie on the dashed line so that the pulses exactly overlap.



■ Figure 14-23

85. If you slosh the water in a bathtub at the correct frequency, the water rises first at one end and then at the other. Suppose you can make a standing wave in a 150-cm-long tub with a frequency of 0.30 Hz. What is the velocity of the water wave?
86. **Guitars** The wave speed in a guitar string is 265 m/s. The length of the string is 63 cm. You pluck the center of the string by pulling it up and letting go. Pulses move in both directions and are reflected off the ends of the string.
- How long does it take for the pulse to move to the string end and return to the center?
  - When the pulses return, is the string above or below its resting location?
  - If you plucked the string 15 cm from one end of the string, where would the two pulses meet?

87. Sketch the result for each of the four cases shown in **Figure 14-24**, when the centers of each of the two wave pulses lie on the dashed line so that the pulses exactly overlap.



■ **Figure 14-24**

### Mixed Review

88. What is the period of a pendulum with a length of 1.4 m?
89. The frequency of yellow light is  $5.1 \times 10^{14}$  Hz. Find the wavelength of yellow light. The speed of light is  $3.00 \times 10^8$  m/s.
90. **Radio Wave** AM-radio signals are broadcast at frequencies between 550 kHz (kilohertz) and 1600 kHz and travel  $3.0 \times 10^8$  m/s.
- What is the range of wavelengths for these signals?
  - FM frequencies range between 88 MHz (megahertz) and 108 MHz and travel at the same speed. What is the range of FM wavelengths?
91. You are floating just offshore at the beach. Even though the waves are steadily moving in toward the beach, you don't move any closer to the beach.
- What type of wave are you experiencing as you float in the water?
  - Explain why the energy in the wave does not move you closer to shore.
  - In the course of 15 s you count ten waves that pass you. What is the period of the waves?
  - What is the frequency of the waves?
  - You estimate that the wave crests are 3 m apart. What is the velocity of the waves?
  - After returning to the beach, you learn that the waves are moving at 1.8 m/s. What is the actual wavelength of the waves?
92. **Bungee Jumper** A high-altitude bungee jumper jumps from a hot-air balloon using a 540-m-bungee cord. When the jump is complete and the jumper is just suspended from the cord, it is stretched 1710 m. What is the spring constant of the bungee cord if the jumper has a mass of 68 kg?
93. The time needed for a water wave to change from the equilibrium level to the crest is 0.18 s.
- What fraction of a wavelength is this?
  - What is the period of the wave?
  - What is the frequency of the wave?
94. When a 225-g mass is hung from a spring, the spring stretches 9.4 cm. The spring and mass then are pulled 8.0 cm from this new equilibrium position and released. Find the spring constant of the spring and the maximum speed of the mass.
95. **Amusement Ride** You notice that your favorite amusement-park ride seems bigger. The ride consists of a carriage that is attached to a structure so it swings like a pendulum. You remember that the carriage used to swing from one position to another and back again eight times in exactly 1 min. Now it only swings six times in 1 min. Give your answers to the following questions to two significant digits.
- What was the original period of the ride?
  - What is the new period of the ride?
  - What is the new frequency?
  - How much longer is the arm supporting the carriage on the larger ride?
  - If the park owners wanted to double the period of the ride, what percentage increase would need to be made to the length of the pendulum?
96. **Clocks** The speed at which a grandfather clock runs is controlled by a swinging pendulum.
- If you find that the clock loses time each day, what adjustment would you need to make to the pendulum so it will keep better time?
  - If the pendulum currently is 15.0 cm, by how much would you need to change the length to make the period lessen by 0.0400 s?
97. **Bridge Swinging** In the summer over the New River in West Virginia, several teens swing from bridges with ropes, then drop into the river after a few swings back and forth.
- If Pam is using a 10.0-m length of rope, how long will it take her to reach the peak of her swing at the other end of the bridge?
  - If Mike has a mass that is 20 kg more than Pam, how would you expect the period of his swing to differ from Pam's?
  - At what point in the swing is *KE* at a maximum?
  - At what point in the swing is *PE* at a maximum?
  - At what point in the swing is *KE* at a minimum?
  - At what point in the swing is *PE* at a minimum?

## Chapter 14 Assessment

- 98.** You have a mechanical fish scale that is made with a spring that compresses when weight is added to a hook attached below the scale. Unfortunately, the calibrations have completely worn off of the scale. However, you have one known mass of 500.0 g that displaces the spring 2.0 cm.
- What is the spring constant for the spring?
  - If a fish displaces the spring 4.5 cm, what is the mass of the fish?
- 99. Car Springs** When you add a 45-kg load to the trunk of a new small car, the two rear springs compress an additional 1.0 cm.
- What is the spring constant for each of the springs?
  - How much additional potential energy is stored in each of the car springs after loading the trunk?
- 100.** The velocity of a wave on a string depends on how tightly the string is stretched, and on the mass per unit length of the string. If  $F_T$  is the tension in the string, and  $\mu$  is the mass/unit length, then the velocity,  $v$ , can be determined by the following equation.

$$v = \sqrt{\frac{F_T}{\mu}}$$

A piece of string 5.30-m long has a mass of 15.0 g. What must the tension in the string be to make the wavelength of a 125-Hz wave 120.0 cm?

- 103. Apply Concepts** Gravel roads often develop regularly spaced ridges that are perpendicular to the road, as shown in **Figure 14-25**. This effect, called washboarding, occurs because most cars travel at about the same speed and the springs that connect the wheels to the cars oscillate at about the same frequency. If the ridges on a road are 1.5 m apart and cars travel on it at about 5 m/s, what is the frequency of the springs' oscillation?



■ Figure 14-25

### Thinking Critically

- 101. Analyze and Conclude** A 20-N force is required to stretch a spring by 0.5 m.
- What is the spring constant?
  - How much energy is stored in the spring?
  - Why isn't the work done to stretch the spring equal to the force times the distance, or 10 J?
- 102. Make and Use Graphs** Several weights were suspended from a spring, and the resulting extensions of the spring were measured. **Table 14-1** shows the collected data.

Table 14-1	
Weights on a Spring	
Force, $F$ (N)	Extension, $x$ (m)
2.5	0.12
5.0	0.26
7.5	0.35
10.0	0.50
12.5	0.60
15.0	0.71

### Writing in Physics

- 104. Research** Christiaan Huygens' work on waves and the controversy between him and Newton over the nature of light. Compare and contrast their explanations of such phenomena as reflection and refraction. Whose model would you choose as the best explanation? Explain why.

### Cumulative Review

- 105.** A 1400-kg drag racer automobile can complete a one-quarter mile (402 m) course in 9.8 s. The final speed of the automobile is 250 mi/h (112 m/s). (Chapter 11)
- What is the kinetic energy of the automobile?
  - What is the minimum amount of work that was done by its engine? Why can't you calculate the total amount of work done?
  - What was the average acceleration of the automobile?
- 106.** How much water would a steam engine have to evaporate in 1 s to produce 1 kW of power? Assume that the engine is 20 percent efficient. (Chapter 12)



# Standardized Test Practice

## Multiple Choice

1. What is the value of the spring constant of a spring with a potential energy of 8.67 J when it's stretched 247 mm?

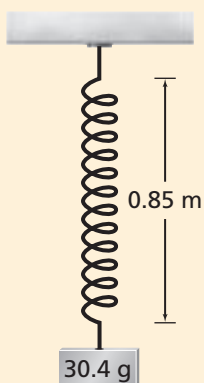
(A) 70.2 N/m                      (C) 142 N/m  
(B) 71.1 N/m                      (D) 284 N/m

2. What is the force acting on a spring with a spring constant of 275 N/m that is stretched 14.3 cm?

(A) 2.81 N                          (C) 39.3 N  
(B) 19.2 N                          (D)  $3.93 \times 10^{30}$  N

3. A mass stretches a spring as it hangs from the spring. What is the spring constant?

(A) 0.25 N/m                      (C) 26 N/m  
(B) 0.35 N/m                      (D)  $3.5 \times 10^2$  N/m



4. A spring with a spring constant of 350 N/m pulls a door closed. How much work is done as the spring pulls the door at a constant velocity from an 85.0-cm stretch to a 5.0-cm stretch?

(A) 112 N·m                          (C) 224 N·m  
(B) 130 J                              (D)  $1.12 \times 10^3$  J

5. What is the correct rearrangement of the formula for the period of a pendulum to find the length of the pendulum?

(A)  $l = \frac{4\pi^2 g}{T^2}$                       (C)  $l = \frac{T^2 g}{(2\pi)^2}$   
(B)  $l = \frac{gT}{4\pi^2}$                       (D)  $l = \frac{Tg}{2\pi}$

6. What is the frequency of a wave with a period of 3 s?

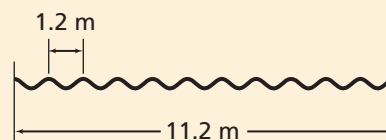
(A) 0.3 Hz                          (C)  $\frac{\pi}{3}$  Hz  
(B)  $\frac{3}{c}$  Hz                          (D) 3 Hz

7. Which option describes a standing wave?

	Waves	Direction	Medium
(A)	Identical	Same	Same
(B)	Nonidentical	Opposite	Different
(C)	Identical	Opposite	Same
(D)	Nonidentical	Same	Different

8. A 1.2-m wave travels 11.2 m to a wall and back again in 4 s. What is the frequency of the wave?

(A) 0.2 Hz                          (C) 5 Hz  
(B) 2 Hz                              (D) 9 Hz



9. What is the length of a pendulum that has a period of 4.89 s?

(A) 5.94 m                          (C) 24.0 m  
(B) 11.9 m                          (D) 37.3 m

## Extended Answer

10. Use dimensional analysis of the equation  $kx = mg$  to derive the units of  $k$ .

### ✓ Test-Taking TIP

#### Practice, Practice, Practice

Practice to improve your performance on standardized tests. Don't compare yourself to anyone else.



**What You'll Learn**

- You will describe sound in terms of wave properties and behavior.
- You will examine some of the sources of sound.
- You will explain properties that differentiate between music and noise.

**Why It's Important**

Sound is an important means of communication and, in the form of music, cultural expression.

**Musical Groups** A small musical group might contain two or three instruments, while a marching band can contain 100 or more. The instruments in these groups form sounds in different ways, but they can create exciting music when they are played together.

**Think About This ►**

How do the instruments in a musical group create the sounds that you hear? Why do various instruments sound different even when they play the same note?





## How can glasses produce musical notes?

### Question

How can you use glasses to produce different musical notes, and how do glasses with stems compare to those without stems?

### Procedure



1. Select a stemmed glass with a thin rim.
2. **Prepare** Carefully inspect the top edge of the glass for sharp edges. Notify your teacher if you observe any sharp edges. Be sure to repeat this inspection every time you select a different glass.
3. Place the glass on the table in front of you. Firmly hold the base with one hand. Wet your finger and slowly rub it around the top edge of the glass. **CAUTION: Glass is fragile. Handle carefully.**
4. Record your observations. Increase or decrease the speed a little. What happens?
5. Select a stemmed glass that is larger or smaller than the first glass. Repeat steps 2-4.
6. Select a glass without a stem and repeat steps 2-4.

### Analysis

Summarize your observations. Which glasses—stemmed, not stemmed, or both—were able to produce ringing tones? What factors affected the tones produced?

### Critical Thinking

Propose a method for producing different notes from the same glass. Test your proposed method. Suggest a test to further investigate the properties of glasses that can produce ringing tones.



## 15.1 Properties and Detection of Sound

**S**ound is an important part of existence for many living things. Animals can use sound to hunt, attract mates, and warn of the approach of predators. In humans, the sound of a siren can heighten our awareness of our surroundings, while the sound of music can soothe and relax us. From your everyday experiences, you already are familiar with several of the characteristics of sound, including volume, tone, and pitch. Without thinking about it, you can use these, and other characteristics, to categorize many of the sounds that you hear; for example, some sound patterns are characteristic of speech, while others are characteristic of a musical group. In this chapter, you will study the physical principles of sound, which is a type of wave.

In Chapter 14, you learned how to describe waves in terms of speed, frequency, wavelength, and amplitude. You also discovered how waves interact with each other and with matter. Knowing that sound is a type of wave allows you to describe some of its properties and interactions. First, however, there is a question that you need to answer: exactly what type of wave is sound?

### ▶ Objectives

- **Demonstrate** the properties that sound shares with other waves.
- **Relate** the physical properties of sound waves to our perception of sound.
- **Identify** some applications of the Doppler effect.

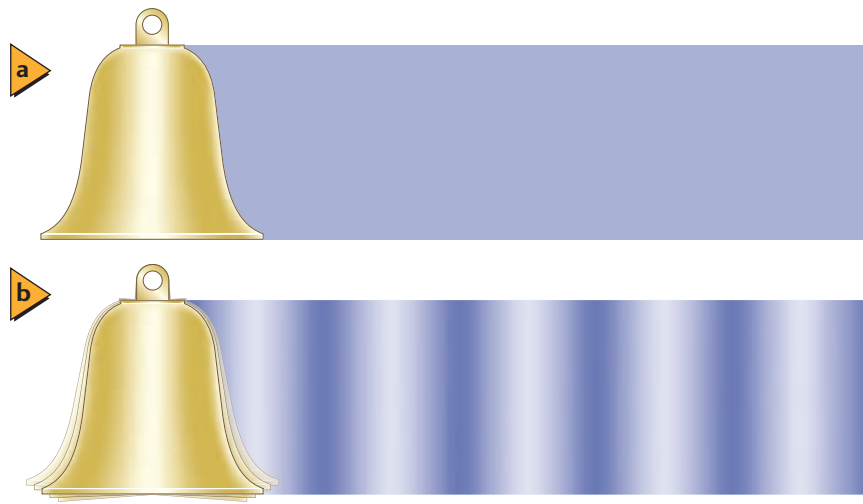
### ▶ Vocabulary

sound wave  
pitch  
loudness  
sound level  
decibel  
Doppler effect





■ **Figure 15-1** Before the bell is struck, the air around it is a region of average pressure (**a**). Once the bell is struck, however, the vibrating edge creates regions of high and low pressure. The dark areas represent regions of higher pressure; the light areas represent regions of lower pressure (**b**). For simplicity, the diagram shows the regions moving in one direction; in reality, the waves move out from the bell in all directions.



## Sound Waves

Put your fingers against your throat as you hum or speak. Can you feel the vibrations? Have you ever put your hand on the loudspeaker of a boom box? **Figure 15-1** shows a vibrating bell that also can represent your vocal cords, a loudspeaker, or any other sound source. As it moves back and forth, the edge of the bell strikes the particles in the air. When the edge moves forward, air particles are driven forward; that is, the air particles bounce off the bell with a greater velocity. When the edge moves backward, air particles bounce off the bell with a lower velocity.

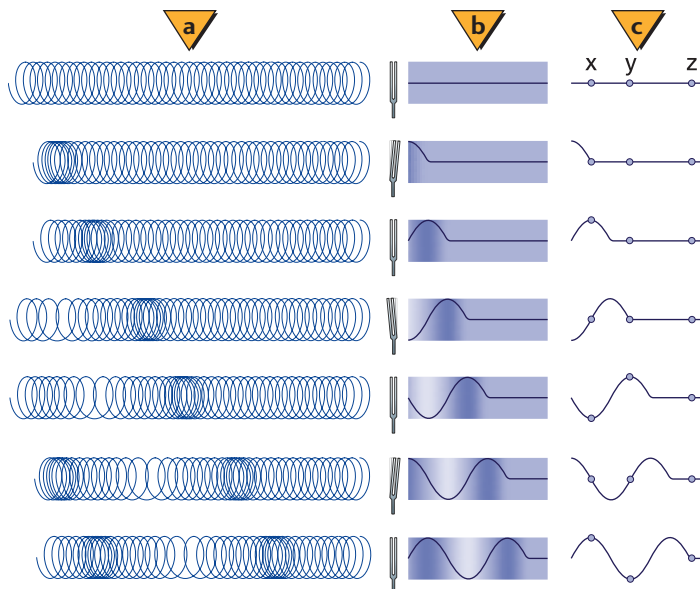
The result of these velocity changes is that the forward motion of the bell produces a region where the air pressure is slightly higher than average. The backward motion produces slightly below-average pressure. Collisions among the air particles cause the pressure variations to move away from the bell in all directions. If you were to focus at one spot, you would see the value of the air pressure rise and fall, not unlike the behavior of a pendulum. In this way, the pressure variations are transmitted through matter.

**Describing sound** A pressure variation that is transmitted through matter is a **sound wave**. Sound waves move through air because a vibrating source produces regular variations, or oscillations, in air pressure. The air

particles collide, transmitting the pressure variations away from the source of the sound. The pressure of the air oscillates about the mean air pressure, as shown in **Figure 15-2**. The frequency of the wave is the number of oscillations in pressure each second. The wavelength is the distance between successive regions of high or low pressure. Because the motion of the particles in air is parallel to the direction of the wave's motion, sound is a longitudinal wave.

The speed of sound in air depends on the temperature, with the speed increasing by about 0.6 m/s for each 1°C increase in air temperature. At room temperature (20°C), sound moves through air at sea level at a speed of 343 m/s. Sound also travels through solids and liquids. In general, the speed of sound is greater in

■ **Figure 15-2** A coiled spring models the compressions and rarefactions of a sound wave (**a**). The pressure of the air rises and falls as the sound wave propagates through the atmosphere (**b**). You can use a sine curve alone to model changes in pressure. Note that the positions of  $x$ ,  $y$ , and  $z$  show that the wave, not matter, moves forward (**c**).



solids and liquids than in gases. **Table 15-1** lists the speeds of sound waves in various media. Sound cannot travel in a vacuum because there are no particles to collide.

Sound waves share the general properties of other waves. For example, they reflect off hard objects, such as the walls of a room. Reflected sound waves are called echoes. The time required for an echo to return to the source of the sound can be used to find the distance between the source and the reflective object. This principle is used by bats, by some cameras, and by ships that employ sonar. Two sound waves can interfere, causing dead spots at nodes where little sound can be heard. As you learned in Chapter 14, the frequency and wavelength of a wave are related to the speed of the wave by the equation  $\lambda = v/f$ .

Table 15-1	
Speed of Sound in Various Media	
Medium	m/s
Air (0°)	331
Air (20°)	343
Helium (0°)	972
Water (25°)	1493
Seawater (25°)	1533
Copper (25°)	3560
Iron (25°)	5130

### PRACTICE Problems

Additional Problems, Appendix B

1. Find the wavelength in air at 20°C of an 18-Hz sound wave, which is one of the lowest frequencies that is detectable by the human ear.
2. What is the wavelength of an 18-Hz sound wave in seawater at 25°C?
3. Find the frequency of a sound wave moving through iron at 25°C with a wavelength of 1.25 m.
4. If you shout across a canyon and hear the echo 0.80 s later, how wide is the canyon?
5. A 2280-Hz sound wave has a wavelength of 0.655 m in an unknown medium. Identify the medium.

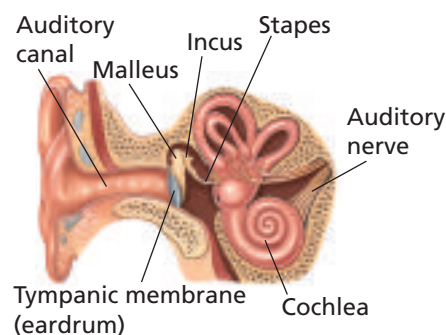
## Detection of Pressure Waves

Sound detectors convert sound energy—the kinetic energy of the vibrating air particles—into another form of energy. A common detector is a microphone, which converts sound waves into electrical energy. A microphone consists of a thin disk that vibrates in response to sound waves and produces an electrical signal. You will learn about this transformation process in Chapter 25, during your study of electricity and magnetism.

**The human ear** As shown in **Figure 15-3**, the human ear is a detector that receives pressure waves and converts them to electrical impulses. Sound waves entering the auditory canal cause vibrations of the tympanic membrane. Three tiny bones then transfer these vibrations to fluid in the cochlea. Tiny hairs lining the spiral-shaped cochlea detect certain frequencies in the vibrating fluid. These hairs stimulate nerve cells, which send impulses to the brain and produce the sensation of sound.

The ear detects sound waves over a wide range of frequencies and is sensitive to an enormous range of amplitudes. In addition, human hearing can distinguish many different qualities of sound. Knowledge of both physics and biology is required to understand the complexities of the ear. The interpretation of sounds by the brain is even more complex, and it is not totally understood.

■ **Figure 15-3** The human ear is a complex sense organ that translates sound vibrations into nerve impulses that are sent to the brain for interpretation. The malleus, incus, and stapes are the three bones of the middle ear that sometimes are referred to as the hammer, anvil, and stirrup.





## Perceiving Sound

**Pitch** Marin Mersenne and Galileo first determined that the **pitch** we hear depends on the frequency of vibration. Pitch can be given a name on the musical scale. For instance, the middle C note has a frequency of 262 Hz. The ear is not equally sensitive to all frequencies. Most people cannot hear sounds with frequencies below 20 Hz or above 16,000 Hz. Older people are less sensitive to frequencies above 10,000 Hz than are young people. By age 70, most people cannot hear sounds with frequencies above 8000 Hz. This loss affects the ability to understand speech.

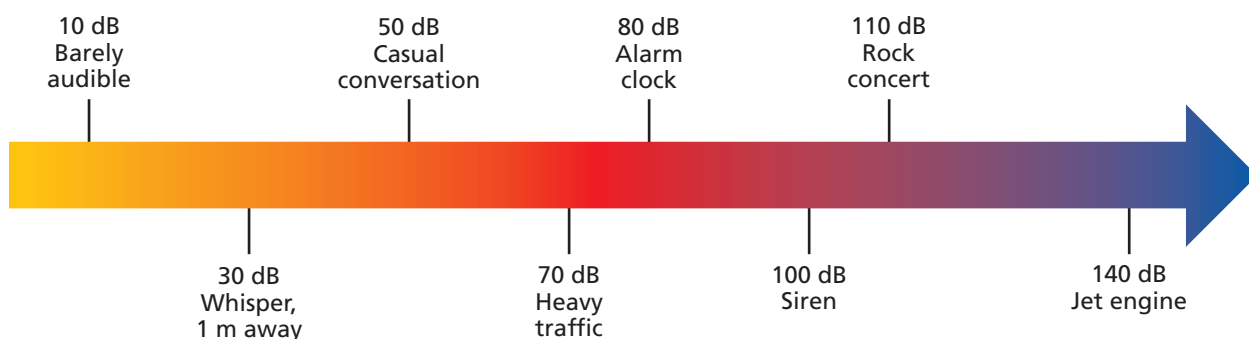
**Loudness** Frequency and wavelength are two physical characteristics of sound waves. Another physical characteristic of sound waves is amplitude. Amplitude is the measure of the variation in pressure along a wave. In humans, sound is detected by the ear and interpreted by the brain. The **loudness** of a sound, as perceived by our sense of hearing, depends primarily on the amplitude of the pressure wave.

The human ear is extremely sensitive to pressure variations in sound waves, which is the amplitude of the wave. Recall from Chapter 13 that 1 atm of pressure equals  $1.01 \times 10^5$  Pa. The ear can detect pressure-wave amplitudes of less than one-billionth of an atmosphere, or  $2 \times 10^{-5}$  Pa. At the other end of the audible range, pressure variations of approximately 20 Pa or greater cause pain. It is important to remember that the ear detects only pressure variations at certain frequencies. Driving over a mountain pass changes the pressure on your ears by thousands of pascals, but this change does not take place at audible frequencies.

Because humans can detect a wide range in pressure variations, these amplitudes are measured on a logarithmic scale called the **sound level**. The unit of measurement for sound level is the **decibel (dB)**. The sound level depends on the ratio of the pressure variation of a given sound wave to the pressure variation in the most faintly heard sound,  $2 \times 10^{-5}$  Pa. Such an amplitude has a sound level of 0 dB. A sound with a pressure amplitude ten times larger ( $2 \times 10^{-4}$  Pa) is 20 dB. A pressure amplitude ten times larger than this is 40 dB. Most people perceive a 10-dB increase in sound level as about twice as loud as the original level. **Figure 15-4** shows the sound level for a variety of sounds. In addition to pressure variations, power and intensity of sound waves can be described by decibel scales.

Exposure to loud sounds, in the form of noise or music, has been shown to cause the ear to lose its sensitivity, especially to high frequencies. The longer a person is exposed to loud sounds, the greater the effect. A person can recover from short-term exposure in a period of hours, but the effects

■ **Figure 15-4** This decibel scale shows the sound levels of some familiar sounds.



of long-term exposure can last for days or weeks. Long exposure to 100-dB or greater sound levels can produce permanent damage. Many rock musicians have suffered serious hearing loss, some as much as 40 percent. Hearing loss also can result from loud music being transmitted to stereo headphones from personal radios and CD players. In some cases, the listeners are unaware of just how high the sound levels really are. Cotton earplugs reduce the sound level only by about 10 dB. Special ear inserts can provide a 25-dB reduction. Specifically designed earmuffs and inserts as shown in **Figure 15-5**, can reduce the sound level by up to 45 dB.

Loudness, as perceived by the human ear, is not directly proportional to the pressure variations in a sound wave. The ear's sensitivity depends on both pitch and amplitude. Also, perception of pure tones is different from perception of a mixture of tones.

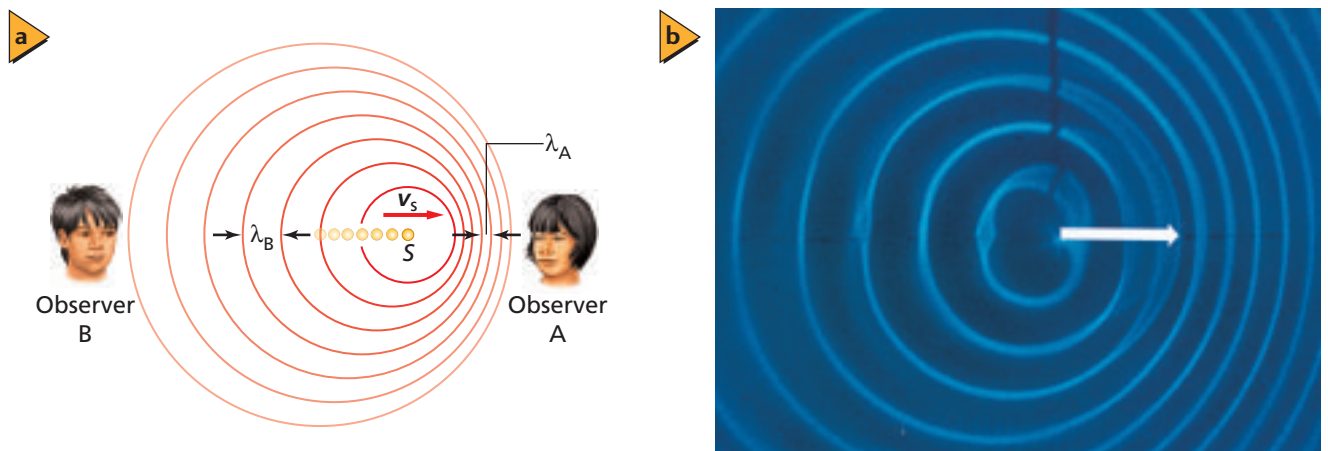
## The Doppler Effect

Have you ever noticed that the pitch of an ambulance, fire, or police siren changed as the vehicle sped past you? The pitch was higher when the vehicle was moving toward you, then it dropped to a lower pitch as the source moved away. This frequency shift is called the **Doppler effect** and is shown in **Figure 15-6**. The sound source,  $S$ , is moving to the right with a speed of  $v_s$ . The waves that it emits spread in circles centered on the source at the time it produced the waves. As the source moves toward the sound detector, Observer A in **Figure 15-6a**, more waves are crowded into the space between them. The wavelength is shortened to  $\lambda_A$ . Because the speed of sound is not changed, more crests reach the ear per second, which means that the frequency of the detected sound increases. When the source is moving away from the detector, Observer B in **Figure 15-6a**, the wavelength is lengthened to  $\lambda_B$  and the detected frequency is lower. **Figure 15-6b** illustrates the Doppler effect for a moving source of sound on water waves in a ripple tank.

A Doppler shift also occurs if the detector is moving and the source is stationary. In this case, the Doppler shift results from the relative velocity of the sound waves and the detector. As the detector approaches the stationary source, the relative velocity is larger, resulting in an increase in the wave crests reaching the detector each second. As the detector recedes from the source, the relative velocity is smaller, resulting in a decrease in the wave crests reaching the detector each second.



■ **Figure 15-5** Continuous exposure to loud sounds can cause serious hearing loss. In many occupations, workers, such as this flight controller, must wear ear protection.



■ **Figure 15-6** As a sound-producing source moves toward an observer, the wavelength is shortened to  $\lambda_A$ ; the wavelength is  $\lambda_B$  for waves produced by a source moving away from an observer (a). A moving wave-producing source illustrates the Doppler effect in a ripple tank (b).

For both a moving source and a moving observer, the frequency that the observer hears can be calculated using the equation below.

**Doppler Effect**  $f_d = f_s \left( \frac{v - v_d}{v - v_s} \right)$

The frequency perceived by a detector is equal to the velocity of the detector relative to the velocity of the wave, divided by the velocity of the source relative to the velocity of the wave, multiplied by the wave's frequency.

In the Doppler effect equation,  $v$  is the velocity of the sound wave,  $v_d$  is the velocity of the detector,  $v_s$  is the velocity of the sound's source,  $f_s$  is the frequency of the wave emitted by the source, and  $f_d$  is the frequency received by the detector. This equation applies when the source is moving, when the observer is moving, and when both are moving.

As you solve problems using the above equation, be sure to define the coordinate system so that the positive direction is from the source to the detector. The sound waves will be approaching the detector from the source, so the velocity of sound is always positive. Try drawing diagrams to confirm that the term  $(v - v_d)/(v - v_s)$  behaves as you would predict based on what you have learned about the Doppler effect. Notice that for a source moving toward the detector (positive direction, which results in a smaller denominator compared to a stationary source) and for a detector moving toward the source (negative direction and increased numerator compared to a stationary detector), the detected frequency,  $f_d$ , increases. Similarly, if the source moves away from the detector or if the detector moves away from the source, then  $f_d$  decreases. Read the Connecting Math to Physics feature below to see how the Doppler effect equation reduces when the source or observer is stationary.

### ▶ Connecting Math to Physics

**Reducing Equations** When an element in a complex equation is equal to zero, the equation might reduce to a form that is easier to use.

Stationary detector, source in motion: $v_d = 0$	Stationary source, detector in motion: $v_s = 0$
$f_d = f_s \left( \frac{v - v_d}{v - v_s} \right)$	$f_d = f_s \left( \frac{v - v_d}{v - v_s} \right)$
$= f_s \left( \frac{v}{v - v_s} \right)$	$= f_s \left( \frac{v - v_d}{v} \right)$
$= f_s \left( \frac{\frac{v}{v}}{\frac{v}{v} - \frac{v_s}{v}} \right)$	$= f_s \left( \frac{\frac{v}{v} - \frac{v_d}{v}}{\frac{v}{v}} \right)$
$= f_s \left( \frac{1}{1 - \frac{v_s}{v}} \right)$	$= f_s \left( \frac{1 - \frac{v_d}{v}}{1} \right)$
	$= f_s \left( 1 - \frac{v_d}{v} \right)$



## ▶ EXAMPLE Problem 1

**The Doppler Effect** A trumpet player sounds C above middle C (524 Hz) while traveling in a convertible at 24.6 m/s. If the car is coming toward you, what frequency would you hear? Assume that the temperature is 20°C.

### 1 Analyze and Sketch the Problem

- Sketch the situation.
- Establish a coordinate axis. Make sure that the positive direction is from the source to the detector.
- Show the velocities of the source and detector.

**Known:**

$$v = +343 \text{ m/s}$$

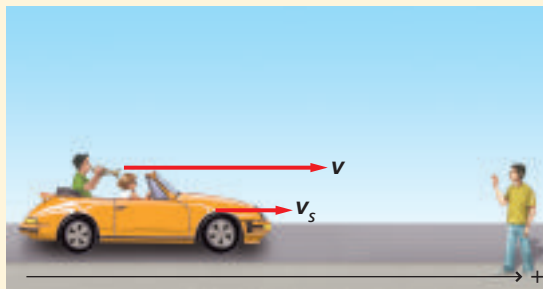
$$v_s = +24.6 \text{ m/s}$$

$$v_d = 0 \text{ m/s}$$

$$f_s = 524 \text{ Hz}$$

**Unknown:**

$$f_d = ?$$



### 2 Solve for the Unknown

Use  $f_d = f_s \left( \frac{v - v_d}{v - v_s} \right)$  with  $v_d = 0 \text{ m/s}$ .

$$f_d = f_s \left( \frac{1}{1 - \frac{v_s}{v}} \right)$$

$$= 524 \text{ Hz} \left( \frac{1}{1 - \frac{24.6 \text{ m/s}}{343 \text{ m/s}}} \right)$$

$$= 564 \text{ Hz}$$

Substitute  $v = +343 \text{ m/s}$ ,  $v_s = +24.6 \text{ m/s}$ , and  $f_s = 524 \text{ Hz}$

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### 3 Evaluate the Answer

- **Are the units correct?** Frequency is measured in hertz.
- **Is the magnitude realistic?** The source is moving toward you, so the frequency should be increased.

## ▶ PRACTICE Problems

Additional Problems, Appendix B

- Repeat Example Problem 1, but with the car moving away from you. What frequency would you hear?
- You are in an auto traveling at 25.0 m/s toward a pole-mounted warning siren. If the siren's frequency is 365 Hz, what frequency do you hear? Use 343 m/s as the speed of sound.
- You are in an auto traveling at 55 mph (24.6 m/s). A second auto is moving toward you at the same speed. Its horn is sounding at 475 Hz. What frequency do you hear? Use 343 m/s as the speed of sound.
- A submarine is moving toward another submarine at 9.20 m/s. It emits a 3.50-MHz ultrasound. What frequency would the second sub, at rest, detect? The speed of sound in water is 1482 m/s.
- A sound source plays middle C (262 Hz). How fast would the source have to go to raise the pitch to C sharp (271 Hz)? Use 343 m/s as the speed of sound.



■ **Figure 15-7** In a process called echolocation, bats use the Doppler effect to locate prey.



The Doppler effect occurs in all wave motion, both mechanical and electromagnetic. It has many applications. Radar detectors use the Doppler effect to measure the speed of baseballs and automobiles. Astronomers observe light from distant galaxies and use the Doppler effect to measure their speeds and infer their distances. Physicians can detect the speed of the moving heart wall in a fetus by means of the Doppler effect in ultrasound. Bats use the Doppler effect to detect and catch flying insects. When an insect is flying faster than a bat, the reflected frequency is lower, but when the bat is catching up to the insect, as in **Figure 15-7**, the reflected frequency is higher. Not only do bats use sound waves to navigate and locate their prey, but they often must do so in the presence of other bats. This means they must discriminate their own calls and reflections against a background of many other sounds of many frequencies. Scientists continue to study bats and their amazing abilities to use sound waves.

### Biology Connection

## 15.1 Section Review

- 11. Graph** The eardrum moves back and forth in response to the pressure variations of a sound wave. Sketch a graph of the displacement of the eardrum versus time for two cycles of a 1.0-kHz tone and for two cycles of a 2.0-kHz tone.
- 12. Effect of Medium** List two sound characteristics that are affected by the medium through which the sound passes and two characteristics that are not affected.
- 13. Sound Properties** What physical characteristic of a sound wave should be changed to change the pitch of the sound? To change the loudness?
- 14. Decibel Scale** How much greater is the sound pressure level of a typical rock band's music (110 dB) than a normal conversation (50 dB)?
- 15. Early Detection** In the nineteenth century, people put their ears to a railroad track to get an early warning of an approaching train. Why did this work?
- 16. Bats** A bat emits short pulses of high-frequency sound and detects the echoes.
  - In what way would the echoes from large and small insects compare if they were the same distance from the bat?
  - In what way would the echo from an insect flying toward the bat differ from that of an insect flying away from the bat?
- 17. Critical Thinking** Can a trooper using a radar detector at the side of the road determine the speed of a car at the instant the car passes the trooper? Explain.



## 15.2 The Physics of Music

In the middle of the nineteenth century, German physicist Hermann Helmholtz studied sound production in musical instruments and the human voice. In the twentieth century, scientists and engineers developed electronic equipment that permits not only a detailed study of sound, but also the creation of electronic musical instruments and recording devices that allow us to listen to music whenever and wherever we wish.

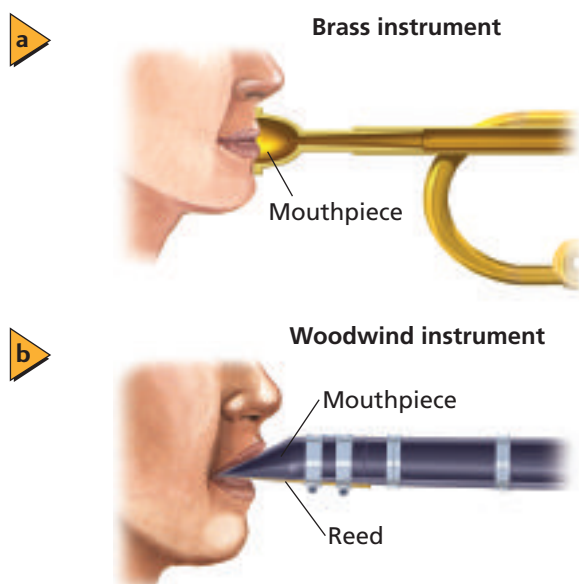
### Sources of Sound

Sound is produced by a vibrating object. The vibrations of the object create particle motions that cause pressure oscillations in the air. A loudspeaker has a cone that is made to vibrate by electrical currents. The surface of the cone creates the sound waves that travel to your ear and allow you to hear music. Musical instruments such as gongs, cymbals, and drums are other examples of vibrating surfaces that are sources of sound.

The human voice is produced by vibrations of the vocal cords, which are two membranes located in the throat. Air from the lungs rushing through the throat starts the vocal cords vibrating. The frequency of vibration is controlled by the muscular tension placed on the vocal cords.

In brass instruments, such as the trumpet and tuba, the lips of the performer vibrate, as shown in **Figure 15-8a**. Reed instruments, such as the clarinet and saxophone, have a thin wooden strip, or reed, that vibrates as a result of air blown across it, as shown in **Figure 15-8b**. In flutes and organ pipes, air is forced across an opening in a pipe. Air moving past the opening sets the column of air in the instrument into vibration.

In stringed instruments, such as the piano, guitar, and violin, wires or strings are set into vibration. In the piano, the wires are struck; in the guitar, they are plucked; and in the violin, the friction of the bow causes the strings to vibrate. Often, the strings are attached to a sounding board that vibrates with the strings. The vibrations of the sounding board cause the pressure oscillations in the air that we hear as sound. Electric guitars use electronic devices to detect and amplify the vibrations of the guitar strings.



#### ► Objectives

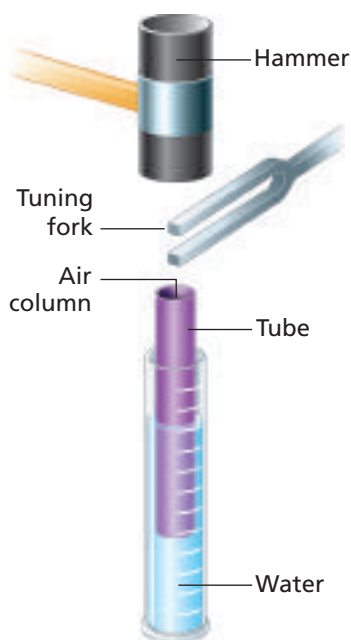
- **Describe** the origin of sound.
- **Demonstrate** an understanding of resonance, especially as applied to air columns and strings.
- **Explain** why there are variations in sound among instruments and among voices.

#### ► Vocabulary

closed-pipe resonator  
open-pipe resonator  
fundamental  
harmonics  
dissonance  
consonance  
beat

■ **Figure 15-8** The shapes of the mouthpieces of a brass instrument **(a)** and a reed instrument **(b)** help determine the characteristics of the sound each instrument produces.

## Resonance in Air Columns



■ **Figure 15-9** Raising or lowering the tube changes the length of the air column. When the column is in resonance with the tuning fork, the sound is loudest.

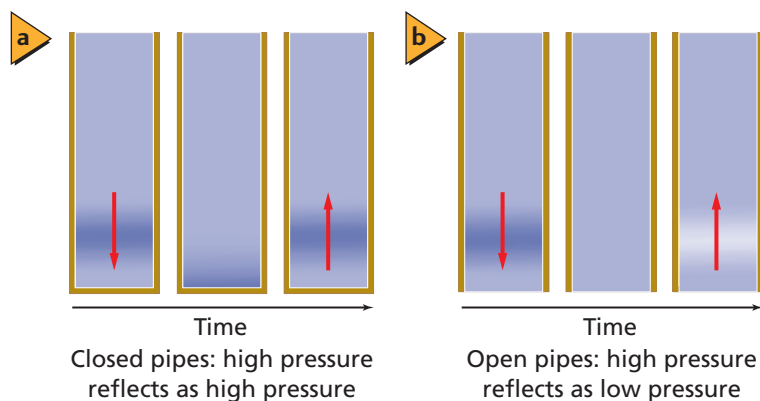
If you have ever used just the mouthpiece of a brass or reed instrument, you know that the vibration of your lips or the reed alone does not make a sound with any particular pitch. The long tube that makes up the instrument must be attached if music is to result. When the instrument is played, the air within this tube vibrates at the same frequency, or in resonance, with a particular vibration of the lips or reed. Remember that resonance increases the amplitude of a vibration by repeatedly applying a small external force at the same natural frequency. The length of the air column determines the frequencies of the vibrating air that will be set into resonance. For many instruments, such as flutes, saxophones, and trombones, changing the length of the column of vibrating air varies the pitch of the instrument. The mouthpiece simply creates a mixture of different frequencies, and the resonating air column acts on a particular set of frequencies to amplify a single note, turning noise into music.

A tuning fork above a hollow tube can provide resonance in an air column, as shown in **Figure 15-9**. The tube is placed in water so that the bottom end of the tube is below the water surface. A resonating tube with one end closed to air is called a **closed-pipe resonator**. The length of the air column is changed by adjusting the height of the tube above the water. If the tuning fork is struck with a rubber hammer and the length of the air column is varied as the tube is lifted up and down in the water, the sound alternately becomes louder and softer. The sound is loud when the air column is in resonance with the tuning fork. A resonating air column intensifies the sound of the tuning fork.

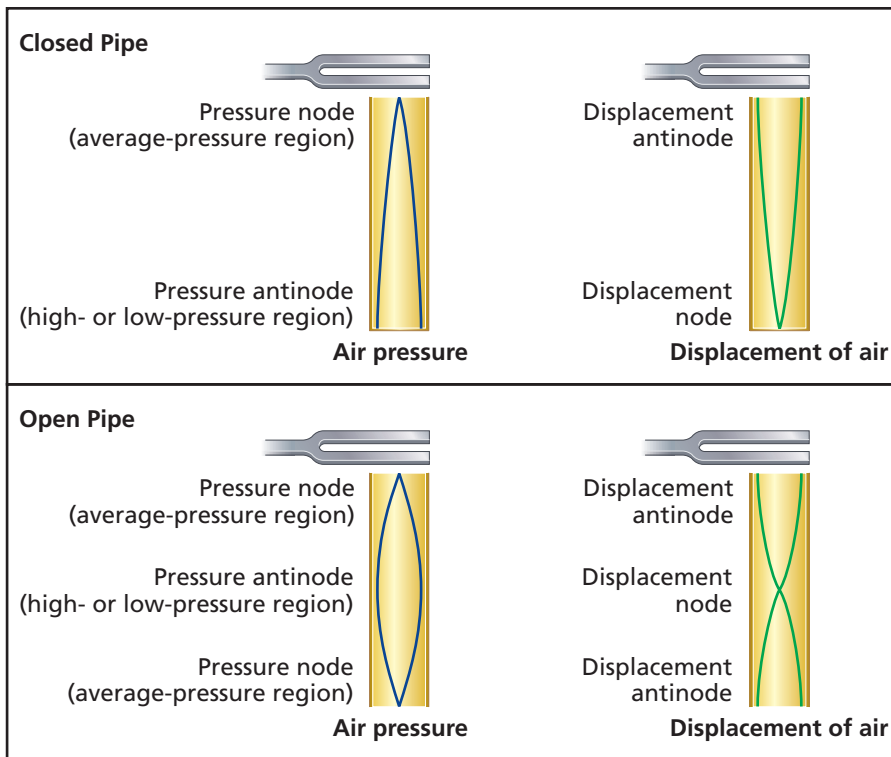
**Standing pressure wave** How does resonance occur? The vibrating tuning fork produces a sound wave. This wave of alternate high- and low-pressure variations moves down the air column. When the wave hits the water surface, it is reflected back up to the tuning fork, as indicated in **Figure 15-10a**. If the reflected high-pressure wave reaches the tuning fork at the same moment that the fork produces another high-pressure wave, then the emitted and returning waves reinforce each other. This reinforcement of waves produces a standing wave, and resonance is achieved.

An **open-pipe resonator** is a resonating tube with both ends open that also will resonate with a sound source. In this case, the sound wave does not reflect off a closed end, but rather off an open end. The pressure of the reflected wave is inverted; for example, if a high-pressure wave strikes the open end, a low-pressure wave will rebound, as shown in **Figure 15-10b**.

■ **Figure 15-10** A tube placed in water is a closed-pipe resonator. In closed pipes, high pressure waves reflect as high pressure **(a)**. In open pipes, the reflected waves are inverted **(b)**.



**Resonance lengths** A standing sound wave in a pipe can be represented by a sine wave, as shown in **Figure 15-11**. Sine waves can represent either the air pressure or the displacement of the air particles. You can see that standing waves have nodes and antinodes. In the pressure graphs, the nodes are regions of mean atmospheric pressure, and at the antinodes, the pressure oscillates between its maximum and minimum values.

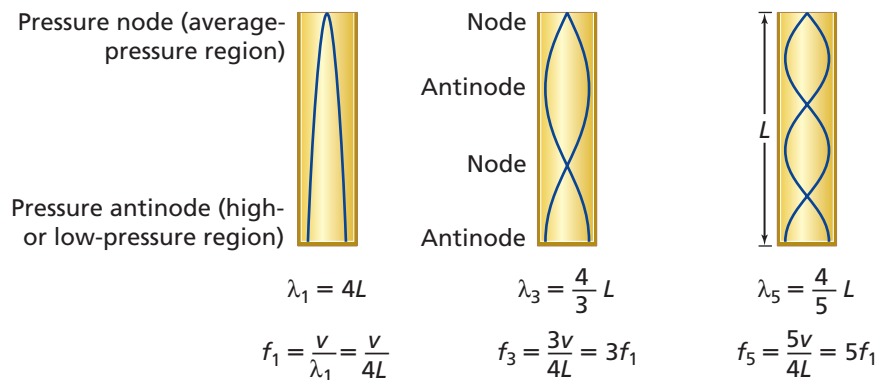


■ **Figure 15-11** Sine waves represent standing waves in pipes.

In the case of the displacement graph, the antinodes are regions of high displacement and the nodes are regions of low displacement. In both cases, two antinodes (or two nodes) are separated by one-half wavelength.

**Resonance frequencies in a closed pipe** The shortest column of air that can have an antinode at the closed end and a node at the open end is one-fourth of a wavelength long, as shown in **Figure 15-12**. As the frequency is increased, additional resonance lengths are found at half-wavelength intervals. Thus, columns of length  $\lambda/4$ ,  $3\lambda/4$ ,  $5\lambda/4$ ,  $7\lambda/4$ , and so on will all be in resonance with a tuning fork.

In practice, the first resonance length is slightly longer than one-fourth of a wavelength. This is because the pressure variations do not drop to zero exactly at the open end of the pipe. Actually, the node is approximately 0.4 pipe diameters beyond the end. Additional resonance lengths, however, are spaced by exactly one-half of a wavelength. Measurements of the spacing between resonances can be used to find the velocity of sound in air, as shown in the next Example Problem.



## APPLYING PHYSICS

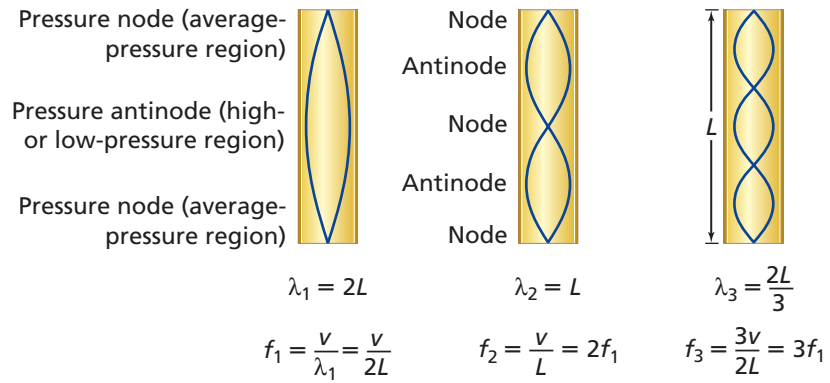
### ► Hearing and Frequency

The human auditory canal acts as a closed-pipe resonator that increases the ear's sensitivity for frequencies between 2000 and 5000 Hz, but the full range of frequencies that people hear extends from 20 to 20,000 Hz. A dog's hearing extends to frequencies as high as 45,000 Hz, and a cat's extends to frequencies as high as 100,000 Hz. ◀

■ **Figure 15-12** A closed pipe resonates when its length is an odd number of quarter wavelengths.



■ **Figure 15-13** An open pipe resonates when its length is an even number of quarter wavelengths.



**Resonance frequencies in an open pipe** The shortest column of air that can have nodes at both ends is one-half of a wavelength long, as shown in **Figure 15-13**. As the frequency is increased, additional resonance lengths are found at half-wavelength intervals. Thus, columns of length  $\lambda/2$ ,  $\lambda$ ,  $3\lambda/2$ ,  $2\lambda$ , and so on will be in resonance with a tuning fork.

If open and closed pipes of the same length are used as resonators, the wavelength of the resonant sound for the open pipe will be half as long as that for the closed pipe. Therefore, the frequency will be twice as high for the open pipe as for the closed pipe. For both pipes, resonance lengths are spaced by half-wavelength intervals.

**Hearing resonance** Musical instruments use resonance to increase the loudness of particular notes. Open-pipe resonators include flutes and saxophones. Clarinets and the hanging pipes under marimbas and xylophones are examples of closed-pipe resonators. If you shout into a long tunnel, the booming sound you hear is the tunnel acting as a resonator. The seashell in **Figure 15-14** acts as a closed-pipe resonator.

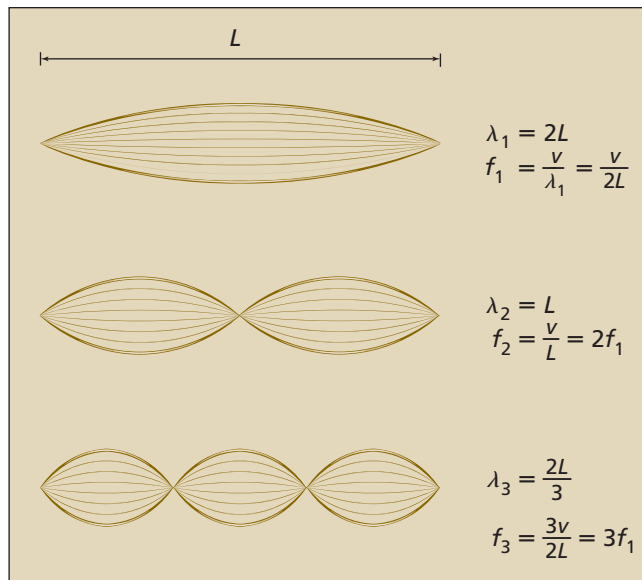
## Resonance on Strings

Although the waveforms on vibrating strings vary in shape, depending upon how they are produced, such as by plucking, bowing, or striking, they have many characteristics in common with standing waves on springs and ropes, which you studied in Chapter 14. A string on an instrument is clamped at both ends, and therefore, the string must have a node at each end when it vibrates. In **Figure 15-15**, you can see that the first mode of vibration has an antinode at the center and is one-half of a wavelength long. The next resonance occurs when one wavelength fits on the string, and additional standing waves arise when the string length is  $3\lambda/2$ ,  $2\lambda$ ,  $5\lambda/2$ , and so on. As with an open pipe, the resonant frequencies are whole-number multiples of the lowest frequency.

The speed of a wave on a string depends on the tension of the string, as well as its mass per unit length. This makes it possible to tune a stringed instrument by changing the tension of its strings. The tighter the string, the faster the wave moves along it, and therefore, the higher the frequency of its standing waves.

■ **Figure 15-14** A seashell acts as a closed-pipe resonator to amplify certain frequencies from the background noise.



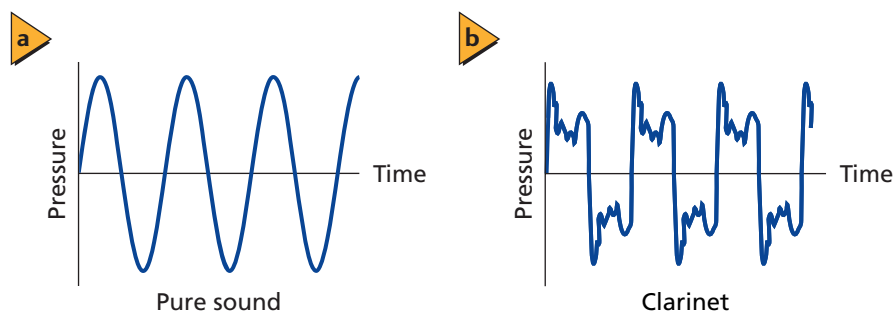


■ **Figure 15-15** A string resonates with standing waves when its length is a whole number of half wavelengths.

Because strings are so small in cross-sectional area, they move very little air when they vibrate. This makes it necessary to attach them to a sounding board, which transfers their vibrations to the air and produces a stronger sound wave. Unlike the strings themselves, the sounding board should not resonate at any single frequency. Its purpose is to convey the vibrations of all the strings to the air, and therefore it should vibrate well at all frequencies produced by the instrument. Because of the complicated interactions among the strings, the sounding board, and the air, the design and construction of stringed instruments are complex processes, considered by many to be as much an art as a science.

## Sound Quality

A tuning fork produces a soft and uninteresting sound. This is because its tines vibrate like simple harmonic oscillators and produce the simple sine wave shown in **Figure 15-16a**. Sounds made by the human voice and musical instruments are much more complex, like the wave in **Figure 15-16b**. Both waves have the same frequency, or pitch, but they sound very different. The complex wave is produced by using the principle of superposition to add waves of many frequencies. The shape of the wave depends on the relative amplitudes of these frequencies. In musical terms, the difference between the two waves is called timbre, tone color, or tone quality.



■ **Figure 15-16** A graph of pure sound versus time (**a**) and a graph of clarinet sound waves versus time (**b**) are shown.

## ▶ EXAMPLE Problem 2

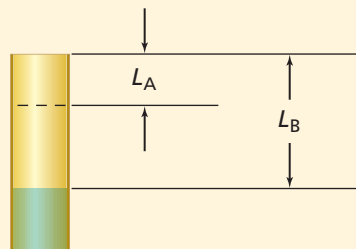
**Finding the Speed of Sound Using Resonance** When a tuning fork with a frequency of 392 Hz is used with a closed-pipe resonator, the loudest sound is heard when the column is 21.0 cm and 65.3 cm long. What is the speed of sound in this case? Is the temperature warmer or cooler than normal room temperature, which is 20°C? Explain your answer.

### 1 Analyze and Sketch the Problem

- Sketch the closed-pipe resonator.
- Mark the resonance lengths.

**Known:**  $f = 392 \text{ Hz}$       **Unknown:**  $v = ?$

$$L_A = 21.0 \text{ cm}$$
$$L_B = 65.3 \text{ cm}$$



### 2 Solve for the Unknown

Solve for the length of the wave using the length-wavelength relationship for a closed pipe.

$$L_B - L_A = \frac{1}{2}\lambda$$
$$\lambda = 2(L_B - L_A)$$
$$= 2(0.653 \text{ m} - 0.210 \text{ m})$$
$$= 0.886 \text{ m}$$

Rearrange the equation for  $\lambda$ .

Substitute  $L_B = 0.653 \text{ m}$ ,  $L_A = 0.210 \text{ m}$

Use  $\lambda = \frac{v}{f}$ .

$$v = f\lambda$$
$$= (392 \text{ Hz})(0.886 \text{ m})$$
$$= 347 \text{ m/s}$$

Rearrange the equation for  $v$ .

Substitute  $f = 392 \text{ Hz}$ ,  $\lambda = 0.886 \text{ m}$

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The speed is slightly greater than the speed of sound at 20°C, indicating that the temperature is slightly higher than normal room temperature.

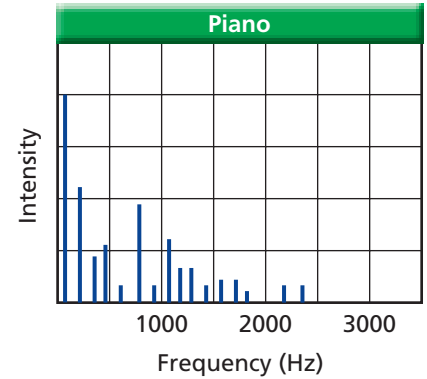
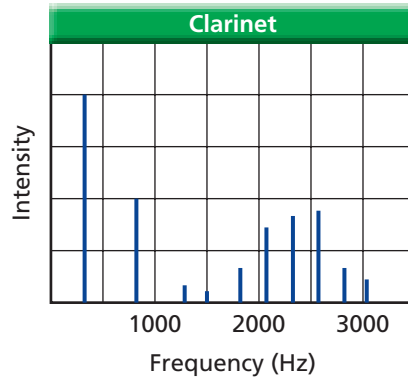
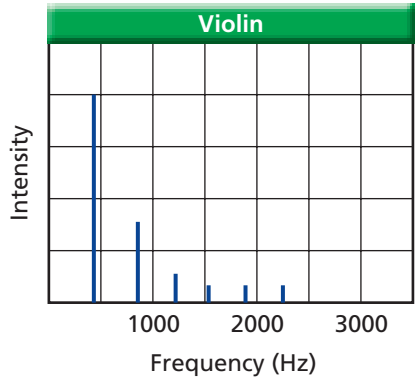
### 3 Evaluate the Answer

- **Are the units correct?**  $(\text{Hz})(\text{m}) = (1/\text{s})(\text{m}) = \text{m/s}$ . The answer's units are correct.
- **Is the magnitude realistic?** The speed is slightly greater than 343 m/s, which is the speed of sound at 20°C.

## ▶ PRACTICE Problems

Additional Problems, Appendix B

18. A 440-Hz tuning fork is held above a closed pipe. Find the spacing between the resonances when the air temperature is 20°C.
19. A 440-Hz tuning fork is used with a resonating column to determine the velocity of sound in helium gas. If the spacings between resonances are 110 cm, what is the velocity of sound in helium gas?
20. The frequency of a tuning fork is unknown. A student uses an air column at 27°C and finds resonances spaced by 20.2 cm. What is the frequency of the tuning fork? Use the speed calculated in Example Problem 2 for the speed of sound in air at 27°C.
21. A bugle can be thought of as an open pipe. If a bugle were straightened out, it would be 2.65-m long.
  - a. If the speed of sound is 343 m/s, find the lowest frequency that is resonant for a bugle (ignoring end corrections).
  - b. Find the next two resonant frequencies for the bugle.



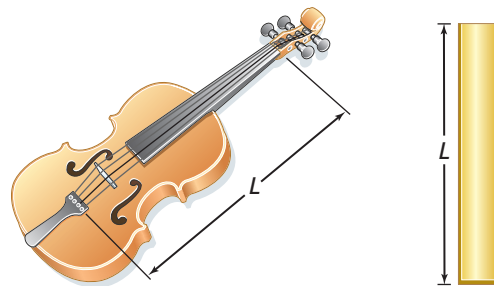
**The sound spectrum: fundamental and harmonics** The complex sound wave in Figure 15-16b was made by a clarinet. Why does the clarinet produce such a sound wave? The air column in a clarinet acts as a closed pipe. Look back at Figure 15-12, which shows three resonant frequencies for a closed pipe. Because the clarinet acts as a closed pipe, for a clarinet of length  $L$  the lowest frequency,  $f_1$ , that will be resonant is  $v/4L$ . This lowest frequency is called the **fundamental**. A closed pipe also will resonate at  $3f_1$ ,  $5f_1$ , and so on. These higher frequencies, which are odd-number multiples of the fundamental frequency, are called **harmonics**. It is the addition of these harmonics that gives a clarinet its distinctive timbre.

Some instruments, such as an oboe, act as open-pipe resonators. Their fundamental frequency, which is also the first harmonic, is  $f_1 = v/2L$  with subsequent harmonics at  $2f_1$ ,  $3f_1$ ,  $4f_1$ , and so on. Different combinations and amplitudes of these harmonics give each instrument its own unique timbre. A graph of the amplitude of a wave versus its frequency is called a sound spectrum. The spectra of three instruments are shown in **Figure 15-17**.

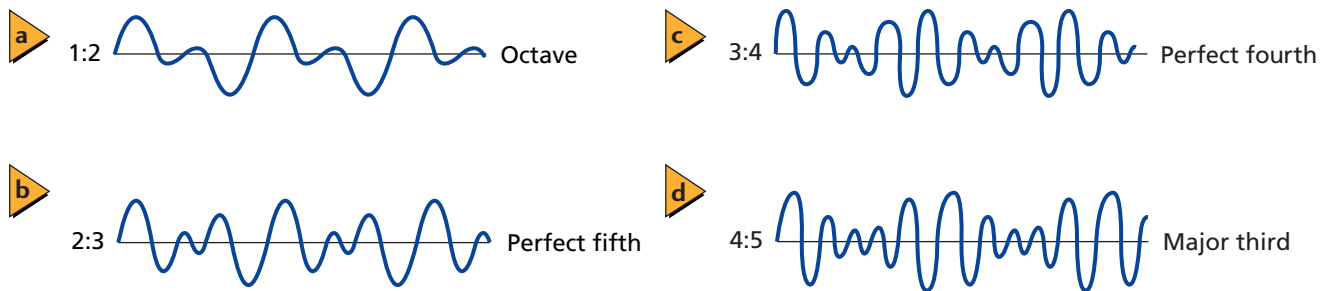
■ **Figure 15-17** A violin, a clarinet, and a piano produce characteristic sound spectra. Each spectrum is unique, as is the timbre of the instrument.

## ● CHALLENGE PROBLEM

- Determine the tension,  $F_T$ , in a violin string of mass  $m$  and length  $L$  that will play the fundamental note at the same frequency as a closed pipe also of length  $L$ . Express your answer in terms of  $m$ ,  $L$ , and the speed of sound in air,  $v$ . The equation for the speed of a wave on a string is  $u = \sqrt{\frac{F_T}{\mu}}$ , where  $F_T$  is the tension in the string and  $\mu$  is the mass per unit length of the string.
- What is the tension in a string of mass 1.0 g and 40.0 cm long that plays the same note as a closed pipe of the same length?







■ **Figure 15-18** These time graphs show the superposition of two waves having the ratios of 1:2, 2:3, 3:4, and 4:5.

**Consonance and dissonance** When sounds that have two different pitches are played at the same time, the resulting sound can be either pleasant or jarring. In musical terms, several pitches played together are called a chord. An unpleasant set of pitches is called **dissonance**. If the combination is pleasant, the sounds are said to be in **consonance**.

What makes a sound pleasant to listen to? Different cultures have different definitions, but most Western cultures accept the definitions of Pythagoras, who lived in ancient Greece. Pythagoras experimented by plucking two strings at the same time. He noted that pleasing sounds resulted when the strings had lengths in small, whole-number ratios, for example 1:2, 2:3, or 3:4. This means that their pitches (frequencies) will also have small, whole-number ratios.

**Musical intervals** Two notes with frequencies related by the ratio 1:2 are said to differ by an octave. For example, if a note has a frequency of 440 Hz, a note that is one octave higher has a frequency of 880 Hz. The fundamental and its harmonics are related by octaves; the first harmonic is one octave higher than the fundamental, the second is two octaves higher, and so on. The sum of the fundamental and the first harmonic is shown in **Figure 15-18a**. It is the ratio of two frequencies, not the size of the interval between them, that determines the musical interval.

In other musical intervals, two pitches may be close together. For example, the ratio of frequencies for a “major third” is 4:5. A typical major third is made up of the notes C and E. The note C has a frequency of 262 Hz, so E has a frequency of  $(5/4)(262 \text{ Hz}) = 327 \text{ Hz}$ . In the same way, notes in a “fourth” (C and F) have a frequency ratio of 3:4, and those in a “fifth” (C and G) have a ratio of 2:3. Graphs of these pleasant sounds are shown in Figure 15-18. More than two notes sounded together also can produce consonance. The three notes called do, mi, and sol make a major chord. For at least 2500 years, this has been recognized as the sweetest of the three-note chords; it has the frequency ratio of 4:5:6.

## Beats

You have seen that consonance is defined in terms of the ratio of frequencies. When the ratio becomes nearly 1:1, the frequencies become very close. Two frequencies that are nearly identical interfere to produce high and low sound levels, as illustrated in **Figure 15-19**. This oscillation of wave amplitude is called a **beat**. The frequency of a beat is the magnitude of difference between the frequencies of the two waves,  $f_{\text{beat}} = |f_A - f_B|$ . When the difference is less than 7 Hz, the ear detects this as a pulsation of loudness. Musical instruments often are tuned by sounding one against another and adjusting the frequency of one until the beat disappears.

## MINI LAB

### Sounds Good

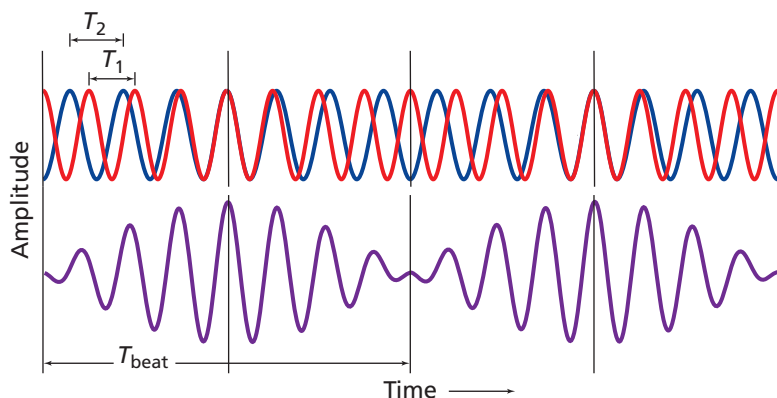
Sometimes it can be pretty tough to tell just by looking whether an instrument will act as an open-pipe resonator or as a closed-pipe resonator. Ask a musician who plays a wind instrument to bring it to class.

- 1. Measure** the length of the instrument.
- Have the musician play the lowest possible note on the instrument.
- 3. Determine** the frequency of the note using a frequency generator.

### Analyze and Conclude

**4. Draw Conclusions** Did the tested instrument behave most like a closed-pipe resonator or most like an open-pipe resonator? Was the frequency the fundamental or one of the subsequent harmonics?

**5. Determine** the frequencies of the notes that would form an octave, a perfect fifth, a perfect fourth, and a major third with your observed note.

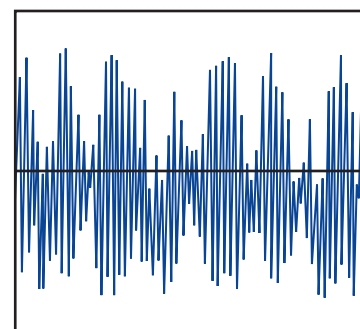


■ **Figure 15-19** Beats occur as a result of the superposition of two sound waves of slightly different frequencies.

## Sound Reproduction and Noise

How often do you listen to music produced directly by a human voice or musical instrument? Most of the time, the music has been recorded and played through electronic systems. To reproduce the sound faithfully, the system must accommodate all frequencies equally. A good stereo system keeps the amplitudes of all frequencies between 20 and 20,000 Hz the same to within 3 dB.

A telephone system, on the other hand, needs only to transmit the information in spoken language. Frequencies between 300 and 3000 Hz are sufficient. Reducing the number of frequencies present helps reduce the noise. A noise wave is shown in **Figure 15-20**. Many frequencies are present with approximately the same amplitude. While noise is not helpful in a telephone system, some people claim that listening to noise has a calming effect. For this reason, some dentists use noise to help their patients relax.



■ **Figure 15-20** Noise is composed of several frequencies and involves random changes in frequency and amplitude.

## 15.2 Section Review

22. **Origins of Sound** What is the vibrating object that produces sounds in each of the following?
  - a. a human voice
  - b. a clarinet
  - c. a tuba
  - d. a violin
23. **Resonance in Air Columns** Why is the tube from which a tuba is made much longer than that of a cornet?
24. **Resonance in Open Tubes** How must the length of an open tube compare to the wavelength of the sound to produce the strongest resonance?
25. **Resonance on Strings** A violin sounds a note of F sharp, with a pitch of 370 Hz. What are the frequencies of the next three harmonics produced with this note?
26. **Resonance in Closed Pipes** One closed organ pipe has a length of 2.40 m.
  - a. What is the frequency of the note played by this pipe?
  - b. When a second pipe is played at the same time, a 1.40-Hz beat note is heard. By how much is the second pipe too long?
27. **Timbre** Why do various instruments sound different even when they play the same note?
28. **Beats** A tuning fork produces three beats per second with a second, 392-Hz tuning fork. What is the frequency of the first tuning fork?
29. **Critical Thinking** Strike a tuning fork with a rubber hammer and hold it at arm's length. Then press its handle against a desk, a door, a filing cabinet, and other objects. What do you hear? Why?

## Speed of Sound

Alternate CBL instructions can be found on the Web site.  
[physicspp.com](http://physicspp.com)

If a vibrating tuning fork is held above a closed pipe of the proper length, the air in the pipe will vibrate at the same frequency,  $f$ , as the tuning fork. By placing a glass tube in a large, water-filled graduated cylinder, the length of the glass tube can be changed by raising or lowering it in the water. The shortest column of air that will resonate occurs when the tube is one-fourth of a wavelength long. This resonance will produce the loudest sound, and the wavelength at this resonance is described by  $\lambda = 4L$ , where  $L$  is the length from the water to the open end of the pipe. In this lab, you will determine  $L$ , calculate  $\lambda$ , and calculate the speed of sound.

### QUESTION

How can you use a closed-pipe resonator to determine the speed of sound?

#### Objectives

- **Collect and organize data** to obtain resonant points in a closed pipe.
- **Measure** the length of a closed-pipe resonator.
- **Analyze** the data to determine the speed of sound.

#### Safety Precautions



- **Immediately wipe up any spilled liquids.**
- **Glass is fragile. Handle with care.**

#### Materials

three tuning forks of known frequencies  
 graduated cylinder (1000-mL)  
 water  
 tuning fork mallet

metric ruler  
 thermometer (non-mercury)  
 glass tube (approximately 40 cm in length and 3.5 cm in diameter)

#### Procedure

1. Put on your safety goggles. Fill the graduated cylinder nearly to the top with water.
2. Measure the room air temperature and record it in Data Table 1.
3. Select a tuning fork and record its frequency in Data Tables 2 and 3.
4. Measure and record the diameter of the glass tube in Data Table 2.
5. Carefully place the glass tube into the water-filled graduated cylinder.
6. Hold the tuning fork by the base. Swiftly strike it on the side with the tuning fork mallet. Do not strike the tuning fork on the laboratory table or other hard surface.
7. Hold the vibrating fork over the open end of the glass tube and slowly raise the tube and the fork until the loudest sound is heard. Once this point is located, move the tube up and down slightly to determine the exact point of resonance. Measure the distance from the water to the top of the glass tube and record this distance in Data Table 2.
8. Repeat steps 3, 6, and 7 for two additional tuning forks and record your results as trials 2 and 3. The three tuning forks that you test should resonate at three different frequencies.
9. Empty the water from the graduated cylinder.



Data Table 1			
Trial	Temperature (°C)	Accepted Speed of Sound (m/s)	Experimental Speed of Sound (m/s)
1			
2			
3			

Data Table 2				
Trial	Tuning Fork Frequency (Hz)	Diameter (m)	Length of Tube Above Water (m)	Calculated Wavelength (m)
1				
2				
3				

### Analyze

- Calculate the accepted speed of sound using the relationship  $v = 331 \text{ m/s} + 0.60T$ , where  $v$  is the speed of sound at temperature  $T$ , and  $T$  is the air temperature in degrees Celsius. Record this as the accepted speed of sound in Data Tables 1 and 3 for all the trials.
- Since the first resonant point is located when the tube is one-fourth of a wavelength above the water, use the measured length of the tube to determine the calculated wavelength for each trial. Record the calculated wavelengths in Data Table 2.
- Multiply the values in Data Table 2 of wavelength and frequency to determine the experimental speed of sound and record this in Data Table 1 for each of the trials.
- Error Analysis** For each trial in Data Table 1, determine the relative error between the experimental and accepted speed of sound.

Relative error =

$$\frac{|\text{Accepted value} - \text{Experimental value}|}{\text{Accepted value}} \times 100$$

- Critique** To improve the accuracy of your calculations, the tube diameter must be taken into consideration. The following relationship provides a more accurate calculation of wavelength:  $\lambda = 4(L + 0.4d)$ , where  $\lambda$  is the wavelength,  $L$  is the length of the tube above the water, and  $d$  is the inside diameter of the tube. Using the values in Data Table 1 for length and diameter, recalculate  $\lambda$  and record it in Data Table 3 as the corrected wavelength. Calculate the corrected experimental speed of sound by multiplying the tuning fork frequency and corrected wavelength and record the new value for the corrected experimental speed of sound in Data Table 3.

Data Table 3				
Trial	Tuning Fork Frequency (Hz)	Accepted Speed of Sound (m/s)	Corrected Calculated Wavelength (m)	Corrected Experimental Speed of Sound (m/s)
1				
2				
3				

- Error Analysis** For each trial in Data Table 3, determine the relative error between the corrected experimental speed and the accepted speed of sound. Use the same formula that you used in step 4, above.

### Conclude and Apply

- Infer** In general, the first resonant point occurs when the tube length =  $\lambda/4$ . What are the next two lengths where resonance will occur?
- Think Critically** If you had a longer tube, would it be possible to locate another position where resonance occurs? Explain your answer.

### Going Further

Which result produced the more accurate speed of sound?

### Real-World Physics

Explain the relationship between the size of organ pipes and their resonant frequencies.



To find out more about the properties of sound waves, visit the Web site: [physicspp.com](http://physicspp.com)



## Sound Waves in the Sun

**The study** of wave oscillations in the Sun is called helioseismology. Naturally occurring sound waves (p waves), gravity waves, and surface gravity waves all occur in the Sun. All of these waves are composed of oscillating particles, but different forces cause the oscillations.

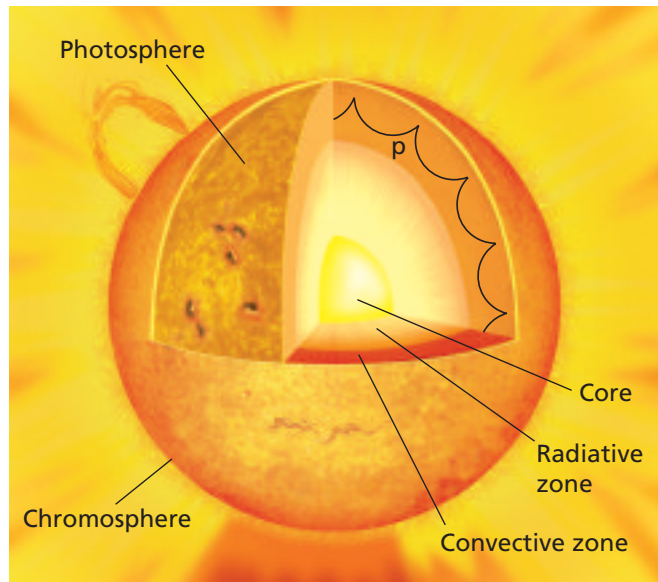
For sound waves, pressure differences cause the particles to oscillate. In the Sun, sound waves travel through the convective zone, which is just under the surface, or photosphere. The sound waves do not travel in a straight line, as shown in the image.

**Ringling like a Bell** The sound waves in the Sun cause the surface of the Sun to vibrate in the radial direction, much like a ringing bell vibrates. When a bell is rung, a clapper hits the bell in one place and standing waves are created. The surface of the Sun does have standing waves, but they are not caused by one large event. Instead, scientists hypothesize that many smaller disruptions in the convective zone start most of the sound waves in the Sun. Just like boiling water in a pot can be noisy, bubbles that are larger than the state of Texas form on the surface of the Sun and start sound waves.

Unlike a pot of boiling water, the sound coming from the Sun is much too low for us to hear. The A above middle C on the piano has a period of 0.00227 s ( $f = 440$  Hz). The middle mode of oscillation of the waves in the Sun has a period of 5 min ( $f = 0.003$  Hz).

Because we cannot hear the sound waves from the Sun, scientists measure the motion of the surface of the Sun to learn about its sound waves. Because a sound wave takes 2 h to travel from one side of the Sun to the other, the Sun must be observed for long time periods. This necessity makes observations from Earth difficult because the Sun is not visible during the night. In 1995, the *Solar and Heliospheric Observatory* (SOHO) was launched by NASA. This satellite orbits Earth such that it always can observe the Sun.

The motion of the surface of the Sun is measured by observing Doppler shifts in sunlight. The measured vibrations are a complicated pattern that equals the sum of all of the standing waves present in the Sun. Just like a ringing bell, many overtones are present in the Sun. Through careful analysis, the individual standing waves in the Sun and their intensities can be calculated.



Sound waves (p waves) travel through the Sun's convective zone.

**Results** Because composition, temperature, and density affect the propagation of sound waves, the Sun's wave oscillations provide information about its interior. SOHO results have given insight into the rotation rate of the Sun as a function of latitude and depth, as well as the density and temperature of the Sun. These results are compared to theoretical calculations to improve our understanding of the Sun.

### Going Further

- Hypothesize** How do scientists separate the surface motion due to sound waves from the motion due to the rotation of the Sun?
- Critical Thinking** Would sound waves in another star, similar to the Sun but different in size, have the same wavelength as sound waves in the Sun?

## 15.1 Properties and Detection of Sound

**Vocabulary**

- sound wave (p. 404)
- pitch (p. 406)
- loudness (p. 406)
- sound level (p. 406)
- decibel (p. 406)
- Doppler effect (p. 407)

**Key Concepts**

- Sound is a pressure variation transmitted through matter as a longitudinal wave.
- A sound wave has frequency, wavelength, speed, and amplitude. Sound waves reflect and interfere.
- The speed of sound in air at room temperature (20°C) is 343 m/s. The speed increases roughly 0.6 m/s with each 1°C increase in temperature.
- Sound detectors convert the energy carried by a sound wave into another form of energy. The human ear is a highly efficient and sensitive detector of sound waves.
- The frequency of a sound wave is heard as its pitch.
- The pressure amplitude of a sound wave can be measured in decibels (dB).
- The loudness of sound as perceived by the ear and brain depends mainly on its amplitude.
- The Doppler effect is the change in frequency of sound caused by the motion of either the source or the detector. It can be calculated with the following equation.

$$f_d = f_s \left( \frac{v - v_d}{v - v_s} \right)$$

## 15.2 The Physics of Music

**Vocabulary**

- closed-pipe resonator (p. 412)
- open-pipe resonator (p. 412)
- fundamental (p. 417)
- harmonics (p. 417)
- dissonance (p. 418)
- consonance (p. 418)
- beat (p. 418)

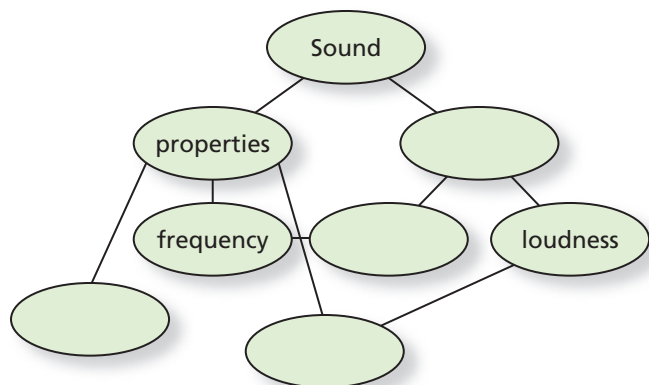
**Key Concepts**

- Sound is produced by a vibrating object in a material medium.
- Most sounds are complex waves that are composed of more than one frequency.
- An air column can resonate with a sound source, thereby increasing the amplitude of its resonant frequency.
- A closed pipe resonates when its length is  $\lambda/4$ ,  $3\lambda/4$ ,  $5\lambda/4$ , and so on. Its resonant frequencies are odd-numbered multiples of the fundamental.
- An open pipe resonates when its length is  $\lambda/2$ ,  $2\lambda/2$ ,  $3\lambda/2$ , and so on. Its resonant frequencies are whole-number multiples of the fundamental.
- A clamped string has a node at each end and resonates when its length is  $\lambda/2$ ,  $2\lambda/2$ ,  $3\lambda/2$ , and so on, just as with an open pipe. The string's resonant frequencies are also whole-number multiples of the fundamental.
- The frequencies and intensities of the complex waves produced by a musical instrument determine the timbre that is characteristic of that instrument.
- The fundamental frequency and harmonics can be described in terms of resonance.
- Notes on a musical scale differ in frequency by small, whole-number ratios. An octave has a frequency ratio of 1:2.
- Two waves with almost the same frequency interfere to produce beats.



## Concept Mapping

30. Complete the concept map below using the following terms: *amplitude*, *perception*, *pitch*, *speed*.



## Mastering Concepts

31. What are the physical characteristics of sound waves? (15.1)
32. When timing the 100-m run, officials at the finish line are instructed to start their stopwatches at the sight of smoke from the starter's pistol and not at the sound of its firing. Explain. What would happen to the times for the runners if the timing started when sound was heard? (15.1)
33. Name two types of perception of sound and the physical characteristics of sound waves that correspond to them. (15.1)
34. Does the Doppler shift occur for only some types of waves or for all types of waves? (15.1)
35. Sound waves with frequencies higher than can be heard by humans, called ultrasound, can be transmitted through the human body. How could ultrasound be used to measure the speed of blood flowing in veins or arteries? Explain how the waves change to make this measurement possible. (15.1)
36. What is necessary for the production and transmission of sound? (15.2)
37. **Singing** How can a certain note sung by an opera singer cause a crystal glass to shatter? (15.2)
38. **Marching** In the military, as marching soldiers approach a bridge, the command "route step" is given. The soldiers then walk out-of-step with each other as they cross the bridge. Explain. (15.2)
39. **Musical Instruments** Why don't most musical instruments sound like tuning forks? (15.2)
40. **Musical Instruments** What property distinguishes notes played on both a trumpet and a clarinet if they have the same pitch and loudness? (15.2)

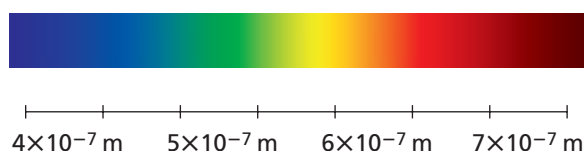
41. **Trombones** Explain how the slide of a trombone, shown in **Figure 15-21**, changes the pitch of the sound in terms of a trombone being a resonance tube. (15.2)



■ Figure 15-21

## Applying Concepts

42. **Estimation** To estimate the distance in kilometers between you and a lightning flash, count the seconds between the flash and the thunder and divide by 3. Explain how this rule works. Devise a similar rule for miles.
43. The speed of sound increases by about 0.6 m/s for each degree Celsius when the air temperature rises. For a given sound, as the temperature increases, what happens to the following?  
 a. the frequency  
 b. the wavelength
44. **Movies** In a science-fiction movie, a satellite blows up. The crew of a nearby ship immediately hears and sees the explosion. If you had been hired as an advisor, what two physics errors would you have noticed and corrected?
45. **The Redshift** Astronomers have observed that the light coming from distant galaxies appears redder than light coming from nearer galaxies. With the help of **Figure 15-22**, which shows the visible spectrum, explain why astronomers conclude that distant galaxies are moving away from Earth.



■ Figure 15-22

46. Does a sound of 40 dB have a factor of 100 ( $10^2$ ) times greater pressure variation than the threshold of hearing, or a factor of 40 times greater?

47. If the pitch of sound is increased, what are the changes in the following?
- the frequency
  - the wavelength
  - the wave velocity
  - the amplitude of the wave
48. The speed of sound increases with temperature. Would the pitch of a closed pipe increase or decrease when the temperature of the air rises? Assume that the length of the pipe does not change.
49. **Marching Bands** Two flutists are tuning up. If the conductor hears the beat frequency increasing, are the two flute frequencies getting closer together or farther apart?
50. **Musical Instruments** A covered organ pipe plays a certain note. If the cover is removed to make it an open pipe, is the pitch increased or decreased?
51. **Stringed Instruments** On a harp, **Figure 15-23a**, long strings produce low notes and short strings produce high notes. On a guitar, **Figure 15-23b**, the strings are all the same length. How can they produce notes of different pitches?

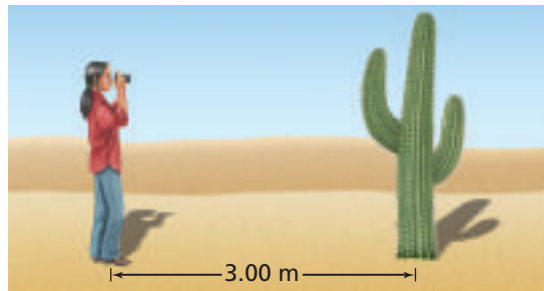


■ Figure 15-23

## Mastering Problems

### 15.1 Properties and Detection of Sound

52. You hear the sound of the firing of a distant cannon 5.0 s after seeing the flash. How far are you from the cannon?
53. If you shout across a canyon and hear an echo 3.0 s later, how wide is the canyon?
54. A sound wave has a frequency of 4700 Hz and travels along a steel rod. If the distance between compressions, or regions of high pressure, is 1.1 m, what is the speed of the wave?
55. **Bats** The sound emitted by bats has a wavelength of 3.5 mm. What is the sound's frequency in air?
56. **Photography** As shown in **Figure 15-24**, some cameras determine the distance to the subject by sending out a sound wave and measuring the time needed for the echo to return to the camera. How long would it take the sound wave to return to such a camera if the subject were 3.00 m away?



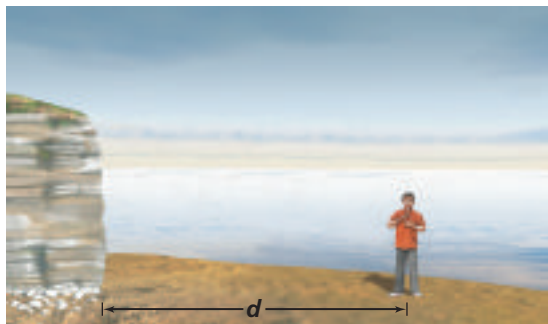
■ Figure 15-24

57. Sound with a frequency of 261.6 Hz travels through water at 25°C. Find the sound's wavelength in water. Do not confuse sound waves moving through water with surface waves moving through water.
58. If the wavelength of a  $4.40 \times 10^2$ -Hz sound in freshwater is 3.30 m, what is the speed of sound in freshwater?
59. Sound with a frequency of 442 Hz travels through an iron beam. Find the wavelength of the sound in iron.
60. **Aircraft** Adam, an airport employee, is working near a jet plane taking off. He experiences a sound level of 150 dB.
- If Adam wears ear protectors that reduce the sound level to that of a typical rock concert, what decrease in dB is provided?
  - If Adam then hears something that sounds like a barely audible whisper, what will a person not wearing the ear protectors hear?
61. **Rock Music** A rock band plays at an 80-dB sound level. How many times greater is the sound pressure from another rock band playing at each of the following sound levels?
- 100 dB
  - 120 dB
62. A coiled-spring toy is shaken at a frequency of 4.0 Hz such that standing waves are observed with a wavelength of 0.50 m. What is the speed of propagation of the wave?
63. A baseball fan on a warm summer day (30°C) sits in the bleachers 152 m away from home plate.
- What is the speed of sound in air at 30°C?
  - How long after seeing the ball hit the bat does the fan hear the crack of the bat?



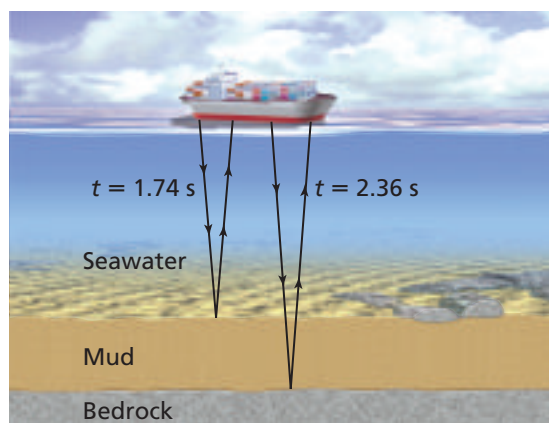
## Chapter 15 Assessment

64. On a day when the temperature is  $15^{\circ}\text{C}$ , a person stands some distance,  $d$ , as shown in **Figure 15-25**, from a cliff and claps his hands. The echo returns in 2.0 s. How far away is the cliff?



■ **Figure 15-25** (Not to scale)

65. **Medical Imaging** Ultrasound with a frequency of 4.25 MHz can be used to produce images of the human body. If the speed of sound in the body is the same as in salt water, 1.50 km/s, what is the length of a 4.25-MHz pressure wave in the body?
66. **Sonar** A ship surveying the ocean bottom sends sonar waves straight down into the seawater from the surface. As illustrated in **Figure 15-26**, the first reflection, off of the mud at the sea floor, is received 1.74 s after it was sent. The second reflection, from the bedrock beneath the mud, returns after 2.36 s. The seawater is at a temperature of  $25^{\circ}\text{C}$ , and the speed of sound in mud is 1875 m/s.
- How deep is the water?
  - How thick is the mud?



■ **Figure 15-26** (Not to scale)

67. Determine the variation in sound pressure of a conversation being held at a sound level of 60 dB.
68. A fire truck is moving at 35 m/s, and a car in front of the truck is moving in the same direction at 15 m/s. If a 327-Hz siren blares from the truck, what frequency is heard by the driver of the car?
69. A train moving toward a sound detector at 31.0 m/s blows a 305-Hz whistle. What frequency is detected on each of the following?
- a stationary train
  - a train moving toward the first train at 21.0 m/s
70. The train in the previous problem is moving away from the detector. What frequency is now detected on each of the following?
- a stationary train
  - a train moving away from the first train at a speed of 21 m/s

### 15.2 The Physics of Music

71. A vertical tube with a tap at the base is filled with water, and a tuning fork vibrates over its mouth. As the water level is lowered in the tube, resonance is heard when the water level has dropped 17 cm, and again after 49 cm of distance exists from the water to the top of the tube. What is the frequency of the tuning fork?
72. **Human Hearing** The auditory canal leading to the eardrum is a closed pipe that is 3.0 cm long. Find the approximate value (ignoring end correction) of the lowest resonance frequency.
73. If you hold a 1.2-m aluminum rod in the center and hit one end with a hammer, it will oscillate like an open pipe. Antinodes of pressure correspond to nodes of molecular motion, so there is a pressure antinode in the center of the bar. The speed of sound in aluminum is 5150 m/s. What would be the bar's lowest frequency of oscillation?
74. One tuning fork has a 445-Hz pitch. When a second fork is struck, beat notes occur with a frequency of 3 Hz. What are the two possible frequencies of the second fork?
75. **Flutes** A flute acts as an open pipe. If a flute sounds a note with a 370-Hz pitch, what are the frequencies of the second, third, and fourth harmonics of this pitch?
76. **Clarinets** A clarinet sounds the same note, with a pitch of 370 Hz, as in the previous problem. The clarinet, however, acts as a closed pipe. What are the frequencies of the lowest three harmonics produced by this instrument?
77. **String Instruments** A guitar string is 65.0 cm long and is tuned to produce a lowest frequency of 196 Hz.
- What is the speed of the wave on the string?
  - What are the next two higher resonant frequencies for this string?

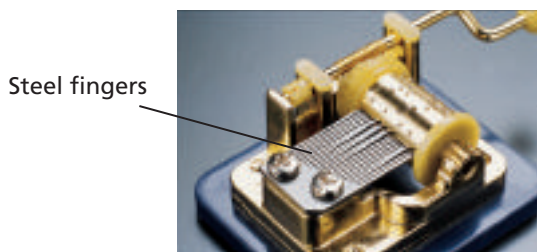


78. **Musical Instruments** The lowest note on an organ is 16.4 Hz.
- What is the shortest open organ pipe that will resonate at this frequency?
  - What is the pitch if the same organ pipe is closed?
79. **Musical Instruments** Two instruments are playing musical A (440.0 Hz). A beat note with a frequency of 2.5 Hz is heard. Assuming that one instrument is playing the correct pitch, what is the frequency of the pitch played by the second instrument?
80. A flexible, corrugated, plastic tube, shown in **Figure 15-27**, is 0.85 m long. When it is swung around, it creates a tone that is the lowest pitch for an open pipe of this length. What is the frequency?



■ Figure 15-27

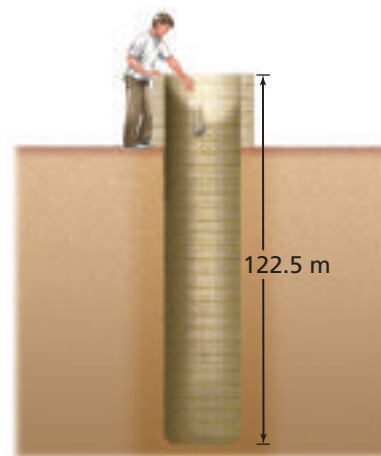
81. The tube from the previous problem is swung faster, producing a higher pitch. What is the new frequency?
82. During normal conversation, the amplitude of a pressure wave is 0.020 Pa.
- If the area of an eardrum is  $0.52 \text{ cm}^2$ , what is the force on the eardrum?
  - The mechanical advantage of the three bones in the middle ear is 1.5. If the force in part a is transmitted undiminished to the bones, what force do the bones exert on the oval window, the membrane to which the third bone is attached?
  - The area of the oval window is  $0.026 \text{ cm}^2$ . What is the pressure increase transmitted to the liquid in the cochlea?
83. **Musical Instruments** One open organ pipe has a length of 836 mm. A second open pipe should have a pitch that is one major third higher. How long should the second pipe be?
84. As shown in **Figure 15-28**, a music box contains a set of steel fingers clamped at one end and plucked on the other end by pins on a rotating drum. What is the speed of a wave on a finger that is 2.4 cm long and plays a note of 1760 Hz?



■ Figure 15-28

## Mixed Review

85. An open organ pipe is 1.65 m long. What fundamental frequency note will it produce if it is played in helium at  $0^\circ\text{C}$ ?
86. If you drop a stone into a well that is 122.5 m deep, as illustrated in **Figure 15-29**, how soon after you drop the stone will you hear it hit the bottom of the well?



■ Figure 15-29 (Not to scale)

87. A bird on a newly discovered planet flies toward a surprised astronaut at a speed of 19.5 m/s while singing at a pitch of 945 Hz. The astronaut hears a tone of 985 Hz. What is the speed of sound in the atmosphere of this planet?
88. In North America, one of the hottest outdoor temperatures ever recorded is  $57^\circ\text{C}$  and one of the coldest is  $-62^\circ\text{C}$ . What are the speeds of sound at those two temperatures?
89. A ship's sonar uses a frequency of 22.5 kHz. The speed of sound in seawater is 1533 m/s. What is the frequency received on the ship that was reflected from a whale traveling at 4.15 m/s away from the ship? Assume that the ship is at rest.
90. When a wet finger is rubbed around the rim of a glass, a loud tone of frequency 2100 Hz is produced. If the glass has a diameter of 6.2 cm and the vibration contains one wavelength around its rim, what is the speed of the wave in the glass?
91. **History of Science** In 1845, Dutch scientist Christoph Buys-Ballot developed a test of the Doppler effect. He had a trumpet player sound an A note at 440 Hz while riding on a flatcar pulled by a locomotive. At the same time, a stationary trumpeter played the same note. Buys-Ballot heard 3.0 beats per second. How fast was the train moving toward him?

## Chapter 15 Assessment

- 92.** You try to repeat Buys-Ballot's experiment from the previous problem. You plan to have a trumpet played in a rapidly moving car. Rather than listening for beat notes, however, you want to have the car move fast enough so that the moving trumpet sounds one major third above a stationary trumpet.
- How fast would the car have to move?
  - Should you try the experiment? Explain.
- 93. Guitar Strings** The equation for the speed of a wave on a string is  $v = \sqrt{\frac{F_T}{\mu}}$ , where  $F_T$  is the tension in the string and  $\mu$  is the mass per unit length of the string. A guitar string has a mass of 3.2 g and is 65 cm long. What must be the tension in the string to produce a note whose fundamental frequency is 147 Hz?
- 94.** A train speeding toward a tunnel at 37.5 m/s sounds its horn at 327 Hz. The sound bounces off the tunnel mouth. What is the frequency of the reflected sound heard on the train? *Hint: Solve the problem in two parts. First, assume that the tunnel is a stationary observer and find the frequency. Then, assume that the tunnel is a stationary source and find the frequency measured on the train.*

### Thinking Critically

- 95. Make and Use Graphs** The wavelengths of the sound waves produced by a set of tuning forks with given frequencies are shown in **Table 15-2** below.
- Plot a graph of wavelength versus the frequency (controlled variable). What type of relationship does the graph show?
  - Plot a graph of wavelength versus the inverse of the frequency ( $1/f$ ). What kind of graph is this? Determine the speed of sound from this graph.

Frequency (Hz)	Wavelength (m)
131	2.62
147	2.33
165	2.08
196	1.75
220	1.56
247	1.39

- 96. Make Graphs** Suppose that the frequency of a car horn is 300 Hz when it is stationary. What would the graph of the frequency versus time look like as the car approached and then moved past you? Complete a rough sketch.
- 97. Analyze and Conclude** Describe how you could use a stopwatch to estimate the speed of sound if you were near the green on a 200-m golf hole as another group of golfers hit their tee shots. Would your estimate of the speed of sound be too large or too small?
- 98. Apply Concepts** A light wave coming from a point on the left edge of the Sun is found by astronomers to have a slightly higher frequency than light from the right side. What do these measurements tell you about the Sun's motion?
- 99. Design an Experiment** Design an experiment that could test the formula for the speed of a wave on a string. Explain what measurements you would make, how you would make them, and how you would use them to test the formula.

### Writing in Physics

- 100.** Research the construction of a musical instrument, such as a violin or French horn. What factors must be considered besides the length of the strings or tube? What is the difference between a quality instrument and a cheaper one? How are they tested for tone quality?
- 101.** Research the use of the Doppler effect in the study of astronomy. What is its role in the big bang theory? How is it used to detect planets around other stars? To study the motions of galaxies?

### Cumulative Review

- 102.** Ball A, rolling west at 3.0 m/s, has a mass of 1.0 kg. Ball B has a mass of 2.0 kg and is stationary. After colliding with ball B, ball A moves south at 2.0 m/s. (Chapter 9)
- Sketch the system, showing the velocities and momenta before and after the collision.
  - Calculate the momentum and velocity of ball B after the collision.
- 103.** Chris carries a 10-N carton of milk along a level hall to the kitchen, a distance of 3.5 m. How much work does Chris do? (Chapter 10)
- 104.** A movie stunt person jumps from a five-story building (22 m high) onto a large pillow at ground level. The pillow cushions her fall so that she feels a deceleration of no more than  $3.0 \text{ m/s}^2$ . If she weighs 480 N, how much energy does the pillow have to absorb? How much force does the pillow exert on her? (Chapter 11)

