McGraw-Hill Ryerson

Physics 12

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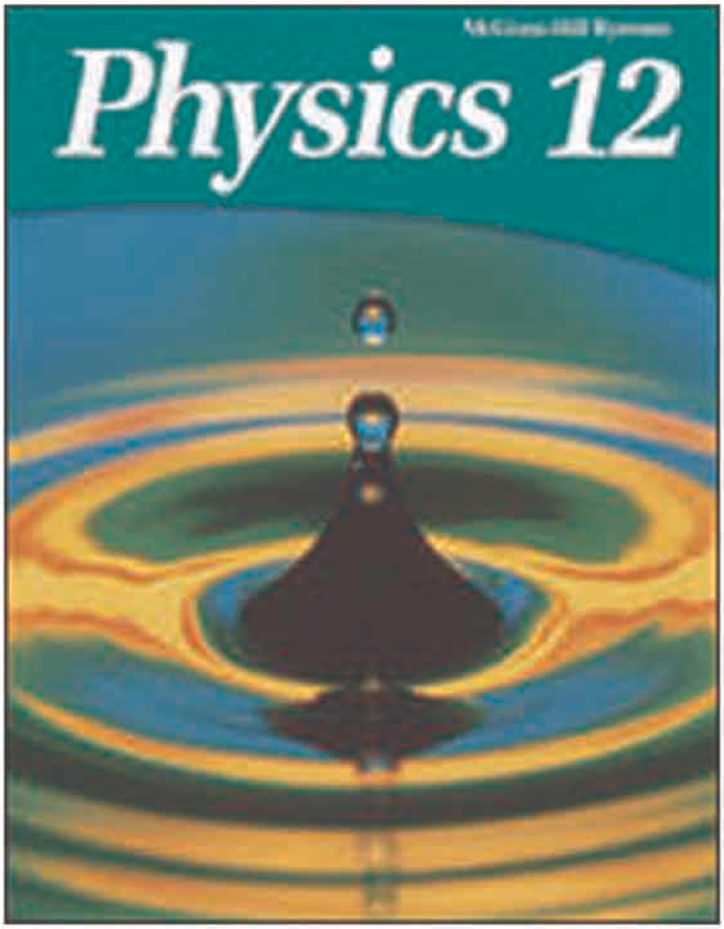


UNIT 1

Forces and Motion: Dynamics

OVERALL EXPECTATIONS

ANALYZE, predict, and explain the motion of selected objects in vertical, horizontal, and inclined planes.

INVESTIGATE, represent, and analyze motion and forces in linear, projectile, and circular motion.

RELATE your understanding of dynamics to the development and use of motion technologies.

UNIT CONTENTS

Spectators are mesmerized by trapeze artists making perfectly timed releases, gliding through graceful arcs, and intersecting the paths of their partners. An error in timing and a graceful arc could become a trajectory of panic. Trapeze artists know that tiny differences in height, velocity, and timing are critical. Swinging from a trapeze, the performer forces his body from its natural straight-line path. Gliding freely through the air, he is subject only to gravity. Then, the outstretched hands of his partner make contact, and the performer is acutely aware of the forces that change his speed and direction.

In this unit, you will explore the relationship between motion and the forces that cause it and investigate how different perspectives of the same motion are related. You will learn how to analyze forces and motion, not only in a straight line, but also in circular paths, in parabolic trajectories, and on inclined surfaces. You will discover how the motion of planets and satellites is caused, described, and analyzed.

UNIT PROJECT PREP

Refer to pages 126–127 before beginning this unit. In the unit project, you will design and build a working catapult to launch small objects through the air.

- What launching devices have you used, watched, or read about? How do they develop and control the force needed to propel an object?
- What projectiles have you launched? How do you direct their flight so that they reach a maximum height or stay in the air for the longest possible time?

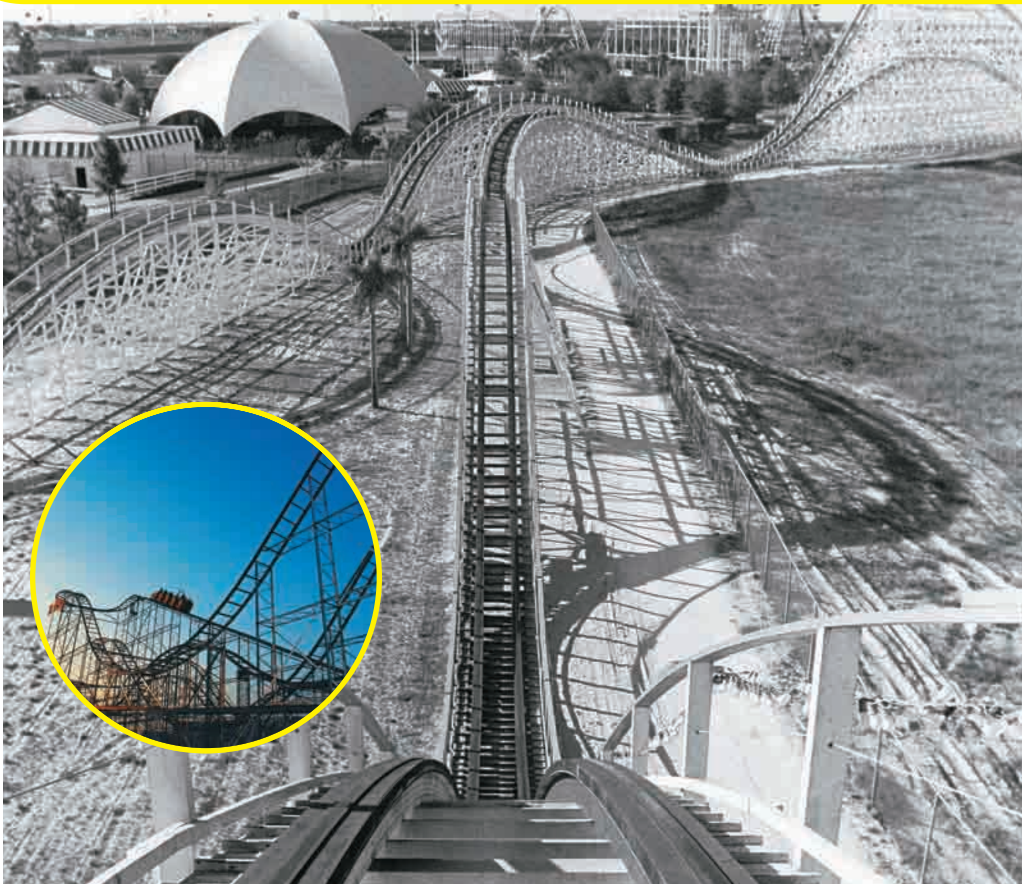
CHAPTER 1

Fundamentals of Dynamics

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Using the kinematic equations for uniformly accelerated motion.



How many times have you heard the saying, "It all depends on your perspective"? The photographers who took the two pictures of the roller coaster shown here certainly had different perspectives. When you are on a roller coaster, the world looks and feels very different than it does when you are observing the motion from a distance. Now imagine doing a physics experiment from these two perspectives, studying the motion of a pendulum, for example. Your results would definitely depend on your perspective or frame of reference. You can describe motion from any frame of reference, but some frames of reference simplify the process of describing the motion and the laws that determine that motion.

In previous courses, you learned techniques for measuring and describing motion, and you studied and applied the laws of motion. In this chapter, you will study in more detail how to choose and define frames of reference. Then, you will extend your knowledge of the dynamics of motion in a straight line.

MULTI LAB

Thinking Physics

TARGET SKILLS
- Predicting
- Identifying variables
- Analyzing and interpreting

Suspended Spring

Tape a plastic cup to one end of a short section of a large-diameter spring, such as a Slinky™. Hold the other end of the spring high enough so that the plastic cup is at least 1 m above the floor. Before you release the spring, predict the exact motion of the cup from the instant that it is released until the moment that it hits the floor. While your partner watches the cup closely from a kneeling position, release the top of the spring. Observe the motion of the cup.

Analyze and Conclude

1. Describe the motion of the cup and the lower end of the spring. Compare the motion to your prediction and describe any differences.
2. Is it possible for any unsupported object to be suspended in midair for any length of time? Create a detailed explanation to account for the behaviour of the cup at the moment at which you released the top of the spring.
3. Athletes and dancers sometimes seem to be momentarily suspended in the air. How might the motion of these athletes be related to the spring's movement in this lab?

Thought Experiments

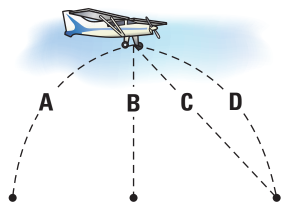
Without discussing the following questions with anyone else, write down your answers.

1. Student A and Student B sit in identical office chairs facing each other, as illustrated. Student A, who is heavier than Student B, suddenly pushes with his feet, causing both chairs to move. Which of the following occurs?

   (a) Neither student applies a force to the other.
   (b) A exerts a force that is applied to B, but A experiences no force.
   (c) Each student applies a force to the other, but A exerts the larger force.
   (d) The students exert the same amount of force on each other.

2. A golf pro drives a ball through the air. What force(s) is/are acting on the golf ball for the *entirety* of its flight?
   (a) force of gravity only
   (b) force of gravity and the force of the "hit"
   (c) force of gravity and the force of air resistance
   (d) force of gravity, the force of the "hit," and the force of air resistance

3. A photographer accidentally drops a camera out of a small airplane as it flies horizontally. As seen from the ground, which path would the camera most closely follow as it fell?



Analyze and Conclude

Tally the class results. As a class, discuss the answers to the questions.

1.1 Inertia and Frames of Reference

SECTION EXPECTATIONS

- Describe and distinguish between inertial and non-inertial frames of reference.
- Define and describe the concept and units of mass.
- Investigate and analyze linear motion, using vectors, graphs, and free-body diagrams.

KEY TERMS

- inertia
- inertial mass
- gravitational mass
- coordinate system
- frame of reference
- inertial frame of reference
- non-inertial frame of reference
- fictitious force

Imagine watching a bowling ball sitting still in the rack. Nothing moves; the ball remains totally at rest until someone picks it up and hurls it down the alley. Galileo Galilei (1564–1642) and later Sir Isaac Newton (1642–1727) attributed this behaviour to the property of matter now called **inertia**, meaning resistance to changes in motion. Stationary objects such as the bowling ball remain motionless due to their inertia.

Now picture a bowling ball rumbling down the alley. Experience tells you that the ball might change direction and, if the alley was long enough, it would slow down and eventually stop. Galileo realized that these changes in motion were due to factors that interfere with the ball's "natural" motion. Hundreds of years of experiments and observations clearly show that Galileo was correct. Moving objects continue moving in the same direction, at the same speed, due to their inertia, unless some external force interferes with their motion.

Figure 1.1 You assume that an inanimate object such as a bowling ball will remain stationary until someone exerts a force on it. Galileo and Newton realized that this "lack of motion" is a very important property of matter.

Analyzing Forces

Newton refined and extended Galileo's ideas about inertia and straight-line motion at constant speed — now called "uniform motion."

NEWTON'S FIRST LAW: THE LAW OF INERTIA

An object at rest or in uniform motion will remain at rest or in uniform motion unless acted on by an external force.

Newton's first law states that a force is required to *change* an object's uniform motion or velocity. Newton's second law then permits you to determine how great a force is needed in order to change an object's velocity by a given amount. Recalling that acceleration is defined as the change in velocity, you can state Newton's second law by saying, "The *net* force ($\vec{F}$) required to accelerate an object of mass m by an amount ($\vec{a}$) is the product of the mass and acceleration."

NEWTON'S SECOND LAW

The word equation for Newton's second law is: Net force is the product of mass and acceleration.

$$\vec{F} = m\vec{a}$$

Quantity	Symbol	SI unit
force	$\vec{F}$	N (newtons)
mass	m	kg (kilograms)
acceleration	$\vec{a}$	$\frac{\text{m}}{\text{s}^2}$ (metres per second squared)

Unit analysis

$$(\text{mass})(\text{acceleration}) = (\text{kilogram})\left(\frac{\text{metres}}{\text{second}^2}\right) \text{kg}\frac{\text{m}}{\text{s}^2} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = \text{N}$$

Note: The force ($\vec{F}$) in Newton's second law refers to the vector sum of all of the forces acting on the object.

Inertial Mass

When you compare the two laws of motion, you discover that the first law identifies inertia as the property of matter that resists a change in its motion; that is, it resists acceleration. The second law gives a quantitative method of finding acceleration, but it does not seem to mention inertia. Instead, the second law indicates that the property that relates force and acceleration is mass.

Actually, the mass (m) used in the second law is correctly described as the **inertial mass** of the object, the property that resists a change in motion. As you know, matter has another property — it experiences a gravitational attractive force. Physicists refer to this property of matter as its **gravitational mass.** Physicists never assume that two seemingly different properties are related without thoroughly studying them. In the next investigation, you will examine the relationship between inertial mass and gravitational mass.

LANGUAGE LINK

The Latin root of *inertia* means "sluggish" or "inactive." An *inertial guidance system* relies on a gyroscope, a "sluggish" mechanical device that resists a change in the direction of motion. What does this suggest about the chemical properties of an *inert gas*?

INVESTIGATION 1-A

Measuring Inertial Mass

TARGET SKILLS
- Hypothesizing
- Performing and recording
- Analyzing and interpreting

Problem

Is there a direct relationship between an object's inertial mass and its gravitational mass?

Hypothesis

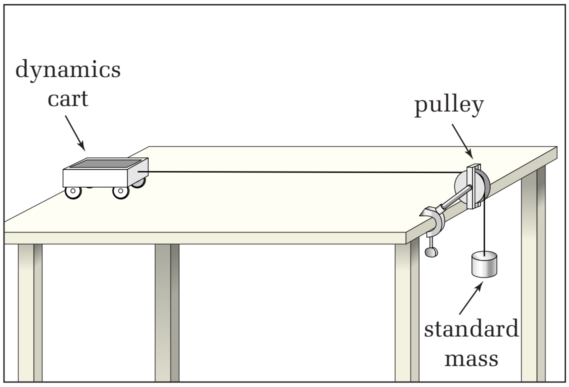
Formulate an hypothesis about the relationship between inertial mass and its gravitational mass.

Equipment

- dynamics cart
- pulley and string
- laboratory balance
- standard mass (about 500 g)
- metre stick and stopwatch *or* motion sensor
- unit masses (six identical objects, such as small C-clamps)
- unknown mass (measuring between one and six unit masses, such as a stone)

Procedure

1. Arrange the pulley, string, standard mass, and dynamics cart on a table, as illustrated.



2. Set up your measuring instruments to determine the acceleration of the cart when it is pulled by the falling standard mass. Find the acceleration directly by using computer software, or calculate it from measurements of displacement and time.
3. Measure the acceleration of the empty cart.
4. Add unit masses one at a time and measure the acceleration several times after each addition. Average your results.
5. Graph the acceleration versus the number of unit inertial masses on the cart.
6. Remove the unit masses from the cart and replace them with the unknown mass, then measure the acceleration of the cart.
7. Use the graph to find the inertial mass of the unknown mass (in unit inertial masses).
8. Find the gravitational mass of one unit of inertial mass, using a laboratory balance.
9. Add a second scale to the horizontal axis of your graph, using standard gravitational mass units (kilograms).
10. Use the second scale on the graph to predict the gravitational mass of the unknown mass.
11. Verify your prediction: Find the unknown's gravitational mass on a laboratory balance.

Analyze and Conclude

1. Based on your data, are inertial and gravitational masses equal, proportional, or independent?
2. Does your graph fit a linear, inverse, exponential, or radical relationship? Write the relationship as a proportion ($a \propto ?$).
3. Write Newton's second law. Solve the expression for acceleration. Compare this expression to your answer to question 2. What inferences can you make?
4. Extrapolate your graph back to the vertical axis. What is the significance of the point at which your graph now crosses the axis?
5. Verify the relationship you identified in question 2 by using curve-straightening techniques (see Skill Set 4, Mathematical Modelling and Curve Straightening). Write a specific equation for the line in your graph.

Over many years of observations and investigations, physicists concluded that inertial mass and gravitational mass were two different manifestations of the same property of matter. Therefore, when you write *m* for mass, you do not have to specify what type of mass it is.

Action-Reaction Forces

Newton's first and second laws are sufficient for explaining and predicting motion in many situations. However, you will discover that, in some cases, you will need Newton's third law. Unlike the first two laws that focus on the forces acting on one object, Newton's third law considers two objects exerting forces on each other. For example, when you push on a wall, you can feel the wall pushing back on you. Newton's third law states that this condition always exists — when one object exerts a force on another, the second force always exerts a force on the first. The third law is sometimes called the "law of action-reaction forces."

NEWTON'S THIRD LAW

For every action force on an object (B) due to another object (A), there is a reaction force, equal in magnitude but opposite in direction, on object A, due to object B.

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

To avoid confusion, be sure to note that the forces described in Newton's third law refer to two different objects. When you apply Newton's second law to an object, you consider only one of these forces — the force that acts *on* the object. You do *not* include any forces that the object itself exerts on something else. If this concept is clear to you, you will be able to solve the "horse-cart paradox" described below.

Conceptual Problem

- The famous horse-cart paradox asks, "If the cart is pulling on the horse with a force that is equal in magnitude and opposite in direction to the force that the horse is exerting on the cart, how can the horse make the cart move?" Discuss the answer with a classmate, then write a clear explanation of the paradox.

QUICK LAB

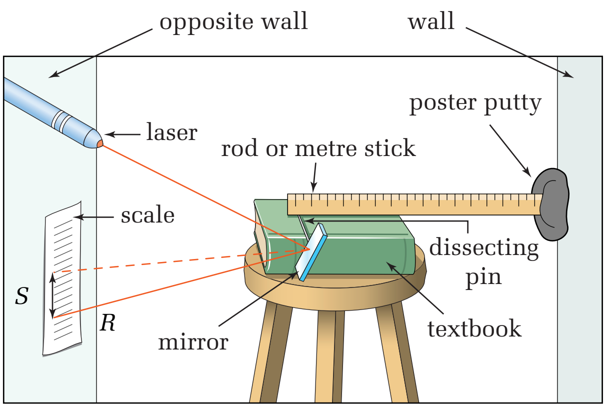
Bend a Wall

TARGET SKILLS
- Initiating and planning
- Performing and recording
- Analyzing and interpreting

Sometimes it might not seem as though an object on which you are pushing is exhibiting any type of motion. However, the proper apparatus might detect some motion. Prove that you can move — or at least, bend — a wall.

CAUTION Do not look into the laser.

Glue a small mirror to a 5 cm T-head dissecting pin. Put a textbook on a stool beside the wall that you will attempt to bend. Place the pin-mirror assembly on the edge of the textbook. As shown in the diagram, attach a metre stick to the wall with putty or modelling clay and rest the other end on the pin-mirror assembly. The pin-mirror should act as a roller, so that any movement of the metre stick turns the mirror slightly. Place a laser pointer so that its beam reflects off the mirror and onto the opposite wall. Prepare a linear scale on a sheet of paper and fasten it to the opposite wall, so that you can make the required measurements.



Push hard on the wall near the metre stick and observe the deflection of the laser spot. Measure
- the radius of the pin (r)
- the deflection of the laser spot (S)
- the distance from the mirror to the opposite wall (R)

Analyze and Conclude

1. Calculate the extent of the movement (s) — or how much the wall "bent" — using the formula $s = \frac{rS}{2R}$.
2. If other surfaces behave as the wall does, list other situations in which an apparently inflexible surface or object is probably moving slightly to generate a resisting or supporting force.
3. Do your observations "prove" that the wall bent? Suppose a literal-minded observer questioned your results by claiming that you did not actually see the wall bend, but that you actually observed movement of the laser spot. How would you counter this objection?
4. Is it scientifically acceptable to use a mathematical formula, such as the one above, without having derived or proved it? Justify your response.
5. If you have studied the arc length formula in mathematics, try to derive the formula above. (Hint: Use the fact that the angular displacement of the laser beam is actually twice the angular displacement of the mirror.)

Apply and Extend

6. Imagine that you are explaining this experiment to a friend who has not yet taken a physics course. You tell your friend that "When I pushed on the wall, the wall pushed back on me." Your friend says, "That's silly. Walls don't push on people." Use the laws of physics to justify your original statement.
7. Why is it logical to expect that a wall will move when you push on it?
8. Dentists sometimes check the health of your teeth and gums by measuring tooth mobility. Design an apparatus that could be used to measure tooth mobility.

Frames of Reference

In order to use Newton's laws to analyze and predict the motion of an object, you need a reference point and definitions of distance and direction. In other words, you need a **coordinate system.** One of the most commonly used systems is the Cartesian coordinate system, which has an origin and three mutually perpendicular axes to define direction.

Once you have chosen a coordinate system, you must decide where to place it. For example, imagine that you were studying the motion of objects inside a car. You might begin by gluing metre sticks to the inside of the vehicle so you could precisely express the positions of passengers and objects relative to an origin. You might choose the centre of the rearview mirror as the origin and then you could locate any object by finding its height above or below the origin, its distance left or right of the origin, and its position in front of or behind the origin. The metre sticks would define a coordinate system for measurements within the car, as shown in Figure 1.2. The car itself could be called the **frame of reference** for the measurements. Coordinate systems are always attached to or located on a frame of reference.

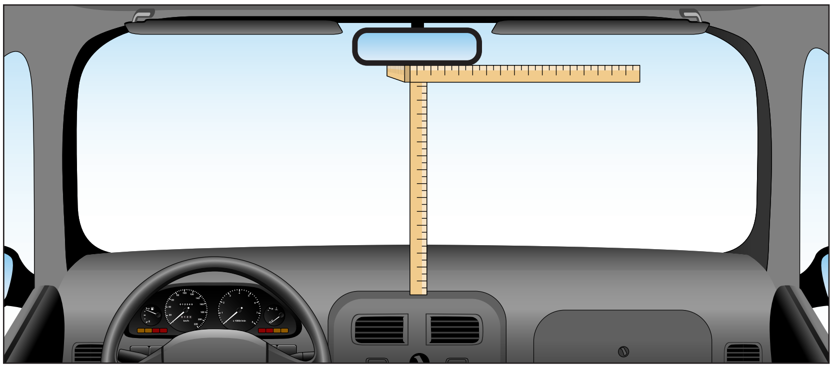


Figure 1.2 Establishing a coordinate system and defining a frame of reference are fundamental steps in motion experiments.

An observer in the car's frame of reference might describe the motion of a person in the car by stating that "The passenger did not move during the entire trip." An observer who chose Earth's surface as a frame of reference, however, would describe the passenger's motion quite differently: "During the trip, the passenger moved 12.86 km." Clearly, descriptions of motion depend very much on the chosen frame of reference. Is there a right or wrong way to choose a frame of reference?

The answer to the above question is no, there is no right or wrong choice for a frame of reference. However, some frames of reference make calculations and predictions much easier than do others. Think again about the coordinate system in the car. Imagine that you are riding along a straight, smooth road at a constant velocity. You are almost unaware of any motion. Then

COURSE CHALLENGE

Reference Frames

A desire to know your location on Earth has made GPS receivers very popular. Discussion about location requires the use of frames of reference concepts. Ideas about frames of reference and your *Course Challenge* are cued on page 603 of this text.

PHYSICS FILE

Albert Einstein used the equivalence of inertial and gravitational mass as a foundation of his general theory of relativity, published in 1916. According to Einstein's principle of equivalence, if you were in a laboratory from which you could not see outside, you could not make any measurements that would indicate whether the laboratory (your frame of reference) was stationary on Earth's surface or in space and accelerating at a value that was locally equal to *g*.

the driver suddenly slams on the brakes and your upper body falls forward until the seat belt stops you. In the frame of reference of the car, you were initially at rest and then suddenly began to accelerate.

According to Newton's first law, a force is necessary to cause a mass — your body — to accelerate. However, in this situation you cannot attribute your acceleration to any observable force: No object has exerted a force on you. The seat belt stopped your motion relative to the car, but what started your motion? It would appear that your motion relative to the car did not conform to Newton's laws.

The two stages of motion during the ride in a car — moving with a constant velocity or accelerating — illustrate two classes of frames of reference. A frame of reference that is at rest or moving at a constant velocity is called an **inertial frame of reference**.

When you are riding in a car that is moving at a constant velocity, motion inside the car seems similar to motion inside a parked car or even in a room in a building. In fact, imagine that you are in a laboratory inside a truck's semitrailer and you cannot see what is happening outside. If the truck and trailer ran perfectly smoothly, preventing you from feeling any bumps or vibrations, there are no experiments that you could conduct that would allow you to determine whether the truck and trailer were at rest or moving at a constant velocity. The law of inertia and Newton's second and third laws apply in exactly the same way in all inertial frames of reference.

Now think about the point at which the driver of the car abruptly applied the brakes and the car began to slow. The velocity was changing, so the car was accelerating. An accelerating frame of reference is called a **non-inertial frame of reference**. Newton's laws of motion do *not* apply to a non-inertial frame of reference. By observing the motion of the car and its occupant from outside the car (that is, from an inertial frame of reference, as shown in Figure 1.3), you can see why the law of inertia cannot apply.

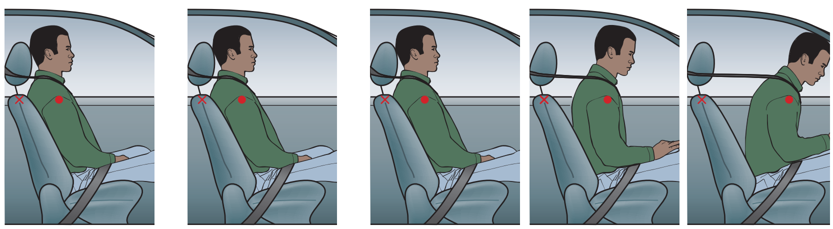


Figure 1.3 The crosses on the car seat and the dots on the passenger's shoulder represent the changing locations of the car and the passenger at equal time intervals. In the first three frames, the distances are equal, indicating that the car and passenger are moving at the same velocity. In the last two frames, the crosses are closer together, indicating that the car is slowing. The passenger, however, continues to move at the same velocity until stopped by a seat belt.

In the first three frames, the passenger's body and the car are moving at the same velocity, as shown by the cross on the car seat and the dot on the passenger's shoulder. When the car first begins to slow, no force has yet acted on the passenger. Therefore, his body continues to move with the same constant velocity until a force, such as a seat belt, acts on him. When you are a passenger, you feel as though you are being thrown forward. In reality, the car has slowed down but, due to its own inertia, your body tries to continue to move with a constant velocity.

Since a change in direction is also an acceleration, the same situation occurs when a car turns. You feel as though you are being pushed to the side, but in reality, your body is attempting to continue in a straight line, while the car is changing its direction.

INERTIAL AND NON-INERTIAL FRAMES OF REFERENCE

An inertial frame of reference is one in which Newton's first and second laws are valid. Inertial frames of reference are at rest or in uniform motion, but they are not accelerating.

A non-inertial frame of reference is one in which Newton's first and second laws are not valid. Accelerating frames of reference are always non-inertial.

Clearly, in most cases, it is easier to work in an inertial frame of reference so that you can use Newton's laws of motion. However, if a physicist chooses to work in a non-inertial frame of reference and still apply Newton's laws of motion, it is necessary to invoke hypothetical quantities that are often called **fictitious forces:** inertial effects that are perceived as "forces" in non-inertial frames of reference, but do not exist in inertial frames of reference.

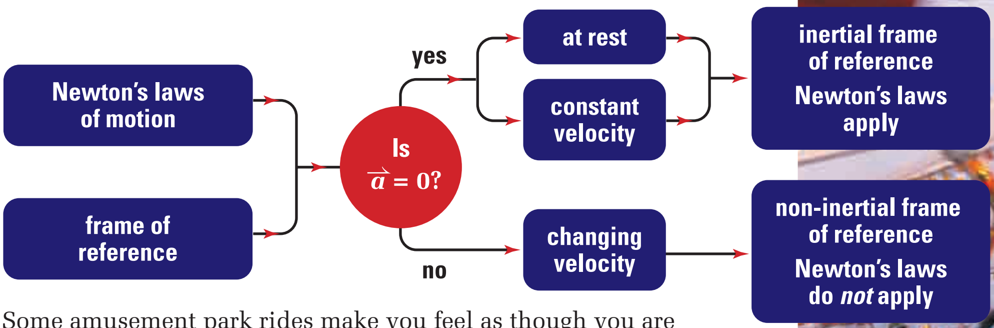
Conceptual Problem

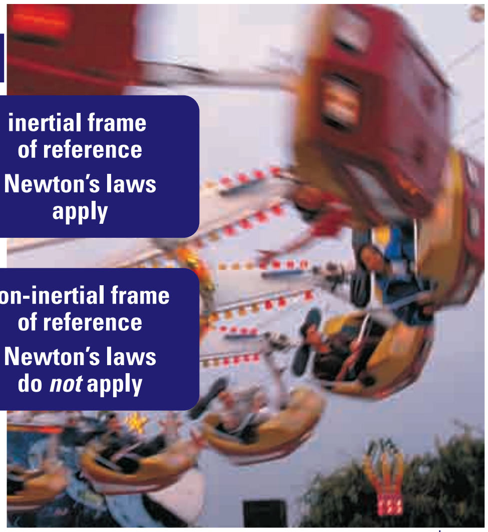
- Passengers in a high-speed elevator feel as though they are being pressed heavily against the floor when the elevator starts moving up. After the elevator reaches its maximum speed, the feeling disappears.
  - **(a)** When do the elevator and passengers form an inertial frame of reference? A non-inertial frame of reference?
  - **(b)** Before the elevator starts moving, what forces are acting on the passengers? How large is the external (unbalanced) force? How do you know?
  - **(c)** Is a person standing outside the elevator in an inertial or non-inertial frame of reference?
  - **(d)** Suggest the cause of the pressure the passengers feel when the elevator starts to move upward. Sketch a free-body diagram to illustrate your answer.
  - **(e)** Is the pressure that the passengers feel in part (d) a fictitious force? Justify your answer.

PHYSICS FILE

Earth and everything on it are in continual circular motion. Earth is rotating on its axis, travelling around the Sun and circling the centre of the galaxy along with the rest of the solar system. The direction of motion is constantly changing, which means the motion is accelerated. Earth is a non-inertial frame of reference, and large-scale phenomena such as atmospheric circulation are greatly affected by Earth's continual acceleration. In laboratory experiments with moving objects, however, the effects of Earth's rotation are usually not detectable.

Concept Organizer





Some amusement park rides make you feel as though you are being thrown to the side, although no force is pushing you outward from the centre. Your frame of reference is moving rapidly along a curved path and therefore it is accelerating. You are in a non-inertial frame of reference, so it seems as though your motion is not following Newton's laws of motion.

Figure 1.4 You can determine the nature of a frame of reference by analyzing its acceleration.

1.1 Section Review

1. **K/U** State Newton's first law in two different ways.

2. **C** Identify the two basic situations that Newton's first law describes and explain how one statement can cover both situations.

3. **K/U** State Newton's second law in words and symbols.

4. **MC** A stage trick involves covering a table with a smooth cloth and then placing dinnerware on the cloth. When the cloth is suddenly pulled horizontally, the dishes "magically" stay in position and drop onto the table.
   - **(a)** Identify all forces acting on the dishes during the trick.
   - **(b)** Explain how inertia and frictional forces are involved in the trick.

5. **K/U** Give an example of an unusual frame of reference used in a movie or a television program. Suggest why this viewpoint was chosen.

6. **K/U** Identify the defining characteristic of inertial and non-inertial frames of reference. Give an example of each type of frame of reference.

7. **C** In what circumstances is it necessary to invoke ficticious forces in order to explain motion? Why is this term appropriate to describe these forces?

8. **C** Compare inertial mass and gravitational mass, giving similarities and differences.

9. **C** Why do physicists, who take pride in precise, unambiguous terminology, usually speak just of "mass," rather than distinguishing between inertial and gravitational mass?

UNIT PROJECT PREP

- What frame of reference would be the best choice for measuring and analyzing the performance of your catapult?
- What forces will be acting on the payload of your catapult when it is being accelerated? When it is flying through the air?
- How will the inertia of the payload affect its behaviour? How will the mass of the payload affect its behaviour?

Test your ideas using a simple elastic band or slingshot.

CAUTION Take appropriate safety precautions before any tests. Use eye protection.

1.2 Analyzing Motion

The deafening roar of the engine of a competitor's tractor conveys the magnitude of the force that is applied to the sled in a tractor-pull contest. As the sled begins to move, weights shift to increase frictional forces. Despite the power of their engines, most tractors are slowed to a standstill before reaching the end of the 91 m track. In contrast to the brute strength of the tractors, dragsters "sprint" to the finish line. Many elements of the two situations are identical, however, since forces applied to masses change the linear (straight-line) motion of a vehicle.

In the previous section, you focussed on basic **dynamics** — the cause of changes in motion. In this section, you will analyze **kinematics** — the motion itself — in more detail. You will consider objects moving horizontally in straight lines.

SECTION EXPECTATIONS

- Analyze, predict, and explain linear motion of objects in horizontal planes.
- Analyze experimental data to determine the net force acting on an object and its resulting motion.

KEY TERMS

- dynamics
- kinematics
- uniform motion
- uniformly accelerated motion
- free-body diagram
- frictional forces
- coefficient of static friction
- coefficient of kinetic friction

Kinematic Equations

To analyze the motion of objects quantitatively, you will use the kinematic equations (or equations of motion) that you learned in previous courses. The two types of motion that you will analyze are **uniform motion** — motion with a constant velocity — and **uniformly accelerated motion** — motion under constant acceleration. When you use these equations, you will apply them to only one dimension at a time. Therefore, vector notations will not be necessary, because positive and negative signs are all that you will need to indicate direction. The kinematic equations are summarized on the next page, and apply only to the type of motion indicated.

Figure 1.5 In a tractor pull, vehicles develop up to 9000 horsepower to accelerate a sled, until they can no longer overcome the constantly increasing frictional forces. Dragsters, on the other hand, accelerate right up to the finish line.

Uniform motion

- definition of velocity $v = \frac{\Delta d}{\Delta t}$
- Solve for displacement in terms of velocity and time. $\Delta d = v\Delta t$

Uniformly accelerated motion

- definition of acceleration $a = \frac{\Delta v}{\Delta t}$ or $a = \frac{v_2 - v_1}{\Delta t}$
- Solve for final velocity in terms of initial velocity, acceleration, and time interval. $v_2 = v_1 + a\Delta t$
- displacement in terms of initial velocity, final velocity, and time interval $\Delta d = \frac{(v_1 + v_2)}{2}\Delta t$
- displacement in terms of initial velocity, acceleration, and time interval $\Delta d = v_1\Delta t + \frac{1}{2}a\Delta t^2$
- final velocity in terms of initial velocity, acceleration, and displacement $v_2^2 = v_1^2 + 2a\Delta d$

Conceptual Problem

- The equations above are the most fundamental kinematic equations. You can derive many more equations by making combinations of the above equations. For example, it is sometimes useful to use the relationship $\Delta d = v_2\Delta t - \frac{1}{2}a\Delta t^2$. Derive this equation by manipulating two or more of the equations above. (Hint: Notice that the equation you need to derive is very similar to one of the equations in the list, with the exception that it has the final velocity instead of the initial velocity. What other equation can you use to eliminate the initial velocity from the equation that is similar to the desired equation?)

Combining Dynamics and Kinematics

When analyzing motion, you often need to solve a problem in two steps. You might have information about the forces acting on an object, which you would use to find the acceleration. In the next step, you would use the acceleration that you determined in order to calculate some other property of the motion. In other cases, you might analyze the motion to find the acceleration and then use the acceleration to calculate the force applied to a mass. The following sample problem will illustrate this process.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner to enhance your understanding of acceleration and velocity.

SAMPLE PROBLEM

Finding Velocity from Dynamics Data

In television picture tubes and computer monitors (cathode ray tubes), light is produced when fast-moving electrons collide with phosphor molecules on the surface of the screen. The electrons (mass 9.1×10^{-31} kg) are accelerated from rest in the electron "gun" at the back of the vacuum tube. Find the velocity of an electron when it exits the gun after experiencing an electric force of 5.8×10^{-15} N over a distance of 3.5 mm.

Conceptualize the Problem

- The electrons are moving *horizontally*, from the back to the front of the tube, under an *electric force*.
- The *force of gravity* on an electron is exceedingly small, due to the electron's small mass. Since the electrons move so quickly, the time interval of the entire flight is very short. Therefore, the *effect of the force of gravity is too small to be detected* and you can consider the electric force to be the only force affecting the electrons.
- Information about *dynamics data* allows you to find the electrons' *acceleration*.
- Each electron is *initially at rest*, meaning that the *initial velocity is zero*.
- Given the acceleration, the equations of motion lead to other variables of motion.
- Let the direction of the force, and therefore the direction of the acceleration, be positive.

Identify the Goal

The final velocity, v_2, of an electron when exiting the electron gun

Identify the Variables and Constants

Known	Implied	Unknown
$m_e = 9.1 \times 10^{-31}$ kg	$v_1 = 0\ \frac{\text{m}}{\text{s}}$	a
$F = 5.8 \times 10^{-15}$ N		v_2
$\Delta d = 3.5 \times 10^{-3}$ m		

Develop a Strategy

Apply Newton's second law to find the net force.

$$\vec{F} = m\vec{a}$$

Write Newton's second law in terms of acceleration.

$$\vec{a} = \frac{\vec{F}}{m}$$

Substitute and solve.

$$\vec{a} = \frac{+5.8 \times 10^{-15}\ \text{N}}{9.1 \times 10^{-31}\ \text{kg}}$$

$\frac{\text{N}}{\text{kg}}$ is equivalent to $\frac{\text{m}}{\text{s}^2}$.

$$\vec{a} = 6.374 \times 10^{15}\ \frac{\text{m}}{\text{s}^2}\text{[toward the front of tube]}$$

continued ►

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Apply the kinematic equation that relates initial velocity, acceleration, and displacement to final velocity.

$$v_2^2 = v_1^2 + 2a\Delta d$$
$$v_2^2 = 0 + 2\left(6.374 \times 10^{15}\ \frac{\text{m}}{\text{s}^2}\right)(3.5 \times 10^{-3}\ \text{m})$$
$$v_2 = 6.67\,967 \times 10^{6}\ \frac{\text{m}}{\text{s}}$$
$$v_2 \cong 6.7 \times 10^{6}\ \frac{\text{m}}{\text{s}}$$

The final velocity of the electrons is about 6.7×10^6 m/s in the direction of the applied force.

Validate the Solution

Electrons, with their very small inertial mass, could be expected to reach high speeds. You can also solve the problem using the concepts of work and energy that you learned in previous courses. The work done on the electrons was converted into kinetic energy, so $W = F\Delta d = \frac{1}{2}mv^2$. Therefore,

$$v = \sqrt{\frac{2F\Delta d}{m}} = \sqrt{\frac{2(5.8 \times 10^{-15}\ \text{N})(3.5 \times 10^{-3}\ \text{m})}{9.1 \times 10^{-31}\ \text{kg}}} = 6.679 \times 10^{6}\ \frac{\text{m}}{\text{s}} \cong 6.7 \times 10^{6}\ \frac{\text{m}}{\text{s}}.$$

Obtaining the same answer by two different methods is a strong validation of the results.

PRACTICE PROBLEMS

1. A linear accelerator accelerated a germanium ion ($m = 7.2 \times 10^{-25}$ kg) from rest to a velocity of 7.3×10^6 m/s over a time interval of 5.5×10^{-6} s. What was the magnitude of the force that was required to accelerate the ion?

2. A hockey stick exerts an average force of 39 N on a 0.20 kg hockey puck over a displacement of 0.22 m. If the hockey puck started from rest, what is the final velocity of the puck? Assume that the friction between the puck and the ice is negligible.

Determining the Net Force

In almost every instance of motion, more than one force is acting on the object of interest. To apply Newton's second law, you need to find the resultant force. A free-body diagram is an excellent tool that will help to ensure that you have correctly identified and combined the forces.

To draw a **free-body diagram**, start with a dot that represents the object of interest. Then draw one vector to represent each force acting on the object. The tails of the vector arrows should all start at the dot and indicate the direction of the force, with the arrowhead pointing away from the dot. Study Figure 1.6 to see how a free-body diagram is constructed. Figure 1.6 (A) illustrates a crate being pulled across a floor by a rope attached to the edge of the crate. Figure 1.6 (B) is a free-body diagram representing the forces acting on the crate.

Two of the most common types of forces that influence the motion of familiar objects are frictional forces and the force of gravity. You will probably recall from previous studies that the magnitude of the force of gravity acting on objects on or near Earth's surface can be expressed as $F = mg$, where g (which is often called the acceleration due to gravity) has a value 9.81 m/s^2. Near Earth's surface, the force of gravity always points toward the centre of Earth.

Whenever two surfaces are in contact, **frictional forces** oppose any motion between them. Therefore, the direction of the frictional force is always opposite to the direction of the motion. You might recall from previous studies that the magnitudes of frictional forces can be calculated by using the equation $F_f = \mu F_N$. The normal force in this relationship (F_N) is the force perpendicular to the surfaces in contact. You might think of the normal force as the force that is pressing the two surfaces together. The nature of the surfaces and their relative motion determines the value of the coefficient of friction (μ). These values must be determined experimentally. Some typical values are listed in Table 1.1.

Table 1.1 Coefficients of Friction for Some Common Surfaces

Surface	Coefficient of static friction (μ_s)	Coefficient of kinetic friction (μ_k)
rubber on dry, solid surfaces	1–4	1
rubber on dry concrete	1.00	0.80
rubber on wet concrete	0.70	0.50
glass on glass	0.94	0.40
steel on steel (unlubricated)	0.74	0.57
steel on steel (lubricated)	0.15	0.06
wood on wood	0.40	0.20
ice on ice	0.10	0.03
Teflon™ on steel in air	0.04	0.04
ball bearings (lubricated)	<0.01	<0.01
joint in humans	0.01	0.003

If the objects are not moving relative to each other, you would use the **coefficient of static friction** (μ_s). If the objects are moving, the somewhat smaller **coefficient of kinetic friction** (μ_k) applies to the motion.

As you begin to solve problems involving several forces, you will be working in one dimension at a time. You will select a coordinate system and resolve the forces into their components in each dimension. Note that the components of a force are not vectors themselves. Positive and negative signs completely describe the motion in one dimension. Thus, when you apply Newton's laws to the components of the forces in one dimension, you will not use vector notations.

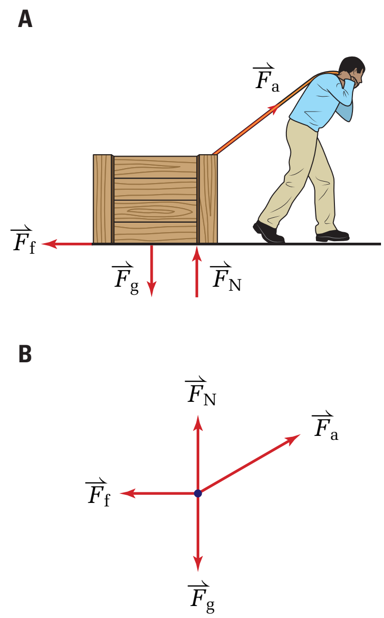


Figure 1.6 **(A)** The forces of gravity ($\vec{F}_g$), friction ($\vec{F}_f$), the normal force of the floor ($\vec{F}_N$), and the applied force of the rope ($\vec{F}_a$) all act on the crate at the same time. **(B)** The free-body diagram includes *only* those forces acting *on* the crate and none of the forces that the crate exerts on other objects.

ELECTRONIC LEARNING PARTNER

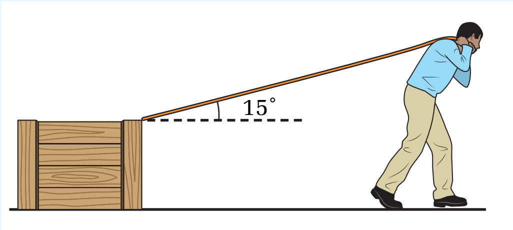
Refer to your Electronic Learning Partner to enhance your understanding of forces and vectors.

Another convention used in this textbook involves writing the sum of all of the forces in one dimension. In the first step, when the forces are identified as, for example, gravitational, frictional, or applied, only plus signs will be used. Then, when information about that specific force is inserted into the calculation, a positive or negative sign will be included to indicate the direction of that specific force. Watch for these conventions in sample problems.

SAMPLE PROBLEM

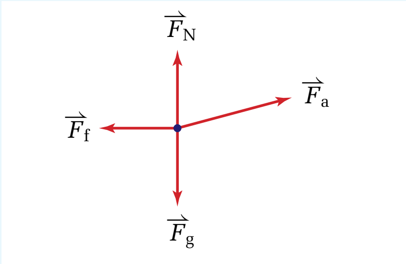
Working with Three Forces

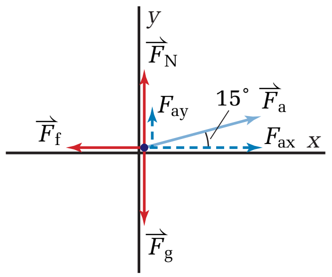
To move a 45 kg wooden crate across a wooden floor (μ = 0.20), you tie a rope onto the crate and pull on the rope. While you are pulling the rope with a force of 115 N, it makes an angle of 15° with the horizontal. How much time elapses between the time at which the crate just starts to move and the time at which you are pulling it with a velocity of 1.4 m/s?



Conceptualize the Problem

- To start framing this problem, draw a free-body diagram.
- *Motion* is in the horizontal direction, so the net *horizontal force* is causing the crate to *accelerate*.
- Let the *direction* of the motion be the *positive* horizontal direction.
- There is no motion in the vertical direction, so the *vertical acceleration is zero*. If the acceleration is zero, the *net vertical force* must be *zero*. This information leads to the value of the *normal force*. Let "up" be the *positive* vertical direction.
- Since the beginning of the time interval in question is the instant at which the crate begins to move, the *coefficient of kinetic friction* applies to the motion.
- Once the *acceleration* is found, the *kinematic equations* allow you to determine the values of other quantities involved in the motion.





Identify the Goal

The time, Δt, required to reach a velocity of 1.4 m/s

Identify the Variables

Known		Implied	Unknown	
$\vec{F}_a = +115\ \text{N}$	$m = 45\ \text{kg}$	$v_i = 0\ \frac{\text{m}}{\text{s}}$	$\vec{F}_N$	$\vec{a}$
$\theta = 15°$	$v_f = 1.4\ \frac{\text{m}}{\text{s}}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_g$	Δx
$\mu = 0.20$			$\vec{F}_f$	

Develop a Strategy

To find the normal force, apply Newton's second law to the vertical forces. Analyze the free-body diagram to find all of the vertical forces that act on the crate.

$$\vec{F} = m\vec{a}$$
$$F_{a(vertical)} + F_g + F_N = ma$$
$$F_g = -mg$$
$$F_{a(vertical)} - mg + F_N = ma$$
$$F_N = ma + mg - F_{a(vertical)}$$
$$F_N = 0 + (45\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) - (115\ \text{N})\sin 15^\circ$$
$$F_N = 441.45\ \text{N} - 29.76\ \text{N}$$
$$F_N = 411.69\ \text{N}$$

To find the acceleration, apply Newton's second law to the horizontal forces. Analyze the free-body diagram to find all of the horizontal forces that act on the crate.

$$\vec{F} = m\vec{a}$$
$$F_{a(horizontal)} + F_f = ma$$
$$F_f = -\mu F_N$$
$$a = \frac{F_{a(horizontal)} - \mu F_N}{m}$$
$$a = \frac{(115\ \text{N})\cos 15^\circ - (0.20)(411.69\ \text{N})}{45\ \text{kg}}$$
$$a = \frac{111.08\ \text{N} - 82.34\ \text{N}}{45\ \text{kg}}$$
$$a = 0.6387\ \frac{\text{m}}{\text{s}^2}$$

To find the time interval, use the kinematic equation that relates acceleration, initial velocity, final velocity, and time.

$$a = \frac{v_f - v_i}{\Delta t}$$
$$\Delta t = \frac{v_f - v_i}{a}$$
$$\Delta t = \frac{1.4\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}}}{0.6387\ \frac{\text{m}}{\text{s}^2}}$$
$$\Delta t = 2.19\ \text{s}$$
$$\Delta t \cong 2.2\ \text{s}$$

You will be pulling the crate at 1.4 m/s at 2.2 s after the crate begins to move.

Validate the Solution

Check the units for acceleration: $\frac{\text{N}}{\text{kg}} = \frac{\frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}}{\cancel{\text{kg}}} = \frac{\text{m}}{\text{s}^2}$. The units are correct. A velocity of 1.4 m/s is not very fast, so you would expect that the time interval required to reach that velocity would be short. The answer of 2.2 s is very reasonable.

PRACTICE PROBLEMS

3. In a tractor-pull competition, a tractor applies a force of 1.3 kN to the sled, which has mass 1.1×10^4 kg. At that point, the coefficient of kinetic friction between the sled and the ground has increased to 0.80. What is the acceleration of the sled? Explain the significance of the sign of the acceleration.

4. A curling stone with mass 20.0 kg leaves the curler's hand at a speed of 0.885 m/s. It slides 31.5 m down the rink before coming to rest.

(a) Find the average force of friction acting on the stone.

(b) Find the coefficient of kinetic friction between the ice and the stone.

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5. Pushing a grocery cart with a force of 95 N, applied at an angle of 35° down from the horizontal, makes the cart travel at a constant speed of 1.2 m/s. What is the frictional force acting on the cart?

6. A man walking with the aid of a cane approaches a skateboard (mass 3.5 kg) lying on the sidewalk. Pushing with an angle of 60° down from the horizontal with his cane, he applies a force of 115 N, which is enough to roll the skateboard out of his way.
   (a) Calculate the horizontal force acting on the skateboard.
   (b) Calculate the initial acceleration of the skateboard.

7. A mountain bike with mass 13.5 kg, with a rider having mass 63.5 kg, is travelling at 32 km/h when the rider applies the brakes, locking the wheels. How far does the bike travel before coming to a stop if the coefficient of friction between the rubber tires and the asphalt road is 0.60?

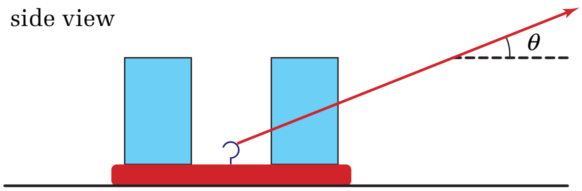
QUICK LAB

Best Angle for Pulling a Block

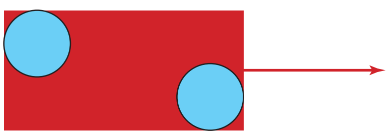
TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

Set two 500 g masses on a block of wood. Attach a rope and drag the block along a table. If the rope makes a steeper angle with the surface, friction will be reduced (why?) and the block will slide more easily. Predict the angle at which the block will move with least effort. Attach a force sensor to the rope and measure the force needed to drag the block at a constant speed at a variety of different angles. Graph your results to test your prediction.

side view



top view



Analyze and Conclude

1. Identify from your graph the "best" angle at which to move the block.
2. How close did your prediction come to the experimental value?
3. Identify any uncontrolled variables in the experiment that could be responsible for some error in your results.
4. In theory, the "best" angle is related to the coefficient of static friction between the surface and the block: $\tan \theta_{best} = \mu_s$. Use your results to calculate the coefficient of static friction between the block and the table.
5. What effect does the horizontal component of the force have on the block? What effect does the vertical component have on the block?
6. Are the results of this experiment relevant to competitors in a tractor pull, such as the one described in the text and photograph caption at the beginning of this section? Explain your answer in detail.

Applying Newton's Third Law

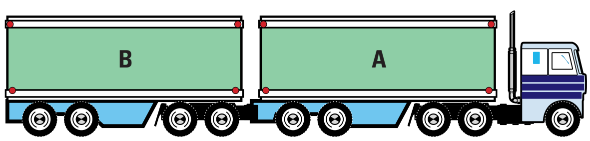
Examine the photograph of the tractor-trailer in Figure 1.7 and think about all of the forces exerted on each of the three sections of the vehicle. Automotive engineers must know how much force each trailer hitch needs to withstand. Is the hitch holding the second trailer subjected to as great a force as the hitch that attaches the first trailer to the truck?

Figure 1.7 This truck and its two trailers move as one unit. The velocity and acceleration of each of the three sections are the same. However, each section is experiencing a different net force.

To analyze the individual forces acting on each part of a train of objects, you need to apply Newton's third law to determine the force that each section exerts on the adjacent section. Study the following sample problem to learn how to determine all of the forces on the truck and on each trailer. These techniques will apply to any type of train problem in which the first of several sections of a moving set of objects is pulling all of the sections behind it.

SAMPLE PROBLEM

Forces on Connected Objects



A tractor-trailer pulling two trailers starts from rest and accelerates to a speed of 16.2 km/h in 15 s on a straight, level section of highway. The mass of the truck itself (T) is 5450 kg, the mass of the first trailer (A) is 31 500 kg, and the mass of the second trailer (B) is 19 600 kg. What magnitude of force must the truck generate in order to accelerate the entire vehicle? What magnitude of force must each of the trailer hitches withstand while the vehicle is accelerating? (Assume that frictional forces are negligible in comparison with the forces needed to accelerate the large masses.)

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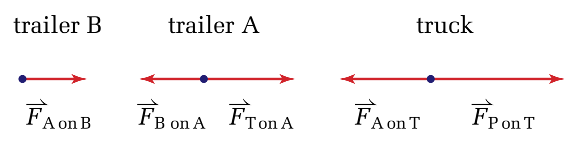
Conceptualize the Problem

- The truck engine generates energy to turn the wheels. When the wheels turn, they exert a *frictional force on the pavement*. According to Newton's third law, the *pavement* exerts a reaction *force* that is equal in magnitude and opposite in direction to the force exerted by the tires. The force of the pavement on the truck tires, $\vec{F}_{\text{P on T}}$, *accelerates* the entire system.
- The truck exerts a *force* on trailer A. According to Newton's third law, the trailer exerts a *force of equal magnitude* on the truck.
- Trailer A exerts a *force* on trailer B, and trailer B therefore must exert a force of equal magnitude on trailer A.
- Summarize all of the *forces* by drawing *free-body diagrams* of each section of the vehicle.



- The *kinematic equations* allow you to calculate the *acceleration* of the system.
- Since each section of the system has the *same acceleration*, this value, along with the masses and *Newton's second law*, lead to all of the *forces*.
- Since the motion is in a straight line and the question asks for only the magnitudes of the forces, vector notations are not needed.

Identify the Goal

The force, $F_{\text{P on T}}$, that the pavement exerts on the truck tires; the force, $F_{\text{T on A}}$, that the truck exerts on trailer A; the force, $F_{\text{A on B}}$, that trailer A exerts on trailer B

Identify the Variables

Known

$v_f = 16.2\ \frac{\text{km}}{\text{h}}$ $\quad m_T = 5450\text{ kg}$

$\Delta t = 15\text{ s}$ $\quad m_A = 31\,500\text{ kg}$

$m_B = 19\,600\text{ kg}$

Implied

$v_i = 0\ \frac{\text{km}}{\text{h}}$

Unknown

a $\quad F_{\text{T on A}}$ $\quad F_{\text{A on T}}$ $\quad m_{\text{total}}$

$F_{\text{P on T}}$ $\quad F_{\text{A on B}}$ $\quad F_{\text{B on A}}$

Develop a Strategy

Use the kinematic equation that relates the initial velocity, final velocity, time interval, and acceleration to find the acceleration.

$$a = \frac{v_2 - v_1}{\Delta t}$$

$$a = \frac{\left(16.2\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} - 0\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{1\ \cancel{\text{km}}}\right)}{15\text{ s}}$$

$$a = 0.30\ \frac{\text{m}}{\text{s}^2}$$

Find the total mass of the truck plus trailers.

$$m_{\text{total}} = m_T + m_A + m_B$$

$$m_{\text{total}} = 5450\text{ kg} + 31\,500\text{ kg} + 19\,600\text{ kg}$$

$$m_{\text{total}} = 56\,550\text{ kg}$$

Use Newton's second law to find the force required to accelerate the total mass. This will be the force that the pavement must exert on the truck tires.

$$\vec{F} = m\vec{a}$$

$$F_{\text{P on T}} = (56\,550\text{ kg})\left(0.30\ \frac{\text{m}}{\text{s}^2}\right)$$

$$F_{\text{P on T}} = 16\,965\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

$$F_{\text{P on T}} \cong 1.7 \times 10^4\text{ N}$$

The pavement exerts 1.7×10^4 N on the truck tires.

Use Newton's second law to find the force necessary to accelerate trailer B at 0.30 m/s². This is the force that the second trailer hitch must withstand.

$$F_{\text{A on B}} = m_{\text{B}}a$$
$$F_{\text{A on B}} = (19\ 600\ \text{kg})\left(0.30\ \frac{\text{m}}{\text{s}^2}\right)$$
$$F_{\text{A on B}} = 5.88 \times 10^3\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$F_{\text{A on B}} \cong 5.9 \times 10^3\ \text{N}$$

The force that the second hitch must withstand is 5.9×10^3 N.

Use Newton's second law to find the total force necessary to accelerate trailer A at 0.30 m/s².

$$F_{\text{total on A}} = m_{\text{A}}a$$
$$F_{\text{total on A}} = (31\ 500\ \text{kg})\left(0.30\ \frac{\text{m}}{\text{s}^2}\right)$$
$$F_{\text{total on A}} = 9.45 \times 10^3\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$F_{\text{total on A}} \cong 9.5 \times 10^3\ \text{N}$$

Use the free-body diagram to help write the expression for total (horizontal) force on trailer A.

$$F_{\text{total}} = F_{\text{T on A}} + F_{\text{B on A}}$$

The force that the first hitch must withstand is the force that the truck exerts on trailer A. Solve the force equation above for $F_{\text{T on A}}$ and calculate the value. According to Newton's third law, $F_{\text{B on A}} = -F_{\text{A on B}}$.

$$F_{\text{T on A}} = F_{\text{total on A}} - F_{\text{B on A}}$$
$$F_{\text{T on A}} = 9.45 \times 10^3\ \text{N} - (-5.88 \times 10^3\ \text{N})$$
$$F_{\text{T on A}} = 1.533 \times 10^4\ \text{N}$$
$$F_{\text{T on A}} \cong 1.5 \times 10^4\ \text{N}$$

The force that the first hitch must withstand is 1.5×10^4 N.

Validate the Solution

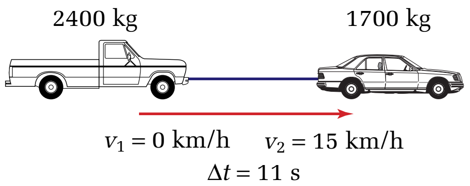
You would expect that $F_{\text{P on T}} > F_{\text{T on A}} > F_{\text{A on B}}$. The calculated forces agree with this relationship. You would also expect that the force exerted by the tractor on trailer A would be the force necessary to accelerate the sum of the masses of trailers A and B at 0.30 m/s².

$$F_{\text{T on A}} = (31\ 500\ \text{kg} + 19\ 600\ \text{kg})\left(0.30\ \frac{\text{m}}{\text{s}^2}\right) = 15\ 330\ \text{N} \cong 1.5 \times 10^4\ \text{N}$$

This value agrees with the value above.

PRACTICE PROBLEMS

8. A 1700 kg car is towing a larger vehicle with mass 2400 kg. The two vehicles accelerate uniformly from a stoplight, reaching a speed of 15 km/h in 11 s. Find the force needed to accelerate the connected vehicles, as well as the minimum strength of the rope between them.



9. An ice skater pulls three small children, one behind the other, with masses 25 kg, 31 kg, and 35 kg. Assume that the ice is smooth enough to be considered frictionless.

(a) Find the total force applied to the "train" of children if they reach a speed of 3.5 m/s in 15 s.

(b) If the skater is holding onto the 25 kg child, find the tension in the arms of the next child in line.

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10. A solo Arctic adventurer pulls a string of two toboggans of supplies across level, snowy ground. The toboggans have masses of 95 kg and 55 kg. Applying a force of 165 N causes the toboggans to accelerate at 0.61 m/s^2.

(a) Calculate the frictional force acting on the toboggans.

(b) Find the tension in the rope attached to the second (55 kg) toboggan.

1.2 Section Review

1. K/U How is direction represented when analyzing linear motion?

2. K/U When you pull on a rope, the rope pulls back on you. Describe how the rope creates this reaction force.

3. K/U Explain how to calculate

(a) the horizontal component (F_x) of a force F

(b) the vertical component (F_y) of a force F

(c) the coefficient of friction (μ) between two surfaces

(d) the gravitational force (F_g) acting on an object

4. K/U Define (a) a *normal* force and (b) the weight of an object.

5. K/U An object is being propelled horizontally by a force F. If the force doubles, use Newton's second law and kinematic equations to determine the change in

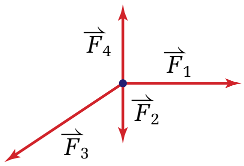
(a) the acceleration of the object

(b) the velocity of the object after 10 s

6. K/U A 0.30 kg lab cart is observed to accelerate twice as fast as a 0.60 kg cart. Does that mean that the net force on the more massive cart is twice as large as the force on the smaller cart? Explain.

7. K/U A force F produces an acceleration a when applied to a certain body. If the mass of the body is doubled and the force is increased fivefold, what will be the effect on the acceleration of the body?

8. K/U An object is being acted on by forces pictured in the diagram.



(a) Could the object be accelerating horizontally? Explain.

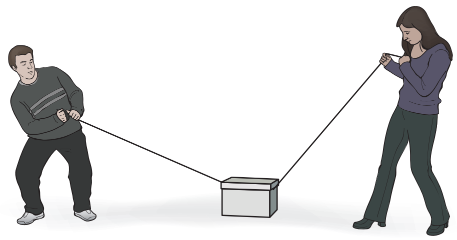
(b) Could the object be moving horizontally? Explain.

9. C Three identical blocks, fastened together by a string, are pulled across a frictionless surface by a constant force, F.

(a) Compare the tension in string A to the magnitude of the applied force, F.

(b) Draw a free-body diagram of the forces acting on block 2.

10. K/U A tall person and a short person pull on a load at different angles but with equal force, as shown.



(a) Which person applies the greater *horizontal* force to the load? What effect does this have on the motion of the load?

(b) Which person applies the greater *vertical* force to the load? What effect does this have on frictional forces? On the motion of the load?

1.3 Vertical Motion

Catapulting a diver high into the air requires a force. How large a force? How hard must the board push up on the diver to overcome her weight and accelerate her upward? After the diver leaves the board, how long will it take before her ascent stops and she turns and plunges toward the water? In this section, you will investigate the dynamics of diving and other motions involving rising and falling or straight-line motion in a vertical plane.

SECTION EXPECTATIONS

- Analyze the motion of objects in vertical planes.
- Explain linear vertical motion in terms of forces.
- Solve problems and predict the motion of objects in vertical planes.

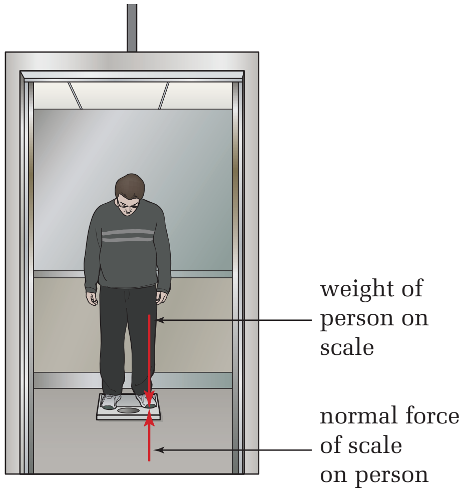
KEY TERMS

- apparent weight
- tension
- counterweight
- free fall
- air resistance
- terminal velocity

Figure 1.8 After the diver leaves the diving board and before she hits the water, the most important force acting on her is the gravitational force directed downward. Gravity affects all forms of vertical motion.

Weight versus Apparent Weight

One of the most common examples of linear vertical motion is riding in an elevator. You experience some strange sensations when the elevator begins to rise or descend or when it slows and comes to a stop. For example, if you get on at the first floor and start to go up, you feel heavier for a moment. In fact, if you are carrying a book bag or a suitcase, it feels heavier, too. When the elevator slows and eventually stops, you and anything you are carrying feels lighter. When the elevator is moving at a constant velocity, however, you feel normal. Are these just sensations that living organisms feel or, if you were standing on a scale in the elevator, would the scale indicate that you *were* heavier? You can answer that question by applying Newton's laws of motion to a person riding in an elevator.



Imagine that you are standing on a scale in an elevator, as shown in Figure 1.9. When the elevator is standing still, the reading on the scale is your weight. Recall that your weight is the force of gravity acting on your mass. Your weight can be calculated by using the equation $F_g = mg$, where g is the acceleration due to gravity. Vector notations are sometimes omitted because the force due to gravity is always directed toward the centre of Earth. Find out what happens to the reading on the scale by studying the following sample problem.

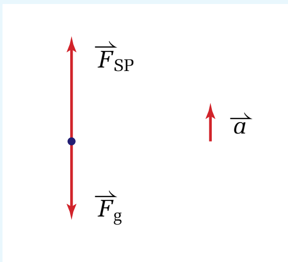
Figure 1.9 When you are standing on a scale, you exert a force on the scale. According to Newton's third law, the scale must exert an equal and opposite force on you. Therefore, the reading on the scale is equal to the force that you exert on it.

SAMPLE PROBLEM

Apparent Weight

A 55 kg person is standing on a scale in an elevator. If the scale is calibrated in newtons, what is the reading on the scale when the elevator is not moving? If the elevator begins to accelerate upward at 0.75 m/s², what will be the reading on the scale?

Conceptualize the Problem



- Start framing the problem by drawing a *free-body diagram* of the person on the scale. A free-body diagram includes all of the *forces acting on the person*.
- The *forces* acting on the person are *gravity* ($\vec{F}_g$) and the *normal force* of the scale.
- According to *Newton's third law*, when the person exerts a *force* ($\vec{F}_{PS}$) on the scale, it exerts an *equal and opposite force* ($\vec{F}_{SP}$) on the person. Therefore, the reading on the scale is the same as the force that the person exerts on the scale.
- When the elevator is *standing still*, the person's *acceleration is zero*.
- When the elevator begins to *rise*, the person is *accelerating* at the same rate as the elevator.
- Since the motion is in one dimension, use only positive and negative signs to indicate direction. Let "*up*" *be positive* and "*down*" *be negative*.
- Apply *Newton's second law* to find the magnitude of $\vec{F}_{SP}$.
- By *Newton's third law*, the magnitudes of $\vec{F}_{PS}$ and $\vec{F}_{SP}$ are equal to each other, and therefore to the reading on the scale.

Identify the Goal

The reading on the scale, $|\vec{F}_{SP}|$, when the elevator is standing still and when it is accelerating upward

Identify the Variables

Known	Implied	Unknown
$m = 55\text{ kg}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_{PS}$ $\vec{F}_{SP}$
$\vec{a} = +0.75\ \frac{\text{m}}{\text{s}^2}$		$\vec{F}_g$

Develop a Strategy

Apply Newton's second law and solve for the force that the scale exerts on the person. The force in Newton's second law is the vector sum of all of the forces acting *on* the person.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_g + \vec{F}_{SP} = m\vec{a}$$
$$\vec{F}_{SP} = -\vec{F}_g + m\vec{a}$$
$$\vec{F}_{SP} = -(-mg) + m\vec{a}$$

In the first part of the problem, the acceleration is zero.

$$\vec{F}_{SP} = (55\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + 0$$
$$\vec{F}_{SP} = 539.55\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$\vec{F}_{SP} \cong 5.4 \times 10^2\text{ N}$$

When the elevator is not moving, the reading on the scale is 5.4×10^2 N, which is the person's weight.

Apply Newton's second law to the case in which the elevator is accelerating upward. The acceleration is positive.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_g + \vec{F}_{SP} = m\vec{a}$$
$$\vec{F}_{SP} = -\vec{F}_g + m\vec{a}$$
$$\vec{F}_{SP} = -(-mg) + m\vec{a}$$
$$\vec{F}_{SP} = (55\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + (55\text{ kg})\left(+0.75\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_{SP} = 580.8\text{ N}$$
$$\vec{F}_{SP} \cong 5.8 \times 10^2\text{ N[up]}$$

When the elevator is accelerating upward, the reading on the scale is 5.8×10^2 N.

Validate the Solution

When an elevator first starts moving upward, it must exert a force that is greater than the person's weight so that, as well as supporting the person, an additional force causes the person to accelerate. The reading on the scale should reflect this larger force. It does. The acceleration of the elevator was small, so you would expect that the increase in the reading on the scale would not increase by a large amount. It increased by only about 7%.

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PRACTICE PROBLEMS

11. A 64 kg person is standing on a scale in an elevator. The elevator is rising at a constant velocity but then begins to slow, with an acceleration of 0.59 m/s^2. What is the sign of the acceleration? What is the reading on the scale while the elevator is accelerating?
12. A 75 kg man is standing on a scale in an elevator when the elevator begins to descend with an acceleration of 0.66 m/s^2. What is the direction of the acceleration? What is the reading on the scale while the elevator is accelerating?
13. A 549 N woman is standing on a scale in an elevator that is going down at a constant velocity. Then, the elevator begins to slow and eventually comes to a stop. The magnitude of the acceleration is 0.73 m/s^2. What is the direction of the acceleration? What is the reading on the scale while the elevator is accelerating?

As you saw in the problems, when you are standing on a scale in an elevator that is accelerating, the reading on the scale is not the same as your true weight. This reading is called your **apparent weight**.

When the direction of the acceleration of the elevator is positive — it starts to ascend or stops while descending — your apparent weight is greater than your true weight. You feel heavier because the floor of the elevator is pushing on you with a greater force than it is when the elevator is stationary or moving with a constant velocity.

When the direction of the acceleration is negative — when the elevator is rising and slows to a stop or begins to descend — your apparent weight is smaller than your true weight. The floor of the elevator is exerting a force on you that is smaller than your weight, so you feel lighter.

Figure 1.10 Mobile construction cranes can withstand the tension necessary to lift loads of up to 1000 t.

Tension in Ropes and Cables

While an elevator is supporting or lifting you, what is supporting the elevator? The simple answer is cables — exceedingly strong steel cables. Construction cranes such the one in Figure 1.10 also use steel cables to lift building materials to the top of skyscrapers under construction. When a crane exerts a force on one end of a cable, each particle in the cable exerts an equal force on the next particle in the cable, creating tension throughout the cable. The cable then exerts a force on its load. **Tension** is the magnitude of the force exerted on and by a cable, rope, or string. How do engineers determine the amount of tension that these cables must be able to withstand? They apply Newton's laws of motion.

To avoid using complex mathematical analyses, you can make several assumptions about cables and ropes that support loads. Your results will be quite close to the values calculated by computers that are programmed to take into account all of the non-ideal conditions. The simplifying assumptions are as follows.

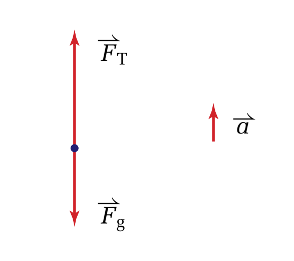
- The mass of the rope or cable is so much smaller than the mass of the load that it does not significantly affect the motion or forces involved.
- The tension is the same at every point in the rope or cable.
- If a rope or cable passes over a pulley, the direction of the tension forces changes, but the magnitude remains the same. This statement is the same as saying that the pulley is frictionless and its mass is negligible.

SAMPLE PROBLEM

Tension in a Cable

An elevator filled with people has a total mass of 2245 kg. As the elevator begins to rise, the acceleration is 0.55 m/s^2. What is the tension in the cable that is lifting the elevator?

Conceptualize the Problem



- To begin framing the problem, draw a free-body diagram.
- The *tension* in the cable has the *same magnitude* as the *force* it exerts on the elevator.
- *Two forces* are acting on the elevator: the *cable* ($\vec{F}_T$) and gravity ($\vec{F}_g$).
- The elevator is *rising* and speeding up, so the *acceleration* is *upward*.
- *Newton's second law* applies to the problem.
- The motion is in *one dimension*, so let positive and negative *signs* indicate *direction*. Let "*up*" *be positive* and "*down*" *be negative*.

Identify the Goal

The tension, F_T, in the rope

Identify the Variables

Known	Implied	Unknown
$m = 2245$ kg	$g = 9.81 \dfrac{m}{s^2}$	$\vec{F}_T$
$\vec{a} = 0.55 \dfrac{m}{s^2}$[up]		$\vec{F}_g$

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Develop a Strategy

Apply Newton's second law and insert all of the forces acting on the elevator. Then solve for the tension.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_T + \vec{F}_g = m\vec{a}$$
$$\vec{F}_T = -\vec{F}_g + m\vec{a}$$
$$\vec{F}_T = -(-mg) + m\vec{a}$$

Substitute values and solve.

$$\vec{F} = (2245\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + (2245\text{ kg})\left(0.55\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = 23\ 258.2\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$\vec{F}_T \cong 2.3 \times 10^4\text{ N[up]}$$

The magnitude of the tension in the cable is 2.3×10^4 N[up].

Validate the Solution

The weight of the elevator is $(2245\text{ kg})(9.81\text{ m/s}^2) \cong 2.2 \times 10^4$ N. The tension in the cable must support the weight of the elevator and exert an additional force to accelerate the elevator. Therefore, you would expect the tension to be a little larger than the weight of the elevator, which it is.

PRACTICE PROBLEMS

14. A 32 kg child is practising climbing skills on a climbing wall, while being belayed (secured at the end of a rope) by a parent. The child loses her grip and dangles from the belay rope. When the parent starts lowering the child, the tension in the rope is 253 N. Find the acceleration of the child when she is first being lowered.

15. A 92 kg mountain climber rappels down a rope, applying friction with a figure eight (a piece of climbing equipment) to reduce his downward acceleration. The rope, which is damaged, can withstand a tension of only 675 N. Can the climber limit his descent to a constant speed without breaking the rope? If not, to what value can he limit his downward acceleration?

16. A 10.0 kg mass is hooked on a spring scale fastened to a hoist rope. As the hoist starts moving the mass, the scale momentarily reads 87 N. Find

(a) the direction of motion

(b) the acceleration of the mass

(c) the tension in the hoist rope

17. Pulling on the strap of a 15 kg backpack, a student accelerates it upward at 1.3 m/s^2. How hard is the student pulling on the strap?

18. A 485 kg elevator is rated to hold 15 people of average mass (75 kg). The elevator cable can withstand a maximum tension of 3.74×10^4 N, which is twice the maximum force that the load will create (a 200% safety factor). What is the greatest acceleration that the elevator can have with the maximum load?

Connected Objects

Imagine how much energy it would require to lift an elevator carrying 20 people to the main deck of the CN Tower in Toronto, 346 m high. A rough calculation using the equation for gravitational potential energy ($E_g = mg\Delta h$), which you learned in previous science courses, would yield a value of about 10 million joules of energy. Is there a way to avoid using so much energy?

Elevators are not usually simply suspended from cables. Instead, the supporting cable passes up over a pulley and then back down to a heavy, movable **counterweight**, as shown in Figure 1.11. Gravitational forces acting *downward* on the counterweight create tension in the cable. The cable then exerts an *upward* force on the elevator cage. Most of the weight of the elevator and passengers is balanced by the counterweight. Only relatively small additional forces from the elevator motors are needed to raise and lower the elevator and its counterweight. Although the elevator and counterweight move in different directions, they are connected by a cable, so they accelerate at the same rate.

Elevators are only one of many examples of machines that have large masses connected by a cable that runs over a pulley. In fact, in 1784, mathematician George Atwood (1745–1807) built a machine similar to the simplified illustration in Figure 1.12. He used his machine to test and demonstrate the laws of uniformly accelerated motion and to determine the value of g, the acceleration due to gravity. The acceleration of Atwood's machine depended on g, but was small enough to measure accurately. In the following investigation, you will use an Atwood machine to measure g.

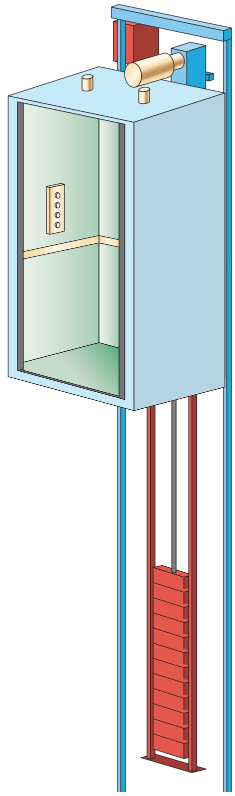


Figure 1.11 Most elevators are connected by a cable to a counterweight that moves in the opposite direction to the elevator. A typical counterweight has a mass that is the same as the mass of the empty elevator plus about half the mass of a full load of passengers.

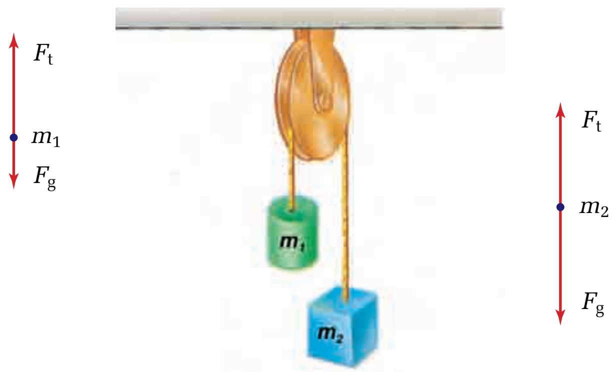


Figure 1.12 An Atwood machine uses a counterweight to reduce acceleration due to gravity.

INVESTIGATION 1-B

Atwood's Machine

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

George Atwood designed his machine to demonstrate the laws of motion. In this investigation, you will demonstrate those laws and determine the value of *g*.

Problem

How can you determine the value of *g*, the acceleration due to gravity, by using an Atwood machine?

Prediction

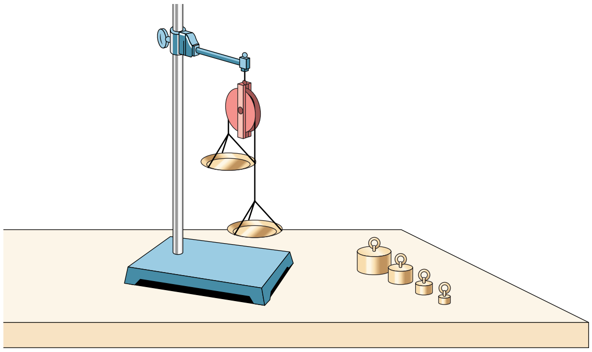
- Predict how changes in the *difference* between the two masses will affect the acceleration of the Atwood machine if the sum of the masses is held constant.
- When the difference between the two masses in an Atwood machine is held constant, predict how increasing the total mass (sum of the two masses) will affect their acceleration.

Equipment

- retort stand
- clamps
- masses: 100 g (2), 20 g (1), 10 g (10), or similar identical masses, such as 1 inch plate washers
- 2 plastic cups to hold masses
- light string

Traditional instrumentation
lab pulley
lab timer
metre stick

Probeware
Smart Pulley® or photogates or ultrasonic range finder
motion analysis software
computer



Procedure

Constant Mass Difference

1. Set up a data table to record m_1, m_2, total mass, Δd and Δt (if you use traditional equipment), and a.
2. Set up an Atwood machine at the edge of a table, so that $m_1 = 120$ g and $m_2 = 100$ g.
3. Lift the heavier mass as close as possible to the pulley. Release the mass and make the measurements necessary for finding its downward acceleration. Catch the mass before it hits the floor.
   - Using traditional equipment, find displacement (Δd) and the time interval (Δt) while the mass descends smoothly.
   - Using probeware, measure velocity (v) and graph velocity versus time. Find acceleration from the slope of the line during an interval when velocity was increasing steadily.
4. Increase each mass by 10 g and repeat the observations. Continue increasing mass and finding acceleration until you have five total mass-acceleration data pairs.
5. Graph acceleration versus total mass. Draw a best-fit line through your data points.

Constant Total Mass

6. Set up a data table to record m_1, m_2, mass difference (Δm), Δd and Δt (if you use traditional equipment), and a.
7. Make $m_1 = 150$ g and $m_2 = 160$ g. Make observations to find the downward acceleration, using the same method as in step 3.
8. Transfer one 10 g mass from m_1 to m_2. The mass difference will now be 30 g, but the total mass will not have changed. Repeat your measurements.
9. Repeat step 8 until you have data for five mass difference-acceleration pairs.
10. Graph acceleration versus mass difference. Draw a best-fit line or curve through your data points.

Analyze and Conclude

1. Based on your graphs for step 5, what type of relationship exists between total mass and acceleration in an Atwood machine? Use appropriate curve-straightening techniques to support your answer (see Skill Set 4, Mathematical Modelling and Curve Straightening). Write the relationship symbolically.
2. Based on your graphs for step 10, what type of relationship exists between mass difference and acceleration in an Atwood machine? Write the relationship symbolically.
3. How well do your results support your prediction?
4. String that is equal in length to the string connecting the masses over the pulley is sometimes tied to the bottoms of the two masses, where it hangs suspended between them. Explain why this would reduce experimental errors. Hint: Consider the mass of the string as the apparatus moves and how that affects m_1 and m_2.
5. Mathematical analysis shows that the acceleration of an ideal (frictionless) Atwood machine is given by $a = g\frac{m_1 - m_2}{m_1 + m_2}$. Use this relationship and your experimental results to find an experimental result for g.
6. Calculate experimental error in your value of g. Suggest the most likely causes of experimental error in your apparatus and procedure.

Apply and Extend

7. Start with Newton's second law in the form $\vec{a} = \frac{\vec{F}}{m}$ and derive the equation for a in question 5 above. Hint: Write $\vec{F}$ and m in terms of the forces and masses in the Atwood machine.
8. Using the formula $a = g\frac{m_1 - m_2}{m_1 + m_2}$ for an Atwood machine, find the acceleration when $m_1 = 2m_2$.
9. Under what circumstances would the acceleration of the Atwood machine be zero?
10. What combination of masses would make the acceleration of an Atwood machine equal to $\frac{1}{2}g$?

WEB LINK

www.mcgrawhill.ca/links/physics12

For some interactive activities involving the Atwood machine, go to the above Internet site and click on **Web Links**.

Assigning Direction to the Motion of Connected Objects

When two objects are connected by a flexible cable or rope that runs over a pulley, such as the masses in an Atwood machine, they are moving in different directions. However, as you learned when working with trains of objects, connected objects move as a unit. For some calculations, you need to work with the forces acting on the combined objects and the acceleration of the combined objects. How can you treat the pair of objects as a unit when two objects are moving in different directions?

Since the connecting cable or rope changes only the direction of the forces acting on the objects and has no effect on the magnitude of the forces, you can assign the direction of the motion as being from one end of the cable or rope to the other. You can call one end "negative" and the other end "positive," as shown in Figure 1.13.

When you have assigned the directions to a pair of connected objects, you can apply Newton's laws to the objects as a unit or to each object independently. When you treat the objects as one unit, you must ignore the tension in the rope because it does not affect the movement of the combined objects. Notice that the force exerted by the rope on one object is equal in magnitude and opposite in direction to the force exerted on the other object. However, when you apply the laws of motion to one object at a time, you must include the tension in the rope, as shown in the following sample problem.

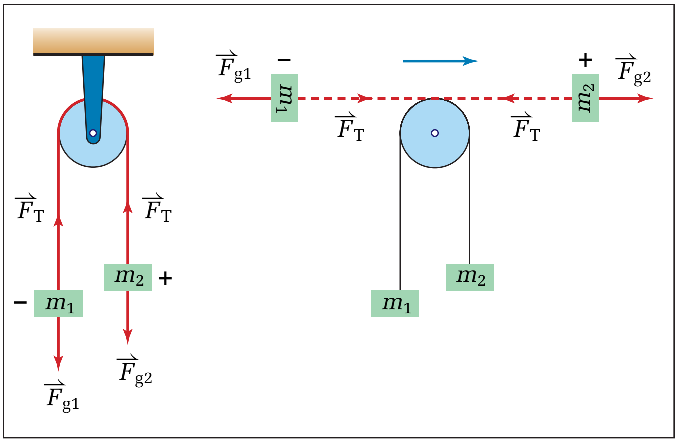


Figure 1.13 You can assign the bottom of the left-hand side of the machine to be negative and the bottom of the right-hand side to be positive. You can then imagine the connected objects as forming a straight line, with left as negative and right as positive. When you picture the objects as a linear train, make sure that you keep the force arrows in the same *relative* directions in relation to the individual objects.

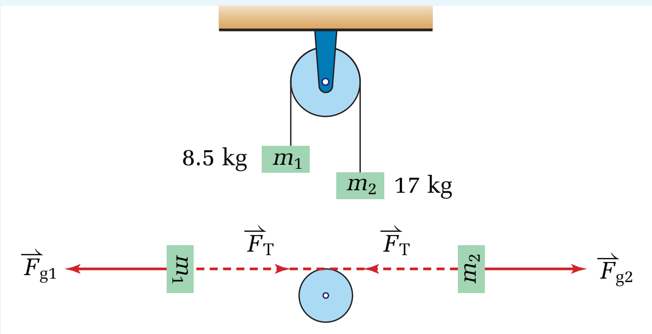
SAMPLE PROBLEM

Motion of Connected Objects

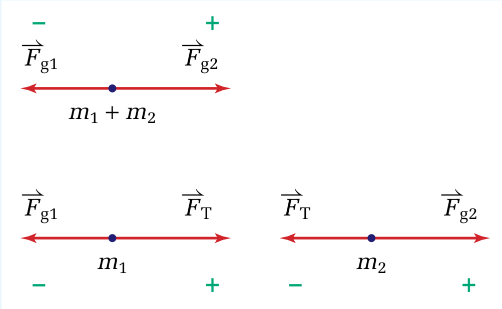
An Atwood machine is made of two objects connected by a rope that runs over a pulley. The object on the left (m_1) has a mass of 8.5 kg and the object on the right (m_2) has a mass of 17 kg.

(a) What is the acceleration of the masses?

(b) What is the tension in the rope?



Conceptualize the Problem



- To start framing the problem, draw free-body diagrams. Draw one diagram of the system moving as a unit and diagrams of each of the two individual objects.
- Let the *negative* direction point from the centre to the 8.5 kg mass and the *positive* direction point from the centre to the 17 kg mass.
- Both objects move with the same acceleration.
- The force of gravity acts on both objects.
- The tension is constant throughout the rope.
- The rope exerts a force of equal magnitude and opposite direction on each object.
- When you isolate the individual objects, the tension in the rope is one of the forces acting on the object.
- Newton's second law applies to the combination of the two objects and to each individual object.

Identify the Goal

(a) The acceleration, $\vec{a}$, of the two objects

(b) The tension, $|\vec{F}_T|$, in the rope

Identify the Variables

Known	Implied	Unknown
$m_1 = 8.5$ kg	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_{g1}$ $\vec{F}_T$
$m_2 = 17$ kg		$\vec{F}_{g2}$

Develop a Strategy

Apply Newton's second law to the combination of masses to find the acceleration.

$$\vec{F} = m\vec{a}$$

The mass of the combination is the sum of the individual masses.

$$\vec{F}_{g1} + \vec{F}_{g2} = (m_1 + m_2)\vec{a}$$

$$-m_1 g + m_2 g = (m_1 + m_2)\vec{a}$$

$$\vec{a} = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$$\vec{a} = \frac{(17\text{ kg} - 8.5\text{ kg})9.8\ \frac{\text{m}}{\text{s}^2}}{8.5\text{ kg} + 17\text{ kg}}$$

$$\vec{a} = 3.27\ \frac{\text{m}}{\text{s}^2}$$

$$\vec{a} \cong 3.3\ \frac{\text{m}}{\text{s}^2}\text{[to the right]}$$

(a) The acceleration of the combination of objects is 3.3 m/s^2 to the right.

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Apply Newton's second law to m_1 and solve for tension.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_{g1} + \vec{F}_T = m_1\vec{a}$$
$$-m_1 g + \vec{F}_T = m_1\vec{a}$$
$$\vec{F}_T = m_1 g + m_1\vec{a}$$
$$\vec{F}_T = (8.5\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + (8.5\text{ kg})\left(3.27\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = 111.18\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$\vec{F}_T \cong 1.1 \times 10^2\text{ N}$$

(b) The tension in the rope is 1.1×10^2 N.

Validate the Solution

You can test your solution by applying Newton's second law to the second mass.

$$\vec{F}_{g2} + \vec{F}_T = m_2\vec{a}$$
$$m_2 g + \vec{F}_T = m_2\vec{a}$$
$$\vec{F}_T = m_2\vec{a} - m_2 g$$
$$\vec{F}_T = (17\text{ kg})\left(3.27\ \frac{\text{m}}{\text{s}^2}\right) - (17\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = -111.18\text{ N}$$
$$\vec{F}_T = -1.1 \times 10^2\text{ N}$$

The magnitudes of the tensions calculated from the two masses independently agree. Also, notice that the application of Newton's second law correctly gave the direction of the force on the second mass.

PRACTICE PROBLEMS

19. An Atwood machine consists of masses of 3.8 kg and 4.2 kg. What is the acceleration of the masses? What is the tension in the rope?

20. The smaller mass on an Atwood machine is 5.2 kg. If the masses accelerate at 4.6 m/s^2, what is the mass of the second object? What is the tension in the rope?

21. The smaller mass on an Atwood machine is 45 kg. If the tension in the rope is 512 N, what is the mass of the second object? What is the acceleration of the objects?

22. A 3.0 kg counterweight is connected to a 4.5 kg window that freely slides vertically in its frame. How much force must you exert to start the window opening with an acceleration of 0.25 m/s^2?

23. Two gymnasts of identical 37 kg mass dangle from opposite sides of a rope that passes over a frictionless, weightless pulley. If one of the gymnasts starts to pull herself up the rope with an acceleration of 1.0 m/s^2, what happens to her? What happens to the other gymnast?

Objects Connected at Right Angles

In the lab, a falling weight is often used to provide a constant force to accelerate dynamics carts. Gravitational forces acting *downward* on the weight create tension in the connecting string. The pulley changes the direction of the forces, so the string exerts a *horizontal* force on the cart. Both masses experience the same acceleration because they are connected, but the cart and weight move at right angles to each other.

You can approach problems with connected objects such as the lab cart and weight in the same way that you solved problems involving the Atwood machine. Even if a block is sliding, with friction, over a surface, the mathematical treatment is much the same. Study Figure 1.14 and follow the directions below to learn how to treat connected objects that are moving both horizontally and vertically.

- Analyze the forces on each individual object, then label the diagram with the forces.
- Assign a direction to the motion.
- Draw the connecting string or rope as though it was a straight line. Be sure that the force vectors are in the same direction relative to each mass.
- Draw a free-body diagram of the combination and of each individual mass.
- Apply Newton's second law to each free-body diagram.

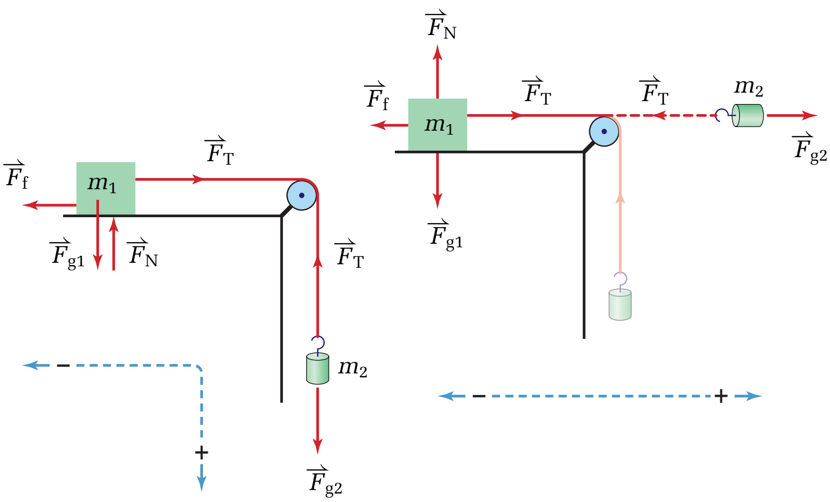


Figure 1.14 When you visualize the string "straightened," the force of gravity appears to pull down on mass 1, but to the side on mass 2. Although it might look strange, be assured that these directions are correct regarding the way in which the forces affect the motion of the objects.

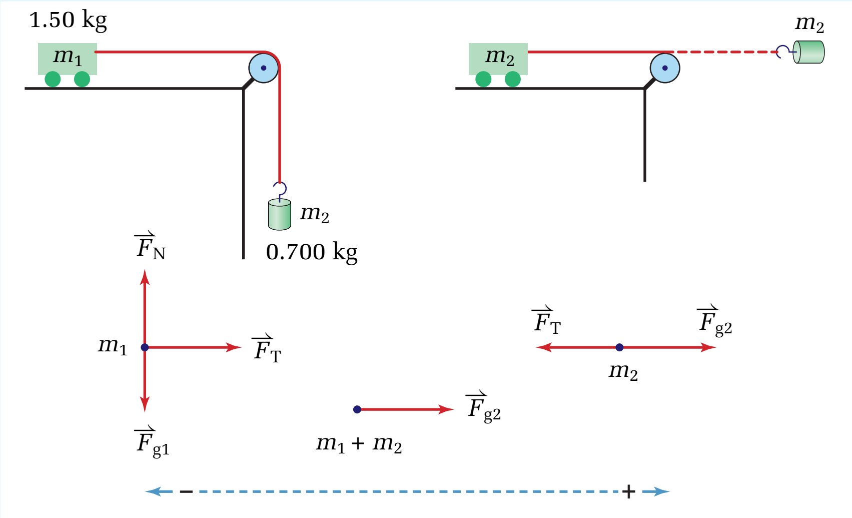
SAMPLE PROBLEM

Connected Objects

A 0.700 kg mass is connected to a 1.50 kg lab cart by a lightweight cable passing over a low-friction pulley. How fast does the cart accelerate and what is the tension in the cable? (Assume that the cart rolls without friction.)

Conceptualize the Problem

- Make a simplified diagram of the connected masses and assign forces.
- Visualize the cable in a straight configuration.
- Sketch free-body diagrams of the forces acting on each object and of the forces acting on the combined objects.



- The force causing the *acceleration* of both masses is the *force* of *gravity* acting on mass 2.
- Newton's second law applies to the combined masses and to each individual mass.
- Let left be the *negative* direction and right be the *positive* direction.

Identify the Goal

The acceleration of the cart, $\vec{a}$, and the magnitude of the tension force in the cable, F_T

Identify the Variables and Constants

Known	Implied	Unknown	
$m_1 = 1.50$ kg	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{a}$	$\vec{F}_{g1}$
$m_2 = 0.700$ kg		$\vec{F}_T$	$\vec{F}_{g2}$

Develop a Strategy

Apply Newton's second law to the combined masses and solve for acceleration.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_{g2} = (m_1 + m_2)\vec{a}$$
$$m_2 g = (m_1 + m_2)\vec{a}$$
$$\vec{a} = \frac{m_2 g}{m_1 + m_2}$$

Substitute values and solve.

$$\vec{a} = \frac{(0.700\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{0.700\ \text{kg} + 1.5\ \text{kg}}$$
$$\vec{a} = 3.121\ 36\ \frac{\text{m}}{\text{s}^2}$$
$$\vec{a} \cong 3.1\ \frac{\text{m}}{\text{s}^2}$$

The cart accelerates at about 3.1 m/s^2. Since the sign is positive, it accelerates to the right.

Apply Newton's second law to mass 1 to find the tension in the rope.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_T = m_1\vec{a}$$
$$\vec{F}_T = (1.5\ \text{kg})\left(3.121\ 36\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = 4.682\ 04\ \text{N}$$
$$\vec{F}_T \cong 4.7\ \text{N}$$

The tension in the cable is about 4.7 N.

Validate the Solution

The acceleration of the combined masses is less than 9.81 m/s^2, which is reasonable since only part of the mass is subject to unbalanced gravitational forces. Also, the tension calculated at m_2 is also about 4.7 N.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_g + \vec{F}_T = m_2\vec{a}$$
$$\vec{F}_T = m_2\vec{a} - \vec{F}_g$$
$$\vec{F}_T = (0.700\ \text{kg})\left(3.121\ 36\ \frac{\text{m}}{\text{s}^2}\right) - (0.700\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T \cong -4.7\ \text{N}$$

PRACTICE PROBLEMS

24. A Fletcher's trolley apparatus consists of a 1.90 kg cart on a level track attached to a light string passing over a pulley and holding a 0.500 kg mass suspended in the air. Neglecting friction, calculate

(a) the tension in the string when the suspended mass is released

(b) the acceleration of the trolley

25. A 40.0 g glider on an air track is connected to a suspended 25.0 g mass by a string passing over a frictionless pulley. When the mass is released, how long will it take the glider to travel the 0.85 m to the other end of the track? (Assume the mass does not hit the floor, so there is constant acceleration during the experiment.)

Free Fall

Have you ever dared to take an amusement park ride that lets you fall with almost no support for a short time? A roller coaster as it drops from a high point in its track can bring you close to the same feeling of **free fall**, a condition in which gravity is the only force acting on you. To investigate free fall quantitatively, imagine, once again, that you are standing on a scale in an elevator. If the cable was to break, there were no safety devices, and friction was negligible, what would be your apparent weight?

Figure 1.15 When you are on a free-fall amusement park ride, you feel weightless.

If gravity is the only force acting on the elevator, it will accelerate downward at the acceleration due to gravity, or g. Substitute this value into Newton's second law and solve for your apparent weight.

- Write Newton's second law. $\vec{F} = m\vec{a}$
- Let "up" be positive and "down" be negative. The total force acting on you is the downward force of gravity and the upward normal force of the scale. Your acceleration is g downward. $F_N + F_g = -mg$
- The force of gravity is $-mg$. $F_N - mg = -mg$
- Solve for the normal force. $F_N = mg - mg$
  $F_N = 0$

The reading on the scale is zero. Your apparent weight is zero. This condition is often called "weightlessness." Your mass has not changed, but you feel weightless because nothing is pushing up on you, preventing you from accelerating at the acceleration due to gravity.

Conceptual Problem

- How would a person on a scale in a freely falling elevator analyze the forces that were acting? Make a free-body analysis similar to the one in the sample problem (Apparent Weight) on page 28, using the elevator as your frame of reference. Consider these points.
  - **(a)** To an observer in the elevator, the person on the scale would not appear to be moving.
  - **(b)** The reading on the scale (the normal force) would be zero.

Close to Earth's surface, weightlessness is rarely experienced, due to the resistance of the atmosphere. As an object collides with molecules of the gases and particles in the air, the collisions act as a force opposing the force of gravity. **Air resistance** or air friction is quite different from the surface friction that you have studied. When an object moves through a fluid such as air, the force of friction increases as the velocity of the object increases.

A falling object eventually reaches a velocity at which the force of friction is equal to the force of gravity. At that point, the net force acting on the object is zero and it no longer accelerates but maintains a constant velocity called **terminal velocity**. The shape and orientation of an object affects its terminal velocity. For example, skydivers control their velocity by their position, as illustrated in Figure 1.16. Table 1.2 lists the approximate terminal velocities for some common objects.

Figure 1.16 Gravity is not the only force affecting these sky-divers, who have become experts at manipulating air friction and controlling their descent.

Table 1.2 Approximate Terminal Velocities

Object	Terminal velocity (m/s downward)
large feather	0.4
fluffy snowflake	1
parachutist	7
penny	9
skydiver (spread-eagled)	58

PHYSICS FILE

In 1942, Soviet air force pilot I. M. Chisov was forced to parachute from a height of almost 6700 m. To escape being shot by enemy fighters, Chisov started to free fall, but soon lost consciousness and never opened his parachute. Air resistance slowed his descent, so he probably hit the ground at about 193 km/h, plowing through a metre of snow as he skidded down the side of a steep ravine. Amazingly, Chisov survived with relatively minor injuries and returned to work in less than four months.

TECHNOLOGY LINK

Air resistance is of great concern to vehicle designers, who can increase fuel efficiency by using body shapes that reduce the amount of air friction or drag that is slowing the vehicle. Athletes such as racing cyclists and speed skaters use body position and specially designed clothing to minimize drag and gain a competitive advantage. Advanced computer hardware and modelling software are making computerized simulations of air resistance a practical alternative to traditional experimental studies using scale models in wind tunnels.

CANADIANS IN PHYSICS

Father of the Canadian Space Program

Can you imagine sending one of the very first satellites into space? How about writing a report that changed the entire direction of Canada's space efforts, or being involved in a telecommunications program that won an Emmy award? These are just a few of the accomplishments that earned John H. Chapman the nickname "Father of the Canadian Space Program."

Chapman, who was born in 1921, was a science graduate of McGill University in Montréal. In 1951, the London, Ontario, native became section leader of the Defence Research Board's unit at Shirley's Bay, Ontario. While there, he played a key role in several ground-breaking projects.

John Herbert Chapman

Lift Off!

Early in the history of space exploration, Canadian space scientists focussed on the study of Earth's upper atmosphere and ionosphere. They wanted to understand the behaviour of radio waves in these lofty regions, especially above Canada's North. As head of the government team researching this area, Chapman was a moving force in the Alouette/International Satellites for Ionospheric Studies (ISIS) program.

With the successful launch of *Alouette I* in 1962, Canada became the third nation to reach space, following the Soviet Union and the United States. Designed to last for one year, *Alouette I* functioned for ten. It has been hailed as one of the greatest achievements in Canadian engineering in the past century. The ISIS satellites lasted for 20 years, earning Canada an international reputation for excellence in satellite design and engineering.

During this time, Chapman brought Canadian industry into the space age. He argued that private companies, not just government laboratories, had the "right stuff" to design and build space hardware. As a result, Canadian industry was given a steadily increasing role in the manufacture of *Alouette II* and the ISIS satellites.

Connecting Canada and the World

Chapman also influenced the very purpose for which Canada's satellites were built. *The Chapman Report*, issued in 1967, helped turn Canada's space program away from space science and toward telecommunications. Chapman believed that satellites could deliver signals to rural and remote regions of the country. This was achieved in 1972, when Canada placed the *Anik A1* satellite into stationary orbit above the equator and became the first country to have its own communications satellite system of this type.

Today, live news reports can be delivered from remote locations, due to technology that Chapman and his team helped pioneer in co-operation with NASA and the European Space Agency. Before the *Hermes* satellite was launched in 1976, videotapes of news events were flown to a production centre and distributed. This was a time-consuming process. With *Hermes* in place, a telecommunications dish on location could beam news up to the satellite and, from there, to anywhere in the world. *Hermes* was also revolutionary because it sent and received television signals on high frequencies that did not interfere with frequencies already in use. For this innovation, the *Hermes* satellite program won an Emmy in 1987.

At the time of his death in 1979, John Chapman was the Assistant Deputy Minister for Space in the Canadian Department of Communications. On October 2, 1996, in recognition of his distinguished career, the headquarters of the Canadian Space Agency was dedicated as the John H. Chapman Space Centre.

WEB LINK

www.mcgrawhill.ca/links/physics12

For more information about the Canadian Space Agency and the *Alouette*, *Hermes*, and ISIS space programs, visit the above Internet site and click on **Web Links**.

QUICK LAB

Descending Drops

TARGET SKILLS

- Performing and recording
- Modelling concepts
- Analyzing and interpreting

You can observe drops of water falling at terminal velocity through cooking oil in a test tube. Use an eye-dropper to carefully "inject" drops of cold water below the surface of the cooking oil. Measure the diameter of the drops and the speed of their descent.

Analyze and Conclude

1. Assume that the drops are spherical and are pure water with density 1.0 g/cm^3. Using the formulas for volume of a sphere $\left(V = \frac{4}{3}\pi r^3\right)$ and density $\left(D = \frac{m}{V}\right)$, calculate the mass of each drop.
2. Calculate the gravitational force and the retarding force on each drop.
3. What force(s) are retarding the downward force of gravity acting on the drops? Compare these forces to those acting on an object falling through air.
4. The curved sides of the test tube act like a lens, producing some optical magnification of objects inside. Describe in detail how this might be affecting your results.
5. How well does this activity model the movement of an object through air and the phenomenon of terminal velocity? Justify your answer.

1.3 Section Review

1. K/U Explain why your apparent weight is sometimes not the same as your true weight.
2. K/U Explain how Newton's third law applies to connected objects that are all pulled by one end.
3. C How does an Atwood machine make it easier to determine *g* (the acceleration due to gravity), rather than by just measuring the acceleration of a free-falling object?
4. C Suppose you are standing on a scale in a moving elevator and notice that the scale reading is *less* than your true weight.
   (a) Draw a free-body diagram to represent the forces acting on you.
   (b) Describe the elevator motion that would produce the effect.
5. K/U List the simplifying assumptions usually made about supporting cables and ropes. Why are simplifying assumptions needed?
6. K/U Two objects are moving in different directions. Under what circumstances can you treat this as a one-dimensional problem?
7. MC By the mid-1800s, steam-driven elevators with counterweights had been developed. However, they were not in common use until 1852, when Elisha Otis invented an elevator with a safety device that prevented the elevator from falling if the cable broke. How do you think that the invention of a safe elevator changed modern society?
8. C Describe a situation in which you could be standing on a scale and the reading on the scale would be zero. (**Note:** The scale is functioning properly and is accurate.) What is the name of this condition?

1.4 Motion along an Incline

SECTION EXPECTATIONS

- Analyze the motion of objects along inclined planes.
- Use vector and free-body diagrams to analyze forces.
- Predict and explain motion along inclined planes.

When you watch speed skiers, it appears as though there is no limit to the rate at which they can accelerate. In reality, their acceleration is always less that that of a free-falling object, because the skier is being accelerated by only a component of the force of gravity and not by the total force. Using the principles of dynamics and the forces affecting the motion, you can predict details of motion along an inclined plane.

Figure 1.17 Gravitational forces acting on downhill skiers have produced speeds greater than 241 km/h, even though only part of the total gravitational force accelerates a skier.

Choosing a Coordinate System for an Incline

The key to analyzing the dynamics and motion of objects on an inclined plane is choosing a coordinate system that simplifies the procedure. Since all of the motion is along the plane, it is convenient to place the *x*-axis of the coordinate system parallel to the plane, making the *y*-axis perpendicular to the plane, as shown in Figure 1.18.

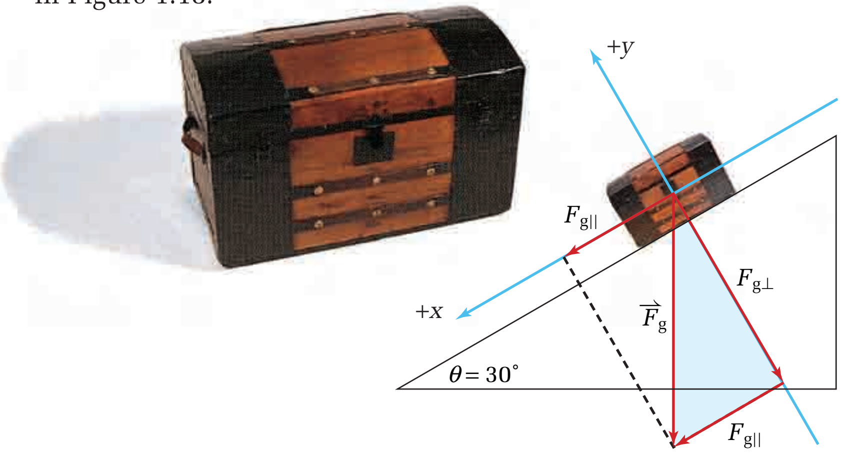


Figure 1.18 To find the components of the gravitational force vector, use the shaded triangle. Note that $\vec{F}_g$ is perpendicular to the horizontal line at the bottom and $F_{g\perp}$ is perpendicular to the plane of the ramp. Since the angles between two sets of perpendicular lines must be equal, the angle (θ) in the triangle is equal to the angle that the inclined plane makes with the horizontal.

The force of gravity affects motion on inclined planes, but the force vector is at an angle to the plane. Therefore, you must resolve the gravitational force vector into components parallel to and perpendicular to the plane, as shown in Figure 1.18. The component of force parallel to the plane influences the acceleration of the object and the perpendicular component affects the magnitude of the friction. Since several forces in addition to the gravitational force can affect the motion on an inclined plane, free-body diagrams are essential in solving problems, as shown in the sample problem below.

SAMPLE PROBLEM

Sliding Down an Inclined Plane

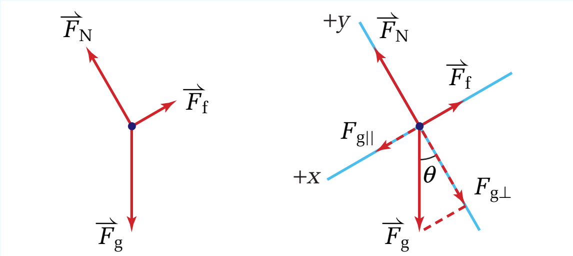
You are holding an 85 kg trunk at the top of a ramp that slopes from a moving van to the ground, making an angle of 35° with the ground. You lose your grip and the trunk begins to slide.

(a) If the coefficient of friction between the trunk and the ramp is 0.42, what is the acceleration of the trunk?

(b) If the trunk slides 1.3 m before reaching the bottom of the ramp, for what time interval did it slide?

Conceptualize the Problem

- To start framing the problem, draw a *free-body diagram*.
- Beside the free-body diagram, draw a coordinate system with the *x-axis parallel* to the ramp. On the coordinate system, draw the forces and *components of forces* acting on the trunk.



- Let the direction pointing *down* the slope be the *positive* direction.
- To find the *normal force* that is needed to determine the magnitude of the *frictional force*, apply *Newton's second law* to the forces or components of forces that are *perpendicular* to the ramp.
- The *acceleration perpendicular* to the ramp is *zero*.
- The *component of gravity parallel* to the trunk causes the trunk to *accelerate* down the ramp.
- *Friction* between the trunk and the ramp *opposes* the motion.
- If the *net force* along the ramp is *positive*, the trunk will *accelerate* down the ramp.
- To find the *acceleration* of the trunk down the ramp, apply *Newton's second law* to the forces or components of forces *parallel* to the ramp.
- Given the *acceleration* of the trunk, you can use the *kinematic equations* to find *other quantities* of motion.

Identify the Goal

(a) The acceleration, $a_{\parallel}$, of the trunk along the ramp

(b) The time interval, Δt, for the trunk to reach the end of the ramp

continued ►

continued from previous page

Identify the Variables

Known		Implied	Unknown		
$m = 85$ kg	$\theta = 35^\circ$	$g = 9.81 \frac{\text{m}}{\text{s}^2}$	$\vec{F}_g$	$\vec{F}_f$	v_f
$\mu = 0.42$	$\Delta d = 1.3$ m	$v_i = 0$	$F_{g\parallel}$	$\vec{F}_N$	$a_\parallel$
		$a_\perp = 0$	$F_{g\perp}$		

Develop a Strategy

Apply Newton's second law to the forces perpendicular to the ramp. Refer to the diagram to find all of the forces that are perpendicular to the ramp. Solve for the normal force.

$$\vec{F} = m\vec{a}$$
$$F_N + F_{g\perp} = ma_\perp$$
$$F_N - mg\cos\theta = ma_\perp$$
$$F_N = mg\cos\theta + ma_\perp$$

Insert values and solve. Note that the acceleration perpendicular to the ramp ($a_\perp$) is zero.

$$F_N = (85\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\cos 35^\circ + 0$$
$$F_N = 683.05\text{ N}$$

Apply Newton's second law to the forces parallel to the ramp. Refer to the diagram to find all of the forces that are parallel to the ramp. Solve for the acceleration parallel to the ramp.

$$\vec{F} = m\vec{a}$$
$$F_{g\parallel} + F_f = ma_\parallel$$
$$F_f = \mu F_N \text{ in negative direction}$$
$$mg\sin\theta - \mu F_N = ma_\parallel$$
$$a_\parallel = \frac{mg\sin\theta - \mu F_N}{m}$$

Insert values and solve.

$$a_\parallel = \frac{(85\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 35^\circ - (0.42)(683.05\text{ N})}{85\text{ kg}}$$
$$a_\parallel = 2.251\,71\ \frac{\text{m}}{\text{s}^2}$$
$$a_\parallel \cong 2.3\ \frac{\text{m}}{\text{s}^2}$$

(a) The acceleration of the trunk down the ramp is 2.3 m/s^2.

Apply the kinematic equation that relates displacement, acceleration, initial velocity, and time interval. Given that the initial velocity was zero, solve the equation for the time interval.

$$\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$$
$$\Delta t^2 = \frac{2\Delta d}{a}$$
$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

Insert values and solve.

$$\Delta t = \sqrt{\frac{2(1.3\text{ m})}{2.251\,71\ \frac{\text{m}}{\text{s}^2}}}$$
$$\Delta t = 1.075\text{ s}$$
$$\Delta t \cong 1.1\text{ s}$$

(b) The trunk slid for 1.1 s before reaching the end of the ramp.

Validate the Solution

(a) Since the ramp is not at an extremely steep slope and since there is a significant amount of friction, you would expect that the acceleration would be much smaller than 9.81 m/s^2, which it is.

(b) The ramp is very short, so you would expect that it would not take long for the trunk to reach the bottom of the ramp. A time of 1.1 s is quite reasonable.

PRACTICE PROBLEMS

26. A 1975 kg car is parked at the top of a steep 42 m long hill inclined at an angle of 15°. If the car starts rolling down the hill, how fast will it be going when it reaches the bottom of the hill? (Neglect friction.)

27. Starting from rest, a cyclist coasts down the starting ramp at a professional biking track. If the ramp has the minimum legal dimensions (1.5 m high and 12 m long), find

(a) the acceleration of the cyclist, ignoring friction

(b) the acceleration of the cyclist if all sources of friction yield an effective coefficient of friction of $\mu = 0.11$

(c) the time taken to reach the bottom of the ramp, if friction acts as in (b)

28. A skier coasts down a 3.5° slope at constant speed. Find the coefficient of kinetic friction between the skis and the snow covering the slope.

QUICK LAB

The Slippery Slope

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting

You can determine the coefficients of static and kinetic friction experimentally. Use a coin or small block of wood as the object and a textbook as a ramp. Find the mass of the object. Experiment to find the maximum angle of inclination possible before the object begins to slide down the ramp (θ_1). Then, use a slightly greater angle (θ_2), so that the object slides down the ramp. Make appropriate measurements of displacement and time, so that you can calculate the average acceleration. If the distance is too short to make accurate timings, use a longer ramp, such as a length of smooth wood or metal.

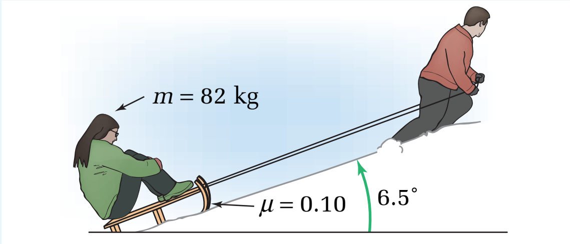
Analyze and Conclude

1. Calculate the gravitational force on the object (weight). Resolve the gravitational force into parallel and perpendicular components.
2. Draw a free-body diagram of the forces acting on the object and use it to find the magnitude of all forces acting on the object just before it started to slide (at angle θ_1). **Note:** If the object is not accelerating, no net force is acting on it, so every force must be balanced by an equal and opposite force.
3. Calculate the coefficient of static friction, μ_s, between the object and the ramp, using your answer to question 2.
4. Use the data you collected when the ramp was inclined at θ_2 to calculate the acceleration of the object. Find the net force necessary to cause this acceleration.
5. Use the net force and the parallel component of the object's weight to find the force of friction between the object and the ramp.
6. Calculate the coefficient of kinetic friction, μ_k, between the object and the ramp.
7. Compare μ_s and μ_k. Are they in the expected relationship to each other? How well do your experimental values agree with standard values for the materials that you used for your object and ramp? (Obtain coefficients of friction from reference materials.)

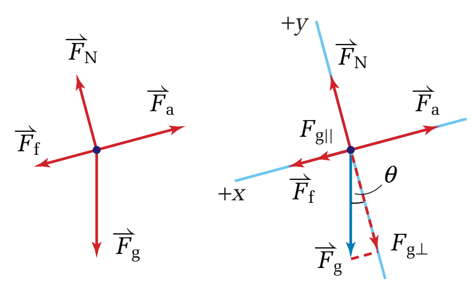
SAMPLE PROBLEM

Pushing or Pulling an Object Up an Incline

You are pulling a sled and rider with combined mass of 82 kg up a 6.5° slope at a steady speed. If the coefficient of kinetic friction between the sled and snow is 0.10, what is the tension in the rope?



Conceptualize the Problem



- Sketch a *free-body diagram* of the forces acting on the sled. Beside it, sketch the *components of the forces* that are *parallel* and *perpendicular* to the slope.
- Since the sled is moving at a *constant velocity*, the *acceleration is zero*.
- The *parallel component* of the sled's weight and the *force of friction* are acting *down* the slope (*positive* direction).
- The *applied force* of the rope acts up the slope on the sled (*negative* direction).
- The *tension* in the rope is the magnitude of the *force* that the *rope exerts* on the sled.
- *Newton's second law* applies independently to the forces perpendicular and parallel to the slope.

Identify the Goal

The magnitude of the tension, $|\vec{F}_a|$, in the rope

Identify the Variables and Constants

Known	Implied	Unknown		
$m = 82$ kg	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_g$	$F_{g\|\|}$	$F_{g\perp}$
$\mu = 0.10$	$a_{\|\|} = 0\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_N$	$\vec{F}_f$	$\vec{F}_a$
$\theta = 6.5°$				
v = constant				

Develop a Strategy

Apply Newton's second law to the forces perpendicular to the slope. Refer to the diagram to find all of the forces that are perpendicular to the slope. Solve for the normal force.

$$\vec{F} = m\vec{a}$$
$$F_N + F_{g\perp} = ma_\perp$$
$$F_N - mg\cos\theta = ma_\perp$$
$$F_N = mg\cos\theta + ma_\perp$$

Insert values and solve. Note that the acceleration perpendicular to the slope ($a_\perp$) is zero.

$$F_N = (82\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\cos 6.5° + 0$$
$$F_N = 799.25\ \text{N}$$

Apply Newton's second law to the forces parallel to the slope. Refer to the diagram to find all of the forces that are parallel to the slope. Solve for the force that the rope exerts on the sled.

$$\vec{F} = m\vec{a}$$
$$F_f + F_a + F_{g\|\|} = ma_{\|\|}$$
$$\mu F_N + F_a + mg\sin\theta = ma_{\|\|}$$
$$F_a = ma_{\|\|} - \mu F_N - mg\sin\theta$$

Insert values and solve.

$$F_a = (82\text{ kg})\left(0\ \frac{\text{m}}{\text{s}^2}\right) - (0.10)(799.25\text{ N}) - (82\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\sin 6.5^\circ$$

$$F_a = -79.925\text{ N} - 91.063\text{ N}$$

$$F_a = -170.988\text{ N}$$

$$|\vec{F}_a| \cong 1.7 \times 10^2\text{ N}$$

The tension force in the rope is about 1.7×10^2 N.

Validate the Solution

The tension is much less than the force of gravity on the sled, since most of the weight of the sled is being supported by the ground. The tension is also greater than the parallel component of the sled's weight, because the rope must balance both the force of friction and the component of the force of gravity parallel to the slope.

PRACTICE PROBLEMS

29. You flick a 5.5 g coin up a smooth board propped at an angle of 25° to the floor. If the initial velocity of the coin is 2.3 m/s up the board and the coefficient of kinetic friction between the coin and the board is 0.40, how far does the coin travel before stopping?

30. You are pushing a 53 kg crate at a constant velocity up a ramp onto a truck. The ramp makes an angle of 22° with the horizontal. If your applied force is 373 N, what is the coefficient of friction between the crate and the ramp?

1.4 Section Review

1. K/U Sketch a free-body diagram and an additional diagram showing the parallel and perpendicular components of gravitational force acting on an object on a ramp inclined at an angle of θ to the horizontal. State the equation used to calculate each force component.

2. K/U Which component of gravitational force affects each of the following?

(a) acceleration down a frictionless incline

(b) the force of friction acting on an object on a ramp

(c) the tension in a rope holding the object motionless

(d) the tension in a rope pulling the object up the ramp

3. C Why is it necessary to use two coefficients (kinetic and static) to describe the frictional forces between two surfaces? How do you decide which coefficient to use when solving a problem?

4. C Suppose you are pulling a heavy box up a ramp into a moving van. Why is it much harder to *start* the box moving than it is to *keep it moving*?

CHAPTER 1 Review

REFLECTING ON CHAPTER 1

- Dynamics relates the motion of objects to the forces acting on them.
- Inertia is the tendency of objects to resist changes in motion.
- In an inertial frame of reference, Newton's laws of motion describe motion correctly. Inertial frames of reference might be stationary or moving at constant velocity.
- In non-inertial frames of reference, Newton's laws of motion do not accurately describe motion. Accelerating frames of reference are non-inertial.
- Fictitious forces are needed to explain motion in non-inertial frames of reference. If the same motion is observed from an inertial frame of reference, the motion can be explained without the use of fictitious forces.
- Inertial mass is equivalent to gravitational mass.
- Frictional forces are described by the equation $F_f = \mu F_N$, where μ is the coefficient of friction between two surfaces and F_N is the normal force pressing the surfaces together. The coefficient of kinetic friction (μ_k) applies when the object is moving. The coefficient of static friction (μ_s) applies when the object is motionless.
- The weight of an object is the gravitational force on it ($F_g = mg$).
- Free fall is vertical motion that is affected by gravitational forces only. In free fall, all objects accelerate at the same rate.
- Terminal velocity is the maximum downward speed reached by a falling object when the force of air friction becomes equal to the force of gravity.
- Air resistance depends on the surface area, shape, and speed of an object relative to the air around it.

Knowledge/Understanding

1. Identify and provide examples of what physicists consider to be the two "natural" types of motion.
2. What is the term used to describe the tendency for objects to have differing amounts of "persistence" in maintaining their natural motion?
3. What concept is used to quantify the inertia of an object?
4. Distinguish between, and provide examples of, inertial and non-inertial frames of reference.
5. Imagine that you are looking sideways out of a car that is stopped at a stoplight. The light turns green and your driver accelerates until the car is travelling with uniform motion at the speed limit.
   (a) Sketch a velocity-time graph of your motion, illustrating the time intervals during which you were stopped, accelerating, and travelling with a constant velocity.
   (b) Identify the time interval(s) during which you were observing objects at the side of the road from an inertial frame of reference or from a non-inertial frame of reference.
   (c) Use this example to explain why Newton's first and second laws do not accurately predict the motion of the objects you are observing at the side of the road while you are accelerating.
6. You know that if you drop two balls from rest from the top of a building, they will accelerate uniformly and strike the ground at the same time (ignoring air resistance). Consider these variations.
   (a) Suppose you drop a ball from rest from the top of a building and it strikes the ground with a final velocity v_f. At the same time that the first ball is dropped, your friend launches a second ball from the ground with a velocity v_f, the same velocity with which the first ball strikes the ground. Will the second ball reach the top of the building at

the same time that the first ball strikes the ground? Explain where the balls cross paths, at half the height of the building, above the halfway point or below the halfway point. Ignore air resistance.

(b) You launch a ball from the edge of the top of a building with an initial velocity of 25 m/s [upward]. The ball rises to a certain height and then falls down and strikes the ground next to the building. Your friend on the ground measures the velocity with which the ball strikes the ground. Next, you launch a second ball from the edge of the building with a velocity of 25 m/s [downward]. Ignoring air resistance, will the second ball strike the ground with greater, smaller, or the same velocity as the first ball?

Hint: what is the velocity of the first ball when it is at the height of the top of the building (after falling from its maximum height) and on its way down?

7. You are having a debate with your lab partner about the correct solution to a physics problem. He says that the normal force acting on an object moving along a surface is *always* equal and opposite to the force of gravity. You disagree with this definition.

(a) Provide the proper definition for the normal force acting on an object.

(b) Describe, with the aid of free-body diagrams, three situations in which the normal force acting on an object cannot be determined using your lab partner's definition.

(c) Describe, with the aid of a free-body diagram, a situation in which your lab partner's definition could apply.

Inquiry

8. You are given two bowling balls. One is pure wood, while the other has an iron core. Your task is to verify Newton's laws. Accordingly, you set up an inclined plane in such a way that you can let the balls roll down the plane and along the floor.

(a) Design an experiment to determine which ball has more inertia.

(b) Sketch a velocity-time graph to illustrate your predictions of the motion of each ball.

(c) Explain how Newton's first law affects the motion of the ball during each phase of its motion. Explain your reasoning.

(d) Draw a free-body diagram for each ball as it descends the ramp. Write equations to predict the acceleration of each. Provide an analysis of the equations to show that each ball's acceleration down the ramp should be the same.

(e) The analysis in **(d)** seems to defy Newton's first law. Initially, you might predict that the ball with more inertia would have a different acceleration. Provide an explanation, based on Newton's laws, of why the ball with more inertia does not experience a greater acceleration.

9. Recall the apparatus set-up you used for Investigation 1-A to explore inertial mass. In this case, you are given a dynamics cart that has a mass of 500 g and you use a falling mass of 200 g.

(a) Assume that the coefficient of friction between the cart and the ramp is 0.12. Calculate theoretical predictions for the acceleration of the system when incremental masses of 100 g, 200 g, 300 g, 400 g, and 500 g are added to the cart.

(b) Plot an acceleration-versus-incremental-mass graph for your theoretical values.

(c) Does the line on this graph pass through the origin? Explain your reasoning.

(d) What acceleration is indicated at the point where the line on the graph crosses the y-axis?
(e) Describe two different modifications you could make to this set-up so that the cart would have zero acceleration.

Communication

10. Draw a free-body diagram of a diver being lowered into the water from a hovering helicopter to make a sea rescue. His downward speed is decreasing. Label all forces and show them with correct scale lengths.

11. Use free-body diagrams to show that the tension in the rope is the same for both of the following situations.
(a) Two horses are pulling in opposite directions on the same rope, with equal and opposite forces of 800 N.
(b) One horse is pulling on a rope, which is tied to a tree, with a force of 800 N.

12. A toy rocket is shot straight into the air and reaches a height of 162 m. It begins its descent in free fall for 2 s before its parachute opens. The rocket then quickly reaches terminal velocity.
(a) Sketch a velocity-time graph for the descent.
(b) Draw a free-body diagram for each of the three passes of the descent: the free fall, the parachute opening and slowing the descent, and terminal velocity

13. A large crate sits on the floor of an elevator. The force of static friction keeps the crate from moving. However, the magnitude of this force changes when the elevator **(a)** is stationary, **(b)** accelerates downward and **(c)** accelerates upward. Explain how the three forces should be ranked from weakest to strongest.

14. Two blocks, of mass M and mass m, are in contact on a horizontal frictionless table (with the block of mass M on the left and the block of mass m on the right). A force F_1 is applied to the block of mass M and the two blocks accelerate together to the right.
(a) Draw a free-body diagram for each block.
(b) Suppose the larger block M exerts a force F_2 on the smaller mass m. By Newton's third law, the smaller block m exerts a force F_2 on the larger block M. Argue whether $F_1 = F_2$ or not. Justify your reasoning.
(c) Derive an expression for the acceleration of the system.
(d) Derive an expression for the magnitude of the force F_2 that the larger block exerts on the smaller block.
(e) Choose different values of M and m (e.g. $M = 2m$, $M = 5m$, including the case $M = m$) and compare the magnitudes of F_1 and F_2.
(f) Comment on the above results.

Making Connections

15. Car tires are designed to optimize the amount of friction between the tire surface and the road. If there is too little friction, the car will be hard to control. Too much friction will negatively affect the car's performance and fuel efficiency.
(a) List the different types of road conditions under which cars are operated. Research the different types of tread designs that have been developed to respond to these conditions. Explain how the different designs are intended to increase or decrease the coefficient of friction between a car's tires and the road.
(b) Compare the positive and negative factors of using "all-season" tires rather than changing car tires to suit the season (e.g., changing to special winter tires). Do a cost analysis of the two systems and recommend your choice for the climatic conditions in your own community.

Problems for Understanding

16. Suppose a marble is rolling with a velocity of 3.0 m/s[N] and no horizontal force is acting on it. What will be its velocity at 10.0 s?

17. What is the mass of a sack of potatoes that weighs 110 N (1.1×10^2 N)?

18. A physics teacher is in an elevator moving upward at a velocity of 3.5 m/s when he drops his watch. What are the initial velocity and acceleration of the watch in a frame that is attached to **(a)** the elevator and **(b)** the building?

19. **(a)** What is the acceleration of a 68.0 kg crate that is pushed across the floor by a 425 N force, if the coefficient of kinetic friction between the box and floor is 0.500?

(b) What force would be required to push the crate across the floor with constant velocity?

20. A red ball that weighs 24.5 N and a blue ball that weighs 39.2 N are connected by a piece of elastic of negligible mass. The balls are pulled apart, stretching the elastic. If the balls are released at exactly the same time, the initial acceleration of the red ball is 1.8 m/s^2 eastward. What is the initial acceleration of the blue ball?

21. If a 0.24 kg ball is accelerated at 5.0 m/s^2, what is the magnitude of the force acting on it?

22. A 10.0 kg brick is pulled from rest along a horizontal bench by a constant force of 4.0 N. It is observed to move a distance of 2.0 m in 8.0 s.

(a) What is the acceleration of the brick?

(b) What is the ratio of the applied force to the mass?

(c) Explain why your two answers above do not agree. Use numerical calculations to support your explanation.

23. A football is thrown deep into the end zone for a touchdown. If the ball was in the air for 2.1 s and air friction is neglected, to what vertical height must it have risen?

24. A 2200 kg car is travelling at 45 km/h when its brakes are applied and it skids to a stop. If the coefficient of friction between the road and the tires is 0.70, how far does the car go before stopping?

25. A 55.0 kg woman jumps to the floor from a height of 1.5 m.

(a) What is her velocity at the instant before her feet touch the floor?

(b) If her body comes to rest during a time interval of 8.00×10^{-3} s, what is the force of the floor on her feet?

26. You are pushing horizontally on a 3.0 kg block of wood, pressing it against a wall. If the coefficient of static friction between the block and the wall is 0.60, how much force must you exert on the block to prevent it from sliding down?

27. The maximum acceleration of a truck is 2.6 m/s^2. If the truck tows another truck with a mass the same as its own, what is its maximum acceleration?

28. A force F produces an acceleration a when applied to a certain body. If the mass of the body is doubled and the force is increased fivefold, what will be the effect on the following?

(a) the acceleration of the body

(b) the distance travelled by the body in a given time

29. A 45.0 kg box is pulled with a force of 205 N by a rope held at an angle of 46.5° to the horizontal. The velocity of the box increases from 1.00 m/s to 1.50 m/s in 2.50 s. Calculate

(a) the net force acting horizontally on the box.

(b) the frictional force acting on the box.

(c) the horizontal component of the applied force.

(d) the coefficient of kinetic friction between the box and the floor.

30. A Fletcher's trolley apparatus consists of a 4.0 kg cart and a 2.0 kg mass attached by a string that runs over a pulley. Find the acceleration of the trolley and the tension in the string when the suspended mass is released.

31. You are a passenger on an airplane and you decide to measure its acceleration as it travels down the runway before taking off. You take out a yo-yo and notice that when you suspend it, it makes an angle of 25° with the vertical. Assume the plane's mass is 4.0×10^3 kg.

(a) What is the acceleration of the airplane?

(b) If the yo-yo mass is 65 g, what is the tension in the string?

CHAPTER 2 Dynamics in Two Dimensions

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Resolving vectors into components
- Combining vector components

Dancers spin, twist, and swing through the air. Athletes move in constantly changing directions. The details of these complex motions are studied by kinesiologists by tracking the position of sensors fastened to knees, elbows, or other joints. Position-time data gathered from such experiments can be used to produce photo-realistic animations for movies and video games. This data can also be used to study the details of the motion from a physicist's point of view, providing a basis for measurement of changes in speed and direction — accelerations and the forces that cause them.

When you turn while you run, walk, or dance, you are moving in two dimensions. You might change direction suddenly or do it over several steps, following a curved path. In either case, changes in direction are accelerations, and accelerations require an unbalanced force. In this chapter, you will examine the accelerations and forces involved in two types of two-dimensional motion — objects following a curved path after being launched into the air and objects moving in a circle or part of a circle.

MULTI LAB

Motion in Two Dimensions

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting

CAUTION Wear impact-resistant safety goggles. Also, do not stand close to other people or equipment while doing these activities.

Race to the Ground

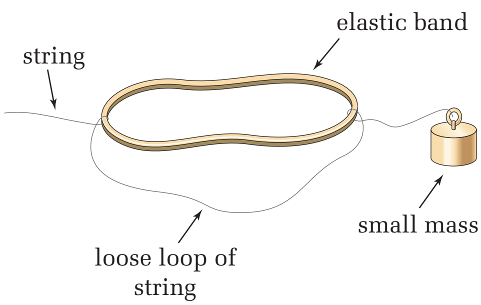
If your school has a vertical acceleration demonstrator, set it up to make observations. If you do not have a demonstrator, devise a method for launching one object, such as a small metal ball, in the horizontal direction, while at the same instant dropping a second object from exactly the same height. Perform several trials, observing the paths of the objects very carefully.

Analyze and Conclude

1. Describe in detail the paths of the two objects. Compare the motion of the two objects.
2. Which object hit the floor first?
3. Did the horizontal motion of the first object appear to affect its vertical motion? Explain your reasoning for your conclusion.

Feel the Force

According to the law of inertia, objects must experience an unbalanced force to change the direction of their motion. What does this suggest about an object moving in a circle? Assemble the apparatus as shown in the diagram to obtain information on the forces involved in circular motion. Gently swing the mass in a horizontal circle. Carefully increase the speed of rotation and observe the effect on the elastic band and the path of the object. Change the angle so that the object moves first in an inclined plane and then in a vertical plane and repeat your observations.



Analyze and Conclude

1. How does the force exerted on the object by the elastic band change as the elastic band stretches?
2. How does the force exerted on the object change as the speed of the mass increases?
3. Sketch free-body diagrams showing the forces acting on the object as it moves in a
   (a) horizontal plane
   (b) vertical plane (at the top of the swing, the bottom of the swing, and when it is at one side of the circle)
4. Describe and attempt to explain any other changes you observed in the object's motion as its speed varied.
5. Was there any difference in the force exerted by the elastic band at the highest and lowest points of the mass's path when it moved in a vertical plane? If so, suggest an explanation.

2.1 Projectile Motion

SECTION EXPECTATIONS

- Describe qualitatively and quantitatively the path of a projectile.
- Analyze, predict, and explain projectile motion in terms of horizontal and vertical components.
- Design and conduct experiments to test the predictions about the motion of a projectile.

KEY TERMS

- trajectory
- projectile
- range
- parabola

HISTORY LINK

Calculating actual trajectories of artillery shells was an enormous task before the advent of electronic computers. Human experts required up to 20 h to do the job, even with mechanical calculators. ENIAC, the first electronic computer that stored its program instructions, was built in 1946 and could calculate the trajectory of an artillery shell in about 30 s. The vacuum tubes in ENIAC's processing unit required 174 kW of electric power, so the energy required to calculate one trajectory was comparable to the energy needed to actually fire the artillery shell!

Figure 2.1 After the water leaves the pipes in this fountain, the only forces acting on the water are gravity and air friction.

A tourist visiting Monte Carlo, Monaco, would probably stand and admire the beauty of the fountain shown in the photograph, and might even toss a coin into the fountain and make a wish. A physics student, however, might admire the symmetry of the water jets. He or she might estimate the highest point that the water reaches and the angle at which it leaves the fountain, and then mentally calculate the initial velocity the water must have in order to reach that height.

The student might then try to think of as many examples of this type of motion as possible. For example, a golf ball hit off the tee, a leaping frog, a punted football, and a show-jumping horse all follow the same type of path or **trajectory** as the water from a fountain. Any object given an initial thrust and then allowed to soar through the air under the force of gravity only is called a **projectile**. The horizontal distance that the projectile travels is called its **range**.

Air friction does, of course, affect the trajectory of a projectile and therefore the range of the projectile, but the mathematics needed to account for air friction is complex. You can learn a great deal about the trajectory of projectiles by neglecting friction, while keeping in mind that air friction will modify the actual motion.

You do not need to learn any new concepts in order to analyze and predict the motion of projectiles. All you need are data that will provide you with the velocity of the projectile at the moment it is launched and the kinematic equations for uniformly accelerated motion. You observed projectile motion in the Race to the Ground segment of the Multi-Lab and identified a feature of the motion that simplifies the analysis. The horizontal motion of the projectile does not influence the vertical motion, nor does the vertical motion affect the horizontal motion. *You can treat the motion in the two directions independently.* The following points will help you analyze all instances of projectile motion.

- Gravity is the only force influencing ideal projectile motion. (Neglect air friction.)
- Gravity affects only the vertical motion, so equations for uniformly accelerated motion apply.
- No forces affect horizontal motion, so equations for uniform motion apply.
- The horizontal and vertical motions are taking place during the same time interval, thus providing a link between the motion in these dimensions.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of two-dimensional motion, go to your Electronic Learning Partner for an interactive activity.

Projectiles Launched Horizontally

If you had taken a picture with a strobe light of your Race to the Ground lab, you would have obtained a photograph similar to the one in Figure 2.2. The ball on the right was given an initial horizontal velocity while, at the same moment, the ball on the left was dropped. As you can see in the photograph, the two balls were the same distance from the floor at any given time — the vertical motion of the two balls was identical. This observation verifies that horizontal motion does not influence vertical motion. Examine the following sample problem to learn how to make use of this feature of projectile motion.

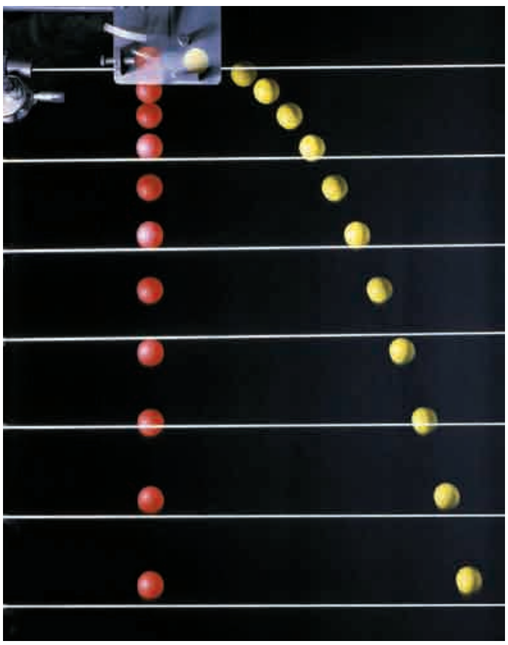


Figure 2.2 You can see that the balls are accelerating downward, because the distances they have travelled between flashes of the strobe light are increasing. If you inspected the horizontal motion of the ball on the right, you would find that it travelled the same horizontal distance between each flash of the strobe light.

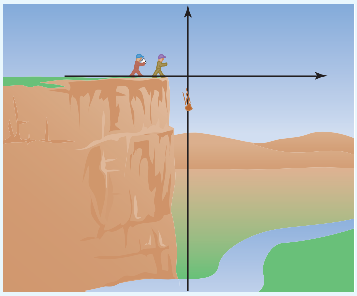
SAMPLE PROBLEM

Analyzing a Horizontal Projectile

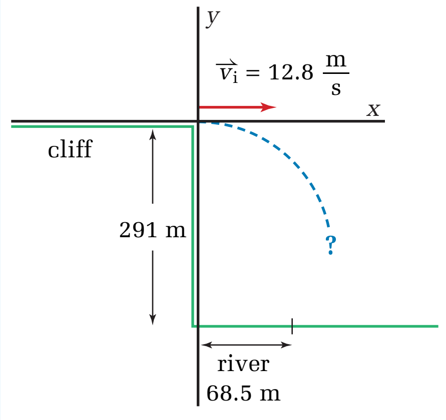
While hiking in the wilderness, you come to a cliff overlooking a river. A topographical map shows that the cliff is 291 m high and the river is 68.5 m wide at that point. You throw a rock directly forward from the top of the cliff, giving the rock a horizontal velocity of 12.8 m/s.

(a) Did the rock make it across the river?

(b) With what velocity did the rock hit the ground or water?



Conceptualize the Problem



- Start to frame the problem by making a rough sketch of the cliff with a *coordinate system* superimposed on it. Write the initial conditions on the sketch.
- The rock *initially* has *no vertical velocity*. It falls, from *rest*, with the *acceleration due to gravity*. Since "down" was chosen as negative, the *acceleration* of the rock is *negative*. (Neglect air friction.)
- Since the coordinate system was placed at the top of the cliff, the *vertical component* of the *displacement* of the rock is *negative*.
- The *displacement* that the rock falls determines the *time interval* during which it falls, according to the kinematic equations.
- The rock moves *horizontally* with a *constant velocity* until it hits the ground or water at the end of the time interval.
- The *final velocity* of the rock at the instant before it hits the ground or water is the *vector sum* of the horizontal velocity and the final vertical velocity.
- Use x to represent the horizontal component of displacement and y for the vertical component of displacement. Use x and y subscripts to identify the horizontal and vertical components of the velocity.

Identify the Goal

(a) Whether the horizontal distance, Δx, travelled by the rock was greater than 68.5 m, the width of the river

(b) The final velocity, $\vec{v}_f$, of the rock the instant before it hit the ground

Identify the Variables

Known		Implied	Unknown
$\Delta y = -291$ m	river width = 68.5 m	$a_y = -9.81\ \frac{m}{s^2}$	Δx
$v_x = 12.8\ \frac{m}{s}$		$v_{iy} = 0.0\ \frac{m}{s}$	$\vec{v}_f$

Develop a Strategy

Find the time interval during which the rock was falling by using the kinematic equation that relates displacement, initial velocity, acceleration, and time interval. Note that the vertical component of the initial velocity is zero and solve for the time interval.

$$\Delta y = v_{yi}\Delta t + \frac{1}{2}a\Delta t^2$$
$$\Delta y = \frac{1}{2}a\Delta t^2$$
$$\frac{2\Delta y}{a} = \Delta t^2$$
$$\Delta t = \sqrt{\frac{2\Delta y}{a}}$$

Insert numerical values and solve.

$$\Delta t = \sqrt{\frac{2(-291\ \cancel{\text{m}})}{-9.81\ \frac{\cancel{\text{m}}}{\text{s}^2}}}$$
$$\Delta t = \sqrt{59.327\ \text{s}^2}$$
$$\Delta t = \pm 7.7024\ \text{s}$$

Find the horizontal displacement of the rock by using the equation for uniform motion (constant velocity) that relates velocity, distance, and time interval. Solve for displacement.

$$v_x = \frac{\Delta x}{\Delta t}$$
$$\Delta x = v_x \Delta t$$

Use the time calculated above and initial velocity to calculate the horizontal distance travelled by the rock. Choose the positive value for time, since negative time has no meaning in this application.

$$\Delta x = \left(12.8\ \frac{\text{m}}{\cancel{\text{s}}}\right)(7.7024\ \cancel{\text{s}})$$
$$\Delta x = 98.591\ \text{m}$$
$$\Delta x \cong 98.6\ \text{m}$$

(a) Since the horizontal distance travelled by the rock (98.6 m) was much greater than the width of the river (68.5 m), the rock hit the ground on the far side of the river.

Find the vertical component of the final velocity by using the kinematic equation that relates initial velocity, final velocity, acceleration, and time.

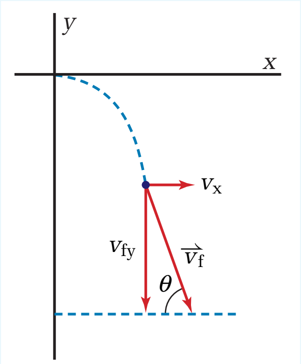
$$v_{fy} = v_{iy} + a\Delta t$$

Insert the numerical values and solve.

$$v_{fy} = 0.0\ \frac{\text{m}}{\text{s}} + \left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)(7.7024\ \text{s})$$
$$v_{fy} = -75.561\ \frac{\text{m}}{\text{s}}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity.

$$|\vec{v}_f| = \sqrt{(v_x)^2 + (v_{fy})^2}$$
$$|\vec{v}_f| = \sqrt{\left(12.8\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-75.561\ \frac{\text{m}}{\text{s}}\right)^2}$$
$$|\vec{v}_f| = \sqrt{5873.30\ \frac{\text{m}^2}{\text{s}^2}}$$
$$|\vec{v}_f| = 76.637\ \frac{\text{m}}{\text{s}}$$
$$|\vec{v}_f| \cong 76.6\ \frac{\text{m}}{\text{s}}$$



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Use trigonometry to find the angle that the rock made with the horizontal when it struck the ground.

$$\tan\theta = \frac{v_{fy}}{v_x}$$
$$\theta = \tan^{-1}\frac{v_{fy}}{v_x}$$
$$\theta = \tan^{-1}\frac{75.561\ \frac{m}{s}}{12.8\ \frac{m}{s}}$$
$$\theta = \tan^{-1}5.9032$$
$$\theta = 80.385^\circ$$
$$\theta \cong 80.4^\circ$$

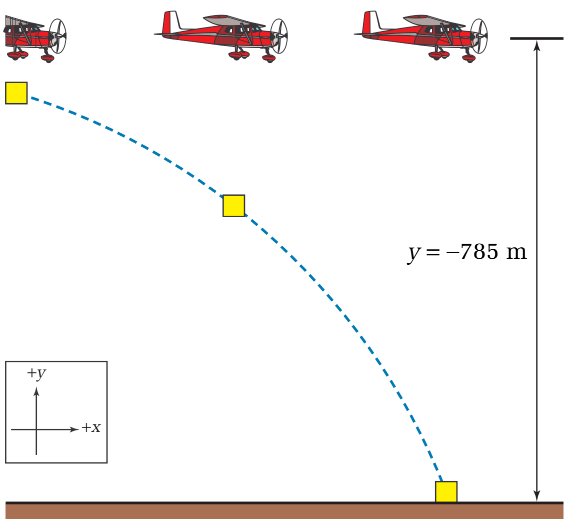
(b) The rock hit the ground with a velocity of 76.6 m/s at an angle of 80.4° with the horizontal.

Validate the Solution

The distance that the rock fell vertically was very large, so you would expect that the rock would be travelling very fast and that it would hit the ground at an angle that was nearly perpendicular to the ground. Both conditions were observed.

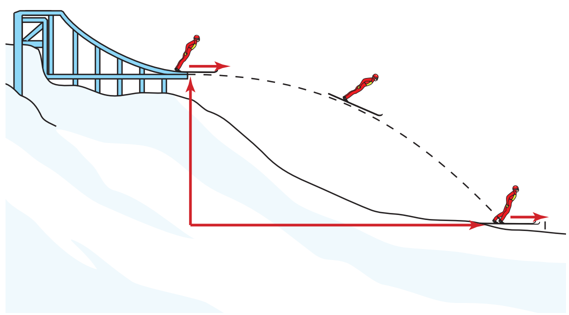
PRACTICE PROBLEMS

1. An airplane is dropping supplies to northern villages that are isolated by severe blizzards and cannot be reached by land vehicles. The airplane is flying at an altitude of 785 m and at a constant horizontal velocity of 53.5 m/s. At what horizontal distance before the drop point should the co-pilot drop the supplies so that they will land at the drop point? (Neglect air friction.)



2. A cougar is crouched on the branch of a tree that is 3.82 m above the ground. He sees an unsuspecting rabbit on the ground, sitting 4.12 m from the spot directly below the branch on which he is crouched. At what horizontal velocity should the cougar jump from the branch in order to land at the point at which the rabbit is sitting?

3. A skier leaves a jump with a horizontal velocity of 22.4 m/s. If the landing point is 78.5 m lower than the end of the ski jump, what horizontal distance did the skier jump? What was the skier's velocity when she landed? (Neglect air friction.)



4. An archer shoots an arrow toward a target, giving it a horizontal velocity of 70.1 m/s. If the target is 12.5 m away from the archer, at what vertical distance below the point of release will the arrow hit the target? (Neglect air friction.)

5. In a physics experiment, you are rolling a golf ball off a table. If the tabletop is 1.22 m above the floor and the golf ball hits the floor 1.52 m horizontally from the table, what was the initial velocity of the golf ball?

6. As you sit at your desk at home, your favourite autographed baseball rolls across a shelf at 1.0 m/s and falls 1.5 m to the floor. How far does it land from the base of the shelf?

7. A stone is thrown horizontally at 22 m/s from a canyon wall that is 55 m high. At what distance from the base of the canyon wall will the stone land?

8. A sharpshooter shoots a bullet horizontally over level ground with a velocity of 3.00×10^2 m/s. At the instant that the bullet leaves the barrel, its empty shell casing falls vertically and strikes the ground with a vertical velocity of 5.00 m/s.
   (a) How far does the bullet travel?
   (b) What is the vertical component of the bullet's velocity at the instant before it hits the ground?

Projectiles Launched at an Angle

Most projectiles, including living ones such as the playful dolphins in Figure 2.3, do not start their trajectory horizontally. Most projectiles, from footballs to frogs, start at an angle with the horizontal. Consequently, they have an initial velocity in both the horizontal and vertical directions. These trajectories are described mathematically as **parabolas**. The only additional step required to analyze the motion of projectiles launched at an angle is to determine the magnitude of the horizontal and vertical components of the initial velocity.

Mathematically, the path of any ideal projectile lies along a parabola. In the following investigation, you will develop some mathematical relationships that describe parabolas. Then, the sample problems that follow will help you apply mathematical techniques for analyzing projectiles.

Figure 2.3 Dolphins have been seen jumping as high as 4.9 m from the surface of the water in a behaviour called a "breach."

INVESTIGATION 2-A

The Components of Projectile Motion

TARGET SKILLS
- Analyzing and interpreting
- Modelling concepts
- Communicating results

A heavy steel ball rolling up and down a ramp follows the same type of trajectory that a projectile follows. You will obtain a permanent record of the steel ball's path by placing a set of white paper and carbon paper in its path. You will then analyze the vertical and horizontal motion of the ball and find mathematical relationships that describe the path.

Problem

What patterns exist in the horizontal and vertical components of projectile velocity?

Hypothesis

Formulate a hypothesis about the relationships between time and the vertical distance travelled by the steel ball.

Equipment

- large sheet of plywood
- very heavy steel ball
- metre stick, graph paper, tape
- set of white paper and carbon paper (or pressure-sensitive paper)

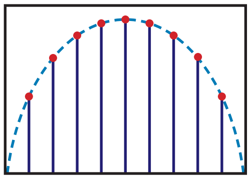
Procedure

1. Set up the apparatus as illustrated.
   CAUTION Wear impact-resistant safety goggles. Also, do not stand close to other people or equipment while doing these activities.

2. Practise rolling the steel ball up the slope at an angle, so that it follows a curved path that will fit the size of your set of white paper and carbon paper.
3. Tape the carbon paper and white paper onto the plywood so that, when the steel ball rolls over it, the carbon paper will leave marks on the white paper.
4. Roll the steel ball up the slope at an angle, as you practised, so that it will roll over the paper and leave a record of its path.
5. Remove the white paper from the plywood. Draw approximately nine or more equally spaced lines vertically through the trajectory.

Analyze and Conclude

1. Measure the vertical displacement in each segment of the path of the steel ball, as shown in the diagram.



2. Assuming that the motion of the ball was uniform in the horizontal direction, each equally spaced vertical line represents the same amount of time. Call it one unit of time.
3. Separate your data into two parts: (a) the period of time that the ball was rolling upward and (b) the period of time that the ball was rolling downward. For each set of data, make a graph of vertical-distance-versus-time units.
4. Use curve-straightening techniques to convert your graphs to straight lines. (See Skill Set 4.)
5. Write equations to describe your graphs.
6. Is the vertical motion of the steel ball uniform or uniformly accelerated?
7. How does it compare to the vertical motion of a freely falling object?
8. Was your hypothesis valid or invalid?
9. Is this lab an appropriate model for actual projectile motion? Explain why or why not.

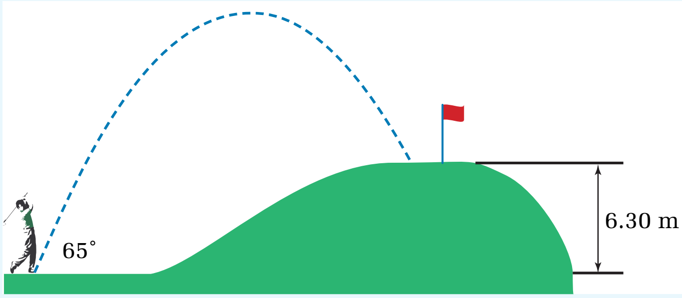
Analyzing Parabolic Trajectories

1. A golfer hits the golf ball off the tee, giving it an initial velocity of 32.6 m/s at an angle of 65° with the horizontal. The green where the golf ball lands is 6.30 m higher than the tee, as shown in the illustration. Find

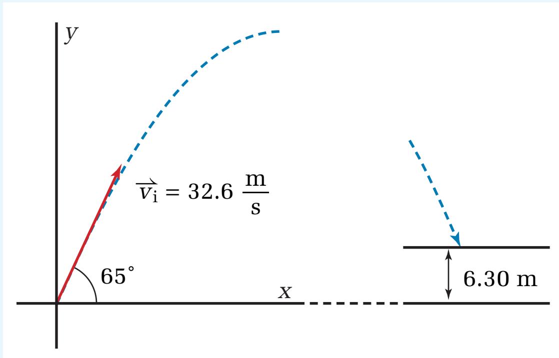
(a) the time interval during which the golf ball was in the air

(b) the horizontal distance that it travelled

(c) the velocity of the ball just before it hit the ground (neglect air friction)



Conceptualize the Problem



- Start to frame the problem by making a sketch that includes a *coordinate system*, the *initial conditions*, and all of the known information.
- The golf ball has a *positive initial velocity* in the *vertical* direction. It will rise and then fall according to the kinematic equations.
- The *vertical acceleration* of the golf ball is *negative* and has the *magnitude* of the acceleration due to *gravity*.
- The *time interval* is determined by the *vertical* motion. The *time interval ends* when the golf ball is at a height equal to the *height of the green*.
- The golf ball will be at the height of the green *twice*, once while it is *rising* and once while it is *falling*.
- Motion in the *horizontal* direction is *uniform*; that is, it has a *constant velocity*.
- The horizontal *displacement of the ball* depends on the *horizontal component* of the *initial velocity* and on the duration of the flight.

Identify the Goal

(a) The time interval, Δt, that the golf ball was in the air

(b) The horizontal distance, Δx, that the golf ball travelled

(c) The final velocity of the golf ball, $\vec{v}_f$

Identify the Variables

Known		Implied	Unknown	
$\lvert\vec{v}_i\rvert = 32.6\ \frac{m}{s}$	$\Delta y = 6.30$ m	$a_y = -9.81\ \frac{m}{s^2}$	Δt	$\vec{v}_f$
$\theta_i = 65°$			Δx	θ_f
			v_{ix}	v_{iy}

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Develop a Strategy

Find the horizontal and vertical components of the initial velocity.

$$v_{ix} = |\vec{v}_i| \cos\theta \qquad v_{iy} = |\vec{v}_i| \sin\theta$$

$$v_{ix} = 32.6\ \frac{m}{s} \cos 65^\circ \qquad v_{iy} = 32.6\ \frac{m}{s} \sin 65^\circ$$

$$v_{ix} = 13.78\ \frac{m}{s} \qquad v_{iy} = 29.55\ \frac{m}{s}$$

Find the time interval at which the ball is at a vertical position of 6.30 m by using the kinematic equation that relates displacement, initial velocity, acceleration, and the time interval.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$

You cannot solve directly for the time interval, because you have a quadratic equation.
Substitute in the numerical values.

$$6.30\text{ m} = 29.55\ \frac{m}{s}\Delta t + \frac{1}{2}\left(-9.81\ \frac{m}{s^2}\right)\Delta t^2$$

Rearrange the equation into the general form of a quadratic equation and solve using the quadratic formula $\Delta t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$4.905\Delta t^2 - 29.55\Delta t + 6.30 = 0$$

$$\Delta t = \frac{29.55 \pm \sqrt{(29.55)^2 - 4(4.905)(6.30)}}{2(4.905)}$$

$$\Delta t = \frac{29.55 \pm \sqrt{749.597}}{9.81}$$

$$\Delta t = 0.2213\text{ s (or) } 5.803\text{ s}$$

$$\Delta t \cong 5.8\text{ s}$$

(a) The smaller value is the time that the ball reached a height of 6.30 m when it was rising. The golf ball hit the green 5.8 s after it was hit off the tee.

Use 5.803 s and the equation for constant velocity to determine the horizontal distance travelled by the golf ball.

$$v = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v\Delta t$$

$$\Delta x = \left(13.78\ \frac{m}{\cancel{s}}\right)(5.803\ \cancel{s})$$

$$\Delta x = 79.965\text{ m}$$

$$\Delta x \cong 8.0 \times 10^1\text{ m}$$

(b) The golf ball travelled 80 m in the horizontal direction.

Find the vertical component of the final velocity by using the kinematic equation that relates the initial and final velocities to the acceleration and the time interval.

$$v_{fy} = v_{iy} + a_y\Delta t$$

$$v_{fy} = 29.55\ \frac{m}{s} + \left(-9.81\ \frac{m}{s^{\cancel{2}}}\right)(5.803\ \cancel{s})$$

$$v_{fy} = -27.38\ \frac{m}{s}$$

Use the Pythagorean theorem to find the magnitude of the final velocity.

$$|\vec{v}_f| = \sqrt{\left(13.78\ \frac{m}{s}\right)^2 + \left(-27.38\ \frac{m}{s}\right)^2}$$

$$|\vec{v}_f| = \sqrt{939.55\ \frac{m^2}{s^2}}$$

$$|\vec{v}_f| = 30.65\ \frac{m}{s}$$

$$|\vec{v}_f| \cong 31\ \frac{m}{s}$$

Use trigonometry to find the angle that the final velocity makes with the horizontal.

$$\tan\theta = \left(\frac{v_{fy}}{v_x}\right)$$

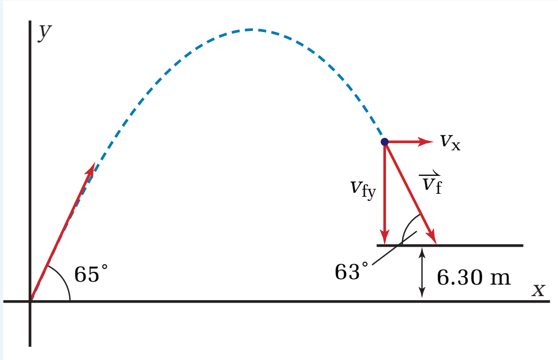
$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_x}\right)$$

$$\theta = \tan^{-1}\frac{|-27.38\ \frac{m}{s}|}{|13.78\ \frac{m}{s}|}$$

$$\theta = \tan^{-1} 1.9869$$

$$\theta = 63.28^\circ$$

$$\theta \cong 63^\circ$$



(c) The final velocity of the golf ball just before it hit the ground was 31 m/s at 63° with the horizontal.

Validate the Solution

Since the golf ball hit the ground at a level slightly higher than the level at which it started, you would expect the final velocity to be slightly smaller than the initial velocity and the angle to be a little smaller than the initial angle. These results were obtained. All of the units cancelled properly.

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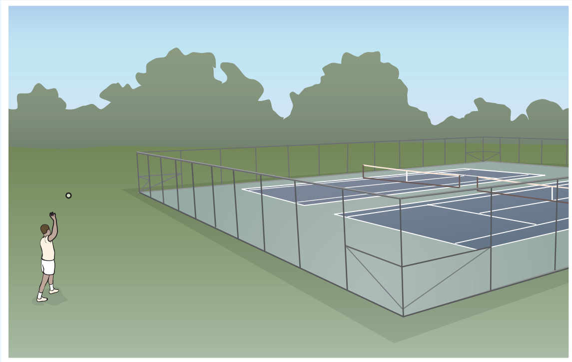
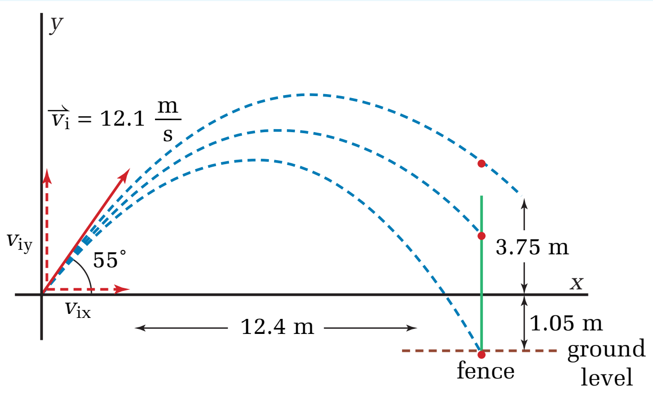
2. You are playing tennis with a friend on tennis courts that are surrounded by a 4.8 m fence. Your opponent hits the ball over the fence and you offer to retrieve it. You find the ball at a distance of 12.4 m on the other side of the fence. You throw the ball at an angle of 55.0° with the horizontal, giving it an initial velocity of 12.1 m/s. The ball is 1.05 m above the ground when you release it. Did the ball go over the fence, hit the fence, or hit the ground before it reached the fence? (Ignore air friction.)



Conceptualize the Problem

- Make a sketch of the *initial conditions* and the three options listed in the question.
- Choose the *origin* of the coordinate system to be at the point at which the *ball left your hand*.
- The equations for *uniformly accelerated motion* apply to the *vertical* motion.
- The definition for *constant velocity* applies to the *horizontal* motion.
- Because the x-axis is above ground level, you will have to determine where the top of the fence is relative to the x-axis.
- The *time interval* is the *link* between the *vertical* motion and the *horizontal* motion. Finding the time interval required for the ball to reach the position of the fence will allow you to determine the *height* of the ball when it reaches the fence.



Identify the Goal

Whether the ball went over the fence, hit the fence, or hit the ground before reaching the fence

Identify the Variables

Known		Implied	Unknown	
$\lvert\vec{v}_i\rvert = 12.1\ \frac{m}{s}$	$\Delta x = 12.4$ m	$a_y = -9.81\ \frac{m}{s^2}$	Δt	$\vec{v}_{iy}$
$\theta = 55°$	$h = 4.8$ m		$\vec{v}_{ix}$	Δy

Develop a Strategy

Find the x- and y-components of the initial velocity.

$$v_{ix} = \lvert\vec{v}_i\rvert \cos\theta \qquad v_{iy} = \lvert\vec{v}_i\rvert \sin\theta$$

$$v_{ix} = 12.1\ \frac{m}{s}\cos 55° \qquad v_{iy} = 12.1\ \frac{m}{s}\sin 55°$$

$$v_{ix} = 6.940\ \frac{m}{s} \qquad v_{iy} = 9.912\ \frac{m}{s}$$

To find the time interval, use the equation for the definition of constant velocity and the data for motion in the horizontal direction.

$$v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta t = \frac{12.4\ \text{m}}{6.940\ \frac{\text{m}}{\text{s}}}$$

$$\Delta t = 1.787\ \text{s}$$

To find the height of the ball at the time that it reaches the fence, use the kinematic equation that relates displacement, acceleration, initial velocity, and time interval.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$

$$\Delta y = \left(9.912\ \frac{\text{m}}{\text{s}}\right)(1.787\ \text{s}) + \frac{1}{2}\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.787\ \text{s})^2$$

$$\Delta y = 2.05\ \text{m}$$

Determine the position of the top of the fence in the chosen coordinate system.

$$y_{\text{fence}} = h - y_{\text{ground to } x\text{–axis}}$$

$$y_{\text{fence}} = 4.8\ \text{m} - 1.05\ \text{m}$$

$$y_{\text{fence}} = 3.75\ \text{m}$$

The ball hit the fence. The fence is 3.75 m above the horizontal axis of the chosen coordinate system, but the ball was only 2.05 m above the horizontal axis when it reached the fence.

Validate the Solution

The units all cancel correctly. The time of flight (about 1.8 s) and the height of the ball (about 2 m) are reasonable values.

PRACTICE PROBLEMS

9. While hiking in the wilderness, you come to the top of a cliff that is 60.0 m high. You throw a stone from the cliff, giving it an initial velocity of 21 m/s at 35° above the horizontal. How far from the base of the cliff does the stone land?

10. A batter hits a baseball, giving it an initial velocity of 41 m/s at 47° above the horizontal. It is a home run, and the ball is caught by a fan in the stands. The vertical component of the velocity of the ball when the fan caught it was –11 m/s. How high is the fan seated above the field?

11. During baseball practice, you go up into the bleachers to retrieve a ball. You throw the ball back into the playing field at an angle of 42° above the horizontal, giving it an initial velocity of 15 m/s. If the ball is 5.3 m above the level of the playing field when you throw it, what will be the velocity of the ball when it hits the ground of the playing field?

12. Large insects such as locusts can jump as far as 75 cm horizontally on a level surface. An entomologist analyzed a photograph and found that the insect's launch angle was 55°. What was the insect's initial velocity?

You have learned to make predictions about projectile motion by doing calculations, but can you make any predictions about patterns of motion without doing calculations? In the following Quick Lab, you will make and test some qualitative predictions.

QUICK LAB

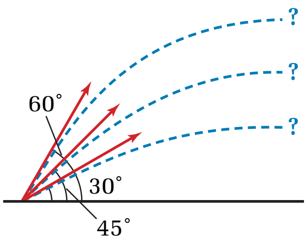
Maximum Range of a Projectile

TARGET SKILLS

- Initiating and planning
- Predicting
- Performing and recording

Football punters try to maximize "hang time" to give their teammates an opportunity to rush downfield while the ball is in the air. Small variations in the initial velocity, especially the angle, make the difference between a great kick and good field position for the opposition.

What launch angle above the horizontal do you predict would maximize the range of an ideal projectile? Make a prediction and then, if your school has a projectile launcher, test your prediction by launching the same projectile several times at the same speed, but at a variety of different angles. If you do not have a projectile launcher, try to devise a system that will allow you to launch a projectile consistently with the same speed but at different angles. Carry out enough trials so you can be confident that you have found the launch angle that gives the projectile the longest range. Always consult with your teacher before using a launch system.



Analyze and Conclude

1. What effect do very large launch angles have on the following quantities?
   - **(a)** maximum height
   - **(b)** vertical velocity component
   - **(c)** horizontal velocity component
   - **(d)** range
2. What effect do very small launch angles have on the above quantities?
3. Did you see any patterns in the relationship between the launch angle and the range of the projectile? If so, describe these patterns.
4. How well did your experimental results match your prediction?
5. What factors might be causing your projectile to deviate from the ideal?
6. Suppose your experimental results were quite different from your prediction. In which number would you place more confidence, your theoretical prediction or your experimental results? Why?

Symmetrical Trajectories

If a projectile lands at exactly the same level from which it was launched and air friction is neglected, the trajectory is a perfectly symmetrical parabola, as shown in Figure 2.4. You can derive some general relationships that apply to all symmetrical trajectories and use them to analyze these trajectories. Follow the steps in the next series of derivations to see how to determine the time of flight, the range, and the maximum height for projectiles that have symmetrical trajectories.

ELECTRONIC LEARNING PARTNER

Enhance your knowledge and test your predictive skills by doing the projectile motion interactive activity provided by your Electronic Learning Partner.

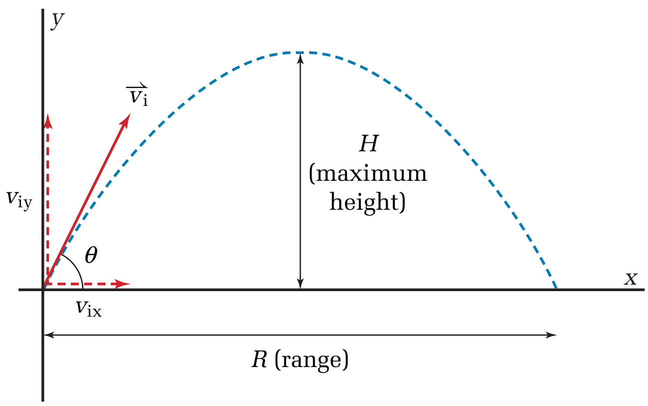


Figure 2.4 The maximum height, *H*, and the range, *R*, as well as the time of flight, *T*, are functions of the initial velocity, $\vec{v}_i$, the angle, θ, and the acceleration due to gravity, *g*.

Time of flight

- The time of flight ends when the projectile hits the ground. Since the height of the projectile is zero when it hits the ground, you can express this position as $\Delta y = 0$. Write the kinematic equation for vertical displacement and set $\Delta y = 0$.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$$
$$0 = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$$

- Write the vertical component of the velocity in terms of the initial velocity and the angle θ. Then, substitute the expression into the equation above. Also, substitute $-g$ for the acceleration, a.

$$v_{iy} = |\vec{v}_i|\sin\theta$$
$$0 = (|\vec{v}_i|\sin\theta)\Delta t + \frac{1}{2}(-g)\Delta t^2$$

- Rearrange the equation to put the zero on the right-hand side and factor out a Δt.

$$\frac{1}{2}g\Delta t^2 - |\vec{v}_i|\Delta t\sin\theta = 0$$
$$\Delta t\left(\frac{1}{2}g\Delta t - |\vec{v}_i|\sin\theta\right) = 0$$

- If either factor is zero, the equation above is satisfied. Write the two solutions.

$$\Delta t = 0 \quad \text{or} \quad \frac{1}{2}g\Delta t = |\vec{v}_i|\sin\theta$$
$$\Delta t = \frac{2|\vec{v}_i|\sin\theta}{g}$$

- $\Delta t = 0$ represents the instant that the projectile was launched. Therefore, the second expression represents the time of flight, T, that the projectile spent in the air before it landed. Since T is a scalar, write the initial velocity without a vector symbol.

$$T = \frac{2v_i\sin\theta}{g}$$

Range

- The range is the horizontal distance that the projectile has travelled when it hits the ground. Write the equation for displacement in the horizontal direction.

$$\Delta x = v_{ix}\Delta t$$

- Write the expression for the horizontal component of velocity in terms of the initial velocity and the launch angle θ. Substitute this expression into the equation for the displacement above.

$$v_{ix} = |\vec{v}_i| \cos\theta$$
$$\Delta x = (|\vec{v}_i| \cos\theta)\Delta t$$

- Since the projectile is at the endpoint of its range, R, when $\Delta t = T$, substitute the expression for T into the equation and simplify. Since R is always in one dimension, omit the vector symbol for the initial velocity.

$$R = \frac{(v_i \cos\theta)(2v_i \sin\theta)}{g}$$
$$R = \frac{v_i^2 2\sin\theta\cos\theta}{g}$$

- Write the trigonometric identity for $2\sin\theta\cos\theta$ and substitute the simpler form into the equation.

$$2\sin\theta\cos\theta = \sin 2\theta$$
$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Maximum height

- As a projectile rises, it slows its upward motion, stops, and then starts downward. Therefore, at its maximum height, its vertical component of velocity is zero. Write the kinematic equation that relates initial and final velocities, acceleration, and displacement and solve for displacement, Δy.

$$v_{fy}^2 = v_{iy}^2 + 2a\Delta y$$
$$0 = v_{iy}^2 + 2a\Delta y$$
$$2a\Delta y = -v_{iy}^2$$
$$\Delta y = \frac{-v_{iy}^2}{2a}$$

- Substitute the expression for initial vertical velocity in terms of the initial velocity and the launch angle, θ. Substitute $-g$ for a. Now Δy is the maximum height, H. Since H is always in one dimension, omit the vector symbol for the initial velocity.

$$H = \frac{-(|\vec{v}_i| \sin\theta)^2}{2(-g)}$$
$$H = \frac{v_i^2 \sin^2\theta}{2g}$$

These three relationships — time of flight, range, and the maximum height — allow you to make important predictions about projectile motion without performing calculations. For example, you can determine the launch angle that will give you the maximum range by inspecting the equation for range. Study the logic of the following steps.

- Inspect the equation for range.

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

- For a given initial velocity on the surface of Earth, the only variable is θ. Therefore, the term "$\sin 2\theta$" determines the maximum range. The largest value that the sine of any angle can achieve is 1.

$$\sin 2\theta = 1$$
$$R_{max} = \frac{v_i^2(1)}{g}$$

- For what angle, θ, is $\sin 2\theta = 1$? Recall that the angle for which the sine is 1 is 90°. Use this information to find θ.

$$\sin 2\theta = 1$$
$$2\theta = \sin^{-1} 1$$
$$2\theta = 90^\circ$$
$$\theta = 45^\circ$$

For any symmetrical trajectory, neglecting air friction, the launch angle that yields the greatest range is 45°. Draw some conclusions of your own by answering the questions in the Conceptual Problems that follow.

Conceptual Problems

- Examine the equation for maximum height. For a given initial velocity, what launch angle would give a projectile the greatest height? What would be the shape of its trajectory?
- Examine the equation for time of flight. For a given initial velocity, what launch angle would give a projectile the greatest time of flight? Would this be a good angle for a football punter? Why?
- Consider the equation for range and a launch angle of 30°. What other launch angle would yield a range exactly equal to that of the range for an angle of 30°?
- Find another pair of launch angles (in addition to your answer to the above question) that would yield identical ranges.
- The acceleration due to gravity on the Moon is roughly one sixth of that on Earth ($g_{\text{moon}} = \frac{1}{6}g$). For a projectile with a given initial velocity, determine the time of flight, range, and maximum height on the Moon relative to those values on Earth.
- The general equation for a parabola is $y = \text{A}x^2 + \text{B}x + \text{C}$, where A, B, and C are constants. Start with the following equations of motion for a projectile and develop one equation in terms of Δx and Δy by eliminating Δt. Show that the resulting equation, in which Δy is a function of Δx, describes a parabola. Note that the values for the initial velocity (v_{i}) and launch angle (θ) are constants for a given trajectory.

$$\Delta x = v_{\text{i}}\Delta t \cos\theta$$
$$\Delta y = v_{\text{i}}\Delta t \sin\theta - \frac{1}{2}g\Delta t^2$$

SAMPLE PROBLEM

Analyzing a Kickoff

A player kicks a football for the opening kickoff. He gives the ball an initial velocity of 29 m/s at an angle of 69° with the horizontal. Neglecting friction, determine the ball's maximum height, hang time, and range.

Conceptualize the Problem

- A football field is level, so the trajectory of the ball is a *symmetrical parabola*.
- You can use the equations that were developed for symmetrical trajectories.
- "Hang time" is the time of flight of the ball.

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Identify the Goal

The maximum height, H, of the football
The time of flight, T, of the football
The range, R, of the football

Identify the Variables and Constants

Known	Implied	Unknown
$\lvert\vec{v}_i\rvert = 29\ \frac{\text{m}}{\text{s}}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	H
$\theta = 69°$		T
		R

Develop a Strategy

Use the equation for the maximum height of a symmetrical trajectory.

$$H = \frac{v_i^2 \sin^2\theta}{2g}$$

Substitute the numerical values and solve.

$$H = \frac{\left(29\ \frac{\text{m}}{\text{s}}\right)^2(\sin 69°)^2}{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$H = \frac{\left(841\ \frac{\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}\right)(0.871\ 57)}{19.62\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}$$

$$H = 37.359\ \text{m}$$

$$H \cong 37\ \text{m}$$

The maximum height the football reached was 37 m.

Use the equation for the time of flight of a symmetrical trajectory.

$$T = \frac{2v_i \sin\theta}{g}$$

Substitute the numerical values and solve.

$$T = \frac{2\left(29\ \frac{\text{m}}{\text{s}}\right)(\sin 69°)}{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$T = \frac{\left(58\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)(0.933\ 58)}{9.81\ \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}}$$

$$T = 5.5196\ \text{s}$$

$$T \cong 5.5\ \text{s}$$

The time of flight, or hang time, of the football was 5.5 s.

Use the equation for the range of a symmetrical trajectory.

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Substitute the numerical values and solve.

$$R = \frac{\left(29\ \frac{\text{m}}{\text{s}}\right)^2(\sin 2(69°))}{9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$R = \frac{\left(841\ \frac{\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}\right)(0.669\ 13)}{9.81\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}$$

$$R = 57.3637\ \text{m}$$

$$R \cong 57\ \text{m}$$

The football travelled 57 m.

Validate the Solution

All of the values are reasonable for a football kickoff. In every case, the units cancel properly to give metres for the range and maximum height and seconds for the time of flight, or hang time.

PRACTICE PROBLEMS

13. A circus stunt person was launched as a human cannon ball over a Ferris wheel. His initial velocity was 24.8 m/s at an angle of 55˚. (Neglect friction)
   - **(a)** Where should the safety net be positioned?
   - **(b)** If the Ferris wheel was placed halfway between the launch position and the safety net, what is the maximum height of the Ferris wheel over which the stunt person could travel?
   - **(c)** How much time did the stunt person spend in the air?

14. You want to shoot a stone with a slingshot and hit a target on the ground 14.6 m away. If you give the stone an initial velocity of 12.5 m/s, neglecting friction, what must be the launch angle in order for the stone to hit the target? What would be the maximum height reached by the stone? What would be its time of flight?

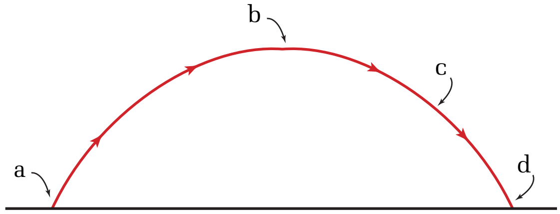
2.1 Section Review

1. K/U Projectiles travel in two dimensions at the same time. Why is it possible to apply kinematic equations for one dimension to projectile motion?

2. K/U How does the analysis of projectiles launched at an angle differ from the analysis of projectiles launched horizontally?

3. C Explain why time is a particularly significant parameter when analyzing projectile motion.

4. C What can you infer about the velocity at each labelled point on the trajectory in this diagram?



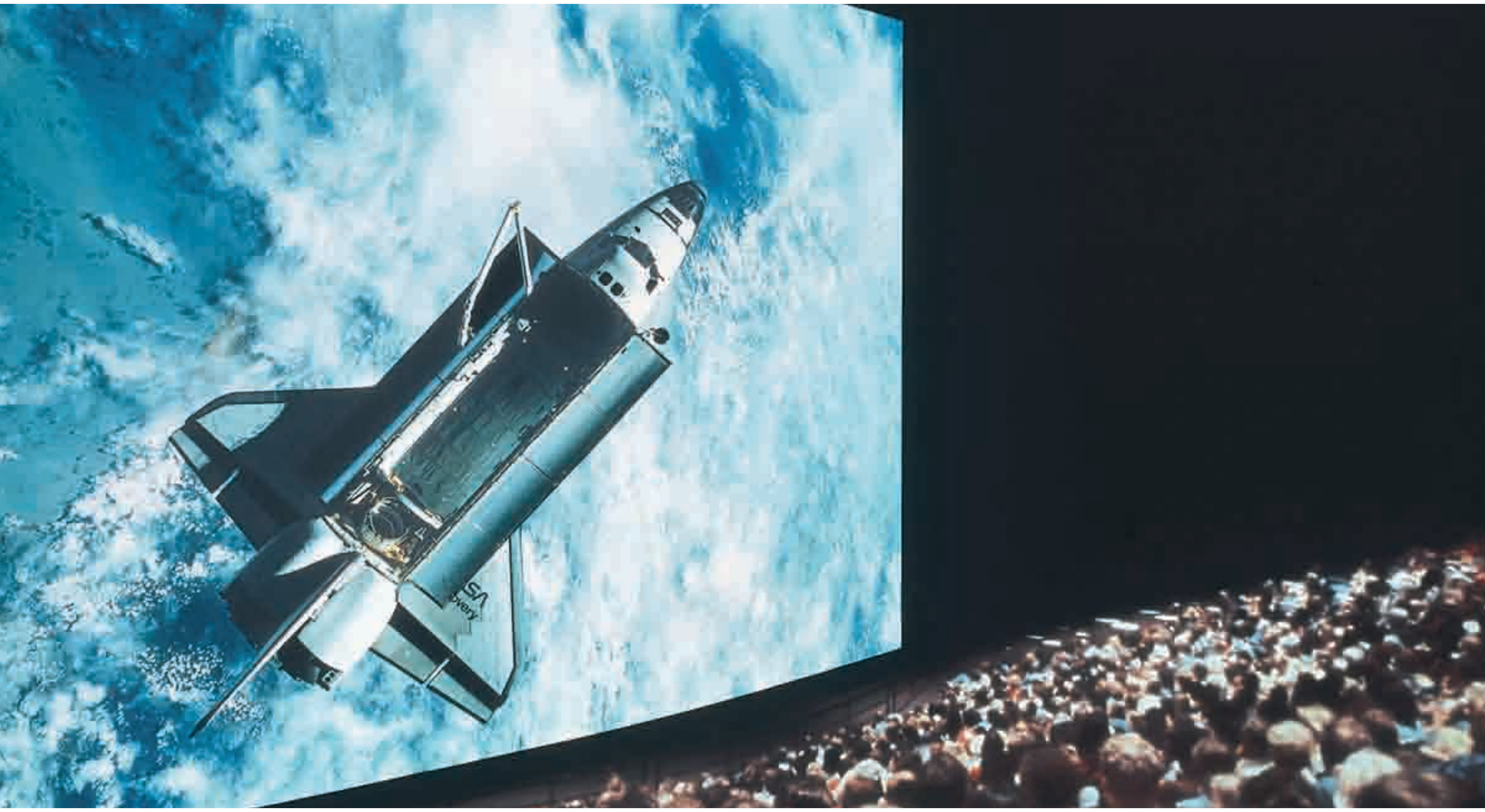
5. C Imagine that you are solving a problem in projectile motion in which you are asked to find the time at which a projectile reaches a certain vertical position. When you solve the problem, you find two different positive values for time that both satisfy the conditions of the problem. Explain how this result is not only possible, but also logical.

6. K/U What properties of projectile motion must you apply when deriving an equation for the maximum height of a projectile?

7. K/U What properties of projectile motion must you apply when deriving an equation for the range of a projectile?

8. I Suppose you knew the maximum height reached by a projectile. Could you find its launch angle from this information alone? If not, what additional information would be required?

The BIG Motion Picture: An IMAX Interview



"Filling people's peripheral vision with image to the point that they lose the sense of actually watching a picture and become totally absorbed in the medium" is the goal of the IMAX Corporation, which has been making and screening large-format films since 1970. Former IMAX executive vice-president of technology Michael Gibbon went on to say in a recent interview, "If you give people a very large image, you can almost disconnect them from reality. They become very involved with the 'thing' they are seeing."

IMAX develops and supplies all of the equipment used by filmmakers and theatres to create an exciting and enthralling film experience — the camera, the projector, and even the enormous movie screen.

The roots of the IMAX system go back to Montréal's Expo '67, where films shown simultaneously on multiple wide screens by several standard 35 mm movie theatre projectors became very popular. A small group of Canadians involved in making some of those films decided to design a new system using a single, powerful projector, rather than the cumbersome multiple projectors. The resulting IMAX system premiered at Expo '70 in Osaka, Japan, and the first permanent IMAX projection system was installed at Toronto's Ontario Place in 1971. In 1997, IMAX Corporation won an Oscar, the highest award of the Academy of Motion Picture Arts and Sciences, for scientific and technical achievement.

We spoke to Gibbon, who joined IMAX in 1986 and is now a consultant to the corporation, about the technical challenges IMAX faces when producing its large-screen films.

Q: How did IMAX create the technology to give people this sense of total immersion in the image?

A: Sensibly, IMAX chose the largest film format that was commercially available, rather than have Kodak produce something new. It was 70 mm, but IMAX turned it on its side and advanced it 15 perforations at a time.

Q: Can you explain a bit more about the film stock and film frames?

A: A filmstrip is a series of individual frames with perforations that run along the sides to help feed film through the projector. Today's cinemas show films with a frame size of 35 mm and advance it four perforations at a time. 70 mm existed when our corporation was starting up, but IMAX's choice of advancing it 15 perforations at a time was fairly revolutionary.

Q: What was the first challenge?

A: There were a number of 70 mm projectors in existence from quite early in the history of cinema, advancing five perforations of film at a time. There were many more 35 mm projectors advancing four perforations of film at a time. Our challenge was to move a format three times larger than 70 mm/5 perforations, and to do that in such a way that the film and film frame not only survived the process, but also were steady when projected. We needed steadiness because we were going to sit people very close to a very large image.

Q: In terms of film motion, what was the problem, exactly?

A: It's the sheer dynamics of the film. You're trying to advance the film quickly. You need to run it at 24 frames per second. That's the standard rate of film advancement. Also, the 35 mm mechanism is fairly rough on a film. It has a high acceleration rate, so the stresses on the perforations are not minor — you can damage the film. When the frame comes to a rest, it can deform, particularly around the perforations, so that you're not absolutely sure where you're going to finish up.

Q: So the frame "overshoots" too far or "undershoots" not far enough?

A: Yes, this was a known problem in the 35 mm projectors. The IMAX Rolling Loop projector was created to solve the problem of advancing more film quickly, yet making sure that the film was firmly in place for exposure in the aperture. The fundamental advantage it has over the 35 mm projectors is fixed registration pins.

Q: Where are registration pins? How do they work?

A: They are pins that are fixed on either side of the aperture. They simply hold the film in place when it is being illuminated.

Q: How does the Rolling Loop work?

A: There is a rotor and on the periphery of that rotor are a total of eight gaps. The film is induced to build or loop up into the gaps, and the rotor rotates. Essentially, what it's doing is lifting up the film, putting it into the gap, and rolling it along.

Q: Did you experiment with different interior motions inside the projector?

A: What evolved after a number of experiments was a deceleration cam. This takes the film, which is coming in at almost two metres per second, grabs it in the last part of its travel, slowly brings it down in a controlled manner and then puts it onto the registration pins at a very low final velocity. The pins hold the frame in a very precise location and then it's vacuumed up against the lens to keep the entire frame in focus.

Q: What are you looking at for the future?

A: Digital is an obvious consideration, but the image quality is not at the level of film. It doesn't yet have the ability to depict fine detail or to produce the same amount of light. But we're working on it.

Making Connections

1. IMAX films are known for creating a sense of motion, instead of simply showing a picture of it. How does IMAX do this?
2. Research the differences between the IMAX Rolling Loop projector and a standard 35 mm projector.

2.2 Uniform Circular Motion

SECTION EXPECTATIONS

- Analyze, predict, and explain uniform circular motion.
- Explain forces involved in uniform circular motion in horizontal and vertical planes.
- Investigate relationships between period and frequency of an object in uniform circular motion.

KEY TERMS

- uniform circular motion
- centripetal acceleration
- centripetal force

Have you ever ridden on the Round Up at the Canadian National Exhibition, the ride shown in the photograph? From a distance, it might not look exciting, but the sensations could surprise you.

Everyone lines up around the outer edge and the ride slowly begins to turn. Not very exciting yet, but soon, the ride is spinning quite fast and you feel as though you are being pressed tightly against the wall. The rotations begin to make you feel disoriented and your stomach starts to feel a little queasy. Then, suddenly, the floor drops away, but you stay helplessly "stuck" to the wall. Just as you realize that you are not going to fall, the entire ride begins to tilt. At one point during each rotation, you find yourself looking toward the ground, which is almost directly in front of you. You do not feel as though you are going to fall, though, because you are literally stuck to the wall.

Figure 2.5 If this ride stopped turning, the people would start to fall. What feature of circular motion prevents people from falling when the ride is in motion and they are facing the ground?

What is unique about moving in a circle that allows you to apparently defy gravity? What causes people on the Round Up to stick to the wall? As you study this section, you will be able to answer these questions and many more.

Centripetal Acceleration

Amusement park rides are only one of a very large number of examples of circular motion. Motors, generators, vehicle wheels, fans, air in a tornado or hurricane, or a car going around a curve are other examples of circular motion. When an object is moving in a circle and its speed — the magnitude of its velocity — is constant, it is said to be moving with **uniform circular motion**. The direction of the object's velocity is always tangent to the circle. Since the direction of the motion is always changing, the object is always accelerating.

Figure 2.6 shows the how the velocity of the object changes when it is undergoing uniform circular motion. As an object moves from point P to point Q, its velocity changes from $\vec{v}_1$ to $\vec{v}_2$. Since the direction of the acceleration is the same as the direction of the *change* in the velocity, you need to find $\Delta\vec{v}$ or $\vec{v}_2 - \vec{v}_1$. Vectors and $\vec{v}_1$ and $\vec{v}_2$ are subtracted graphically under the circle. To develop an equation for centripetal acceleration, you will first need to show that the triangle OPQ is similar to the triangle formed by the velocity vectors, as shown in the following points.

- $r_1 = r_2$ because they are radii of the same circle. Therefore, triangle OPQ is an isosceles triangle.
- $|\vec{v}_1| = |\vec{v}_2|$ because the speed is constant. Therefore, the triangle formed by $-\vec{v}_1$, $\vec{v}_2$, and $\Delta\vec{v}$ is an isosceles triangle.
- $r_1 \perp \vec{v}_1$ and $r_2 \perp \vec{v}_2$ because the radius of a circle is perpendicular to the tangent to the point where the radius contacts the circle.
- $\theta_r = \theta_v$ because the angle between corresponding members of sets of perpendicular lines are equal.
- Since the angles between the equal sides of two isosceles triangles are equal, the triangles are similar.

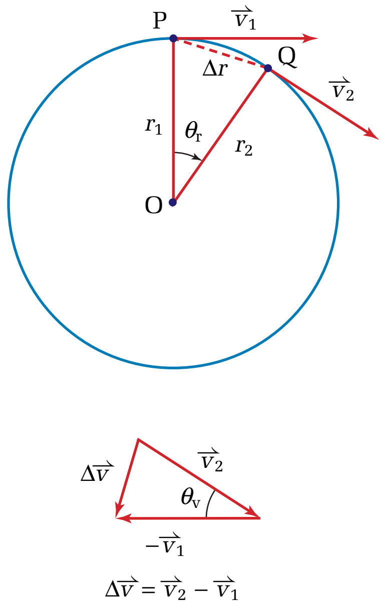


Figure 2.6 The direction of the change in velocity is found by defining the vector $-\vec{v}_1$ and then adding $\vec{v}_2$ and $-\vec{v}_1$. Place the tail of $-\vec{v}_1$ at the tip of $\vec{v}_2$ and draw the resultant vector, $\Delta\vec{v}$, from the tail of $\vec{v}_2$ to the tip of $-\vec{v}_1$.

Now use the two similar triangles to find the magnitude of the acceleration. Since the derivation involves only magnitudes, omit vector notations.

- The ratios of the corresponding sides of similar triangles are equal. There is no need to distinguish between the sides r_1 and r_2 or v_1 and v_2, because the radii are equal and the magnitudes of the velocities are equal.
$$\frac{\Delta r}{r} = \frac{\Delta v}{v}$$
- The object travelled from point P to point Q in the time interval Δt. Therefore, the magnitude of the object's displacement along the arc from P to Q is
$$\Delta d = v\Delta t$$
- The length of the arc from point P to point Q is almost equal to Δr. As the angle becomes very small, the lengths become more nearly identical.
$$\Delta r = v\Delta t$$
- Substitute this value of Δr into the first equation.
$$\frac{v\Delta t}{r} = \frac{\Delta v}{v}$$

MATH LINK

Mathematicians have developed a unique system for defining components of vectors such as force, acceleration, and velocity for movement on curved paths, even when the magnitude of the velocity is changing. Any curve can be treated as an arc of a circle. So, instead of using the *x*- and *y*-components of the typical Cartesian coordinate system, the vectors are divided into tangential and radial components. The tangential component is the component of the vector that is tangent to the curved path at the point at which the object is momentarily located. The radial component is perpendicular to the path and points to the centre of the circle defined by the arc or curved section of the path. Radial components are the same as centripetal components.

- Divide both sides of the equation by Δt. $\quad \frac{v}{r} = \frac{\Delta v}{v\Delta t}$
- Recall the definition of acceleration. $\quad a = \frac{\Delta v}{\Delta t}$
- Substitute a into the equation for $\frac{\Delta v}{\Delta t}$. $\quad \frac{v}{r} = \frac{a}{v}$
- Multiply both sides of the equation by v. $\quad a = \frac{v^2}{r}$

The magnitude of the acceleration of an object moving with uniform circular motion is $a = v^2/r$. To determine its direction, again inspect the triangle formed by the velocity vectors in Figure 2.6. The acceleration is changing constantly, so imagine a vector $\vec{v}_2$ as close to $\vec{v}_1$ as possible. The angle θ is extremely small. In this case, $\Delta\vec{v}$ is almost exactly perpendicular to both $\vec{v}_1$ and $\vec{v}_2$. Since $\vec{v}_1$ and $\vec{v}_2$ are tangent to the circle and therefore are perpendicular to the associated radii of the circle, the acceleration vector points directly toward the centre of the circle.

Describing the acceleration vector in a typical Cartesian coordinate system would be extremely difficult, because the direction is always changing and, therefore, the magnitude of the *x*- and *y*-components would always be changing. It is much simpler to specify only the magnitude of the acceleration, which is constant for uniform circular motion, and to note that the direction is always toward the centre of the circle. To indicate this, physicists speak of a "centre-seeking acceleration" or **centripetal acceleration**, which is denoted as a_c, without a vector notation.

CENTRIPETAL ACCELERATION

Centripetal acceleration is the quotient of the square of the velocity and the radius of the circle.

$$a_c = \frac{v^2}{r}$$

Quantity	Symbol	SI unit
centripetal acceleration	a_c	$\frac{\text{m}}{\text{s}^2}$ (metres per second squared)
velocity (magnitude)	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
radius (of circle)	r	m (metres)

Unit Analysis

$$\frac{\text{metre}}{\text{second}^2} = \frac{\left(\frac{\text{metre}}{\text{second}}\right)^2}{\text{metre}} \qquad \frac{\left(\frac{\text{m}}{\text{s}}\right)^2}{\text{m}} = \frac{\frac{\text{m}^{\not 2}}{\text{s}^2}}{\not\text{m}} = \frac{\text{m}}{\text{s}^2}$$

Note: The direction of the centripetal acceleration is always along a radius pointing toward the centre of the circle.

Centripetal Force

According to Newton's laws of motion, an object will accelerate only if a force is exerted on it. Since an object moving with uniform circular motion is always accelerating, there must always be a force exerted on it in the same direction as the acceleration, as illustrated in Figure 2.7. If at any instant the force is withdrawn, the object will stop moving along the circular path and will proceed to move with uniform motion, that is, in a straight line that is tangent to the circular path on which it had been moving.

Since the force causing a centripetal acceleration is always pointing toward the centre of the circular path, it is called a **centripetal force**. The concept of centripetal force differs greatly from that of other forces that you have encountered. It is not a type of force such as friction or gravity. It is, instead, a force that is *required* in order for an object to move in a circular path.

A centripetal force can be supplied by any type of force. For example, as illustrated in Figure 2.8, gravity provides the centripetal force that keeps the Moon on a roughly circular path around Earth, friction provides a centripetal force that causes a car to move in a circular path on a flat road, and the tension in a string tied to a ball will cause the ball to move in a circular path when you twirl it around. In fact, two different types of force could act together to provide a centripetal force.

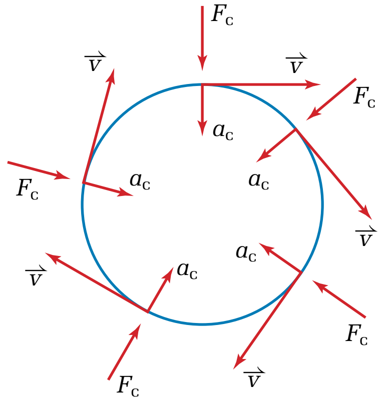


Figure 2.7 A force acting perpendicular to the direction of the velocity is always required in order for any object to move continuously along a circular path.

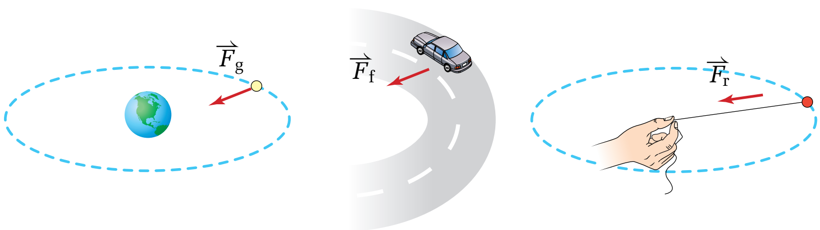


Figure 2.8 Any force that is directed toward the centre of a circle can provide a centripetal force.

You can determine the magnitude of a centripetal force required to cause an object to travel in a circular path by applying Newton's second law to a mass moving with a centripetal acceleration.

- Write Newton's second law. $\vec{F} = m\vec{a}$
- Write the equation describing centripetal acceleration. $a_c = \dfrac{v^2}{r}$
- Substitute into Newton's second law. Omit vector notations because the force and acceleration always point toward the centre of the circular path. $F_c = \dfrac{mv^2}{r}$

The equation for the centripetal force required to cause a mass m moving with a velocity v to follow a circular path of radius r is summarized in the following box.

CENTRIPETAL FORCE

The magnitude of the centripetal force is the quotient of the mass times the square of the velocity and the radius of the circle.

$$F_c = \frac{mv^2}{r}$$

Quantity	**Symbol**	**SI unit**
centripetal force	F_c	N (newtons)
mass	m	kg (kilograms)
velocity	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
radius of circular path	r	m (metres)

Unit Analysis

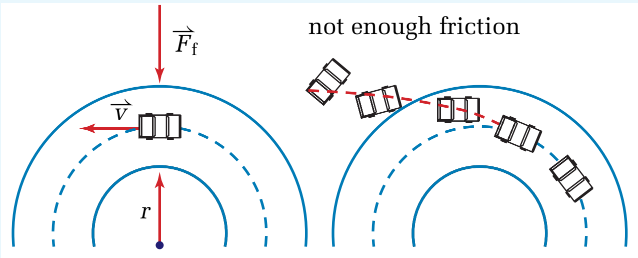
$$(\text{newtons}) = \left(\frac{\text{kilogram}\left(\frac{\text{metres}}{\text{second}}\right)^2}{\text{metres}}\right)$$

$$N = \frac{\text{kg}\left(\frac{\text{m}}{\text{s}}\right)^2}{\text{m}} = \frac{\text{kg}\frac{\cancel{\text{m}^2}}{\text{s}^2}}{\cancel{\text{m}}} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \text{N}$$

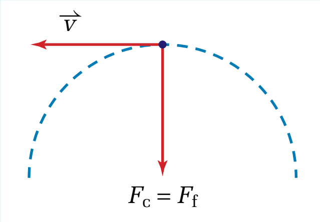
SAMPLE PROBLEMS

Centripetal Force in a Horizontal and a Vertical Plane

1. A car with a mass of 2135 kg is rounding a curve on a level road. If the radius of curvature of the road is 52 m and the coefficient of friction between the tires and the road is 0.70, what is the maximum speed at which the car can make the curve without skidding off the road?



Conceptualize the Problem



- Make a sketch of the motion of the car and the forces acting on it.
- The *force of friction* must provide a sufficient *centripetal force* to cause the car to follow the curved road.
- The magnitude of *force* required to keep the car on the road depends on the *velocity* of the car, its *mass*, and the *radius of curvature* of the road.
- Since r is in the denominator of the expression for centripetal force, as the *radius* becomes *smaller*, the amount of *force* required becomes *greater*.
- Since v is in the numerator, as the *velocity* becomes *larger*, the *force* required to keep the car on the road becomes *greater*.

Identify the Goal

The maximum speed, v, at which the car can make the turn

Identify the Variables

Known		Implied	Unknown	
$m = 2135$ kg	$\mu = 0.70$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	F_f	F_N
$r = 52$ m			v	

Develop a Strategy

Set the frictional force equal to the centripetal force.

$$F_f = F_c$$
$$\mu F_N = \frac{mv^2}{r}$$

Since the car is moving on a level road, the normal force of the road is equal to the weight of the car. Substitute mg for F_N.

$$\mu mg = \frac{mv^2}{r}$$

Solve for the velocity.

$$v^2 = \mu \cancel{m} g\left(\frac{r}{\cancel{m}}\right)$$
$$v = \sqrt{\mu rg}$$

Substitute in the numerical values and solve.

$$v = \sqrt{(0.70)(52\ \text{m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$v = \sqrt{357.08\ \frac{\text{m}^2}{\text{s}^2}}$$
$$v = 18.897\ \frac{\text{m}}{\text{s}}$$
$$v \cong 19\ \frac{\text{m}}{\text{s}}$$

If the car is going faster than 19 m/s, it will skid off the road.

Validate the Solution

A radius of curvature of 52 m is a sharp curve. A speed of 19 m/s is equivalent to 68 km/h, which is a high speed at which to take a sharp curve. The answer is reasonable. The units cancelled properly to give metres per second for velocity.

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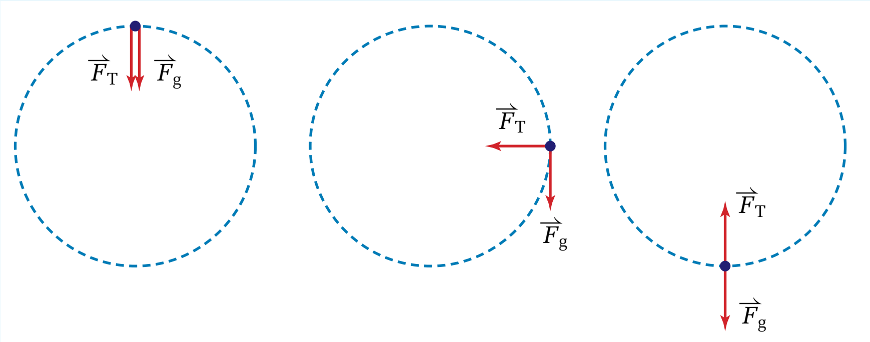
2. **You are playing with a yo-yo with a mass of 225 g. The full length of the string is 1.2 m. You decide to see how slowly you can swing it in a vertical circle while keeping the string fully extended, even when the yo-yo is at the top of its swing.**

(a) **Calculate the minimum speed at which you can swing the yo-yo while keeping it on a circular path.**

(b) **At the speed that you determine in part (a), find the tension in the string when the yo-yo is at the side and at the bottom of its swing.**

Conceptualize the Problem

- Draw *free-body diagrams* of the yo-yo at the *top, bottom, and one side* of the swing.



- At the *top* of the swing, both *tension and the force of gravity* are acting *toward the centre* of the circle.
- If the required *centripetal force* is *less than the force of gravity*, the yo-yo will *fall* away from the circular path.
- If the required *centripetal force* is *greater than the force of gravity*, the *tension* in the string *will* have to *contribute* to the centripetal force.
- Therefore, the *smallest possible velocity* would be the case where the required *centripetal force* is exactly *equal* to the *force of gravity*.
- At the *side* of the swing, the force of *gravity is perpendicular* to the direction of the required centripetal force and therefore contributes *nothing*. The centripetal force must all be supplied by the tension in the string.
- At the *bottom* of the swing, the force of *gravity* is in the *opposite* direction from the required *centripetal force*. Therefore, the *tension* in the string must *balance* the force of *gravity* and *supply* the required *centripetal force*.

Identify the Goal

The minimum speed, v, at which the yo-yo will stay on a circular path
The tension, F_T, in the string when the yo-yo is at the side of its circular path
The tension, F_T, in the string when the yo-yo is at the bottom of its circular path

Identify the Variables

Known	Implied	Unknown
$m = 225$ kg	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	v_{min}
$r = 1.2$ m		$F_{\text{T(side)}}$
		$F_{\text{T(bottom)}}$

Develop a Strategy

Set the force of gravity on the yo-yo equal to the centripetal force and solve for the velocity.

$$F_g = F_c$$
$$mg = \frac{mv^2}{r}$$
$$\cancel{m}g\left(\frac{r}{\cancel{m}}\right) = v^2$$
$$v = \sqrt{gr}$$

Substitute numerical values and solve.

$$v = \sqrt{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.2\ \text{m})}$$
$$v = \sqrt{11.772\ \frac{\text{m}^2}{\text{s}^2}}$$

A negative answer has no meaning in this application.

$$v = \pm 3.431\ \frac{\text{m}}{\text{s}}$$
$$v \cong 3.4\ \frac{\text{m}}{\text{s}}$$

(a) The minimum speed at which the yo-yo can move is 3.4 m/s.

Set the force of tension in the string equal to the centripetal force. Insert numerical values and solve.

$$F_T = F_c$$
$$F_T = \frac{mv^2}{r}$$
$$F_T = \frac{(225\ \cancel{\text{g}})\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)\left(3.431\ \frac{\text{m}}{\text{s}}\right)^2}{1.2\ \text{m}}$$
$$F_T = 2.207\ \frac{\text{kg}\frac{\text{m}^{\cancel{2}}}{\text{s}^2}}{\cancel{\text{m}}}$$
$$F_T \cong 2.2\ \text{N}$$

(b): Side – When the yo-yo is at the side of its swing, the tension in the string is 2.2 N.

Set the centripetal force equal to the vector sum of the force of tension in the string and the gravitational force. Solve for the force due to the tension in the string.

$$F_c = F_T + F_g$$
$$\frac{mv^2}{r} = F_T - mg$$
$$F_T = \frac{mv^2}{r} + mg$$

Substitute numerical values and solve.

$$F_T = \frac{(225\ \cancel{\text{g}})\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)\left(3.431\ \frac{\text{m}}{\text{s}}\right)^2}{1.2\ \text{m}} + (225\ \cancel{\text{g}})\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)$$
$$F_T = 2.207\ \frac{\frac{\text{kg}\cdot\text{m}^{\cancel{2}}}{\text{s}^2}}{\cancel{\text{m}}} + 2.207\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$F_T = 4.414\ \text{N}$$
$$F_T \cong 4.4\ \text{N}$$

(b): Bottom – When the yo-yo is at the bottom of its swing, the tension in the string is 4.4 N.

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Validate the Solution

The force of gravity (weight) of the yo-yo is 2.2 N. At the top of the swing, the weight supplies the entire centripetal force and the speed of the yo-yo is determined by this value. At the side of the swing, the tension must provide the centripetal force and the problem was set up so that the centripetal force had to be equal to the weight of the yo-yo, or 2.2 N. At the bottom of the swing, the tension must support the weight (2.2 N) and, in addition, provide the required centripetal force (2.2 N). You would therefore expect that the tension would be twice the weight of the yo-yo. The units cancel properly to give newtons for force.

PRACTICE PROBLEMS

15. A boy is twirling a 155 g ball on a 1.65 m string in a horizontal circle. The string will break if the tension reaches 208 N. What is the maximum speed at which the ball can move without breaking the string?

16. An electron (mass 9.11×10^{-31} kg) orbits a hydrogen nucleus at a radius of 5.3×10^{-11} m at a speed of 2.2×10^{6} m/s. Find the centripetal force acting on the electron. What type of force supplies the centripetal force?

17. A stone of mass 284 g is twirled at a constant speed of 12.4 m/s in a vertical circle of radius 0.850 m. Find the tension in the string **(a)** at the top and **(b)** at the bottom of the revolution. **(c)** What is the maximum speed the stone can have if the string will break when the tension reaches 33.7 N?

18. You are driving a 1654 kg car on a level road surface and start to round a curve at 77 km/h. If the radius of curvature is 129 m, what must be the frictional force between the tires and the road so that you can safely make the turn?

19. A stunt driver for a movie needs to make a 2545 kg car begin to skid on a large, flat, parking lot surface. The force of friction between his tires and the concrete surface is 1.75×10^{4} N and he is driving at a speed of 24 m/s. As he turns more and more sharply, what radius of curvature will he reach when the car just begins to skid?

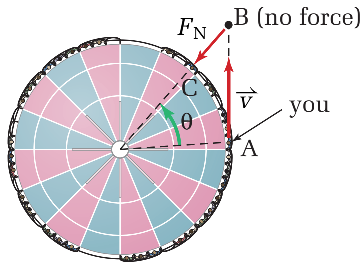


Figure 2.9 Assume that the Round Up ride is rotating at a constant speed and you are at point A. After a short time interval, in the *absence* of a force acting on you, you would move to point B, radially outward from point C. A centripetal force is required to change the direction of your velocity and place you at point C.

Centripetal Force versus Centrifugal Force

You read in Chapter 1, Fundamentals of Dynamics, that a centrifugal force is a fictitious force. Now that you have learned about centripetal forces, you can understand more clearly why a centrifugal force is classed as fictitious.

Analyze the motion of and the force on a person who is riding the Round Up. Imagine that Figure 2.9 is a view of the Round Up ride from above and at some instant you are at point A on the ride. At that moment, your velocity ($\vec{v}$) is tangent to the path of the ride. If no force was acting on you at all, you would soon be located at point B. However, the solid cylindrical structure of the ride exerts a normal force on you, pushing you to point C. There is no force pushing you outward, just a centripetal force pushing you toward the centre of the circular ride.

CAREERS IN PHYSICS

An Amusing Side of Physics

Your roller-coaster car is heading up the first and highest incline. As you turn around to wave to your faint-of-heart friends on the ground, you realize that six large players from the Hamilton Tiger Cats football team have piled into the three cars immediately behind you! Now you're poised at the top, ready to drop, and hoping fervently that the roller-coaster manufacturer designed the cars to stay on the track, even when carrying exceptionally heavy loads.

Perhaps your anxiety will lessen if you are aware that a highly qualified mechanical engineer, such as Matthew Chan, has checked out the performance specifications of the roller coaster, as well as all of the other rides at the amusement park. They meet Canada's safety codes, or the park is not allowed to operate them.

Chan has worked for the Technical Standards and Safety Authority (TSSA) for the past 11 years. As special devices engineer for the Elevating and Amusement Devices Safety Division, he spends most of his time in the office, verifying design submissions from elevator, ski lift, and amusement park ride manufacturers.

When some unique piece of machinery comes along or when he wants to verify how equipment will perform in reality versus on paper, Chan goes into the field. He confesses a special fondness for going on-site to test amusement park rides, including at Canada's Wonderland, north of Toronto, the Western Fair in London, Ontario, and Toronto's Canadian National Exhibition.

His favourite challenge is analyzing the performance of reverse bungees, amusement park rides that look like giant slingshots and that hurl people hundreds of feet into the air. Chan says that most reverse bungees are unique devices — rarely are two designed and made exactly the same. "It takes all of your engineering knowledge to analyze these devices," claims Chan. "There are no hard and fast rules for how reverse bungees are made, and the industry is always changing."

A reverse bungee ride

A university degree in mechanical engineering is required to become a special devices engineer, and Chan says that an understanding of the dynamics of motion is a must. You have to be able to predict how a device will behave in a variety of "what if" scenarios. The details of balance, mass placement, overload situations, restraint systems, and footings are all worked through carefully.

In short, you are in good hands. Now, sit back, relax, and enjoy the ride!

Going Further

1. According to Chan, one of the best ways to get a hands-on feel for the complex dynamic forces at work in amusement park rides is to build and operate scale models. Form a study group with some of your friends and investigate designs for your favourite ride. Build a scale model and test it to determine the smallest and greatest weights it can carry safely.

2. Visit a science centre that has exhibits demonstrating the principles of dynamics.

WEB LINK

www.mcgrawhill.ca/links/physics12

The Internet has sites that allow you to design and "run" your own amusement park rides. Go to the Internet site shown above and click on **Web Links**.

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and follow the links for an in-depth activity on circular motion.

Describing Rotational Motion

When an object is constantly rotating, physicists sometimes find it more convenient to describe the motion in terms of the frequency — the number of complete rotations per unit time — or the period — the time required for one complete rotation — instead of the velocity of the object. You can express the centripetal acceleration and the centripetal force in these terms by finding the relationship between the magnitude of the velocity of an object in uniform circular motion and its frequency and period.

- Write the definition of velocity. Since period and frequency are scalar quantities, omit vector notations.

$$v = \frac{\Delta d}{\Delta t}$$

- The distance that an object travels in one rotation is the circumference of the circle.

$$\Delta d = 2\pi r$$

- The time interval for one cycle is the period, T.

$$\Delta t = T$$

- Substitute the distance and period into the equation for velocity, v.

$$v = \frac{2\pi r}{T}$$

- Substitute the above value for v into the equation for centripetal acceleration, a, and simplify.

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$a_c = \frac{\frac{4\pi^2 r^{\cancel{2}}}{T^2}}{\cancel{r}}$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

- Substitute the above value for a into the equation for centripetal force and simplify.

$$F_c = ma_c$$

$$F_c = m\left(\frac{4\pi^2 r}{T^2}\right)$$

$$F_c = \frac{4\pi^2 mr}{T^2}$$

- The frequency is the inverse of the period.

$$f = \frac{1}{T} \text{ or } T = \frac{1}{f}$$

- Substitute the above value for the period into the equation for centripetal acceleration and simplify.

$$a_c = \frac{4\pi^2 r}{\left(\frac{1}{f}\right)^2}$$

$$a_c = 4\pi^2 rf^2$$

- Substitute the above value for acceleration into the equation for the centripetal force.

$$F_c = m(4\pi^2 rf^2)$$

$$F_c = 4\pi^2 mrf^2$$

INVESTIGATION 2-B

Verifying the Circular Motion Equation

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

You have seen the derivation of the equation for circular motion and solved problems by using it. However, it is always hard to accept a theoretical concept until you test it for yourself. In this investigation, you will obtain experimental data for uniform circular motion and compare your data to the theory.

Problem

How well does the equation describe actual experimental results?

Equipment

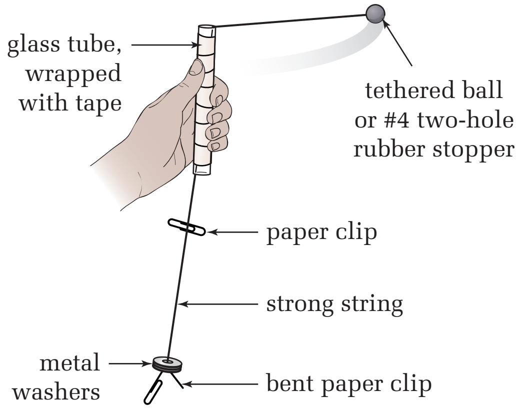
- laboratory balance
- force probeware or stopwatch
- ball on the end of a strong string
- glass tube (15 cm long with fire-polished ends, wrapped in tape)
- metre stick
- 12 metal washers
- tape
- paper clips

CAUTION Wear impact-resistant safety goggles. Also, do not stand close to other people and equipment while doing this activity.

Procedure

Alternative A: Using Traditional Apparatus

1. Measure the mass of the ball.
2. Choose a convenient radius for swinging the ball in a circle. Use the paper clip or tape as a marker, as shown in the diagram at the top of the next column, so you can keep the ball circling within your chosen radius.
3. Measure the mass of one washer.
4. Fasten three washers to the free end of the string, using a bent paper clip to hold them in place. Swing the string at a velocity that will maintain the chosen radius. Measure the time for several revolutions and use it to calculate the period of rotation.



5. Calculate the gravitational force on the washers (weight), which creates tension in the string. This force provides the centripetal force to keep the ball moving on the circular path.
6. Repeat for at least four more radii.

Alternative B: Using Probeware

1. Measure the mass of the ball.
2. Attach the free end of the string to a swivel on a force probe, as shown in the diagram on the next page.
3. Set the software to collect force-time data approximately 50 times per second. Start the ball rotating at constant velocity, keeping the radius at the proper value, and collect data for at least 10 revolutions.

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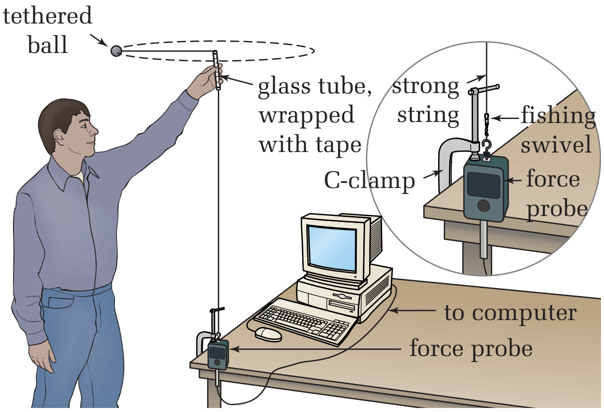
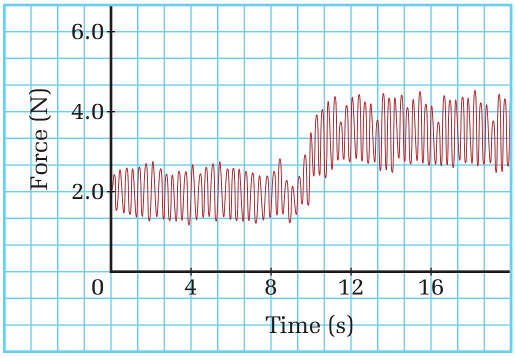
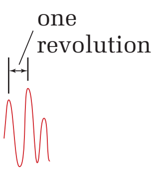


4. Examination of the graph will show regular variations from which you can calculate the period of one revolution, as well as the average force.





5. Repeat for at least five different radii.

Analyze and Conclude

1. For each radius, calculate and record in your data table the velocity of the ball. Use the period and the distance the ball travels in one revolution (the circumference of its circular path).
2. For each radius, calculate and record in your data table $\frac{mv^2}{r}$.
3. Graph F_c against $\frac{mv^2}{r}$. Each radius will produce one data point on your graph.
4. Draw the best-fit line through your data points. How can you tell from the position of the points whether the relationship being tested, $F_c = \frac{mv^2}{r}$, actually describes the data reasonably well?
5. Calculate the slope of the line. What does the slope tell you about the validity of the mathematical relationship?
6. Identify the most likely sources of error in the experiment. That is, what facet of the experiment might have been ignored, even though it could have a significant effect on the results?

Apply and Extend

Based on the experience you have gained in this investigation and the theory that you have learned, answer the following questions about circular motion. Support your answers in each case by describing how you would experimentally determine the answer to the question and how you would use the equations to support your answer.

7. How is the required centripetal force affected when everything else remains the same but the frequency of rotation increases?
8. How is the required centripetal force affected when everything else remains the same but the period of rotation increases?
9. If the radius of the circular path of an object increases and the frequency remains the same, how will the centripetal force change?
10. How can you keep the velocity of the object constant while the radius of the circular path decreases?

Banked Curves

Have you ever wondered why airplanes tilt or bank so much when they turn, as the airplanes in the photograph are doing? Now that you have learned that a centripetal force is required in order to follow a curved path or turn, you probably realize that banking the airplane has something to do with creating a centripetal force. Land vehicles can use friction between the tires and the road surface to obtain a centripetal force, but air friction (or drag) acts opposite to the direction of the motion of the airplane and cannot act perpendicular to the direction of motion. What force could possibly be used to provide a centripetal force for an airplane?

When an airplane is flying straight and horizontally, the design of the wings and the flow of air over them creates a lift force (L) that keeps the airplane in the air, as shown in Figure 2.11. The lift must be equal in magnitude and opposite in direction to the weight of the airplane in order for the airplane to remain on a level path. When an airplane banks, the lift force is still perpendicular to the wings. The vertical component of the lift now must balance the gravitational force, while the horizontal component of the lift provides a centripetal force. The free-body diagram on the right-hand side of Figure 2.11 helps you to see the relationship of the forces more clearly.

Figure 2.10 When an airplane follows a curved path, it must tilt or bank to generate a centripetal force.

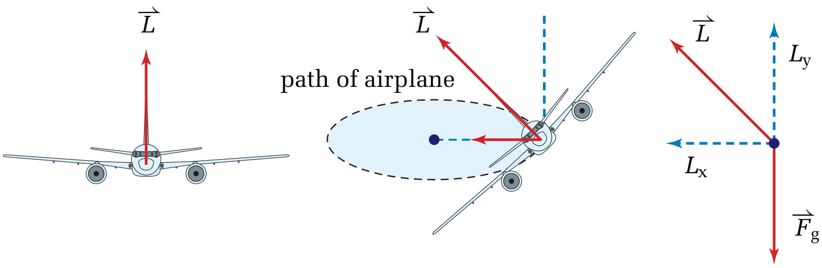


Figure 2.11 When a pilot banks an airplane, the forces of gravity and lift are not balanced. The resultant force is perpendicular to the direction that the airplane is flying, thus creating a centripetal force.

Cars and trucks can use friction as a centripetal force. However, the amount of friction changes with road conditions and can become very small when the roads are icy. As well, friction causes wear and tear on tires and causes them to wear out faster. For these reasons, the engineers who design highways where speeds are high and large centripetal forces are required incorporate another source of a centripetal force — banked curves. Banked curves on a road function in a way that is similar to the banking of airplanes.

Figure 2.12 shows you that the normal force of the road on a car provides a centripetal force when the road is banked, since a normal force is always perpendicular to the road surface.

You can use the following logic to develop an equation relating the angle of banking to the speed of a vehicle rounding a curve. Since an angle is a scalar quantity, omit vector notations and use only magnitudes. Assume that you wanted to know what angle of banking would allow a vehicle to move around a curve with a radius of curvature r at a speed v, without needing any friction to supply part of the centripetal force.

- Since a car does not move in a vertical direction, the vertical component of the normal force must be equal in magnitude to the force of gravity.

$$F_N \cos\theta = F_g$$
$$F_N \cos\theta = mg$$

- The horizontal component of the normal force must supply the centripetal force.

$$F_N \sin\theta = F_c$$
$$F_N \sin\theta = \frac{mv^2}{r}$$

- Divide the second equation by the first and simplify.

$$\frac{\cancel{F_N} \sin\theta}{\cancel{F_N} \cos\theta} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g}$$
$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{v^2}{r}}{g}$$
$$\tan\theta = \frac{v^2}{rg}$$

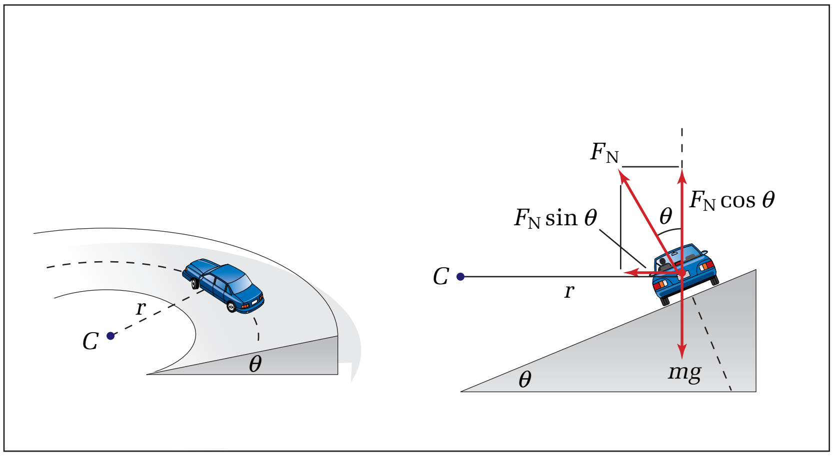
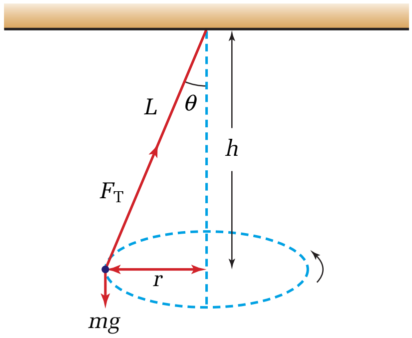


Figure 2.12 When you look at a cross section of a car rounding a curve, you can see that the only two forces in a vertical plane that are acting on the car are the force of gravity and the normal force of the road. The resultant force is horizontal and perpendicular to the direction in which the car is moving. This resultant force supplies a centripetal force that causes the car to follow a curved path.

Notice that the mass of the vehicle does not affect the amount of banking that is needed to drive safely around a curve. A semitrailer and truck could take a curve at the same speed as a motorcycle without relying on friction to supply any of the required centripetal force. Apply what you have learned about banking to the following problems.

Conceptual Problem

- A conical pendulum swings in a circle, as shown in the diagram. Show that the form of the equation relating the angle that the string of the pendulum makes with the vertical to the speed of the pendulum bob is identical to the equation for the banking of curves. The pendulum has a length L, an angle θ with the vertical, a force of tension F_T in the string, a weight mg, and swings in a circular path of radius r. The plane of the circle is a distance h from the ceiling from which the pendulum hangs.



SAMPLE PROBLEM

Banked Curves and Centripetal Force

Canadian Indy racing car driver Paul Tracy set the speed record for time trials at the Michigan International Speedway (MIS) in the year 2000. Tracy averaged 378.11 km/h in the time trials. The ends of the 3 km oval track at MIS are banked at 18.0˚ and the radius of curvature is 382 m.

(a) At what speed can the cars round the curves without needing to rely on friction to provide a centripetal force?

(b) Did Tracy rely on friction for some of his required centripetal force?

Conceptualize the Problem

- The *normal force* of a *banked curve provides* a *centripetal force* to help cars turn without requiring an excessive amount of friction.
- For a given *radius of curvature* and *angle of banking*, there is *one speed* at which the normal force provides precisely the amount of centripetal force that is needed.

Identify the Goal

(a) The speed, v, for which the normal force provides exactly the required amount of centripetal force for driving around the curve

(b) Whether Tracy needed friction to provide an additional amount of centripetal force

Identify the Variables and Constant

Known	Implied	Unknown
$r = 382\ \text{m}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	v
$\theta = 18.0˚$		
$v_{PT} = 378.11\ \frac{\text{km}}{\text{h}}$		

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Develop a Strategy

Write the equation that relates angle of banking, speed, and radius of curvature, and solve for speed, v.

$$\tan\theta = \frac{v^2}{rg}$$
$$v^2 = rg\tan\theta$$
$$v = \sqrt{rg\tan\theta}$$

Substitute the numerical values and solve.

$$v = \sqrt{(382\ \text{m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(\tan 18.0°)}$$
$$v = \sqrt{1217.61\ \frac{\text{m}^2}{\text{s}^2}}$$
$$v = 34.894\ \frac{\text{m}}{\text{s}}$$
$$v \cong 34.9\ \frac{\text{m}}{\text{s}}$$

(a) A vehicle driving at 34.9 m/s could round the curve without needing any friction for centripetal force.

Convert the velocity in m/s into km/h.

$$v = \left(34.894\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{3600\ \cancel{\text{s}}}{\text{h}}\right)\left(\frac{1\ \text{km}}{1000\ \cancel{\text{m}}}\right)$$
$$v = 125.619\ \frac{\text{km}}{\text{h}}$$
$$v \cong 126\ \frac{\text{km}}{\text{h}}$$

(b) Tracy was driving three times as fast as the speed of 126 km/h at which the normal force provides the needed centripetal force. Paul had to rely on friction for a large part of the needed centripetal force.

Validate the Solution

An angle of banking of 18° is very large compared to the banking on normal highway curves. You would expect that it was designed for speeds much higher than the highway speed limit. A speed of 126 km/h is higher than highway speed limits.

PRACTICE PROBLEMS

20. An engineer designed a turn on a road so that a 1225 kg car would need 4825 N of centripetal force when travelling around the curve at 72.5 km/h. What is the radius of curvature of the road?

21. A car exits a highway on a ramp that is banked at 15° to the horizontal. The exit ramp has a radius of curvature of 65 m. If the conditions are extremely icy and the driver cannot depend on any friction to help make the turn, at what speed should the driver travel so that the car will not skid off the ramp?

22. An icy curve with a radius of curvature of 175 m is banked at 12°. At what speed must a car travel to ensure that it does not leave the road?

23. An engineer must design a highway curve with a radius of curvature of 155 m that can accommodate cars travelling at 85 km/h. At what angle should the curve be banked?

You have studied just a few examples of circular motion that you observe or experience nearly every day. Although you rarely think about it, you have been experiencing several forms of circular motion every minute of your life. Simply existing on Earth's surface places you in uniform circular motion as Earth rotates. In addition, Earth is revolving around the Sun. In the next chapter, you will apply many of the concepts you have just learned about force and motion to the motion of planets, moons, and stars, as well as to artificial satellites.

2.2 Section Review

1. K/U Define uniform circular motion and describe the type of acceleration that is associated with it.
2. K/U Study the diagram in Figure 2.6 on page 79. Explain what approximation was made in the derivation that requires you to imagine what occurs as the angle becomes smaller and smaller.
3. C What are the benefits of using the concept of centripetal acceleration rather than working on a traditional Cartesian coordinate system?
4. K/U Explain how centripetal force differs from common forces, such as the forces of friction and gravity.
5. K/U If you were swinging a ball on a string around in a circle in a vertical plane, at what point in the path would the string be the most likely to break? Explain why. In what direction would the ball fly when the string broke?
6. C Explain why gravity does *not* affect circular motion in a horizontal plane, and why it *does* affect a similar motion in a vertical plane.
7. C Describe three examples in which different forces are contributing the centripetal force that is causing an object to follow a circular path.
8. MC When airplane pilots make very sharp turns, they are subjected to very large *g* forces. Based on your knowledge of centripetal force, explain why this occurs.
9. C A centrifugal force, if it existed, would be directed radially outward from the centre of a circle during circular motion. Explain why it feels as though you are being thrown outward when you are riding on an amusement park ride that causes you to spin in a circle.
10. K/U On a highway, why are sharp turns banked more steeply than gentle turns? Use vector diagrams to clarify your answer.
11. I Imagine that you are in a car on a major highway. When going around a curve, the car starts to slide sideways down the banking of the curve. Describe conditions that could cause this to happen.

UNIT PROJECT PREP

Parts of your catapult launch mechanism will move in part of a circle. The payload, once launched, will be a projectile.

- How will your launch mechanism apply enough centripetal force to the payload to move it in a circle, while still allowing the payload to be released?
- How will you ensure that the payload is launched at the optimum angle for maximum range?
- What data will you need to gather from a launch to produce the most complete possible analysis of the payload's actual path and flight parameters?

CHAPTER 2 Review

REFLECTING ON CHAPTER 2

- Ideal projectiles move in a parabolic trajectory in a vertical plane under the influence of gravity alone.
- The path of a projectile is determined by the initial velocity.
- The range of a projectile is its horizontal displacement.
- For a given magnitude of velocity, the maximum range of an ideal projectile occurs when the projectile is launched at an angle of 45°.
- Projectile trajectories are computed by separately analyzing horizontal and vertical components of velocity during a common time interval.
- Objects in uniform circular motion experience a centripetal (centre-seeking) acceleration: $a_c = v^2/r$.
- A centripetal (centre-seeking) force is required to keep an object in uniform circular motion: $F_c = mv^2/r$.
- Centripetal force can be supplied by any type of force, such as tension, gravitational forces, friction, and electrostatic force, or by a combination of forces.
- The force of gravity has no effect on circular motion in a horizontal plane, but does affect circular motion in a vertical plane.
- Objects moving around banked curves experience a centripetal force due to the horizontal component of the normal force exerted by the surfaces on which they travel.

Knowledge/Understanding

1. Differentiate between the terms "one-dimensional motion" and "two-dimensional motion." Provide examples of each.
2. Explain what physicists mean by the "two-dimensional nature of motion in a plane," when common sense suggests that an object can be travelling in only one direction at any particular instant in time.
3. Describe and explain two specific examples that illustrate how the vertical and horizontal components of projectile motion are independent of each other.
4. When analyzing ideal projectiles, what type of motion is the horizontal component? What type of motion is the vertical component?
5. Standing on the school roof, a physics student swings a rubber stopper tied to a string in a circle in a vertical plane. He releases the string so that the stopper flies outward in a horizontal direction.
   - **(a)** Draw a sketch of this situation. Draw and label the velocity and acceleration vectors at the instant at which he releases the string in order to produce the horizontal motion.
   - **(b)** Explain at what point the horizontal component of the stopper's motion becomes uniform.
6. Explain why an object with uniform circular motion is accelerating.
7. Draw a free-body diagram of a ball on the end of a string that is in uniform circular motion in a horizontal plane. Explain why the weight of the ball *does not* affect the value of the tension of the string that is providing the centripetal force required to maintain the motion.
8. Draw a free-body diagram of a car rounding a banked curve. Explain why the weight of the car *does* affect the value of the centripetal force required to keep the car in a circular path.
9. A rubber stopper tied to a string is being swung in a vertical loop.
   - **(a)** Draw free-body diagrams of the stopper at its highest and lowest points.
   - **(b)** Write equations to show the relationships among the centripetal force, the tension in

the string, and the weight of the stopper for each location.

10. Outline the conditions under which an object will travel in uniform circular motion and explain why a centripetal force is considered to be the *net* force required to maintain this motion.

Inquiry

11. Design and conduct a simple experiment to test the independence of the horizontal and vertical components of projectiles. Analyze your data to determine the percent deviation between your theoretical predictions and your actual results. Identify factors that could explain any deviations.
12. Design and construct a model of a vertical loop-the-loop section of a roller coaster. Refine your model, and your skill at operating it, until the vehicle will consistently round the loop without falling. Determine the minimum speed at which the vehicle must travel in order to complete the vertical loop without falling. Given the radius of your loop, calculate the theoretical value of the speed at which your vehicle would need to be travelling. Explain any deviation between the theoretical prediction and your actual results.

Communication

13. A stone is thrown off a cliff that has a vertical height of 45 m above the ocean. The initial horizontal velocity component is 15 m/s. The initial vertical velocity component is 10 m/s upward. Draw a scale diagram of the stone's trajectory by locating its position at one-second time intervals. At each of these points, draw a velocity vector to show the horizontal velocity component, the vertical velocity component, and the resultant velocity in the frame of reference of a person standing on the cliff. Assume that the stone is an ideal projectile and use 10 m/s^2 for the value of g to simplify calculations.
14. Draw a diagram to represent an object moving with uniform circular motion by constructing a rectangular x–y-coordinate system and drawing a circle with radius of 5 cm centred on the origin. Label the point where the circle crosses the positive y-axis, A; the positive x-axis, B; the negative y-axis, C; and the negative x-axis, D.
    - **(a)** At each of the four labelled points, draw a 2 cm vector to represent the object's instantaneous velocity.
    - **(b)** Construct a series of scale vector diagrams to determine the average acceleration between A and B, B and C, C and D, and D and A. Assume the direction of motion to be clockwise.
    - **(c)** Designate on the diagram at which points the average accelerations would occur and draw in the respective acceleration vectors.
    - **(d)** Write a general statement about the direction of the acceleration of an object in circular motion.
15. The mass of an object does not affect the angle at which a curve must be banked. The law of inertia, however, states that the motion of any object is affected by its inertia, which depends on its mass. How can objects rounding banked curves obey the law of inertia if the amount of banking required for a curve of a given radius of curvature and speed is independent of mass?
16. You are facing north, twirling a tethered ball in a horizontal circle above your head. At what point in the circle must you release the string in order to hit a target directly to the east? Sketch the situation, indicating the correct velocity vector.
17. A transport truck is rounding a curve in the highway. The curve is banked at an angle of 10˚ to the horizontal.
    - **(a)** Draw a free-body diagram to show all of the forces acting on the truck.
    - **(b)** Write an equation in terms of the weight of a truck, that will express the value of the centripetal force needed to keep the truck turning in a circle.

Making Connections

18. A pitched baseball is subject to the forces of gravity, air resistance, and lift. The lift force is produced by the ball's spin. Do research to find out

(a) how a spinning baseball creates lift

(b) how a pitcher can create trajectories for different types of pitches, such as fastballs, curve balls, knuckle balls, and sliders

(c) why these pitches are often effective in tricking the batter

19. Discuss similarities between a banked curve in a road and the tilt or banking of an airplane as it makes a turn. Draw free-body diagrams for each situation. What is the direction of the lift force on the airplane before and during the turn? Explain how tilting the airplane creates a centripetal force. What must the pilot do to make a sharper turn?

Problems for Understanding

20. You throw a rock off a 68 m cliff, giving it a horizontal velocity of 8.0 m/s.

(a) How far from the base of the cliff will it land?

(b) How long will the rock be in the air?

21. A physics student is demonstrating how the horizontal and vertical components of projectile motion are independent of each other. At the same instant as she rolls a wooden ball along the floor, her lab partner rolls an identical wooden ball from the edge of a platform directly above the first ball. Both balls have an initial horizontal velocity of 6.0 m/s. The platform is 3.0 m above the ground.

(a) When will the second ball strike the ground?

(b) Where, relative to the first ball, will the second ball hit the ground?

(c) At what distance from the base of the platform will the second ball land?

(d) With what velocity will the second ball land?

22. **(a)** A 350 g baseball is thrown horizontally at 22 m/s[forward] from a roof that is 18 m high. How far does it travel before hitting the ground?

(b) If the baseball is thrown with the same velocity but at an angle of 25° above the horizontal, how far does it travel? (Neglect air friction.)

23. A rescue plane flying horizontally at 175 km/h[N], at an altitude of 150 m, drops a 25 kg emergency package to a group of explorers. Where will the package land relative to the point above which it was released? (Neglect friction.)

24. You throw a ball with a velocity of 18 m/s at 24° above the horizontal from the top of your garage, 5.8 m above the ground. Calculate the

(a) time of flight

(b) horizontal range

(c) maximum height

(d) velocity when the ball is 2.0 m above the roof

(e) angle at which the ball hits the ground

25. Using a slingshot, you fire a stone horizontally from a tower that is 27 m tall. It lands 122 m from the base of the tower. What was its initial velocity?

26. At a ballpark, a batter hits a baseball at an angle of 37° to the horizontal with an initial velocity of 58 m/s. If the outfield fence is 3.15 m high and 323 m away, will the hit be a home run?

27. An archer shoots a 4.0 g arrow into the air, giving it a velocity of 40.0 m/s at an elevation angle of 65°. Find

(a) its time of flight

(b) its maximum height

(c) its range

(d) its horizontal and vertical distance from the starting point at 2.0 s after it leaves the bow

(e) the horizontal and vertical components of its velocity at 6.0 s after it leaves the bow

(f) its direction at 6.0 s after leaving the bow

Plot the trajectory on a displacement-versus-time graph.

28. A hang-glider, diving at an angle of 57.0° with the vertical, drops a water balloon at an altitude of 680.0 m. The water balloon hits the ground 5.20 s after being released.
   (a) What was the velocity of the hang-glider?
   (b) How far did the water balloon travel during its flight?
   (c) What were the horizontal and vertical components of its velocity just before striking the ground?
   (d) At what angle does it hit the ground?

29. A ball moving in a circular path with a constant speed of 3.0 m/s changes direction by 40.0° in 1.75 s.
   (a) What is its change in velocity?
   (b) What is the acceleration during this time?

30. You rotate a 450 g ball on the end of a string in a horizontal circle of radius 2.5 m. The ball completes eight rotations in 2.0 s. What is the centripetal force of the string on the ball?

31. A beam of electrons is caused to move in a circular path of radius 3.00 m at a velocity of 2.00×10^{7} m/s. The electron mass is 9.11×10^{-31} kg.
   (a) What is the centripetal acceleration of one of the electrons?
   (b) What is the centripetal force on one electron?

32. A car travelling on a curved road will skid if the road does not supply enough friction. Calculate the centripetal force required to keep a 1500 kg car travelling at 65 km/h on a flat curve of radius 1.0×10^{2} m. What must be the coefficient of friction between the car's wheels and the ground?

33. Consider an icy curved road, banked 6.2° to the horizontal, with a radius of curvature of 75.0 m. At what speed must a 1200 kg car travel to stay on the road?

34. You want to design a curve, with a radius of curvature of 350 m, so that a car can turn at a velocity of 15 m/s on it without depending on friction. At what angle must the road be banked?

35. **(a)** A motorcycle stunt rider wants to do a loop-the-loop within a vertical circular track. If the radius of the circular track is 10.0 m, what minimum speed must the motorcyclist maintain to stay on the track?
   (b) Suppose the radius of the track was doubled. By what factor will the motorcyclist need to increase her speed to loop-the-loop on the new track?

36. An amusement park ride consists of a large cylinder that rotates around a vertical axis. People stand on a ledge inside. When the rotational speed is high enough, the ledge drops away and people "stick" to the wall. If the period of rotation is 2.5 s and the radius is 2.5 m, what is the minimum coefficient of friction required to keep the riders from sliding down?

37. Use your understanding of the physics of circular motion to explain why we are not thrown off Earth like heavy particles in a centrifuge or mud off a tire, even though Earth is spinning at an incredible rate of speed. To make some relevant calculations, assume that you are standing in the central square of Quito, a city in Ecuador that is located on Earth's equator.
   (a) Calculate your average speed around the centre of Earth.
   (b) Determine the centripetal force needed to move you in a circle with Earth's radius at the speed that you calculated in part (a).
   (c) In what direction does the centripetal force act? What actual force is providing the amount of centripetal force that is required to keep you in uniform circular motion on Earth's surface?
   (d) What is your weight?
   (e) What is the normal force exerted on you by Earth's surface?
   (f) Use the calculations just made and other concepts about circular motion that you have been studying to explain why you are not thrown off Earth as it spins around its axis.

CHAPTER 3

Planetary and Satellite Dynamics

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Centripetal force
- Centripetal acceleration

On April 13, 1970, almost 56 h and 333 000 km into their flight to the Moon, the crew of *Apollo 13* heard a loud bang and felt the spacecraft shudder. Astronaut Jack Swigert radioed NASA Ground Control: "Houston, we've had a problem here." The above photograph, taken by the astronauts after they jettisoned the service module, shows how serious that problem was — an oxygen tank had exploded and damaged the only other oxygen tank. After assessing the situation, the astronauts climbed into the lunar landing module, where the oxygen and supplies were designed to support two people for two days. They would have to support the three astronauts for four days.

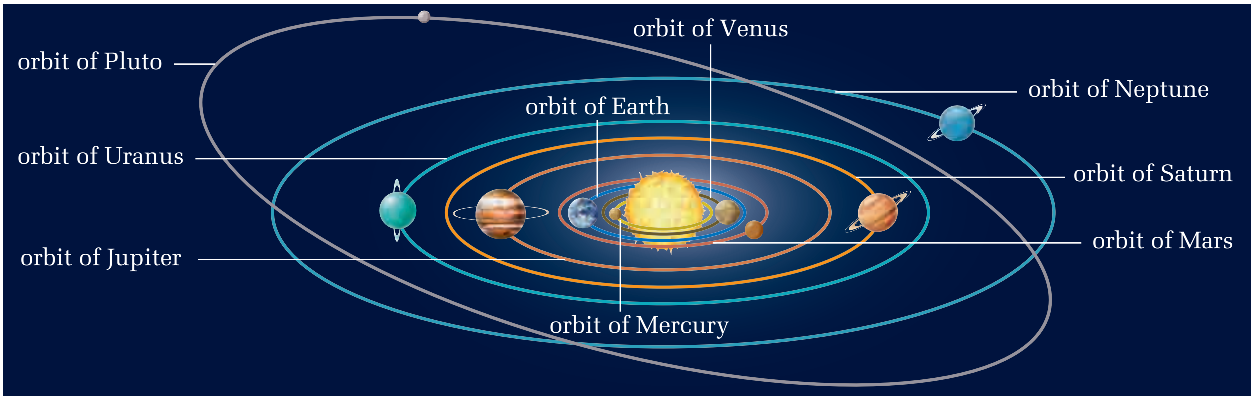
The spacecraft was still hurtling toward the Moon at more than 5000 km/h, and the engines of the lunar landing module could certainly not provide the force necessary to turn the craft back toward Earth. The only available force that could send the astronauts home was the gravitational force of the Moon, which swung the crippled spacecraft around behind the Moon and hurled it back toward Earth. With the engines of the lunar landing module, the crew made two small course corrections that prevented the craft from careening past Earth into deep space. Exactly 5 days, 22 h, and 54 min after lift-off, the astronauts, back inside the command module, landed in the Pacific Ocean, less than 800 m from the rescue ship.

In this chapter, you will learn about Newton's law of universal gravitation and how it guides the motion of planets and satellites — and damaged spacecraft.

QUICK LAB

Kepler's Empirical Equations

TARGET SKILLS
- Initiating and planning
- Analyzing and interpreting
- Communicating results



The famous German astronomer Johannes Kepler (1571–1630) studied a vast amount of detailed astronomical data and found three empirical mathematical relationships within these data. Empirical equations are based solely on data and have no theoretical foundation. Often, however, an empirical equation will provide scientists with insights that will lead to a hypothesis that can be tested further.

In this chapter, you will learn the significance of Kepler's empirical equations. First, however, you will examine the data below, which is similar to the data that Kepler used, and look for a relationship.

Planet	Orbital radius R (AU)*	Orbital period T (days)
Mercury	0.389	87.77
Venus	0.724	224.70
Earth	1.000	365.25
Mars	1.524	686.98
Jupiter	5.200	4332.62
Saturn	9.150	10 759.20

* One astronomical unit (AU) is the average distance from Earth to the Sun, so distances expressed in AU are fractions or multiples of the Earth's average orbital radius.

From the data, make a graph of radius (R) versus period (T). Study the graph. Does the curve look like an inverse relationship, a logarithmic relationship, or an exponential relationship? Choose the type of mathematical relationship that you think is the most likely. Review Skill Set 4, Mathematical Modelling and Curve Straightening, and make at least four attempts to manipulate the data and plot the results. If you found a relationship that gives you a straight-line plot, write the mathematical relationship between radius and period. If you did not find the correct relationship, confer with your classmates to see if anyone found the correct relationship. As a class, agree on the final mathematical relationship.

Analyze and Conclude

1. When Kepler worked with astronomical data, he did not know whether a relationship existed between specific pairs of variables. In addition, Kepler had no calculator — he had to do all of his calculations by hand. Comment on the effort that he exerted in order to find his relationships.
2. Think about the relationship between the radius of an orbit and the period of an orbit on which your class agreed. Try to think of a theoretical basis for this relationship.
3. What type of additional information do you think that you would need in order to give a physical meaning to your mathematical relationship?

3.1 Newton's Law of Universal Gravitation

SECTION EXPECTATIONS

- Describe Newton's law of universal gravitation.
- Apply Newton's law of universal gravitation quantitatively.

KEY TERMS

- Tychonic system
- Kepler's laws
- law of universal gravitation

In previous science courses, you learned about the Ptolemaic system for describing the motion of the planets and the Sun. The system developed by Ptolemy (151–127 B.C.E.) was very complex because it was geocentric, that is, it placed Earth at the centre of the universe. In 1543, Nicholas Copernicus (1473–1543) proposed a much simpler, heliocentric system for the universe in which Earth and all of the other planets revolved around the Sun. The Copernican system was rejected by the clergy, however, because the religious belief system at the time placed great importance on humans and Earth as being central to a physically perfect universe. You probably remember learning that the clergy put Galileo Galilei (1564–1642) on trial for supporting the Copernican system.

Have you ever heard of the **Tychonic system**? A famous Danish nobleman and astronomer, Tycho Brahe (1546–1601), proposed a system, shown in Figure 3.1, that was intermediate between the Ptolemaic and Copernican systems. In Brahe's system, Earth is still and is the centre of the universe; the Sun and Moon revolve around Earth, but the other planets revolve around the Sun. Brahe's system captured the interest of many scientists, but never assumed the prominence of either the Ptolemaic or Copernican systems. Nevertheless, Tycho Brahe contributed a vast amount of detailed, accurate information to the field of astronomy.

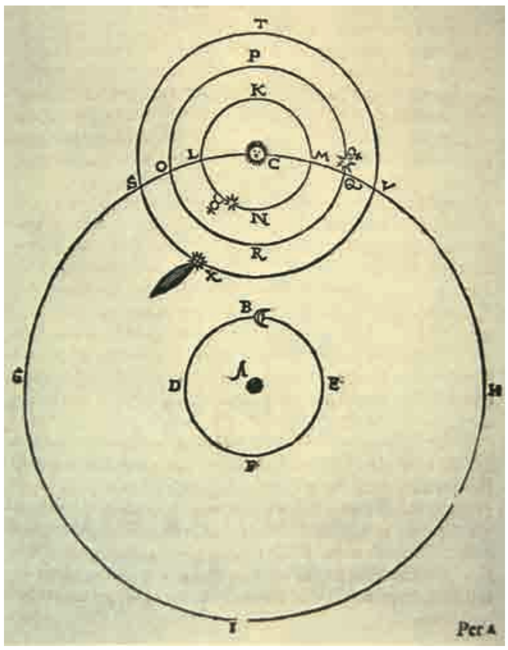


Figure 3.1 The Tychonic universe was acceptable to the clergy, because it maintained that Earth was the centre of the universe. The system was somewhat satisfying for scientists, because it was simpler than the Ptolemaic system.

Laying the Groundwork for Newton

Astronomy began to come of age as an exact science with the detailed and accurate observations of Tycho Brahe. For more than 20 years, Brahe kept detailed records of the positions of the planets and stars. He catalogued more than 777 stars and, in 1572, discovered a new star that he named "Nova." Brahe's star was one of very few supernovae ever found in the Milky Way galaxy.

In 1577, Brahe discovered a comet and demonstrated that it was not an atmospheric phenomenon as some scientists had believed, but rather that its orbit lay beyond the Moon. In addition to making observations and collecting data, Brahe designed and built the most accurate astronomical instruments of the day (see Figure 3.2). In addition, he was the first astronomer to make corrections for the refraction of light by the atmosphere.

HISTORY LINK

Tycho Brahe was a brilliant astronomer who led an unusual and tumultuous life. At age 19, he was involved in a duel with another student and part of his nose was cut off. For the rest of his life, Brahe wore an artificial metal nose.

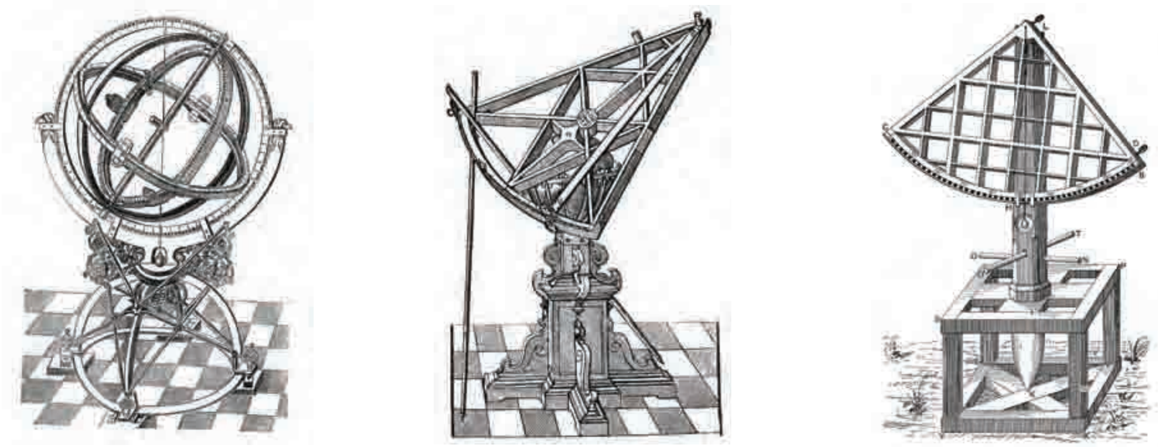


Figure 3.2 Brahe's observatory in Hveen, Denmark, contained gigantic instruments that, without magnification, were precise to 1/30 of a degree.

In 1600, Brahe invited Kepler to be one of his assistants. Brahe died suddenly the following year, leaving all of his detailed data to Kepler. With this wealth of astronomical data and his ability to perform meticulous mathematical analyses, Kepler discovered three empirical relationships that describe the motion of the planets. These relationships are known today as **Kepler's laws**.

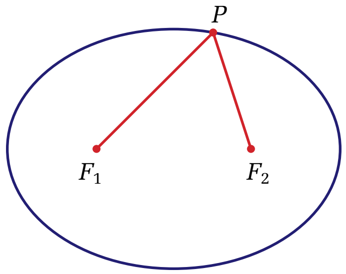
KEPLER'S LAWS

1. Planets move in elliptical orbits, with the Sun at one focus of the ellipse.
2. An imaginary line between the Sun and a planet sweeps out equal areas in equal time intervals.
3. The quotient of the square of the period of a planet's revolution around the Sun and the cube of the average distance from the Sun is constant and the same for all planets.

$\frac{T^2}{r^3} = k$ or $\frac{T_A^2}{r_A^3} = \frac{T_B^2}{r_B^3}$, where A and B are two planets.

MATH LINK

A circle is a special case of an ellipse. An ellipse is defined by two focuses and the relationship $\overline{F_1P} + \overline{F_2P} = k$, where k is a constant and is the same for every point on the ellipse. If the two focuses of an ellipse are brought closer and closer together until they are superimposed on each other, the ellipse becomes a circle.



Kepler's first law does not sound terribly profound, but he was contending not only with scientific observations of the day, but also with religious and philosophical views. For centuries, the perfection of "celestial spheres" was of extreme importance in religious beliefs. Ellipses were not considered to be "perfect," so many astronomers resisted accepting any orbit other than a "perfect" circle that fit on the surface of a sphere. However, since Kepler published his laws, there has never been a case in which the data for the movement of a satellite, either natural or artificial, did not fit an ellipse.

Kepler's second law is illustrated in Figure 3.3. Each of the shaded sections of the ellipse has an equal area. According to Kepler's second law, therefore, the planet moves along the arc of each section in the same period of time. Since the arcs close to the Sun are longer than the arcs more distant from the Sun, the planet must be moving more rapidly when it is close to the Sun.

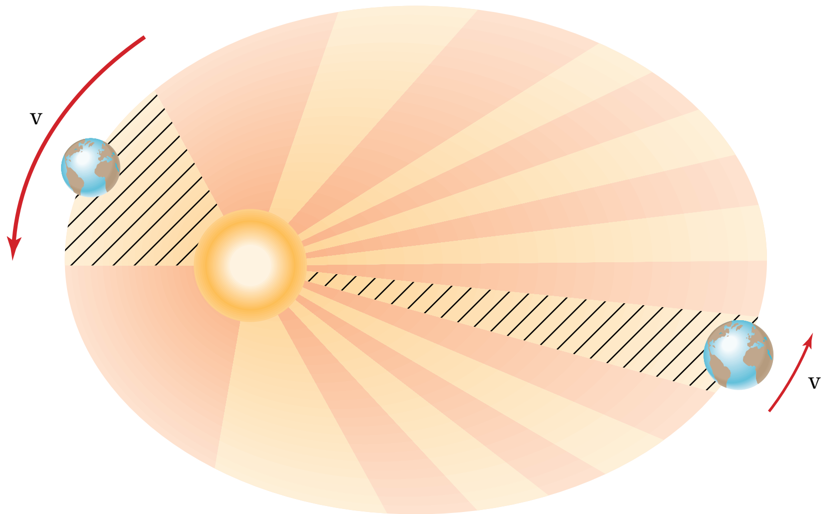


Figure 3.3 According to Kepler's second law, the same length of time was required for a planet to move along each of the arcs at the ends of the segments of the ellipse. Kepler could not explain why planets moved faster when they were close to the Sun than when they were farther away.

When Kepler published his third law, he had no way of knowing the significance of the constant in the mathematical expression $T^2/r^3 = k$. All he knew was that the data fit the equation. Kepler suspected that the Sun was in some way influencing the motion of the planets, but he did not know how or why this would lead to the mathematical relationship. The numerical value of the constant in Kepler's third law and its relationship to the interaction between the Sun and the planets would take on significance only when Sir Isaac Newton (1642–1727) presented his law of universal gravitation.

Universal Gravitation

Typically in research, the scientist makes some observations that lead to an hypothesis. The scientist then tests the hypothesis by planning experiments, accumulating data, and then comparing the results to the hypothesis. The development of Newton's law of universal gravitation happened in reverse. Brahe's data and Kepler's analysis of the data were ready and waiting for Newton to use to test his hypothesis about gravity.

Newton was not the only scientist of his time who was searching for an explanation for the motion, or orbital dynamics, of the planets. In fact, several scientists were racing to see who could find the correct explanation first. One of those scientists was astronomer Edmond Halley (1656–1742). Halley and others, based on their calculations, had proposed that the force between the planets and the Sun decreased with the square of the distance between a planet and the Sun. However, they did not know how to apply that concept to predict the shape of an orbit.

Halley decided to put the question to Newton. Halley first met Newton in 1684, when he visited Cambridge. He asked Newton what type of path a planet would take if the force attracting it to the Sun decreased with the square of the distance from the Sun. Newton quickly answered, "An elliptical path." When Halley asked him how he knew, Newton replied that he had made that calculation many years ago, but he did not know where his calculations were. Halley urged Newton to repeat the calculations and send them to him.

Three months later, Halley's urging paid off. He received an article from Newton entitled "De Motu" ("On Motion"). Newton continued to improve and expand his article and in less than three years, he produced one of the most famous and fundamental scientific works: *Philosophiae Naturalis Principia Mathematica* (*The Mathematical Principles of Natural Philosophy*). The treatise contained not only the law of universal gravitation, but also Newton's three laws of motion.

Possibly, Newton was successful in finding the law of universal gravitation because he extended the concept beyond the motion of planets and applied it to all masses in all situations. While other scientists were looking at the motion of planets, Newton was watching an apple fall from a tree to the ground. He reasoned that the same attractive force that existed between the Sun and Earth was also responsible for attracting the apple to Earth. He also reasoned that the force of gravity acting on a falling object was proportional to the mass of the object. Then, using his own third law of action-reaction forces, if a falling object such as an apple was attracted to Earth, then Earth must also be attracted to the apple, so the force of gravity must also be proportional to the mass of Earth. Newton therefore proposed that *the force of gravity between any two objects is proportional to the product of their*

HISTORY LINK

Sir Edmond Halley, the astronomer who prompted Newton to publish his work on gravitation, is the same astronomer who discovered the comet that was named in his honour — Halley's Comet. Without Halley's urging, Newton might never have published his famous *Principia*, greatly slowing the progress of physics.

masses and inversely proportional to the square of the distance between their centres — the **law of universal gravitation**. The mathematical equation for the law of universal gravitation is given in the following box.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

The force of gravity is proportional to the product of the two masses that are interacting and inversely proportional to the square of the distance between their centres.

$$F_g = G\frac{m_1 m_2}{r^2}$$

Quantity	Symbol	SI unit
force of gravity	F_g	N (newtons)
first mass	m_1	kg (kilograms)
second mass	m_2	kg (kilograms)
distance between the centres of the masses	r	m (metres)
universal gravitational constant	G	$\frac{N \cdot m^2}{kg^2}$ (newton · metre squared per kilogram squared)

Unit Analysis

$$\text{newton} = \left(\frac{\text{newton} \cdot \text{metre}^2}{\text{kilogram}^2}\right)\left(\frac{\text{kilogram} \cdot \text{kilogram}}{\text{metre}^2}\right)$$

$$\left(\frac{N \cdot \cancel{m^2}}{\cancel{kg^2}}\right)\left(\frac{\cancel{kg} \cdot \cancel{kg}}{\cancel{m^2}}\right) = N$$

Note: The value of the universal gravitational constant is

$$G = 6.67 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}.$$

Conceptual Problem

- You have used the equation $F_g = mg$ many times to calculate the weight of an object on Earth's surface. Now, you have learned that the weight of an object on Earth's surface is $F_g = G\frac{m_E m_o}{r_{E\text{-}o}^2}$, where m_E is the mass of Earth, m_o is the mass of the object, and $r_{E\text{-}o}$ is the distance between the centres of Earth and the object. Explain how the two equations are related. Express g in terms of the variables and constant in Newton's law of universal gravitation.

SAMPLE PROBLEM

Weighing an Astronaut

A 65.0 kg astronaut is walking on the surface of the Moon, which has a mean radius of 1.74×10^3 km and a mass of 7.35×10^{22} kg. What is the weight of the astronaut?

Conceptualize the Problem

- The weight of the astronaut is the gravitational force on her.
- The relationship $F_g = mg$, where $g = 9.81 \frac{m}{s^2}$, *cannot* be used in this problem, since the astronaut is not on Earth's surface.
- The law of universal gravitation applies to this problem.

Identify the Goal

The gravitational force, F_g, on the astronaut

Identify the Variables and Constants

Known	Implied	Unknown
$m_M = 7.35 \times 10^{22}$ kg	$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$	F_g
$m_a = 65.0$ kg		
$r = 1.74 \times 10^3$ km $(1.74 \times 10^6$ m$)$		

Develop a Strategy

Apply the law of universal gravitation.

$$F_g = G\frac{m_1 m_2}{r^2}$$

Substitute the numerical values and solve.

$$F_g = \left(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}\right)\frac{(7.35 \times 10^{22}\ kg)(65.0\ kg)}{(1.74 \times 10^6\ m)^2}$$

$$F_g = 105.25\ N$$

$$F_g \cong 105\ N$$

The weight of the astronaut is approximately 105 N.

Validate the Solution

Weight on the Moon is known to be much less than that on Earth. The astronaut's weight on the Moon is about one sixth of her weight on Earth $\left(65.0\ kg \times 9.81 \frac{m}{s^2} \cong 638\ N\right)$, which is consistent with this common knowledge.

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PRACTICE PROBLEMS

1. Find the gravitational force between Earth and the Sun. (See Appendix B, Physical Constants and Data.)
2. Find the gravitational force between Earth and the Moon. (See Appendix B, Physical Constants and Data.)
3. How far apart would you have to place two 7.0 kg bowling balls so that the force of gravity between them would be 1.25×10^{-4} N? Would it be possible to place them at this distance? Why or why not?
4. Find the gravitational force between the electron and the proton in a hydrogen atom if they are 5.30×10^{-11} m apart. (See Appendix B, Physical Constants and Data.)
5. On Venus, a person with mass 68 kg would weigh 572 N. Find the mass of Venus from this data, given that the planet's radius is 6.31×10^6 m.
6. In an experiment, an 8.0 kg lead sphere is brought close to a 1.5 kg mass. The gravitational force between the two objects is 1.28×10^{-8} N. How far apart are the centres of the objects?
7. The radius of the planet Uranus is 4.3 times the radius of earth. The mass of Uranus is 14.7 times Earth's mass. How does the gravitational force on Uranus' surface compare to that on Earth's surface?
8. Along a line connecting Earth and the Moon, at what distance from Earth's centre would an object have to be located so that the gravitational attractive force of Earth on the object was equal in magnitude and opposite in direction from the gravitational attractive force of the Moon on the object?

Gravity and Kepler's Laws

The numerical value of G, the universal gravitational constant, was not determined experimentally until more than 70 years after Newton's death. Nevertheless, Newton could work with concepts and proportionalities to verify his law.

Newton had already shown that the inverse square relationship between gravitational force and the distance between masses was supported by Kepler's first law — that planets follow elliptical paths.

Kepler's second law showed that planets move more rapidly when they are close to the Sun and more slowly when they are farther from the Sun. The mathematics of elliptical orbits in combination with an inverse square relationship to yield the speed of the planets is somewhat complex. However, you can test the concepts graphically by completing the following investigation.

INVESTIGATION 3-A

Orbital Speed of Planets

TARGET SKILLS
- Modelling concepts
- Analyzing and interpreting
- Communicating results

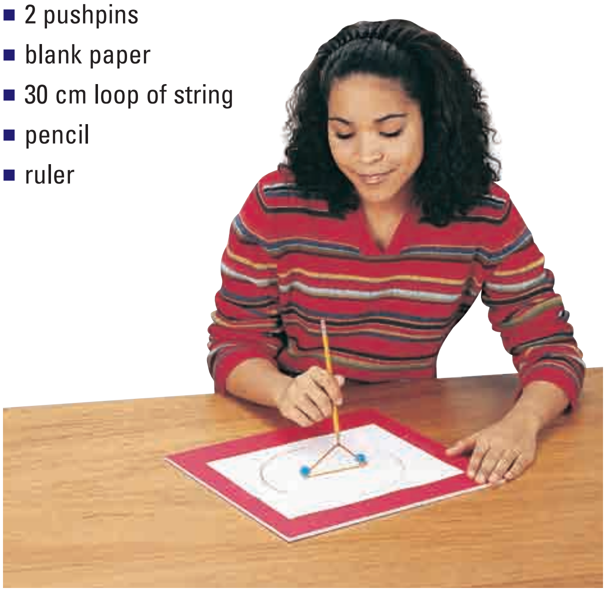
Can you show diagrammatically that a force directed along the line between the centres of the Sun and a planet would cause the planet's speed to increase as it approached the Sun and decrease as it moved away? If you can, you have demonstrated that Kepler's second law supports Newton's proposed law of universal gravitation.

Problem

How does a force that follows an inverse square relationship affect the orbital speed of a planet in an elliptical orbit?

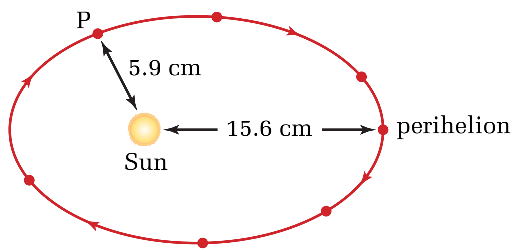
Equipment

- corkboard or large, thick piece of cardboard
- 2 pushpins
- blank paper
- 30 cm loop of string
- pencil
- ruler



Procedure

1. Place the paper on the corkboard or cardboard. Insert two pushpins into the paper about 8 to 10 cm apart.
2. Loop the string around the pushpins, as shown in the illustration. With your pencil, pull the string so that it is taut and draw an ellipse by pulling the string all the way around the pushpins.
3. Remove the string and pushpins and label one of the pinholes "Sun."
4. Choose a direction around the elliptical orbit in which your planet will be moving. Make about four small arrowheads on the ellipse to indicate the direction of motion of the planet.
5. Make a dot for the planet at the point that is most distant from the Sun (the perihelion). Measure and record the distance on the paper from the perihelion to the Sun. From that point, draw a 1 cm vector directed straight toward the Sun.
6. This vector represents the force of gravity on the planet at that point: $F_{g(per)} = 1$ unit. ($F_{g(per)}$ is the force of gravity when the planet is at perihelion.)
7. Select and label at least three more points on each side of the ellipse at which you will analyze the force and motion of the planet.
8. For each point, measure and record, on a separate piece of paper, the distance from the Sun to point P, as indicated in the diagram. Do not write on your diagram, because it will become too cluttered.



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9. Follow the steps in the table to see how to determine the length of the force vector at each point.

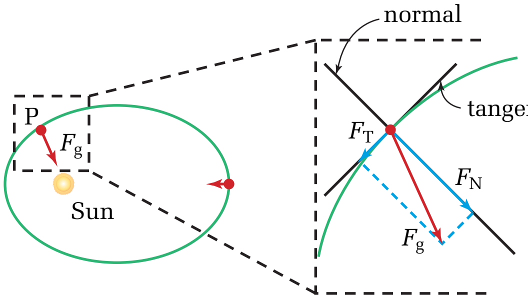
Procedure	Equation
■ The masses of the Sun and planet remain the same, so the value Gm_Sm_p is constant. Therefore, the expression F_gr^2 for any point on the orbit is equal to the same value.	$F_g = G\frac{m_Sm_p}{r^2}$ $F_gr^2 = Gm_Sm_p$
■ Consequently, you can set the expression F_gr^2 for any one point equal to F_gr^2 for any other point. Use the values at perihelion as a reference and set $F_{g(P)}r^2$ equal to $F_{g(peri)}r^2_{peri}$. Then solve for the $F_{g(P)}$.	$F_{g(peri)}\,r^2_{peri} = F_{g(P)}\,r^2_P$ $F_{g(P)} = \frac{F_{g(peri)}\,r^2_{peri}}{r^2_P}$
■ You can now find the relative magnitude of the gravitational force on the planet at any point on the orbit by substituting the magnitudes of the radii into the above equation. For example, the magnitude of the force at point P in step 8 is 6.99 units.	$F_{g(P)} = \frac{(1\text{ unit})(15.6\text{ cm})^2}{(5.9\text{ cm})^2}$ $F_{g(P)} = 6.99$ units

10. Calculate the length of the force vector from each of the points that you have selected on your orbit.

11. On your diagram, draw force vectors from each point directly toward the Sun, making the lengths of the vectors equal to the values that you calculated in step 10.

12. At each point at which you have a force vector, draw a very light pencil line tangent to the ellipse. Then, draw a line that is perpendicular (normal) to the tangent line.

13. Graphically draw components of the force vector along the tangent (F_T) and normal (F_N) lines, as shown in the diagram.



Analyze and Conclude

1. The tangential component of the force vector (F_T) is parallel to the direction of the velocity of the planet when it passes point P. What effect will the tangential component of force have on the velocity of the planet?

2. The normal component of the force vector (F_N) is perpendicular to the direction of the velocity of the planet when it passes point P. What effect will the normal component of force have on the velocity of the planet?

3. Analyze the change in the motion of the planet caused by the tangential and normal components of the gravitational force at each point where you have drawn force vectors. Be sure to note the direction of the velocity of the planet as you analyze the effect of the components of force at each point.

4. Summarize the changes in the velocity of the planet as it makes one complete orbit around the Sun.

5. The force vectors and components that you drew were predictions based on Newton's law of universal gravitation. How well do these predictions agree with Kepler's observations as summarized in his second law? Would you say that Kepler's data supports Newton's predictions?

Kepler's third law simply states that the ratio T^2/r^3 is constant and the same for each planet orbiting the Sun. At first glance, it would appear to have little relationship to Newton's law of universal gravitation, but a mathematical analysis will yield a relationship. To keep the mathematics simple, you will consider only circular orbits. The final result obtained by considering elliptical orbits is the same, although the math is more complex. Follow the steps below to see how Newton's law of universal gravitation yields the same ratio as given by Kepler's third law.

- Write Newton's law of universal gravitation, using m_S for the mass of the Sun and m_p for the mass of a planet.

$$F_g = G\frac{m_S m_p}{r^2}$$

- Since the force of gravity must provide a centripetal force for the planets, set the gravitational force equal to the required centripetal force.

$$G\frac{m_S m_p}{r^2} = \frac{m_p v^2}{r}$$

  Simplify the equation.

$$G\frac{m_S}{r} = v^2$$

- Since Kepler's third law includes the period, T, as a variable, find an expression for the velocity, v, of the planet in terms of its period.

$$v = \frac{\Delta d}{\Delta t}$$
$$\Delta d = 2\pi r$$
$$\Delta t = T$$

  A planet travels a distance equal to the circumference of the orbit during a time interval equal to its period.

$$v = \frac{2\pi r}{T}$$

- Substitute the expression for the velocity of the planet into the above equation.

$$G\frac{m_S}{r} = \left(\frac{2\pi r}{T}\right)^2$$
$$G\frac{m_S}{r} = \frac{4\pi^2 r^2}{T^2}$$

- Multiply each side of the equation by T^2/r^2.

$$\left(G\frac{m_S}{r}\right)\left(\frac{T^2}{r^2}\right) = \left(\frac{4\pi^2 \cancel{r^2}}{\cancel{T^2}}\right)\left(\frac{\cancel{T^2}}{\cancel{r^2}}\right)$$
$$\frac{Gm_S T^2}{r^3} = 4\pi^2$$

- Solve for T^2/r^3.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{Gm_S}$$

As you can see, Newton's law of universal gravitation indicates not only that the ratio T^2/r^3 is constant, but also that the constant is $4\pi^2/Gm_S$. All of Kepler's laws, developed prior to the time when

Newton did his work, support Newton's law of universal gravitation. Kepler had focussed only on the Sun and planets, but Newton proposed that the laws applied to all types of orbital motion, such as moons around planets. Today, we know that all of the artificial satellites orbiting Earth, as well as the Moon, follow Kepler's laws.

Mass of the Sun and Planets

HISTORY LINK

Henry Cavendish was a very wealthy and brilliant man, but he also was very reclusive. He was rarely seen in public places, other than at scientific meetings. His work was meticulous, yet he published only a very small part of it. After his death, other scientists discovered his notebooks and finally published his results. Cavendish had performed the same experiments and obtained the same results for some experiments that were later done by Coulomb, Faraday, and Ohm, who received the credit for the work.

Have you ever looked at tables that contain data for the mass of the Sun and planets and wondered how anyone could "weigh" the Sun and planets or determine their masses? English physicist and chemist Henry Cavendish (1731–1810) realized that if he could determine the universal gravitational constant, *G*, he could use the mathematical relationship in Kepler's third law to calculate the mass of the Sun. A brilliant experimentalist, Cavendish designed a torsion balance, similar to the system in Figure 3.4, that allowed him to measure *G*.

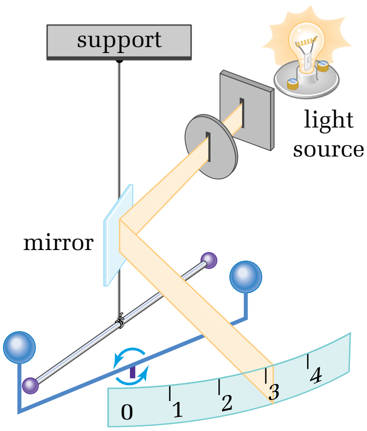


Figure 3.4 In the torsion balance that Cavendish designed and used, the spheres were made of lead. The small spheres were about 5 cm in diameter and were attached by a thin but rigid rod about 1.83 m long. The large spheres were about 20 cm in diameter.

A torsion balance can measure extremely small amounts of the rotation of a wire. First, the torsion balance must be calibrated to determine the amount of force that causes the wire to twist by a specific amount. Then, the large spheres are positioned so that the bar supporting them is perpendicular to the rod supporting the small spheres. In this position, the large spheres are exerting equal gravitational attractive forces on each of the small spheres. The system is in equilibrium and the scale can be set to zero. The large spheres are then moved close to the small spheres and the amount of twisting of the wire is determined. From the amount of twisting and the calibration, the mutual attractive force between the large and small spheres is calculated.

Using his torsion balance, Cavendish calculated the value of *G* to be $6.75 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$. The best-known figure today is $6.672\ 59 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$. Cavendish's measurement was within approximately 1% of the correct value. As Cavendish did, you can now calculate the mass of the Sun and other celestial bodies.

PHYSICS FILE

The value of *G* shows that the force of gravity is extremely small. For example, use unit amounts of each of the variables and substitute them into Newton's law of universal gravitation. You will find that the mutual attractive force between two 1 kg masses that are 1 m apart is $6.672\ 59 \times 10^{-11}$ N.

SAMPLE PROBLEM

The Mass of the Sun

Find the mass of the Sun, using Earth's orbital radius and period of revolution.

Conceptualize the Problem

- Kepler's third law, combined with Newton's law of universal gravitation, yields an equation that relates the period and orbital radius of a satellite to the mass of the body around which the satellite is orbiting.
- Earth orbits the Sun once per year.
- Let R_E represent the radius of Earth's orbit around the Sun. This value can be found in Appendix B, Physical Constants and Data.

Identify the Goal

The mass of the Sun, m_S

Identify the Variables and Constants

Known	Implied	Unknown
Sun	$G = 6.67 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}$	m_S
	$T = 365.25$ days	
	$R_{E(orbit)} = 1.49 \times 10^{11}$ m	

Develop a Strategy

Write Kepler's third law, using the constant derived from Newton's law of universal gravitation.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{Gm_S}$$

Solve for the mass of the Sun.

$$\frac{T^2}{r^3} m_S = \frac{4\pi^2}{G\cancel{m_S}}\cancel{m_S}$$

$$m_S = \left(\frac{4\pi^2}{G}\right)\left(\frac{r^3}{T^2}\right)$$

Convert the period into SI units.

$$365.25\ \cancel{days}\left(\frac{24\ \cancel{h}}{\cancel{day}}\right)\left(\frac{60\ \cancel{min}}{\cancel{h}}\right)\left(\frac{60\ s}{\cancel{min}}\right) = 3.1558 \times 10^7\ s$$

Substitute the numerical values into the equation and solve.

$$m_S = \left(\frac{4\pi^2}{6.67 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}}\right)\frac{(1.49 \times 10^{11}\ m)^3}{(3.1558 \times 10^7\ s)^2}$$

$$m_S = 1.9660 \times 10^{30}\ kg$$

$$m_S \cong 1.97 \times 10^{30}\ kg$$

The mass of the Sun is approximately 1.97×10^{30} kg.

Validate the Solution

The Sun is much more massive than any of the planets. The value sounds reasonable.

Check the units: $\left(\frac{1}{\frac{N \cdot m^2}{kg^2}}\right)\left(\frac{m^3}{s^2}\right) = \left(\frac{kg^2}{N \cdot \cancel{m^2}}\right)\left(\frac{\cancel{m^3}}{s^2}\right) = \left(\frac{kg^{\cancel{2}}}{\frac{\cancel{kg} \cdot \cancel{m}}{\cancel{s^2}} \cdot \cancel{m^2}}\right)\left(\frac{\cancel{m^3}}{\cancel{s^2}}\right) = kg$.

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PRACTICE PROBLEMS

9. Jupiter's moon Io orbits Jupiter once every 1.769 days. Its average orbital radius is 4.216×10^8 m. What is Jupiter's mass?
10. Charon, the only known moon of the planet Pluto, has an orbital period of 6.387 days at an average distance of 1.9640×10^7 m from Pluto. Use Newton's form of Kepler's third law to find the mass of Pluto from this data.
11. Some weather satellites orbit Earth every 90.0 min. How far above Earth's surface is their orbit? (Hint: Remember that the centre of the orbit is the centre of Earth.)
12. How fast is the moon moving as it orbits Earth at a distance of 3.84×10^5 km?
13. On each of the *Apollo* lunar missions, the command module was placed in a very low, approximately circular orbit above the Moon. Assume that the average height was 60.0 km above the surface and that the Moon's radius is 7738 km.
    (a) What was the command module's orbital period?
    (b) How fast was the command module moving in its orbit?
14. A star at the edge of the Andromeda galaxy appears to be orbiting the centre of that galaxy at a speed of about 2.0×10^2 km/s. The star is about 5×10^9 AU from the centre of the galaxy. Calculate a rough estimate of the mass of the Andromeda galaxy. Earth's orbital radius (1 AU) is 1.49×10^8 km.

Newton's law of universal gravitation has stood the test of time and the extended limits of space. As far into space as astronomers can observe, celestial bodies move according to Newton's law. As well, the astronauts of the crippled *Apollo 13* spacecraft owe their lives to the dependability and predictability of the Moon's gravity. Although Albert Einstein (1879–1955) took a different approach in describing gravity in his general theory of relativity, most calculations that need to be made can use Newton's law of universal gravitation and make accurate predictions.

3.1 Section Review

1. K/U Explain the meaning of the term "empirical" as it applies to empirical equations.
2. K/U What did Tycho Brahe contribute to the development of the law of universal gravitation?
3. K/U Describe how Newton used each of the following phenomena to support the law of universal gravitation.
    (a) the orbit of the moon
    (b) Kepler's third law
4. K/U How did Newton's concepts about gravity and his development of the law of universal gravitation differ from the ideas of other scientists and astronomers who were attempting to find a relationship that could explain the motion of the planets?
5. K/U Describe the objective, apparatus, and results of the Cavendish experiment.
6. C Explain how you can "weigh" a planet.
7. I Suppose the distance between two objects is doubled and the mass of one is tripled. What effect does this have on the gravitational force between the objects?

3.2 Planetary and Satellite Motion

A perfectly executed football or hockey pass is an amazing achievement. A fast-moving player launches an object toward the place where a fast-moving receiver will most likely be when the ball or puck arrives. The direction and force of the pass are guided by intuition and skill acquired from long practice.

Launching a spacecraft has the same objective, but extreme precision is required — the outcome is more critical than that of an incomplete pass. Scientists and engineers calculate every detail of the trajectories and orbits in advance. The magnitude and direction of the forces and the time interval for firing the rockets are analyzed and specified in minute detail. Even last-minute adjustments are calculated exhaustively. The process can be extremely complex, but it is based on principles that you have already studied — the dynamics of circular motion and the law of universal gravitation.

SECTION EXPECTATION

- Use Newton's law of universal gravitation to explain planetary and satellite motion.

KEY TERMS

- geostationary orbit
- microgravity
- perturbation

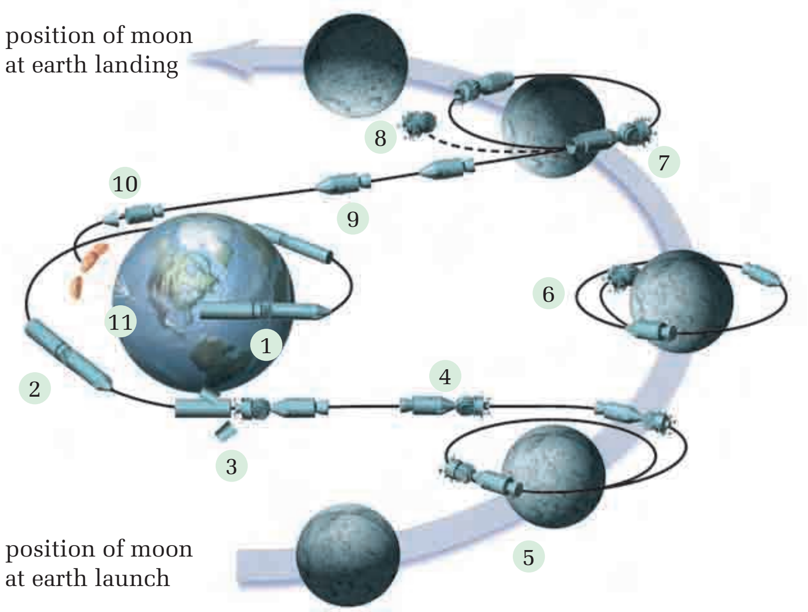

Figure 3.5 The steps in a typical *Apollo* lunar mission are: **(1)** lift off and enter Earth orbit, **(2)** leave Earth orbit, **(3)** release booster rocket, turn, and dock with lunar module that is stored between the booster rocket and the service module, **(4)** make a mid-course correction, **(5)** enter lunar orbit, **(6)** command module continues to orbit the Moon, while the lunar module descends to the lunar surface, carries out tasks, ascends, and reconnects with the command module, **(7)** leave lunar orbit, **(8)** eject lunar module, **(9)** mid-course correction, **(10)** eject service module, and **(11)** command module lands in the Pacific Ocean.

Newton's Mountain

The planets in our solar system appear to have been "orbiting" the Sun while they were forming. Great swirling dust clouds in space began to condense around a newly formed Sun until they finally became the planets. How, then, do artificial satellites begin orbiting Earth?

Soon after Newton formulated his law of universal gravitation, he began thought experiments about artificial satellites. He reasoned that you could put a cannon at the top of an extremely high mountain and shoot a cannon ball horizontally, as shown in Figure 3.6. The cannon ball would certainly fall toward Earth. If the cannon ball travelled far enough horizontally while it fell, however, the curvature of Earth would be such that Earth's surface would "fall away" as fast as the cannon ball fell.

You can determine how far the cannon ball will fall in one second by using the kinematic equation $\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$. If a cannon ball had zero vertical velocity at time zero, in one second it would fall a distance $\Delta d = 0 + \frac{1}{2}\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)(1\ \text{s})^2 = -4.9\ \text{m}$. From the size and curvature of Earth, Newton knew that Earth's surface would drop by 4.9 m over a horizontal distance of 8 km.

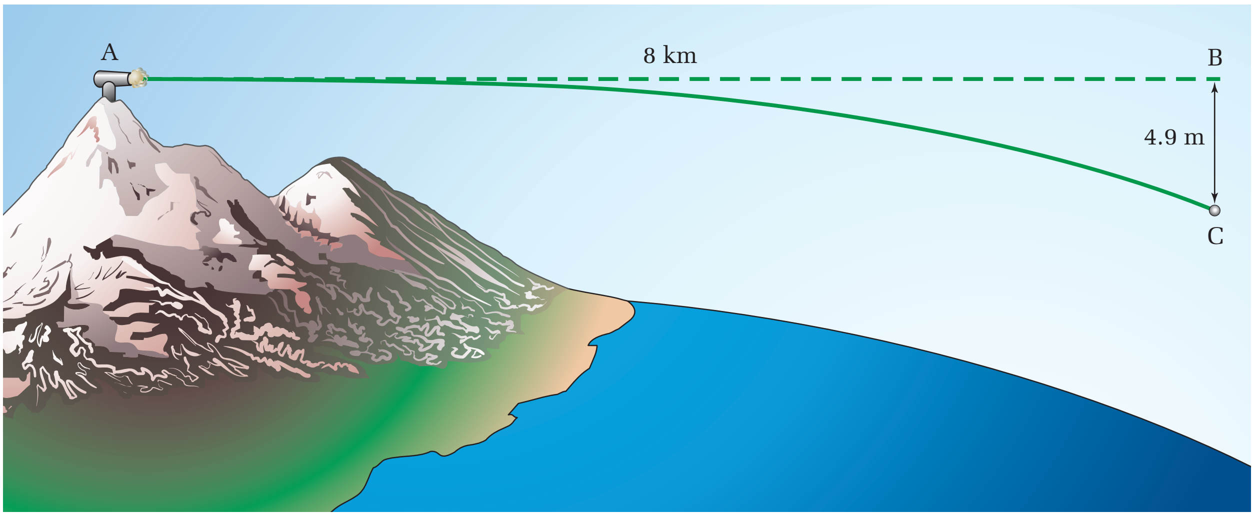

Figure 3.6 The values shown here represent the distance that the cannon ball would have to go in one second in order to go into orbit.

Newton's reasoning was absolutely correct, but he did not account for air friction. Although the air is too thin to breathe easily on top of Mount Everest, Earth's highest mountain, it would still exert a large amount of air friction on an object moving at 8 km/s. If you could take the cannon to 150 km above Earth's surface, the atmosphere would be so thin that air friction would be negligible. Newton understood how to put an artificial satellite into orbit, but he did not have the technology.

Today, launching satellites into orbit is almost routine, but the scientists and engineers must still carefully select an orbit and perform detailed calculations to ensure that the orbit will fulfil the purpose of the satellite. For example, some weather satellites orbit over the Poles at a relatively low altitude in order to collect data in detail. Since a satellite is constantly moving in relation to a ground observer, the satellite receiver has to track the satellite continually so that it can capture the signals that the satellite is sending. In addition, the satellite is on the opposite side of Earth for long periods of time, so several receivers must be located around the globe to collect data at all times.

Communication satellites and some weather satellites travel in a **geostationary orbit** over the equator, which means that they appear to hover over one spot on Earth's surface at all times. Consequently, a receiver can be aimed in the same direction at all times and constantly receive a signal from the satellite. The following problem will help you to find out how these types of orbits are attained.

PHYSICS FILE

Arthur C. Clarke (1917–), scientist and science fiction writer, wrote a technical paper in 1945, setting out the principles of geostationary satellites for communications. Many scientists in the field at the time did not believe that it was possible. Today, geostationary orbits are sometimes called "Clarke orbits." Clarke also co-authored the book and movie *2001: A Space Odyssey*.

SAMPLE PROBLEM

Geostationary Orbits

At what velocity and altitude must a satellite orbit in order to be geostationary?

Conceptualize the Problem

- A satellite in a *geostationary orbit* must remain over the *same point* on Earth at all times.
- To be geostationary, the satellite must make *one complete orbit* in exactly the same time that Earth rotates on its axis. Therefore, the period must be *24 h*.
- The *period* is related to the *velocity* of the satellite.
- The *velocity* and *altitude* of the satellite are determined by the amount of *centripetal force* that is causing the satellite to remain on a circular path.
- Earth's *gravity* provides the *centripetal force* for satellite motion.
- The values for the mass and radius of Earth are listed in Appendix B, Physical Constants and Data.

Identify the Goal

(a) The velocity, v, of a geostationary satellite

(b) The altitude, h, of a geostationary satellite

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Identify the Variables and Constants

Known	**Implied**		**Unknown**
Orbit is geostationary.	$T = 24$ h	$m_E = 5.98 \times 10^{24}$ kg	r (radius of satellite's orbit)
	$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$	$r_E = 6.38 \times 10^6$ m	h

Develop a Strategy

To find the velocity, start by setting the gravitational force equal to the centripetal force.

$$F_g = F_c$$

$$G\frac{m_E m_s}{r^2} = \frac{m_s v^2}{r}$$

Simplify the expression.

$$G\frac{m_E}{r} = v^2$$

To eliminate r from the equation, use the equation for the definition of velocity and solve for r. Recall that the period, T, is known.

$$v = \frac{\Delta d}{\Delta t}$$

$$\Delta d = 2\pi r$$

$$\Delta t = T$$

$$v = \frac{2\pi r}{T}$$

$$r = \frac{vT}{2\pi}$$

Substitute the expression for r into the equation for v above and solve for v.

$$G\frac{m_E}{\frac{vT}{2\pi}} = v^2$$

$$v^2 = \frac{2\pi G m_E}{vT}$$

$$v^3 = \frac{2\pi G m_E}{T}$$

$$v = \sqrt[3]{\frac{2\pi G m_E}{T}}$$

Convert T to SI units.

$$T = (24\ \cancel{h})\left(\frac{60\ \cancel{min}}{\cancel{h}}\right)\left(\frac{60\ s}{\cancel{min}}\right) = 8.64 \times 10^4\ s$$

Substitute numerical values and solve.

$$v = \sqrt[3]{\frac{2\pi\left(6.67 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}\right)(5.98 \times 10^{24}\ kg)}{8.64 \times 10^4\ s}}$$

$$v = \sqrt[3]{2.9006 \times 10^{10}\ \frac{m^3}{s^3}}$$

$$v = 3.0724 \times 10^3\ \frac{m}{s}$$

$$v \cong 3.07 \times 10^3\ \frac{m}{s}$$

(a) The orbital velocity of the satellite is about 3.07×10^3 m/s, which is approximately 11 000 km/h.

To find the altitude of the satellite, substitute the value for velocity into the expression above that you developed to find r in terms of v.

$$r = \frac{vT}{2\pi}$$

$$r = \frac{\left(3.0724 \times 10^3\ \frac{\text{m}}{\text{s}}\right)(8.64 \times 10^4\ \text{s})}{2\pi}$$

$$r = 4.2249 \times 10^7\ \text{m}$$

The calculated value for r is the distance from Earth's centre to the satellite. To find the altitude of the satellite, you must subtract Earth's radius from r.

$$r = r_{\text{E}} + h$$

$$h = r - r_{\text{E}}$$

$$h = 4.2249 \times 10^7\ \text{m} - 6.38 \times 10^6\ \text{m}$$

$$h = 3.5869 \times 10^7\ \text{m}$$

$$h \cong 3.59 \times 10^7\ \text{m}$$

(b) The altitude of all geostationary satellites must be 3.59×10^7 m, or 3.59×10^4 km, above Earth's surface.

Validate the Solution

A velocity of 11 000 km/h seems extremely fast to us, but the satellite is circling Earth once per day, so the velocity is reasonable.

Check the units for velocity. $$\sqrt[3]{\frac{\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\text{kg}}{\text{s}}} = \sqrt[3]{\frac{\text{N}\cdot\text{m}^2\cdot\cancel{\text{kg}}}{\text{s}\cdot\text{kg}^{\cancel{2}}}} = \sqrt[3]{\frac{\frac{\cancel{\text{kg}}\cdot\text{m}^3}{\text{s}^2}}{\text{s}\cdot\cancel{\text{kg}}}} = \sqrt[3]{\frac{\text{m}^3}{\text{s}^3}} = \frac{\text{m}}{\text{s}}$$

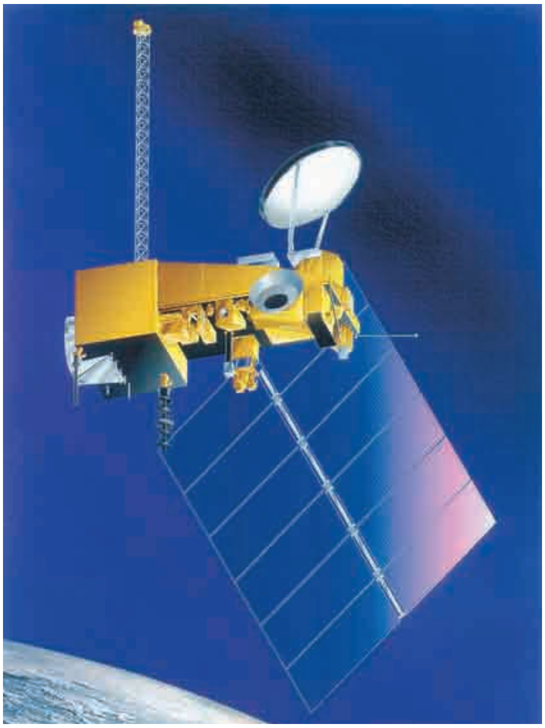
PRACTICE PROBLEMS

15. The polar-orbiting environmental satellites (POES) and some military satellites orbit at a much lower level in order to obtain more detailed information. POES complete an Earth orbit 14.1 times per day. What are the orbital speed and the altitude of POES?

16. The International Space Station orbits at an altitude of approximately 226 km. What is its orbital speed and period?

17. (a) The planet Neptune has an orbital radius around the Sun of about 4.50×10^{12} m. What are its period and its orbital speed?

(b) Neptune was discovered in 1846. How many orbits has it completed since its discovery?



NASA operates two polar-orbiting environmental satellites (POES) designed to collect global data on cloud cover; surface conditions such as ice, snow, and vegetation; atmospheric temperatures; and moisture, aerosol, and ozone distributions.

"Weightlessness" in Orbit

You have probably seen pictures of astronauts in a space capsule, space shuttle, or space station similar to Figure 3.7. The astronauts appear to float freely in the spacecraft and they describe the condition as being "weightless." Is a 65 kg astronaut in a space station weightless? Check it out, using the following calculation.

Figure 3.7 If an astronaut in a spacecraft dropped an apple, it would fall toward Earth, but it would not look as though it was falling.

- The space station orbits at an altitude of approximately 226 km. Find the radius of its orbit by adding the altitude to Earth's radius.

$$r = r_E + h$$

$$r = 6.38 \times 10^6\ \text{m} + 226\ \cancel{\text{km}}\left(\frac{1000\ \text{m}}{\cancel{\text{km}}}\right)$$

$$r = 6.606 \times 10^6\ \text{m}$$

- Use Newton's law of universal gravitation to find the astronaut's weight.

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$F_g = \left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(5.98 \times 10^{24}\ \text{kg})(65\ \text{kg})}{(6.606 \times 10^6\ \text{m})^2}$$

$$F_g = 594\ \text{N}$$

The astronaut's weight on Earth would be $F_g = mg = (65\ \text{kg})(9.81\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}) \cong 638\ \text{N}$. There is very little difference between the astronaut's weight on Earth and in the space station. Why do astronauts appear "weightless" in space? Think back to your calculations of apparent weight in Chapter 1, Fundamentals of Dynamics. When the imaginary elevator that you were riding in was falling with an acceleration of 9.81 m/s^2, the scale you were standing on read zero newtons. Your apparent weight was zero because you, the scale, and the elevator were falling with the same acceleration. You and the scale were not exerting any force on each other.

The same situation exists in the space station and all orbiting spacecraft all of the time. As in the case of Newton's cannon ball, everything falls to Earth with the same acceleration, but Earth is "falling away" equally as fast. You could say that a satellite is "falling around Earth." Some physicists object to the term "weightlessness" because, as you saw, there is no such condition. NASA coined the term **microgravity** to describe the condition of apparent weightlessness.

WEB LINK

www.mcgrawhill.ca/links/physics12

To learn about the effects of weightlessness on the human body, go to the above Internet site and click on **Web Links**.

PHYSICS & TECHNOLOGY

TARGET SKILLS
- Analyzing and interpreting
- Communicating results

Simulating Microgravity

In preparation for space flights, astronauts benefit by practising movements in microgravity conditions. As you recall from your study of projectile motion, an object will follow a parabolic trajectory if the only force acting on it is gravity. Large jet aircraft can fly on a perfect parabolic path by exerting a force to overcome air friction. Thus, inside the aircraft, all objects move as though they were following a path determined only by gravity. Objects inside the aircraft, including people, exert no forces on each other, because they are all "falling" with the same acceleration. Astronauts can experience 20 s of microgravity on each parabolic trajectory.

Scientists have also found that certain chemical and physical reactions occur in a different way in microgravity. They believe that some manufacturing processes can be carried out more efficiently under these conditions — a concept that might lead to manufacturing in space. To test some of these reactions without going into orbit, researchers sometimes use drop towers, as shown in the photograph. A drop tower has a very long shaft that can be evacuated to eliminate air friction. A sample object or, sometimes, an entire experiment is dropped and the reaction can proceed for up to 10 s in microgravity conditions. These experiments provide critical information for future processing in space.

Analyze

1. Explain in detail why an airplane must use energy to follow an accurate parabolic trajectory, but everything inside the airplane appears to be weightless.
2. Do research to learn about some chemical or physical processes that might be improved by carrying them out in microgravity.

WEB LINK

www.mcgrawhill.ca/links/physics12

To learn more about experiments carried out in drop towers, go to the above Internet site and click on **Web Links**.

Perturbing Orbits

In all of the examples that you have studied to this point, you have considered only perfectly circular or perfectly elliptical orbits. Such perfect orbits would occur only if the central body and satellite were totally isolated from all other objects. Since this is essentially never achieved, all orbits, such as those of planets around the Sun and moons and satellites around planets, are slightly distorted ellipses. For example, when an artificial satellite is between the Moon and Earth, the Moon's gravity pulls in an opposite direction to that of Earth's gravity. When a satellite is on the side of Earth opposite to the Moon, the Moon and Earth exert their forces in the same direction. The overall effect is a very slight change in the satellite's orbit. Engineers must take these effects into account.

In the solar system, each planet exerts a gravitational force on every other planet, so each planet perturbs the orbit of the other planets. In some cases, the effects are so small that they cannot be measured. Astronomers can, however, observe these **perturbations** in the paths of the planets when the conditions are right. In fact, in 1845, two astronomers in two different countries individually observed perturbations in the orbit of the planet Uranus. British astronomer and mathematician John Couch Adams (1819–1892) and French astronomer Urbain John Joseph Le Verrier (1811–1877) could not account for their observed perturbations of the planet's orbit, even by calculating the effects of the gravitational force of the other planets. Both astronomers performed detailed calculations and predicted both the existence and the position of a new, as yet undiscovered planet. In September of 1846, at the Berlin Observatory, astronomer J.G. Galle (1812–1910) searched the skies at the location predicted by the two mathematical astronomers. Having excellent star charts for comparison, Galle almost immediately observed the new planet, which is now called "Neptune."

About 50 years later, U.S. astronomer Percival Lowell (1855–1916) performed calculations on the orbits of both Neptune and Uranus, and discovered that these orbits were again perturbed, probably by yet another undiscovered planet. About 14 years after Lowell's death, astronomer Clyde Tombaugh (1906–1997), working in the Lowell Observatory, discovered the planet now called "Pluto."

Since the discovery of two planets that were predicted mathematically by the perturbations of orbits of known planets, several more predictions about undiscovered planets have been made. None have been discovered and most astronomers believe that no more planets exist in our solar system. The laws of Newton and Kepler, however, have provided scientists and astronomers with a solid foundation on which to explain observations and make predictions about planetary motion, as well as send space probes out to observe all of the planets in our solar system.

3.2 Section Review

1. C Explain Newton's thought experiment about "launching a cannon-ball satellite."
2. I Why must a geostationary satellite orbit over the equator? To answer that question, think about the point that is the centre of the orbit. If you launched a satellite that had a period of 24 hr, but it did not start out over the equator, what path would it follow? If you were at the spot on Earth just below the point where the satellite started to orbit, how would the path of the satellite appear to you?
3. K/U What conditions create apparent weightlessness when an astronaut is in an orbiting spacecraft?
4. K/U How could you discover a planet without seeing it with a telescope?

CHAPTER 3 Review

REFLECTING ON CHAPTER 3

- Tycho Brahe collected detailed astronomical data for 20 years.
- Johannes Kepler analyzed Brahe's data and developed three empirical equations that are now called "Kepler's laws."
  1. Planets follow elliptical paths.
  2. The areas swept out by a line from the Sun to a planet during a given time interval are always the same.
  3. $\frac{T^2}{r^3} = k$ or $\frac{T_A^2}{r_A^3} = \frac{T_B^2}{r_B^3}$
- Kepler's laws support Newton's law of universal gravitation.
- Newton extended the concept of gravity and showed that not only does it cause celestial bodies to attract each other, but also that all masses exert a mutually attractive force on each other.
- Newton's law of universal gravitation is written mathematically as $F_g = G\frac{m_1 m_2}{r^2}$.
- Newton's law of universal gravitation shows that the constant in Kepler's third law is given by $\frac{T^2}{r^3} = \frac{4\pi^2}{Gm_s}$.
- You can use the combination of Newton's law of universal gravitation and Kepler's third law to determine the mass of any celestial body that has one or more satellites.
- Cavendish used a torsion balance to determine the universal gravitational constant, G, in Newton's law of universal gravitation: $G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$.
- Newton reasoned that if you shot a cannon ball from the top of a very high mountain and it went fast enough horizontally, it would fall exactly the same distance that Earth's surface would drop, due to the spherical shape. This is the correct theory about satellite motion.
- A satellite in a geostationary orbit appears to hover over one spot on Earth's surface, because it completes one cycle as Earth makes one revolution on its axis.
- Apparent weightlessness in orbiting spacecraft is due to the fact that the spacecraft and everything in it are falling toward Earth with exactly the same acceleration.
- The planets Neptune and Pluto were both discovered because their gravity perturbed the path of other planets.

Knowledge/Understanding

1. Distinguish between mass and weight.
2. Define **(a)** heliocentric and **(b)** geostationary.
3. State Kepler's laws.
4. Several scientists and astronomers had developed the concept that the attractive force on planets orbiting the Sun decreased with the square of the distance between a planet and the Sun. What was Newton's reasoning for including in his law of universal gravitation the magnitude of the masses of the planets? In what other ways did Newton's law of universal gravitation differ conceptually from the ideas of other scientists of his time?
5. Explain how Kepler's third law supports Newton's law of universal gravitation.
6. Is Kepler's constant a universal constant? That is, can it be applied to Jupiter's system of satellites or to other planetary systems? Explain.
7. How does a torsion balance work?
8. Explain whether it is possible to place a satellite into geosynchronous orbit above Earth's North Pole.

Inquiry

9. No reliable evidence supports the astrological claim that the motions of the stars and planets affect human activities. However, belief in astrology remains strong. Create an astrology defence or opposition kit. Include any or all of the following: arguments based on Newton's laws to refute or support astrological claims,

an experiment that tests the validity of birth horoscopes, a report on scientific studies of astrological claims and any findings, a summary of the success and failure of astrological predictions and a comparison of these to the success of predictions based on chance.

10. Devise an observational test (which will require a telescope) that will convince a doubting friend that Earth orbits the Sun.

11. Demonstrate the inverse square law form of the universal law of gravitation by calculating the force on a 100.0 kg astronaut who is placed at a range of distances from Earth's surface, out to several Earth radii. Make a graph of force versus position and comment on the results.

12. You often hear that the Moon's gravity, as opposed to the Sun's gravity, is responsible for the tides.

(a) Calculate the force of gravity that the Moon and the Sun exert on Earth. How does this appear to conflict with the concept stated above?

(b) Calculate the force of the Moon's gravity on 1.00×10^4 kg of water at the surface of an ocean on the same side of Earth as the Moon and on the opposite side of Earth from the Moon. Also, calculate the Moon's gravity on 1.00×10^4 kg of matter at Earth's centre. (Assume that the distance between the Moon's centre and Earth's centre is 3.84×10^8 m and that Earth's radius is 6.38×10^6 m.)

(c) Perform the same calculations for the Sun's gravity on these masses. (Use 1.49×10^{11} m for the distance between the centres of Earth and the Sun.)

(d) Examine your results from parts (b) and (c) and use them to justify the claim that the Moon's gravity is responsible for tides.

13. Many comets have been identified and the regularity of their return to the centre of the solar system is very predictable.

(a) From what you have learned about satellite motion, provide a logical explanation for the disappearance and reappearance of comets.

(b) Make a rough sketch of the solar system and add to it a probable comet path.

(c) What is the nature of the path taken by comets?

Communication

14. Consider a marble of mass m accelerating in free fall in Earth's gravity. Neglect air resistance and show that the marble's acceleration due to gravity is independent of its mass. (That is, you could use a bowling ball, a feather, or any other object in free fall and obtain the same result.) Hint: Equate Newton's universal law of gravitation to Newton's second law. Look up the values for Earth's mass and radius and use them in your acceleration equation to calculate the marble's acceleration. This number should be familiar to you!

15. Suppose that the Sun's mass was four times greater than it is now and that the radius of Earth's orbit was unchanged. Explain whether a year would be longer or shorter. By what factor would the period change? Explain in detail how you determined your answer.

16. (a) At what distance from Earth would an astronaut have to travel to actually experience a zero gravitational force, or "zero g"?

(b) Are astronauts in a space shuttle orbiting Earth subject to a gravitational force?

(c) How can they appear to be "weightless"?

17. A cow attempted to jump over the Moon but ended up in orbit around the Moon, instead. Describe how the cow could be used to determine the mass of the Moon.

18. Discuss what would happen to Earth's motion if the Sun's gravity was magically turned off.

19. The Sun gravitationally attracts Earth. Explain why Earth does not fall into the Sun.

Making Connections

20. Examine some Olympic records, such as those for the long-jump, shot-put, weightlifting, high-jump, javelin throw, 100 m dash, 400 m hurdles, and the marathon. How would you expect these records to change if the events were performed under an athletic dome on the Moon?

21. Einstein once recalled his inspiration for the theory of general relativity from a sudden thought that occurred to him: "If a person falls freely, he will not feel his own weight.'' He said he was startled by the simple thought and that it impelled him toward a theory of gravitation. Although the mathematics of the general theory of relativity is advanced, its concepts are fascinating and have been described in several popular books. Research and write a short essay on the general theory of relativity, including a discussion of its predictions and tests, and how it supersedes (but does not replace) Newton's theory of gravitation.

Problems for Understanding

22. The gravitational force between two objects is 80.0 N. What would the force become if the mass of one object was halved and the distance between the two objects was doubled?

23. Two stars of masses $m*$ and $3m*$ are 7.5×10^{11} m apart. If the force on the large star is F, which of the following is the force on the small star?
(a) $F/9$ **(b)** $F/3$ **(c)** F **(d)** $3F$ **(e)** $9F$

24. For the above situation, if the acceleration of the small star is a, what is the acceleration of the large star?
(a) $a/9$ **(b)** $a/3$ **(c)** a **(d)** $3a$ **(e)** $9a$

25. **(a)** Use Newton's law of universal gravitation and the centripetal force of the Sun to determine Earth's orbital speed. Assume that Earth orbits in a circle.
(b) What is Earth's centripetal acceleration around the Sun?

26. Calculate the Sun's acceleration caused by the force of Earth.

27. A space shuttle is orbiting Earth at an altitude of 295 km. Calculate its acceleration and compare it to the acceleration at Earth's surface.

28. Orbital motions are routinely used by astronomers to calculate masses. A ring of high-velocity gas, orbiting at approximately 3.4×10^4 m/s at a distance of 25 light-years from the centre of the Milky Way, is considered to be evidence for a black hole at the centre. Calculate the mass of this putative black hole. How many times greater than the Sun's mass is it?

29. In a Cavendish experiment, two 1.0 kg spheres are placed 50.0 cm apart. Using the known value of G, calculate the gravitational force between these spheres. Compare this force to the weight of a flea.

30. The Hubble space telescope orbits Earth with an orbital speed of 7.6×10^3 m/s.
(a) Calculate its altitude above Earth's surface.
(b) What is its period?

31. The Moon orbits Earth at a distance of 3.84×10^8 m. What are its orbital velocity and period?

32. The following table gives orbital information for five of Saturn's largest satellites.

Satellite	Mean orbital radius (m)	Period (days)
Tethys	2.95×10^8	1.888
Dione	3.78×10^8	2.737
Rhea	5.26×10^8	4.517
Titan	1.221×10^9	15.945
Iapetus	3.561×10^9	79.331

(a) Determine whether these satellites obey Kepler's third law.
(b) If they obey Kepler's third law, use the data for the satellites to calculate an average value for the mass of Saturn.

33. Suppose the Oort cloud of comets contains 10^{12} comets, which have an average diameter of 10 km each.
(a) Assume that a comet is composed mostly of water-ice with a density of 1.00 g/cm^3 and calculate the mass of a comet.
(b) Calculate the total mass of the Oort cloud.
(c) Compare your mass of the Oort cloud to the mass of Earth and of Jupiter.

UNIT 1 PROJECT

Catapult Machine

Background

Humans have built machines for launching projectiles since ancient times. The Romans constructed *ballista* to hurl stones and *catapulta* to shoot arrows. In one design, stretched or twisted ropes were suddenly released to launch the projectile. Other machines bent and then released wooden beams. The medieval *trebuchet* harnessed the energy of a falling counterweight. More recently, catapults powered by compressed air provided the first effective method of launching aircraft from ships.

A medieval catapult

Challenge

In a small group, design, construct, test, and evaluate a catapult that launches a standard projectile to meet specified flight criteria. Your class as a whole will decide on the criteria for the flight and any restrictions on building materials and cost. Each catapult will be evaluated by comparing its performance to the expected results for an ideal projectile. As part of the project, you will prepare a report that outlines the design features of your catapult, provides an analysis of its operation, and makes recommendations for its improvement.

Materials

- construction materials, such as wood, plastic, cardboard, metal
- elastic materials, such as elastic bands, springs, or a mousetrap
- materials to attach parts together, such as fasteners, tape, and glue
- materials for the projectile
  - foam plastic egg carton
  - plastic sandwich bags
  - sand

Safety Precautions

- Wear eye protection when using power tools.
- Ensure that electrical equipment, such as power tools, is properly grounded.
- Take appropriate precautions when using electrical equipment.
- Take appropriate precautions when using knives, saws, and other sharp tools.
- Handle glue guns with care to avoid burns. Hot glue guns take several minutes to cool after they are disconnected.
- Wear eye protection at all times when testing your catapult.
- Test your catapult in a large, clear space, away from other people and from equipment and windows that could be damaged.
- Ensure that all spectators are behind the catapult before firing it.

Project Criteria

A. As a class, decide on the criteria for evaluating your catapults. Possible challenges are to construct catapults that launch projectiles to hit a specific target, or to achieve maximum range or a specified flight time, or to reach a particular height or go over a wall.

B. As a class, decide whether your catapults are to be made entirely from recycled materials or to be constructed from other materials within a set cost limit.

C. Research, design, and construct your catapult to launch egg-carton projectiles. These projectiles are to be made from a single section of a foam plastic egg carton, filled with 25 grams of sand in a plastic bag. Tape the bag of sand securely inside the egg-carton section. The total mass of the projectile is not to exceed 30 grams.

D. Prepare a written report about your project that includes

- an appropriate title and the identification of group members
- a labelled design drawing of the catapult
- an overview of the physics involved that includes calculations of
  - the average force applied to the projectile
  - the distance through which the force is applied
  - the time for which the force is applied
- a theoretical prediction for the performance of your projectile that includes
  - its range
  - its maximum height
  - its flight time
- an analysis of the catapult's performance that includes calculations or measurements of the
  - average launch velocity
  - launch angle
  - flight time
  - range
  - maximum height achieved
- an evaluation of the catapult's performance and recommendations for its refinement

ASSESSMENT

After you complete this project

- **assess the performance of your catapult. How closely did the projectile meet your challenge criteria?**
- **assess the design of your catapult. What physics and engineering principles did you incorporate into its design?**
- **assess the problem-solving effectiveness of your group. What design and construction obstacles did you face during this project? How did you overcome them?**

Action Plan

1. Work in groups of two to four people.
2. Establish a time line for the design, construction, testing, and evaluation phases of this project.
3. Research possible designs and energy sources for your catapult. Select one that can be adapted to be feasible and meet the design criteria.
4. Construct and test your catapult, measuring the quantities specified above.
5. Prepare the written report and enter the competition.

Evaluate

1. Compare the average performance of your catapult with your theoretical predictions and the challenge criteria set out by the class.
2. Recommend refinements to your catapult. Indicate specifically how performance was affected by each design feature that you have identified for improvement.

WEB LINK

www.mcgrawhill.ca/links/physics12

For a lot of pictures and some stories about "leverage artillery" used in times past and for plans for various types of catapults, go to the above Internet site and click on **Web Links**.

UNIT 1 Review

Knowledge/Understanding

Multiple Choice

In your notebook, choose the most correct answer for each of the following questions. Outline your reasons for your choice.

1. A ball is thrown upward. After it is released, its acceleration
- **(a)** is zero
- **(b)** increases
- **(c)** decreases
- **(d)** remains constant
- **(e)** increases, then decreases

2. You drop a 1.0 kg stone off the roof of a 10-storey building. Just as the stone passes the fifth floor, your friend drops a 1.0 kg ball out of a fifth-floor window. If air resistance is neglected, which of the following statements is true? Explain your reasoning.
- **(a)** The stone and the ball hit the ground at the same time and at the same speed.
- **(b)** The stone hits the ground first and with a greater speed than the ball does.
- **(c)** The stone and the ball hit the ground at the same time, but the speed of the stone is greater.
- **(d)** The ball hits the ground before the stone.

3. A football is thrown by a quarterback to a receiver deep in the end zone. The acceleration of the football during the flight
- **(a)** depends on how the ball was thrown
- **(b)** depends on whether the ball is going up or coming down
- **(c)** is the same during the entire flight
- **(d)** is greatest at the top of its trajectory
- **(e)** is greatest at the beginning and end of its trajectory

4. A ball of mass m_1 is dropped from the roof of a 10-storey building. At the same instant, another ball of mass m_2 is dropped out of a ninth-storey window, 10 m below the roof. The distance between the balls during the flight
- **(a)** remains at 10 m throughout
- **(b)** decreases
- **(c)** increases
- **(d)** depends on the ratio m_1/m_2

5. On a position-time graph, a straight horizontal line corresponds to motion at
- **(a)** zero speed
- **(b)** constant speed
- **(c)** increasing speed
- **(d)** decreasing speed

Short Answer

6. **(a)** Define what is meant by a *net* or *unbalanced force* acting on an object.
- **(b)** Explain, with the aid of a free-body diagram, how an object can be experiencing no net force when it has at least three forces acting on it.
- **(c)** Describe, with the aid of a free-body diagram, an object that is experiencing a net force. Identify in which direction the object will move and with what type of motion. Relate the direction and type of motion to the direction of the net force.

7. A child is riding a merry-go-round that is travelling at a constant speed.
- **(a)** Is he viewing the world from an inertial or non-inertial frame of reference? Explain your reasoning.
- **(b)** What type of force does his horse exert on him to keep him travelling in a circle? In which direction does this force act?
- **(c)** In what direction does the child feel that a force is pushing him? Explain why this perceived force is called a "fictitious force."

8. A football is kicked into the air. Where in its trajectory is the velocity at a minimum? Where is it at a maximum?

9. A bright orange ball is dropped from a hot-air balloon that is travelling with a constant velocity.
- **(a)** Draw a sketch of the path the ball will travel from the perspective of a person standing on the ground from the instant in time at which the ball was dropped until the instant it lands.
- **(b)** From the ground, what type of motion is observed in the horizontal dimension? Identify the mathematical equations that can be used to model this motion.

(c) From the ground, what type of motion is observed in the vertical dimension? Identify the mathematical equations that can be used to model this motion.
(d) Identify the variable that is common to the equations that describe the horizontal motion and those that describe the vertical motion.
(e) Describe, with the aid of sketches, how motion on a plane can be modelled by considering its component motion along two directions that are perpendicular to each other.

Inquiry

10. A rope and pulley are often used to assist in lifting heavy loads. Demonstrate with the use of free-body diagrams and equations that, using the same force, a heavier load can be lifted with a rope and pulley system than with a rope alone.
11. A wooden T-bar attached to a cable is used at many ski hills to tow skiers and snowboarders up the hill in pairs. Design a T-bar lift for a ski hill. Estimate how much tension the cable for an individual T-bar should be able to withstand, assuming that it transports two adults, the slope is 10.0˚, and the T-bar cable pulls the people at an angle of 25.0˚ to the slope. Determine how the tension is affected when the steepness of the slope, the angle of the T-bar cable to the slope, or the coefficient of friction of the snow changes.
12. Examine three different ways of suspending signs (for example, for stores) in front of buildings or above sidewalks by using cables or rods (that is, the sign is not attached directly to the building). Determine which method can support the heaviest sign.
13. Review the meaning of the kinematics equations for constant acceleration by deriving them for yourself. Begin with the following situation. In a time interval, Δt, a car accelerates uniformly from an initial velocity, v_i, to a final velocity, v_f. Sketch the situation in a velocity-versus-time graph. By determining the slope of the graph and the area under the graph (Hint: What quantities do these represent?), see how many of the kinematics equations you can derive.

Communication

14. According to Newton's third law, for every action force, there is an equal and opposite reaction force. How, then, can a team win a tug-of-war contest?
15. Consider a block of wood on an incline. Determine the acceleration of the block and the normal force of the incline on the block for the two extreme cases where $\theta = 0˚$ and $\theta = 90˚$, and for the general case of $0˚ < \theta < 90˚$. Discuss the results, particularly why an inclined plane could be described as a way of "diluting" gravity. (**Note:** Galileo recognized this.)
16. You probably have a working understanding of mass and velocity, but what about force and acceleration? At what rate can a person accelerate on a bicycle? What average force does a tennis racquet exert on a tennis ball?
(a) Construct examples of everyday situations involving accelerations of approximately $0.5\ m/s^2$, $2.0\ m/s^2$, $5.0\ m/s^2$, and $20\ m/s^2$.
(b) Construct examples of everyday situations involving forces of 1 N, 10 N, 50 N, 100 N, 1000 N, and 1.0×10^4 N.
17. A ball rolls down an inclined plane, across a horizontal surface, and then up another inclined plane. Assume there is no friction.
(a) What forces act on the ball at the beginning, middle, and end of its roll?
(b) If the angles of the inclined planes are equal and the ball begins its roll from a vertical height of 10 cm, how high will the ball roll up the second inclined plane?
(c) If the first inclined plane is twice as steep as the second and the ball begins its roll from a vertical height of 10 cm, to what height will the ball roll up the second inclined plane?
(d) If the ball begins its roll from a vertical height of 10 cm on the first inclined plane and the second inclined plane is removed, how far will the ball roll across the horizontal surface?

(e) Explain how the above four situations are explained by using the law of inertia.

18. A car turns left off the highway onto a curved exit ramp.
   (a) What type of motion does the passengers' frame of reference experience relative to the ground?
   (b) Explain why the passengers feel a force to the right as the car turns.
   (c) How would an observer on an overpass describe the motion of the passengers and the car at the beginning of the curve?
   (d) Suppose that in the middle of the turn, the car hits a patch of ice. Sketch the path of the car as it slides.
   (e) Determine the magnitude and direction of the force that the road exerts in dry and icy road conditions and discuss the results for the two situations.
19. Suppose you could place a satellite above Earth's atmosphere with a gigantic crane. In which direction would the satellite travel when the crane released it? Explain your answer.
20. Explain why the kinematics equations, which describe the motion of an object that has constant acceleration, cannot be applied to uniform circular motion.

Making Connections

21. Choose an Olympic sport and estimate the magnitude of realistic accelerations and forces involved in the motion. For example, approximately how fast do Olympic athletes accelerate during the first 10 m of the 100 m dash? What average force is applied during this time? What average force do shot-putters exert on the shot-put as they propel it? How does this compare to the force exerted by discus throwers?
22. In automobiles, antilock braking systems were developed to slow down a car without letting the wheels skid and thus reduce the stopping distance, as compared to a braking system in which the wheels lock and skid. Explain the physics behind this technology, using the concepts of static friction and kinetic friction. Develop and solve a problem that demonstrates this situation. In which case, stopping without skidding or stopping with skidding, do you use the coefficient of kinetic friction and in which case do you use the coefficient of static friction?
23. "Natural motion" is difficult to explore experimentally on Earth because of the inherent presence of friction. Research the history of friction experiments. Examine the relationship between static and kinetic friction. Explain why a fundamental theory of friction eludes physicists.
24. Aristotle's theory of dynamics differed from Newton's and Galileo's theories partly because Aristotle tried to develop common sense explanations for real-life situations, whereas Newton and Galileo imagined ideal situations and tested them by experiment. Outline in an essay the differences in these two approaches and their results.
25. Railroads are typically built on level land, but in mountainous regions, inclines are unavoidable. In 1909, to improve the Canadian Pacific Railway through the Rocky Mountains, near Field, British Columbia, engineers significantly reduced the grade of the old track by building a spiral tunnel through a mountain. Research this engineering feat and answer the following questions.
   (a) What is the grade of the incline?
   (b) How much force must the train exert going up through the tunnel, as compared to when it goes down or travels on level track, or as compared to what it required for travel on the old track?
   (c) What is the elevation of the train before entering and after leaving the tunnel?
   (d) What is the typical acceleration of the train in the tunnel? Make some rough calculations, if necessary, to support your answers.
26. Tycho Brahe built two observatories and had his assistants observe the same things independently. He also repeated observations in order to understand his errors. He is recognized as the greatest astronomical observer prior to development of the telescope. Research the

contribution he made to observational astronomy and the role his methods played in developing the scientific method.

27. In the solar system, objects at greater distances from the Sun have slower orbital velocities because of the decrease in the gravitational force from the Sun. This pattern is expected to be observed in the Milky Way galaxy also. However, some objects that are more distant from the centre of the galaxy than the Sun (such as star clusters) have higher orbital velocities than the Sun. This is considered to be evidence for dark matter in the galaxy. Review some recent articles in astronomy magazines and research the nature of this problem. Why are the above observations considered to be evidence for dark matter? How strong is this evidence? What are some of the candidates?

28. Volcanoes on Mars, such as Olympus Mons, are much taller than those on Earth. Compare the sizes of volcanoes on different bodies in the solar system and discuss the role that gravity plays in determining the size of volcanoes.

Problems for Understanding

29. A 1.2×10^3 kg car is pulled along level ground by a tow rope. The tow rope will break if the tension exceeds 1.7×10^3 N. What is the largest acceleration the rope can give to the car? Assume that there is no friction.

30. Two objects, m_1 and m_2, are accelerated independently by forces of equal magnitude. Object m_1 accelerates at 10.0 m/s^2 and m_2 at 20.0 m/s^2. What is the ratio of **(a)** their inertial masses? **(b)** their gravitational masses?

31. A 720 kg rocket is to be launched vertically from the surface of Earth. What force is needed to give the rocket an initial upward acceleration of 12 m/s^2? Explain what happens to the acceleration of the rocket during the first few minutes after lift-off if the force propelling it remains constant.

32. A 42.0 kg girl jumps on a trampoline. After stretching to its bottom limit, the trampoline exerts an average upward force on the girl over a displacement of 0.50 m. During the time that the trampoline is pushing her up, she experiences an average acceleration of 65.0 m/s^2. Her velocity at the moment that she leaves the trampoline is 9.4 m/s[up].

(a) What is the average force that the trampoline exerts on the girl?

(b) How high does she bounce?

33. An 8.0 g bullet moving at 350 m/s penetrates a wood beam to a distance of 4.5 cm before coming to rest. Determine the magnitude of the average force that the bullet exerts on the beam.

34. Soon after blast-off, the acceleration of the *Saturn V* rocket is 80.0 m/s^2[up].

(a) What is the apparent weight of a 78.0 kg astronaut during this time?

(b) What is the ratio of the astronaut's apparent weight to true weight?

35. A 1500 kg car stands at rest on a hill that has an incline of 15°. If the brakes are suddenly released, describe the dynamics of the car's motion by calculating the following: **(a)** the car's weight, **(b)** the component of the weight parallel to the incline, **(c)** the car's acceleration, **(d)** the velocity acquired after travelling 100.0 m (in m/s and km/h), and **(e)** the time for the car to travel 100.0 m.

36. A 2.5 kg brick is placed on an adjustable inclined plane. If the coefficient of static friction between the brick and the plane is 0.30, calculate the maximum angle to which the plane can be raised before the brick begins to slip.

37. Superman tries to stop a speeding truck before it crashes through a store window. He stands in front of it and extends his arm to stop it. If the force he exerts is limited only by the frictional force between his feet and the ground, and $\mu_s = \mu_k = 1.0$, **(a)** what is the maximum force he can exert? (Let Superman's mass be 1.00×10^2 kg, the truck's mass 4.0×10^4 kg, and the truck's velocity 25 m/s.) **(b)** What is the minimum distance over which he can stop the truck?

38. Two bricks, with masses 1.75 kg and 3.5 kg, are suspended from a string on either side of a pulley. Calculate the acceleration of the masses and the tension in the string when the masses are released. Assume that the pulley is massless and frictionless.

39. A helicopter is flying horizontally at 8.0 m/s when it drops a package.
   (a) How much time elapses before the velocity of the package doubles?
   (b) How much additional time is required for the velocity of the package to double again?
   (c) At what altitude is the helicopter flying if the package strikes the ground just as its velocity doubles the second time?

40. A soccer player redirects a pass, hitting the ball toward the goal 21.0 m in front of him. The ball takes off with an initial velocity of 22.0 m/s at an angle of 17.0° above the ground.
   (a) With what velocity does the goalie catch the ball in front of the goal line?
   (b) At what height does the goalie catch the ball?
   (c) Is the ball on its way up or down when it is caught?

41. A wheelchair basketball player made a basket by shooting the ball at an angle of 62°, with an initial velocity of 6.87 m/s. The ball was 1.25 m above the floor when the player released it and the basket was 3.05 m above the floor. How far from the basket was the player when making the shot?

42. A 10.0 g arrow is fired horizontally at a target 25 m away. If it is fired from a height of 2.0 m with an initial velocity of 40.0 m/s, at what height should the target be placed above the ground for the arrow to hit it?

43. A Ferris wheel of radius 10.0 m rotates in a vertical circle of 7.0 rev/min. A 45.0 kg girl rides in a car alone. What (vertical) normal force would she experience when she is:
   (a) halfway towards the top, on her way up?
   (b) at the top?
   (c) halfway towards the bottom?
   (d) at the bottom?
   Compare this to her weight in each case.

44. How much force is needed to push a 75.0 kg trunk at constant velocity across a floor, if the coefficient of friction between the floor and the crate is 0.27?

45. A car can accelerate from rest to 100 km/h (1.00×10^2 km/h) in 6.0 s. If its mass is 1.5×10^3 kg, what is the magnitude and direction of the applied force?

46. A 62.4 kg woman stands on a scale in an elevator. What is the scale reading (in newtons) for the following situations.
   (a) The elevator is at rest.
   (b) The elevator has a downward acceleration of 2.80 m/s^2.
   (c) The elevator has an upward acceleration of 2.80 m/s^2.
   (d) The elevator is moving upward with a constant velocity of 2.80 m/s.

47. Suppose you attach a rope to a 5.0 kg brick and lift it straight up. If the rope is capable of holding a 20.0 kg mass at rest, what is the maximum upward acceleration you can give to the brick?

48. A 52.0 kg parachutist is gliding to Earth with a constant velocity of 6.0 m/s[down]. The parachute has a mass of 5.0 kg.
   (a) How much does the parachutist weigh?
   (b) How much upward force does the air exert on the parachutist and parachute?

49. **(a)** If you want to give an 8.0 g bullet an acceleration of 2.1×10^4 m/s^2, what average net force must be exerted on the bullet as it is propelled through the barrel of the gun?

(b) With this acceleration, how fast will the bullet be travelling after it has moved 2.00 cm from rest?

50. A snowboarder, whose mass including the board is 51 kg, stands on a steep 55° slope and wants to go straight down without turning. What will be his acceleration if **(a)** there is no friction and **(b)** the coefficient of kinetic friction is 0.20? **(c)** In each case, starting from rest, what will be his velocity after 7.5 s?

51. Replace the cart in a Fletcher's trolley apparatus (see page 8) with a block of wood of mass 4.0 kg and use a suspended mass of 2.0 kg. Calculate the acceleration of the system and the tension in the string when the mass is released, if the coefficient of friction between the block of wood and the table is

(a) 0.60

(b) 0.20

(c) What is the maximum value of the coefficient of friction that will allow the system to move?

52. Calculate the acceleration of two different satellites that orbit Earth. One is located at 2.0 Earth radii and the other at 4.0 Earth radii.

53. **(a)** Calculate your velocity on the surface of Earth (at the equator) due to Earth's rotation.

(b) What velocity would you require to orbit Earth at this distance? (Neglect air resistance and obstructions.)

54. Two galaxies are orbiting each other at a separation of 1×10^{11} AU and the orbital period is estimated to be 30 billion years. Use Kepler's third law to find the total mass of the pair of galaxies. Calculate how many times larger the mass of the pair of galaxies is than the Sun's mass, which is 1.99×10^{30} kg.

COURSE CHALLENGE

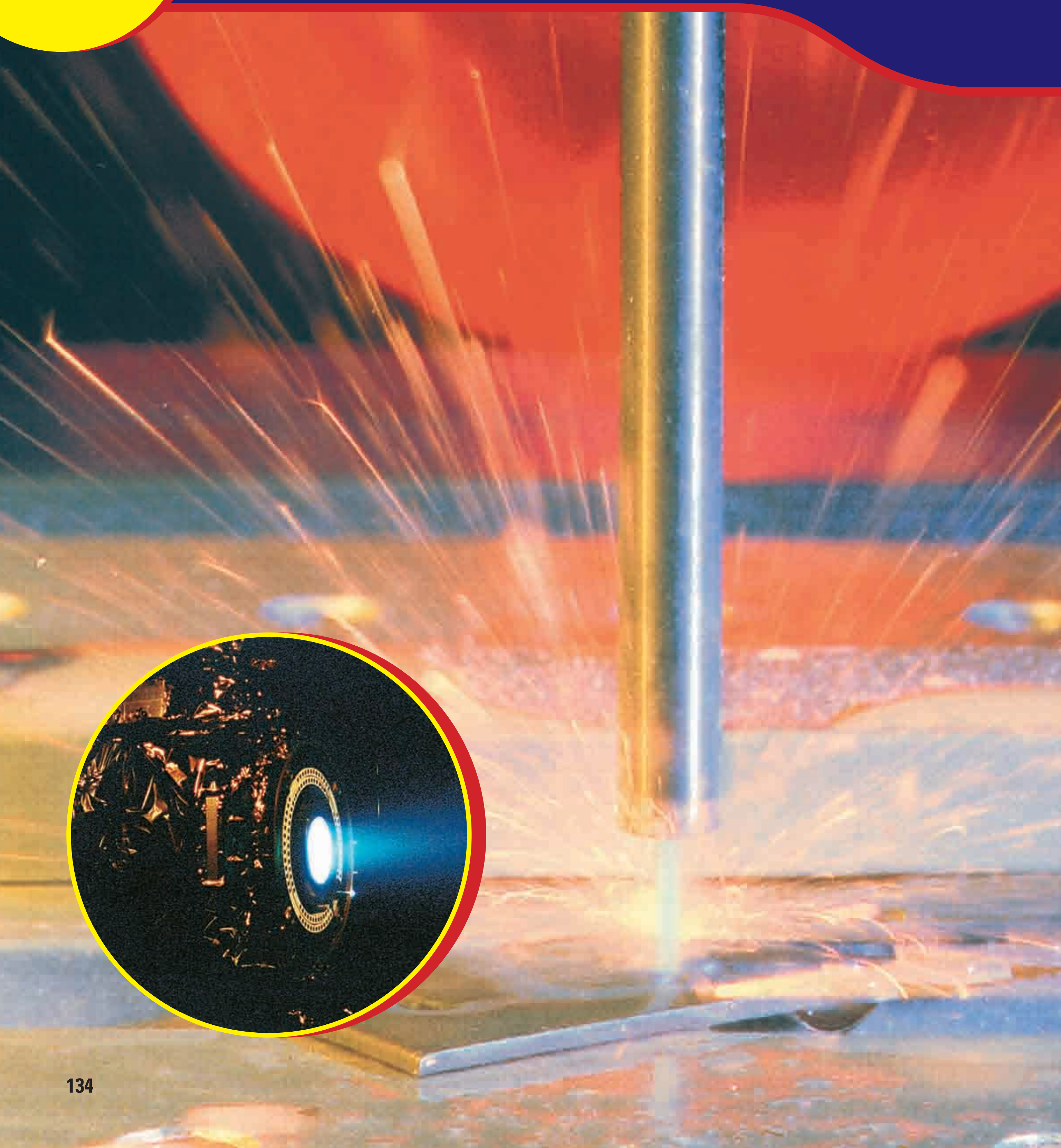
Scanning Technologies: Today and Tomorrow

Consider the following as you begin gathering information for your end-of-course project.

- Analyze the contents of this unit and begin recording concepts, diagrams, and equations that might be useful.
- Collect information in a variety of ways, including concept organizers, useful Internet sites, experimental data, and perhaps unanswered questions to help you create your final presentation.
- Scan magazines, newspapers, and the Internet for interesting information to enhance your project.

UNIT 2

Energy and Momentum



OVERALL EXPECTATIONS

DEMONSTRATE an understanding of work, energy, impulse, momentum, and conservation of energy and of momentum.

INVESTIGATE and analyze two-dimensional situations involving conservation of energy and of momentum.

DESCRIBE and analyze how common impact-absorbing devices apply concepts of energy and momentum.

UNIT CONTENTS

The motion of water, subjected to 4000 kPa of pressure, has sufficient energy to cut through steel. The motion of electrically charged particles, propelled at speeds in excess of 3.0×10^4 m/s, could provide the energy needed to power space probes in the near future. In the small photograph of an experimental ion engine, the blue glow is composed of electrically charged ions of xenon gas, travelling at speeds in excess of 3.0×10^4 m/s. An ion engine emits even smaller high-speed particles than a water-jet cutting tool.

In this unit, you will build on previous studies of energy to include another concept: momentum. Momentum considers the amount of motion in an object and the effect of moving objects — large or small, solid, liquid, or gas — on each other. You will use the concepts of energy and momentum to analyze physical interactions, such as collisions and propulsion systems. You will also examine two great theoretical foundations of physics: the law of conservation of momentum and the law of conservation of energy.

UNIT PROJECT PREP

Refer to pages 262–263 before beginning this unit. In this unit project, you will create a presentation that explores the importance of scientific theories.

- What theories do you already know that are helpful in analyzing changes in the motion of an object?
- In what career fields are ideas, principles, or mathematical techniques used to analyze the motion and interactions of objects?

CHAPTER 4

Momentum and Impulse

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Newton's laws of motion
- Kinetic energy
- Gravitational potential energy
- Conservation of mechanical energy

The driver of the race car in the above photograph walked away from the crash without a scratch. Luck had little to do with this fortunate outcome, though — a practical application of Newton's laws of motion by the engineers who designed the car and its safety equipment protected the driver from injury.

You learned in Unit 1 that Newton's laws can explain and predict a wide variety of patterns of motion, such as the motion of a projectile and the orbits of planets. How can some of the same laws that guide the stars and planets protect a race car driver who is in a crash?

When Newton originally formulated his laws of motion, he expressed them in a somewhat different form than you see in most textbooks today. Newton emphasized a concept called a "quantity of motion," which is defined as the product of an object's mass and its velocity. Today, we call this quantity "momentum." In this chapter, you will see how the use of momentum allows you to analyze and predict the motion of objects in countless situations that you might not yet have encountered in your study of physics.

INVESTIGATION 4-A

Newton's Cradle

TARGET SKILLS
- Hypothesizing
- Performing and recording
- Analyzing and interpreting

Newton's cradle, also called a "Newtonian demonstrator," looks like a simple child's toy. However, explaining the motion of Newton's cradle requires the application of more than one important physical principle.

Problem

Explain the motion of a Newton's cradle.

Equipment

- Newton's cradle
- modelling clay

Procedure

1. Pull to the side one sphere at the end of the row of the Newton's cradle, keeping the supporting cords taut. Then, release the sphere. Observe and record the resulting motion of the spheres.
2. Pull two spheres to the side, keeping all of the supporting cords taut and keeping the spheres in contact. Release the spheres and observe and record the resulting motion.
3. Repeat step 2, using first three spheres and then four spheres.
4. Pull back two spheres from one end and one sphere from the other end. Release all of the spheres at the same time. Observe and record the motion.
5. Pull one of the end spheres aside and put a small piece of modelling clay on the second sphere at the point where the first sphere will hit it. Release the first sphere and observe and record the motion of the spheres.
6. Leaving the clay in place between the two spheres, pull back one sphere from the opposite end of the row. Release the sphere and observe and record the resulting motion.

Analyze and Conclude

1. Summarize any patterns of motion that you observed for the various trials with the Newton's cradle.
2. Imagine that an end sphere was moving at 0.16 m/s when it hit the row and that two spheres bounced off the other end. What would the speed of the two spheres have to be in order to conserve kinetic energy? Assume that each sphere has a mass of 0.050 kg.
3. Could kinetic energy be conserved in the pattern described in question 2? During your trials, did you ever observe the pattern described in question 2?
4. Did you ever observe a pattern in which more than one sphere was released and only one sphere bounced off the far end?
5. Propose a possible explanation for the motion you observed in Procedure steps 5 and 6.
6. Momentum is involved in the motion of the spheres. Write a definition of momentum as you now understand it.
7. Formulate an hypothesis that could explain why some patterns that would *not* violate the law of conservation of energy were, however, *not* observed.
8. As you study this chapter, look for explanations for the patterns of motion that you observed. Reread your hypothesis and make any necessary corrections.

4.1 Defining Momentum and Impulse

SECTION EXPECTATIONS

- Define and describe the concepts and units related to momentum and impulse.
- Analyze and describe practical applications of momentum, using the concepts of momentum.
- Identify and analyze social issues that relate to the development of safety devices for automobiles.

KEY TERMS

- momentum
- impulse
- impulse-momentum theorem

By now, you have become quite familiar with a wide variety of situations to which Newton's laws apply. Frequently, you have been cautioned to remember that when you apply Newton's second law, you must use only the forces acting on one specific object. Then, by applying Newton's laws, you can predict precisely the motion of that object. However, there are a few types of interactions for which it is difficult to determine or describe the forces acting on an object or on a group of objects. These interactions include collisions, explosions, and recoil. For these more complex scenarios, it is easier to observe the motion of the objects before and after the interaction and then analyze the interaction by using Newton's concept of a quantity of motion.

Defining Momentum

Although you have not used the mathematical expression for momentum, you probably have a qualitative sense of its meaning. For example, when you look at the photographs in Figure 4.1, you could easily list the objects in order of their momentum. Becoming familiar with the mathematical expression for momentum will help you to analyze interactions between objects.

Momentum is the product of an object's mass and its velocity, and is symbolized by $\vec{p}$. Since it is the product of a vector and a scalar, momentum is a vector quantity. The direction of the momentum is the same as the direction of the velocity.

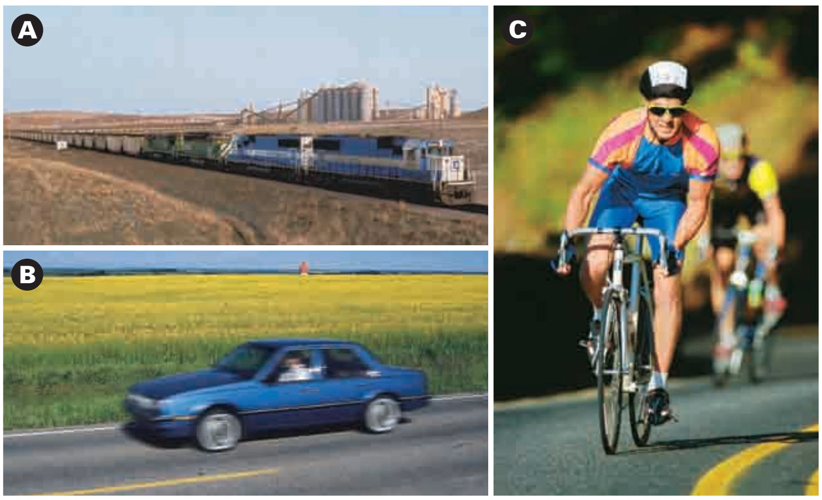


Figure 4.1 If the operator of each of these vehicles was suddenly to slam on the brakes, which vehicle would take the longest time to stop?

DEFINITION OF MOMENTUM

Momentum is the product of an object's mass and its velocity.

$$\vec{p} = m\vec{v}$$

Quantity	Symbol	SI unit
momentum	$\vec{p}$	$\frac{\text{kg} \cdot \text{m}}{\text{s}}$ (kilogram metres per second)
mass	m	kg (kilograms)
velocity	$\vec{v}$	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$(\text{mass})(\text{velocity}) = \text{kg} \cdot \frac{\text{m}}{\text{s}} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$

Note: Momentum does not have a unique unit of its own.

SAMPLE PROBLEM

Momentum of a Hockey Puck

Determine the momentum of a 0.300 kg hockey puck travelling across the ice at a velocity of 5.55 m/s[N].

Conceptualize the Problem

- The *mass* is *moving*; therefore, it has *momentum*.
- The *direction* of an object's *momentum* is the same as the *direction* of its *velocity*.

Identify the Goal

The momentum, $\vec{p}$, of the hockey puck

Identify the Variables and Constants

Known	Unknown
$m = 0.300$ kg	$\vec{p}$
$\vec{v} = 5.55 \frac{\text{m}}{\text{s}}[\text{N}]$	

Develop a Strategy

Use the equation that defines momentum.

$$\vec{p} = m\vec{v}$$

$$\vec{p} = (0.300 \text{ kg}) \times \left(5.55 \frac{\text{m}}{\text{s}}[\text{N}]\right)$$

$$\vec{p} = 1.665 \frac{\text{kg} \cdot \text{m}}{\text{s}}[\text{N}]$$

$$\vec{p} \cong 1.67 \frac{\text{kg} \cdot \text{m}}{\text{s}}[\text{N}]$$

The momentum of the hockey puck was $1.67 \frac{\text{kg} \cdot \text{m}}{\text{s}}[\text{N}]$.

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Validate the Solution

Approximate the solution by multiplying 0.3 kg times 6 m/s. The magnitude of the momentum should be slightly less than this product, which is 1.8 kg · m/s. The value, 1.67 kg · m/s, fits the approximation very well. The direction of the momentum is always the same as the velocity of the object.

PRACTICE PROBLEM

1. Determine the momentum of the following objects.

(a) 0.250 kg baseball travelling at 46.1 m/s[E]

(b) 7.5×10^6 kg train travelling west at 125 km/h

(c) 4.00×10^5 kg jet travelling south at 755 km/h

(d) electron (9.11×10^{-31} kg) travelling north at 6.45×10^6 m/s

Defining Impulse

Originally, Newton expressed his second law by stating that the change in an object's motion (rate of change of momentum) is proportional to the force impressed on it. Expressed mathematically, his second law can be written as follows.

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

To show that this expression is fundamentally equivalent to the equation that you have learned in the past, take the following steps.

- Write the change in momentum as the difference of the final and initial momenta. $$\vec{F} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$
- Write momentum in terms of mass and velocity. $$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$
- If you assume that m is constant (that is, does not change for the duration of the time interval), you can factor out the mass, m. $$\vec{F} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$$ $$\vec{F} = \frac{m\Delta\vec{v}}{\Delta t}$$
- Recall that the definition of average acceleration is the rate of change of velocity, and substitute an $\vec{a}$ into the above expression. $$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$ $$\vec{F} = m\vec{a}$$

Knowing that $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ is a valid expression of Newton's second law, you can mathematically rearrange the expression to demonstrate some very useful relationships involving momentum. When you multiply both sides of the equation by the time interval, you derive a new quantity, $\vec{F}\Delta t$, called "impulse."

$$\vec{F}\Delta t = \Delta\vec{p}$$

MATH LINK

In reality, Newton expressed his second law using the calculus that he invented. The procedure involves allowing the time interval to become smaller and smaller, until it becomes "infinitesimally small." The result allows you to find the instantaneous change in momentum at each instant in time. The formulation of Newton's second law using calculus looks like this.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Impulse is the product of the force exerted on an object and the time interval over which the force acts, and is often given the symbol $\vec{J}$. Impulse is a vector quantity, and the direction of the impulse is the same as the direction of the force that causes it.

DEFINITION OF IMPULSE

Impulse is the product of force and the time interval.

$$\vec{J} = \vec{F}\Delta t$$

Quantity	Symbol	SI unit
impulse	$\vec{J}$	N · s (newton seconds)
force	$\vec{F}$	N (newtons)
time interval	Δt	s (seconds)

Unit Analysis

(impulse) = (force)(time interval) = N · s

Note: Impulse is equal to the change in momentum, which has units of $\frac{\text{kg}\cdot\text{m}}{\text{s}}$. To show that these units are equivalent to the N · s, express N in terms of the base units.

$$\text{N}\cdot\text{s} = \frac{\text{kg}\cdot\text{m}}{\text{s}^{\cancel{2}}}\cdot\cancel{\text{s}} = \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

MISCONCEPTION

$\vec{F} = m\vec{a}$ Is Correct!

When students read the sentence "If you assume that *m* is constant (that is, does not change for the duration of the time interval), you can factor out the mass, *m*," they sometimes think that the result of the derivation, $\vec{F} = m\vec{a}$, is wrong. However, this equation is a special case of Newton's second law that is correct for all cases in which the mass, *m*, is constant. Since the mass is constant in a very large number of situations, it is acceptable to consider $\vec{F} = m\vec{a}$ as a valid statement of Newton's second law.

SAMPLE PROBLEM

Impulse on a Golf Ball

If a golf club exerts an average force of 5.25×10^3 N[W] on a golf ball over a time interval of 5.45×10^{-4} s, what is the impulse of the interaction?

Conceptualize the Problem

- The golf club exerts an *average force* on the golf ball for a period of *time*. The product of these quantities is defined as *impulse*.
- Impulse is a *vector* quantity.
- The *direction* of the impulse is the same as the *direction of its average force*.

Identify the Goal

The impulse, $\vec{J}$, of the interaction

Identify the Variables and Constants

Known	Unknown
$\vec{F} = 5.25 \times 10^3$ N[W]	$\vec{J}$
$\Delta t = 5.45 \times 10^{-4}$ s	

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Develop a Strategy

Apply the equation that defines impulse.

$$\vec{J} = \vec{F}\Delta t$$
$$\vec{J} = (5.25 \times 10^3 \text{ N[W]})(5.45 \times 10^{-4} \text{ s})$$
$$\vec{J} = 2.8612 \text{ N} \cdot \text{s[W]}$$
$$\vec{J} \cong 2.86 \text{ N} \cdot \text{s[W]}$$

When the golf club strikes the golf ball, the impulse to drive the ball down the fairway is 2.86 N · s[W].

Validate the Solution

Round the values in the data to 5000 N[W] and 0.0006 s and do mental multiplication. The product is 3 N · s[W]. The answer, 2.86 N · s[W], is very close to the estimate.

PROBLEM TIP

When you want to use mental math to approximate an answer to validate your calculations, you can usually find the best approximation by rounding one value up and the other value down before multiplying.

PRACTICE PROBLEMS

2. A sledgehammer strikes a spike with an average force of 2125 N[down] over a time interval of 0.0205 s. Calculate the impulse of the interaction.

3. In a crash test, a car strikes a wall with an average force of 1.23×10^7 N[S] over an interval of 21.0 ms. Calculate the impulse.

4. In a crash test similar to the one described in problem 3, another car, with the same mass and velocity as the first car, experiences an impulse identical to the value you calculated in problem 3. However, the second car was designed to crumple more slowly than the first. As a result, the duration of the interaction was 57.1 ms. Determine the average force exerted on the second car.

The Impulse-Momentum Theorem

You probably noticed that the sample and practice problems above always referred to "average force" and not simply to "force." Average force must be used to calculate impulse in these short, intense interactions, because the force changes continually throughout the few milliseconds of contact of the two objects. For example, when a golf club first contacts a golf ball, the force is very small. Within milliseconds, the force is great enough to deform the ball. The ball then begins to move and return to its original shape and the force soon drops back to zero. Figure 4.2 shows how the force changes with time. You could find the impulse by determining the area under the curve of force versus time.

In many collisions, it is exceedingly difficult to make the precise measurements of force and time that you need in order to calculate the impulse. The relationship between impulse and momentum provides an alternative approach to analyzing such collisions, as well as other interactions. By analyzing the momentum before and after an interaction between two objects, you can determine the impulse.

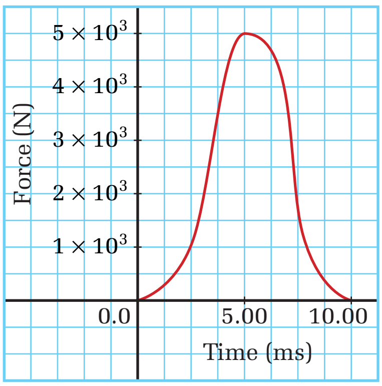

Figure 4.2 You can find the impulse of an interaction (area under the curve) by using the same mathematical methods that you used to find work done from a force-versus-position curve.

When you first rearranged the expression for Newton's second law, you focussed only on the concept of impulse, $\vec{F}\Delta t$. By taking another look at the equation $\vec{F}\Delta t = \Delta\vec{p}$, you can see that impulse is equal to the *change* in the momentum of an object. This relationship is called the **impulse-momentum theorem** and is often expressed as shown in the box below.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner to enhance your understanding of momentum.

IMPULSE-MOMENTUM THEOREM

Impulse is the difference of the final momentum and initial momentum of an object involved in an interaction.

$$\vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1$$

Quantity	Symbol	SI unit
force	$\vec{F}$	N (newtons)
time interval	Δt	s (seconds)
mass	m	kg (kilograms)
initial velocity	$\vec{v}_1$	$\frac{\text{m}}{\text{s}}$ (metres per second)
final velocity	$\vec{v}_2$	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

(force)(time interval) = (mass)(velocity)

$\text{N}\cdot\text{s} = \text{kg}\frac{\text{m}}{\text{s}}$ $\qquad \frac{\text{kg}\cdot\text{m}}{\text{s}^{\cancel{2}}}\cancel{\text{s}} = \frac{\text{kg}\cdot\text{m}}{\text{s}}$

Note: Impulse is a vector quantity. The direction of the impulse is the same as the direction of the *change* in the momentum.

SAMPLE PROBLEM

Impulse and Average Force of a Tennis Ball

A student practises her tennis volleys by hitting a tennis ball against a wall.

(a) If the 0.060 kg ball travels 48 m/s before hitting the wall and then bounces directly backward at 35 m/s, what is the impulse of the interaction?

(b) If the duration of the interaction is 25 ms, what is the average force exerted on the ball by the wall?

Conceptualize the Problem

- The *mass* and *velocities* before and after the interaction are known, so it is possible to calculate the *momentum* before and after the interaction.
- Momentum is a *vector* quantity, so all calculations must include *directions*.

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- Since the motion is all in *one dimension*, use plus and minus to denote direction. Let the *initial direction* be the *positive direction*.
- You can find the impulse from the *change* in momentum.

Identify the Goal

The impulse, $\vec{J}$, of the interaction

The average force, $\vec{F}$, on the tennis ball

> **PROBLEM TIP**
>
> Whenever you use a result from one step in a problem as data for the next step, use the unrounded form of the data.

Identify the Variables and Constants

Known		Unknown
$m = 0.060\ \text{kg}$	$\vec{v}_1 = 48\ \dfrac{\text{m}}{\text{s}}$	$\vec{J}$
$\Delta t = 25\ \text{ms} = 0.025\ \text{s}$	$\vec{v}_2 = -35\ \dfrac{\text{m}}{\text{s}}$	$\vec{F}$

Develop a Strategy

Use the impulse-momentum theorem to calculate the impulse.

$$\vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1$$

$$\vec{F}\Delta t = 0.060\ \text{kg}\left(-35\ \frac{\text{m}}{\text{s}}\right) - 0.060\ \text{kg}\left(48\ \frac{\text{m}}{\text{s}}\right)$$

$$\vec{F}\Delta t = -2.1\ \frac{\text{kg}\cdot\text{m}}{\text{s}} - 2.88\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$\vec{F}\Delta t = -4.98\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$\vec{F}\Delta t \cong -5.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

(a) The impulse was 5.0 kg · m/s in a direction opposite to the initial direction of the motion of the ball.

Use the definition of impulse to find the average force.

$$\vec{F}\Delta t = -4.98\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$\vec{F} = \frac{-4.98\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}{\Delta t}$$

$$\vec{F} = \frac{-4.98\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}{0.025\ \text{s}}$$

$$\vec{F} = -199.2\ \text{N}$$

$$\vec{F} \cong -2.0 \times 10^2\ \text{N}$$

(b) The average force of the wall on the tennis ball was 2.0×10^2 N in the direction opposite to the initial direction of the ball.

Validate the Solution

Use an alternative mathematical technique for the impulse calculation by factoring out the mass, subtracting the velocities, then multiplying to see if you get the same answer.

$$\vec{F}\Delta t = m(\vec{v}_2 - \vec{v}_1)$$

$$\vec{F}\Delta t = 0.060\ \text{kg}\left(-35\frac{\text{m}}{\text{s}} - 48\frac{\text{m}}{\text{s}}\right)$$

$$\vec{F}\Delta t = (0.060\ \text{kg})\left(-83\ \frac{\text{m}}{\text{s}}\right)$$

$$\vec{F}\Delta t = -4.98\ \frac{\text{kg}\cdot\text{m}}{\text{s}} \cong -5.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Check the units for the second part of the problem.

$$\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}}}{\text{s}} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \text{N}$$

PRACTICE PROBLEMS

5. The velocity of the serve of some professional tennis players has been clocked at 43 m/s horizontally. (Hint: Assume that any vertical motion of the ball is negligible and consider only the horizontal direction of the ball after it was struck by the racquet.) If the mass of the ball was 0.060 kg, what was the impulse of the racquet on the ball?

6. A 0.35 kg baseball is travelling at 46 m/s toward the batter. After the batter hits the ball, it is travelling 62 m/s in the opposite direction. Calculate the impulse of the bat on the ball.

7. A student dropped a 1.5 kg book from a height of 1.75 m. Determine the impulse that the floor exerted on the book when the book hit the floor.

Impulse and Auto Safety

One of the most practical and important applications of impulse is in the design of automobiles and their safety equipment. When a car hits another car or a solid wall, little can be done to reduce the change in momentum. The mass of the car certainly does not change, while the velocity changes to zero at the moment of impact. Since you cannot reduce the change in momentum, you cannot reduce the impulse. However, since impulse ($\vec{F}\Delta t$) depends on both force and time, engineers have found ways to reduce the force exerted on car occupants by extending the time interval of the interaction. Think about how the design of a car can expand the duration of a crash.

In the early days of auto manufacturing, engineers and designers thought that a very strong, solid car would be ideal. As the number of cars on the road and the speed of the cars increased, the number and seriousness of accident injuries made it clear that the very sturdy cars were not protecting car occupants. By the late 1950s and early 1960s, engineers were designing cars with very rigid passenger cells that would not collapse onto the passengers, but with less rigid “crumple zones” in the front and rear, as shown in Figure 4.3.

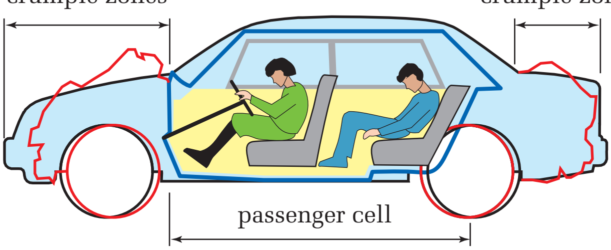


Figure 4.3 Although a car crash seems almost instantaneous, the time taken for the front or rear of the car to “crumple” is great enough to significantly reduce the average force of the impact and, therefore, the average force on the passenger cell and the passengers.

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and follow the links for an in-depth activity on impulse and momentum.

ELECTRONIC LEARNING PARTNER

Use the crash test provided by your Electronic Learning Partner to enhance your understanding of momentum.

QUICK LAB

Designing Crumple Zones

TARGET SKILLS
- Hypothesizing
- Performing and recording
- Analyzing and interpreting
- Communicating results

How soft is too soft and how rigid is too rigid for an effective vehicle crumple zone? In this lab, you will design and test several materials to determine the optimum conditions for passengers in a vehicle.

Obtain a rigid (preferably metal) toy vehicle to simulate the passenger cell of an automobile. The vehicle must have an open space in the centre for the "passenger." Make a passenger out of putty, modelling clay, or some material that will easily show "injuries" in the form of dents and deformations.

Design and build some type of device that will propel your vehicle rapidly into a solid wall (or stack of bricks) with nearly the same speed in all trials. The wall must be solid, but you will need to ensure that you do not damage the wall. Perform several crash tests with your vehicle and passenger and observe the types of injuries and the extent of injuries caused by the collision.

Select a variety of materials, from very soft to very hard, from which to build crumple zones. For example, you could use very soft foam rubber for the soft material. The thickness of each crumple zone must be approximately one third the length of your vehicle.

One at a time, attach your various crumple zones to your vehicle and test the effectiveness of the material in reducing the severity of injury to the passenger. Be sure that the vehicle travels at the same speed with the crumple zone attached as it did in the original crash tests without a crumple zone. Also, be sure that the materials you use to attach the crumple zones do not influence the performance of the crumple zones. Formulate an hypothesis about the relative effectiveness of each of the various crumple zones that you designed.

Analyze and Conclude

1. How do the injuries to the passenger that occurred with a very soft crumple zone compare to the injuries in the original crash tests?

2. How do the injuries to the passenger that occurred with a very rigid crumple zone compare to the injuries in the original crash tests?

3. Describe the difference in the passenger's injuries between the original crash tests and the test using the most effective crumple zone material.

Apply and Extend

4. The optimal crumple zone for a very massive car would be much more rigid than one for a small, lightweight car. However, a crash between a large and a small car would result in much greater damage to the small car. Write a paragraph responding to the question "Should car manufacturers consider other cars on the road when they design their own cars, or should they ignore what might happen to other manufacturers' cars?"

5. Crumple zones are just one of many types of safety systems designed for cars. Should the government regulate the incorporation of safety systems into cars? Give a rationale for your answer.

6. Some safety systems are very costly. Who should absorb the extra cost — the buyer, the manufacturer, or the government? For example, should the government provide a tax break or some other monetary incentive for manufacturers to build or consumers to buy cars with highly effective safety systems? Give a rationale for your answer.

When a rigid car hits a wall, a huge force stops the car almost instantaneously. The car might even look as though it was only slightly damaged. However, parts of the car, such as the steering wheel, windshield, or dashboard, exert an equally large force on the passengers, stopping them exceedingly rapidly and possibly causing very serious injuries.

When a car with well-designed crumple zones hits a wall, the force of the wall on the car causes the front of the car to collapse over a slightly longer time interval than it would in the absence of a crumple zone. Since $\vec{F}\Delta t$ is constant and Δt is larger, the average force, $\vec{F}$, is smaller than it would be for a rigid car. Although many other factors must be considered to reduce injury in collisions, the presence of crumple zones has had a significant effect in reducing the severity of injuries in automobile accidents.

The concept of increasing the duration of an impact applies to many forms of safety equipment. For example, the linings of safety helmets are designed to compress relatively slowly. If the lining was extremely soft, it would compress so rapidly that the hard outer layer of the helmet would impact on the head very quickly. If the lining did not compress at all, it would collide with the head over an extremely short time interval and cause serious injury. Each type of sport helmet is designed to compress in a way that compensates for the type of impacts expected in that sport.

WEB LINK

www.mcgrawhill.ca/links/physics12

To learn more about the design and testing of helmets and other safety equipment in sports, go to the above Internet site and click on **Web Links**.

4.1 Section Review

1. **K/U** Define momentum qualitatively and quantitatively.

2. **K/U** What assumption do you have to make in order to show that the two forms of Newton's second law ($\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ and $\vec{F} = m\vec{a}$) are equivalent?

3. **I** Try to imagine a situation in which the form $\vec{F} = m\vec{a}$ would not apply, but the form $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ could be used. Describe that situation. How could you test your prediction?

4. **C** State the impulse-momentum theorem and give one example of its use.

5. **MC** A bungee jumper jumps from a very high tower with bungee cords attached to his ankles. As he reaches the end of the bungee cord, it begins to stretch. The cord stretches for a relatively long period of time and then it recoils, pulling him back up. After several bounces, he dangles unhurt from the bungee cord (if he carried out the jump with all of the proper safety precautions). If he jumped from the same point with an ordinary rope attached to his ankles, he would be very severely injured. Use the concept of impulse to explain the difference in the results of a jump using a proper bungee cord and a jump using an ordinary rope.

UNIT PROJECT PREP

Can environmentally responsible transportation be the product of properly applying scientific models and theories?

- Is the theory of momentum and impulse currently used in vehicle design?
- Can you envision using momentum and impulse theory to design more environmentally responsible transportation systems?

4.2 Conservation of Momentum

SECTION EXPECTATIONS

- Define and describe the concepts related to elastic and inelastic collisions and to open and closed energy systems.
- Analyze situations involving the conservation of momentum and apply them quantitatively.
- Investigate the law of conservation of momentum in one and two dimensions.

KEY TERMS

- conservation of momentum
- system of particles
- internal force
- external force
- open system
- closed system
- isolated system
- recoil

When the cue ball hits the eight ball in billiards, the eight ball hits the cue ball. When a rock hits the ground, the ground hits the rock. In any collision, two objects exert forces on each other. You can learn more about momentum by analyzing the motion of both objects in a collision.

Figure 4.4 The game of billiards offers many excellent examples of collisions.

Newton's Third Law and Momentum

Newton's third law states that "For every action force on object B due to object A, there is a reaction force, equal in magnitude but opposite in direction, acting on object A due to object B." Unlike Newton's second law, which focusses on the motion of one specific object, his third law deals with the interaction between *two* objects. When you apply Newton's third law to collisions, you discover one of the most important laws of physics — the law of conservation of momentum. The following steps, along with the diagram in Figure 4.5, show you how to derive the law of conservation of momentum by applying Newton's third law to a collision between two objects.

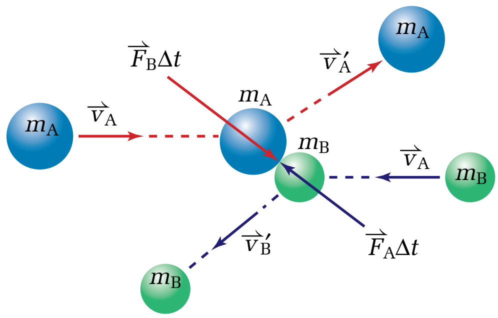


Figure 4.5 Object A exerts a force on object B, causing a change in B's momentum. At the same time, object B exerts a force equal in size and opposite in direction on object A, changing A's momentum.

Write the impulse-momentum theorem for each of two objects, A and B, that collide with each other.	$\vec{F}_A \Delta t = m_A \vec{v}_{A2} - m_A \vec{v}_{A1}$ $\vec{F}_B \Delta t = m_B \vec{v}_{B2} - m_B \vec{v}_{B1}$
Apply Newton's third law to the forces that A and B exert on each other.	$\vec{F}_A = -\vec{F}_B$
The duration of the collision is the same for both objects. Therefore, you can multiply both sides of the equation above by Δt.	$\vec{F}_A \Delta t = -\vec{F}_B \Delta t$
Substitute the expressions for change in momentum in the first step into the equation in the third step and then simplify.	$m_A \vec{v}_{A2} - m_A \vec{v}_{A1} = -(m_B \vec{v}_{B2} - m_B \vec{v}_{B1})$
Algebraically rearrange the last equation so that (1) the terms representing the before-collision conditions precede the equals sign and (2) the terms for the after-collision conditions follow the equals sign.	$m_A \vec{v}_{A2} - m_A \vec{v}_{A1} = -m_B \vec{v}_{B2} + m_B \vec{v}_{B1}$ $m_A \vec{v}_{A1} + m_B \vec{v}_{B1} = m_A \vec{v}_{A2} + m_B \vec{v}_{B2}$

The last equation is a mathematical expression of the law of **conservation of momentum,** which states that the total momentum of two objects before a collision is the same as the total momentum of the same two objects after they collide.

LAW OF CONSERVATION OF MOMENTUM

The sum of the momenta of two objects before collision is equal to the sum of their momenta after they collide.

$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}'_A + m_B \vec{v}'_B$$

Quantity	Symbol	SI unit
mass of object A	m_A	kg (kilograms)
mass of object B	m_B	kg (kilograms)
velocity of object A before the collision	$\vec{v}_A$	$\frac{m}{s}$ (metres per second)
velocity of object B before the collision	$\vec{v}_B$	$\frac{m}{s}$ (metres per second)
velocity of object A after the collision	$\vec{v}'_A$	$\frac{m}{s}$ (metres per second)
velocity of object B after the collision	$\vec{v}'_B$	$\frac{m}{s}$ (metres per second)

PHYSICS FILE

When working with collisions, instead of using subscripts such as "2," physicists often use a superscript symbol called a "prime," which looks like an apostrophe, to represent the variables *after* a collision. The variable is said to be "primed." Look for this notation in the box on the left.

MISCONCEPTION

A Closed System Is Not Isolated

Many people confuse the terms "closed" and "isolated" as they apply to systems. Although it might sound as though closed systems would not exchange anything with their surroundings, they do allow energy to enter or leave. Only isolated systems prevent the exchange of energy with the surroundings.

The law of conservation of momentum can be broadened to more than two objects by defining a **system of particles**. Any group of objects can be defined as a system of particles. Once a system is defined, forces are classified as internal or external forces. An **internal force** is any force exerted on any object in the system due to another object in the system. An **external force** is any force exerted by an object that is not part of the system on an object within the system.

Scientists classify systems according to their interaction with their surroundings, as illustrated in Figure 4.6. An **open system** can exchange both matter and energy with its surroundings. Matter does not enter or leave a **closed system,** but energy can enter or leave. Neither matter nor energy can enter or leave an **isolated system.**

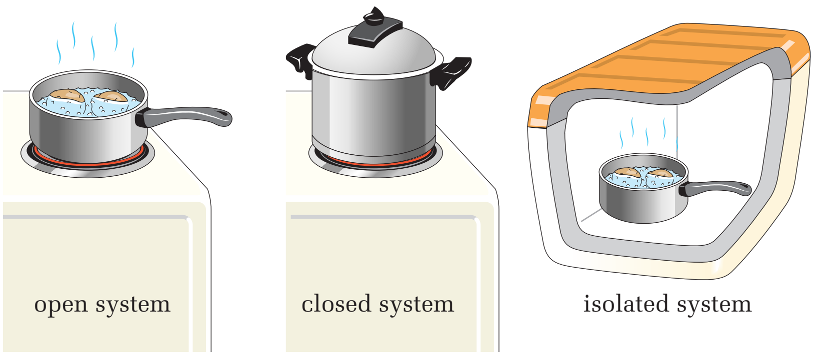


Figure 4.6 An open pot of potatoes boiling on the stove represents an open system, because heat is entering the pot and water vapour is leaving the system. A pressure cooker prevents any matter from escaping but heat is entering, so the pressure cooker represents a closed system. If the pot is placed inside a perfect insulator, neither heat nor water can enter or leave the system, making it an isolated system.

A force can do work on a closed system, thus increasing the energy of the system. Clearly, if no external forces can act on a system, it is isolated. To demonstrate that the momentum of an isolated system is conserved, start with the impulse-momentum theorem, where $\vec{p}_{sys}$ represents the total momentum of all of the objects within the system.

- An impulse on a system due to an external force causes a change in the momentum of the system. $\vec{F}_{ext}\Delta t = \Delta\vec{p}_{sys}$
- If a system is isolated, the net external force acting on the system is zero. $(0.0 \text{ N})\ \Delta t = |\Delta\vec{p}_{sys}|$
- If the impulse is zero, the *change* in momentum must be zero. $|\Delta\vec{p}_{sys}| = 0.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$

COURSE CHALLENGE

Momentum

A moving planet, bowling ball, or an electron share the property of momentum. Learn more about momentum conservation as it relates to your *Course Challenge* on page 603 of this text.

The last expression is an alternative form of the equation for the conservation of momentum. The equation states that the change in momentum of an isolated system is zero. The particles or objects within the system might interact with each other and exchange momentum, but the total momentum of the isolated system does not change.

In reality, systems are rarely perfectly isolated. In nearly all real situations, immediately after a collision, frictional forces and interactions with other objects change the momentum of the objects involved in the collision. Therefore, it might appear that the law of conservation of momentum is not very useful. However, the law always applies to a system from the instant before to the instant after a collision. If you know the conditions just before a collision, you can always use conservation of momentum to determine the momentum and, thus, velocity of an object at the instant after a collision. Often, these values are all that you need to know.

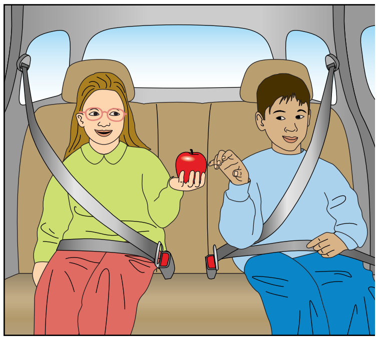


Figure 4.7 A moving car and its occupants can be defined as being a system. The children in the car might be exerting forces on each other or on objects that they are handling. Although they are exchanging momentum between themselves and the objects, these changes have no effect on the total momentum of the system.

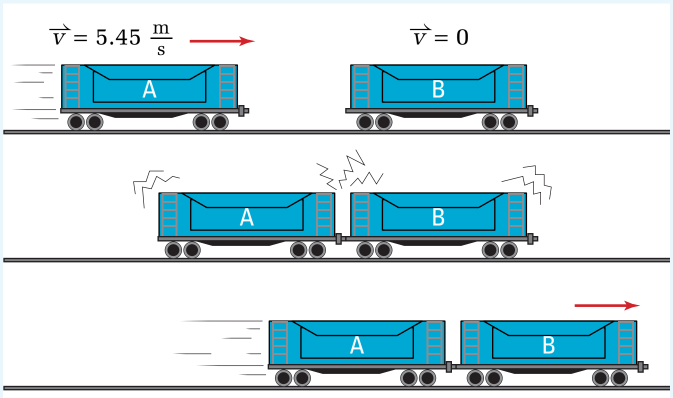
Collisions in One Dimension

Since momentum is a vector quantity, both the magnitude and the direction of the momentum must be conserved. Therefore, momentum is conserved in each dimension, *independently*. For complex situations, it is often convenient to separate the momentum into its components and work with each dimension separately. Then you can combine the results and find the resultant momentum of the objects in question. Solving problems that involve only one dimension is good practice for tackling more complex problems.

SAMPLE PROBLEM

Analyzing a Collision between Boxcars

A 1.75×10^4 kg boxcar is rolling down a track toward a stationary boxcar that has a mass of 2.00×10^4 kg. Just before the collision, the first boxcar is moving east at 5.45 m/s. When the boxcars collide, they lock together and continue down the track. What is the velocity of the two boxcars immediately after the collision?



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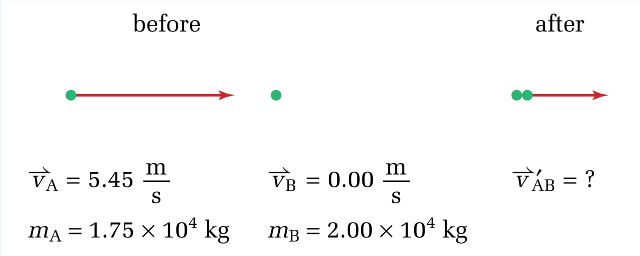
Conceptualize the Problem

- Make a sketch of the *momentum vectors* representing conditions just *before* and just *after* the collision.
- Before the collision, only *one boxcar* (A) is *moving* and therefore has *momentum*.
- At the instant of the collision, *momentum is conserved*.
- After the collision, the *two boxcars* (A and B) *move as one mass*, with the *same velocity*.



Identify the Goal

The velocity, $\vec{v}'_{AB}$, of the combined boxcars immediately after the collision

Identify the Variables and Constants

Known	Implied	Unknown
$m_A = 1.75 \times 10^4$ kg	$\vec{v}_B = 0.00\ \frac{m}{s}$	$\vec{v}'_{AB}$
$m_B = 2.00 \times 10^4$ kg		
$\vec{v}_A = 5.45\ \frac{m}{s}[E]$		

Develop a Strategy

Apply the law of conservation of momentum.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

After the collision, the two masses act as one, with one velocity. Rewrite the equation to show this condition.

$$m_A\vec{v}_A + m_B\vec{v}_B = (m_A + m_B)\vec{v}'_{AB}$$

Solve for $\vec{v}'_{AB}$.

$$\vec{v}'_{AB} = \frac{m_A\vec{v}_A + m_B\vec{v}_B}{(m_A + m_B)}$$

Substitute values and solve.

$$\vec{v}'_{AB} = \frac{(1.75 \times 10^4\ \text{kg})(5.45\ \frac{m}{s}[E]) + (2.00 \times 10^4\ \text{kg})(0.00\ \frac{m}{s})}{(1.75 \times 10^4\ \text{kg} + 2.00 \times 10^4\ \text{kg})}$$

$$\vec{v}'_{AB} = \frac{9.53\ 75 \times 10^4\ \frac{\cancel{kg}\cdot m}{s}[E]}{3.75 \times 10^4\ \cancel{kg}}$$

$$\vec{v}'_{AB} = 2.543\ \frac{m}{s}[E]$$

$$\vec{v}'_{AB} \cong 2.54\ \frac{m}{s}[E]$$

The locked boxcars were rolling *east* down the track at 2.54 m/s.

Validate the Solution

The combined mass of the boxcars was nearly double the mass of the boxcar that was moving before the collision. Since the exponents of mass and velocity are always one, making the relationships linear, you would expect that the velocity of the combined boxcars would be just under half of the velocity of the single boxcar before the collision. Half of 5.45 m/s is approximately 2.7 m/s. The calculated value of 2.54 m/s is very close to what you would expect.

PRACTICE PROBLEMS

8. Claude and Heather are practising pairs skating for a competition. Heather (47 kg) is skating with a velocity of 2.2 m/s. Claude (72 kg) is directly behind her, skating with a velocity of 3.1 m/s. When he reaches her, he holds her waist and they skate together. At the instant after he takes hold of her waist, what is their velocity?

9. Two amusement park "wrecker cars" are heading directly toward each other. The combined mass of car A plus driver is 375 kg and it is moving with a velocity of +1.8 m/s. The combined mass of car B plus driver is 422 kg and it is moving with a velocity of −1.4 m/s. When they collide, they attach and continue moving along the same straight line. What is their velocity immediately after they collide?

Recoil

Imagine yourself in the situation illustrated in Figure 4.8. You are in a small canoe with a friend and you decide to change places. Assume that the friction between the canoe and the water is negligible. While the canoe is not moving in the water, you very carefully stand up and start to take a step. You suddenly have the sense that the boat is moving under your feet. Why?

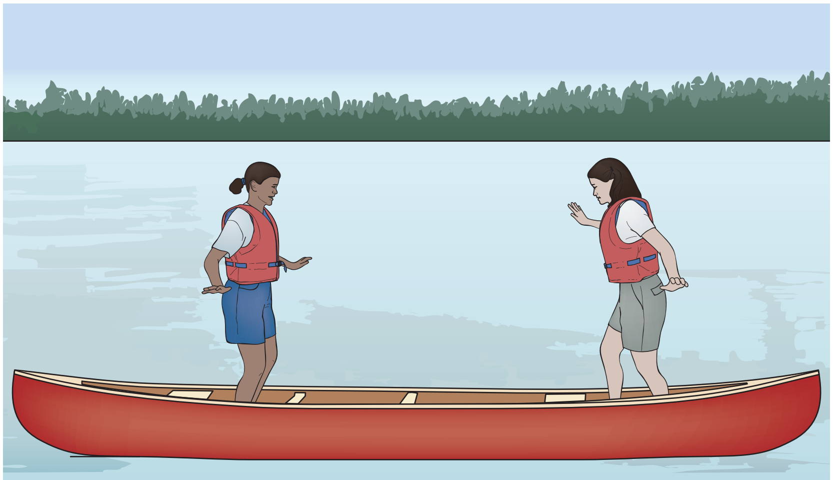


Figure 4.8 If you start to step forward in a canoe, the canoe recoils under your feet.

When you stepped forward, your foot pushed against the bottom of the canoe and you started to move. You gained momentum due to your velocity. Momentum of the system — you, your friend, and the canoe — must be conserved, so the canoe started to move in the opposite direction. The interaction that occurs when two stationary objects push against each other and then move apart is called **recoil.** You can use the equation for conservation of momentum to solve recoil problems, as the following problem illustrates.

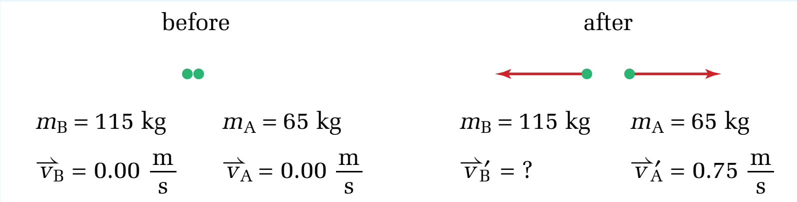
SAMPLE PROBLEM

Recoil of a Canoe

For the case described in the text, find the velocity of the canoe and your friend at the instant that you start to take a step, if your velocity is 0.75 m/s[forward]. Assume that your mass is 65 kg and the combined mass of the canoe and your friend is 115 kg.

Conceptualize the Problem

- Make a simple sketch of the conditions before and after you took a step.



- The canoe was *not moving* when you started to take a step.
- You gained *momentum* when you started to *move*. Label yourself "A" and consider the direction of your motion to be *positive*.
- The canoe had to *move* in a *negative direction* in order to conserve momentum. Label the canoe and your friend "B."

Identify the Goal

The initial velocity, $\vec{v}'_B$ of the canoe and your friend

Identify the Variables and Constants

Known

$m_A = 65$ kg

$m_B = 115$ kg

$\vec{v}'_A = 0.75 \frac{m}{s}$

Implied

$\vec{v}_A = 0.00 \frac{m}{s}$

$\vec{v}_B = 0.00 \frac{m}{s}$

Unknown

$\vec{v}'_B$

Develop a Strategy

Apply conservation of momentum.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

Velocities before the interaction were zero; therefore, the total momentum before the interaction was zero. Set these values equal to zero and solve for the velocity of B after the reaction.

$$0.0 \frac{kg \cdot m}{s} = m_A\vec{v}'_A + m_B\vec{v}'_B$$

$$m_A\vec{v}'_A = -m_B\vec{v}'_B$$

$$\vec{v}'_B = -\frac{m_A\vec{v}'_A}{m_B}$$

Substitute values and solve.

$$\vec{v}'_B = -\frac{(65\ \cancel{kg})(0.75\ \frac{m}{s})}{115\ \cancel{kg}}$$

$$\vec{v}'_B = -0.4239 \frac{m}{s}$$

$$\vec{v}'_B \cong -0.42 \frac{m}{s}$$

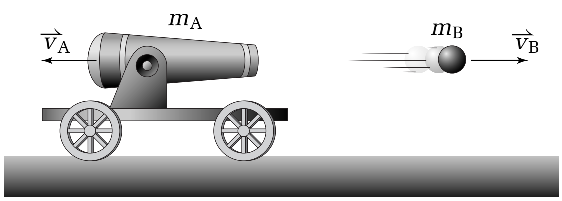
The velocity of the canoe and your friend, immediately after you started moving, was –0.42 m/s.

Validate the Solution

Since the mass of the canoe plus your friend was larger than your mass, you would expect that the magnitude of their velocity would be smaller, which it was. Also, the direction of the velocity of the canoe plus your friend must be negative, that is, in a direction opposite to your direction. Again, it was.

PRACTICE PROBLEMS

10. A 1385 kg cannon containing a 58.5 kg cannon ball is on wheels. The cannon fires the cannon ball, giving it a velocity of 49.8 m/s north. What is the initial velocity of the cannon the instant after it fires the cannon ball?



11. While you are wearing in-line skates, you are standing still and holding a 1.7 kg rock. Assume that your mass is 57 kg. If you throw the rock directly west with a velocity of 3.8 m/s, what will be your recoil velocity?

12. The mass of a uranium-238 atom is 3.95×10^{-25} kg. A stationary uranium atom emits an alpha particle with a mass of 6.64×10^{-27} kg. If the alpha particle has a velocity of 1.42×10^4 m/s, what is the recoil velocity of the uranium atom?

Collisions in Two Dimensions

Very few collisions are confined to one dimension, as anyone who has played billiards knows. Nevertheless, you can work in one dimension at a time, because momentum is conserved in each dimension independently. For example, consider the car crash illustrated in Figure 4.9. Car A is heading north and car B is heading east when they collide at the intersection. The cars lock together and move off at an angle. You can find the total momentum of the entangled cars because the component of the momentum to the north must be the same as car A's original momentum. The eastward component of the momentum must be the same as car B's original momentum. You can use the Pythagorean theorem to find the resultant momentum, as shown in the following problems.

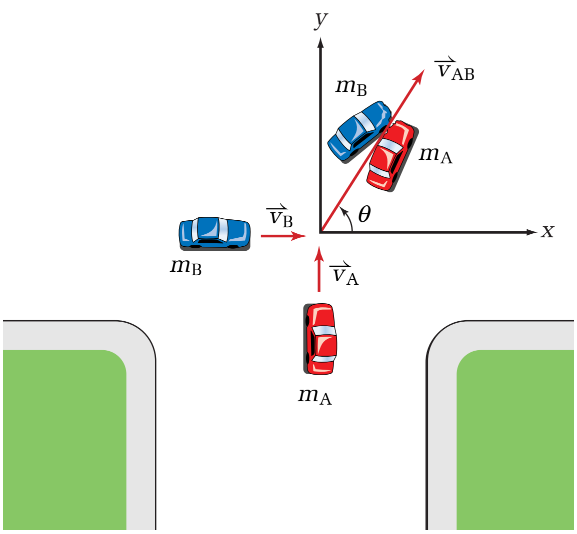
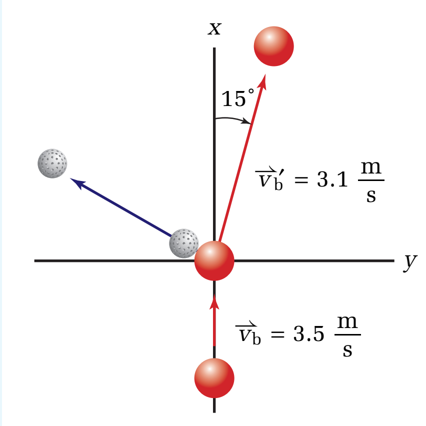


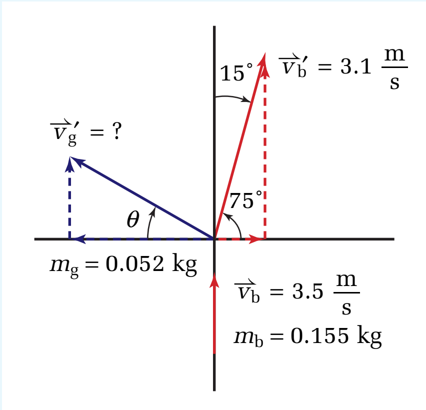
Figure 4.9 Momentum is conserved independently in both the north-south dimension and the east-west dimension.

SAMPLE PROBLEMS

Applying Conservation of Momentum in Two Dimensions

1. A billiard ball of mass 0.155 kg is rolling directly away from you at 3.5 m/s. It collides with a stationary golf ball of mass 0.052 kg. The billiard ball rolls off at an angle of 15° clockwise from its original direction with a velocity of 3.1 m/s. What is the velocity of the golf ball?





Conceptualize the Problem

- Sketch the vectors representing the momentum of the billiard ball and the golf ball immediately before and just after the collision. It is always helpful to superimpose an *x*–*y*-coordinate system on the vectors so that the origin is at the point of the contact of the two balls. For calculations, use the angles that the vectors make with the *x*-axis.
- Momentum is *conserved* in the *x* and *y* directions *independently*.
- The *total momentum* of the system (billiard ball and golf ball) *before* the collision is carried by the *billiard ball* and is all in the positive *y* direction.
- *After* the collision, both balls have *momentum* in both the *y* direction and the *x* direction.
- Since the *momentum* in the *x direction* was *zero* before the collision, it must be *zero* after the collision. Therefore, the *x-components of the momentum* of the two balls after the collision must be *equal in magnitude* and *opposite in direction*.
- The *sum* of the *y*-components of the two balls *after* the collision must equal the *momentum* of the billiard ball *before* the collision.
- Use subscript "b" for the billiard ball and subscript "g" for the golf ball.

Identify the Goal

The velocity, $\vec{v}'_g$, of the golf ball after the collision

Identify the Variables and Constants

Known		Implied	Unknown
$m_b = 0.155$ kg	$\vec{v}_b = 3.5\ \frac{m}{s}$[forward]	$\vec{v}_g = 0.00\ \frac{m}{s}$	$\vec{v}'_g$
$m_g = 0.052$ kg	$\vec{v}'_b = 3.1\ \frac{m}{s}$[15° clockwise from original]		

PROBLEM TIP

When you are working with many bits of data in one problem, it is often helpful to organize the data in a table such as the one shown here.

Object		P_x	P_y
before	A		
	B		
	total		
after	A		
	B		
	total		

Develop a Strategy

Write the expression for the conservation of momentum in the x direction.

$$m_b v_{bx} + m_g v_{gx} = m_b v'_{bx} + m_g v'_{gx}$$

Note that the x-component of the momentum of both balls was zero before the collision. Then solve for the x-component of the velocity of the golf ball after the collision.

$$0.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = m_b v'_{bx} + m_g v'_{gx}$$
$$m_g v'_{gx} = -m_b v'_{bx}$$
$$v'_{gx} = -\frac{m_b v'_{bx}}{m_g}$$

Substitute values and solve.

$$v'_{gx} = -\frac{(0.155\ \text{kg})(3.1\ \frac{\text{m}}{\text{s}}\cos 75°)}{0.052\ \text{kg}}$$
$$v'_{gx} = -2.3916\ \frac{\text{m}}{\text{s}}$$

Carry out the same procedure for the y-components.

$$m_b v_{by} + m_g v_{gy} = m_b v'_{by} + m_g v'_{gy}$$
$$m_b v_{by} + 0.0\ \frac{\text{kg}\cdot\text{m}}{s} = m_b v'_{by} + m_g v'_{gy}$$
$$m_g v'_{gy} = m_b v'_{by} - m_b v_{by}$$
$$v'_{gy} = \frac{m_b v_{by} - m_b v'_{by}}{m_g}$$
$$v'_{gy} = \frac{(0.155\ \text{kg})(3.5\ \frac{\text{m}}{\text{s}}) - (0.155\ \text{kg})(3.1\ \frac{\text{m}}{\text{s}}\sin 75°)}{0.052\ \text{kg}}$$
$$v'_{gy} = 1.507\ \frac{\text{m}}{\text{s}}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity vector of the golf ball.

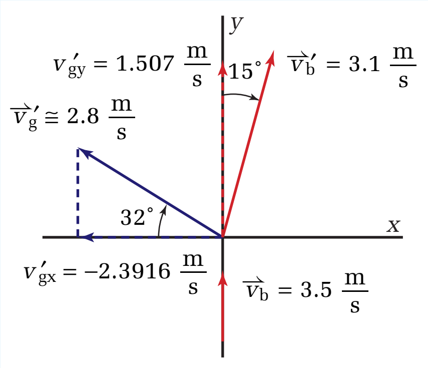
$$|\vec{v}'_g|^2 = v'^2_{gx} + v'^2_{gy}$$
$$|\vec{v}'_g|^2 = \left(-2.3916\ \frac{\text{m}}{\text{s}}\right)^2 + \left(1.507\ \frac{\text{m}}{\text{s}}\right)^2$$
$$|\vec{v}'_g|^2 = 5.7198\ \frac{\text{m}^2}{\text{s}^2} + 2.271\ \frac{\text{m}^2}{\text{s}^2}$$
$$|\vec{v}'_g|^2 = 7.9908\ \frac{\text{m}^2}{\text{s}^2}$$
$$|\vec{v}'_g| = 2.8268\ \frac{\text{m}}{\text{s}}$$
$$|\vec{v}'_g| \cong 2.8\ \frac{\text{m}}{\text{s}}$$

Use the tangent function to find the direction of the velocity vector.

$$\tan\theta = \frac{v_{gy}}{v_{gx}}$$

Since the x-component is negative and the y-component is positive, the vector is in the second quadrant. Use positive values to find the magnitude of the angle from the x-axis.

$$\tan\theta = \frac{1.507\ \frac{\text{m}}{\text{s}}}{2.3916\ \frac{\text{m}}{\text{s}}}$$
$$\theta = \tan^{-1} 0.6301$$
$$\theta = 32.22°$$
$$\theta \cong 32°$$



Since the x-component is negative and the y-component is positive, the resultant vector lies in the second quadrant and the angle is measured clockwise from the x-axis.

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The velocity of the golf ball after the collision is 2.8 m/s at 32° clockwise from the negative x-axis. (At more advanced levels, you will be expected to report angles counterclockwise from the positive x-axis. In this case, the angle would be 180° − 32° = 148° counterclockwise from the x-axis.)

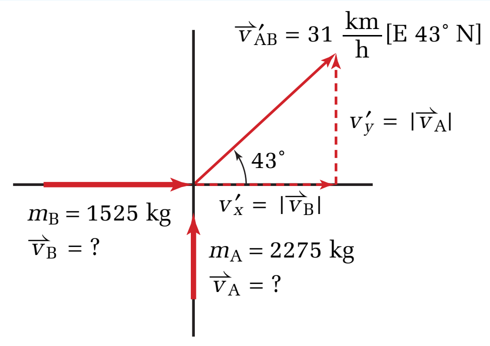
Validate the Solution

Since all of the momentum before the collision was in the positive y direction, the y-component of momentum after the collision had to be in the positive y direction, which it was. Since there was no momentum in the x direction before the collision, the x-components of the momentum after the collision had to be in opposite directions, which they were.

---

2. The police are investigating an accident similar to the one pictured in Figure 4.9. Using data tables, they have determined that the mass of car A is 2275 kg and the mass of car B is 1525 kg. From the skid marks and data for the friction between tires and concrete, the police determined that the cars, when they were locked together, had a velocity of 31 km/h at an angle of 43° north of the eastbound street. If the speed limit was 35 km/h on both streets, should one or both cars be ticketed for speeding? Which car had the right of way at the intersection? Was one driver or were both drivers at fault for the accident?

Conceptualize the Problem



- Sketch a vector diagram of the momentum before and after the collision.
- Consider the two cars to be a "system." Before the collision, the *north component* of the *momentum* of the system was carried by car A and the *east component* was carried by car B.
- Momentum is conserved in the north-south direction and in the east-west direction independently.
- After the collision, the cars form *one mass* with all of the *momentum*.

Identify the Goal

The velocities, $\vec{v}_A$ and $\vec{v}_B$, of the two cars before the collision (in order to determine who should be ticketed)

Identify the Variables and Constants

Known		Unknown
$m_A = 2275$ kg	$\vec{v}'_{AB} = 31 \frac{km}{h}$ [E43°N]	$\vec{v}_A$
$m_B = 1525$ kg		$\vec{v}_B$

PROBLEM TIP

In this problem, you have two unknown values, the velocity of car A and the velocity of car B before the collision. To find two unknown values, you need at least two equations. Since momentum is a vector quantity, conservation of momentum provides three equations, one for each dimension. Remember, use as many dimensions as you have unknowns and you will be able to solve momentum problems with as many as three unknowns.

Develop a Strategy

Write the equation for conservation of momentum.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

Work with the north-south direction only. Modify the equation to show that car B was moving directly east before the crash; its north-south momentum was zero. After the crash, the cars were combined.

$$m_A v_A[\text{N}] = (m_A + m_B)v'_{A/B}[\text{N}]$$

(Note that vector notations are not included, because you are considering only the north-south component of the velocities.)

Solve the equation for the original velocity of car A.

$$v_A[\text{N}] = \frac{(m_A + m_B)v'_{AB}[\text{N}]}{m_A}$$

Substitute the values and solve.

$$v_A[\text{N}] = \frac{(2275\text{ kg} + 1525\text{ kg})(31\ \frac{\text{km}}{\text{h}}\sin 43°)}{2275\text{ kg}}$$

$$v_A[\text{N}] = \frac{(3800\text{ kg})(31\ \frac{\text{km}}{\text{h}})(0.681\ 998\ 4)}{2275\text{ kg}}$$

$$v_A[\text{N}] = 35.3\ \frac{\text{km}}{\text{h}}$$

$$\vec{v}_A \cong 35\ \frac{\text{km}}{\text{h}}[\text{N}]$$

(Note that the north component of car A's velocity before the crash was the total velocity.)

Carry out the same procedure for the east-west direction of the momentum.

$$m_B v_B[\text{E}] = (m_A + m_B)v'_{AB}[\text{E}]$$

$$v_B[\text{E}] = \frac{(m_A + m_B)v'_{AB}[\text{E}]}{m_B}$$

$$v_B[\text{E}] = \frac{(2275\text{ kg} + 1525\text{ kg})(31\ \frac{\text{km}}{\text{h}}\cos 43°)}{1525\text{ kg}}$$

$$v_B[\text{E}] = 56.49\ \frac{\text{km}}{\text{h}}$$

$$\vec{v}_B \cong 56\ \frac{\text{km}}{\text{h}}[\text{E}]$$

Car A was travelling 35 km/h north and car B was travelling 56 km/h east at the instant before the crash. Therefore, car B was speeding and the driver should be ticketed. As well, the driver on the right has the right of way, giving car A the right of way at the intersection. The driver of car B was at fault for the collision. Nevertheless, the driver of car A would have benefited if he or she could have prevented the crash.

Validate the Solution

The angle at which the locked cars moved after the crash was very close to 45°, which means that the momentum of the two cars before the crash was nearly the same. Car B had a smaller mass than car A, so car B must have moving at a greater speed (magnitude of the velocity), which agrees with the results. Also, the units all cancelled to give km/h, which is correct for velocity.

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PRACTICE PROBLEMS

13. A 0.150 kg billiard ball (A) is rolling toward a stationary billiard ball (B) at 10.0 m/s. After the collision, ball A rolls off at 7.7 m/s at an angle of 40.0˚ clockwise from its original direction. What is the speed and direction of ball B after the collision?

14. A bowling ball with a mass of 6.00 kg rolls with a velocity of 1.20 m/s toward a single standing bowling pin that has a mass of 0.220 kg. When the ball strikes the bowling pin, the pin flies off at an angle of 70.0˚ counterclockwise from the original direction of the ball, with a velocity of 3.60 m/s. What was the velocity of the bowling ball after it hit the pin?

15. Car A (1750 kg) is travelling due south and car B (1450 kg) is travelling due east. They reach the same intersection at the same time and collide. The cars lock together and move off at 35.8 km/h[E31.6˚S]. What was the velocity of each car before they collided?

Angular Momentum

Why is a bicycle easy to balance when you are riding, but falls over when you come to a stop? Why does a toy gyroscope, like the one in Figure 4.10, balance on a pointed pedestal when it is spinning, but falls off the pedestal when it stops spinning? The answer lies in the conservation of angular momentum.

Figure 4.10 When a spinning object begins to fall, its angular momentum resists the direction of the fall.

When an object is moving on a curved path or rotating, it has angular momentum. Angular momentum and linear (or translational) momentum are similar in that they are both dependent on an object's mass and velocity. Analyze Figure 4.11 to find the third quantity that affects angular momentum.

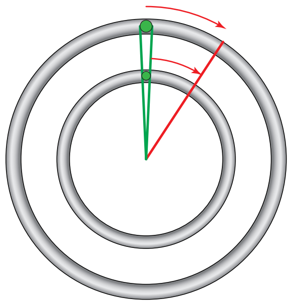


Figure 4.11 As the distance from the centre of rotation increases, a unit of mass must move faster in order to maintain a constant rate of rotation.

WEB LINK

www.mcgrawhill.ca/links/physics12

For information on current accident-investigation research topics and technological developments related to vehicle safety, go to the above Internet site and click on **Web Links**.

PHYSICS & TECHNOLOGY

TARGET SKILLS

- Predicting
- Hypothesizing
- Communicating results
- Conducting research

The Physics of a Car Crash

Skid marks, broken glass, mangled pieces of metal — these telltale signs of an automobile crash are stark reminders of the dangers of road travel. For accident investigators, sometimes referred to as "crash analysts" or "reconstructionists," these remnants of a collision can also provide valuable clues that will help in under-standing the cause and the nature of an accident.

The reasons for studying a car crash can vary, depending on who is conducting the investigation. Police officers might be interested in determining how fast a vehicle was being driven prior to a collision in order to know whether to lay criminal charges. Insurance companies might require proof that the occupants of a car were wearing seat belts to make decisions on insurance claims.

Government agencies, meanwhile, conduct large-scale research projects (based on both real-world accidents and staged collisions) that guide in the establishment of safety standards and regulations for the manufacture of automobiles.

Regardless of the purpose, however, most car crash investigations share some common elements. First, they typically draw on the same fundamental concepts and principles of physics that you are learning in this chapter — especially those related to energy and momentum. As well, these investigations often incorporate an array of technological resources to help with both the data-collection and data-analysis phases of the process. Investigators make use of a variety of data-collection tools, ranging from everyday hardware, such as a measuring tape and a camera, to more sophisticated instruments, such as brake-activated chalk guns and laser-operated surveyors' transits.

Customized computer programs, designed with algorithms based on Newtonian mechanics, are used to analyze data collected from the scene of the accident. The length and direction of skid marks, "crush" measurements and stiffness coefficients associated with the damaged vehicles, coefficients of friction specific to the tires and road surface — known or estimated values of these and other relevant parameters are fed into the computer programs. The programs then generate estimates of important variables, such as the speed at the time of impact or the change in speed over the duration of a collision. Quantitative and qualitative data obtained from car accidents is also often coded and added to large computerized databases that can be accessed for future investigations and for research purposes.

Recently, some automobile manufacturers have started installing event data recorders (EDRs) in the vehicles they build. Like the cockpit data recorder or "black box" commonly used in the aviation industry, EDRs in road vehicles record valuable data such as vehicle speed, engine revolutions per minute, brake-switch status, throttle position, and seat-belt use. This information is processed by a vehicle's central computer system to monitor and regulate the operation of such safety components as air bags and antilock brakes. It also provides accident investigators with an additional source of data about the conditions that existed immediately before and during a collision. As a result, EDRs promise to provide significant enhancement to the field of automobile accident investigation.

Analyze

1. Why is it important for car accident investigators to take into account the weather conditions that existed at the time of a car crash? List some specific weather conditions and predict the effects that they might have on the calculations carried out as part of an investigation.

2. Explore the Internet to learn about current research topics and technological developments related to vehicle safety. Prepare a one-page report on your research results.

PHYSICS FILE

Although Kepler knew nothing about angular momentum, his second law, the law of areas, is an excellent example of the conservation of angular momentum. With somewhat complex mathematics, it is possible to write the law of conservation of angular momentum for a planet in orbit and show that it is equivalent to Kepler's second law.

Picture the movement of a unit of mass in each of the two wheels illustrated in Figure 4.11. If the two wheels are rotating at the same rate, each unit of mass in the large wheel is moving faster than a unit of mass in the small wheel. Thus, r, the distance of a mass from the centre of rotation, affects the angular momentum. The magnitude of the angular momentum, L, of a particle that is moving in a circle is equal to the product of its mass, velocity, and distance from the centre of rotation, or $L = mvr$. You will not pursue a quantitative study of angular momentum any further in this course, but it is essential to be aware of the law of conservation of angular momentum in order to have a complete picture of the important conservation laws of physics. Similar to conservation of linear momentum, the angular momentum of an isolated system is conserved.

4.2 Section Review

1. **C** Explain qualitatively how Newton's third law is related to the law of conservation of momentum.

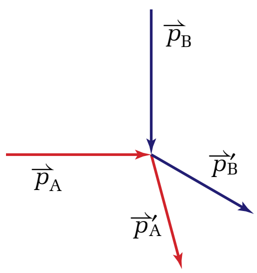
2. **K/U** What is the difference between an internal force and an external force?

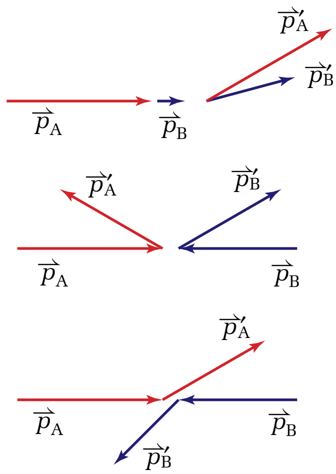
3. **K/U** How does a closed system differ from an isolated system?

4. **K/U** Under what circumstances is the change in momentum of a system equal to zero?

5. **K/U** Define and give an example of recoil.

6. **I** The vectors in the following diagrams represent the momentum of objects before and after a collision. Which of the diagrams (there might be more than one) does *not* represent real collisions? Explain your reasoning.





7. **C** Some collision problems have two unknown variables, such as the velocities of two cars before a collision. Explain how it is possible to find two unknowns by using only the law of conservation of momentum.

8. **MC** Two cars of identical mass are approaching the same intersection, one from the south and one from the west. They reach the intersection at the same time and collide. The cars lock together and move away at an angle of 22° counterclockwise from the road, heading east. Which car was travelling faster than the other before the collision? Explain your reasoning.

4.3 Elastic and Inelastic Collisions

Momentum is conserved in the two collisions pictured in Figure 4.12, but the two cases are quite different. When the metal spheres in the Newton's cradle collided, both momentum and kinetic energy were conserved. When the cars in the photograph crashed, kinetic energy was *not* conserved. This feature divides all collisions into two classes. Collisions in which kinetic energy is conserved are said to be **elastic.** When kinetic energy is *not* conserved, the collisions are **inelastic.**

SECTION EXPECTATIONS

- Distinguish between elastic and inelastic collisions.
- Define and describe the concepts related to momentum, energy, and elastic and inelastic collisions.
- Investigate the laws of conservation of momentum and of energy in one and two dimensions.

KEY TERMS

- elastic
- inelastic

Figure 4.12 How do the collisions pictured here differ from each other?

Analyzing Collisions

You can determine whether a collision is elastic or inelastic by calculating both the momentum and the kinetic energy before and after the collision. Since momentum is always conserved at the instant of the collision, you can use the law of conservation of momentum to find unknown values for velocity. Then, use the known and calculated values for velocity to calculate the total kinetic energy before and after the collision. You will probably recall that the equation for kinetic energy is $E = \frac{1}{2}mv^2$.

INVESTIGATION 4-B

Examining Collisions

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

You have learned the definition of elastic and inelastic collisions, but are there characteristics that allow you to predict whether a collision will be elastic? In this investigation, you will observe and analyze several collisions and draw conclusions regarding whether a type of collision will be elastic or inelastic.

Problem

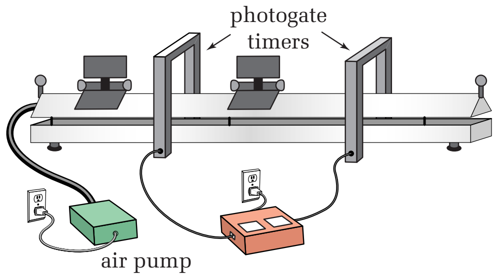
What are the characteristics of elastic and inelastic collisions?

Equipment

- air track (with source of compressed air)
- 2 gliders (identical, either middle- or large-sized)
- 2 photogate timers
- laboratory balance
- 4 glider bumper springs
- 2 Velcro™ bumpers (or a needle and a piece of wax)
- 2 velocity flags (10 cm) (or file cards cut to a 10 cm length)
- modelling clay

Procedure

1. Set up the air track and adjust the levelling screw to ensure that the track is horizontal. You can test whether the track is level by turning on the air pressure and placing a glider on the track. Hold the glider still and then release it. If the track is level, the glider will remain in place. If the glider gradually starts moving, the air track is not level.
2. Attach a velocity flag (or 10 cm card) and two bumper springs to each glider. If only one bumper spring is attached, the glider might not be properly balanced.
3. Position the photogates about one fourth the length of the track from each end, as shown in the diagram. Adjust the height of the photogates so that the velocity flags will pass through the gates smoothly but will trigger the gates.



4. Label one glider "A" and the other glider "B." Use the laboratory balance to determine accurately the mass of each glider.
5. With the air flowing, place glider A on the left end of the air track and glider B in the centre.
6. Perform a test run by pushing glider A so that it collides with glider B. Ensure that the photogates are placed properly so that the flags are not inside the gates when the gliders are in contact. Adjust the positions of the photogates, if necessary.
7. The first set of trials will be like the test run, with glider A on the left end of the track and glider B in the centre. Turn on the photogates and press the reset button. Push glider A and allow it to collide with glider B. Allow both gliders to pass through a photogate after the collision, then catch them before they bounce back and pass through a photogate again. Record the data in a table similar to the one shown on the next page. Since all of the motion will be in one dimension, only positive and negative signs will be needed to indicate direction. Vector notations will not be necessary.
The displacement, Δd, is the distance that the gliders travelled while passing through the photogates. This displacement is the length of the flag. Time Δt_i is the time that a glider spent in the photogate before the

collision, while Δt_f is the time the glider took to pass through the photogate after the collision. Calculate velocity, v, from the displacement and the time interval. Be sure to include positive and negative signs.

	Glider A (mass = ?)				
Trial	$\Delta\vec{d}$	Δt_i	$\vec{v}_i$	Δt_f	$\vec{v}_f$
1					
2					
3					

	Glider B (mass = ?)		
Trial	$\Delta\vec{d}$	Δt_f	$\vec{v}_f$
1			
2			
3			

8. Increase the mass of glider B by attaching some modelling clay to it. Be sure that the clay is evenly distributed along the glider. If the glider tips to the side or to the front or back, the motion will not be smooth. Determine the mass of glider B. Repeat step 7 for the two gliders, which are now of unequal mass.

9. Exchange the gliders and their labels. That is, the glider with the extra mass is on the left and becomes glider "A." The glider with no extra mass should be in the centre and labelled "B." Repeat step 7 for the new arrangement of gliders.

10. Remove the clay from the glider. Place one glider at each end of the track. Practise starting both gliders at the same time, so that they collide near the middle of the track. The collision must not take place while either glider is in a photogate. When you have demonstrated that you can carry out the collision correctly, perform three trials and record the data in a table similar to the ones shown here. This table will need two additional columns — one for the initial time for glider B and a second for the initial velocity of glider B.

11. Remove the bumper springs from one end of each glider and attach the Velcro™ bumpers. (If you do not have Velcro™ bumpers, you can attach a large needle to one glider and a piece of wax to the other. Test to ensure that the needle will hit the wax when the gliders collide.)

12. With the Velcro™ bumpers attached, perform three sets of trials similar to those in steps 7, 8, and 9. You might need to perform trial runs and adjust the position of the photogates so that both gliders can pass through the photogate before reaching the right-hand end of the air track. Record the data in tables similar to those you used previously.

Analyze and Conclude

1. For each glider in each trial, calculate the initial momentum (before the collision) and the final momentum (after the collision).

2. For each trial, calculate the total momentum of both gliders before the collision and the total momentum of both after the collision.

3. For each trial, compare the momentum before and after the collision. Describe how well the collisions demonstrated conservation of momentum.

4. In any case for which momentum did not seem to be conserved, provide possible explanations for errors.

5. Calculate the kinetic energy of each glider in each trial. Then calculate the total kinetic energy of both gliders before and after the collision for each trial.

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6. Compare the kinetic energies before and after the collisions and decide which collisions were elastic and which were inelastic. Due to measurement errors, do not expect the kinetic energies to be identical before and after a collision. Decide if the values appear to be close enough that the differences could be attributed to measurement errors.

7. Examine the nature of the collisions that you considered to be elastic and those that you classed as inelastic. Look for a trend that would permit you to predict whether a collision would be elastic or inelastic. Discuss your conclusions with the rest of class. How well did your conclusions agree with those of other class members?

Apply and Extend

8. If you have access to an air table, a strobe light, and a Polaroid™ camera, you can observe and collect data for collisions in two dimensions. (If you do not have the equipment, your teacher might be able to provide you with simulated photographs.) Using the laboratory balance, determine the mass of each of two pucks. If the pucks have nearly the same mass, add some mass to one of them, using modelling clay.

CAUTION People with certain medical conditions, such as epilepsy, can experience seizures if exposed to strobe lighting.

9. With the air pressure on, place a puck in the centre of the table. Direct the strobe light onto the table and set up the camera so that it is above the table and pointing down. Turn on the strobe light and set the camera for a long exposure time. At the moment that one partner pushes the other puck toward the stationary puck in the centre of the table, the other partner should take a picture. Take enough photographs to provide each pair of partners with a photograph.

10. Determine the scale of the photograph by determining the ratio of the size of the air table to its apparent size in the photograph. Measure two or three distances before and after the collision. Correct the distances by using the scale that you determined. Measure the angles that the pucks took after the collision in relation to the original direction.

11. Using the rate at which the strobe light was flashing, determine the time between flashes. Calculate the velocity, momentum, and kinetic energy of each puck before and after the collision.

12. Compare the total momentum before and after the collision and comment on how well the motion seemed to obey the law of conservation of momentum.

13. Compare the total kinetic energies before and after the collision. Decide whether the collisions were elastic or nearly so.

14. Compare your results from your one-dimensional data and two-dimensional data, and comment on any differences that you noticed.

15. Review the results you obtained in Investigation 4-A, Newton's Cradle. Do you think the collisions were elastic or inelastic? Explain why.

16. Support your answer to question 15 by performing trial calculations. Assume that the spheres in Newton's cradle each has a mass of 0.200 kg and that, when one sphere collided with the row, it was moving at 0.100 m/s. Imagine that, when one sphere collided with the row, two spheres bounced up from the opposite end. Calculate the velocity that the two spheres would have to have in order to conserve momentum. Calculate the kinetic energy before and after. Use these calculations to explain why you did not observe certain patterns in the motion of Newton's cradle.

SAMPLE PROBLEM

Classifying a Collision

A 0.0520 kg golf ball is moving east with a velocity of 2.10 m/s when it collides, head on, with a 0.155 kg billiard ball. If the golf ball rolls directly backward with a velocity of –1.04 m/s, was the collision elastic?

Conceptualize the Problem

- *Momentum* is always conserved in a collision.
- If the collision is *elastic*, kinetic energy must also be conserved.
- The motion is in one dimension, so only positive and negative signs are necessary to indicate directions.

Identify the Goal

Is the total kinetic energy of the system before the collision, E_{kg}, equal to the total kinetic of the system after the collision, $E'_{kg} + E'_{kb}$?

Identify the Variables and Constants

Known		Implied	Unknown	
$m_g = 0.0520\ \text{kg}$	$v_g = +2.10\ \frac{\text{m}}{\text{s}}$	$v_b = 0.0\ \frac{\text{m}}{\text{s}}$	v'_b	E'_{kg}
$m_b = 0.155\ \text{kg}$	$v'_g = -1.04\ \frac{\text{m}}{\text{s}}$		E_{kg}	E'_{kb}

Develop a Strategy

Since momentum is always conserved, use the law of conservation of momentum to find the velocity of the billiard ball after the collision.

$$m_g v_g + m_b v_b = m_g v'_g + m_b v'_b$$
$$m_g v_g + 0.0 - m_g v'_g = m_b v'_b$$
$$v'_b = \frac{m_g v_g - m_g v'_g}{m_b}$$
$$v'_b = \frac{(0.0520\ \text{kg})(2.10\ \frac{\text{m}}{\text{s}}) - (0.0520\ \text{kg})(-1.04\ \frac{\text{m}}{\text{s}})}{0.155\ \text{kg}}$$
$$v'_b = 1.0534\ \frac{\text{m}}{\text{s}}$$

Calculate the kinetic energy of the golf ball before the collision.

$$E_{kg} = \frac{1}{2} m_g v_g^2$$
$$E_{kg} = \frac{1}{2}(0.0520\ \text{kg})\left(2.10\ \frac{\text{m}}{\text{s}}\right)^2$$
$$E_{kg} = 0.114\ 66\ \text{J}$$

Calculate the sum of the kinetic energies of the balls after the collision.

$$E'_{kg} = \frac{1}{2} m_g v'^2_g$$
$$E'_{kg} = \frac{1}{2}(0.0520\ \text{kg})\left(-1.04\ \frac{\text{m}}{\text{s}}\right)^2$$
$$E'_{kg} = 0.028\ 12\ \text{J}$$
$$E'_{kb} = \frac{1}{2} m_b v'^2_b$$
$$E'_{kb} = \frac{1}{2}(0.155\ \text{kg})\left(1.0534\ \frac{\text{m}}{\text{s}}\right)^2$$
$$E'_{kb} = 0.086\ 00\ \text{J}$$
$$E'_{kg} + E'_{kb} = 0.028\ 12\ \text{J} + 0.085\ 99\ \text{J}$$
$$E'_{kg} + E'_{kb} = 0.114\ 12\ \text{J}$$

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The kinetic energies before and after the collision are the same to the third decimal place. Therefore, the collision was probably elastic.

Validate the Solution

Although the kinetic energies before and after the collision differ in the fourth decimal place, the difference is less than 1%. Since the data contained only three significant digits, this difference could easily be due to the precision of the measurement. Therefore, it is fair to say that the collision was elastic.

PRACTICE PROBLEMS

16. A billiard ball of mass 0.155 kg moves with a velocity of 12.5 m/s toward a stationary billiard ball of identical mass and strikes it with a glancing blow. The first billiard ball moves off at an angle of 29.7˚ clockwise from its original direction, with a velocity of 9.56 m/s. Determine whether the collision was elastic.

17. Car A, with a mass of 1735 kg, was travelling north at 45.5 km/h and Car B, with a mass of 2540 kg, was travelling west at 37.7 km/h when they collided at an intersection. If the cars stuck together after the collision, what was their combined momentum? Was the collision elastic or inelastic?

Elastic Collisions

By now, you have probably concluded that when objects collide, become deformed, and stick together, the collision is inelastic. Physicists say that such a collision is *completely inelastic*. Conversely, when hard objects such as billiard balls collide, bounce off each other, and return to their original shape, they have undergone elastic collisions. Very few collisions are perfectly elastic, but in many cases, the loss of kinetic energy is so small that it can be neglected.

Since both kinetic energy and momentum are conserved in perfectly elastic collisions, as many as four independent equations can be used to solve problems. Since you have two equations, you can solve for up to four unknown quantities. When combining these equations, however, the math becomes quite complex for all cases except head-on collisions, for which all motion is in one dimension.

An analysis of head-on collisions yields some very informative results, however. For example, if you know the velocities of the two masses before a collision, you can determine what the velocities will be after the collision. The following derivation applies to a mass, m_1, that is rolling toward a stationary mass, m_2. Follow the steps to find the velocities of the two objects after the collision in terms of their masses and the velocity of the first mass before the collision. Since the motion in head-on collisions is in one dimension, vector notations will not be used.

LANGUAGE LINK

In science, the word "elastic" does not mean "easily stretched." In fact, it can mean exactly the opposite. For example, glass is very elastic, up to its breaking point. Also, "elastic" is the opposite of "plastic." Find the correct meanings of the words "elastic" and "plastic" and then explain why "elastic" is an appropriate term to apply to collisions in which kinetic energy is conserved.

- Write the equations for the conservation of momentum and kinetic energy for a perfectly elastic collision, inserting zero for the velocity of the second mass before the collision.

$$m_1v_1 + 0 = m_1v_1' + m_2v_2'$$
$$\frac{1}{2}m_1v_1^2 + 0 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

- Multiply by 2 both sides of the equation for conservation of kinetic energy.

$$m_1v_1^2 = m_1v_1'^2 + m_2v_2'^2$$

- Algebraically rearrange both equations so that terms describing mass 1 are on the left-hand side of the equations and terms describing mass 2 are on the right-hand side.

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2$$

- Factor m_1 out of the left-hand side of both equations.

$$m_1(v_1 - v_1') = m_2v_2'$$
$$m_1(v_1^2 - v_1'^2) = m_2v_2'^2$$

- Divide the first equation by the second equation.

$$\frac{\cancel{m_1}(v_1 - v_1')}{\cancel{m_1}(v_1^2 - v_1'^2)} = \frac{\cancel{m_2}v_2'}{\cancel{m_2}v_2'^2}$$

- Notice that the masses cancel. Expand the expression in the denominator on the left. Notice that it is the difference of perfect squares.

$$\frac{\cancel{(v_1 - v_1')}}{\cancel{(v_1 - v_1')}(v_1 + v_1')} = \frac{\cancel{v_2'}}{v_2'^{\cancel{2}}}$$

- Simplify. Solve the equation for v_2' by inverting. Also, solve the equation for v_1'.

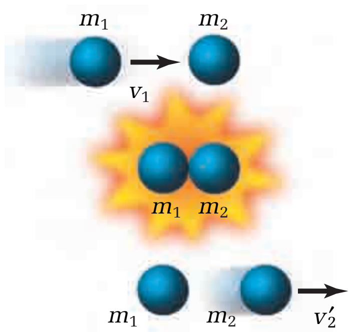
$$\frac{1}{(v_1 + v_1')} = \frac{1}{v_2'}$$
$$v_2' = v_1 + v_1'$$
$$v_1' = v_2' - v_1$$

- Develop two separate equations by substituting the values for v_1' and v_2' above into the equation for conservation of momentum, $m_1v_1 - m_1v_1' = m_2v_2'$. Expand and rearrange the equations and then solve for v_1' (left) and v_2' (right).

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1 - m_1v_1' = m_2(v_1 + v_1')$$
$$m_1v_1 - m_1v_1' = m_2v_1 + m_2v_1'$$
$$m_1v_1' + m_2v_1' = m_1v_1 - m_2v_1$$
$$v_1'(m_1 + m_2) = v_1(m_1 - m_2)$$
$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1$$

$$m_1v_1 - m_1v_1' = m_2v_2'$$
$$m_1v_1 - m_1(v_2' - v_1) = m_2v_2'$$
$$m_1v_1 - m_1v_2' + m_1v_1 = m_2v_2'$$
$$2m_1v_1 = m_1v_2' + m_2v_2'$$
$$2m_1v_1 = (m_1 + m_2)v_2'$$
$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

The two equations derived above allow you to find the velocities of two masses after a head-on collision in which a moving mass collides with a stationary mass. Without doing any calculations, however, you can draw some general conclusions. First, consider the case in which the two masses are identical.



When one moving mass collides head on with an identical stationary mass, the first mass stops. The second mass then moves with a velocity identical to the original velocity of the first mass.

Case 1: $m_1 = m_2$

Since the masses are equal, call them both "m." Substitute m into the two equations for the velocities of the two masses after the collision. Then, mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$$v_1' = \left(\frac{m - m}{m + m}\right) v_1 \qquad v_2' = \left(\frac{2m}{m + m}\right) v_1$$

$$v_1' = \left(\frac{0}{m + m}\right) v_1 \qquad v_2' = \left(\frac{\cancel{2m}}{\cancel{2m}}\right) v_1$$

$$v_1' = 0 \qquad v_2' = v_1$$

Case 2: $m_1 >>> m_2$

Since mass 1 is much larger than mass 2, you can almost ignore the mass of the second object in your calculations. You can therefore make the following approximations.

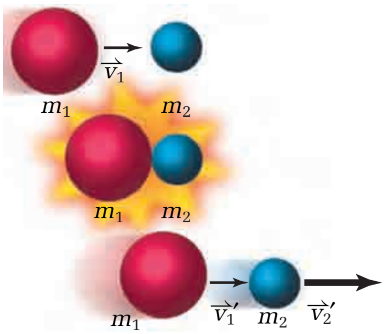
$$m_1 - m_2 \cong m_1 \quad \text{and} \quad m_1 + m_2 \cong m_1$$

Substitute these approximations into the two equations for the velocities of the two masses after the collision. Then, mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$$v_1' \cong \left(\frac{\cancel{m_1}}{\cancel{m_1}}\right) v_1 \qquad v_2' \cong \left(\frac{2\cancel{m_1}}{\cancel{m_1}}\right) v_1$$

$$v_1' \cong v_1 \qquad v_2' \cong 2v_1$$



When one moving mass collides head on with a much smaller stationary mass, the first mass continues at nearly the same speed. The second mass then moves with a velocity that is approximately twice the original velocity of the first mass.

Case 3: $m_1 <<< m_2$

Since mass 1 is much smaller than mass 2, you can ignore the mass of the first object in your calculations. You can therefore make the following approximations.

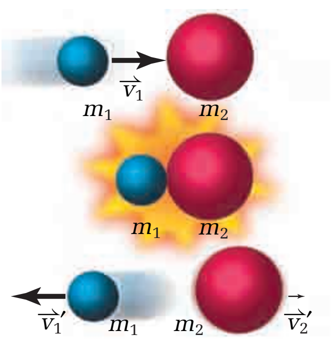
$$m_1 - m_2 \cong -m_2 \quad \text{and} \quad m_1 + m_2 \cong m_2 \quad \text{and} \quad m_1 \cong 0$$

Substitute these approximations into the two equations for the velocities of the two masses after the collision. Then mathematically simplify the equations.

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 \qquad v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$$v_1' \cong \left(\frac{-\cancel{m_2}}{\cancel{m_2}}\right) v_1 \qquad v_2' \cong \left(\frac{0}{m_2}\right) v_1$$

$$v_1' \cong -v_1 \qquad v_2' \cong 0$$



When one moving mass collides head on with a much larger stationary mass, the first mass bounces backward with a velocity opposite in direction and almost the same in magnitude as its original velocity. The motion of the second mass is almost imperceptible.

Conceptual Problems

- Using the special cases of elastic collisions, qualitatively explain what would happen in each of the following situations.
  (a) A bowling ball collides head on with a single bowling pin.
  (b) A golf ball hits a tree.
  (c) A marble collides head on with another marble that is not moving.
- Cars, trucks, and motorcycles do not undergo elastic collisions, but the general trend of the motion is similar to the motion of objects involved in elastic collisions. Describe, in very general terms, what would happen in each of the following cases. In each case, assume that the vehicles did not become attached to each other.
  (a) A very small car runs into the back of a parked tractor-trailer.
  (b) A mid-sized car runs into the back of another mid-sized car that has stopped at a traffic light.
  (c) A pickup truck runs into a parked motorcycle.

ELECTRONIC LEARNING PARTNER

Study the effects of the variable elasticity of a bouncing ball by using the interactive activity in your Electronic Learning Partner.

Inelastic Collisions

When you are working with inelastic collisions, you can apply only the law of conservation of momentum to the motion of the objects at the instant of the collision. Depending on the situation, however, you might be able to apply the laws of conservation of energy to motion just before or just after the collision. For example, a ballistic pendulum can be used to measure the velocity of a projectile such as a bullet, as illustrated in Figure 4.13. When the bullet collides with the wooden block of the ballistic pendulum, it becomes embedded in the wood, making the collision completely inelastic.

After the collision, you can apply the law of conservation of mechanical energy to the motion of the pendulum. The kinetic energy of the pendulum at the instant after the collision is converted into potential energy of the pendulum bob. By measuring the height to which the pendulum rises, you can calculate the velocity of the bullet just before it hit the pendulum, as shown in the following sample problem.

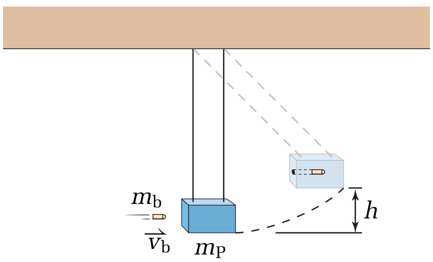


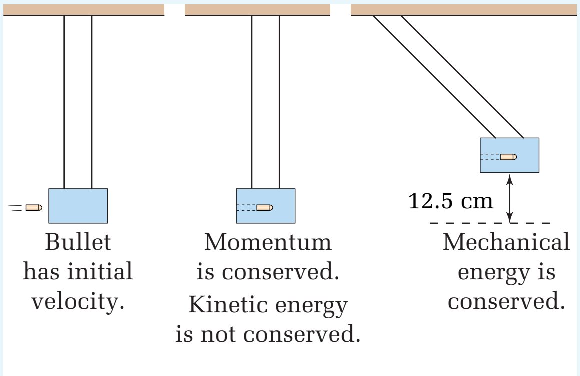
Figure 4.13 A ballistic pendulum is designed to have as little friction as possible. Therefore, you can assume that, at the top of its swing, the gravitational potential energy of the pendulum bob is equal to the kinetic energy of the pendulum bob at the lowest point of its motion.

SAMPLE PROBLEMS

Energy Conservation Before and After a Collision

1. A forensic expert needed to find the velocity of a bullet fired from a gun in order to predict the trajectory of a bullet. He fired a 5.50 g bullet into a ballistic pendulum with a bob that had of mass 1.75 kg. The pendulum swung to a height of 12.5 cm above its rest position before dropping back down. What was the velocity of the bullet just before it hit and became embedded in the pendulum bob?

Conceptualize the Problem



- Sketch the positions of the bullet and pendulum bob just before the collision, just after the collision, and with the pendulum at its highest point.
- When the bullet hit the pendulum, *momentum* was *conserved*.
- If you can find the *velocity* of the combined bullet and pendulum bob after the collision, you can use conservation of momentum to find the *velocity* of the bullet before the collision.
- The collision was completely inelastic so *kinetic energy* was *not* conserved.
- However, you can assume that the friction of the pendulum is negligible, so *mechanical energy* of the pendulum was *conserved*.
- The *gravitational potential energy* of the combined masses at the highest point of the pendulum is equal to the *kinetic energy* of the combined masses at the lowest point of the pendulum.
- If you know the kinetic energy of the combined masses just after the collision, you can find the *velocity* of the masses just *after* the collision.
- Use the subscripts "b" for the bullet and "p" for the pendulum.

Identify the Goal

The velocity, v_b, of the bullet just before it hit the ballistic pendulum

Identify the Variables and Constants

Known		Implied	Unknown	
$m_b = 5.50$ g	$\Delta h = 12.5$ cm	$g = 9.81\ \frac{m}{s^2}$	$\vec{v}_b$	E_g
$m_p = 1.75$ kg			$\vec{v}_p$	E_k

Develop a Strategy

To find the velocity of the combined masses of the bullet and pendulum bob just after the collision, use the relationship that describes the conservation of mechanical energy of the pendulum.

$$E_{k(bottom)} = E_{g(top)}$$

Substitute the expressions for kinetic energy and gravitational potential energy that you learned in previous physics courses. Solve for velocity. Convert all units to SI units.

$$\frac{1}{2}\cancel{m}v_{\text{bottom}}^2 = \cancel{m}g\Delta h$$

$$v_{\text{bottom}}^2 = 2g\Delta h$$

$$v_{\text{bottom}} = \sqrt{2g\Delta h}$$

$$v_{\text{bottom}} = \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(12.5\ \cancel{\text{cm}})\left(\frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right)}$$

$$v_{\text{bottom}} = \sqrt{2.4525\ \frac{\text{m}^2}{\text{s}^2}}$$

$$v_{\text{bottom}} = \pm 1.566\ \frac{\text{m}}{\text{s}}$$

Define the direction of the bullet as positive during and immediately after the collision.

Apply the conservation of momentum to find the velocity of the bullet before the collision. Convert all units to SI units.

$$m_{\text{b}}\vec{v}_{\text{b}} + m_{\text{p}}\vec{v}_{\text{p}} = m_{\text{b}}\vec{v}\,'_{\text{b}} + m_{\text{p}}\vec{v}\,'_{\text{p}}$$

$$m_{\text{b}}\vec{v}_{\text{b}} + 0 = (m_{\text{b}} + m_{\text{p}})\vec{v}\,'_{\text{b/p}}$$

$$\vec{v}_{\text{b}} = \frac{(m_{\text{b}} + m_{\text{p}})v'_{\text{b/p}}}{m_{\text{b}}}$$

$$\vec{v}_{\text{b}} = \frac{\left[5.50\ \cancel{\text{g}}\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right) + 1.75\ \text{kg}\right]1.566\ \frac{\text{m}}{\text{s}}}{5.50\ \text{g}\left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)}$$

$$\vec{v}_{\text{b}} = \frac{(1.7555\ \cancel{\text{kg}})1.566\ \frac{\text{m}}{\text{s}}}{0.005\ 50\ \cancel{\text{kg}}}$$

$$\vec{v}_{\text{b}} = 499.8387\ \frac{\text{m}}{\text{s}}$$

$$\vec{v}_{\text{b}} \cong 5.00 \times 10^2\ \frac{\text{m}}{\text{s}}\text{[in positive direction]}$$

The velocity of the bullet just before the collision was about 500 m/s in the positive direction.

Validate the Solution

In both calculations, the units cancelled to give metres per second, which is correct for velocity. The velocity of 500 m/s is a reasonable velocity for a bullet.

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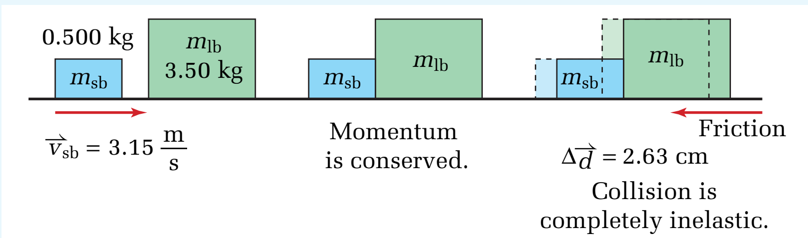
2. A block of wood with a mass of 0.500 kg slides across the floor toward a 3.50 kg block of wood. Just before the collision, the small block is travelling at 3.15 m/s. Because some nails are sticking out of the blocks, the blocks stick together when they collide. Scratch marks on the floor show that they slid 2.63 cm before coming to a stop. What is the coefficient of friction between the wooden blocks and the floor?

Conceptualize the Problem

- Sketch the blocks just before, at the moment of, and after the collision, when they came to a stop.
- *Momentum* is conserved during the collision.



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- Since the blocks stuck together, the collision was *completely inelastic*, so *kinetic energy* was *not* conserved. Some kinetic energy was lost to sound, heat, and deformation of the wood during the collision.
- Some *kinetic energy* remained after the collision.
- The force of *friction* did *work* on the moving blocks, converting the remaining kinetic energy into heat.
- Due to the *law of conservation of energy*, you know that the *work* done by the force of *friction* was equal to the *kinetic energy* of the blocks at the instant after the collision.
- Since the motion is in one direction, use a plus sign to symbolize direction.
- Use the subscripts "sb" for the small block, "lb" for the large block, and "cb" for connected blocks.

Identify the Goal

The coefficient of friction, μ, between the wooden blocks and the floor

Identify the Variables and Constants

Known		Implied	Unknown		
$m_{sb} = 0.500\text{ kg}$	$\vec{v}_{sb} = 3.15\ \frac{\text{m}}{\text{s}}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	μ	W	$\vec{F}_N$
$m_{lb} = 3.50\text{ kg}$	$\Delta\vec{d} = 2.63\text{ cm}$	$\vec{v}_{lb} = 0.00\ \frac{\text{m}}{\text{s}}$	$\vec{F}_f$	E_k	$\vec{v}'_{cb}$

Develop a Strategy

Apply the law of conservation of energy to find the velocity of the connected blocks of wood after the collision.

$$m_{sb}\vec{v}_{sb} + m_{lb}\vec{v}_{lb} = m_{sb}\vec{v}'_{sb} + m_{lb}\vec{v}'_{lb}$$

$$m_{sb}\vec{v}_{sb} + 0 = (m_{sb} + m_{lb})\vec{v}'_{cb}$$

$$\vec{v}'_{cb} = \frac{m_{sb}\vec{v}_{sb}}{m_{sb} + m_{lb}}$$

$$\vec{v}'_{cb} = \frac{(0.500\text{ kg})(3.15\ \frac{\text{m}}{\text{s}})}{0.500\text{ kg} + 3.50\text{ kg}}$$

$$\vec{v}'_{cb} = \frac{1.575\ \cancel{\text{kg}}\frac{\text{m}}{\text{s}}}{4.00\ \cancel{\text{kg}}}$$

$$\vec{v}'_{cb} = 0.393\ 75\ \frac{\text{m}}{\text{s}}\text{[to the right]}$$

Due to the law of conservation of energy, the work done on the blocks by the force of friction is equal to the kinetic energy of the connected blocks after the collision.

$$W_{\text{(to stop blocks)}} = E_{k\text{ (after collision)}}$$

Substitute the expressions for work and kinetic energy into the equations.

$$F_{||}\Delta d = \frac{1}{2}mv^2$$

Friction is the force doing the work, and it is always parallel to the direction of motion. Substitute the formula for the force of friction.

$$F_f\Delta d = \frac{1}{2}mv^2$$

$$\mu F_N\Delta d = \frac{1}{2}mv^2$$

Since the blocks are moving horizontally, the normal force is the weight of the blocks. Substitute the weight into the expression and solve for the coefficient of friction.

$$\mu mg\Delta d = \frac{1}{2}mv^2$$

$$\mu = \frac{\frac{1}{2}\cancel{m}v^2}{\cancel{m}g\Delta d}$$

$$\mu = \frac{v^2}{2g\Delta d}$$

$$\mu = \frac{(0.393\ 75\ \frac{\text{m}}{\text{s}})^2}{2(9.81\ \frac{\text{m}}{\text{s}^2})(2.63\ \cancel{\text{cm}})(\frac{1\ \text{m}}{100\ \cancel{\text{cm}}})}$$

$$\mu = \frac{0.15\ 504\ \cancel{\frac{\text{m}^2}{\text{s}^2}}}{0.5160\ \cancel{\frac{\text{m}^2}{\text{s}^2}}}$$

$$\mu = 0.300\ 46$$

$$\mu \cong 0.300$$

The coefficient of friction between the blocks and the floor is 0.300.

Validate the Solution

All of the units cancel, which is correct because the coefficient of friction is unitless. The value of 0.300 is quite reasonable for a coefficient of friction between wood and another similar surface.

PRACTICE PROBLEMS

18. A 12.5 g bullet is shot into a ballistic pendulum that has a mass of 2.37 kg. The pendulum rises a distance of 9.55 cm above its resting position. What was the speed of the bullet?

19. A student flings a 23 g ball of putty at a 225 g cart sitting on a slanted air track that is 1.5 m long. The track is slanted at an angle of 25° with the horizontal. If the putty is travelling at 4.2 m/s when it hits the cart, will the cart reach the end of the track before it stops and slides back down? Support your answer with calculations.

20. A car with a mass of 1875 kg is travelling along a country road when the driver sees a deer dart out onto the road. The driver slams on the brakes and manages to stop before hitting the deer. The driver of a second car (mass of 2135 kg) is driving too close and does not see the deer. When the driver realizes that the car ahead is stopping, he hits the brakes but is unable to stop. The cars lock together and skid another 4.58 m. All of the motion is along a straight line. If the coefficient of friction between the dry concrete and rubber tires is 0.750, what was the speed of the second car when it hit the stopped car?

21. You and some classmates read that the record for the speed of a pitched baseball is 46.0 m/s. You wanted to know how fast your school's star baseball pitcher could throw. Not having a radar gun, you used the concepts you learned in physics class. You made a pendulum with a rope and a small box lined with a thick layer of soft clay, so that the baseball would stick to the inside of the box. You drew a large protractor on a piece of paper and placed it at the top, so that one student could read the maximum angle of the rope when the pendulum swung up. The rope was 0.955 m long, the box with clay had a mass of 5.64 kg, and the baseball had a mass of 0.350 kg. Your star pitcher pitched a fastball into the box and the student reading

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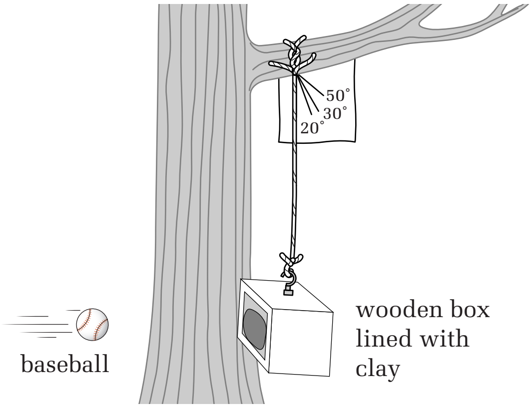
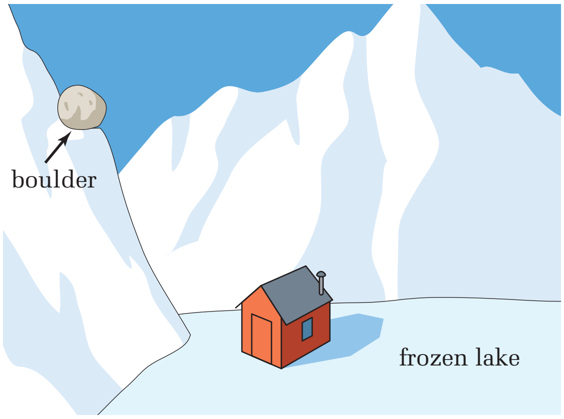
the angle recorded a value of 20.0° from the resting, vertical position. How fast did your star pitcher pitch the ball?



22. A 55.6 kg boulder sat on the side of a mountain beside a lake. The boulder was 14.6 m above the surface of the lake. One winter night, the boulder rolled down the mountain, directly into a 204 kg ice-fishing shack that was sitting on the frozen lake. What was the velocity of the boulder and shack at the instant that they began to slide across the ice? If the coefficient of friction between the shack and the rough ice was 0.392, how far did the shack and boulder slide?



4.3 Section Review

1. K/U What is the difference between an elastic collision and an inelastic collision?
2. C Describe an example of an elastic collision and an example of an inelastic collision that were not discussed in the text.
3. C Given a set of data for a collision, describe a step-by-step procedure that you could use to determine whether the collision was elastic.
4. I The results of the head-on collision in which the moving mass was much larger than the stationary mass ($m_1 >>> m_2$) showed that (a) that the velocity of mass 1 after the collision was almost the same as it had been before the collision and (b) that mass 2, which was stationary before the collision, attained a velocity nearly double that of mass 1 after the collision. Explain how it is possible for kinetic energy ($\frac{1}{2}mv^2$) to be conserved in such a collision, when there was a negligible change in the velocity of mass 1 and a large increase in the velocity of mass 2.
5. MC Imagine that you have a very powerful water pistol. Describe in detail an experiment that you could perform, including the measurements that you would make, to determine the velocity of the water as it leaves the pistol.

CAREERS IN PHYSICS

The Invisible Universe

Go outside on a cloudless night and look up. You might see the Moon, a few planets, and many stars. The universe stretches before you, but your eyes are not taking in the full picture. Astronomer Dr. Samar Safi-Harb and her colleagues see a very different universe by using instruments that detect X rays and several other wavelengths of electromagnetic radiation that are invisible to the human eye.

Dr. Safi-Harb is an assistant professor with the Department of Physics and Astronomy at the University of Manitoba. She uses the instruments aboard satellites such as NASA's Chandra X-ray Observatory to research the death throes of super-massive stars. When a super-massive star runs out of nuclear energy, it collapses under its own weight and its outer layers burst into space in a violent explosion called a "supernova." In some cases, the mass left behind compacts into a neutron star. This astounding type of star is so dense that all of its matter fits into a volume no larger than that of a city. A neutron star, along with its strong magnetic field, spins incredibly fast — up to several dozen times per second!

Dr. Samar Safi-Harb

A neutron star is a remarkable source of electromagnetic radiation. As its magnetic field spins through space, it creates an electric field that generates powerful beams of electromagnetic waves, ranging from radio waves to gamma rays. If the beams sweeps past Earth, astronomers detect them as pulses, like a lighthouse beacon flashing past. Such neutron stars are called "pulsars."

The Crab Nebula is one of Dr. Safi-Harb's favourite objects in the sky. It is the remains of a star that went supernova in 1054 A.D. The Crab Nebula is energized by fast-moving particles emitted from its central pulsar. "It looks different at different wavelengths," she explains. "The radio image reveals a nebula a few light-years across that harbours low-energy electrons. The diffuse optical nebula shines by intermediate energy particles, showing a web of filaments that trace the debris of the explosion. The X-ray image reveals a smaller nebula — the central powerhouse — containing very energetic particles. Its jets, rings, and wisp-like structures unveil the way pulsars dump energy into their surroundings."

In part, the ground-breaking work of Jocelyn Bell, the discoverer of pulsars, inspired Dr. Safi-Harb to follow this line of research. Although an astronomer, she has a doctorate in physics from the University of Wisconsin, Madison. Few universities today have astronomy programs that stand alone from physics.

Going Further

1. Earth is orbited by a wide array of satellites that explore the sky at high-energy and low-energy wavelengths. Research two or more of these satellites and describe how images taken by them enhance our understanding of the universe.

2. When two objects in space approach or recede from one another at great speed, light emitted from either object appears altered by the time it reaches the other object. Research and describe what astronomers mean when they talk about a "red shift" or a "blue shift" in light.

WEB LINK

www.mcgrawhill.ca/links/physics12

Radio, infrared, optical, X-ray, and gamma-ray images of our galaxy, the Milky Way, can be found on the Internet. You can also learn more about Dr. Safi-Harb's work and see images of several of her favourite objects in space by going to the above Internet site and clicking on **Web Links**.

CHAPTER 4 Review

REFLECTING ON CHAPTER 4

- Newton expressed his second law in terms of momentum: $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$.
- Rearrangement of Newton's form of the second law yields the quantities of impulse, $\vec{F}\Delta t$, and a change in momentum, $\Delta \vec{p}$, and shows that impulse is equal to the change in momentum: $\vec{F}\Delta t = \Delta \vec{p}$. This expression is called the "impulse-momentum theorem."
- Momentum is mass times velocity $\vec{p} = m\vec{v}$.
- The concept of impulse plays a significant role in the design of safety systems. By extending the time, Δt, of a collision, you can reduce the amount of force, $\vec{F}$, exerted.
- By applying Newton's third law, you can show that momentum is conserved in a collision.
- The momentum of an isolated system is conserved.
- Recoil is the interaction of two objects that are in contact with each other and exert a force on each other. Momentum is conserved during recoil.
- Kinetic energy is conserved in elastic collisions.
- Kinetic energy is *not* conserved in inelastic collisions.

Knowledge/Understanding

1. Write Newton's second law in terms of momentum. Show how this expression of Newton's law leads to the definition of impulse and to the impulse-momentum theorem.
2. Give an example of a situation in which it is easier to measure data that you can use to calculate a change in momentum than it is to determine forces and time intervals.
3. In many professional auto races, stacks of old tires are placed in front of walls that are close to turns in the racetrack. Explain in detail, using the concept of impulse, why the tires are stacked there.
4. Define "internal force" and "external force" and explain how these terms relate to the law of conservation of momentum.
5. At the instant after a car crash, the force of friction acts on the cars, causing a change in their momentum. Explain how you can apply the law of conservation of momentum to car crashes.
6. When two objects recoil, they start at rest and then push against each other and begin to move. Initially, since their velocities are zero, they have no momentum. When they begin to move, they have momentum. Explain how momentum can be conserved during recoil, when the objects start with no momentum and then acquire momentum.
7. An object that is moving in a circle has angular momentum. Explain why the amount of angular momentum depends on the object's distance from the centre of rotation.
8. Assume that you are provided with data for a collision between two masses that undergo a collision. Describe, step by step, how you would determine whether the collision was elastic or inelastic.
9. Under what conditions is a collision *completely inelastic*?
10. The equations $v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1$ and $v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$ were developed for perfectly elastic head-on collisions between two masses, when mass 1 was moving and mass 2 was stationary before the collision. Use these equations to show that, after the collision, the first mass will come to a complete stop and the second mass will move away with a velocity identical to the original velocity of the first mass.

11. Explain how a forensic expert can determine the velocity of a bullet by using a ballistic pendulum.

Inquiry

12. The International Tennis Federation (ITF) approaches you, a physics student, complaining that too many games in tournaments are being won on the strength of either player's serve. They ask you to examine ways to slow down the serve and thus make the game more interesting to watch. Devise a series of experiments to test the ball, the type of racquet used, and the surface of the courts, from which you could make recommendations to the ITF about how to improve the game of tennis.
13. Design a small wooden cart, with several raw eggs as passengers. Incorporate elements into your design to ensure that the passengers suffer no injury if the cart was involved in a collision while travelling at 5.0 m/s. If possible, test your design.
14. Design and carry out an experiment in which an object initially had gravitational potential energy that is then converted into kinetic energy. The object then collides with another object that is stationary. Include in your design a method for testing whether mechanical energy is conserved in the first part of the experiment. If possible, test your design.

Communication

15. A car and a bicycle are travelling with the same velocity. Which vehicle has greater momentum? Explain your reasoning.
16. It is a calm day on a lake and you and a friend are on a sailboat. Your friend suggests attaching a fan to the sailboat and blowing air into the sails to propel the sailboat ahead. Explain whether this would work.
17. Imagine you are standing, at rest, in the middle of a pond on *perfectly frictionless* ice. Explain what will happen when you try to walk back to shore. Describe a possible method that you could use to start moving. Would this method allow you to reach shore? Explain.
18. You and a friend arrive at the scene of a car crash. The cars were both severely mangled. Your friend is appalled at the damage to the cars and says that cars ought to be made to be sturdier. Explain to your friend why this reaction to the crash is unwarranted.
19. Start with expressions that apply the impulse-momentum theorem to two objects and use Newton's third law to derive the conservation of momentum for a collision between the two objects. Explain and justify every substitution and mathematical step in detail.
20. Write the units for impulse and for momentum. Show that these combinations of units are equivalent and explain your reasoning in detail.
21. Movie stunt people can fall from great heights and land safely on giant air bags. Using the principles of conservation of momentum and impulse, explain how this is possible.
22. Why is a "follow-through" important in sports in, for example, hitting, kicking, or throwing a ball?
23. A boy jumps from a boat onto a dock. Explain why he would have to jump with more energy than he would need if he was to jump the same distance from one dock to another.
24. In soccer, goalkeepers need to jump slightly forward to avoid being knocked into the goal by a fast-moving soccer ball as they jump up to catch it. If the ball and goalkeeper are momentarily at rest after the catch, what must have been the relative momenta of the goalkeeper and ball just before the catch?

Making Connections

25. Many automobiles are now equipped with air bags that are designed to prevent injuries to passengers if the vehicle is involved in a collision. Research the properties of air bags. In terms of impulse, how do they work? How quickly do they inflate? Do they remain inflated? What force do they exert on the driver or passenger? What are the safety concerns of using air bags?

26. A patient lies flat on a table that is supported by air bearings so that it is, effectively, a frictionless platform. As the patient lies there, the table moves slightly, due to the pulsating motion of the blood from her heart. By examining the subtle motion of the table, a ballistocardiograph is obtained, which is used to diagnose certain potential deficiencies of the patient's heart. Research the details of how this device works and the information that can be obtained from the data.

27. Particle physicists investigate the properties of elementary particles by examining the tracks these particles make during collisions in "bubble chambers." Examine some bubble chamber photographs (check the Internet) and research the information that can be obtained. Include a discussion of how the law of conservation of linear momentum is applied.

Problems for Understanding

28. Determine the momentum of a 5.0 kg bowling ball rolling with a velocity of 3.5 m/s[N] toward a set of bowling pins.

29. What is the mass of a car that is travelling with a velocity of 28 m/s[W] and a momentum of 4.2×10^4 kg · m/s[W]?

30. The momentum of a 55.0 kg in-line skater is 66.0 kg m/s[S]. What is his velocity?

31. How fast would a 5.0×10^{-3} kg golf ball have to travel to have the same momentum as a 5.0 kg bowling ball that is rolling at 6.0 m/s[forward]?

32. Calculate the impulse for the following interactions.
   (a) A person knocks at the door with an average force of 9.1 N[E] over a time interval of 2.5×10^{-3} s.
   (b) A wooden mallet strikes a large iron gong with an average force of 4.2 N[S] over a time interval of 8.6×10^{-3} s.

33. A volleyball player spikes the ball with an impulse of 8.8 kg · m/s over a duration of 2.3×10^{-3} s. What was the average applied force?

34. If a tennis racquet exerts an average force of 55 N and an impulse of 2.0 N · s on a tennis ball, what is the duration of the contact?

35. (a) What is the impulse of a 0.300 kg hockey puck slapshot that strikes the goal post at a velocity of 44 m/s[N] and rebounds straight back with a velocity of 9.2 m/s[S]?
   (b) If the average force of the interaction was -2.5×10^3 N, what was the duration of the interaction?

36. A 2.5 kg curling stone is moving down the ice at 3.5 m/s[W]. What force would be needed to stop the stone in a time of 3.5×10^{-4} s?

37. At an automobile test facility, a car with a 75.0 kg crash-test dummy is travelling 28 m/s[forward] when it hits a wall. Calculate the force that the seat belt exerts on the dummy on impact. Assume that the car and dummy travel about 1.0 m as the car comes to rest and that the acceleration is constant during the crash.

38. A 0.0120 kg bullet is fired horizontally into a stationary 5.00 kg block of wood and becomes embedded in the wood. After the impact, the block and bullet begin to move with an initial velocity of 0.320 m/s[E]. What was the velocity of the bullet just before it hit the wood?

39. A 48.0 kg skateboarder kicks his 7.0 kg board ahead with a velocity of 2.6 m/s[E]. If he runs with a velocity of 3.2 m/s[E] and jumps onto the skateboard, what is the velocity of the skateboard and skateboarder immediately after he jumps on the board?

40. Astrid, who has a mass of 37.0 kg, steps off a stationary 8.0 kg toboggan onto the snow. If her forward velocity is 0.50 m/s, what is the recoil velocity of the toboggan? (Assume that the snow is level and the friction is negligible.)

41. A 60.0 t submarine, initially travelling forward at 1.5 m/s, fires a 5.0×10^2 kg torpedo straight ahead with a velocity of 21 m/s in relation to the submarine. What is the velocity of the submarine immediately after it fires the torpedo?

42. Suppose that a 75.0 kg goalkeeper catches a 0.40 kg ball that is moving at 32 m/s. With what forward velocity must the goalkeeper jump when she catches the ball so that the goalkeeper and the ball have a horizontal velocity of zero?

43. In billiards, the 0.165 kg cue ball is hit toward the 0.155 kg eight ball, which is stationary. The cue ball travels at 6.2 m/s forward and, after impact, rolls away at an angle of 40.0˚ counterclockwise from its initial direction, with a velocity of 3.7 m/s. What are the velocity and direction of the eight ball?

44. Consider a nuclear reaction in which a neutron travelling 1.0×10^7 m/s in the $+x$ direction collides with a proton travelling 5.0×10^6 m/s in the $+y$ direction. They combine at impact to form a new particle called a "deuteron." What is the magnitude and direction of the deuteron velocity? Assume for simplicity that the proton and neutron have the same mass.

45. A ball of mass m_1 strikes a stationary ball of mass m_2 in a head-on, elastic collision.

(a) Show that the final velocities of the two balls have the form

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \qquad v_2' = \frac{2m_1}{m_1 + m_2} v_1$$

(b) Examine three cases for the masses

$m_1 <<< m_2 \qquad m_1 \cong m_2 \qquad m_1 >>> m_2$

(c) Comment on the results.

46. In a demonstration, two identical 0.0520 kg golf balls collide head on. If the initial velocity of one ball is 1.25 m/s[N] and the other is 0.860 m/s[S], what is the final velocity of each ball?

47. A 750 g red ball travelling 0.30 m/s[E] approaches a 550 g blue ball travelling 0.50 m/s[W]. They suffer a glancing collision. The red ball moves away at 0.15 m/s[E30.0˚S] and the blue ball moves away in a north-westerly direction.

(a) What is the final velocity of the blue ball?

(b) What percentage of the total kinetic energy is lost in the collision?

48. You and a colleague are on a spacewalk, repairing your spacecraft that has stalled in deep space. Your 60.0 kg colleague, initially at rest, asks you to throw her a hammer, which has a mass of 3.0 kg. You throw it to her with a velocity of 4.5 m/s[forward].

(a) What is her velocity after catching the hammer?

(b) What impulse does the hammer exert on her?

(c) What percentage of kinetic energy is lost in the collision?

CHAPTER 5 Conservation of Energy

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Concept of work
- Kinetic energy
- Potential energy
- Thermal energy and heat

With a "whump," the fireworks shell is lofted upward into the darkness. As the shell rises, it slows; kinetic energy transforms into gravitational potential energy. Then, the shell explodes and chemical potential energy rapidly converts into heat, light, and sound. The darkness gives way to brilliant colour and loud bangs startle the crowd below.

There is a balance in all of these transformations and effects. Energy gained in one form comes at the expense of another. This is the law of conservation of energy.

In this chapter, you will examine these energy transformations and balances and investigate a few of their applications.

MULTI LAB

Energy Transformations in Springs

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

A Spring Pendulum

Hang a spring (at least 50 cm long) from a rigid support and attach a mass to the lower end of the spring. Ensure that the mass is heavy enough to extend the spring noticeably without overstretching it. The hanging mass should not come close to the desktop. Pull the mass to the side and release it, allowing the spring to swing from side to side. Observe the motion of the mass and the spring.

Analyze and Conclude

1. What types of periodic (repeating) motion did you observe?
2. When the amplitude of one type of vibration was at a maximum, what happened to the amplitude of the other type of vibration?
3. When the spring was stretched to a greater length than it was when the mass was at rest, was the mass moving rapidly or slowly?
4. What was being transferred between **(a)** the different types of vibration, and **(b)** the mass and the spring?
5. The law of conservation of energy states that the total energy of an isolated system remains constant. How was that law illustrated by the action of the spring and the mass in this case?
6. What eventually happened to the motion of the spring and the mass? Suggest why this occurred.
7. If the system regularly switched back and forth between the two patterns of motion, was the time taken for the change consistent?

Slinky Motion

Place a Slinky™ toy several steps up from the bottom of a set of stairs, with the axis of the spring vertical. Take the top coil and arc it over and down to touch the next lower step. Release the Slinky™ and observe its motion.

Analyze and Conclude

1. What is the condition of the coils of the spring when energy is stored in the spring?
2. At what stage or stages in the action of the spring is kinetic energy being converted into elastic potential energy in the spring?
3. At what stage or stages in the action of the spring is elastic potential energy in the spring being converted into kinetic energy?
4. Is there any instant during the motion of the spring when both the kinetic energy and the elastic potential energy are at a maximum? Would you expect this to be possible? Give a reason for your answer.
5. Any system loses energy due to friction, which converts mechanical energy into thermal energy. Why then does the spring continue going down the stairs? From where is it getting its energy?

5.1 Work and the Transformation of Energy

SECTION EXPECTATIONS

- Define and describe concepts and units related to energy forms and conservation.
- Analyze and explain common situations using the work-energy theorem.
- Investigate the law of conservation of energy experimentally.

KEY TERMS

- energy
- work
- work-kinetic energy theorem
- work-energy theorem
- conservation of mechanical energy

In the introduction to this chapter, you read about some different types of energy transfers and transformations. You might recall from previous science courses that the two mechanisms by which energy is transferred from one system to another are work and heat. In fact, **energy** is often defined as the ability to do work. In this section, you will focus on work, extending your knowledge and your ability to make predictions about work and solve problems involving work as the transfer of mechanical energy from one system to another.

Characterizing Work

What is work? How do you know if one object or system is doing work on another? If work is being done, how much work is done? In physics, these questions are easier to answer than in everyday life. If an object or system, such as your body, exerts a force on an object and that force causes the object's position to change, you are doing **work** on the object.

The most direct way to express work mathematically is with the equation $W = F\Delta d \cos\theta$, where F is the magnitude of the force doing work on an object, Δd is the magnitude of the displacement caused by the force, and θ is the angle between the vectors for force and displacement. Notice that the force, F, and displacement, Δd, do not have vector notations. The reason for the omission of the vector symbols is that work, W, is a scalar quantity and is the scalar product of the vectors $\vec{F}$ and $\Delta\vec{d}$. Since the product of the vectors is a scalar quantity, the directions of the force and displacement do not determine a final direction of their product. To understand why $\cos\theta$ is included in the equation, study Figure 5.1.

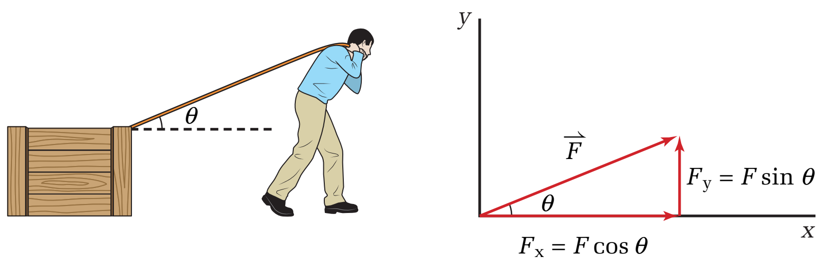


Figure 5.1 The only component of the force acting on an object that does work is the component that is parallel to the direction of the displacement.

In Figure 5.1, you see a person pulling a crate with a force that is at an angle, θ, relative to the direction of the motion. Only part of that force is actually doing work on the crate. In the diagram beside the sketch, you can see that the *x*-component (horizontal) of the force has a magnitude $F\cos\theta$. This component is in the direction of the motion and is the only component that is doing work. The *y*-component (vertical) is perpendicular to the direction of the motion and does no work on the crate.

Figure 5.2 shows four special cases that will clarify the question of whether work is being done by a force. In part (A), a person is pushing a cart with a force ($\vec{F}$) that is in the same direction as the motion of the cart. The angle between the force and the displacement is zero, so $\cos\theta = \cos 0 = 1$ and the work is $W = F\Delta d$. When the force and the displacement are in the same direction, the entire force is doing work. In this case, the cart is speeding up, so its kinetic energy is increasing. The work that the person is doing on the cart is transferring energy to the cart, so positive work is being done on the cart.

In part (B) of Figure 5.2, the cart has kinetic energy and is moving forward. The person is pulling on the cart to slow it down. Notice that the direction of the force that the person is exerting on the cart is opposite to the direction of the motion. The angle θ is 180°, so $\cos\theta = -1$ and, therefore, $W = -F\Delta d$. Just as the results indicate, the person is doing negative work on the cart by slowing it down and reducing its kinetic energy.

MATH LINK

In mathematics, a scalar product is also called a "dot product" and the equation for work is written as $W = \vec{F} \cdot \Delta\vec{d}$ The magnitude of a dot product is always the product of the magnitudes of the two vectors and the cosine of the angle between them.

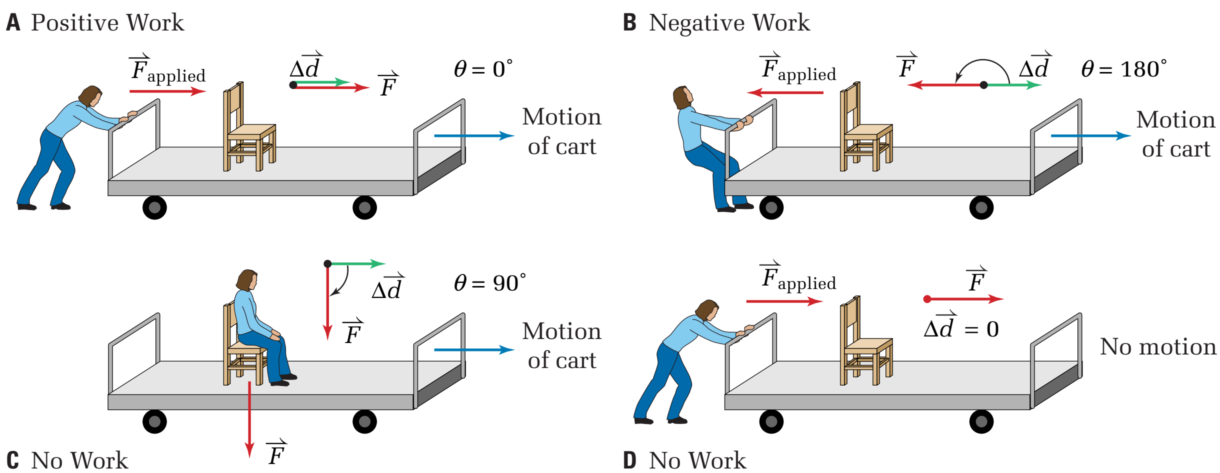


Figure 5.2 In order to do work, the force must be acting in a direction parallel to the displacement.

In part (C) of Figure 5.2, the person is sitting on the cart, exerting a downward force on it. The angle θ is 90°, so $\cos\theta = \cos 90° = 0$ and the work is $W = F\Delta d(0) = 0$. Even though the cart is moving, the force that the person is exerting is not doing work, because it is not directly affecting the horizontal motion of the cart. Notice that if you use the equation for work properly, the term $\cos\theta$ will tell you whether the work is positive, negative, or zero.

Finally, in part (D), the person is pushing on the cart, but the cart is stuck and will not move. Even though the person is exerting a force on the cart, the person is not doing work on the cart, because the displacement is zero.

DEFINING WORK

Work is the product of the force, the displacement, and the cosine of the angle between the force and displacement vectors.

$$W = F\Delta d \cos\theta$$

Quantity	Symbol	SI unit
work	W	J (joules)
force	F	N (newtons)
displacement	Δd	m (metres)
angle between force and displacement	θ	degrees (The cosine of an angle is a number and has no units.)

Unit Analysis

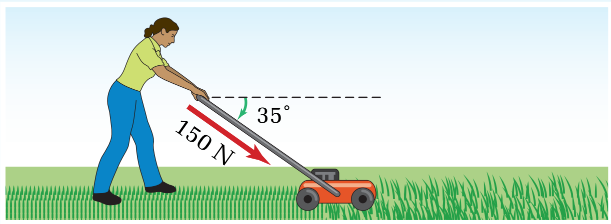
(joule) = (newton)(metre) $\quad$ $J = N \cdot m$

A newton · metre is equivalent to a joule.

SAMPLE PROBLEM

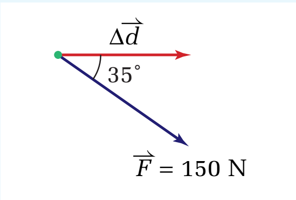
Working on the Lawn

A woman pushes a lawnmower with a force of 150 N at an angle of 35° down from the horizontal. The lawn is 10.0 m wide and requires 15 complete trips across and back. How much work does she do?



Conceptualize the Problem

- Draw a sketch to show the relationship between the force and the motion.
- A *force* is acting at an *angle* to the *direction of motion*.
- Since a *component* of the force is in the *direction of the motion*, the force is *doing work* on the lawnmower.
- *Work* done can be determined from the general work equation.



Identify the Goal

The work, W, done by the woman on the lawnmower

Identify the Variables and Constants

Known	Unknown
$\vec{F} = 150$ N	Δd
$\theta = 35^\circ$	W
width of lawn = 10.0 m	
15 trips	

Develop a Strategy

Determine the total distance over which the force acted.

$$\Delta d = 2(10.0 \text{ m})(15)$$
$$\Delta d = 300 \text{ m}$$

Use the general work equation.

$$W = F\Delta d \cos\theta$$

Substitute and solve.

$$W = (150 \text{ N})(300 \text{ m}) \cos 35^\circ$$
$$W = 36\,861.8 \text{ J}$$
$$W \cong 3.7 \times 10^4 \text{ J}$$

The work done by the woman on the lawnmower is 3.7×10^4 J.

Validate the Solution

If the force had been horizontal, then the work done would have been $300 \text{ m} \times 150 \text{ N}$, which equals 45 000 J. Because the force is at an angle to the direction of motion, the work done is less than this value.

PRACTICE PROBLEMS

1. A man pulls with a force of 100.0 N at an angle of 25° up from the horizontal on a sled that is moving horizontally. If the sled moves a distance of 200.0 m, how much work does the man do on the sled?
2. A tow truck does 42.0 MJ of work on a car while pulling it with a force of 3.50 kN exerted upward at 10.0° to the horizontal. If the car moves horizontally, how far was it towed?
3. A kite moves 14.0 m horizontally while pulled by a string. If the string did 60.0 J of work on the kite while exerting a force of 8.2 N, what angle did the string make with the vertical?

Work and Kinetic Energy

In the examples you just examined, you determined the amount of work that was done on several objects. Now, consider the form of the energy that is given to an object on which work is done and the relationship to the work that was done. First, look at a situation in which all of the work done on a cart transfers only kinetic energy to the cart. Imagine that a cart is rolling horizontally to the right with a speed of v_i when a force is exerted on it in the direction of motion, as shown in Figure 5.3. The force acts over a displacement of $\Delta\vec{d}$. Since all of the motion is horizontal, there will be no changes in gravitational potential energy. Assume also that friction is negligible. Study Figure 5.3 and then examine the steps that follow the illustration.

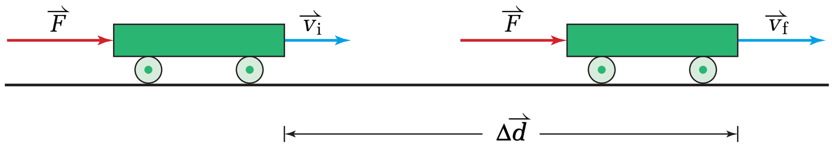


Figure 5.3 All of the work being done on the cart is increasing the cart's kinetic energy.

- Write the equation for work. From Figure 5.3, you can see that the angle between the force vector and the displacement vector is zero. Use this information to simplify the equation.

$$W = F\Delta d \cos\theta$$
$$W = F\Delta d \cos 0^\circ$$
$$\cos 0^\circ = 1$$
$$W = F\Delta d$$

- Recall the expression for force from Newton's second law. Substitute the expression for F into the equation for work. Since work is a scalar quantity, do not use vector symbols.

$$F = ma$$
$$W = ma\Delta d$$

- Write the kinematic equation that relates initial velocity, final velocity, displacement, and acceleration. Solve that equation for displacement.

$$v_f^2 = v_i^2 + 2a\Delta d$$
$$\Delta d = \frac{v_f^2 - v_i^2}{2a}$$

- Substitute the expression for displacement into the equation for work. Simplify the expression.

$$W = ma\left(\frac{v_f^2 - v_i^2}{2a}\right)$$
$$W = \frac{m(v_f^2 - v_i^2)}{2}$$

- Expand the equation.

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

- Recognize the expression $\frac{1}{2}mv^2$ as kinetic energy.

$$W = E_{kf} - E_{ki}$$
$$W = \Delta E_k$$

The work done on the cart is equal to the change in the kinetic energy of the cart. You can now generalize and state that, when work is done on an object in which the force and displacement are horizontal and friction is negligible, the work done is equal to the change in the kinetic energy of the object. The expression $W = \Delta E_k$ is often called the **work-kinetic energy theorem**.

SAMPLE PROBLEM

The Hammer and Nail

You drive a nail horizontally into a wall, using a 0.448 kg hammerhead. If the hammerhead is moving horizontally at 5.5 m/s and in one blow drives the nail into the wall a distance of 3.4 cm, determine the average force acting on

(a) the hammerhead

(b) the nail

Conceptualize the Problem

- The hammer possesses *kinetic energy*.
- The *backward force* exerted by the nail on the hammer *removes* all of the *kinetic energy*.
- The *magnitude of the force* exerted by the hammer on the nail equals the *magnitude of the force* exerted by the nail on the hammer, according to Newton's third law of motion.

Identify the Goal

(a) The force acting on the hammer, $\vec{F}_h$, by the nail

(b) The force applied to the nail, $\vec{F}_n$, by the hammer

Identify the Variables and Constants

Known	Implied	Unknown
$m = 0.448$ kg	$\vec{v}_f = 0\ \frac{m}{s}$	$\vec{F}_h$
$\vec{v}_i = 5.5\ \frac{m}{s}$[forward]		$\vec{F}_n$
$\Delta\vec{d} = 0.034$ m[forward]		

PROBLEM TIP

When solving problems involving work and energy, be sure to express all quantities in SI units. For example, a speed of 80 km/h must be converted into 22.2 m/s, and a mass of 25 g must be expressed as 0.025 kg.

Develop a Strategy

With only horizontal motion, work done equals the change in kinetic energy.

$$|\vec{F}_h||\Delta\vec{d}| = \Delta E_k$$

$$|\vec{F}_h||\Delta\vec{d}| = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$|\vec{F}_h| = \frac{\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2}{\Delta d}$$

$$|\vec{F}_h| = \frac{\tfrac{1}{2}(0.448\text{ kg})\left(0\ \tfrac{m}{s}\right)^2 - \tfrac{1}{2}(0.448\text{ kg})\left(5.5\ \tfrac{m}{s}\right)^2}{0.034\text{ m}}$$

$$\vec{F}_h = -199.2941\text{ N}$$

$$\vec{F}_h \cong -2.0 \times 10^2\text{ N}$$

(a) The average force exerted on the hammer by the nail was 2.0×10^2 N[backward].

Apply Newton's third law to the forces between the hammer and the nail.

$$\vec{F}_n = -\vec{F}_h$$

$$\vec{F}_n \cong -(-2.0 \times 10^2\text{ N})$$

$$\vec{F}_n \cong 2.0 \times 10^2\text{ N}$$

(b) The force exerted on the nail by the hammer was 2.0×10^2 N[forward].

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Validate the Solution

Since kinetic energy must be transferred out of the hammerhead, the force on the hammer must be in the opposite direction to its motion and so the force must be negative.

PRACTICE PROBLEMS

4. A car with a mass of 2.00×10^3 kg is travelling at 22.2 m/s (80 km/h) when the driver applies the brakes. If the force of static friction between the tires and the road is 8.00×10^3 N[backward], determine the stopping distance of the car. Use the concepts of work and energy in solving this problem.

5. A 1.00×10^2 g arrow is fired horizontally from a bow. If the average applied force on the arrow is 150.0 N and it acts over a displacement of 40.0 cm, with what speed will the arrow leave the bow? Use the concepts of work and energy in solving this problem.

6. A 12.0 kg sled is sliding at 8.0 m/s over ice when it encounters a patch of snow. If it comes to rest in 1.5 m, determine the magnitude of the average force acting on the sled.

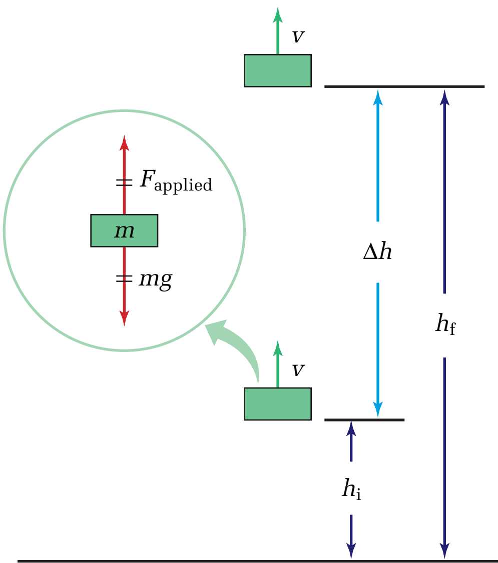

Figure 5.4 The upward force that you apply to the mass is equal in magnitude and opposite in direction to the force of gravity. Therefore, the velocity of the mass is constant and only its height changes.

Work and Gravitational Potential Energy

Consider, now, a contrasting situation — the motion of the object on which work is being done and the force that does the work are vertical and there is no change in the object's velocity. For example, imagine that you are lifting a mass at constant speed, so there is no change in its kinetic energy. The only energy that the mass will gain will be due to its position in the gravitational field — gravitational potential energy (E_g). The relationship between the quantities is shown in Figure 5.4.

Because neither the speed nor the direction of the mass is changing, it is not accelerating. If the acceleration of the mass is zero, then the net force must be zero and therefore the magnitude of upward applied force must equal the magnitude of the downward force of gravity.

$$F_{applied} = F$$
$$F_{applied} = mg$$

Examine the following steps to derive the relationship between work done by a vertical force and gravitational potential energy.

- Write the equation for work. From Figure 5.4, you can see that the angle between the force vector and the displacement vector is zero. Use this information to simplify the equation.

$$W = F\Delta d \cos\theta$$
$$W = F\Delta d \cos 0^\circ$$
$$\cos 0^\circ = 1$$
$$W = F\Delta d$$

- To find the amount of work done by the applied force, substitute the applied force into the equation.

$$W = F\Delta d$$
$$W = F_{\text{applied}}\Delta d$$
$$F_{\text{applied}} = mg$$
$$W = mg\Delta d$$

- From Figure 5.4, you can see that the displacement is the difference of the initial and final heights. Substitute this value into the equation for work.

$$\Delta d = h_{\text{f}} - h_{\text{i}}$$
$$W = mg(h_{\text{f}} - h_{\text{i}})$$

- Expand the equation.

$$W = mgh_{\text{f}} - mgh_{\text{i}}$$

- Recognize the expression *mgh* as gravitational potential energy and substitute E_{g} into the equation for work.

$$W = E_{\text{gf}} - E_{\text{gi}}$$
$$W = \Delta E_{\text{g}}$$

When velocity does not change but the object's position changes in height, the work done on an object is equal to the change in the gravitational potential energy of the object.

SAMPLE PROBLEM

A Rescue at Sea

A gas-powered winch on a rescue helicopter does 4.20×10^3 J of work while lifting a 50.0 kg swimmer at a constant speed up from the ocean. Through what height was the swimmer lifted?

Conceptualize the Problem

- The *speed* was *constant*, so there was *no change* in *kinetic energy*.
- Assume that the *work done* by the winch equals the gain in *gravitational potential energy* of the swimmer.

Identify the Goal

The height, Δh, through which the swimmer was lifted

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Identify the Variables and Constants

Known	Implied	Unknown
$W = 4.20 \times 10^3$ J	$g = 9.81 \frac{m}{s^2}$	Δh
$m = 50.0$ kg		

Develop a Strategy

With no change in kinetic energy, work done equals the change in gravitational potential energy. Substitute and solve.

$$W = \Delta E_g$$
$$W = mg\Delta h$$
$$\Delta h = \frac{W}{mg}$$
$$\Delta h = \frac{4.20 \times 10^3 \text{ J}}{(50.0 \text{ kg})\left(9.81 \frac{m}{s^2}\right)}$$
$$\Delta h = 8.562\,69 \text{ m}$$
$$\Delta h \cong 8.56\ m$$

The height through which the swimmer was lifted was 8.56 m.

Validate the Solution

$1 \text{ J} = \frac{1 \text{ kg} \cdot \text{m}^2}{\text{s}^2}$, so the answer has units of $\frac{\frac{1 \cancel{\text{kg}} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}}{\cancel{\text{kg}} \cdot \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}} = \text{m}$.

PRACTICE PROBLEMS

7. A motorized crane did 40.4 kJ of work when slowly lifting a pile driver to a height of 8.00 m. What was the mass of the pile driver?

8. A 4.00×10^2 kg elevator car rose at a constant speed past several floors. If the motor did 58.8 kJ of work, through what height did the car rise?

9. A battery-powered scoop used by a Mars lander lifted a 54 g rock through a height of 24 cm. If $g_{\text{Mars}} = 3.8$ m/s^2, how much work was done by the scoop?

The Work-Energy Theorem and Conservation of Energy

Very few processes are as limited as the two situations that you have just considered — changes in kinetic energy only or in potential energy only. Real processes usually involve more than one form of energy. However, you can combine the two cases that you just considered. For example, if an applied force does work on an object so that both its kinetic energy and its various forms of potential energy change, then the work done by that force equals the total change in both the kinetic energy and the potential energies: $W = \Delta E_k + \Delta E_p$. The relationship shown in this equation is known as the **work-energy theorem**.

Picture an object or a system of objects on which no work is being done by some outside agency. In other words, no energy is being added to the system and no energy is being removed from the system. This is called an "isolated system." A swinging pendulum could be such a system until someone comes along and gives it a shove. A roller coaster in which the car is running freely up and down slopes and around curves is isolated if wind effects and friction are ignored. The flight of an arrow away from its position in a stretched bow can be treated as an isolated system if air friction effects and wind are again ignored.

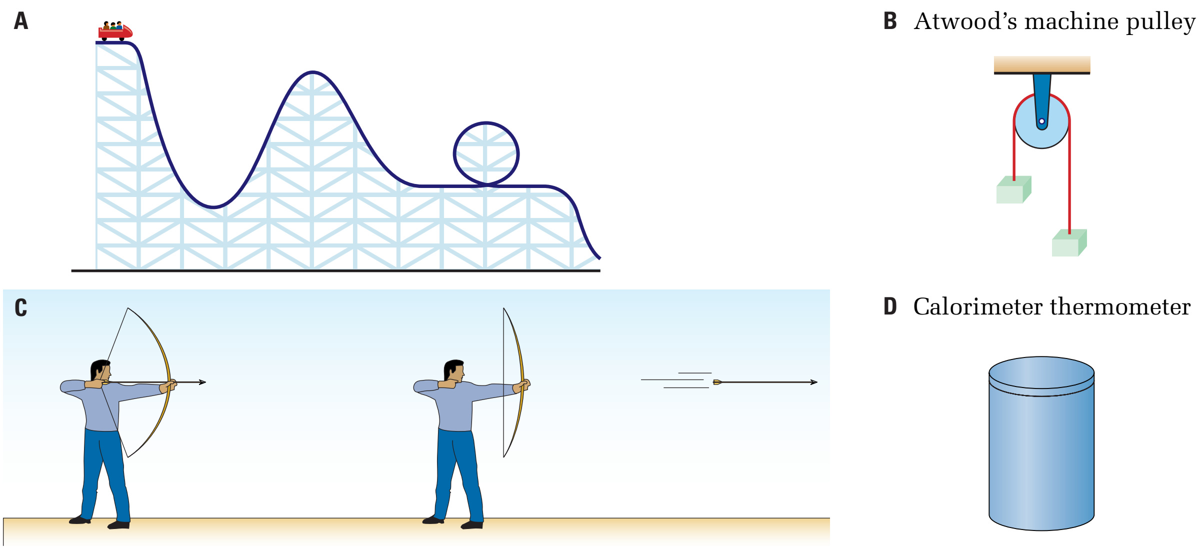


Figure 5.5 Our universe is possibly the only truly isolated system. However, in many applications, such as the ones shown here, you will work on the assumption that no energy enters from the outside and none is lost to the outside.

If a system is isolated in that no outside work is done on it, you can use the work-energy theorem to derive another important relationship, as shown in the following steps.

- Write the work-energy theorem. $W = \Delta E_k + \Delta E_p$
- If the system is isolated, $W = 0$. Substitute zero into the equation above. $0 = \Delta E_k + \Delta E_p$ $\Delta E_k + \Delta E_p = 0$
- Expand the expression and use primes to represent the energies after the process is complete. $(E'_k - E_k) + (E'_p - E_p) = 0$
- Rearrange the equation so that the initial energies are on the left-hand side of the equals sign and all of the final energies are on the right-hand side. $E_k + E_p = E'_k + E'_p$

PHYSICS FILE

The work-energy theorem links two apparently different types of quantities. On the left-hand side is a concept that is extremely concrete in that it deals with readily measured quantities of force and distance. On the right-hand side is the extremely abstract concept of energy. In fact, the right-hand side deals *only* with energy changes and never in absolute amounts of energy.

ELECTRONIC LEARNING PARTNER

Use the interactive pendulum simulation in your Electronic Learning Partner to enhance your understanding of energy transformations.

The last statement in the derivation is known as the law of **conservation of mechanical energy**. The equation $\Delta E_k + \Delta E_p = 0$ says that the change of total mechanical energy in an isolated system is zero. This does not mean, however, that no processes are occurring within the system. This last statement implies that kinetic energy of an object in the system can be transformed into potential energy, or the reverse can happen. In addition, one object in the system might transfer energy to another object in the system. Many processes can occur in an isolated system.

THE LAW OF CONSERVATION OF MECHANICAL ENERGY

The law of conservation of mechanical energy states that the sum of the kinetic and potential energies before a process occurs in an isolated system is equal to the sum of the kinetic and potential energies of the system after the process is complete.

$$E_k + E_p = E'_k + E'_p$$

Quantity	Symbol	SI unit
kinetic energy before the process occurred	E_k	J (joules)
potential energy before the process occurred	E_p	J (joules)
kinetic energy after the process was completed	E'_k	J (joules)
potential energy after the process was completed	E'_p	J (joules)

PROBEWARE

If your school has probeware equipment, visit *www.mcgrawhill.ca/links/physics12* and follow the links for an in-depth activity on energy and momentum in collisions.

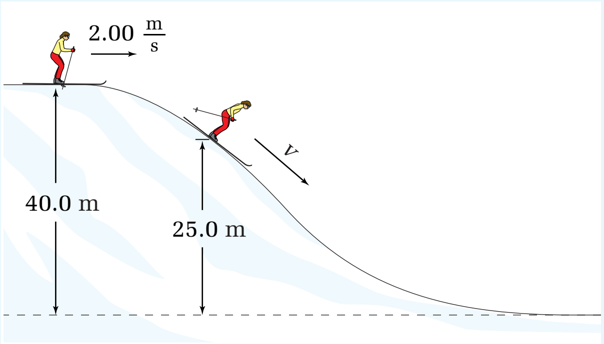
SAMPLE PROBLEM

Conservation of Energy on the Ski Slopes

A skier is gliding along with a speed of 2.00 m/s at the top of a ski hill, 40.0 m high, as shown in the diagram. The skier then begins to slide down the icy (frictionless) hill.

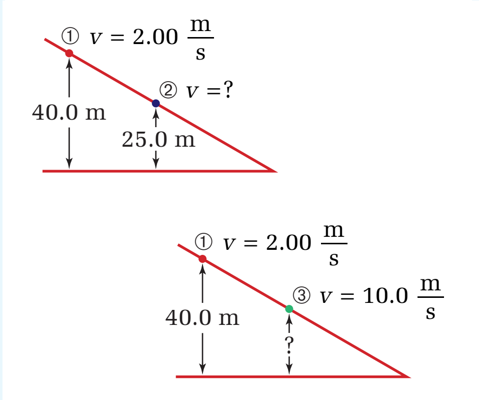
(a) What will be the skier's speed at a height of 25.0 m?

(b) At what height will the skier have a speed of 10.0 m/s?



Conceptualize the Problem

- Sketch the two parts of the problem separately. Label the initial conditions (top of the hill) "1." Label the position when $h = 25$ m as "2." Label the position at which the skier is travelling at 10.0 m/s as "3."
- Use subscripts 1 and 2 to indicate the initial and final conditions in part (a) and use subscripts 1 and 3 to indicate the initial and final conditions in part (b).
- Define the *system* as the *skier and the slope*.
- Assume that the system of skier and slope is *isolated*.
- The law of *conservation of energy* can be applied.



Identify the Goal

(a) the speed, v_2, at a height of 25.0 m

(b) the height, h_3, at which the skier's speed will be 10.0 m/s

Identify the Variables and Constants

Known	Implied	Unknown
$v_1 = 2.00\ \frac{\text{m}}{\text{s}}$	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	v_2
$h_1 = 40.0$ m		h_3
$v_3 = 10.0\ \frac{\text{m}}{\text{s}}$		
$h_2 = 25.0$ m		

Develop a Strategy

State the law of conservation of mechanical energy.

$$E'_\text{k} + E'_\text{p} = E_\text{k} + E_\text{p}$$

Expand by replacing E with the expression that defined the type of energy.

$$\frac{1}{2}mv_2^2 + mgh_2 = \frac{1}{2}mv_1^2 + mgh_1$$

Divide through by m.

$$\frac{1}{2}v_2^2 = \frac{1}{2}v_1^2 + gh_1 - gh_2$$

Simplify and rearrange the equation to solve for v_2.

$$v_2^2 = v_1^2 + 2g(h_1 - h_2)$$

$$v_2 = \sqrt{v_1^2 + 2g(h_1 - h_2)}$$

Substitute and solve.

$$v_2 = \sqrt{\left(2.00\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(40.0\ \text{m} - 25.0\ \text{m})}$$

$$v_2 = \sqrt{298.3\ \frac{\text{m}^2}{\text{s}^2}}$$

Speed does not involve direction, so choose the positive root since speed can never be negative.

$$v_2 = \pm 17.271\ \frac{\text{m}}{\text{s}}$$

$$v_2 \cong 17.3\ \frac{\text{m}}{\text{s}}$$

(a) The speed is 17.3 m/s at a height of 25.0 m.

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Write the expanded version of the conservation of mechanical energy. Rearrange and solve for height.

$$\frac{1}{2}mv_3^2 + mgh_3 = \frac{1}{2}mv_1^2 + mgh_1$$

$$gh_3 = \frac{1}{2}v_1^2 + gh_1 - \frac{1}{2}v_3^2$$

$$h_3 = \frac{\frac{1}{2}v_1^2 + gh_1 - \frac{1}{2}v_3^2}{g}$$

Substitute numerical values and solve.

$$h_3 = \frac{\frac{1}{2}\left(2.00\ \frac{\text{m}}{\text{s}}\right)^2 + \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(40.0\ \text{m}) - \frac{1}{2}\left(10.0\ \frac{\text{m}}{\text{s}}\right)^2}{9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$h_3 = 35.107\ \text{m}$$

$$h_3 \cong 35.1\ \text{m}$$

(b) The height must be 35.1 m when the speed is 10.0 m/s.

Validate the Solution

(a) The units for the right-hand side of the equation for v_2 are

$$\left[\left(\frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{\text{m}}{\text{s}^2}\right)\text{m}\right]^{\frac{1}{2}} = \frac{\text{m}}{\text{s}}.$$

These are the correct units for the speed. In addition, since the skier is going downhill, the final speed must be larger than the initial speed of 2.00 m/s.

(b) The units on the right-hand side of the equation for h_3 are

$$\frac{\left(\frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{\text{m}}{\text{s}^2}\right)(\text{m}) - \left(\frac{\text{m}}{\text{s}}\right)^2}{\frac{\text{m}}{\text{s}^2}} = \text{m}.$$

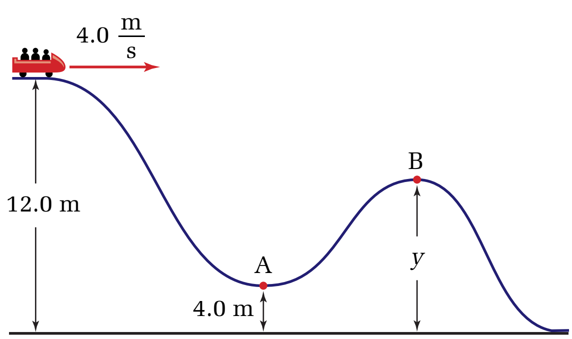
Since the speed in part (b) is less than the speed in part (a), the skier should be higher on the hill than in part (a).

PROBLEM TIP

Students are often tempted to apply the equations for linear motion to the solution of these problems. However, the paths are not always linear, so the equations might not apply. One of the great advantages of using conservation of energy is that you generally do not need to know the exact path between two points or vector directions. You need to know the conditions at only those two points.

PRACTICE PROBLEMS

Use the following diagram for practice problems 10, 11, and 12.



10. A car on a roller coaster is moving along a level section 12.0 m high at 4.0 m/s when it begins to roll down a slope, as shown in the diagram. Determine the speed of the car at point A.

11. What is the height of point B in the roller coaster track if the speed of the car at that point is 10.0 m/s?

12. What is the height of y if the speed of the roller coaster at B is 12.5 m/s?

INVESTIGATION 5-A

Testing the Law of Conservation of Energy

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

When scientists set out to test an hypothesis or challenge a law, they often use the hypothesis or law to make a prediction and then test it. In this investigation, you will make a prediction based on the law of conservation of energy.

Problem

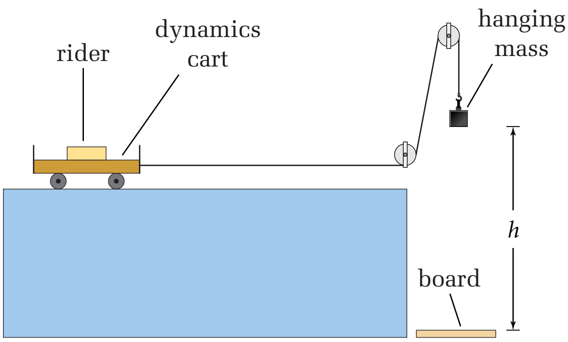
To perform a test of the law of conservation of energy

Equipment

- dynamics cart
- balance capable of measuring a mass of 1 to 2 kg
- 2 pulleys
- retort stand and clamps for the pulleys
- board (or similar material) to protect the floor from dropped masses
- metric measuring tape
- selection of masses, including several that can be suspended on a string
- stopwatch or photogate timers
- string about 4 m long

Procedure

1. Determine the mass of the dynamics cart.
2. Select a mass to be the rider and a mass to be the hanging mass. Decide on the height of the hanging mass above the board.
3. Set up the apparatus on a long desktop, as shown in the diagram.



4. Using the law of conservation of energy, calculate the expected speed of the cart and the hanging mass just before the mass strikes the board. If the cart is to be released from rest, determine the average speed of the system and then the predicted time interval for the hanging mass to drop to the board.
5. With the entire apparatus in place, hold the cart still. Release the cart and measure the time taken for the hanging mass to reach the board. Be sure to catch the cart before it hits the lower pulley.
6. Repeat the measurement several times and average the results.
7. Perform several trial runs with a different pair of masses for the cart and the hanging mass.

Analyze and Conclude

1. Prepare a table to show all of your data, as well as your calculations for the final speed, average speed, and the time interval.
2. What is the percent difference between the time as predicted by the law of conservation of energy and the measured average time?
3. Based on the precision of the timing devices, what range of experimental error would you expect in this investigation? How does this range compare with the percent difference determined in question 2?
4. Do the results of this investigation support the law of conservation of energy?

Apply and Extend

5. Which part of this investigation caused the greatest difficulty? Provide suggestions for overcoming this difficulty.
6. List some of the sources of error in this investigation and suggest how these errors might be reduced.

Work and Energy Change with a Variable Force

Until now, you have assumed that the force in the expression for work was constant. Quite often, however, you must deal with situations in which the force varies with the position, as shown in part (A) of Figure 5.6.

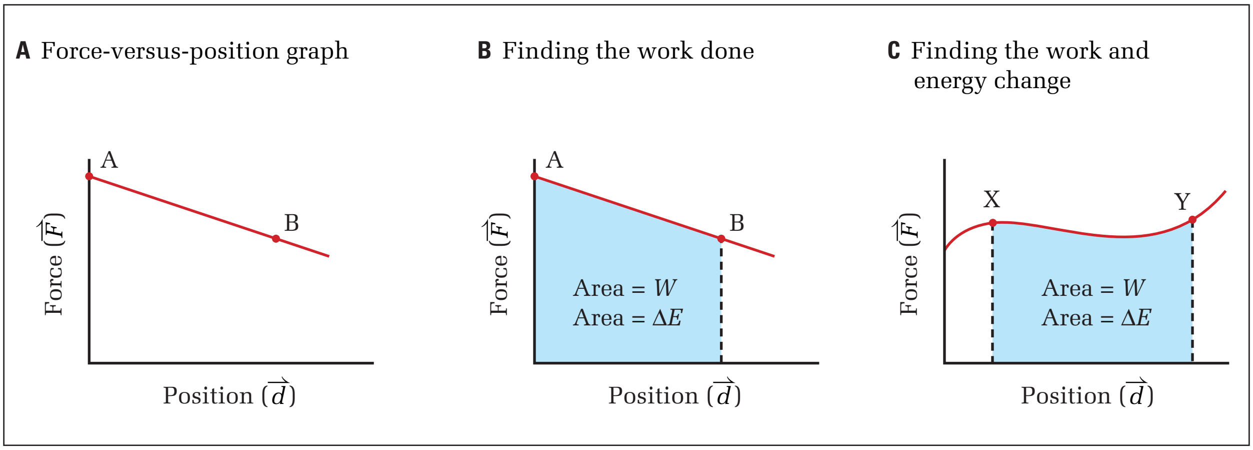


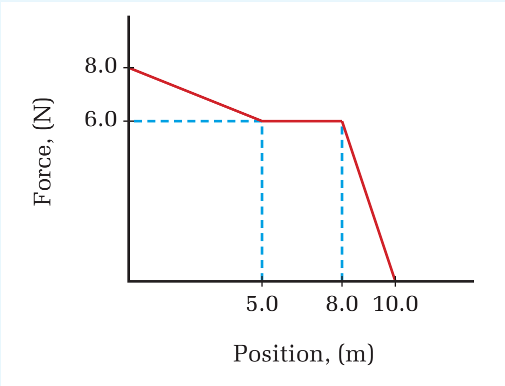
Figure 5.6 The work done, or the change in energy, is equal to the area under the graph

Since work is defined as the product of force times the displacement over which the force acts, then work must be equivalent to the graphical area under a graph of force versus position (displacement is a change in position). Such an area is illustrated in parts (B) and (C) of Figure 5.6. In the simplest cases, the graphical area forms a figure for which the area can be readily determined, as shown in the following sample problem.

SAMPLE PROBLEM

Work Done by a Variable Force

The graph shows the variation of applied force with position. Determine the work done by the force and the total energy change due to that force.



Conceptualize the Problem

- The problem involves a graph of *force versus position*.
- The graphical *area under the curve* provides the *work* done.
- *Work* is equal to the total *change in energy*.

Identify the Goal

- The work, W, done by the applied force
- The total energy change ΔE_{total}, due to the force

Identify the Variables and Constants

Known	**Unknown**
graph of force versus position	W
	ΔE_{total}

Develop a Strategy

Divide the graphical area under the curve up into recognizable shapes.

Determine the area of each region.

The region from the origin to 5.0 m is a trapezoid.

Area of trapezoid

A = (average of parallel sides)(distance between them)

$$A = \left(\frac{8.0\ \text{N} + 6.0\ \text{N}}{2}\right)5.0\ \text{m}$$

$$A = 35\ \text{N} \cdot \text{m}$$

$$A = 35\ \text{J}$$

The region from 5.0 m to 8.0 m is a rectangle.

Area of rectangle = (base)(height)

$$A = (3.0\ \text{m})(6.0\ \text{N})$$

$$A = 18\ \text{N} \cdot \text{m}$$

$$A = 18\ \text{J}$$

The region from 8.0 m to 10.0 m is a triangle.

Area of triangle = $\frac{1}{2}$(base)(height)

$$A = \frac{1}{2}(2.0\ \text{m})(6.0\ \text{N})$$

$$A = 6.0\ \text{N} \cdot \text{m}$$

$$A = 6.0\ \text{J}$$

Find the total area.

Total area = 35 J + 18 J + 6.0 J

Total area = 59 J

$$W = \Delta E_{total}$$

$$\Delta E_{total} = 59\ \text{J}$$

The work done, and therefore the total change in energy, is 59 J.

Validate the Solution

From examination of the graph, an average force would seem to be approximately 5 N and acts over 10 m. An approximate value for the work done would be 50 N. The answer is not far from this value.

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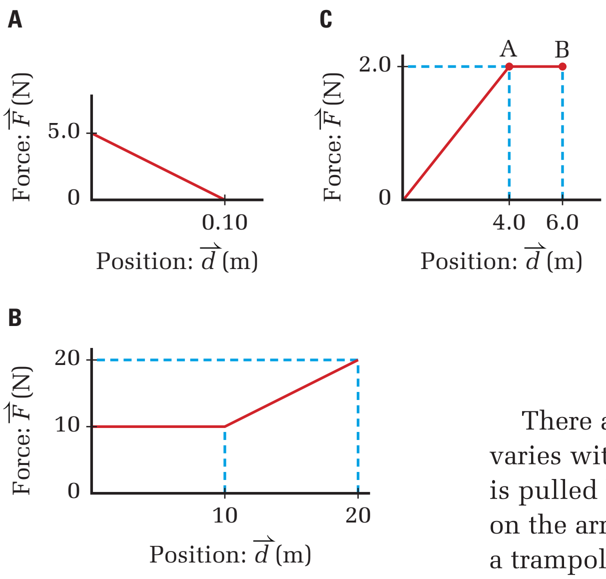
PRACTICE PROBLEMS



13. Calculate the work done by the force depicted in part (A) of the diagram.

14. Determine the magnitude of the energy change produced by the force illustrated in part (B) of the diagram.

15. The force shown in part (C) of the diagram acts horizontally on a 2.0 kg cart, initially at rest on a level surface. Determine the speed of the cart at point A and at point B.

There are many examples in which the applied force varies with displacement. The more an archery bow string is pulled back, the greater the force that the string can exert on the arrow. Force increases with the amount of stretch in a trampoline. Springs are interesting in that they can exert forces when they are stretched or compressed, and the amount of force depends on the amount of extension or compression. You will study this type of relationship in the next section of this chapter.

5.1 Section Review

1. K/U Give examples that were not used in the text to show that no work is done by an applied force that is perpendicular to the direction of the motion of the object.

2. I If the force of friction is constant, prove that the stopping distance of a car on a level road varies directly with the square of the initial speed.

3. K/U When developing the equation for gravitational potential energy, why was it necessary to assume that the mass was rising at a constant speed?

4. I Prove the expressions for gravitational potential energy and kinetic energy have units that are equivalent to the newton · metre.

5. MC The absolute temperature of a gas is a measure of the average kinetic energy of the gas atoms or molecules. What happens to the average speed of these particles when the absolute temperature of the gas is doubled?

6. C

(a) What is meant by the term “isolated system”?

(b) Describe an example of a system that could be considered as being isolated.

(c) Explain why this system is probably not completely isolated.

7. MC A variable force, $\vec{F}$, acts through a displacement, Δd. The magnitude of the force is proportional to the displacement, so $F = k\Delta d$, where k is constant.

(a) Sketch a graph of this force against position up to position x.

(b) According to the equation, what is the value of the force at x?

(c) Determine an expression for the work done by this force in terms of k and x.

5.2 Hooke's Law and Periodic Motion

The diver approaches the end of the board, bounces a couple of times, then arcs out into the air in a graceful dive. The diving board plays an important role in his action. The diver uses chemical energy to jump, gaining kinetic energy. His kinetic energy transforms into gravitational potential energy and then back into kinetic energy. When he returns to the board, slows, and stops, his kinetic energy does not transform into gravitational potential energy. In what form is the energy stored?

Figure 5.7 A diving board transforms a form of potential energy into kinetic energy of the diver.

SECTION EXPECTATIONS

- Analyze and explain common situations using the work-energy theorem.
- State Hooke's law and analyze it in quantitative terms.
- Define and describe the concepts and units related to elastic potential energy.
- Apply Hooke's law and the conservation of energy to periodic motion.

KEY TERMS

- elastic potential energy
- Hooke's law
- spring constant
- restoring force
- periodic motion

Describing Elastic Potential Energy

The diving board in Figure 5.7 is behaving much like a spring. When the diver lands on the board after jumping, the diving board exerts a force on him, doing work on him that reduces his kinetic energy to zero. At the same time, the diver is exerting a force on the diving board, doing work on the board and causing it to bend. It its bent condition, the diving board is storing energy called **elastic potential energy**. Because the diving board is elastic, it returns to its original form, and in doing so, it transfers its elastic potential energy back into kinetic energy.

Springs are commonly used, much like the diving board, to absorb energy, store it as elastic potential energy, then release it in the form of kinetic energy. A bicycle seat has a spring that reduces the jarring effects on the rider of bumps in the road. Springs in a mattress provide a flexible support that allows the surface to match the contours of the sleeper. Pressure is applied evenly over the lower surface of the sleeper, rather than being concentrated at a few points.

In this section, you will examine elastic potential energy in the form of stretched and compressed springs.

Figure 5.8 In what ways do these springs behave the same?

INVESTIGATION 5-B

Testing Hooke's Law

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

Problem

What relationship exists between the force applied to a spring and its extension?

Equipment

- retort stand and C-clamp
- weight hanger and accompanying set of masses
- coil spring
- ring clamp
- metre stick

CAUTION Wear protective eye goggles during this investigation.

Procedure

1. Clamp the retort stand firmly to the desk.
2. Attach the ring clamp close to the top of the retort stand.
3. Hang the spring by one end from the ring clamp.
4. Prepare a data table with the headings: Mass on hanger, m(kg); Applied force, F(N); Height of hanger above desk, h(m); and Extension of spring, x(m).
5. Attach the weight holder and measure its distance above the desktop. Record this value in the first row of the table. This value will be your equilibrium value, h_o, at which you will assign the value of zero to the extension of the spring, x. Put these values in the first line of your table.
6. To create an applied force, add a mass to the weight holder. Wait for the spring to come to rest and measure the height of the weight holder above the desk. Record these values in the table.
7. Complete the second row in the table by calculating the value of the applied force (weight of the mass) and the extension of the spring ($x = h_o - h$).
8. Continue by adding more masses until you have at least five sets of data. Make sure that you do not overextend the spring.

Analyze and Conclude

1. Draw a graph of the applied force versus the extension of the spring. **Note:** Normally, you would put the independent variable (in this case, the applied force) on the x-axis and the dependent variable (in this case, the extension of the spring) on the y-axis. However, the mathematics will be simplified in this case by reversing the position of the variables.
2. Draw a smooth curve through the data points.
3. Describe the curve and write the equation for the curve.
4. State the relationship between the applied force and the extension. This relationship is known as "Hooke's law."
5. When the spring is at rest, what is the relationship between the applied force and the force exerted on the mass by the spring? This force is usually referred to as the "restoring force." Restate the spring relationship in terms of the restoring force of the spring.
6. By finding the area under the graph between the origin and the point of maximum extension, determine the amount of energy stored in the spring.
7. Write an equation for the energy stored in the spring when the slope of the graph is k and the extension is x.
8. Devise and carry out an experiment to determine whether a similar relationship exists for the bending of a metre stick. Obtain your teacher's approval before carrying out the experiment.

Hooke's Law

Investigation 5-B illustrated **Hooke's law**, which states that the amount of extension or compression of a spring varies directly with the applied force. A graphical illustration of this law for an extended spring is shown in Figure 5.9.

Since the data produce a straight line, the equation can be written in the form $y = mx + b$, where m is the slope and b is the y-intercept. The slope of the line describing the properties of a spring, called the **spring constant**, is symbolized by k and has units of newtons/metre. Each spring has its own constant that describes the amount of force that is necessary to stretch (or compress) the spring a given amount. In your investigation, you were directed to assign the reference or zero position of your spring as the position of the spring with no applied force. As a result, x was zero when F was zero. This choice is the accepted convention for working with springs, and it makes the y-intercept equal to zero because the line on the graph passes through the origin. This relationship leads to the mathematical form of Hooke's law (which is summarized in the following box): $F_a = kx$, where F_a is the magnitude of the applied force, x is the magnitude of the extension or compression, and k is the spring constant.

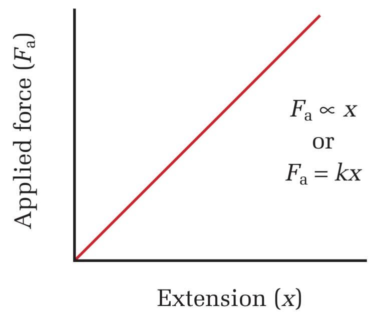


Figure 5.9 The applied force varies directly with the extension of a spring.

HOOKE'S LAW

The applied force is directly proportional to the extension or compression of a spring.

$$F_a = kx$$

Quantity	Symbol	SI unit
applied force	F_a	N (newtons)
spring constant	k	$\frac{N}{m}$ (newtons per metre)
amount of extension or compression of the spring	x	m (metres)

Unit Analysis

newtons $= \left(\frac{\text{newtons}}{\text{metre}}\right)$(metre) $N = \frac{N}{\cancel{m}}\cancel{m} = N$

According to Newton's third law of motion, the force exerted by the object that is applying the force to the spring is equal and opposite to the force that the spring exerts on that object. The force exerted by the spring is called the **restoring force**. Often, Hooke's law is written in terms of the restoring force of the

PHYSICS FILE

The spring constant is closely related to a quantity called the "modulus of elasticity." This is defined as the stress on the object divided by the strain. Stress is defined as the applied force divided by the cross-sectional area, and the strain is the amount of extension or compression per unit length. This quantity is used to predict how structural components, from aircraft wings to steel beams, will behave when under a given load.

spring: $F_s = -kx$. The negative sign shows that the restoring force is always opposite to the direction of the extension or compression of the spring.

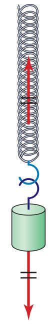


Figure 5.10 The restoring force always opposes the applied force and acts in the direction of the equilibrium position of the spring.

SAMPLE PROBLEM

Hooke's Law in an Archery Bow

A typical compound archery bow requires a force of 133 N to hold an arrow at "full draw" (pulled back 71 cm). Assuming that the bow obeys Hooke's law, what is its spring constant?

Conceptualize the Problem

- When an archer draws a bow, the *applied force* does *work* on the bow, giving it *elastic potential energy*.
- *Hooke's law* applies to this problem.

Identify the Goal

The spring constant, k, of the bow

Identify the Variables and Constants

Known	Unknown
$F_a = 133$ N	k
$x = 71$ cm	

Develop a Strategy

Use Hooke's law (applied force form). $F_a = kx$

Solve for the spring constant. $k = \frac{F_a}{x}$

Substitute numerical values and solve. $k = \frac{133 \text{ N}}{0.71 \text{ m}}$

$$k = 187.32 \frac{\text{N}}{\text{m}}$$

$$k \cong 1.9 \times 10^2 \frac{\text{N}}{\text{m}}$$

The spring constant of the bow is about $1.9 \times 10^2 \frac{\text{N}}{\text{m}}$.

Validate the Solution

When units are carried through the calculation, the final quantity has units of N/m, which are correct for the spring constant.

PRACTICE PROBLEMS

16. A spring scale is marked from 0 to 50 N. The scale is 9.5 cm long. What is the spring constant of the spring in the scale?

17. A slingshot has an elastic cord tied to a Y-shaped frame. The cord has a spring constant of 1.10×10^3 N/m. A force of 455 N is applied to the cord.

(a) How far does the cord stretch?

(b) What is the restoring force from the spring?

18. The spring in a typical Hooke's law apparatus has a force constant of 1.50 N/m and a maximum extension of 10.0 cm. What is the largest mass that can be placed on the spring without damaging it?

Calculating Elastic Potential Energy

The graph of Hooke's law in Figure 5.9 not only gives information about the forces and extensions for a spring (or any elastic substance), you can also use it to determine the quantity of potential energy stored in the spring. As discussed previously, you can find the amount of work done or energy change by calculating the area under a force-versus-position graph. The Hooke's law graph is such a graph, since extension or compression is simply a displacement. The area under the graph, therefore, is equal to the amount of potential energy stored in the spring, as illustrated in Figure 5.11.

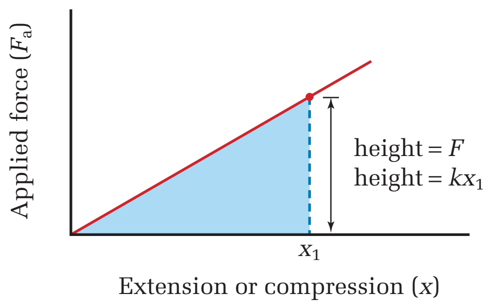


Figure 5.11 The triangular area under the Hooke's law graph gives you the amount of elastic potential energy stored in the spring at any amount of extension.

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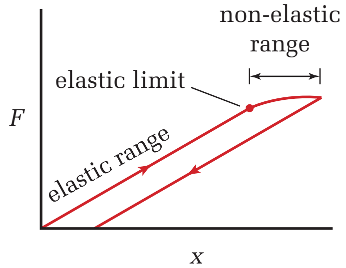
A *perfectly elastic* material will return precisely to its original form after being deformed, such as stretching a spring. No real material is perfectly elastic. Each material has an elastic limit, and when stretched to that limit, will not return to its original shape. The graph below shows that when something reaches its elastic limit, the restoring force does not increase as rapidly as it did in its elastic range.



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Robert Hooke (1635–1703) was one of the most renowned scientists of his time. His studies in elasticity, which resulted in the law being named after him, allowed him to design better balance springs for watches. He also contributed to our understanding of optics and heat. In 1663, he was elected as a Fellow of the Royal Society in London. His studies ranged from the microscopic — he observed and named the cells in cork and investigated the crystal structure of snowflakes — to astronomy — his diagrams of Mars allowed others to measure its rate of rotation. He also proposed the inverse square law for planetary motion. Newton used this relationship in his law of universal gravitation. Hooke felt that he had not been given sufficient credit by Newton for his contribution, and the two men remained antagonistic for the rest of Hooke's life.

As you can see in Figure 5.11, the area under the curve of applied force versus extension of a spring is a triangle. You can use the geometry of the graph to derive an equation for the elastic potential energy stored in a spring.

- Write the equation for the area of a triangle.
  $A = \frac{1}{2}(\text{base})(\text{height})$
- The elastic potential energy stored in a spring is the area under the curve.
  $E_e = A$
  $E_e = \frac{1}{2}(\text{base})(\text{height})$
- The base of the triangle is the amount of extension of the spring, x_1.
  $\text{base} = x_1$
- The height of the triangle is the force at an extension of x_1.
  $\text{height} = F(x_1)$
  $F(x_1) = kx_1$
  $\text{height} = kx_1$
- Substitute the values into the expression for elastic potential energy.
  $E_e = \frac{1}{2}(x_1)(kx_1)$
  $E_e = \frac{1}{2}kx_1^2$
- The expression is valid for any value of x.
  $E_e = \frac{1}{2}kx^2$

The equation you just derived applies to any perfectly elastic system and is summarized in the box below.

ELASTIC POTENTIAL ENERGY

The elastic potential energy of a perfectly elastic material is one half the product of the spring constant and the square of the length of extension or compression.

$$E_e = \frac{1}{2}kx^2$$

Quantity	Symbol	SI unit
elastic potential energy	E_e	J (joules)
spring constant	k	$\frac{\text{N}}{\text{m}}$ (newtons per metre)
length of extension or compression	x	m (metres)

Unit Analysis

$$\text{joule} = \frac{\text{newton}}{\text{metre}}\text{metre}^2 \qquad \text{J} = \left(\frac{\text{N}}{\cancel{\text{m}}}\right)\text{m}^{\cancel{2}} = \text{N}\cdot\text{m} = \text{J}$$

SAMPLE PROBLEM

Elastic Potential Energy of a Spring

A spring with spring constant of 75 N/m is resting on a table.

(a) **If the spring is compressed a distance of 28 cm, what is the increase in its potential energy?**

(b) **What force must be applied to hold the spring in this position?**

Conceptualize the Problem

- There is *no change* in the *gravitational potential energy* of the spring.
- The *elastic potential energy* of the spring *increases* as it is compressed.
- *Hooke's law* and the definition of *elastic potential energy* apply to this problem.

Identify the Goal

The elastic potential energy, E_e, stored in the spring
The applied force, F_a, required to compress the spring

Identify the Variables and Constants

Known	Unknown
$k = 75\ \frac{N}{m}$	E_e
$x = 0.28$ m	F_a

Develop a Strategy

Apply the equation for elastic potential energy.
Substitute and solve.

$$E_e = \frac{1}{2}kx^2$$

$$E_e = \frac{1}{2}\left(75\ \frac{N}{m}\right)(0.28\ m)^2$$

$$E_e = 2.94\ J$$

$$E_e \cong 2.9\ J$$

(a) The potential energy of the spring increases by 2.9 J when it is compressed by 28 cm.

Use Hooke's law to calculate the force at 28 cm compression.

$$F_a = kx$$

$$F_a = \left(75\ \frac{N}{m}\right)(0.28\ m)$$

$$F_a = 21\ N$$

(b) A force of 21 N is required to hold the spring in this position.

Validate the Solution

Round the given information to 80 N and 0.3 m and do mental multiplication. The resulting estimated change in elastic potential energy is 3.6 J and the estimated applied force is 24 N. The exact answers are reasonably close to these estimated values. In addition, a unit analysis of the first part yields an answer in N · m or joules, while the second answer is in newtons.

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PRACTICE PROBLEMS

19. An object is hung from a vertical spring, extending it by 24 cm. If the spring constant is 35 N/m, what is the potential energy of the stretched spring?

20. An unruly student pulls an elastic band that has a spring constant of 48 N/m, producing a 2.2 J increase in its potential energy. How far did the student stretch the elastic band?

21. A force of 18 N compresses a spring by 15 cm. By how much does the spring's potential energy change?

PHYSICS FILE

Gravitational potential energy is energy that an object possesses due to its position in a gravitational field. Elastic potential energy is a bit harder to picture. However, when a material is stretched or twisted, its atoms move relative to each other. Since the atoms are held together by electric forces, elastic potential energy is related to the position of an object, such as an atom in an electric field.

Restoring Force and Periodic Motion

The restoring force exerted by a spring always points toward the equilibrium or rest position for that spring. When the spring is extended, the restoring force pulls it back toward its equilibrium position. When the spring is compressed, the restoring force pushes it outward. The nature of this force makes it possible for a spring to undergo a back-and-forth, or oscillating, motion called **periodic motion**. If the restoring force obeys Hooke's law precisely, the periodic motion is called *simple harmonic motion*.

Periodic motion is closely associated with wave motion, a topic that you have studied in previous physics courses. You might recall that a vibrating or oscillating object often creates a wave. You can make the comparison by imagining that you attached a pen to the end of a spring and allowed it to rest on a long sheet of paper. If you extended the spring and then released it, the pen would oscillate back and forth, drawing a line on the paper. If you pulled the paper under the pen at a steady rate while the pen was in motion, you would create an image like the one in Figure 5.12. You probably recognize the figure as having the same shape as the waves that you studied. Many of the terms that you learned in connection with waves also apply to periodic motion.

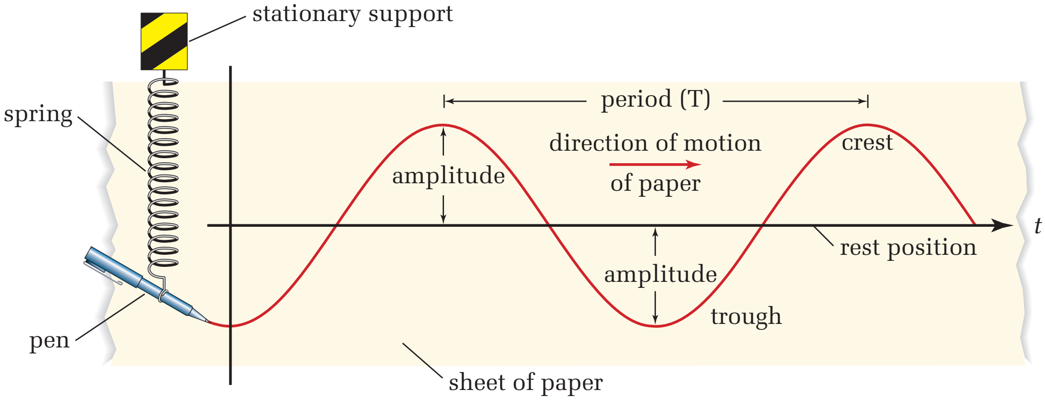


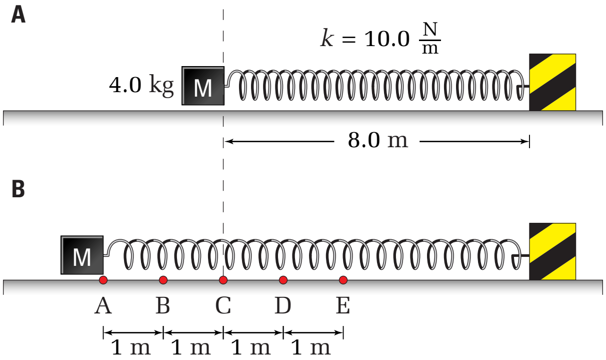
Figure 5.12 This wave shows how the position of the mass at the end of a spring changes with time. Mathematically, this graph is called a sine wave.

INVESTIGATION 5-C

Analyzing Periodic Motion

TARGET SKILLS
- Predicting
- Analyzing and interpreting
- Communicating results

Imagine that a spring is lying on a frictionless surface. One end is fastened to an immovable object and a 4.0 kg mass is attached to the free end, as shown in part (A) of the diagram. Then, the 4.0 kg mass is pulled 2.0 m to the left and held in place. You will follow the mass from the time it is released until it has travelled from Point A to point E, and then back to point A again, by determining the values of several of its variables at each labelled point. Let the positive direction be to the right and negative to the left.



Problem

Why does a spring continue to vibrate when stretched and then released?

Procedure

1. Prepare a table with the headings: Point, Extension or compression x(m), Restoring force F(N), Acceleration a(m/s^2), Elastic potential energy E_e(J), Kinetic energy E_k(J), Total energy E_t(J), Velocity $\vec{v}$(m/s), and displacement of the mass Δd(m). Provide nine rows under the headings.
2. Under the heading "Point," list the letters A through E in the first five rows. In the following four rows, to represent the return trip, list D through A, with a prime on each letter. In the column headed "Extension or compression," write the displacement, x, between the given point and the equilibrium position, C.
3. Use the mathematical relationships that you have learned in this chapter and previous chapters to calculate all of the other values in the table. Be sure to include positive and negative directions in your calculations, where appropriate. (Hint: Note that the mass was released *from rest* at point A.)

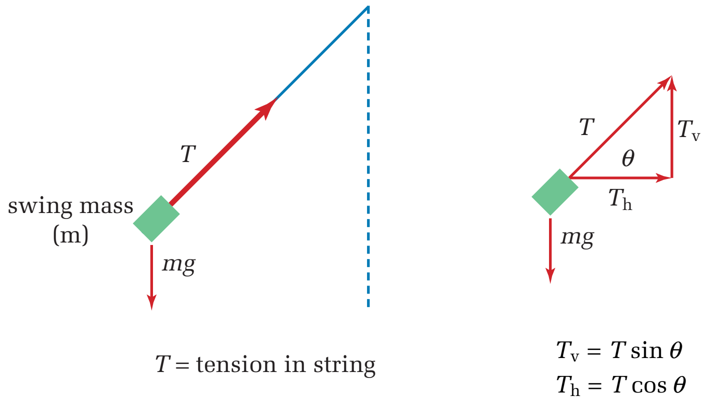
Analyze and Conclude

1. At which points in the cycle does the mass have **(a)** the greatest acceleration and **(b)** the greatest speed?
2. At which points in the cycle does the spring **(a)** exert the greatest restoring force, **(b)** possess the greatest amount of elastic potential energy, and **(c)** possess the least amount of elastic potential energy?
3. Why does the mass reverse direction at point E and then at point A′?
4. If there is no friction, what will happen to the motion of the mass and spring? Give reasons for your answer.
5. Plot both of the following relationships on one graph: acceleration versus position and velocity versus position. Use different colours and scales on the vertical axis for each of the plots.
6. Explain the relationships between velocity and acceleration at those points where velocity is zero and where acceleration is zero.
7. On one graph, again using different colours, plot elastic potential energy versus position, kinetic energy versus position, and total energy versus position. Discuss the significance of the relationships among these graphs.

PHYSICS FILE

The previous investigation asked you to imagine a frictionless surface. Physics courses are littered with frictionless surfaces and massless springs, strings, and ropes. Making these assumptions simplifies calculations and helps to focus on the essential ideas. Once the process becomes one of engineering and design, however, friction and masses of components become extremely important and cannot be ignored.

Periodic motion always requires a restoring force that depends on a displacement from a rest or equilibrium position in order to keep on repeating. In the case of a vibrating spring, this restoring force is provided by the attractive forces between atoms in the spring. In a resonating air column, such as in a sounding trumpet or clarinet, the restoring force is provided by collisions with other air molecules. In an oscillating pendulum, such as a mass swinging on a length of string, the horizontal component of the tension (T) in the string returns the mass to the centre.



$$T_v = T \sin \theta$$
$$T_h = T \cos \theta$$

Figure 5.13 The horizontal component of the tension always acts in the direction of the equilibrium position of the pendulum. At the equilibrium position, the horizontal component of the tension is zero.

Periodic (or nearly periodic) motion is seen everywhere. Playground swings and teeter-totters exhibit periodic motion. Sound waves, water waves, and earthquake waves involve periodic motion. The electromagnetic spectrum from the radio waves that carry signals to our radios and televisions, to the gamma radiation emitted from radioactive materials, all embody periodic motion.

Figure 5.14 Waves on the ocean involve both transverse and longitudinal vibrations.

INVESTIGATION 5-D

Another Test of the Law of Conservation of Energy

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

In Investigation 5-A, you attempted to test the law of conservation of energy by making a prediction involving the transfer of energy from gravitational potential energy into kinetic energy. In this investigation, you will examine the transfer from elastic potential energy into kinetic energy. You will then use the predicted value of kinetic energy to determine the launch velocity of a projectile and, therefore, its range.

Problem

Does the law of conservation of energy make valid predictions when energy is converted from elastic potential energy into kinetic energy?

Equipment

- balance
- retort stand
- ramp or small, smooth board
- set of masses with a mass holder
- protractor
- metre stick or metric tape measure
- utility clamp
- small spring
- small cardboard box
- masking tape

CAUTION Safety goggles must be worn during this activity.

Procedure

Work in small groups for the investigation.

1. Measure the mass of the spring.
2. Using the equipment, determine the spring constant for the spring.
3. Set up the ramp on a desk, or make a ramp by resting one end of the board on a stack of books. Measure the angle that the ramp makes with the desktop. Make sure that there is a long stretch of clear space in front of the ramp.
4. Decide on the amount of extension that you intend to use with the spring and then determine the corresponding elastic potential energy stored in the spring at that extension.
5. Set up the spring by hooking one end over the upper edge of the ramp. Then, pull it backward to extend it the selected distance and release it. Use the law of conservation of energy to determine the velocity with which the spring will leave the ramp.
6. Use the velocity and the height of the end of the ramp to determine the point at which the spring will hit the floor (or the wall).
7. Place the cardboard box at that predicted point and perform the launch.

Analyze and Conclude

1. Provide a summary of your force-extension measurements for the spring.
2. Show your calculation of the spring constant.
3. What extension did the group choose? Show your calculation of the elastic potential energy stored in the spring.
4. Show your calculation of the
   (a) velocity of the spring as it leaves the ramp
   (b) range of the projectile (the spring)
5. How close did the spring come to its predicted landing point?
6. Describe the energy changes that occurred during the launch and flight of the spring.
7. Does this investigation further confirm the law of conservation of energy?

Apply and Extend

8. Spring-loaded dart guns with dart safety tips are available as toys. Decide how you could determine the spring constant and hence the maximum range of the projectile (the dart). If possible, repeat this investigation using one of these toy guns. You might recall from earlier studies that the maximum range occurs when the dart is launched at 45° to the horizontal.

5.2 Section Review

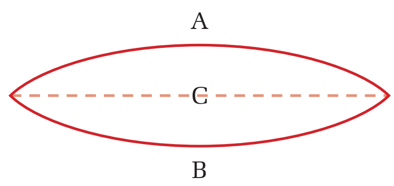
1. K/U Explain how each of the following behave like a spring.
   (a) a pole used in pole-vaulting
   (b) the strings in a tennis racquet
   (c) the string on a bow

2. I Prove that the expression for elastic potential energy has units equivalent to the joule.

3. MC In what way is a spring similar to a chemical bond?

4. MC List three other forms of periodic motion not mentioned in the section.

5. K/U A guitar string is vibrating horizontally, as shown in the diagram. It vibrates between positions A and B, passing through the equilibrium or rest position C. In which positions is the string vibrating with the following?



   (a) greatest speed
   (b) least speed
   (c) greatest kinetic energy
   (d) greatest elastic potential energy

   Give reasons for your choices.

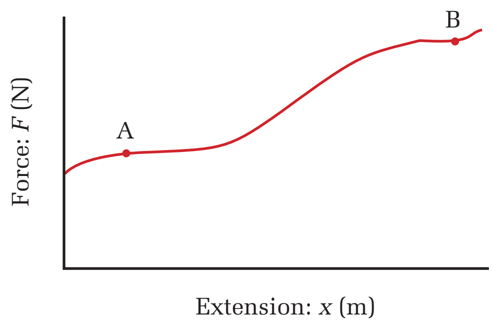
6. MC There are four basic forces in our universe.
   - the weak nuclear force (between particles in the nucleus)
   - the strong nuclear force (between particles in the nucleus)
   - electromagnetic force (between charged particles)
   - gravitational force (between masses)

   Which force is responsible for the potential energy stored in the following?
   (a) a battery
   (b) the water behind a dam
   (c) a stretched spring
   (d) a mound of snow at the top of a slope just before an avalanche

7. I Describe an investigation to determine the force-extension characteristics of an archery bow.

8. C Prepare a diagram to demonstrate the relationships between the gravitational potential energy, and the kinetic energy of the swinging bob in a pendulum.

9. I Given the following graph of applied force against extension, describe a technique for determining the amount of potential energy stored in the object between points A and B.



UNIT PROJECT PREP

Once you understand both periodic motion and the conditions necessary to generate it, you will find that periodic motion frequently appears in both natural and manufactured systems.

- Brainstorm to identify systems that experience periodic motion.
- Attempt to formulate an argument supporting an intrinsic link between understanding the periodic transformation of energy and environmentalism.

5.3 Energy Transformations

The law of conservation of energy is one of the most useful tools in physics. Since work and energy are scalar quantities, directions are not involved, as they are in momentum. As a result, vector diagrams are not needed, and angles do not have to be calculated. In any given event, the problem is usually to identify the types of energy involved and to ensure that the total energy in all its different forms at the end of the event equals the total at the beginning.

The analysis is often easiest when the motion occurs in a horizontal plane. No change in gravitational potential energy is involved. The following sample problem illustrates this feature.

SECTION EXPECTATIONS

- Analyze situations involving the concepts of mechanical energy, thermal energy, and its transfer.
- Analyze situations involving the concept of conservation of energy.

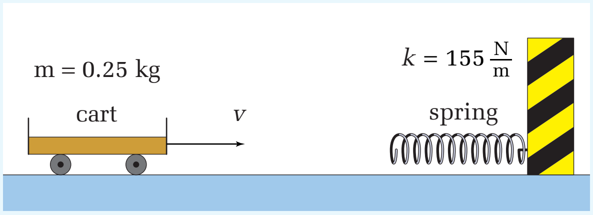
KEY TERMS

- conservative force
- non-conservative force

SAMPLE PROBLEM

Horizontal Elastic Collisions

A low-friction cart with a mass of 0.25 kg travels along a horizontal track and collides head on with a spring that has a spring constant of 155 N/m. If the spring was compressed by 6.0 cm, how fast was the cart initially travelling?



Conceptualize the Problem

- The cart is *moving* so it has *kinetic energy*.
- The spring does *negative work* on the cart, lowering its *kinetic energy*.
- The cart does *work* on the spring, giving it *elastic potential energy*.
- The *height* of the cart does *not change*, so there is no change in *gravitational potential energy*.
- The term *low friction* tells you to neglect the energy lost to work done by friction.
- The law of conservation of energy applies to this problem.

Identify the Goal

The initial speed, v, of the cart

Identify the Variables and Constants

Known		Unknown
$m = 0.25$ kg	$x = 0.060$ m	v
$k = 155\ \frac{\text{N}}{\text{m}}$		

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Develop a Strategy

Write the law of conservation of energy, including the energy quantities associated with the interaction.

$$E'_k + E'_e = E_k + E_e$$

Initially, the spring was not compressed, so the initial elastic potential energy was zero.

$$E_e = 0 \text{ J}$$

After the interaction, the cart stopped, so the kinetic energy was zero.

$$E'_k = 0 \text{ J}$$

Substitute the values for energy listed above.

$$0 \text{ J} + E'_e = E_k + 0 \text{ J}$$
$$E_k = E'_e$$

Expand by substituting the expressions for the energies.

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

Solve for the initial velocity.

$$v = \sqrt{\frac{kx^2}{m}}$$

Substitute numerical values and solve.

$$v = \sqrt{\frac{(155\ \frac{\text{N}}{\text{m}})(0.060\ \text{m})^2}{0.25\ \text{kg}}}$$
$$v = 1.493\ 99\ \frac{\text{m}}{\text{s}}$$
$$v \cong 1.5\ \frac{\text{m}}{\text{s}}$$

The cart was travelling at approximately 1.5 m/s before the collision.

Validate the Solution

Unit analysis of the equation $v = \sqrt{\frac{kx^2}{m}}$ shows that it is equivalent to m/s, the standard units for velocity. $\sqrt{\frac{\frac{\text{N}}{\cancel{\text{m}}}\text{m}^{\cancel{2}}}{\text{kg}}} = \sqrt{\frac{\frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}\text{m}}{\cancel{\text{kg}}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}}$.

A velocity of 1.5 m/s is reasonable for a lab cart.

PRACTICE PROBLEMS

22. A 1.2 kg dynamics cart is rolling to the right along a horizontal lab desk at 3.6 m/s, when it collides head on with a spring bumper that has a spring constant of 2.00×10^2 N/m.

(a) Determine the maximum compression of the spring.

(b) Determine the speed of the cart at the moment that the spring was compressed by 0.10 m.

(c) Determine the acceleration of the cart at the moment that the spring was compressed 0.10 m.

23. A circus car with a clown has a total mass of 150 kg. It is coasting at 6.0 m/s, when it hits a large spring head on. If it is brought to a stop by the time the spring is compressed 2.0 m, what is the spring constant of the spring?

The analysis becomes a bit more complicated when the motion is vertical, since there are now changes in gravitational potential energy along with elastic potential energy and kinetic energy.

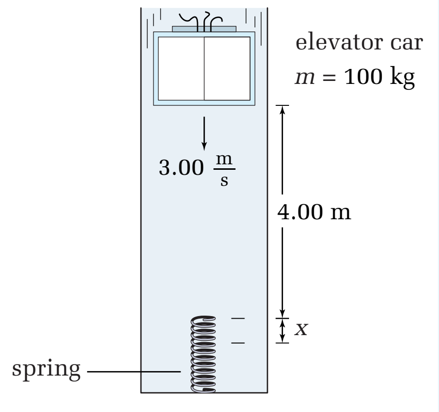
PHYSICS FILE

The energies discussed here are commonly found in mechanical systems with springs and pulleys. As a result, kinetic energy, gravitational potential energy, and elastic potential energy are commonly referred to as "mechanical energy."

SAMPLE PROBLEM

Vertical Elastic Collisions

A freight elevator car with a total mass of 100.0 kg is moving downward at 3.00 m/s, when the cable snaps. The car falls 4.00 m onto a huge spring with a spring constant of 8.000×10^3 N/m. By how much will the spring be compressed when the car reaches zero velocity?



Conceptualize the Problem

- Initially, the car is in *motion* and therefore has *kinetic energy*. It also has *gravitational potential energy*.
- As the car begins to *fall*, the *gravitational potential energy* transforms into *kinetic energy*. When the elevator hits the spring, the elevator *slows*, losing *kinetic energy*, and the spring compresses, gaining *elastic potential energy*.
- When the elevator comes to a complete *stop*, it has *no kinetic* or *gravitational potential energy*. All of the energy is now stored in the spring in the form of *elastic potential energy*.
- Since all of the motion is in a downward direction, define "down" as the positive direction for this problem.

Identify the Goal

The compression of the spring, x, when the car comes to rest

Identify the Variables and Constants

Known	Implied	Unknown
$m_{car} = 100.0$ kg	$g = 9.81\ \frac{m}{s^2}$	x
$v = 3.00\ \frac{m}{s}$[down]		
$k = 8.000 \times 10^3\ \frac{N}{m}$		
$h_{(above\ spring)} = 4.00$ m		

Develop a Strategy

Write the law of conservation of energy for the forms of energy involved in the problem.

$$E'_g + E'_e + E'_k = E_g + E_e + E_k$$

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Choose the lowest level of the elevator (maximum compression of the spring) as the reference level for gravitational potential energy.

$E'_g = 0\ \text{J}$

The car comes to a rest at the lowest point.

$E'_k = 0\ \text{J}$

Initially, the spring is not compressed.

$E_e = 0\ \text{J}$

Substitute these initial and final conditions into the equation for conservation of energy and simplify.

$$0\ \text{J} + E'_e + 0\ \text{J} = E_g + 0\ \text{J} + E_k$$
$$E'_e = E_g + E_k$$

Expand by substituting the expressions for the various forms of energy.

$$\frac{1}{2}kx^2 = mg\Delta h + \frac{1}{2}mv^2$$

The change in height for the gravitational potential is 4.00 m, plus the compression of the spring, x. Substitute this expression into the equation.

$$\frac{1}{2}kx^2 = mg(4.00 + x) + \frac{1}{2}mv^2$$

Since the equation yields a quadratic equation, you cannot solve for x. Substitute in the numerical values and rearrange so that the right-hand side is zero.

$$\frac{1}{2}\left(8.00 \times 10^3\ \frac{\text{N}}{\text{m}}\right)x^2 = (100.0\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(4.00 + x)\ \text{m} + \frac{1}{2}(100.0\ \text{kg})\left(3.00\ \frac{\text{m}}{\text{s}}\right)^2$$

$$4.00 \times 10^3x^2 - 981x - 3924 - 450 = 0$$

$$4.00 \times 10^3x^2 - 981x - 4374 = 0$$

Use the quadratic formula to find the value of x.

$$x = \frac{981 \pm \sqrt{(-981)^2 - 4(4.00 \times 10^3)(-4374)}}{2(4.00 \times 10^3)}$$

$$x = 1.1756\ \text{m} \quad \text{or} \quad -0.930\ 25\ \text{m}$$

$$x \cong 1.18\ \text{m}$$

Compression cannot be negative (or the spring would be stretching), so choose the positive value. The spring was compressed 1.18 m.

Validate the Solution

The units on the left-hand side of the final equation are $\frac{\text{N}}{\text{m}} \cdot \text{m}^2 = \frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{\text{m}} \cdot \text{m}^2 = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$.

On the right-hand side of the equation, the units are $\text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$.

Both sides of the equation have the same units, so you can have confidence in the equation. The answer is also in a range that would be expected with actual springs.

Note: The negative root in this problem is interesting in that it does have meaning. If the car had somehow latched onto the spring during the collision, the negative value would represent the maximum extension of the spring if the car had bounced up from the bottom due to the upward push of the spring.

PRACTICE PROBLEMS

24. A 70.0 kg person steps through the window of a burning building and drops to a rescue net held 8.00 m below. If the surface of the net is 1.40 m above the ground, what must be the value of the spring constant for the net so that the person just touches the ground when the net stretches downward?

25. A 6.0 kg block is falling toward a spring located 1.80 m below. If it has a speed of 4.0 m/s at that instant, what will be the maximum compression of the spring? The spring constant is 2.000×10^3 N/m.

26. In a "head dip" bungee jump from a bridge over a river, the bungee cord is fastened to the jumper's ankles. The jumper then steps off and falls toward the river until the cord becomes taut. At that point, the cord begins to slow the jumper's descent, until his head just touches the water. The bridge is 22.0 m above the river. The unstretched length of the cord is 12.2 m. The jumper is 1.80 m tall and has a mass of 60.0 kg. Determine the

(a) required value of the spring constant for this jump to be successful

(b) acceleration of the jumper at the bottom of the descent

Conservative and Non-Conservative Forces

Until now, you have been asked to assume that objects could move without friction. A pendulum would keep swinging repeatedly with the same amplitude, continuously converting energy between kinetic and gravitational potential forms of energy. A skier could slide down a hill, converting gravitational potential energy into kinetic energy and then, faced with an upward slope, could keep on going, converting the kinetic energy back into potential energy until the original height was reached.

The forces with which you have been dealing are referred to as **conservative forces**. This means that the amount of work that they do on a moving object does not depend on the path taken by that object. In the absence of friction, the boulder in Figure 5.15 will reach the bottom of the hill with the same kinetic energy and speed whether it dropped off the cliff on the left or slid down the slope on the right.

COURSE CHALLENGE

Energy Transformations

Light energy is transformed into stored chemical energy each time you take a photograph. The operation of infrared cameras, ultrasound images, and video cameras also relies on various energy transformations. Refer to page 604 for suggestions on relating energy transformations to your *Course Challenge*.

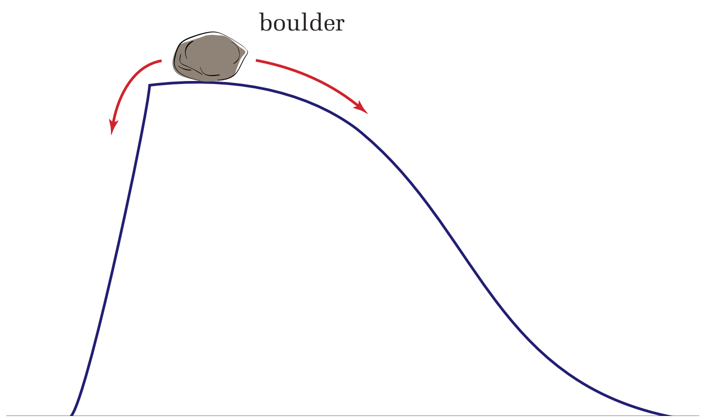


Figure 5.15 Gravity is a conservative force. If the boulder was dropped over the edge of the cliff, all of the gravitational potential energy would be converted into kinetic energy. Friction is not a conservative force. If the boulder slides down the hill, the kinetic energy at the bottom will not be as great as it would if the boulder fell straight down.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of energy transformation, refer to your Electronic Learning Partner.

Friction is a **non-conservative force**. The amount of work done by a non-conservative force depends on the path taken by the force and the object. For example, the amount of energy transferred to the snow in Figure 5.16 depends on the path taken by the skier. The skier going straight down the slope should reach the bottom with a greater speed than the skier who is tracking back and forth across the slope.

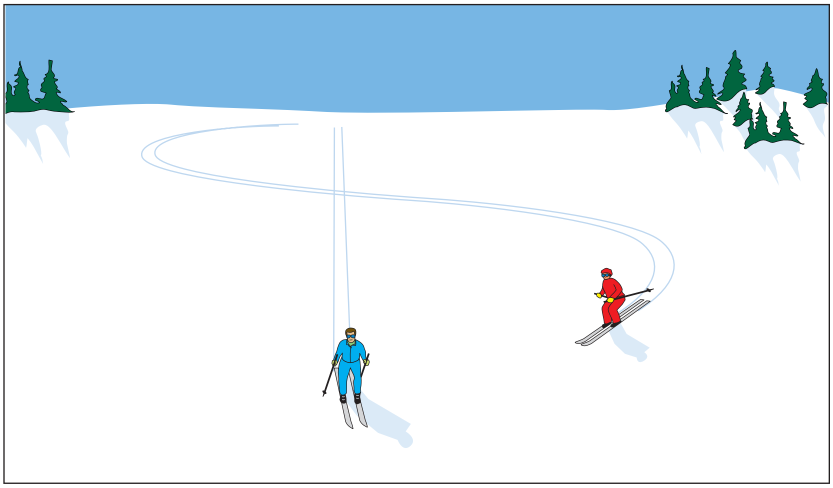


Figure 5.16 Although friction between the skis and the snow is small, friction nevertheless does some work on the skiers, slowing their velocity a little. The work done by friction is greater along the longer of the paths.

Friction causes the skier to do work on the environment. The snow heats up slightly and is moved around. For the skier, this is negative work — the skier is losing energy and cannot regain it as useful kinetic or potential energy. The sum of the skier's kinetic and gravitational potential energy at the end of the run will be less than it was at the start.

Wind pressure is another example of a non-conservative force. If the skier had the wind coming from behind, the wind (the environment) could be doing work on the skier. This would be positive work. The sum of the skier's kinetic and gravitational potential energies could increase beyond the initial total. However, the amount of energy transferred by the wind would depend to a large extent on the path of the skier, so the wind would be a non-conservative force.

When dealing with non-conservative forces, the law of conservation of energy still applies. However, you must account for the energy exchanged between the moving object and its environment. One approach to this type of situation is to define the system as the skier and the local environment; that is, the skier, wind, and snow become the system. The following sample problem illustrates this concept.

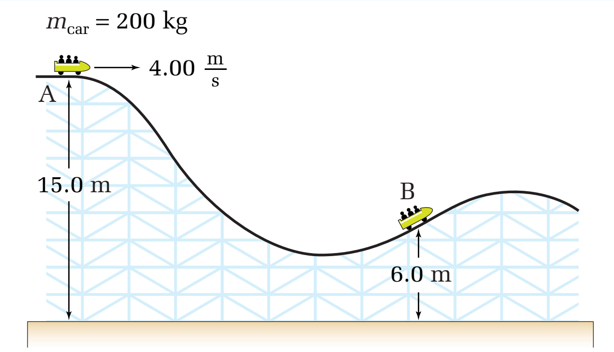
PHYSICS FILE

Quite often, you might not want your forces to be conservative. Without friction, many of your clothes would simply fall apart into strands as you moved. In addition, keep-fit programs would have to be greatly modified. A person who rides an exercise bicycle to lose mass (through chemical reactions that provide the energy) does not want the energy back. It simply is dissipated as sound and heat. Likewise, the weight lifter who does work to lift a bar bell does not expect to receive that energy back when the bar bell is lowered.

SAMPLE PROBLEM

Energy Conversions on a Roller Coaster

A roller-coaster car with a mass of 200.0 kg (including the riders) is moving to the right at a speed of 4.00 m/s at point A in the diagram. This point is 15.00 m above the ground. The car then heads down the slope toward point B, which is 6.00 m above the ground. If 3.40×10^3 J of heat energy are produced through friction between points A and B, determine the speed of the car at point B.



Conceptualize the Problem

- As the roller-coaster car *moves* down the track, most of the *gravitational potential energy* is converted into *kinetic energy*, but some is lost as *heat* due to *friction*.
- The law of conservation of total energy applies.
- *Heat energy* must be included as a *final energy*.

Identify the Goal

The speed of the car at point B, v_B

Identify the Variables and Constants

Known		Implied	Unknown
$h_A = 15.00$ m	$E_{heat} = 3.40 \times 10^3$ J	$g = 9.81 \frac{m}{s^2}$	v_B
$h_B = 6.00$ m	$m = 200.0$ kg		
$v_A = 4.00 \frac{m}{s}$			

Develop a Strategy

Write the law of conservation of energy, including heat as a final energy form.

$$E'_k + E'_g + E_{heat} = E_k + E_g$$

Expand by substituting the expressions for the forms of energy. Solve for the speed of the car at point B

$$\frac{1}{2}mv_B^2 + mgh_B + E_{heat} = \frac{1}{2}mv_A^2 + mgh_A$$

$$\frac{1}{2}mv_B^2 = -mgh_B - E_{heat} + \frac{1}{2}mv_A^2 + mgh_A$$

$$v_B^2 = \frac{2(-mgh_B - E_{heat} + \frac{1}{2}mv_A^2 + mgh_A)}{m}$$

$$v_B = \sqrt{\frac{2(-mgh_B - E_{heat} + \frac{1}{2}m(v_A)^2 + mgh_A)}{m}}$$

$$v_B = \sqrt{\frac{2\left[-\left(1.1772 \times 10^4 \frac{kg \cdot m^2}{s^2}\right) - (3.40 \times 10^3\ J) + \left(1.6 \times 10^3\ kg\frac{m^2}{s^2}\right) + \left(2.943 \times 10^4 \frac{kg \cdot m^2}{s^2}\right)\right]}{200.0\ kg}}$$

$$v_B = \sqrt{\frac{3.1716 \times 10^4 \frac{\cancel{kg} \cdot m^2}{s^2}}{200.0\ \cancel{kg}}}$$

continued ▶

continued from previous page

$$v_B = \sqrt{1.5858 \times 10^2 \ \frac{m^2}{s^2}}$$

$$v_B = 1.2593 \times 10^1 \ \frac{m}{s}$$

$$v_B \cong 12.6 \ \frac{m}{s}$$

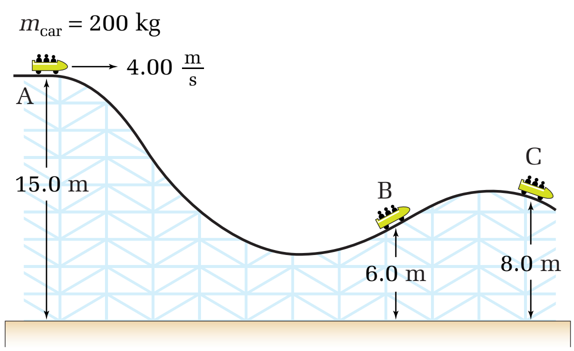
The speed of the car at point B will be 12.6 m/s.

Validate the Solution

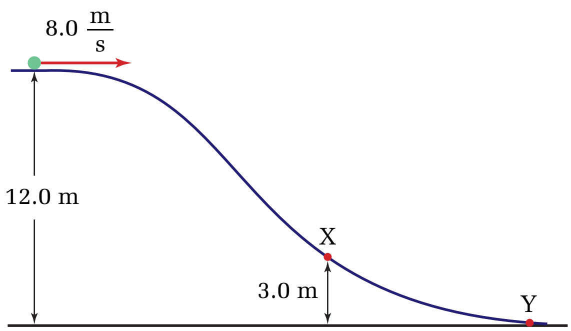
The speed at point B is expected to be larger than its speed at point A.

PRACTICE PROBLEMS

27. Determine the speed of the roller-coaster car in the sample problem at point C if point C is 8.0 m above the ground and another 4.00×10^2 J of heat energy are dissipated by friction between points B and C.



28. A sled at the top of a snowy hill is moving forward at 8.0 m/s, as shown in the diagram. The height of the hill is 12.0 m. The total mass of the sled and rider is 70.0 kg. Determine the speed of the sled at point X, which is 3.0 m above the base of the hill, if the sled does 1.22×10^3 J of work on the snow on the way to point X.



29. If the sled in the previous question reaches the base of the hill with a speed of 15.6 m/s, how much work was done by the snow on the sled between points X and Y?

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and follow the links for an in-depth activity on energy, Hooke's law, and simple harmonic motion.

In solving these problems, you have assumed that the value for the acceleration due to gravity (g) is constant at 9.81 m/s^2. You probably recall reading that this value is valid only for a small region close to Earth's surface. In Chapter 3, you learned that, as you go to the higher altitudes, the acceleration due to gravity decreases. You worked with forces of gravity at any distance from Earth, other planets, and even stars. You learned how to calculate the radii of orbits and orbital speed of satellites.

In the next chapter, you will focus on the energy requirements for sending a satellite into orbit and even for escaping Earth's gravitational pull entirely. You will also learn the importance of the conservation of momentum in navigating through space.

INVESTIGATION 5-E

Mechanical and Thermal Energy

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Identifying variables

Before 1800, physicists and chemists did not know that a relationship existed between mechanical energy and heat. Count Rumford (Benjamin Thompson: 1753–1814) was the first to observe such a relationship, followed by Julius Robert Mayer (1814–1878). Rumford and Mayer made some very important discoveries. Mayer was unable to express himself clearly in writing, however, so his discoveries were overlooked. Eventually, James Prescott Joule (1818–1889) was credited with the determination of the mechanical equivalence of heat. In this investigation you will perform experiments similar to those of Mayer and Joule.

Problem

How much heat is produced when a mass of lead pellets is repeatedly lifted and dropped through a known distance?

Equipment

- balance
- thermometer (°C)
- lead shot
- cardboard or plastic tube with a small hole in the side, close to one end; the ends must be able to be closed
- metre stick
- small amount of masking or duct tape

Procedure

1. Determine the mass of the lead shot.
2. Place the lead shot into the tube and close up the tube. Let the tube sit upright on a desk for several minutes to allow the tube and its contents to come to room temperature. Make sure that the hole is close to the bottom of the tube.
3. Insert the thermometer or temperature probe through the hole in the tube and nestle the end in the lead shot. Measure and record the temperature.
4. Close the hole.
5. Measure the length of the tube.
6. Repeatedly invert the tube for several minutes, waiting only to allow the lead shot to fall to the bottom on each inversion. Keep track of the number of inversions.
7. Finish the inversions with the hole near the bottom of the tube. Remove the tape and measure the temperature of the lead shot. Record the final temperature.

Analyze and Conclude

1. What were the initial and final temperatures of the lead shot? What was the total mass of the lead shot?
2. Determine the quantity of heat gained by the lead shot (the specific heat capacity of lead is 128 J/kg · C°).
3. Determine the total distance through which the lead shot was lifted by the inversions and calculate the total gain in gravitational potential energy of the lead.
4. Determine the percentage of the gravitational potential energy that was converted into heat.
5. If the conversion into heat does not account for all of the gravitational potential energy gained by the lead shot, where else might some of the energy have gone?

Apply and Extend

6. How could this investigation be improved? Try to design a better apparatus and, if possible, carry out the investigation again.
7. Do research and write a summary of the work of Rumford, Mayer, and Joule on the mechanical equivalence of heat.

CANADIANS IN PHYSICS

Follow Your Dreams

One of the great honours in physics is to have a physical law or constant named after you — Newton's laws, Planck's constant, the Heisenberg uncertainty principle. Now, the name of Canadian physicist Dr. Ian Keith Affleck can be added to this list. "Affleck-Dine Baryogenesis" is the name given to a physical mechanism that might have played an important role in the early universe in creating one of the classes of particles that now make up all of the matter that exists today.

For Dr. Affleck, who was born in Vancouver and grew up both there and in Ottawa, understanding nature has always been one of his great interests. "I became rather fascinated at a fairly young age with the idea that deep things about the universe could be understood by using mathematics," he explains. At high school in Ottawa, he was inspired by the intellectual enthusiasm of his physics teachers, and in university decided on a career in theoretical physics. Ironically, at the time, he "was not very optimistic about actually being able to make a career from my interests."

Dr. Ian Affleck

One of the great questions plaguing theoretical physicists is nothing less than the age-old philosophical question: Why are we here? It is believed that at the time of the Big Bang, there were nearly equal amounts of matter and antimatter. Since matter and antimatter annihilate each other, if the amounts of each were *exactly* equal, there would be nothing left after particle annihilation. So, there had to be some excess of matter over antimatter, and it had to be just the right amount of excess to yield the universe and the physics that exist today. Dr. Affleck, together with fellow theoretician Michael Dine, proposed a possible explanation — Affleck-Dine Baryogenesis. Verifying this principle is now an active part of physics research all over the world.

Dr. Affleck has since gone on to bring his mathematical talents to more immediate problems. Specifically, he has been hard at work adapting the mathematics he helped develop for an understanding of the universe to problems of understanding how and why high-temperature superconductors work. "I saw some opportunities to apply the same sort of mathematical ideas more directly," he says. Just as there is a problem with the pairing of particles and antiparticles in the early universe, there appears to be a pairing mechanism at work in the behaviour of high-temperature superconductors, so that it is possible to gain a deeper understanding of these materials through mathematics originally devised for more abstract research.

Such creative thinking has earned the physicist a number of awards, including the Rutherford Medal and the Governor General's Medal. He is also the recipient of many honorary degrees. His advice to aspiring physicists is simple: "They should follow up on what they find interesting, and not be afraid to follow their dreams."

Going Further

1. The astronomer Carl Sagan used to say, "You never know where inspiration will come from." One of Dr. Affleck's great achievements was to adapt what seemed like very abstract and very specific physical theory to a more concrete problem. This is not the first time this has happened in the history of physics; look into a few of the popular books on physics and see if you can find some other examples. (Hint: You can start with Carl Sagan.)
2. Much of Dr. Affleck's recent work has had to do with superconductivity. Superconductors have applications in medicine, engineering, and elsewhere. Research two different present-day applications related to superconductivity.
3. What are some of the difficulties with the superconductors now in use? Report and discuss with the class. Design a poster or a media presentation to present your findings.

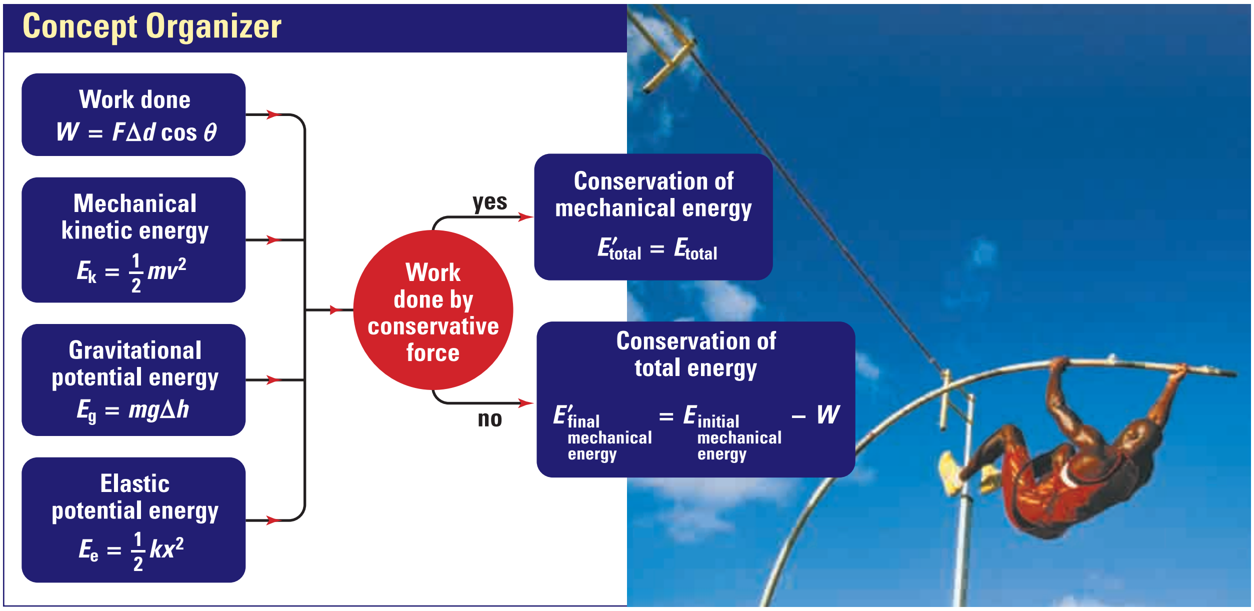


Figure 5.17 How many energy transformations are taking place in the photograph?

5.3 Section Review

1. **K/U**
   (a) Why is the application of the law of conservation of energy often much easier than the application of the law of conservation of momentum?
   (b) What conditions can increase the difficulty of applying the law of conservation of energy?

2. **K/U** Which types of energy are generally referred to as mechanical energy?

3. **C** Using examples not found in the textbook, describe and explain an example in which the forces are
   (a) conservative
   (b) non-conservative

4. **I** A student is sliding down a frictionless water slide at an amusement park.
   (a) Sketch a graph of gravitational potential energy against height for the descent. (No numbers are required on the axes.)
   (b) On the same axes, sketch a graph of the total energy of the student against height for the descent.
   (c) On the same axes, sketch a graph of the kinetic energy of the student against height for the descent.

5. **MC**
   (a) In an amusement park there is a ride on which children sit in a simulated log while it slides rapidly down a water-covered slope. At the bottom, the log slams into a trough of water, which slows it down. Why did the ride designers not simply have the log slam into a large spring?
   (b) Steel or plastic barrels are located along highways to cushion the impact if a car skids into a bridge abutment. These barrels are often filled with energy-absorbing material. Why are these barrels used instead of large springs to bring the cars to a stop?

CHAPTER 5 Review

REFLECTING ON CHAPTER 5

- Work is defined as the product of force times displacement. In general, if the force acts at an angle (θ) to the displacement, the work done by the force is given by $W = F\Delta d \cos\theta$.
- The work done by an applied force equals the change in energy produced by that applied force.
- Kinetic energy is expressed as $E_k = \frac{1}{2}mv^2$.
- Gravitational potential energy for positions near Earth's surface is expressed as $E_g = mg\Delta h$.
- An isolated system is one that neither gains energy from its environment nor loses energy to its environment.
- The law of conservation of energy states that, in an isolated system, the total energy is conserved, but can be transformed from one form to another.
- For an ideal spring, the restoring force is proportional to the amount of extension or compression of the spring. This is expressed as $F = -kx$, where k is the spring constant. The applied force that causes the spring to stretch (or compress) is equal in magnitude and opposite in direction to the restoring force: $F_a = kx$
- The amount of elastic potential energy stored in a spring is equal to the area under the force-extension (or compression) graph for the spring. It can be calculated from $E_e = \frac{1}{2}kx^2$.
- Applied forces are conservative if the amount of work that they do on an object as it moves between two points is independent of the path of the object between those points.
- Applied forces are non-conservative if the amount of work that they do on an object as it moves between two points is dependent on the path of the object between those points.
- Work done by an object on its environment is negative work and decreases the total energy of that object. Work done on an object by its environment is positive work and increases the total energy of the object.

Knowledge/Understanding

1. Explain what happens to the total mechanical energy over a period of time for open systems, closed systems, and isolated systems.
2. Write a general equation that relates the change in mechanical energy in systems to the amount of work done on it and the amount of heat lost by it.
3. You wind up the spring of a toy car and then release it so that it travels up a ramp. Describe all of the energy transformations that take place.
4. Compare how the everyday notion of work as "exerting energy to complete a task" differs from the physical definition of work.
5. Explain the sign convention for designating whether work is being done by an object on its environment or whether the environment is doing work on an object.
6. **(a)** Explain whether work done by a frictional force on an object can be positive.
   (b) Explain when the work done by the restoring force of a spring on a mass is considered to be positive and when it is considered to be negative with respect to the mechanical energy of the mass.
   (c) Discuss your answers to the above questions in terms of conservative and non-conservative forces.
7. Define and give an example of periodic motion.

Inquiry

8. Imagine taking a spring of 10 coils and cutting it in half. Will each smaller spring have a smaller, larger, or the same spring constant as the larger spring? (Hint: Consider the force required to compress the large and small springs by the same amount.)

9. A basic clock consists of an oscillator and a mechanism that is "turned" by the oscillator (to count the oscillations). Design a clock based on a simple pendulum or other oscillating device, using readily available materials. If possible, construct the clock and determine its accuracy. Even if you are not successful in constructing a functional clock, outline the technological challenges that you encountered.

10. The transformation of energy between kinetic and potential forms in an ideal simple harmonic oscillator can be modelled mathematically by writing a total mechanical energy equation for specified points during its motion. Consider a spring attached to a wall at floor level. A block of wood is attached to the other end of the spring so that the block can oscillate across the floor in a horizontal plane. Assume that the floor is frictionless. Set a frame of reference for the spring so that the equilibrium position of the system is $x = 0$ and the maximum displacements of the block is $x = -A$ and $x = +A$. Set the block of wood in motion by pulling it back to position +A.
    - **(a)** Write expressions for the total energy of the system at points −A, +A, and zero.
    - **(b)** At which of the three above points is the kinetic energy at its maximum and at its minimum?
    - **(c)** At which of the three points is the velocity at its maximum and at its minimum?
    - **(d)** Sketch a graph of energy versus position with individual curves for the elastic potential energy and the kinetic energy of the block as it oscillates. What is the geometrical shape of each curve?
    - **(e)** Sketch a velocity-versus-position graph.

11. Bowling balls need to be returned promptly from the end of the alley so that they can be used again. Sketch a ball-return system that requires no external energy source. Explain the energy transformations involved in the operation of your system. Identify the conservative and non-conservative forces that need to be taken into consideration. What features does the design include to minimize wear and tear on the bowling balls, despite their large mass?

Communication

12. Imagine that you are moving a negatively charged sphere toward a Van de Graaff generator. As you bring the sphere closer, does the energy of the system increase or decrease? Explain your reasoning.

13. Each of three stones is displaced to a vertical height of h. Stone R is placed on the top of a ramp, stone P is at the end of a taut pendulum string, and stone G is simply held above the ground. Do each of these stones have the same gravitational potential energy?
    - **(a)** If frictional forces are neglected, will each stone have the same kinetic energy at the instant before it reaches the bottom of its path? Explain your reasoning.
    - **(b)** If you consider likely frictional forces, will each stone have the same kinetic energy at the instant before it reaches the bottom of its path?
    - **(c)** Use the above two examples to differentiate between conservative and non-conservative forces.

14. A child descends a slide in the playground. Write expressions to show the total mechanical energy of the child at the top, halfway down, and at the bottom of the slide. Write a mathematical expression that relates the energy total at the three positions.

Making Connections

15. When stretched or compressed, a spring stores potential energy. Make a list of other common devices that store potential energy when temporarily deformed.

16. Research and write a brief report about how chemists use the concept of ideal springs to model the action of the bonds holding atoms together in molecules.

17. Car bumper systems are designed to absorb the impact of slow-speed collisions in such a way that the vehicles involved sustain no permanent damage. Prepare a presentation on how a bumper system works, including an explanation of the energy transformations involved.

Problems for Understanding

18. A 0.80 kg block of wood has an initial velocity of 0.25 m/s as it begins to slide across a table. The block comes to rest over a distance of 0.72 m.
    - **(a)** What is the average frictional force on the block?
    - **(b)** How much work is done on the block by friction?
    - **(c)** How much work is done on the table by the block?
19. A 1.5 kg book falls 1.12 m from a table to the floor.
    - **(a)** How much work did the gravitational force do on it?
    - **(b)** How much gravitational potential energy did it lose?
20. A 175 kg cart is pushed along level ground for 18 m, with a force of 425 N, and then released.
    - **(a)** How much work did the applied force do on the cart?
    - **(b)** If a frictional force of 53 N was acting on the cart while it was being pushed, how much work did the frictional force do on the cart?
    - **(c)** Determine how fast the cart was travelling when it was released.
    - **(d)** Determine how far the cart will travel after it is released.
21. A man is pushing a 75 kg crate at constant velocity a distance of 12 m across a warehouse. He is pushing with a force of 225 N at an angle of 15° down from the horizontal. The coefficient of friction between the crate and the floor is 0.24. How much work did the man do on the crate?
22. A boy, starting from rest, does 2750 J of work to propel himself on a scooter across level ground. The combined mass of the boy and scooter is 68 kg. Assume friction can be neglected.
    - **(a)** How fast is he travelling?
    - **(b)** What is his kinetic energy?
    - **(c)** If he then coasts up a hill, to what vertical height does he rise before stopping?
23. While coasting on level ground on a bicycle, you notice that your speed decreased from 12 m/s to 7.5 m/s over a distance of 50.0 m. If your mass combined with the bicycle's mass is 65 kg, calculate the average force that opposes your motion.
24. A 0.50 kg air puck is accelerated from rest with a force of 12.0 N. If the force acts over 45 cm and the surface is frictionless, how fast is the puck going when it is released?
25. If 25 N are required to compress a spring 5.5 cm, what is the spring constant of the spring?
26. **(a)** What is the change in elastic potential energy of a spring that has a spring constant of 120 N/m if it is compressed by 8.0 cm?
    - **(b)** What force is required to compress the spring by 8.0 cm?
27. A 0.500 kg mass resting on a frictionless surface is attached to a horizontal spring with a spring constant of 45 N/m. When you are not looking, your lab partner pulls the mass to one side and then releases it. When it passes the equilibrium position, its speed is 3.375 m/s. How far from the equilibrium position did your lab partner pull the mass before releasing it?
28. A mass m_1 is hung on a spring and stretches the spring by $x = 10.0$ cm. What is the spring constant in terms of the variables?
29. A dart gun has a spring with a constant of 74 N/m. An 18 g dart is loaded into the gun, compressing the spring from a resting length of 10.0 cm to a compressed length of 3.5 cm. If the spring transfers 75% of its energy to the dart after the gun is fired, how fast is the dart travelling when it leaves the gun?

30. A 12 g metal bullet (specific heat capacity: $c = 669$ J/kg°C) is moving at 92 m/s when it penetrates a block of wood. If 65% of the work done by the stopping forces goes into heating the metal, how much will the bullet's temperature rise in the process?

31. Consider a waterfall that is 120 m high. How much warmer is the water at the bottom of the waterfall than at the top? (The specific heat of water is 4186 J/kg °C.)

32. A spring with a constant of 555 N/m is attached horizontally to a wall at floor level. A 1.50 kg wooden block is pushed against it, compressing the spring by 12 cm, and then released.
   (a) How fast will the block be travelling at the instant it leaves the spring? (Assume that friction can be ignored and that the mass of the spring is so small that its kinetic energy can be ignored.)
   (b) If the block of wood travels 75 cm after being released and then comes to rest, what friction force opposes its motion?

33. A simple pendulum swings freely and rises at the end of its swing to a position 8.5 cm above its lowest point. What is its speed at its lowest point?

34. A 50.0 g pen has a retractable tip controlled by a button on the other end and an internal spring that has a constant of 1200 N/m. Suppose you hold the pen vertically on a table with the tip pointing up. Clicking the button into the table compresses the spring 0.50 cm. When the pen is released, how fast will it rise from the table? To what vertical height will it rise? (Assume for simplicity that the mass of the pen in concentrated in the button.)

35. A spring with a spring constant of 950 N/m is compressed 0.20 m. What speed can it give to a 1.5 kg ball when it is released?

36. A basketball player dunks the ball and momentarily hangs from the rim of the basket. Assume that the player can be considered as a 95.0 kg point mass at a height of 2.0 m above the floor. If the basket rim has a spring constant of 7.4×10^3 N/m, by how much does the player displace the rim from the horizontal position?

37. A 35 kg child is jumping on a pogo stick. If the spring has a spring constant of 4945 N/m and it is compressed 25 cm, how high will the child bounce? (Assume that the mass of the pogo stick is negligible.)

38. A spring with a spring constant of 450 N/m hangs vertically. You attach a 2.2 kg block to it and allow the mass to fall. What is the maximum distance the block will fall before it begins moving upward?

39. A 48.0 kg in-line skater begins with a speed of 2.2 m/s. Friction also does –150 J of work on her. If her final speed is 5.9 m/s,
   (a) determine the change (final – initial) in her gravitational potential energy.
   (b) By how much, and in which direction (up or down), has her height changed?

CHAPTER 6

Energy and Motion in Space

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Newton's law of universal gravitation
- Potential energy
- Centripetal force
- Kinetic energy

Master jugglers can keep as many as eight plates or seven flaming torches airborne and under perfect control at the same time. Amazing muscle and hand-eye co-ordination enables the launching of each object with precisely the right kinetic energy. Opposing Earth's gravitational attraction, this energy allows the object to free fall for a precise interval, returning to the height of the juggler's hand at just the right time and location to be caught and passed to the other hand for another toss.

Launching a missile or an Earth satellite is much like juggling. Work done against gravitational forces partially overcomes Earth's attraction and allows the object to follow a planned trajectory or to be inserted into a previously defined orbit. With even more initial energy, a space probe can eventually escape from Earth's orbit — or even from the solar system entirely. Successful launches depend on calculating, modelling, and simulating the energies needed to attain orbits or trajectories with specific shapes and sizes.

Your investigations of impulse, momentum, work, and energy have given you many of the mathematical tools needed to analyze energy and motion in space. In this chapter, you will refine your concept of gravitational potential energy, find out how much work must be done to boost an object away from a planet's surface, and investigate the energy of satellites in orbit.

QUICK LAB

Escape from a Planetoid

TARGET SKILLS

- Analyzing and interpreting
- Communicating results

Imagine that you are stationed on a spherical planetoid (a small planet-like object) somewhere in space. The planetoid has a mass of 1.0×10^{22} kg and a radius of 1.0×10^{6} m. You want to send a small 6.0 kg canister off into space so that it will escape the gravity of the planetoid and not fall back to the surface. You can accomplish this task by estimating the amount of work that must be done to lift the canister to 10 times the radius of the planet. You cannot use the formula $W = F\Delta d \cos \theta$, because the force changes with the distance from the centre of the planet. Therefore, you will need to use a graphical method, such as the one described in the following steps.

- Prepare a table with two headings: Distance from the centre of the planetoid (d), and Gravitational force (N). In the first column, write the following distances: 1.0×10^{6} m, 2.0×10^{6} m, 3.0×10^{6} m, and so on, up to 10.0×10^{6} m. (Notice that these values are multiples of the radius of the planetoid, where 1.0×10^{6} m represents the surface of the planetoid.)
- Calculate the force of gravity on the 6.0 kg canister for each of these distances.
- Plot the graph of gravitational force (y-axis) against distance from the centre of the planetoid. Since your graph is of force versus position, the area under the graph represents the amount of work required to move the canister to a separation of 10 radii (9 radii from the surface). Graphically determine the area under the curve and, thus, the amount of work done.

  Note: There are several ways to find the area under the graph. One is to determine the graphical area represented by each square and then count the number of squares under the curve. Where the curve actually crosses a square, include the square if half or more of it is under the curve. Another method is to divide the area up into different regions and approximate their areas by using figures such as trapezoids and triangles.

Analyze and Conclude

1. If the canister has been lifted a distance of 10 radii and remains there, what type of energy does the area under the curve represent?
2. To launch the canister so that it will be able to travel straight out to a separation of 10 radii, how much kinetic energy must it be given at the start? From this kinetic energy, determine the required speed that would allow the canister to reach this separation.
3. The gravitational force that is trying to pull the canister back is extremely small at a separation of 10 radii. With only slightly more speed, the canister would never return to the planetoid, so the speed that you found is essentially the escape speed for the planetoid. What is the escape speed for this planetoid?

Apply and Extend

4. Considering the energies involved, does the canister have to be thrown straight up at its escape speed for it to be able to escape? Give reasons for your answer.

6.1 Energy for Lift-Off

SECTION EXPECTATIONS

- Calculate the generalized gravitational potential energy for an isolated system involving two objects, based on the law of universal gravitation.
- Determine the escape speed for a given celestial object.
- Develop appropriate scientific models for natural phenomena.

KEY TERMS

- escape energy
- binding energy
- escape speed

Did you know that it takes almost 10 t of fuel for a large passenger jet to take off? It is hard to even imagine the amount of energy required for a rocket or space shuttle to lift off. How do the engineers and scientists determine these values?

Figure 6.1 The energy to hurl this spacecraft into orbit comes from the chemical potential energy of the fuel.

MISCONCEPTION

Gravity and Orbiting Spacecraft

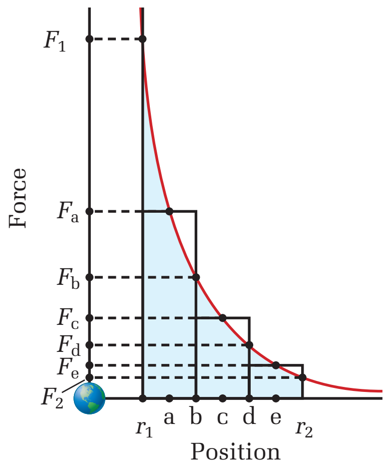
Many people believe that gravity does not act on orbiting spacecraft. In fact, a satellite such as the International Space Station *Freedom* still has about 80% of its initial weight. The impression of weightlessness comes from the fact that the weight is being used to hold the space station in its orbit. If there was no weight, it would simply continue to move off into space.

Work for Lift-Off

One way to determine the amount of energy needed to carry out a particular task is to determine the amount of work that you would have to do. When a spacecraft is lifting off from Earth, the force against which it must do work is the force of gravity.

In Chapter 3, Planetary and Satellite Dynamics, you learned that the equation for the gravitational force is $F_g = G\frac{m_1 m_2}{r^2}$. When working with a planet and a small object, physicists often use M for the planet and m for the small object. You can then write the equation as $F_g = G\frac{Mm}{r^2}$. In the Quick Lab, Escape from a Planetoid, you used this expression for force and multiples of the radius of the planetoid for position, and then estimated the area under the curve of force versus position to estimate the amount of work needed to escape from the planetoid. However, if you were an engineer working for the space program, you would want a much more accurate value before you launched a spacecraft. In the following derivation, you will develop a general expression for the area under the curve of F_g versus r from position r_1 to r_2. This area will be the amount of work needed to raise an object such as a spacecraft of mass m from a distance r_1 to a distance r_2 from the centre of a planet of mass M.

- Draw a graph of gravitational force versus position, where the origin of the graph lies at the centre of the planet.
- Choose points r_1 and r_2. Divide the axis between r_1 and r_2 into six equal spaces and label the end point "a" through "e."
- Draw three rectangles with heights F_a, F_c, and F_e.



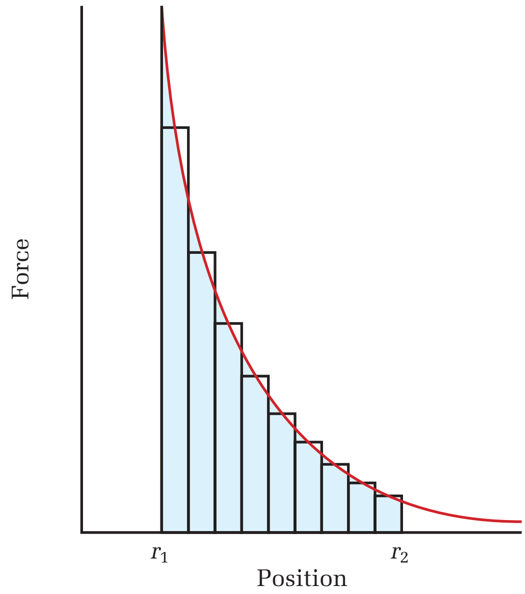
- A first rough estimate of the total work done to move m from r_1 to r_2 will be the sum of the areas of the rectangles.

$$W_{total} = W_e + W_c + W_a$$
$$W_{total} = F_a(b - r_1) + F_c(d - b) + F_e(r_2 - d)$$

- You could simplify this equation if you could express the forces in terms of the points on the curve at the ends of the rectangles, instead of the centre. For example, how can you express F_a in terms of F_1 and F_b? Clearly, F_a is not the average or arithmetic mean of F_1 and F_b, because the curve is an exponential curve. However, it can be accurately expressed as the *geometric* mean, which is expressed as $\sqrt{F_1F_b}$. Substitute the geometric mean of each value for force into the equation for work. Notice that in the last step, all intermediate terms have cancelled each other and only the first and last terms remain.

$$W_{total} = \sqrt{F_1F_b}(b - r_1) + \sqrt{F_bF_d}(d - b) + \sqrt{F_dF_2}(r_2 - d)$$
$$W_{total} = \sqrt{\frac{GMm}{{r_1}^2} \cdot \frac{GMm}{b^2}}(b - r_1) + \sqrt{\frac{GMm}{b^2} \cdot \frac{GMm}{d^2}}(d - b) + \sqrt{\frac{GMm}{d^2} \cdot \frac{GMm}{{r_2}^2}}(r_2 - d)$$
$$W_{total} = \frac{GMm}{r_1b}(b - r_1) + \frac{GMm}{bd}(d - b) + \frac{GMm}{dr_2}(r_2 - d)$$
$$W_{total} = GMm\left(\frac{b - r_1}{r_1b} + \frac{d - b}{bd} + \frac{r_2 - d}{dr_2}\right)$$
$$W_{total} = GMm\left(\frac{1}{r_1} - \frac{1}{b} + \frac{1}{b} - \frac{1}{d} + \frac{1}{d} - \frac{1}{r_2}\right)$$
$$W_{total} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- At first consideration, this result would appear to be a rough estimate. However, consider the fact that you could make as many rectangles as you want. Examination of the figure on the right shows that as the number of rectangles increases, the sum of their areas becomes very close to the true area under the curve. If you drew an infinite number of rectangles, your result would be precise. Now, analyze the last two mathematical steps above. No matter how many rectangles you drew, all of the intermediate terms would cancel and the result would be exactly the same as the result above. In this case, the result above is not an approximation but is, in fact, exact.



MATH LINK

The arithmetic mean of two values, m and n, is $\frac{m+n}{2}$. The geometric mean is $\sqrt{mn}$.

Escape Energy and Speed

You can now use the equation that you just derived — $W_{total} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ — to determine the amount of energy needed by a spacecraft to escape from Earth's gravitational pull. Let r_1 be Earth's radius so that the spacecraft will be sitting on the ground. Let r_2 be so far out into space that the force of gravity is negligible. Notice that as r_2 becomes exceedingly large, $\frac{1}{r_2}$ approaches zero, so the equation for the amount of work that must be done to free the spacecraft from the surface of the planet is $W_{to\ escape} = GMm/r_1$.

Work represents the change in energy that, in this case, is the amount of energy that a spacecraft would need to escape Earth's gravity. When a spacecraft blasts off from Earth, that amount of energy is provided as kinetic energy through the thrust of the engines. If the spacecraft is to escape Earth, therefore, it must be provided with at least GMm/r_1 J of kinetic energy, which probably come from GMm/r_1 J of chemical potential energy in the fuel. For this reason, the quantity GMm/r_1 is known as the **escape energy** for the spacecraft. If a spacecraft has any less energy, you could say that it is *bound* by Earth's gravity. Therefore, you can think of the value GMm/r_1 as the **binding energy** of the spacecraft to Earth.

Typically, when a spacecraft lifts off, rockets fire, the craft lifts off, and the rockets continue to fire, accelerating the spacecraft as it rises. However, you can often obtain important information by considering the extreme case. For example, if all of the escape energy must be provided as initial kinetic energy at the moment of lift-off, what would be the spacecraft's initial speed?

PHYSICS FILE

Escape speed is often referred to as "escape velocity." However, since the direction in which the escaping object is headed has no effect on its ability to escape (unless it is headed into the ground), the correct term is "escape speed."

- The initial kinetic energy of the spacecraft would have to be equal to the escape energy. Let r_p be the radius of the planet.

$$\frac{1}{2}mv^2 = \frac{GMm}{r_p}$$

- Solve for v.

$$v^2 = \frac{2GM\cancel{m}}{\cancel{m}r_p}$$

$$v = \sqrt{\frac{2GM}{r_p}}$$

This equation gives the **escape speed**, the minimum speed at the surface that will allow an object to leave a planet and not return. Notice that the speed does not depend on the mass of the escaping object.

ESCAPE SPEED

The escape speed of an object from the surface of a planet is the square root of two times the product of the universal gravitational constant and the mass of the planet divided by the radius of the planet.

$$v = \sqrt{\frac{2GM}{r_p}}$$

Quantity	Symbol	SI unit
escape speed	v	$\frac{m}{s}$ (metres per second)
mass of planet	M	kg (kilograms)
radius of planet	r_p	m (metres)
universal gravitational constant	G	$\frac{N \cdot m^2}{kg^2}$ (newton metres squared per kilograms squared)

Unit Analysis

$$\frac{m}{s} = \sqrt{\frac{\frac{N \cdot m^{\cancel{2}}}{kg^{\cancel{2}}} \cdot \cancel{kg}}{\cancel{m}}} = \sqrt{\frac{\frac{\cancel{kg} \cdot m}{s^2} \cdot m}{\cancel{kg}}} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s}$$

SAMPLE PROBLEM

Escaping from Earth

Determine the escape energy and escape speed for a 1.60×10^4 kg rocket leaving the surface of Earth.

continued ►

continued from previous page

Conceptualize the Problem

- *Escape speed* is the speed at which a spacecraft would have to be lifting off Earth's surface in order to *escape Earth's gravity* with no additional input of energy.
- You can find the *radius* and *mass* of *Earth* in Appendix B, Physical Constants and Data.

Identify the Goal

The escape energy, E_{escape}, and escape speed, v_{escape}, for a rocket from Earth

Identify the Variables and Constants

Known	Implied	Unknown
$m_{rocket} = 1.60 \times 10^4$ kg	$G = 6.673 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$	E_{escape}
	$r_{Earth} = 6.38 \times 10^6$ m	v_{escape}
	$m_{Earth} = 5.98 \times 10^{24}$ kg	

Develop a Strategy

State the equation for escape energy. Substitute and solve.

$$E_{escape} = \frac{GM_{Earth}m_{object}}{r_{Earth}}$$

$$E_{escape} = \frac{\left(6.673 \times 10^{-11} \frac{N \cdot \cancel{m^2}}{\cancel{kg^2}}\right)(5.98 \times 10^{24}\ \cancel{kg})(1.60 \times 10^4\ \cancel{kg})}{6.38 \times 10^6\ \cancel{m}}$$

$$E_{escape} = 1.00 \times 10^{12}\ J$$

State the equation for escape speed. Substitute and solve.

$$v_{escape} = \sqrt{\frac{2GM_{Earth}}{r_{Earth}}}$$

$$v_{escape} = \sqrt{\frac{2\left(6.673 \times 10^{-11} \frac{N \cdot m^2}{kg^2}\right)(5.98 \times 10^{24}\ kg)}{6.38 \times 10^6\ m}}$$

$$v_{escape} = 1.1184 \times 10^4 \frac{m}{s}$$

$$v_{escape} \cong 1.12 \times 10^4 \frac{m}{s}$$

The escape energy for this rocket is 1.00×10^{12} J and its escape speed is 1.12×10^4 m/s or 11.2 km/s.

Validate the Solution

A unit analysis escape energy shows $\frac{\frac{N \cdot m^2}{kg^2} \cdot kg \cdot kg}{m} = \frac{N \cdot m^{\cancel{2}} \cdot \cancel{kg^2}}{\cancel{kg^2} \cdot \cancel{m}} = N \cdot m = J$

which is correct for energy. A unit analysis for escape speed shows

$$\sqrt{\frac{\frac{N \cdot m^2}{kg^2} \cdot kg}{m}} = \sqrt{\frac{N \cdot m^{\cancel{2}} \cdot \cancel{kg}}{\cancel{m} \cdot kg^{\cancel{2}}}} = \sqrt{\frac{\cancel{kg} \cdot m}{s^2} \cdot \frac{m}{\cancel{kg}}} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s}$$ which is correct

for speed. A value of a few km/s agrees with the types of speeds observed during rocket lift-offs.

PRACTICE PROBLEMS

1. Determine the escape energy and escape speed for an asteroid with a mass of 1.00×10^{22} kg and a radius of 1.00×10^{6} m. How closely does your answer for escape energy compare to the value obtained by finding the area under the force-separation graph in the Quick Lab at the beginning of this chapter?

2. Calculate the escape energy and escape speed for a 15 g stone from Mars. Such stones have been blasted off the surface of Mars by meteor impacts and have fallen to Earth, where they are found preserved in the snow and ice of the Antarctic. The mass of Mars is 6.42×10^{23} kg and the radius of Mars is 3.38×10^{6} m.

3. The *Pioneer 10* spacecraft, shown in the photo, was the first to journey beyond Jupiter and is now well past Pluto. To escape from the solar system, how fast did *Pioneer 10* have to be travelling as it passed the orbit of Jupiter? Assume that the mass of the solar system is essentially concentrated in the Sun. The mass of the Sun is 1.99×10^{30} kg and the radius of Jupiter's orbit is 7.78×10^{11} m.

6.1 Section Review

1. K/U State the equations for escape energy and escape speed. Indicate the meaning of each factor and the appropriate units for each factor.

2. I Prove from basic energy equations that the escape speed for an object from the surface of a planet is independent of the mass of the object.

3. K/U Explain the meaning of (a) escape energy, (b) escape speed, and (c) binding energy.

4. C Sketch graphs to show how the escape speed from a planet varies with
   - **(a)** the mass of the planet for constant planetary radius
   - **(b)** the radius of the planet for constant planetary mass
   - **(c)** the mass of the escaping object from a given planet

5. C What factors would make the actual energy that must be provided in the form of fuel greater than the escape energy? Explain the role of each factor.

6. MC Look up the meaning of the term "bond energy" as it applies to bonds between the atoms in a diatomic molecule. How does the concept of bond energy relate to the concept of escape energy?

UNIT PROJECT PREP

Space-based energy schemes have for a long time been promoted as the environmentally friendly way to provide energy of the future. Understanding the physics concepts of low Earth orbit provides you with a method of judging each scheme's feasibility.

- List environmental factors involved in getting into Earth orbit.
- How do you envision space travel in the near future?
- Do you believe that environmental or other factors will motivate more space-based power initiatives?

6.2 Energy of Orbiting Satellites

SECTION EXPECTATIONS

- Analyze the factors affecting the motion of isolated celestial objects, and calculate the gravitational potential energy for such a system.
- Analyze isolated planetary and satellite motion and describe it in terms of the forms of energy and energy transformations that occur.
- Calculate the kinetic and gravitational potential energy of a satellite that is in a stable circular orbit around a planet.

KEY TERMS

- circular orbit
- total orbital energy

The rockets that launched *Voyager 1* and *Voyager 2,* were designed to escape Earth's gravity and send them into space to search the solar system. The *Voyager* craft have found such things as new moons orbiting Jupiter, Saturn, Uranus, and Neptune. They have also discovered volcanoes on Io and rings around Jupiter. However, the majority of satellites are launched into Earth's orbit and will remain captive in Earth's gravitational field, destined to circle the planet year after year and perform tasks of immediate importance to people on Earth.

Satellites in Earth Orbit

Some of these satellites monitor the weather, the growth and health of crops, the temperature of the oceans, the presence of ice floes, and the status of the ozone layer. Others actively scan Earth's surface with radar to enhance our knowledge of the geography and geology of our planet. These days, many people routinely use satellites to tell them where they are (for example, the Global Positioning System) and to provide them with mobile communication and seemingly limitless television entertainment.

Other satellites look outward, monitoring regions of the electromagnetic spectrum that cannot pass easily through Earth's atmosphere. In doing so, they tell us about our own solar system, as well as other solar systems, stars, and galaxies located many light-years away from us.

The largest artificial satellite in Earth's orbit is the International Space Station. During its construction and lifetime, it has been serviced from other temporary satellites, the space shuttles. For these shuttles to rendezvous successfully with the space station, teams of scientists, engineers, and technicians must solve problems involving the orbital motions and energies that are the subject of this section.

Most satellites are in either a **circular orbit** or a near-circular orbit. In Chapter 3, you learned how the force of gravity acts as a centripetal force, holding each satellite in its own unique orbit. You learned how to calculate the orbital speeds of the satellites that orbit at specific radii. In this section, you will focus on the energies of these satellites.

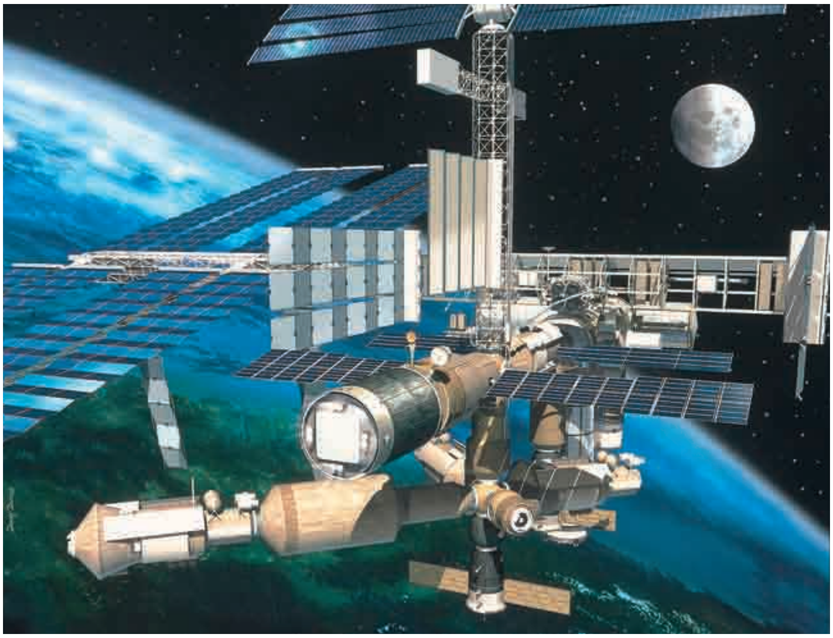


Figure 6.2 A thorough understanding of orbital mechanics is necessary for a successful rendezvous with the space station or with any other satellite.

Orbital Energies

The orbital energy of a satellite consists of two components: its kinetic energy and its gravitational potential energy. To a great extent, the Earth-satellite system can be treated as an isolated system. Subtle effects, such as the pressure of light, the solar wind, and collisions with the few atmospheric molecules that exist at that distance from the surface, can change the energy of the system. In fact, without the occasional boost from a thruster, the orbits of all satellites will decay. However, this generally takes decades. These effects are so tiny over the short run that you will neglect them in the following topics.

- Write the relationship that represents a planet's gravity providing a centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

- Multiply both sides of the equation by r.

$$\frac{mv^2\cancel{r}}{\cancel{r}} = \frac{GMm}{r^{\cancel{2}}}\cancel{r}$$

$$mv^2 = \frac{GMm}{r}$$

- Multiply both sides of the equation by $\frac{1}{2}$.

$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

- Since $\frac{1}{2}mv^2$ is the kinetic energy of any object of mass m, you can substitute E_k for the expression.

$$E_k = \frac{GMm}{2r}$$

The kinetic energy of an orbiting satellite of mass m is $E_k = \frac{GMm}{2r}$.

Gravitational Potential Energy

In Chapter 5, Conservation of Energy, you demonstrated that the change in the gravitational potential energy of an object was equal to the work done in raising the object from one height to another. That relationship ($W = mg\Delta h$) was the special case, where any change in height was very close to Earth's surface. Since you are now dealing with objects being launched into space, you cannot use the special case. You must consider the change in the force of gravity as the distance from Earth increases. Fortunately, however, you have already developed an expression for the amount of work required to lift an object from a distance r_1 to a distance r_2 from Earth's centre. Therefore, the result of your derivation is equal to the change in the gravitational potential energy between those two positions.

$$\Delta E_g = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

As you know, you must choose a reference point for all forms of potential energy. Earth's surface in no longer an appropriate reference, because you are measuring distances from Earth's centre to deep into space. Physicists have accepted the convention of assigning the reference or zero point for gravitational potential energy as an infinite distance from the centre of the planet or other celestial body that is exerting the gravitational force on the object of mass m. This is appropriate because at an infinite distance, the gravitational force goes to zero. You can now state that the gravitational potential energy of an object at a distance r_2 from Earth's centre is the amount of work required to move an object from an infinite distance, r_1, to r_2.

$$E_g = GMm\left(\frac{1}{\infty} - \frac{1}{r_2}\right)$$

$$E_g = GMm\left(0 - \frac{1}{r_2}\right)$$

$$E_g = -\frac{GMm}{r_2}$$

Since there is only one distance (r_2) in the equation, it is often written without a subscript. Notice, also, that the value is negative. This is simply a result of the arbitrary choice of an infinite distance for the reference position. You will discover as you work with the concept that it is a fortunate choice.

GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy of an object is the negative of the product of the universal gravitational constant, the mass of the planet or celestial body, and the mass of the object, divided by the distance from the centre of the planet or celestial body.

$$E_g = -\frac{GMm}{r}$$

Quantity	Symbol	SI unit
gravitational potential energy	E_g	J (joules)
universal gravitational constant	G	$\frac{N \cdot m^2}{kg^2}$ (newton metres squared per kilograms squared)
mass of the planet or celestial body	M	kg (kilograms)
mass of the object	m	kg (kilograms)
distance from centre of planet or celestial body	r	m (metres)

Unit Analysis

$$\text{joule} = \frac{\frac{\text{newton} \cdot \text{metre}^2}{\text{kilogram}^2} \cdot \text{kilogram} \cdot \text{kilogram}}{\text{metre}}$$

$$J = \frac{\frac{N \cdot m^{\cancel{2}}}{\cancel{kg^2}} \cdot \cancel{kg} \cdot \cancel{kg}}{\cancel{m}} = N \cdot m = J$$

Note: Use of this equation implies that the reference or zero position is an infinite distance from the planet or celestial body.

It might seem odd that the potential energy is always negative. Since changes in energy are always of interest, however, these changes will be the same, regardless of the location of the zero level.

To illustrate this concept, consider the houses in the Loire Valley in France that are carved out of the face of limestone cliffs as shown in Figure 4.3(B). To the person on the cobblestone street, everyone on floors A, B, and C in Figure 6.3(A) would have positive gravitational energy, due to their height above the street. However, to a person on floor B, those on floor A are at a negative height, and so have negative gravitational potential energy relative to them. At the same time, the person on floor B would consider that people on floor C would have a positive gravitational potential energy because they are higher up the cliff.

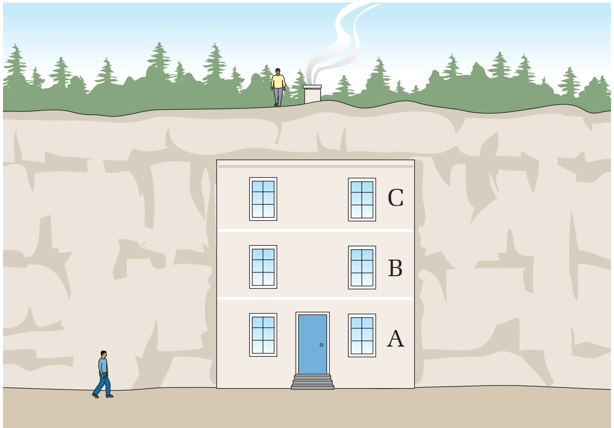


Figure 6.3 Some houses in the Loire Valley are carved out of limestone cliffs.

Naturally, the person standing on the roof beside the chimney would consider that everyone in the house had negative gravitational potential energy. All of the residents would agree, however, on the amount of work that it took to carry a chair up from floor A to floor C, so the energy change would remain the same, regardless of the observer's level. At the same time, a book dropped from a window in floor B would hit the ground with the same kinetic energy, regardless of the location of the zero level for gravitational potential energy.

Figure 6.4 is a graph of the gravitational potential energy of a 1.0 kg object as it moves away from Earth's surface. Since work must be done on that object to increase the separation, the object is often referred to as being in a gravitational potential energy "well."

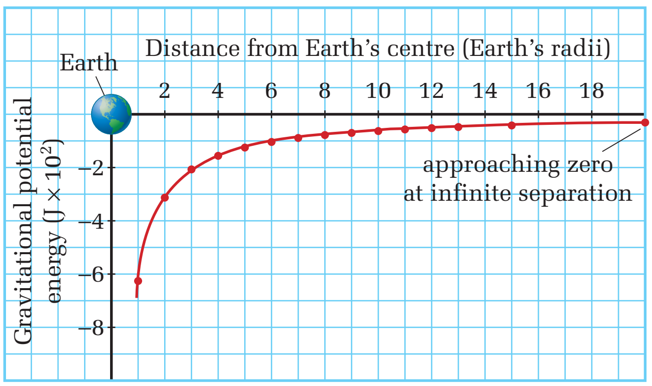


Figure 6.4 Since work must be done on the 1.0 kg object to move it away from Earth, although the gravitational potential energy is always negative, it is increasing (becoming less negative) as it retreats farther and farther from Earth.

In summary, the energies of orbiting objects can be expressed as

$$E_g = -\frac{GMm}{r}$$

$$E_k = \frac{GMm}{2r}$$

Adding, you obtain

$$E_{total} = E_k + E_g$$

$$E_{total} = \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right)$$

$$E_{total} = -\frac{GMm}{2r}$$

The last equation, the **total orbital energy,** involves only the mechanical energies — gravitational potential energy and kinetic energy. Other forms of energy, such as thermal energy, are not considered unless the satellite comes down in flames through the atmosphere.

You can obtain key information by determining whether the total orbital energy of an object is positive, zero, or negative. First, consider the conditions under which an object would have zero total orbital energy around a central object, such as a planet or star.

If an object is so far from Earth that gravity cannot pull it back, its gravitational potential energy is zero. If the object is motionless at that point, its kinetic energy is also zero, which gives a total energy of zero. The total orbital energy could also be zero if the magnitude of the kinetic and potential energies were equal. Under these conditions, the kinetic energy would be just great enough to carry the object to a distance at which gravity could no longer pull it back. It would then have no kinetic energy left and it would be motionless. By a similar analysis, you could draw all of the following conclusions.

- If the total of the kinetic and gravitational potential energies of an object is zero, it can just escape from the central object.
- If the total of the kinetic and gravitational potential energies of an object is greater than zero, it can escape from the central object and keep on going.
- If the total of the kinetic and gravitational potential energies of an object is less than zero, it cannot escape from the central object. It is said to be bound to the object.

The extra energy needed to free the object is called the binding energy. Since the object will be free with a total energy of zero, the binding energy is always the negative of the total energy: $E_{binding} = -E_{total}$.

TECHNOLOGY LINK

Fire a cannon ball into space? In the early 1960s, Project HARP (High Altitude Research Project) did just that. Scientists at McGill University in Montréal welded two U.S. Navy cannon barrels together into a "super-gun" that fired 91 kg instrumentation packages to a height of more than 145 km from a launch site on the Caribbean island of Barbados.

HARP cannon

Photo courtesy from the web sites: http://www.phy6.org/stargaze/Smartlet.htm and http://www-istp.gsfc.nasa.gov/stargaze/Smartlet.htm taken by Peter Millman.

The orbital energy equations also have some informative simple relationships among themselves, as listed below.

- The magnitudes of the kinetic energy, the total energy, and the binding energy of an orbiting object are the same $|E_k| = |E_t| = |E_{binding}|$
- The magnitude of the gravitational potential energy is twice that of the other energies. $|E_g| = 2|E_k| = 2|E_t| = 2|E_{binding}|$
- If a satellite is in an orbit close to the planet, the radius of the orbit is essentially the same as the radius of the planet: $r_{orbit} \cong r_{planet} \cong r$. $$E_k = \frac{GMm}{2r}$$
- At the planet's surface, the energy needed to break free was seen in Section 6.1 to be $$E_{binding} = \frac{GMm}{2r}$$

By comparing the last two equations, you can see that the satellite in a circular orbit close to the planet already has half of the energy it needs to completely escape from that planet. The following problems will help you to develop a deeper understanding of orbital energies.

SAMPLE PROBLEMS

Space Problems

1. On March 6, 2001, the Mir space station was deliberately crashed into Earth. At the time, its mass was 1.39×10^3 kg and its altitude was 220 km.

(a) Prior to the crash, what was its binding energy to Earth?

(b) How much energy was released in the crash? Assume that its orbit was circular.

Conceptualize the Problem

- When Mir was in Earth orbit, it had *kinetic* and *gravitational potential energy*, both of which are determined by its *orbital radius*.
- Mir's *binding energy* is the *negative* of its *total energy*.
- After the crash, Mir had *zero kinetic energy*.

- The law of *conservation of energy* applies; therefore, the energy released in the crash is the *difference* between the *total energy in orbit* and the *total energy* when resting on *Earth's surface*.

Identify the Goals

The binding energy, $E_{binding}$, of the Mir space station to Earth

The energy released during the crash of the Mir space station

Identify the Variables and Constants

Known	Implied	Unknown
$m = 1.39 \times 10^3$ kg (Mir)	$G = 6.673 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$	E_{total} in orbit
$h = 2.20 \times 10^5$ m	$M = 5.978 \times 10^{24}$ kg (Earth)	$E_{binding}$ in orbit
	$r_{Earth} = 6.378 \times 10^6$ m	E_g on ground
		ΔE_{total}

Develop a Strategy

Determine the orbital radius.

$$r_{orbit} = r_{earth} + h_{from\ Earth}$$
$$r_{orbit} = 6.378 \times 10^6\ m + 2.20 \times 10^5\ m$$
$$r_{orbit} = 6.598 \times 10^6\ m$$

Calculate the total orbital energy before the crash.

$$E_t = -\frac{GMm}{2r}$$
$$E_t = -\frac{\left(6.673 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}\right)\left(5.978 \times 10^{24}\ kg\right)\left(1.39 \times 10^3\ kg\right)}{2\left(6.598 \times 10^6\ m\right)}$$
$$E_t = -4.2019 \times 10^{10}\ J$$

Binding energy is the negative of the total energy.

$$E_{binding} = +4.2019 \times 10^{10}\ J$$
$$E_{binding} \cong +4.20 \times 10^{10}\ J$$

(a) The binding energy of the Mir space station in orbit was 4.20×10^{10} J.

Calculate the mechanical energy after the crash. Note that when Mir was on Earth's surface, its kinetic energy was zero.

$$E_k = 0\ J$$
$$E_g = -\frac{GMm}{r}$$
$$E_g = -\frac{\left(6.673 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}\right)\left(5.978 \times 10^{24}\ kg\right)\left(1.39 \times 10^3\ kg\right)}{6.378 \times 10^6\ m}$$
$$E_g = -8.6938 \times 10^{10}\ J$$
$$E_{total} = E_k + E_g$$
$$E_{total} = 0\ J - 8.6938 \times 10^{10}\ J$$
$$E_{total} = -8.6938 \times 10^{10}\ J$$

continued ►

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Determine the difference in total energy before and after the crash.

$$\Delta E = E'_{total} - E_{total}$$
$$\Delta E = -8.6938 \times 10^{10}\ \text{J} - (-4.2019 \times 10^{10}\ \text{J})$$
$$\Delta E = -4.4919 \times 10^{10}\ \text{J}$$
$$\Delta E \cong -4.49 \times 10^{10}\ \text{J}$$

(b) When Mir crashed, 4.49×10^{10} J of energy were released into the environment.

Validate the Solution

The final answer is negative, which indicates a decrease in the energy of the system or a loss of energy to the environment.

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2. A 4025 kg spacecraft (including the astronauts) is in a circular orbit 256 km above the lunar surface. Determine

(a) the kinetic energy of the spacecraft

(b) the total orbital energy of the spacecraft

(c) the binding energy of the spacecraft

(d) the speed required for escape

Conceptualize the Problem

- The spacecraft is in a circular orbit around the Moon, so the *Moon* is the *central* body.
- The spacecraft is *moving*, so it has *kinetic energy*.
- The spacecraft is in *orbit*, so it has *gravitational potential energy*.
- *Binding energy* is the amount of energy necessary to escape the *gravitational* pull of the *central body*.
- To escape a central body, a spacecraft must *increase* its *kinetic energy* until the *total energy* is *zero*.
- If you know the *kinetic energy*, you can find *speed*.

Identify the Goals

(a) The kinetic energy of the spacecraft, E_k

(b) The total orbital energy of the spacecraft, E_t

(c) The binding energy of the spacecraft, $E_{binding}$

(d) The speed required for escape, v_{escape}

Identify the Variables and Constants

Known	Implied	Unknown
$m = 4025$ kg	$G = 6.673 \times 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	r_{orbit}
$h = 256$ km	$r_{Moon} = 1.738 \times 10^6$ m	E_k
	$M_{Moon} = 7.36 \times 10^{22}$ kg	E_t
		$E_{binding}$
		v_{escape}

Develop a Strategy

Determine the orbital radius of the spacecraft.

$$r_{orbit} = r_{Moon} + h$$

$$r_{orbit} = 1.738 \times 10^6 \text{ m} + 0.256 \times 10^6 \text{ m}$$

$$r_{orbit} = 1.994 \times 10^6 \text{ m}$$

Calculate the orbital kinetic energy.

$$E_k = \frac{GMm}{2r}$$

$$E_k = \frac{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.36 \times 10^{22} \text{ kg})(4025 \text{ kg})}{2(1.994 \times 10^6 \text{ m})}$$

$$E_k = 4.9569 \times 10^9 \text{ J}$$

$$E_k \cong 4.96 \times 10^9 \text{ J}$$

(a) The kinetic energy of the spacecraft is 4.96×10^9 J.

Total orbital energy is the negative of the kinetic energy.

$$E_t = -E_k$$

$$E_t = -4.96 \times 10^9 \text{ J}$$

(b) The total energy of the spacecraft is -4.96×10^9 J.

The binding energy is the negative of the total energy

$$E_{binding} = -E_t$$

$$E_{binding} = -(-4.952 \times 10^9 \text{ J})$$

$$E_{binding} = 4.96 \times 10^9 \text{ J}$$

(c) The binding energy of the spacecraft is 4.96×10^9 J.

The binding energy must come through additional kinetic energy

$$E'_k = E_k + E_{binding}$$

$$E'_k = 4.9569 \times 10^9 \text{ J} + 4.9569 \times 10^9 \text{ J}$$

$$E'_k = 9.9138 \times 10^9 \text{ J}$$

Find the speed from the kinetic energy.

$$\frac{1}{2}mv^2 = E_k$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$v = \sqrt{\frac{2(9.9138 \times 10^9 \text{ J})}{4025 \text{ kg}}}$$

$$v = 2.2195 \times 10^3 \frac{\text{m}}{\text{s}}$$

$$v \cong 2.22 \times 10^3 \frac{\text{m}}{\text{s}}$$

(d) The escape speed for the spacecraft is 2.22×10^3 m/s.

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Validate the Solution

The escape speed for the spacecraft could have been determined from the equation $v_{escape} = \sqrt{\frac{2GM}{r}}$. Carrying out this calculation yields the same escape speed. Since it agrees with the results of the energy calculations, it acts as a check on the first three answers.

PRACTICE PROBLEMS

4. A 55 kg satellite is in a circular orbit around Earth with an orbital radius of 7.4×10^6 m. Determine the satellite's

(a) kinetic energy

(b) gravitational potential energy

(c) total energy

(d) binding energy

5. A 125 kg satellite in a circular orbit around Earth has a potential energy of -6.64×10^9 J. Determine the satellite's

(a) kinetic energy

(b) orbital speed

(c) orbital radius

6. A 562 kg satellite is in a circular orbit around Mars. Data: $r_{Mars} = 3.375 \times 10^6$ m; $r_{orbit} = 4.000 \times 10^6$ m; $M_{Mars} = 6.420 \times 10^{23}$ kg

(a) If the satellite is allowed to crash on Mars, how much energy will be released to the Martian environment?

(b) List several of the forms that the released energy might take.

7. From the orbital kinetic energy of the lunar spacecraft in the second sample problem, determine its orbital speed. What increase beyond that speed was required for escape from the Moon?

8. A 60.0 kg space probe is in a circular orbit around Europa, a moon of Jupiter. If the orbital radius is 2.00×10^6 m and the mass of Europa is 4.87×10^{22} kg, determine the

(a) kinetic energy of the probe and its orbital speed

(b) gravitational potential energy of the probe

(c) total orbital energy of the probe

(d) binding energy of the probe

(e) *additional* speed that the probe must gain in order to break free of Europa

9. A 1.00×10^2 kg space probe is in a circular orbit, 25 km above the surface of Titan, a moon of Saturn. If the radius of Titan is 2575 km and its mass is 1.346×10^{23} kg, determine the

(a) orbital kinetic energy and speed of the space probe

(b) gravitational potential energy of the space probe

(c) total orbital energy of the space probe

(d) binding energy of the space probe

(e) *additional* speed required for the space probe to break free from Titan

10. Material has been observed in a circular orbit around a black hole some five thousand light-years away from Earth. Spectroscopic analysis of the material indicates that it is orbiting with a speed of 3.1×10^7 m/s. If the radius of the orbit is 9.8×10^5 m, determine the mass of the black hole.

6.2 Section Review

1. C Explain why the determination of orbital speed does not require knowledge of the satellite's mass, while determination of orbital energies does require knowledge of the satellite's mass.

2. K/U Explain why the binding energy of a satellite is the negative of its total orbital energy. Why does this relationship not depend on the satellite being in a circular orbit?

3. C Draw a concept organizer to show the links between the general equations for work and energy and the orbital energy equations. Indicate in the organizer which equations are joined together to produce the new equation.

4. I A satellite with an orbital speed of v_{orbit} is in a circular orbit around a planet. Prove that the speed for a satellite to escape from orbit and completely leave the planet is given by $v_{escape} = \sqrt{2}(v_{orbit})$.

5. MC The magnitude of the attractive force between an electron and a proton is given by $F = \frac{kq_e q_p}{r^2}$, where q_e is the magnitude of the charge on the electron, q_p is the magnitude of the charge on the proton, r is the separation between them, and k is a constant that plays the same role as G. If the mass of the electron is represented by m_e, derive an equation for the orbital speed of the electron.

Go To Mars With Newton

In his well-known and universally acclaimed book, *The Principia*, Sir Isaac Newton proposed a way to launch an object into orbit. His method was to place a cannon on a mountaintop and fire cannon balls parallel to Earth's surface. By using more gunpowder each time, the cannon ball would fly farther before falling to the ground. Newton imagined increasing the gunpowder until the cannon ball took off with such a great initial speed that it fell all the way around Earth — the cannon ball went into orbit and became a cannon ball satellite. In fact, if the cannon ball was able to make it half-way around the world, it would continue to orbit.

Today, scientists do not use cannons and gunpowder to launch rockets, but Newton's method and his laws still apply. Launching a rocket into orbit simply requires that the trajectory of its "free fall" carry it around the globe. While the rocket continually falls toward Earth's surface, the surface itself continually curves away from the rocket's path. Orbital motion is much like a perpetual game of tag between the satellite and the planet's surface. If the speed of the satellite drops below the orbital speed, then the satellite's trajectory will lead it to an impact on the surface.

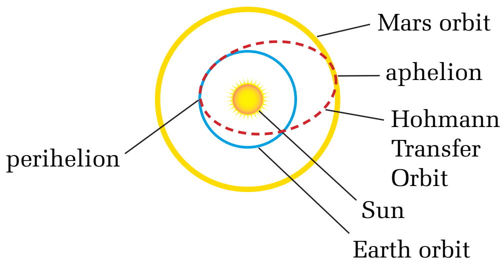
The speed required for an orbit close to Earth is around 28 000 km/h. Part of this can be provided by launching the rocket in an easterly direction from a location near the equator. As Earth revolves, its surface at the equator moves eastward at about 1675 km/h, thus providing a free, although relatively small, boost.

Lift-off involves the ignition of fuel and oxidizer in a reaction which is much like a controlled explosion. Hot, high pressure gas is formed as the product of the burning. When the high-speed molecules of the gas collide with the walls of the combustion chamber, they exert forces on the walls of the chamber. Because of an opening at one end of the chamber, the net force exerted by the gas molecules on the chamber is in the forward direction. By Newton's third law of motion, the chamber walls exert an equal and opposite force on the gas molecules, causing them to stream backward out of the nozzle at the end of the chamber. The nozzle controls the direction and rate of flow of the exhaust gases and so provides control of the direction and magnitude of the thrust. Once the thrust becomes greater than the weight of the rocket, the rocket begins to accelerate upward.

Once the fuel in the first stage has been consumed, that stage can be separated from the rest of the rocket and a second stage compartment is ignited. By letting the first compartment drop away, the rocket has less mass that needs to be propelled. Applying Newton's second law, which states that the acceleration is proportional to the ratio of the applied force and the mass, for the same amount of thrust, the rocket will accelerate more quickly.

Sending a Rocket to Mars

Contrary to popular opinion, the best time to send a spacecraft to Mars is not when Mars and Earth are closest in their orbits around the Sun. Instead, the launch opportunity occurs when the spacecraft can be fired tangentially from Earth's orbit, travelling along an elliptical orbit around the Sun, and arrive at Mars about 259 days later travelling tangentially to the orbit of Mars. The elliptical orbit is called a Hohmann transfer orbit. The Earth–Sun distance represents the closest point to the Sun (the perihelion) and the Mars–Sun distance represents the farthest point from the Sun (the aphelion). Energy is saved in two ways: The first is due to the use of Earth's orbital speed as the starting speed for the spacecraft. In fact, the spacecraft only needs an additional 3 km/s above the orbital speed of Earth around the Sun. The second comes from the fact that the average radius of the orbit of the ellipse is less than the orbital radius of Mars. If the spacecraft was not captured by the gravitational field of Mars, the spacecraft would continue along the ellipse and fall back toward its perihelion. It is thus necessary to launch the spacecraft such that its arrival time at aphelion coincides with the arrival of Mars at the same location. Such launch opportunities come only every 25 to 26 months.



In general then, the Hohmann transfer orbit only requires a burst of thrust when the spacecraft leaves Earth orbit and a second burst to allow it to settle into an orbit around Mars. Further manoeuvring would be required if the spacecraft was then going to land on the surface of the Red Planet. On its return, the spacecraft would drop away from Mars and follow the second half of the Hohmann transfer orbit back to Earth.

Making Connections

1. Describe how Newton's laws apply to a rocket at lift-off, in orbit, and landing.
2. **(a)** What differences would you need to consider to send a rocket to Venus instead of to Mars?

   (b) Draw a Hohmann transfer orbit for a rocket travelling to Venus.

6.3 Energy and Momentum in Space

SECTION EXPECTATIONS

- Apply quantitatively the law of conservation of linear momentum.
- Analyze the factors affecting the motion of isolated celestial objects.

KEY TERMS

- combustion chamber
- exhaust velocity
- thrust
- reaction mass
- gravitational assist or gravitational slingshot

Collisions in space are among the more interesting celestial events. The collision of comet Shoemaker-Levy 9 in July of 1994 created great excitement for both astronomers and the general public. Other collisions in the past have greatly affected Earth. The demise of the dinosaurs, along with about 70% of all other species, is attributed to a collision between Earth and an asteroid some 65 million years ago. Remains of such a collision can be seen on the sea floor of the Gulf of Mexico near the coast of the Yucatan Peninsula.

More recently, on June 30, 1908, an object with a mass of about one hundred thousand tonnes slammed into Earth's atmosphere above Siberia, not far from the Tunguska River. The explosion, which occurred about eight kilometres above the ground, flattened one hundred thousand square kilometres of forest, killing all of the wildlife in the area. Since the region is remote, no humans are thought to have perished.

Figure 6.5 The dark blemishes on the face of Jupiter after the collision with the comet Shoemaker-Levy 9 mark the impact locations of fragments of the comet.

Not all celestial collisions are devastating. Present theories about the formation of our solar system suggest that planets were formed as a result of collisions of smaller rocky objects. If the collisions were energetic enough, they would have generated enough thermal energy to fuse the rocks together into a larger mass.

QUICK LAB

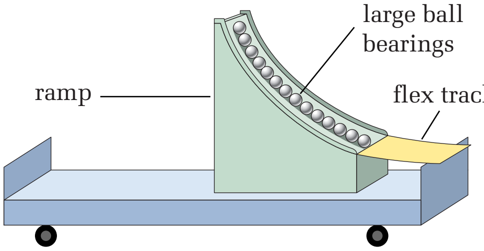
The Reaction Engine

TARGET SKILLS
- Hypothesizing
- Performing and recording
- Analyzing and interpreting

The Reaction Engine

According to Newton's first law of motion, an object requires a net force to push out in order to produce a change in speed or direction. If a rocket is out in space, what is available to provide this push? This activity should give you some ideas.

Set up a light dynamics cart with a ramp which could be made from Hot Wheels™ track as shown in the diagram. The ramp should be as high as possible and curved at the base so that ball bearings will be ejected horizontally from the back of the cart.



Arrange a track for the cart by clamping or taping metre sticks to the demonstration desk or tape them to the floor.

Place as many large ball bearings as possible on the ramp and hold them in place. Release the ball bearings and observe the motion of the ball bearings and the cart.

Analyze and Conclude

1. Describe the motion of the cart and the ball bearings. Did the last ball bearings move as quickly along the desk or floor as the first ones did? Did any actually end up moving in the direction of the cart?
2. Using Newton's laws of motion, explain why the cart accelerated.
3. What was the source of the energy that was transformed into the kinetic energy of the cart and the ball bearings?
4. The ramp with the ball bearings and a rocket are examples of reaction engines. Explain why the term is appropriate. What is the reaction mass in each case?

Apply and Extend

5. Research the topic of magnetohydrodynamic propulsion and prepare a brief report in diagram form. Where is this process mainly used?

Propulsion in Space

Newton's third law of motion states that if you exert a backward force on an object, that object will exert a forward force on you. In Chapter 4, Momentum and Impulse, you learned how Newton's third law led to the law of conservation of momentum. This concept is the basis for all motion and manoeuvring of astronauts and rockets in space. In fact, a spacecraft could be propelled by having an astronaut stand at the rear of the spacecraft and throw objects backward. This process is an example of recoil. As the astronaut pushed the objects backward, they would push just as hard forward on the astronaut.

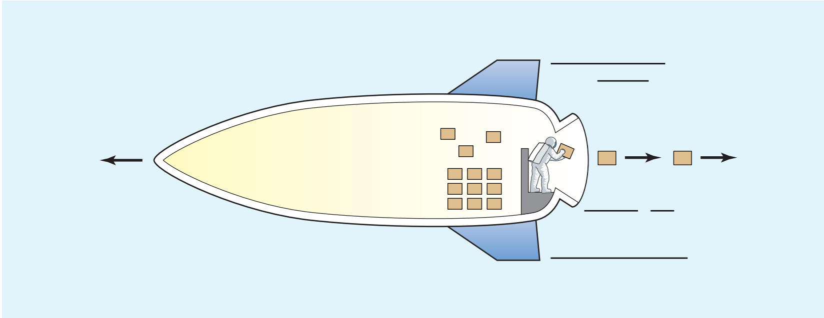
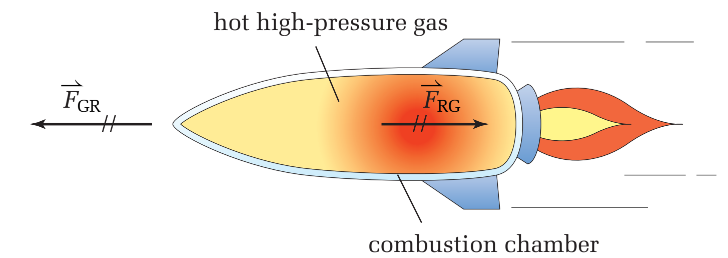


Figure 6.6 Recoil, a result of the conservation of momentum, is the basis of rocket propulsion. The concept is the same as the motion of an ice skater throwing a rock — a problem that you solved in Chapter 4.

Although this is the general principle on which rocket engines operate, most rely on hot, high-pressure gas to provide the reaction mass. The burning of the gas takes place in a **combustion chamber**, as shown in Figure 6.7. The walls of the combustion chamber exert a backward force on the gas, causing it to stream out backward. The gas in turn exerts a force on the walls of the combustion chamber, pushing it and the rocket forward.



$\vec{F}_{GR}$ = force exerted by the gas on the rocket (thrust) [forward]
$\vec{F}_{RG}$ = force exerted by the rocket on the gas [backward]
$\vec{F}_{GR} = -\vec{F}_{RG}$

Figure 6.7 Newton's third law explains how the exhaust of gases creates thrust for a rocket.

The relationship between the motion of the gas and the forces on the gas can be found by applying the impulse momentum theorem: $\vec{F}\Delta t = m\Delta\vec{v}$. When physicists and engineers apply this theorem to rocket exhaust gases, they usually rearrange it as follows.

$$\vec{F}_{(\text{on gas})}\Delta t = m_{\text{gas}}\Delta\vec{v}_{\text{gas}}$$

$$\vec{F}_{(\text{on gas})} = \frac{m_{\text{gas}}\Delta\vec{v}_{\text{gas}}}{\Delta t}$$

$$\vec{F}_{(\text{on gas})} = \left(\frac{m_{\text{gas}}}{\Delta t}\right)\Delta\vec{v}_{\text{gas}}$$

In rocket technology, the term $\left(\frac{m_{\text{gas}}}{\Delta t}\right)$ is important because it represents the rate of flow of a given mass of exhaust in kilograms of gas per second. Because of the law of conservation of mass, it also represents the burn rate of the fuel and oxidizer combined. Since the gas is initially at rest in the combustion chamber, the $\Delta\vec{v}_{\text{gas}}$ represents the backward velocity of the gas relative to the combustion chamber of the rocket. This is also known as the

exhaust velocity. For most chemical propellants, the exhaust velocity ranges from 2 km/s to 5 km/s.

If you know the rate of combustion and the velocity of the exhaust gases, you can calculate the force with which the rocket pushes on the gas: $\vec{F}_{(\text{on gas})} = \left(\frac{m_{\text{gas}}}{\Delta t}\right)\Delta\vec{v}_{\text{gas}}$. According to Newton's third law of motion, this also represents the force with which the gas pushes on the rocket. This force is known as the **thrust** (action force, in Newton's third law). The gas experiences the reaction force and its mass is referred to as **reaction mass**.

The idea that the rocket exerts a force on the gas might seem strange, but when molecules of gas strike the walls, they exert a force on the walls. At the same time, the walls exert a backward force on the molecules of gas, causing them to recoil. The two forces are equal in magnitude, but opposite in direction.

PHYSICS FILE

As early as 1232 A.D., the Chinese were using gunpowder as a propulsive agent for arrows and incendiary bombs.

SAMPLE PROBLEM

Rocket Propulsion

A rocket engine consumes 50.0 kg of hydrogen and 400.0 kg of oxygen during a 5.00 s burn.

(a) If the exhaust speed of the gas is 3.54 km/s, determine the thrust of the engine

(b) If the rocket has a mass of 1.5×10^4 kg, calculate the acceleration of the rocket if no other forces are acting.

Conceptualize the Problem

- Because the hot gases *move* rapidly out of the combustion chamber, they have *momentum*.
- The *total momentum* of the gases plus rocket must be *conserved*; therefore, the *momentum of the rocket* must be *equal* in magnitude and *opposite* in direction to the gases.
- If you know the *change in momentum* of the rocket and the *time interval* over which that change occurs, you can determine the *force* on the rocket.
- From the *force* and the *mass* of the rocket, you can find its *acceleration*.

Identify the Goal

(a) The thrust, $\vec{F}_{\text{gas on rocket}}$, of the engine

(b) The acceleration, $\vec{a}_{\text{rocket}}$, of the rocket

Identify the Variables and Constants

Known	Unknown
$m_{\text{hydrogen}} = 50.0$ kg	$m_{\text{exhaust gas}}$
$m_{\text{oxygen}} = 400.0$ kg	$\vec{F}_{\text{gas on rocket}}$
$t = 5.00$ s	$\vec{a}_{\text{rocket}}$
$\vec{v}_{\text{exhaust}} = 3.54 \times 10^3 \frac{\text{m}}{\text{s}}$[back]	
$m_{\text{rocket}} = 1.5 \times 10^4$ kg	

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Develop a Strategy

Find the total mass of the exhaust gases.

$$m_{\text{exhaust gas}} = m_{\text{hydrogen}} + m_{\text{oxygen}}$$
$$m_{\text{exhaust gas}} = 50.0\text{ kg} + 400.0\text{ kg}$$
$$m_{\text{exhaust gas}} = 450.0\text{ kg}$$

Find the flow rate of the exhaust gas.

$$\text{Flow rate of the exhaust gas} = \frac{m_{\text{exhaust gas}}}{\Delta t}$$
$$\text{Flow rate of the exhaust gas} = \frac{450.0\text{ kg}}{5.00\text{ s}}$$
$$\text{Flow rate of the exhaust gas} = 90.0\ \frac{\text{kg}}{\text{s}}$$

Use impulse equals change in momentum to determine the force on the gas.

$$\vec{F}\Delta t = m\Delta\vec{v}$$
$$\vec{F} = \frac{m\Delta\vec{v}}{\Delta t}$$

Since the gases started from rest relative to the combustion chamber, $\Delta\vec{v} = \vec{v}_{\text{exhaust}}$.

$$\vec{F} = \left(\frac{m}{\Delta t}\right)\Delta\vec{v}$$
$$\vec{F} = \left(90.0\ \frac{\text{kg}}{\text{s}}\right)\left(3.54 \times 10^3\ \frac{\text{m}}{\text{s}}\right)[\text{back}]$$
$$\vec{F} = 3.186 \times 10^5\text{ N}[\text{back}]$$
$$\vec{F} \cong 3.19 \times 10^5\text{ N}[\text{back}]$$

Use Newton's third law to determine the force on the rocket (combustion chamber).

$$\vec{F}_{(\text{gas on rocket})} = -\vec{F}_{(\text{rocket on gas})}$$
$$\vec{F}_{(\text{gas on rocket})} = -(3.186 \times 10^5\text{ N}[\text{back}]$$
$$\vec{F}_{(\text{gas on rocket})} = 3.186 \times 10^5\text{ N}[\text{forward}]$$

(a) The thrust on the rocket is 3.19×10^5 N[forward].

Use Newton's second law to calculate the acceleration of the rocket.

$$\vec{F} = m\vec{a}$$
$$\vec{a} = \frac{\vec{F}}{m}$$
$$\vec{a} = \frac{3.186 \times 10^5\text{ N}[\text{forward}]}{1.5 \times 10^4\text{ kg}}$$
$$\vec{a} = 21.24\ \frac{\text{m}}{\text{s}^2}[\text{forward}]$$
$$\vec{a} \cong 21.2\ \frac{\text{m}}{\text{s}^2}[\text{forward}]$$

(b) The acceleration of the rocket is $21.2\ \frac{\text{m}}{\text{s}^2}$[forward].

Validate the Solution

The magnitude of the change in momentum for the rocket must equal the magnitude of the change in momentum for the gas, so

$$m_{\text{rocket}}\Delta v_{\text{rocket}} = m_{\text{gas}}\Delta v_{\text{gas}}$$
$$\Delta v_{\text{rocket}} = \frac{m_{\text{gas}}\Delta v_{\text{gas}}}{m_{\text{rocket}}}$$

The change in velocity is inversely proportional to the masses, so you would expect that the velocity of the rocket would be much less than the velocity of the gases. This is in agreement with the calculated value of the acceleration.

PRACTICE PROBLEMS

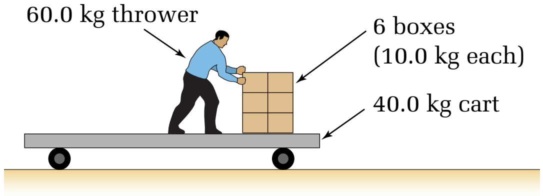
11. Determine the thrust produced if 1.50×10^3 kg of gas exit the combustion chamber each second, with a speed of 4.00×10^3 m/s.

12. What must be the burn rate in kilograms per second if gas with an exhaust speed of 4.15×10^3 m/s is to exert a thrust of 20.8 MN?

13. As an analogy for a reaction engine, imagine that a 60.0 kg person is standing on a 40.0 kg cart, as shown in the diagram. Also on the cart are six boxes, each with a mass of 10.0 kg. The cart is initially at rest. The person then throws the boxes backward, one at a time at 5.0 m/s *relative to the cart*.

(a) Determine the velocity of the cart after each throw, until you have the final velocity of the cart. Keep in mind that the mass on the cart decreases with each throw.

(b) Would the final velocity of the cart be different if the person had thrown all of the boxes at once with a velocity of 5.0 m/s[backward]? If there is a difference, give reasons for it.



The process of burning fuel to provide reaction mass is not the only way to generate a thrust. One extremely efficient method involves an ion engine, such as the one shown in Figure 6.8. In an engine such as this, gas atoms are ionized and the resulting positive ions are driven backward by electrostatic repulsion. The thrust is quite low, but it can act steadily month after month, gradually increasing the velocity of the spacecraft.

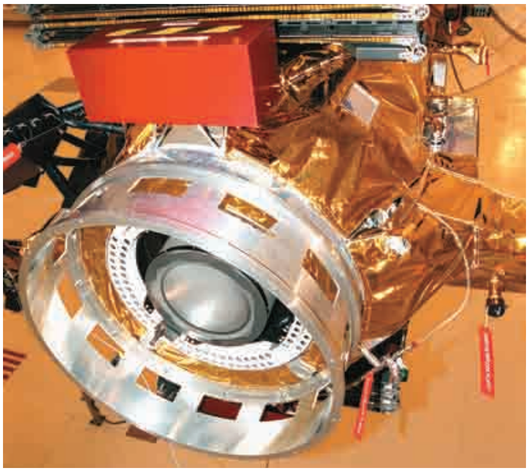


Figure 6.8 Experimental ion engine

Gravitational Assist

Sometimes, free energy seems to be gained for a spacecraft through a manoeuvre known as a **gravitational assist** or a **gravitational slingshot**. The process involves directing a spacecraft to swing around a planet, while keeping far from the atmosphere of the

TECHNOLOGY LINK

The *Deep Space 1* probe, launched on October 15, 1998, was the first spacecraft to use an ion engine. Xenon atoms are ionized and then repelled electrostatically, emerging from the spacecraft at speeds of up to 28 km/s and producing a maximum thrust of 90 mN. The spacecraft has enough propellant to operate continuously for 605 days.

planet. The interaction represents an extremely elastic collision, even through the objects do not actually meet. Figure 6.9 illustrates the process.

Earlier studies of elastic collisions showed that the speed of approach of colliding objects is equal to the speed with which they separate. In this case, the spacecraft is approaching the planet with a relative speed of $(v + V)$, where v is the speed of the spacecraft and V is the orbital speed of the planet. If the collision is elastic, the speed with which the spacecraft moves away from the planet must also be $(v + V)$. Since the planet itself is moving at speed V, the spacecraft must be moving at $V + (v + V)$ or $v + 2V$. As a result, if the spacecraft arcs around the planet and returns parallel to its initial path, it will gain a speed of $2V$, which is twice the orbital speed of the planet.

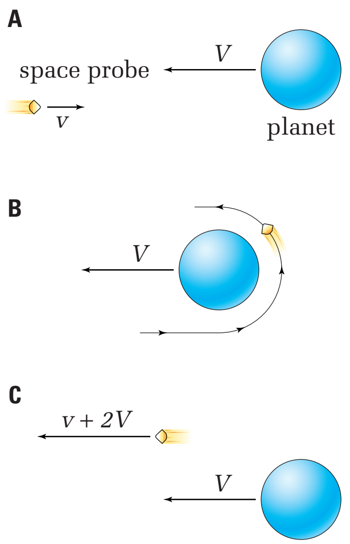


Figure 6.9 A celestial slingshot

A similar effect can be seen on Earth. If a tiny Superball™ is held just above a more massive ball (such as a lacrosse ball) and they are dropped together, the Superball™ will rebound at high speed from the collision. The effect is shown in Figure 6.10.

In part (A) of the diagram, both balls are falling. Since they are close together, their speeds are about the same. In part (B), the large ball has hit the ground and is about to bounce upward. If that collision is elastic, it will rebound with the same speed it had just before hitting the ground, as shown in (C).

The two balls are now approaching each other, closing the gap between them at a speed of $2v$. If their collision is elastic, the speed with which they separate must also be $2v$. Because of the huge difference in their masses, the large ball is only slightly slowed down in the collision, and so is still effectively travelling at speed v. The small ball will therefore rebound with a speed that is $2v$ greater than the larger ball's speed. In other words, it will have a speed of $3v$.

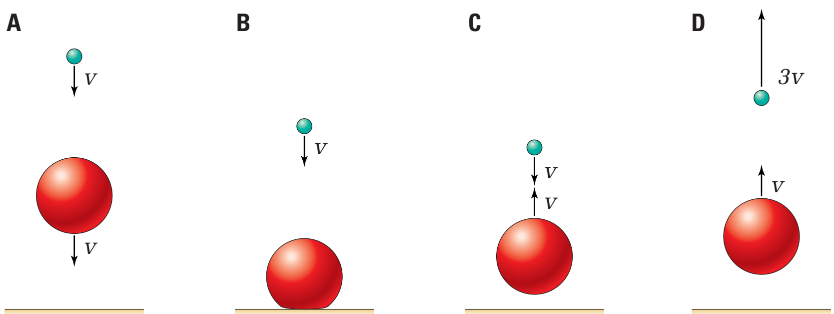


Figure 6.10 In elastic collisions, the speed of approach of colliding objects is equal to the speed with which they separate, as demonstrated by this experiment with a Superball™ and a lacrosse ball.

Because kinetic energy varies with the square of the speed, tripling the speed of the Superball™ will multiply its kinetic energy by a factor of nine. As a result, it will bounce to a height that is nine times its initial height.

This Superball™ discussion assumes that the collision is completely elastic. If there is some energy loss, the ball will not rise as high as predicted. The following investigation looks at just how elastic this collision actually is.

INVESTIGATION 6-A

Superball™ Boost

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

In the discussion in the text, it was predicted that if a tiny Superball™ is held above a far more massive ball and the two are dropped at the same time, the Superball™ should triple its speed. This assumes that the collision is completely elastic and that the large ball does not significantly slow down during the collision. In this investigation, you will determine how valid those assumptions are.

Problem

How does the speed with which a Superball™ leaves a collision with a more massive ball compare with the theoretical speed?

Equipment

- Superball™
- more massive ball, such as a lacrosse ball
- metric measuring tape

CAUTION Wear a face shield if you are conducting this experiment. The other students must wear safety goggles.

Procedure

1. Hold the Superball™ just above the larger ball and at a height of 0.50 m from the floor.
2. Drop the two together so that the Superball™ will land on top of the larger ball.
3. If the Superball™ bounces straight upward, observe how close the ball comes to the ceiling.
4. Adjust the drop height until the upward-bouncing Superball™ just touches the ceiling.
5. Measure and record the drop height, the diameter of the lacrosse ball, and the height of the ceiling.

Analyze and Conclude

1. Determine the actual drop distance for the Superball™ by subtracting the diameter of the lacrosse ball from the initial height of the Superball™ above the floor. (This assumes that the lacrosse ball has not risen significantly before colliding with the Superball™.)
2. Calculate the speed of the Superball™ just before it collided with the lacrosse ball.
3. Determine the actual height through which the Superball™ rose to reach the ceiling.
4. From the maximum height that the Superball™ attained, determine its actual speed just after the collision with the lacrosse ball.
5. What was the theoretical speed of the Superball™ after the collision?
6. How well does the measured speed compare with the theoretical speed? Express your answer as a percentage.
7. Discuss possible reasons for the difference between the actual speed and the theoretical speed.
8. A comparison of the actual height of the bounce to the theoretical height gives a direct comparison between the amount of kinetic energy the ball received and the theoretical amount of kinetic energy. Express the actual kinetic energy as a percentage of the theoretical kinetic energy. How efficient was this process in transferring energy to the Superball™?

Apply and Extend

9. Provide several suggestions for improving the precision of this investigation.

6.3 Section Review

1. K/U Describe how Newton's third law of motion relates to propulsion in space.

2. K/U Show why the mass rate of flow and exhaust velocity are both involved in the development of thrust.

3. K/U
   (a) During the slingshot procedure for increasing the speed of a space probe, what happens to the orbital speed of the planet? Give reasons for your answer.
   (b) Should you be concerned about this? Justify your answer.

4. MC Which planet is most likely to provide the best "slingshot" effect, Jupiter or Mercury? Give reasons for your choice.

5. C Two identical rocks with equal masses and equal speeds collide head-on in space and stick together.
   (a) Explain why there will be no motion of the clump after the collision.
   (b) If all of the initial kinetic energy is changed into thermal energy in the collision, which situation will create the greater amount of thermal energy?
      - doubling the masses of the rocks, but leaving the speeds the same
      - doubling the speeds of the rocks, while leaving the masses the same

      Give reasons for your choice.
   (c) Is it possible that one of those two situations will result in no change in the temperature increase during the collision? Justify your answer.

6. I By means of a series of diagrams, predict the speed at which a Superball™ would bounce if it was falling on top of a much more massive ball, which was in turn falling on top of an extremely massive ball.

7. MC One method of propulsion that does not involve the ejection of reaction mass is the use of a "solar sail." This device consists essentially of a thin film that could cover an area equal to the size of several football fields. It would be stored during lift-off and unfurled out in space. Light from the Sun (or from huge lasers on Earth) would exert pressure on the sail. At the distance that Earth is from the Sun, the pressure of sunlight would be about 3.5 N/km^2. How realistic is this concept for space travel? What are its advantages and disadvantages?

CHAPTER 6 Review

REFLECTING ON CHAPTER 6

- The work done in moving a mass (m) from a separation of r_1 to a separation of r_2 is given by
$$W = \frac{GMm}{r_1} - \frac{GMm}{r_2}$$
$$\text{or } W = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
where M is the mass of the larger object.
- The work that must be done to move a mass m from a separation of r_1 to infinite separation is given by $W_{\text{to escape}} = \frac{GMm}{r_1}$.
- If the escape energy from a planet is to be provided through kinetic energy, the kinetic energy can be expressed as $\frac{1}{2}mv^2 = \frac{GMm}{r_p}$, where r_p is the radius of the planet. This leads to the escape speed being $v = \sqrt{\frac{2GM}{r_p}}$.
- The energies associated with circular orbits are $E_k = \frac{GMm}{2r}$, $E_g = -\frac{GMm}{r}$ and $E_{\text{total}} = -\frac{GMm}{2r}$.
- The gravitational potential energy of an object in the vicinity of a planet is negative, since work must be done on the object to remove it completely from the gravitational influence of the planet. The object is said to be in a gravitational potential energy well.
- For an object to be able to escape from the gravitational field of a planet, its total energy must be equal to or greater than zero. The extra energy that must be added to an object to bring its energy up to zero (thus allowing it to escape) is called the "binding energy" and is given by $E_{\text{binding}} = -E_{\text{total}}$.
- The magnitude of the thrust exerted by the gas being ejected from a rocket can be determined from the equation $F_{\text{thrust}} = \left(\frac{m_{\text{gas}}}{\Delta t}\right)\Delta v_{\text{gas}}$. This gas is known as "reaction mass."
- Gravitational assist is a technique by which a spacecraft gains kinetic energy from a planet around which it swings.

Knowledge/Understanding

1. **(a)** What must be true about the total orbital energy of any planet? Give reasons for your answer.
(b) What does it mean if some comets have total energies less than zero and others have total energies greater than zero?

2. Why does it take more energy to send a satellite into polar orbit (which follows Earth's longitudinal lines and passes over the North and South Poles) than into an equatorial orbit (which follows the equator eastward over Earth)?

3. For a satellite in circular orbit above Earth, state how the following properties depend on radius: **(a)** period; **(b)** kinetic energy; **(c)** speed.

Inquiry

4. Investigate the sizes of black holes by doing some simple escape velocity calculations. As postulated by Einstein's theory of general relativity, black holes are objects with such a strong gravitational field that their escape velocity exceeds the speed of light. Hence, nothing — not even light — can escape from a black hole. They are thought to be caused by massive stars that collapse in on themselves.
(a) What would be the escape velocity if, without losing any mass, Earth shrank to the following percentages of its present radius?
- $\frac{1}{100}$
- $\frac{1}{10000}$
- $\frac{1}{1 \times 10^6}$

(b) To how small a size would Earth have to collapse for its escape velocity to equal the speed of light?
(c) Although a full treatment of black holes requires general relativity, the radius (called the "Schwarzchild radius") can be calculated by using Newtonian theory: by setting $v_{\text{escape}} = c$ and solving for the radius. Suppose the core of a star 8.0 times more

massive than the Sun exhausted its fuel and collapsed. What size of black hole would form? (The Sun's mass is 1.99×10^{30} kg.)

(d) The centres of galaxies, including our own Milky Way, might contain black holes with masses of a million times the Sun's mass, or more. Calculate the size of a black hole with a mass of 1.99×10^{36} kg. Compare this to the size of the Sun's radius (6.96×10^{8} m).

5. Consider a space shuttle in circular orbit around Earth. If the commander briefly fires a forward-pointing thruster so that the speed of the shuttle abruptly decreases, what would be the resulting effects on the kinetic energy, the total mechanical energy, the radius of the orbit, and the orbital period? Sketch the new orbit. Explain whether the new orbit will take the shuttle to the same point at which the thrusters were fired.

Communication

6. Explain how a satellite should be launched so that its orbit takes it over every point on Earth as Earth rotates.

7. What is the reason for choosing the zero of gravitational potential energy at infinity rather than, for example, at Earth's surface?

8. If the Sun shrank to the size of a black hole without losing any mass, what would happen to Earth's orbit?

9. Discuss whether you and your friends on the surface of Earth can be considered to be satellites orbiting Earth at a distance of 1.0 Earth radii.

10. **(a)** Explain whether the gravitational force of the Sun ever does work on a planet in a circular orbit.

(b) Does the gravitational force of the Sun ever do work on a comet in an elliptical orbit?

11. Suppose you launched a projectile in Halifax and it landed in Vancouver. If you assumed that the mass of Earth was concentrated at the centre, could you consider the projectile to be in temporary orbit around the centre of Earth? Explain your reasoning. Sketch the trajectory of the projectile over Earth. What would the complete orbit look like?

Making Connections

12. Investigate some of the details of rocket launches for interplanetary probes. What percentage of the probe's mass at lift-off is fuel? How much of the fuel is consumed at lift-off? If an expendable rocket is used to launch the probe, what options are currently available? How much does a launch cost? Summarize your findings in a report.

13. The United States recently announced plans to send an astronaut to Mars and return him or her safely to Earth. Such space travel is very costly compared to sending remote controlled probes, which can often collect as much information. Investigate the issues involved in interplanetary space travel and stage a debate in your class to examine them.

Problems for Understanding

14. Calculate the binding energy that a 50.0 kg classmate has while on the surface of Earth.

15. **(a)** What is the change in gravitational potential energy of a 6200 kg satellite that lifts off from Earth's surface into a circular orbit of altitude 2500 km?

(b) What percent error is introduced by assuming a constant value of g and calculating the change in gravitational potential energy from $mg\Delta h$?

16. The small ellipticity of Earth's orbit causes Earth's distance from the Sun to vary from 1.47×10^{11} m to 1.52×10^{11} m, with the average distance being 1.49×10^{11} m. The Sun's mass is 1.99×10^{30} kg. What is the change in Earth's gravitational potential energy as it moves from its smallest distance to its greatest distance from the Sun?

17. The mass of Mars is 0.107 times Earth's mass and its radius is 0.532 times Earth's radius. How does the escape velocity on Mars compare to the escape velocity on Earth?

18. The Sun's mass is 1.99×10^{30} kg and its radius is 6.96×10^{8} m.

(a) Calculate the escape velocity from the Sun's surface.
(b) Calculate the escape velocity at the distance of Earth's orbit.
(c) Calculate the escape velocity at the distance of Pluto's orbit, 5.9×10^{12} m.

19. A rocket is launched vertically from Earth's surface with a velocity of 3.4 km/s. How high does it go **(a)** from Earth's centre and **(b)** from Earth's surface?

20. Calculate whether a major league baseball pitcher, who can pitch a fastball at 160 km/h (44 m/s), can pitch it right off of any of the following.
(a) Saturn's moon, Mimas, which has a mass of 3.8×10^{19} kg and a radius of 195 km.
(b) Jupiter's moon, Himalia, which has a mass of 9.5×10^{18} kg and a radius of 93 km.
(c) Mars' moon, Phobos, which has a mass of 1.1×10^{16} kg and a radius of 11 km.
(d) a neutron star, with 2.5 times the Sun's mass crammed into a radius of 8.0 km.

21. Consider an object at rest 1.00×10^2 Earth radii from Earth. With what speed will it hit the surface of Earth? Compare this to Earth's escape speed.

22. To exit from the solar system, the *Pioneer* spacecraft used a gravitational assist from Jupiter, which increased its kinetic energy at the expense of Jupiter's kinetic energy. If the spacecraft did not have this assist, how far out in the solar system would it travel? When it left Earth's vicinity, the spacecraft's speed, relative to the Sun, was 38 km/s.

23. A 650 kg satellite is to be placed into synchronous orbit around Earth.
(a) Calculate the gravitational potential energy of the satellite on Earth's surface.
(b) Calculate the total energy of the satellite while it is in its synchronous orbit with a radius of 4.22×10^7 m.
(c) What amount of work must be done on the satellite to raise it into synchronous orbit?
(d) Suppose that from its orbit you wanted to give the satellite enough energy to escape from Earth. How much energy would be required?

24. The atmosphere can exert a small air-drag force on satellites in low orbits and cause these orbits to decay.
(a) Despite an increased air-drag force as the orbit decays, the speed of the satellite increases. Show this by calculating the speed of a satellite when its altitude is 200 km (2.00×10^5 m) and when its altitude is 100 km (1.00×10^5 m).
(b) If the satellite's mass is 500 kg (5.00×10^2), show that the mechanical energy decreases, despite the increase in the satellite's kinetic energy.

25. One of the interesting things about a collapsed, compact object like a neutron star or a black hole is that, theoretically, a spacecraft could be sent close to it without suffering the effects of intense radiation. Consider a neutron star with a mass of 2.0 times the Sun's mass and a radius of 10.0 km. (**Note:** Think about the density that this implies!)
(a) Calculate the velocity of a spacecraft orbiting 500.0 km above the neutron star. (Note that at closer orbits, the effects of high gravity would need to be considered and the familiar formula used here would not apply.) What is this speed as a fraction of the speed of light?
(b) What is the spacecraft's period?

26. A rocket stands vertically on a launch pad and fires its engines. Gas is ejected at a rate of 1200 kg/s and the molecules have a speed of 40.0 km/s. What is the maximum weight the rocket can have if this thrust is to lift it slowly off the launch pad?

27. A 3.5×10^5 kg rocket stands vertically on a launch pad.
(a) Calculate the minimum thrust required to cause the rocket to rise from the launch pad.
(b) The rocket engines eject fuel at a rate of 28.0 kg/s. What is the velocity of the gas as it leaves the engine? Neglect the small mass change in the rocket due to the ejected fuel.

UNIT 2 PROJECT

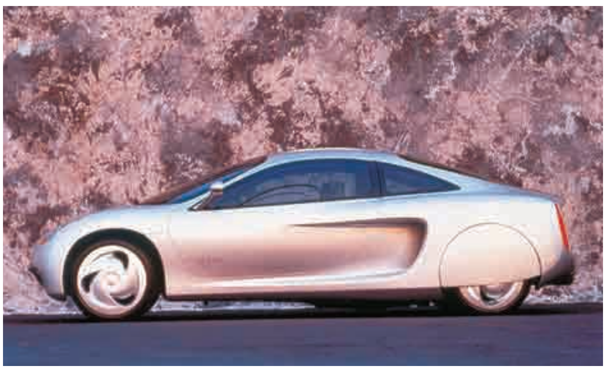
Just a Theory?

Background

Evidence of the deficiencies of existing transportation technologies is everywhere. Limited and increasingly expensive fuel supplies, noise and air pollution, and congested roads and highways are all indications that the existing methods of moving people and goods are less than ideal.

In response, more sustainable, environmentally responsible transportation methods are slowly emerging. Lightweight, low-drag body designs are commonplace. Computerization has improved the efficiency of gasoline engines and has led to the development of "hybrid" vehicles with both gasoline and electric engines. Vehicles powered by fuel cells and improved rechargeable batteries are in limited production. The space shuttle provides a far more efficient method of placing payloads in orbit than do single-use rockets.

Improved transportation technologies are too important to be left to lucky guesses or inspired tinkering. Research projects that apply basic scientific principles to guide and evaluate the development of new vehicles and their components are critically needed. Far from being "just theory," physics principles related to momentum and energy conversion and conservation are key to developing the environmentally friendly, sustainable technologies of the future.



Hybrid cars combine power-generating technologies.

This is a recumbent or "reclining" bicycle.

Scooters offer a transportation alternative.

Challenge

Research the information and prepare a presentation that illustrates how the scientific theories and principles studied in this unit can be used to develop environmentally responsible transportation alternatives. The presentation must include aspects of your study of momentum, energy, and energy transformations. The presentation is to be designed to provide an intelligent adult audience of non-scientists with an understanding of how scientific theories impact everyday life.

ASSESSMENT

After you complete this project

- **assess the clarity of your background summaries about each topic. Can others read your report and formulate specific questions about the topic?**
- **assess the effectiveness of your argument and examples. How well did it persuade audience members of the need for responsible choices of transportation technology and the value of studying basic scientific principles?**
- **assess the impact of your group's presentation as a whole. How well were you able to link the separate examples into a unified, coherent presentation?**

Project Criteria

A. As a class, develop clear, specific criteria for the presentation. Decide on acceptable methods of presentation, sourcing of information, time limits, and time lines for the project.

B. In small groups, brainstorm examples of transportation technologies and related scientific principles. The examples can be very specific, such as a particular type of fuel cell, or quite general, such as an innovative bicycle frame design. Select topics so that your completed presentation will include information that you studied in all three chapters of the unit.

Action Plan

1. Decide on a theme for your presentation, so that each of your examples contributes to the development of your overall thesis.
2. Prepare a one- or two-page background outline to summarize your research, including properly referenced sources.
3. Develop a questionnaire, quiz, or rating scale for your audience so that you can gather feedback on your presentation.
4. Develop and present your project.
5. Prepare a written evaluation of your project that includes a summary of audience feedback and ideas for improving the presentation.

Evaluate

1. What information sources did your group find most useful in this project? How did you ensure that your presentation was free of plagiarism?
2. To what extent has working on this project increased your awareness of alternatives for responsible, sustainable transportation technologies? Explain how your transportation choices in everyday life model these values.
3. Is providing information enough to change people's behaviour? What else can be done so that manufacturers and consumers are encouraged to make responsible choices regarding transportation technology?
4. Suppose a friend questioned the value of studying basic scientific principles by saying, "They are just a bunch of generalizations and mathematical tricks. We need to concentrate on practical ways of solving real problems"? What examples from this project would you use to demonstrate the importance of studying scientific theory?

UNIT 2 Review

Knowledge/Understanding

Multiple Choice

In your notebook, choose the most correct answer for each of the following questions. Outline your reasons for your choice.

1. A hockey puck and a curling stone are at rest on a sheet of ice. If you apply equal impulses to each of them with a hockey stick
   (a) they will have the same acceleration
   (b) the forces applied were equal
   (c) they apply equal reaction forces to the hockey stick
   (d) they will have the same velocity, but different momenta
   (e) they will have the same momentum but different velocities
2. Ball B is moving and collides with a stationary ball A. After the collision, ball B bounces backwards with a velocity of nearly the same magnitude as it had before the collision. Ball A rolls forward very slowly. What is the relationship between the masses of the ball.
   (a) $m_A = m_B/2$
   (b) $m_A = 2\ m_B$
   (c) $m_A = 4\ m_B$
   (d) $m_A = m_B/4$
   (e) $m_A = m_B$
3. You throw a rock straight up into the air. While it rises and falls, its kinetic energy
   (a) remains constant
   (b) increases steadily
   (c) changes direction only
   (d) decreases then increases
   (e) increases then decreases
4. Starting from rest at the top of a hill, a bicyclist pedals furiously on the way down. The kinetic energy of the bicycle and rider at the bottom will be equal to
   (a) lost potential energy
   (b) work done
   (c) work done plus lost potential energy
   (d) work done plus kinetic energy plus potential energy
   (e) zero
5. An astronaut in an orbiting spacecraft is said to be weightless because
   (a) no force of gravity is exerted on the astronaut
   (b) the spacecraft exerts a force opposite to Earth's gravity and acts to suspend the astronaut
   (c) the astronaut and the spacecraft are both in free fall
   (d) the astronaut wears a special gravity-resistant spacesuit
   (e) there is no air resistance in the region where the astronaut is orbiting
6. A rocket launched with a velocity equal to the escape velocity of a planet has
   (a) positive total energy
   (b) negative total energy
   (c) zero total energy
   (d) a total energy that depends on its distance from the planet
   (e) a constantly changing total energy

Short Answer

7. If you throw a ball against a wall, which of the three impulses is the greatest: throw, bounce, or catch?
8. How is it possible for an object to obtain a larger impulse from a smaller force than from a larger force?
9. **(a)** Describe the differences between solving problems for elastic and inelastic collisions.
   (b) How can you tell whether a collision is elastic or not?
   (c) What happens to the kinetic energy of each object in an elastic collision?
10. Distinguish between an open system, a closed system, and an isolated system.
11. Explain why a water hose recoils when the water is turned on.
12. Explain why the first hill of a roller-coaster ride must be the highest hill.
13. **(a)** Under what conditions will a marble of mass m_1 and a rock of mass $3m_1$ have the same gravitational potential energy?
    (b) Under what conditions will a moving marble of mass m_1 and a moving rock of mass $3m_1$ have the same kinetic energy?
14. Write a general equation for the amount of mechanical energy in a system and include

expressions for as many different forms of potential energy as you can locate.

15. A physics wizard is sitting still, puzzling over a homework question. Provide an argument that she is not doing work in the physics sense. Provide a second argument that she *is* doing work in the physics sense.

16. Consider two bodies, A and B, moving in the same direction with the same kinetic energy. A has a mass twice that of B. If the same retarding force is applied to each, how will the stopping distances of the bodies compare?

17. **(a)** Under what circumstances does the work done on a system equal its change in kinetic energy only?
    (b) Under what circumstances does the work done on a system equal the change in gravitational potential energy only?
    (c) Under what circumstances does the change in kinetic energy of a system equal the change in gravitational potential energy?

18. Use the law of conservation of energy to discuss how the speed of an object changes while in an elliptical orbit.

Inquiry

19. The total momentum vector of a projectile is tangential to its path. This vector changes in magnitude and direction because of the action of an internal force (gravity).
    (a) Sketch the path of a projectile and draw momentum vectors at several points along the path.
    (b) The equation $\vec{F} = \Delta\vec{p}/\Delta t$ indicates that a change in momentum is evidence of a net force. Draw vectors that show the change in momentum at several points on the path and thus indicate the direction of the net force. (Neglect air resistance.) Discuss your result.

20. The law of conservation of energy can be written in the form $\Delta E = W + Q$, where ΔE is the change in energy in a system, W is the amount of useful work done, and Q is the amount of heat produced. In this form, it is called the "first law of thermodynamics." For centuries, crafty inventors have tried to violate the law by designing perpetual-motion machines. Such a machine would provide more energy as output than was input. The Canadian Patent Office has shown its faith in the law by refusing to grant patents for such machines based on design only. The inventor must submit a working model. Research and report on some designs for perpetual-motion machines. Include a sketch and use the first law of thermodynamics to discuss why the machine will not work.

21. Design a pogo stick for a child. Designate the age range of the child you hope will enjoy the stick and calculate the required spring constant. Determine other parameters, such as the length of the stick, the size of the spring, and the range of distances that the child will be able to depress the spring. Include a sketch of your design.

22. Insight into simple harmonic motion can be gained by contrasting it with non-simple harmonic motion.
    (a) Consider the simple harmonic motion of an object oscillating on a spring. Does the velocity of the object change smoothly or abruptly when the object changes direction? Sketch a graph of the displacement of the object versus time.
    (b) On the same graph, indicate how the spring force changes with time. Are the restoring force and displacement ever zero at the same time?
    (c) Now consider the motion of a highly elastic rubber ball bouncing up and down on an elastic steel plate, always returning to the same height from which it fell. Set a frame of reference so that you can describe the ball's motion. Does the ball's velocity change smoothly or abruptly at its peak altitude and during impact? Sketch a graph of the displacement of the ball versus time.
    (d) Draw a free-body diagram of the forces acting on the ball when it is in the air. What is the net force on the ball during contact with the steel plate? Are the net force and

displacement ever zero at the same time? On the same graph as (c), sketch how the net force on the ball changes with time.

(e) Contrast the two graphs in terms of the motions they represent.

23. In this question, you will use the kinetic theory of gases to probe the compositions of the atmospheres of four bodies in the solar system. The kinetic theory of gases can be used to relate the average kinetic energy of the molecules in a gas to the temperature, T, of the gas, $E_{k(average)} = \frac{1}{2} mv^2_{average} = \frac{3}{2} kT$, where m is the mass of the molecule and k is the Boltzmann constant.

(a) Calculate the escape velocities of Jupiter, Mars, Earth, and the Moon.

(b) In a table of atomic masses, look up the masses for the following molecules: hydrogen (H_2), helium (He), water vapour (H_2O), methane (CH_4), oxygen (O_2), nitrogen (N_2), and carbon dioxide (CO_2).

(c) Calculate the average speed of each of the above gases of molecules at a temperature of 300.0 K.

(d) Some models of velocity distributions of gases indicate that over the lifetime of the solar system (approximately 5 billion years), a gas will escape from a planet unless its average speed times 10.0 ($v_{average} \times 100$) is less than the escape velocity of the planet. Use this to determine which gases should be present in each of the atmospheres in (a). (The velocity distribution of a gas is described as a Maxwellian velocity distribution — look up this term for further information.)

(e) Compare your results to observations.

(f) Summarize and discuss your results.

24. The orbit of a satellite is often used to determine the mass of the planet or star that it is orbiting. How can the mass of a satellite be determined?

25. Explain whether you could put a satellite in an orbit that kept it stationary over the North or South Pole.

26. Imagine that you found a very unusual spring that did not obey Hooke's law. In fact, you performed experiments on the spring and discovered that the restoring force was proportional to the square distance that the spring was stretched or compressed from its equilibrium or $F = -kx^2$.

(a) Describe an experiment that you might have done to find the expression for the restoring force.

Communication

27. To bunt a baseball effectively, at the instant the ball strikes the bat, the batter moves the bat in the same direction as the moving baseball. Explain what effect this action has.

28. You drop a dish from the table. Explain whether the impulse will be less if the dish lands on a carpet instead of a bare floor.

29. Explain whether it is possible to exert a force and yet not cause a change in kinetic energy.

30. You blow up a balloon and release the open end, causing the balloon to fly around the room as the air is rapidly exhausted. What exerts the force that causes the balloon to accelerate?

31. A jet engine intakes air in the front and mixes it with fuel. The mixture burns and is exhausted from the rear of the engine. Use the concept of momentum to explain how this process results in a force on the airplane that is directed forward.

32. Explain the difference between g and G.

Making Connections

33. When Robert Goddard first proposed sending a rocket to the Moon early in the twentieth century, he was ridiculed in the newspapers. People thought that the rocket would have nothing to push against in the vacuum of space and therefore could not move.

(a) How does a rocket move?

(b) Contrast the rocket's motion with the motion produced by a propeller or a wheel.

(c) A rocket can be considered to represent a case of the inverse of an inelastic collision. Explain this statement.

(d) Develop three analogies that could help explain rocket motion.

(e) To test his idea, Goddard set up a pistol in a bell jar from which the air had been evacuated and fired a blank cartridge. What do you think happened?

34. Analyze any appliance or technical device in terms of its component parts and the energy it consumes. Trace the path of this energy in detail backward through its various forms. How many steps does it typically take before you get to the Sun as the ultimate source of energy?

35. A Foucault pendulum can be used to demonstrate that Earth is rotating. Explain how this is possible. What differences would you notice if you used the pendulum at the North Pole, at Earth's equator, and at latitudes between these two points?

36. Before nuclear energy was postulated as the source of energy for the Sun, other energy-generation processes were considered. At the end of the nineteenth century, one promising method was proposed by Lord Kelvin. It was based on the perfect gas law: If a gas is compressed, it heats up. Heating the gas causes it to radiate energy away, so the gas can be further compressed. The process, gravitational contraction, is now thought to heat protostars (newly forming stars) before they begin nuclear fusion in their cores. Research this process and describe in detail how it could heat a star. How is gravitational potential energy converted into heat? What lifetime did this process predict for the Sun? Also, discuss how Darwin's theory of evolution led astronomers to believe that the lifetime for the Sun predicted by this process was too short.

Problems for Understanding

37. A 1400 kg car travels north at 25 m/s. What is its momentum?

38. What impulse is needed to stop the following?
(a) 150 g baseball travelling at 44 m/s
(b) 5.0 kg bowling ball travelling at 8.0 m/s
(c) 1200 kg car rolling forward at 2.5 m/s

39. A 0.80 kg ball travelling at 12 m/s[N] strikes a wall and rebounds at 9.5 m/s[S]. The impact lasts 0.065 s.
(a) What was the initial momentum of the ball?
(b) What was the change of momentum of the ball?
(c) What was the impulse on the wall?
(d) What was the average force acting on the wall?
(e) What was the average force acting on the ball?

40. A tennis player smashes a serve so that the racquet is in contact with the ball for 0.055 s, giving it an impulse of 2.5 N · s.
(a) What average force was applied during this time?
(b) Assume that the vertical motion of the ball can be ignored. If the ball's mass is 0.060 kg, what will be the ball's horizontal velocity?

41. A hockey player gives a stationary 175 g hockey puck an impulse of 6.3 N · s. At what velocity will the puck move toward the goal?

42. A 550 kg car travelling at 24.0 m/s[E] collides head-on with a 680 kg pickup truck. Both vehicles come to a complete stop on impact.
(a) What is the momentum of the car before the collision?
(b) What is the change in the car's momentum?
(c) What is the change in the truck's momentum?
(d) What is the velocity of the truck before the collision?

43. A rocket is travelling 160 m/s[forward] in outer space. It has a mass of 750 kg, which includes 130 kg of fuel. Burning all of the fuel produces an impulse of 41 600 N · s. What is the new velocity of the rocket?

44. A 19.0 kg curling stone for Team Ontario travels at 3.0 m/s[N] down the centre line of the ice toward an opponent's stone that is at rest. It strikes the opponent's stone and rolls off to the side with a velocity of 1.8 m/s[N22°W]. The opponent's stone moves in a northeasterly direction. What is the final velocity (magnitude and direction) of the opponent's stone?

45. Two balls collide on a horizontal, frictionless table. Ball A has a mass of 0.175 kg and is travelling at 1.20 m/s[E40°S]. Ball B has a mass of 0.225 kg and is travelling at 0.68 m/s[E]. The velocity of ball B after the collision is 0.93 m/s[E37°S].

(a) What is the velocity (magnitude and direction) of ball A after the collision?
(b) What percentage of kinetic energy is lost in the collision?

46. An 8.0 kg stone falls off a 10.0 m cliff.
(a) How much work is done on it by the gravitational force?
(b) How much gravitational potential energy does it lose?

47. Each minute, approximately 5×10^8 kg of water flow over Niagara Falls. The average height of the falls is 65 m.
(a) What is the gravitational potential energy of the water flow?
(b) How much power (in W or J/s) can this water flow generate?

48. A 0.250 kg ball is thrown straight upward with an initial velocity of 38 m/s. If air friction is ignored, calculate the
(a) height of the ball when its speed is 12 m/s
(b) height to which the ball rises before falling
(c) How would your answers to (a) and (b) change if you repeated the exercise with a ball twice as massive?

49. You are in a 1400 kg car, coasting down a 25° slope. When the car's speed is 15 m/s, you apply the brakes. If the car is to stop after travelling 75 m, what constant force (parallel to the road) must be applied?

50. An archery string has a spring constant of 1.9×10^2 N/m. By how much does its elastic potential energy increase if it is stretched (a) 5.0 cm and (b) 71.0 cm?

51. You exert 72 N to compress a spring with a spring constant of 225 N/m a certain distance.
(a) What distance is the spring displaced?
(b) What is the elastic potential energy of the displaced spring?

52. A 2.50 kg mass is attached to one end of a spring on a horizontal, frictionless surface. The other end of the spring is attached to one end of a spring is attached to a solid wall. The spring has a spring constant of 75.0 N/m. The spring is stretched to 25.0 cm from its equilibrium point and released.
(a) What is the total energy of the mass-spring system?
(b) What is the velocity of the mass when it passes the equilibrium position?
(c) What is the elastic potential energy stored in the spring when the mass passes a point that is 15.0 cm from its equilibrium position?
(d) What is the velocity of the spring when it passes a point that is 15.0 cm from its equilibrium position?

53. A 275 g ball is resting on top of a spring that is mounted to the floor. You exert a force of 325 N on the ball and it compresses the spring 44.5 cm. If you release the ball from that position, how high, above the equilibrium position of the spring-ball system will the ball rise?

54. A 186 kg cart is released at the top of a hill.
(a) How much gravitational potential energy is lost after it descends through a vertical height of 8.0 m?
(b) If the amount of friction acting on the cart is negligible, determine the kinetic energy and the speed of the cart after it has descended through a vertical height of 8.0 m.
(c) Explain what variables you would need to know in order to calculate the kinetic energy and the speed of the cart for the same conditions if the frictional forces were significant. Assume some reasonable values and make calculations for the kinetic energy and the speed of the cart influenced by friction.

55. A small 95 g toy consists of a piece of plastic attached to a spring with a spring constant of 365 N/m. You compress the spring against the floor through a displacement of 5.5 cm, then release the toy. How fast is it travelling when it rises to a height of 10.0 cm?

56. Suppose a 1.5 kg block of wood is slid along a floor and it compresses a spring that is attached horizontally to a wall. The spring constant is 555 N/m and the block of wood is travelling 9.0 m/s when it hits the spring. Assume that the floor is frictionless and the spring is ideal.
(a) By how much does the block of wood compress the spring?

(b) If the block of wood attaches to the spring so that the system then oscillates back and forth, what will be the amplitude of the oscillation?

57. A spring with a spring constant of 120 N/m is stretched 5.0 cm from its rest position.
(a) Calculate the average force applied.
(b) Calculate the work done.
(c) If the spring is then stretched from its 5.0 cm position to 8.0 cm, calculate the work done.
(d) Sketch a graph of the applied force versus the spring displacement to show the extension of the spring. Explain how you can determine the amount of work done by analyzing the graph.

58. A 32.0 kg child descends a slide 4.00 m high. She reaches the bottom with a speed of 2.40 m/s. Was the mechanical energy conserved? Explain your reasoning and identify the energy transformations involved.

59. A 2.5 kg wooden block slides from rest down an inclined plane that makes an angle of 30° with the horizontal.
(a) If the plane is frictionless, what is the speed of the block after slipping a distance of 2.0 m?
(b) If the plane has a coefficient of kinetic friction of 0.20, what is the speed of the block after slipping a distance of 2.0 m?

60. (a) Given Earth's radius (6.38×10^6 m) and mass (5.98×10^{24} kg), calculate the escape velocity from Earth's surface.
(b) What is the escape velocity for a satellite orbiting Earth a distance of 2.00 Earth radii from Earth's centre?
(c) How far away do you have to travel from Earth so that the escape velocity at that point is 1% of the escape velocity at Earth's surface? Answer in metres and in Earth radii.

61. A projectile fired vertically from Earth with an initial velocity v reaches a maximum height of 4800 km. Neglecting air friction, what was its initial velocity?

62. An amateur astronomer discovers two new comets with his backyard telescope. If one comet is moving at 38 km/s as it crosses Earth's orbit on its way toward the Sun and the other at 47 km/s, calculate whether each orbit is bound or not.

63. You want to launch a satellite into a circular orbit at an altitude of 16 000 km (above Earth's surface). What orbital speed will it have? What launch speed will be required?

64. In a joint international effort, two rockets are launched from Earth's surface. One has an initial velocity of 13 km/s and the other 19 km/s. How fast is each moving when it crosses the Moon's orbit (3.84×10^8 m)?

65. A 460 kg satellite is launched into a circular orbit and attains an orbital altitude of 850 km above Earth's surface. Calculate the
(a) kinetic energy of the satellite
(b) total energy of the satellite
(c) period of the satellite
(d) binding energy of the satellite
(e) additional energy and speed required for the satellite to escape

66. (a) Calculate the gravitational potential energy of the Earth-Moon system. (Assume that their mean separation is 3.84×10^8 m.)
(b) Calculate the gravitational potential energy of the Earth-Sun system. (Assume that their mean separation is 1.49×10^{11} m.)

67. Proposals for dealing with radioactive waste include shooting it into the Sun. Consider a waste container that is simply dropped from rest in the vicinity of Earth's orbit. With what speed will it hit the Sun?

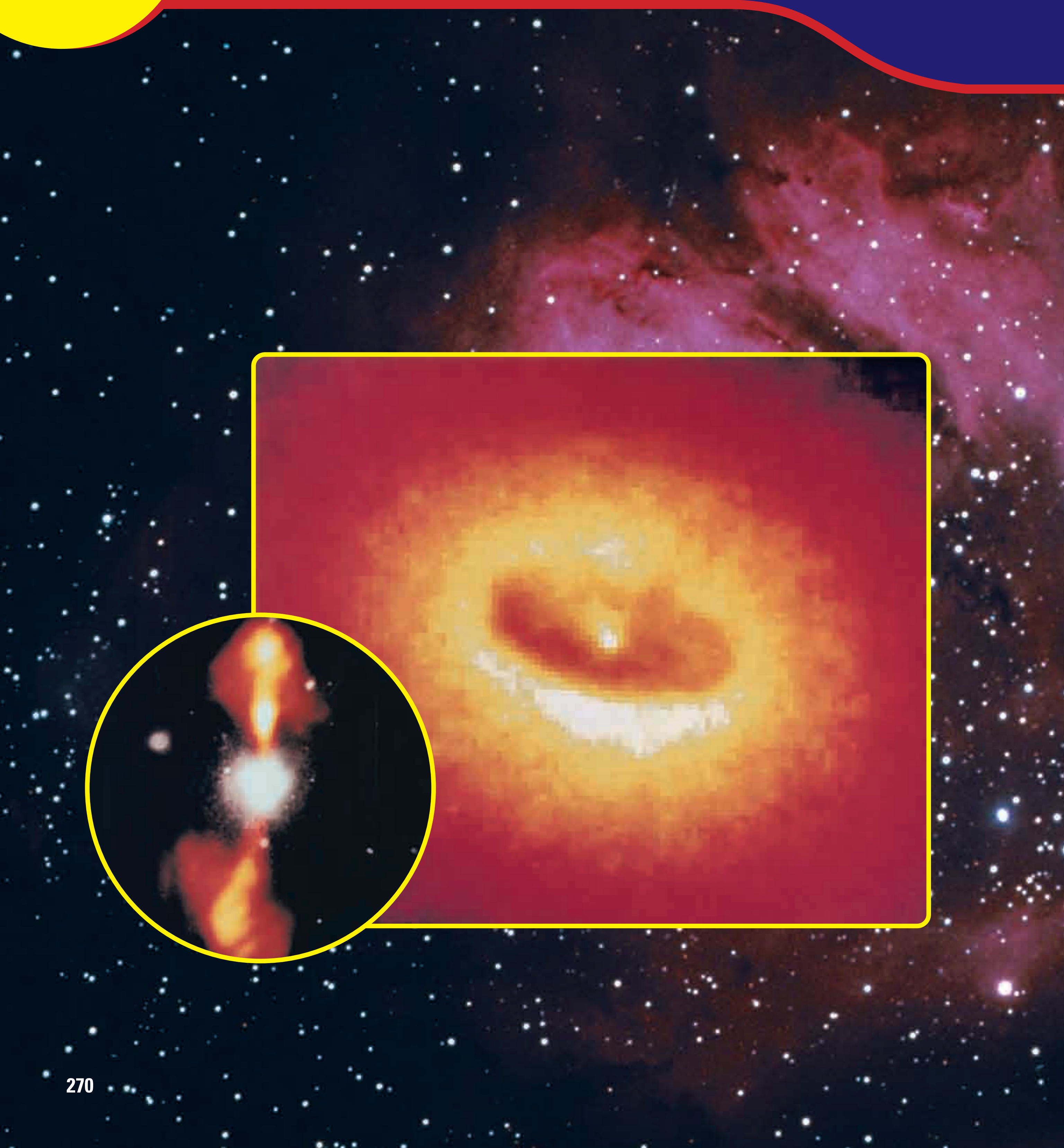
COURSE CHALLENGE

Scanning Technologies: Today and Tomorrow

Consider the following as you continue to build your Course Challenge research portfolio.

- Add important concepts, equations, interesting and disputed facts, and diagrams from this unit.
- Review the information you have gathered in preparation for the end-of-course presentation. Consider any new findings to see if you want to change the focus of your project.
- Scan magazines, newspapers, and the Internet for interesting information to enhance your project.

UNIT 3 Electric, Gravitational, and Magnetic Fields



OVERALL EXPECTATIONS

DEMONSTRATE an understanding of the principles and laws related to electric, gravitational, and magnetic forces and fields.

INVESTIGATE and analyze electric, gravitational, and magnetic fields.

EVALUATE the impact of technological developments related to the concept of fields.

UNIT CONTENTS

What is it about "black holes" that stretches the imagination to the limit? Is it that black holes, such as the artist's conception here, defy reason because both matter and energy seemingly disappear into nothingness?

A major part of understanding the black hole phenomenon lies in the characteristics of fields, regions of space over which a force seemingly acts at a distance. You are already familiar with everyday forces that act in this manner — gravity, magnetism, and electricity. Based on straightforward laboratory studies, you can begin to answer such questions as: "How are these fields formed? How are they related to each other?

Recent research indicates, for example, that black holes are points with almost infinite density. The gravitational field generated by this concentration of mass is so strong that not only objects but even light passing within range can never escape.

This unit provides an examination of the properties of electric, gravitational, and magnetic fields. As our understanding of fields increases, so to do the technological applications that use fields. You will study the fundamental properties of fields, how civilization has harnessed this knowledge, and consider possible directions for future research.

UNIT PROJECT PREP

Refer to pages 370–371. In this unit project you will prepare a report and a debate on particle accelerators and relevant research.

- How can you use electric and magnetic fields to accelerate charged particles to very high speeds?
- What are the costs and benefits to society of the research into particle accelerators and the application of the knowledge gained?

CHAPTER 7 Fields and Forces

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Magnetic fields
- Law of universal gravitation placement

You cannot see electric energy, but the electric eel in the photograph can. It is not really an eel — it is actually a knife fish, or *Electrophorous electricus* — but it *is* electric. This fish can detect and generate an electric potential difference. Nearly half of the knife fish's body consists of specialized muscle cells that function like a series of electric cells. This living "battery" can generate an electric potential difference of up to 600 V. The electric shock caused by the knife fish can kill some small prey and often stuns large prey, which the knife fish then devours.

The pits along the side of the knife fish's head and body, called the "lateral line system," are specialized to detect electric fields. The knife fish uses its ability to generate and detect electric energy to navigate, detect enemies, kill or stun prey, and possibly even communicate with other knife fish. If water is polluted, it modifies the electric field generated by the knife fish. A university in France is studying the possibility of using the knife fish to monitor water quality. As you can see, there are even some areas of research in biology that require a basic understanding of physics.

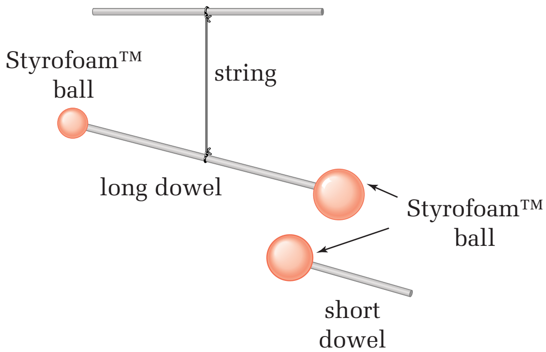
In this chapter, you will learn more about electric energy and fields and compare them with gravitational and magnetic energy and fields.

QUICK LAB

A Torsion Balance

TARGET SKILLS
- Hypothesizing
- Identifying variables
- Performing and recording

The torsion balance was an important tool in early studies of both gravitational force and the electrostatic force. As you know, Henry Cavendish was able to determine the universal gravitational constant, *G*, using a torsion balance. Charles Coulomb, unaware of Cavendish's balance, developed a very similar balance, which he used to develop the law now known as Coulomb's law. This lab will help you to understand the principles of the torsion balance, as well as to develop an appreciation of those who used it.



Attach a string (approximately 1.0 m long) to the centre of a thin wooden dowel (approximately 80 cm long) and suspend it from a retort stand or the ceiling. Wrap four Styrofoam™ balls with aluminum foil. Push one of the balls onto each end of the dowel. Make two probes by pushing one ball onto the end of each of two shorter-length wooden dowels. Charge one of the balls on the longer dowel on the suspended balance and also charge the balls on one of the probes. (Use either an ebonite rod and wool or an electrostatic generator to charge the balls.) Now, hold the charged "probe" ball in the vicinity of the "balance" ball and allow the system to reach equilibrium, with the torsion balance turned a small amount.

Experiment with different-sized charges by holding a charged probe in a fixed position near the "balance" ball, and observe the equilibrium position of the balance. Then, touch the charged probe ball to the uncharged probe ball to reduce (by approximately one half) the quantity of charge it carries. Then hold the probe ball in exactly the same position as before and observe the position of the balance.

Experiment with different types of string, a heavier dowel, protection from air currents, and any other variables that you think might affect the performance of the balance.

Analyze and Conclude

1. Describe the performance of the torsion balance.
2. How did the response of the balance change when you reduced the amount of charge on the probe?
3. How would you calibrate your balance if you wanted to obtain quantitative data?
4. What type of string and weight of dowel seemed to perform best?
5. Comment on the use of a torsion balance as a precision tool by early physicists.

WEB LINK

www.mcgrawhill.ca/links/physics12

To see an illustration of Charles Coulomb's torsion balance, go the the above Internet site and click on **Web Links**.

7.1 Laws of Force

SECTION EXPECTATIONS

- Define and describe the concepts related to electric, gravitational, and magnetic fields.
- Analyze and compare Coulomb's law and Newton's law of universal gravitation.
- Apply quantitatively Coulomb's law and Newton's law of universal gravitation.
- Collect, analyze, and interpret data from experiments on charged particles.

KEY TERMS

- inverse square law
- electrostatic force
- torsion balance
- Coulomb's law
- Coulomb's constant

In science courses over the past several years, you have gained experience in applying the laws of motion of Sir Isaac Newton (1642–1727) and analyzing the motion of many types of objects. The two forces that you encounter most frequently are the forces of gravity and friction. In many cases, you have also dealt with an applied force, in which one object or person exerted a force on another. In this unit, you will focus on the nature of the forces themselves.

Gravity and the Inverse Square Law

Several astronomers and other scientists before Newton developed the concept that the force of gravity obeyed an **inverse square law**. In other words, the magnitude of the force of gravity between two masses is proportional to the inverse of the square of the distance separating their centres: $F \propto \frac{1}{r^2}$. It was Newton, though, who verified the relationship.

Figure 7.1 The centripetal force that keeps the Moon in its orbit is the gravitational force between Earth and the Moon.

Newton reasoned that, since the Moon is revolving around Earth with nearly circular motion, the gravitational force between Earth and the Moon must be providing the centripetal force. His reasoning was similar to the following.

- Write the equation for centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

- Write the equation for speed.

$$v = \frac{\Delta d}{\Delta t}$$

- The Moon travels the circumference of an orbit in one period. Therefore, its speed is

$$\Delta d = 2\pi r = 2\pi(3.84 \times 10^8\ \text{m}) = 2.41 \times 10^9\ \text{m}$$

$$T = 2.36 \times 10^6\ \text{s}$$

$$v = \frac{2\pi r}{T} = \frac{2.41 \times 10^9\ \text{m}}{2.36 \times 10^6\ \text{s}} = 1.02 \times 10^3\ \frac{\text{m}}{\text{s}}$$

- The centripetal acceleration of the Moon is therefore

$$a_c = \frac{v^2}{r} = \frac{\left(1.02 \times 10^3 \frac{m}{s}\right)^2}{3.84 \times 10^8\ m} = 2.71 \times 10^{-3}\ \frac{m}{s^2}$$

- If the force of gravity decreases with the square of the distance between the centre of Earth and the centre of the Moon, then the acceleration due to gravity should also decrease. Write the inverse square relationships and divide the first by the second.

$$a_{g(Moon)} \propto \frac{1}{r^2_{E\text{-}Moon}} \qquad a_{g(Moon)} = \frac{GM_E}{r^2_{E\text{-}Moon}}$$

$$g \propto \frac{1}{r^2_E} \qquad g = \frac{GM_E}{r^2_E}$$

$$\frac{a_{g(Moon)}}{g} = \frac{\frac{1}{r^2_{E\text{-}Moon}}}{\frac{1}{r^2_E}} = \frac{r^2_E}{r^2_{E\text{-}Moon}}$$

- In the ratio above, solve for the acceleration due to gravity at the location of the Moon. Insert the value of g and the distances.

$$a_{c(Moon)} = \frac{gr^2_E}{r^2_{E\text{-}Moon}}$$

$$a_{c(Moon)} = \frac{\left(9.81 \frac{m}{s^2}\right)(6.38 \times 10^6\ m)^2}{(3.84 \times 10^8\ m)^2}$$

$$a_{c(Moon)} = 2.71 \times 10^{-3}\ \frac{m}{s^2}$$

The values of acceleration due to gravity that were calculated in two completely different ways are in full agreement. The centripetal acceleration of the Moon in orbit is exactly what you would expect it to be if that acceleration was provided by the force of gravity and if the force of gravity obeyed an inverse square law.

The force of gravity exerts its influence over very long distances and is the same in all directions, suggesting that the influence extends outward like a spherical surface. The equation relating the surface area of a sphere to its radius is $A = 4\pi r^2$, or the area of a sphere increases as the square of the radius. You can relate the influence of the force of gravity with a portion of a spherical surface, A, at a distance r, as shown in Figure 7.2. When the distance doubles to $2r$, the area increases by 2^2, or four. When the distance increases to $4r$, the area of the sphere increases by 4^2, or 16. The influence of the force of gravity appears to be spreading out over the surface area of a sphere. How does this property of the force of gravity compare to the electromagnetic force?

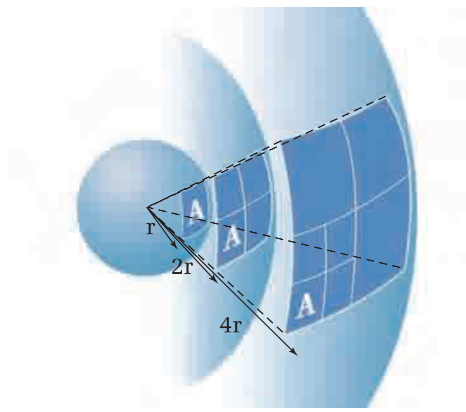


Figure 7.2 The intensity of physical phenomena that obey inverse square laws can be compared to the spreading out of the surface of a sphere.

COURSE CHALLENGE

Contact versus Non-Contact

Action at a distance — something that might have seemed magical to you as a child — lies at the heart of several cutting-edge technologies. Refer to page 604 of this textbook for suggestions about non-contact interactions for your Course Challenge project.

INVESTIGATION 7-A

The Nature of the Electrostatic Force

TARGET SKILLS
- Hypothesizing
- Performing and recording
- Analyzing and interpreting

In this investigation, you will use pith balls to quantitatively analyze the electrostatic force of repulsion.

Problem

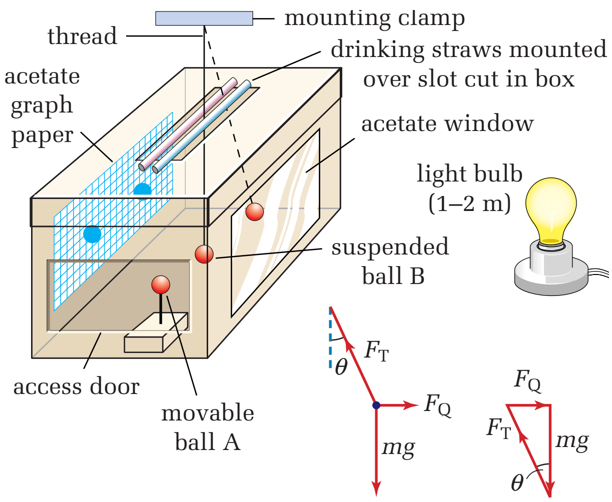
What is the relationship between electrostatic force and the distance of separation between two charged pith balls?

Equipment

- electronic balance
- clear straight filament lamp
- razor knife
- pith ball on thread
- pith ball mounted on wooden base
- acetate graph paper
- clear acetate sheet
- ebonite and fur
- cardboard shoe box
- two drinking straws

Procedure

CAUTION Be careful when using any sharp cutting object.



1. Cut rectangular holes in the front, rear, and side of the box and a slit on top, as shown.
2. Mount the clear acetate in the front hole and the acetate graph sheet in the rear. Mount the drinking straws on either side of the slit on top.
3. Poke the free end of the thread attached to pith ball B up between the drinking straws and mount on a clamp above. Ensure that the thread hangs vertically.
4. Place the pith ball with the wooden base (A) inside the box. Record the rest positions of both pith balls on the acetate grid.
5. Rub the ebonite with fur and reach in and charge both pith balls. Adjust the height of the mount of pith ball B so that it is level with pith ball A. Record the position of both pith balls.
6. Move pith ball A toward pith ball B several times. Adjust the mount of pith ball B each time to keep B level with A. For each trial, read and record the positions of both pith balls.
7. Measure the mass of a large number of balls and take an average to find the mass of one.

Analyze and Conclude

1. For each trial, use the rest positions and the final positions of the pith balls to determine the distance between A and B.
2. For each trial, use the lateral displacement of B, relative to its original rest position, to determine the electrostatic force acting on B. (Prove for yourself that $F_Q = mg \tan \theta$.)
3. Draw a graph with the electrostatic force on the vertical axis and the distance of separation between the charges on the horizontal axis. What does your graph suggest about the relationship between the electrostatic force and the distance of separation?
4. Calculate $1/r^2$ for each of your trials and plot a new graph of F versus $1/r^2$. Does your new graph provide evidence to back up the prediction you made in your original analysis? Discuss.

Electromagnetic Force

The exact nature of frictional forces and applied forces that are due to the electromagnetic force is very complex. How would anyone obtain fundamental information about such complex forces? Physicists start with the simplest cases of such forces, analyze these cases, and then extend them to more and more complex situations. The simplest case of an electromagnetic force is the electrostatic force between two stationary point charges.

Several scientists, including Daniel Bernoulli, Joseph Priestly, and Henry Cavendish, had proposed that the **electrostatic force** obeyed an inverse square relationship, based on a comparison with Newton's inverse square law of universal gravitation.

Coulomb's Experiment

French scientist Charles Augustin Coulomb (1736–1806) carried out experiments in 1785 similar to the investigation that you have just completed. Coulomb had previously developed a **torsion balance** for measuring the twisting forces in metal wires. He used a similar apparatus, shown in Figure 7.3, to analyze the forces between two charged pith balls.

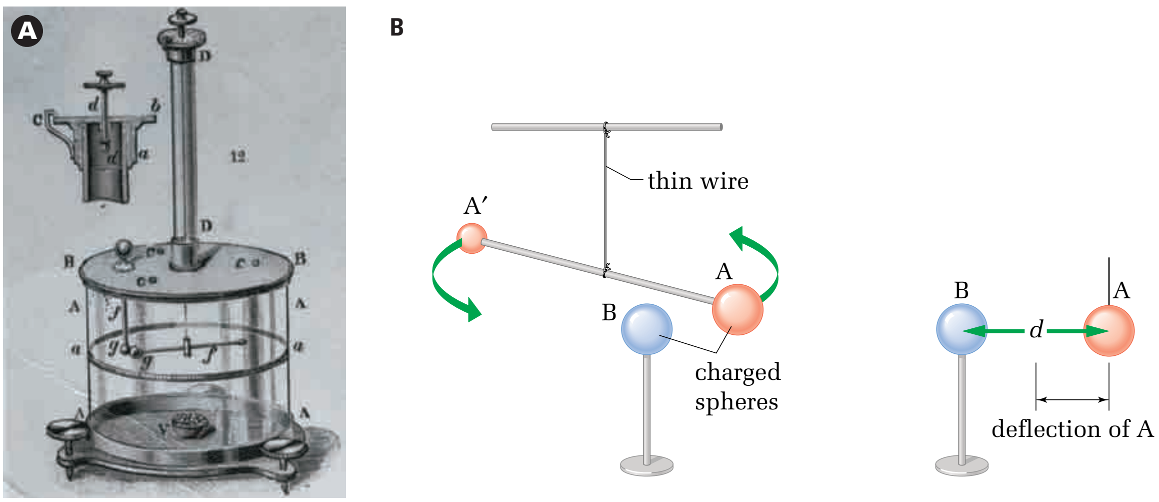


Figure 7.3 Coulomb's torsion balance **(A)** is simplified in **(B)**. Coulomb measured the force required to twist the thread a given angle. He then used this value to determine the force between the two pith balls.

Coulomb charged the two pith balls equally, placed them at precisely measured distances apart. Observing the angle of deflection, he was able to determine the force acting between them for each distance of separation. He found that the electric force, F, varied inversely with the square of the distance between the centres of the pith balls $\left(F_Q \propto \frac{1}{r^2}\right)$.

To investigate the dependence of the force on the magnitude of the charge on the pith balls, Coulomb began with two identically charged pith balls and measured the force between them. He then touched a pith ball with a third identical but uncharged pith ball to reduce the amount of charge on the ball by half. He found that

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You can develop a sense of the meaning of the Coulomb constant by considering two charges that are carrying exactly one unit of charge, a coulomb, and located one metre apart. Substituting ones into Coulomb's law, you would discover that these two charges exert a force of 9.00×10^9 N on each other. This amount of force could lift about 50 000 railroad cars or 2 million elephants. Clearly, one coulomb is an exceedingly large amount of charge. Typical laboratory charges would be much smaller — in the order of μC or millionths of a coulomb.

the force was now only one half the previous value. After several similar modifications of the charges, Coulomb concluded that the electric force varied directly with the magnitude of the charge on each pith ball ($F_Q \propto q_1 q_2$). The two proportion statements can be combined as one $\left(F_Q \propto \frac{q_1 q_2}{r^2}\right)$ and expressed fully as **Coulomb's law**.

Any proportionality can be written as an equality by including a proportionality constant. Although the value of the constant was not known until long after Coulomb's law was accepted, it is now known to be $8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$, in SI units.

The value of the proportionality constant in a vacuum is denoted k and known as the **Coulomb constant**. In fact, air is so close to "free space" — the early expression for a vacuum — that any effect on the value of the constant is beyond the number of significant digits that you will be using. For practical purposes, the Coulomb constant is often rounded to $9.00 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

Coulomb's law can now be written as $F_Q = k\frac{q_1 q_2}{r^2}$. The direction of the force is always along the line between the two point charges. Between charges of like sign, the force is repulsive; between charges of unlike sign, the force is attractive.

COULOMB'S LAW

The electrostatic force between two point charges, q_1 and q_2, distance r apart, is directly proportional to the magnitudes of the charges and inversely proportional to the square of the distance between their centres.

$$F_Q = k\frac{q_1 q_2}{r^2}$$

Quantity	Symbol	SI unit
electrostatic force between charges	F_Q	N (newtons)
Coulomb's constant	k	$\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$ (newton · metres squared per coulomb squared)
electric charge on object 1	q_1	C (coulombs)
electric charge on object 2	q_2	C (coulombs)
distance between object centres	r	m (metres)

Unit Analysis

$$\text{newton} = \frac{(\text{newton})(\text{metre})^2}{(\text{coulomb})^2} \cdot \frac{(\text{coulomb})(\text{coulomb})}{(\text{metre})^2}$$

$$\frac{\mathrm{N \cdot \cancel{m^2}}}{\cancel{\mathrm{C^2}}} \cdot \frac{\cancel{\mathrm{C}} \cdot \cancel{\mathrm{C}}}{\cancel{\mathrm{m^2}}} = \mathrm{N}$$

PHYSICS FILE

Note that not only does the proportionality constant have to validate the numerical relationship, it must also make the units match. Thus, the units for k are obtained by rearranging the Coulomb equation.

$$k = \frac{F \cdot d^2}{q_1 \cdot q_2}$$

$$= \frac{(\text{force})(\text{distance})^2}{(\text{charge})(\text{charge})}$$

$$= \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$$

Strictly speaking, the description of Coulomb's law given on the previous page is meant to apply to point charges. However, just as Newton was able to develop the mathematics (calculus) that proved that the mass of any spherical object can be considered to be concentrated at a point at the centre of the sphere for all locations outside the sphere, so it might also be proven that if charge is uniformly distributed over the surface of a sphere, then the value of the charge can be considered to be acting at the centre for all locations outside the sphere.

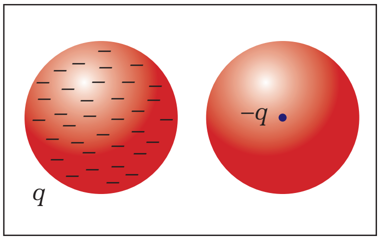

Figure 7.4 A uniformly charged sphere acts as if all of its charge is concentrated at its centre.

SAMPLE PROBLEM

Applying Coulomb's Law

A small sphere, carrying a charge of −8.0 μC, exerts an attractive force of 0.50 N on another sphere carrying a charge with a magnitude of 5.0 μC.

(a) What is the sign of the second charge?

(b) What is the distance of separation of the centres of the spheres?

Conceptualize the Problem

- *Charged spheres* appear to be the same as *point charges* relative to any point *outside* of the sphere.
- The *force*, *charge*, and *distance* are related by *Coulomb's law*.

Identify the Goal

The sign, ±, and separation distance, r, of the charges

Identify the Variables and Constants

Known		Implied	Unknown
$q_1 = -8.0 \times 10^{-6}$ C	$F = 0.50$ N	$k = 9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$	r
$\lvert q_2 \rvert = 5.0 \times 10^{-6}$ C			

Develop a Strategy

Since the spheres are uniformly charged, they can be considered to be points and Coulomb's law can be applied.

$$F = k\frac{q_1 q_2}{r^2}$$

$$r^2 = \frac{k q_1 q_2}{F}$$

$$r = \pm\sqrt{\frac{k q_1 q_2}{F}}$$

Only the positive root is chosen to represent the distance in this situation

$$r = \sqrt{\frac{\left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(8.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{5.0 \times 10^{-1}\ \text{N}}}$$

$$r = 0.84853\ \text{m}$$

$$r \cong 0.85\ \text{m}$$

(a) Since the force is attractive, the second charge must be positive.

(b) The distance between the centres of the charges is 0.85 m.

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Validate the Solution

Charges in the microcoulomb range are expected to exert moderate forces on each other.

PRACTICE PROBLEMS

1. Calculate the electrostatic force between charges of $-2.4\ \mu C$ and $+5.3\ \mu C$, placed 58 cm apart in a vacuum.
2. The electrostatic force of attraction between charges of $+4.0\ \mu C$ and $-3.0\ \mu C$ is 1.7×10^{-1} N. What is the distance of separation of the charges?
3. Two identically charged objects exert a force on each other of 2.0×10^{-2} N when they are placed 34 cm apart. What is the magnitude of the charge on each object?
4. Two oppositely charged objects exert a force of attraction of 8.0 N on each other. What will be the new force of attraction if the objects are moved to a distance four times their original distance of separation?
5. Two identical objects have charges of $+6.0\ \mu C$ and $-2.0\ \mu C$, respectively. When placed a distance d apart, their force of attraction is 2.0 N. If the objects are touched together, then moved to a distance of separation of $2d$, what will be the new force between them?

QUICK LAB Graphical Analysis of Coulomb's Law

TARGET SKILLS

- **Analyzing and interpreting**
- **Communicating results**

In this Quick Lab, you will use sample data to gain practice with the inverse square dependence of the electrostatic force between two point charges on the distance between them. Two equally charged, identical small spheres are placed at measured distances apart and the force between them is determined by using a torsion balance. Prepare a table similar to the one shown here, in which to record your data.

1. Draw a graph of force versus distance for this data. What is the shape of this graph?
2. Rearrange the distance data (use the third column in your table) and draw a graph that shows the relationship as a linear one (refer to Skill Set 4, Mathematical Modelling and Curve Straightening).
3. Measure the slope of the straight line.
4. Using the known value of Coulomb's constant ($k = 9 \times 10^9\ N \cdot m^2/C^2$), calculate the value of the original charge on the spheres.

Force ($\times 10^2$ N)	Distance between centres (cm)	
5.63	1.2	
2.50	1.8	
1.30	2.5	
0.791	3.2	
0.383	4.6	
0.225	6.0	

The Nature of Electric, Magnetic, and Gravitational Forces

All forces, including electrostatic forces, are vector quantities and obey the laws of vector addition. The equation describing Coulomb's law uses only scalar quantities, with the understanding that the direction of the force always lies along the line joining the centre of the two charges. However, when one charge experiences a force from more than one other charge, the direction must be resolved.

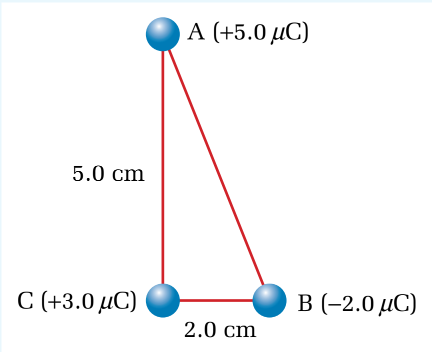
ELECTRONIC LEARNING PARTNER

Go to your Physics 12 Electronic Learning Partner to enhance your knowledge of Coulomb's law.

SAMPLE PROBLEM

Multiple Charges

Three charges, A (+5.0 μC), B (–2.0 μC), and C (+3.0 μC), are arranged at the corners of a right triangle as shown. What is the net force on charge C?



Conceptualize the Problem

- *Charges* A and B both exert a *force* on C.
- Although A and B exert forces on each other, these forces have no effect on the forces that they exert on C.
- The *net force* on charge C is the *vector sum* of the *two forces* exerted by charges A and B.
- The *forces* exerted by A and B are related to the magnitude of the *charges* and the *distance* between the charges, according to *Coulomb's law*.

Identify the Goal

The net force, $\vec{F}_{net}$, on charge C

Identify the Variables and Constants

Known		Implied	Unknown
$q_A = +5.0 \times 10^{-6}$ C	$r_{AC} = 5.0 \times 10^{-2}$ m	$k = 9.0 \times 10^9 \dfrac{N \cdot m^2}{C^2}$	$\vec{F}_{net}$
$q_B = -2.0 \times 10^{-6}$ C	$r_{BC} = 2.0 \times 10^{-2}$ m		
$q_C = +3.0 \times 10^{-6}$ C			

Develop a Strategy

Use Coulomb's law to find the magnitude of the forces acting on C.

Let F_{AC} represent the magnitude of the force of charge A on charge C.

$$F_{AC} = k\frac{q_A q_C}{r^2}$$

$$F_{AC} = \left(9.0 \times 10^9 \frac{N \cdot m^2}{C^2}\right)\frac{(5.0 \times 10^{-6}\ C)(3.0 \times 10^{-6}\ C)}{(0.050\ m)^2}$$

$$F_{AC} = 54\ N$$

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Let F_{BC} represent the magnitude of the force of charge B on charge C (attraction).

$$F_{BC} = k\frac{q_B q_C}{r^2}$$

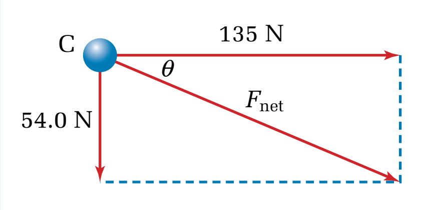
$$F_{BC} = \left(9.0 \times 10^9 \ \frac{N \cdot m^2}{C^2}\right)\frac{(2.0 \times 10^{-6}\ C)(3.0 \times 10^{-6}\ C)}{(0.020\ m)^2}$$

$$F_{BC} = 135\ N$$

Since charges A and C are both positive, the force will be repulsive and point directly downward on C.

Since B and C are oppositely charged, the force will be attractive and will point directly to the right of C.

Draw a diagram of the forces on charge C.



Use the Pythagorean theorem to calculate the magnitude of F_{net}.

$$F_{net}^2 = (135\ N)^2 + (54.0\ N)^2$$

$$F_{net}^2 = 21\ 141\ N^2$$

$$F_{net} = 145.39\ N$$

$$F_{net} \cong 1.5 \times 10^2\ N$$

Use the definition of the tangent function to find the angle, θ.

$$\tan\theta = \frac{54.0}{135}$$

$$\tan\theta = 0.40$$

$$\theta = \tan^{-1} 0.40$$

$$\theta = 21.8^\circ$$

$$\theta \cong 22^\circ$$

The net force on charge C is 1.5×10^2 N at an angle of 22˚ clockwise from the horizontal.

Validate the Solution

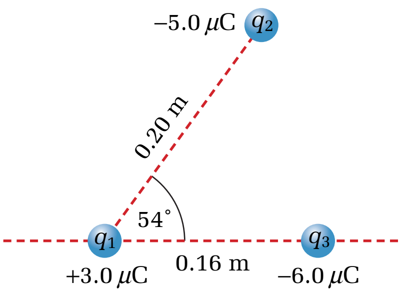
The magnitude and direction of the net force are consistent with the orientation of the three charges.

PRACTICE PROBLEMS

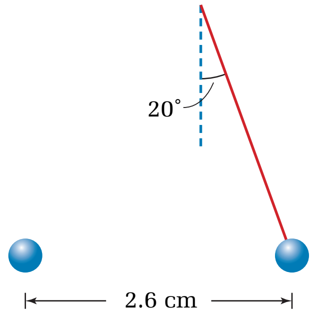
6. A single isolated proton is fixed on a surface. Where must another proton be located in relation to the first in order that the electrostatic force of repulsion would just support its weight?

7. Three charged objects are located at the vertices of a right triangle. Charge A (+5.0 μC) has Cartesian coordinates (0,4); charge B (–5.0 μC) is at the origin; charge C (+4.0 μC) has coordinates (5,0), where the coordinates are in metres. What is the net force on each charge?

8. The diagram shows three charges situated in a plane. What is the net electrostatic force on q_1?



9. The diagram below shows two pith balls, equally charged and each with a mass of 1.5 g. While one ball is suspended by a thread, the other is brought close to it and a state of equilibrium is reached. In that situation, the two balls are separated by 2.6 cm and the thread attached to the suspended ball makes an angle of 20° with the vertical. Calculate the charge on each of the pith balls.



10. Two 2.0 g spheres are attached to each end of a silk thread 1.20 m long. The spheres are given identical charges and the midpoint of the thread is then suspended from a point on the ceiling. The spheres come to rest in equilibrium, with their centres 15 cm apart. What is the magnitude of the charge on each sphere?

Although magnetic forces and electrostatic forces are related and both fit into the category of electromagnetic forces, the strength of a magnetic force cannot be defined in the same way as electrostatic and gravitational forces. The reason for the difference is that magnetic monopoles do not exist or, at least, have never been detected, in spite of the efforts of physicists. Where there is a north pole, you will also find a south pole. Nevertheless, Coulomb was able to approximate isolated magnetic monopoles by measuring the forces between the poles of very long, thin magnets.

If one pole of a long, thin bar magnet is placed in the vicinity of one pole of another long, thin bar magnet, Coulomb's magnetic force law states: The magnetic force F between one pole of magnetic strength p_1 and another pole of magnetic strength p_2 is inversely proportional to the square of the distance r between them, or $F \propto \frac{p_1 p_2}{r^2}$. It is not possible, however, to find a proportionality constant, because it is not possible to define a unit for p, a magnetic monopole.

You have seen that the three different types of forces — electrostatic, gravitational, and magnetic — all exhibit some form of an inverse square distance relationship. Are there any significant differences that you should note?

Probably the greatest difference between gravitational and electromagnetic forces is the strength. Gravitational forces are much weaker than electrostatic and magnetic forces. For example, you do not see uncharged pith balls, nor demagnetized iron bars, moving toward each other under the action of their mutual gravitational attraction.

In summary, the similarities and differences among electrostatic, gravitational, and magnetic forces are listed in Table 7.1.

Table 7.1 Differences among Electrostatic, Gravitational, and Magnetic Forces

Electrostatic force	Gravitational force	Magnetic force
■ can be attractive or repulsive ■ demonstrates an inverse square relationship in terms of distance ■ depends directly on the unit property (charge) ■ law easily verified using point charges (or equivalent charged spheres)	■ can only be attractive ■ demonstrates an inverse square relationship in terms of distance ■ depends directly on the unit property (mass) ■ law easily verified using point masses (or solid spheres) ■ magnitude of the force is much weaker than electrostatic or magnetic force	■ can be attractive or repulsive ■ demonstrates an inverse square relationship in terms of distance (between isolated poles) ■ depends directly on the unit property (pole strength) ■ law cannot be verified using magnetic monopoles as they do not exist independently (must be simulated using long, thin magnets or thin, magnetized wire)

7.1 Section Review

1. K/U What is meant by the statement that Coulomb "quantified" the electric force?

2. K/U In what way did Coulomb determine the dependence of the electrostatic force on different variables?

3. C Explain the similarities and differences between the Coulomb experiment for charge and the Cavendish experiment on mass.

4. C Explain how, in one sense, Coulomb's law is treated as a scalar relationship, but on the other hand, its vector properties must always be considered.

5. K/U State some similarities and some differences between the gravitational force and the electrostatic force.

6. MC Research the role of electrostatic charge in technology and write a brief report on your findings. Examples could include photocopiers and spray-painting equipment.

7. I By what factor would the electrostatic force between two charges change under the following conditions?
   (a) The distance is tripled.
   (b) Each of the charges is halved.
   (c) Both of the above changes are made.

7.2 Describing Fields

In Figure 7.5 (A), a woodcutter exerts a splitting force on a log by direct contact between the axe and the log. In contrast, in Figure 7.5 (B), a charged comb is exerting a force on charged pith balls without coming into contact with the balls. This electrostatic force between the comb and pith balls is an example of an action-at-a-distance force. You have just been studying the characteristics of the three common action-at-a-distance forces: gravitational, electric, and magnetic forces. The phrase **action at a distance** describes some of the characteristics of these forces, but does not really explain how these results are achieved. The critical question now is: How is each mass or charge or magnet "aware" of the other?

Figure 7.5 **(A)** A woodcutter chopping a log is an example of a contact force. **(B)** When a charged comb exerts a force on charged pith balls, the force is acting at a distance and is a non-contact force.

The question of how an object can exert a force on another object without making contact with the object was addressed by Michael Faraday (1792–1867), who proposed the concept of a field. This field concept became quite popular and was extended to explain the gravitational forces between masses.

The fundamental concept is that a **field** is a property of space. An object influences the space around it, setting up either an electric, gravitational, or magnetic field. The object producing the field is called the "source" of the field. This field in turn exerts a force on other objects located within it. This concept is consistent with the inverse square law, which implies that an object influences the space around it.

SECTION EXPECTATIONS

- Define and describe the concepts related to electric, gravitational, and magnetic fields.
- Compare the properties of electric, gravitational, and magnetic fields.
- Analyze the electric field and the electric forces produced by point charges.
- Sketch simple field patterns using field lines.

KEY TERMS

- action at a distance
- field
- test charge
- electric field intensity
- electric field
- gravitational field intensity
- magnetic field intensity
- electric field line
- gravitational field line
- magnetic field line

HISTORY LINK

In 1600, William Gilbert (1540–1603) hypothesized that the rubbing of certain materials, such as amber, removes a fluid or "humour" from the material and releases an "effluvium" into the surroundings. He proposed that the effluvium made contact with other materials and caused the force now known as the "electrostatic force." As you continue to study this section, decide whether Gilbert was on the right track.

Defining Field Intensity

Figure 7.6 illustrates the generation of an electric field by a charge, q_1. The density of the shading designates the strength of the field. If a second charge, q_2, is introduced into the field at point P, for example, it is the *field* that interacts with q_2. Because this is a local interaction, it is not necessary to explain how forces can act between objects separated by any distance.

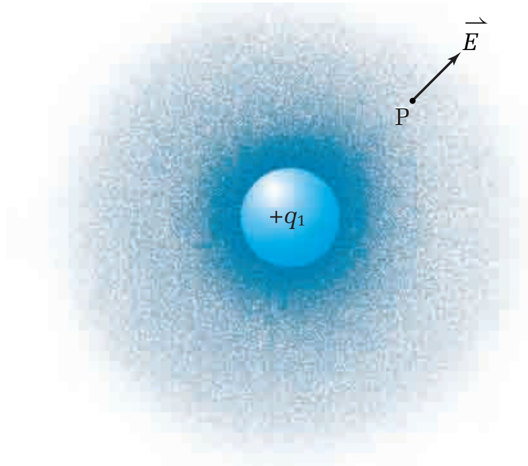


Figure 7.6 Charge q_1 influences the space around it by generating an electric field. The density of the shading indicates the strength of the field.

To describe the field around a charge, q, it is convenient to use the concept of a test charge. By definition, a **test charge** is a point charge with a magnitude so much smaller than the source charge that any field generated by the test charge itself is negligible in relation to the field generated by the source charge. You can place the test charge, q_t, at any point within the field generated by q, and then take the following steps.

- Write Coulomb's law to describe the force between the source charge, q, and the test charge, q_t. $$F = k\frac{qq_t}{r^2}$$
- Divide both sides of the equation by q_t. $$\frac{F}{q_t} = k\frac{q}{r^2}$$

The term on the right-hand side of the equation contains only the source charge and the distance that q_t is from the source charge. Since it is independent of anything that might be located at q_t, it provides a convenient way to describe the condition of space at q_t. Now the term on the left-hand side of this equation is defined as the magnitude of the **electric field intensity**, $\vec{E}$, which is commonly called the **electric field**.

DEFINITION OF ELECTRIC FIELD INTENSITY

The electric field intensity at a point is the quotient of the electric force on a charge and the magnitude of the charge located at the point.

$$\vec{E} = \frac{\vec{F}_Q}{q}$$

Quantity	Symbol	SI unit
electric field intensity	$\vec{E}$	$\frac{N}{C}$ (newtons per coulomb)
electric force	$\vec{F}_Q$	N (newtons)
electric charge	q	C (coulombs)

Unit Analysis

$$\frac{\text{newtons}}{\text{coulomb}} = \frac{N}{C}$$

Note: Electric field intensity has no unit of its own.

Since force is a vector quantity, so also is an electric field. An electric force can be attractive or repulsive, so physicists have accepted the convention that the direction of the electric field vector at any point is given by the direction of the force that would be exerted on a *positive* charge located at that point. Using this concept, you can illustrate an electric field by drawing force vectors at a variety of points in the field. As shown in Figure 7.7, the length of the vector represents the magnitude of the field at the tail of the vector, and the direction of the vector represents the direction of the field at that point.

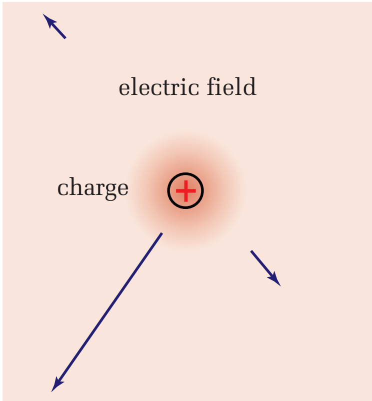


Figure 7.7 Vector arrows can be used to represent the magnitude and direction of the electric field around a charge at various locations.

SAMPLE PROBLEM

Calculating Electric Field Intensity

A positive test charge, $q_t = +2.0 \times 10^{-9}$ C, is placed in an electric field and experiences a force of $\vec{F} = 4.0 \times 10^{-9}$ N[W].

(a) What is the electric field intensity at the location of the test charge?

(b) Predict the force that would be experienced by a charge of $q = +9.0 \times 10^{-6}$ C if it replaced the test charge, q_t.

Conceptualize the Problem

- The *electric field intensity* is related to the *force* and the test *charge*.
- If you know the *electric field intensity* at a point in space, you can determine the *force* on any *charge* that is placed at that point without knowing anything about the source of the field.

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Identify the Goal

The electric field, $\vec{E}$, at a given point in space

The force, $\vec{F}$, on the new charge located at the same point

Identify the Variables

Known	**Unknown**
$\vec{F}_{q_t} = 4.0 \times 10^{-9}$ N[W]	$\vec{E}$
$q_t = 2.0 \times 10^{-9}$ C	$\vec{F}$
$q = 9.0 \times 10^{-6}$ C	

Develop a Strategy

Find the electric field intensity by using the equation that defines electric field.

$$\vec{E} = \frac{\vec{F}}{q_t}$$

$$\vec{E} = \frac{4.0 \times 10^{-9}\ \text{N[W]}}{2.0 \times 10^{-9}\ \text{C}}$$

$$\vec{E} = 2.0\ \frac{\text{N}}{\text{C}}\text{[W]}$$

(a) The electric field intensity is $\vec{E} = 2.0\ \frac{\text{N}}{\text{C}}\text{[W]}$.

Rearrange the equation for electric field to solve for the new force.

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{F} = q\vec{E}$$

Substitute the numerical values and solve.

$$\vec{F} = (9.0 \times 10^{-6}\ \text{C})\left(2.0\ \frac{\text{N}}{\text{C}}\text{[W]}\right)$$

$$\vec{F} = 18 \times 10^{-6}\ \text{N[W]}$$

$$\vec{F} = 1.8 \times 10^{-5}\ \text{N[W]}$$

(b) The force on the 9.0×10^{-6} C charge is $\vec{F} = 1.8 \times 10^{-5}$ N[W].

Validate the Solution

You would expect the electric field to have units N/C and be pointing west. The magnitude of the field seems to be reasonable in relation to the charge and force.

Since the second charge is larger than the first, you would expect the second force to be larger than the first. Charges in the microcoulomb range are considered to be average charges that occur in electrostatic experiments.

PRACTICE PROBLEMS

11. A positive charge of 3.2×10^{-5} C experiences a force of 4.8 N to the right when placed in an electric field. What is the magnitude and direction of the electric field at the location of the charge?

12. An electric field points due east with a magnitude of 3.80×10^{3} N/C at a particular location. If a charge of $-5.0\ \mu$C is placed at this location, what will be the magnitude and the direction of the electric force that it experiences?

13. A negative charge of 2.8×10^{-6} C experiences an electrostatic force of 0.070 N to the right. What is the magnitude and direction of the electric field at the location of the charge?

14. A small charged sphere is placed at a point in an electric field that points due west and has a magnitude of 1.60×10^4 N/C. If the sphere experiences an electrostatic force of 6.4 N east, what is the magnitude and sign of its charge?

A discussion similar to that for the electric field intensity can be made for gravitational field intensity. A mass, such as Earth, can exert a gravitational force on a test mass placed in its vicinity. The ratio of the gravitational force to the test mass depends only on the source and the location in the field. This ratio is called the **gravitational field intensity**, for which the symbol is $\vec{g}$.

DEFINITION OF GRAVITATIONAL FIELD INTENSITY

The gravitational field intensity at a point is the quotient of the gravitational force and the magnitude of the test mass.

$$\vec{g} = \frac{\vec{F}_g}{m}$$

Quantity	Symbol	SI unit
gravitational field intensity	$\vec{g}$	$\frac{N}{kg}$ (newtons per kilogram)
gravitational force	$\vec{F}_g$	N (newtons)
mass	m	kg (kilograms)

Unit Analysis

$$\frac{\text{newtons}}{\text{kilogram}} = \frac{N}{kg}$$

In the past, you have used the symbol g to represent the acceleration due to gravity at Earth's surface. If you analyze the equation that described gravitational field intensity in the box above, you will see that it can be rearranged to give $\vec{F} = m\vec{g}$, which is the same as the equation for the weight of an object at Earth's surface. So, in fact, the g that you have been using is the same as the gravitational field intensity at Earth's surface.

Conceptual Problem

- Show that the units for g, m/s^2, are equivalent to the units for gravitational field intensity, or N/kg.

SAMPLE PROBLEM

Calculating Gravitational Field Intensity

A mass of 4.60 kg is placed 6.37×10^6 m from the centre of a planet and experiences a gravitational force of attraction of 45.1 N.

(a) Calculate the gravitational field intensity at this location.

(b) Discuss the significance of your answer.

Conceptualize the Problem

- The definition of *gravitational field intensity* is the gravitational *force* per unit *mass*.

Identify the Goal

The gravitational field intensity, $\vec{g}$, at this location

Identify the Variables

Known	Unknown
$\vert\vec{F}\vert = 45.1$ N	$\vec{g}$
$m = 4.60$ kg	
$r = 6.37 \times 10^6$ m	

Develop a Strategy

Find the gravitational field intensity by using the equation for field intensity and the given variables.

$$\vec{g} = \frac{\vec{F}}{m}$$

$$|\vec{g}| = \frac{45.1 \text{ N}}{4.60 \text{ kg}}$$

$$\vec{g} = 9.80 \frac{\text{N}}{\text{kg}} \text{[in the direction of the force]}$$

(a) The gravitational field intensity at this location is 9.80 N/kg.

Look for recognizable characteristics, then investigate other data.

The value of the field intensity is identical to that of Earth's near its surface.

The distance given is actually the average radius of Earth.

(b) The location seems to be at the surface of Earth, although another alternative is that it could be *above* the surface of a planet with gravitational field intensity at its surface that is greater than that of Earth.

Validate the Solution

The units are correct for gravitational field. The values for both distance and field intensity provide more validation, because they are identical to the values for the surface of Earth. However, this does not preclude the possibility of the object being above another planet.

PRACTICE PROBLEMS

15. What is the gravitational field intensity at the surface of Mars if a 2.0 kg object experiences a gravitational force of 7.5 N?

16. The gravitational field intensity on the surface of Jupiter is 26 N/kg. What gravitational force would a 2.0 kg object experience on Jupiter?

17. The planet Saturn has a gravitational field intensity at its surface of 10.4 N/kg. What is the mass of an object that weighs 36.0 N on the surface of Saturn?

18. What would be the gravitational field intensity at a location exactly one Saturn radius above the surface of Saturn?

19. What is the centripetal acceleration of a satellite orbiting Saturn at the location described in the previous problem?

The gravitational field can also be mapped in the region of a source mass by drawing the gravitational field vectors at corresponding points in the field. Similar to the electric field, the vector length represents the magnitude of the gravitational field and the direction of the vector represents the direction in which a gravitational force would be exerted on a test mass placed in the field.

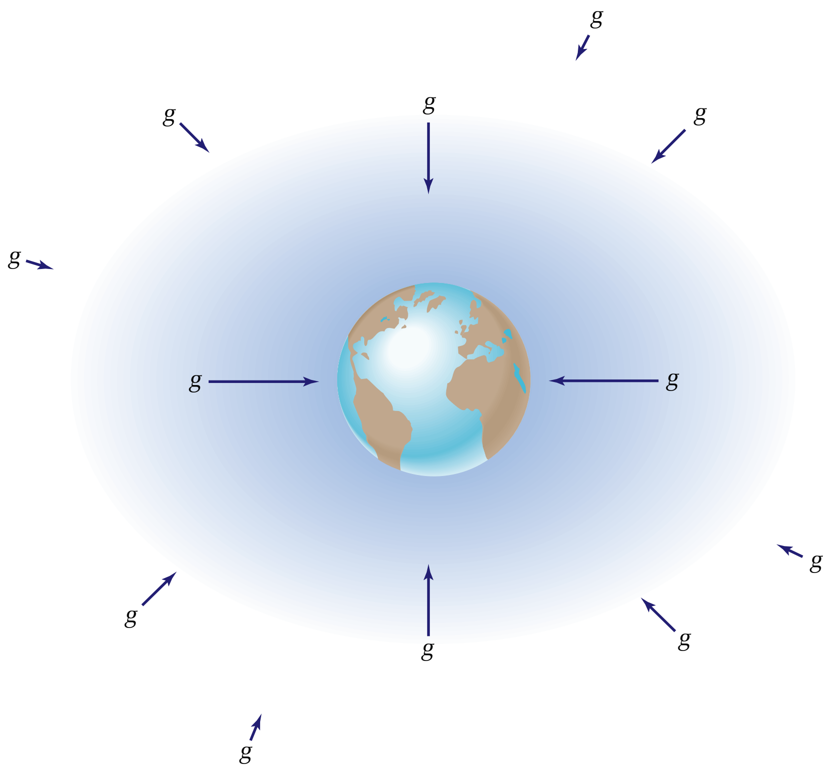
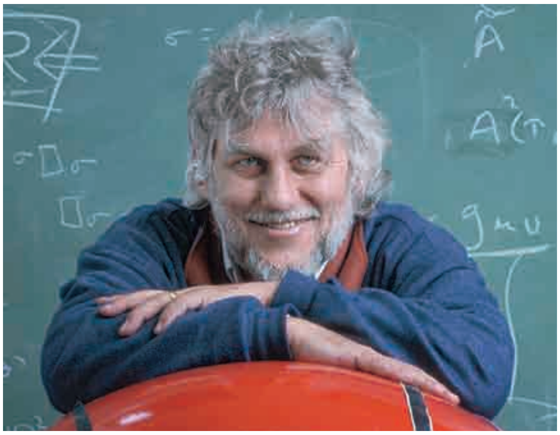


Figure 7.8 Earth's gravitational field can be represented by vectors, with the length of each being proportional to the field intensity at that point.

CANADIANS IN PHYSICS

Gravity: A Matter of Time

William George Unruh was born in Winnipeg, the son of a high school physics teacher. "As a boy I loved looking at the pictures in my father's physics textbooks," he recalls. "They aroused my curiosity about how the world works." He attended the University of Manitoba and then Princeton University, where he received his Ph.D. Today, he is a professor of physics and astronomy at the University of British Columbia and a Fellow of the Canadian Institute for Advanced Research.



Dr. Unruh with the beach ball that he sometimes uses to help explain the concept of gravity.

Dr. Unruh explains that, according to Albert Einstein, "The rate at which time flows can change from place to place, and it is this change in the flow of time that causes the phenomenon we usually refer to as gravity." Dr. Unruh's work focusses on understanding aspects of Einstein's theories. "For example," he says, "Since matter can influence time and matter influences gravity, which is just the variable flow of time, the very measuring instruments we use to measure time can change time. While this is not important in most situations, it becomes very important in trying to decide how the universe operates; for example, in understanding black holes." Dr. Unruh explains that, in black holes, the structures of space and time collaborate, creating regions through which even light cannot travel.

"All of physics is now described in terms of field theories," Dr. Unruh points out. "However, we also experience the world in terms of particles. Since fields exist everywhere at all times, part of my work has been trying to understand the particulate nature of fields. Probably my best known work is showing that the particle nature of fields depends on the observer's state of motion. If an observer is accelerated through a region that seems to be empty of particles to an observer at rest, that region will, to the accelerated observer, appear to be filled with a hot bath of particles. Thus, the existence or non-existence of particles in a field can depend on how the observer moves as he or she observes that field. The effect is extremely small, but it is there."

Another area in which Dr. Unruh works is gravity wave detection. A gravity wave might be called a "vibration of space and time." It is caused by the acceleration of masses; for example, of black holes around each other. The techniques that Dr. Unruh and others have developed will be important to the future refinement of gravity wave detectors now being built in the states of Louisiana and Washington, as well as elsewhere in the world.

Since magnetic monopoles are not known to exist, it is not practical to try to define magnetic field intensities in a way that is analogous to the definitions of electric and gravitational fields. The most practical way to describe magnetic field intensity at this point is to relate it to the effect of a magnetic field on a current-carrying wire, which you studied in previous science courses. The following steps show you how to relate the **magnetic field intensity**, B, to the force, $\vec{F}_B$, exerted by the magnetic field on a length, l, of wire carrying a current, I.

- Write the equation describing the force on a current-carrying conductor in a magnetic field when the direction of the current is perpendicular to the magnetic field.

$$\vec{F}_B = \vec{I}\,l\vec{B}$$

- Rearrange the equation to solve for the magnetic field intensity.

$$\vec{B} = \frac{\vec{F}}{\vec{I}l}$$

- The SI unit of magnetic field intensity is the tesla, T. Substitute SI units for the symbols in the equation above.

$$\mathrm{T} = \frac{\mathrm{N}}{\mathrm{A \cdot m}} \text{ or}$$

$$\text{tesla} = \frac{\text{newton}}{\text{ampere} \cdot \text{metre}}$$

The above relationship states that if each metre of a conductor that is carrying a current of one ampere experiences a force of one newton due to the presence of a magnetic field that is perpendicular to the direction of the current, the magnitude of the magnetic field is one tesla.

Fields near Point Sources

The definition and accompanying equation that you learned for electric field strength, $\vec{E} = \vec{F}_Q/q$, is a general definition. If you know the force on a charge due to an electric field, you can determine the electric field intensity without knowing anything about the source of the field. It is convenient, however, to develop equations that describe the electric field intensity for a few common, special cases, such as point charges.

- Write the equation describing the magnitude of the force on a test charge, q_t, that is a distance, r, from a point charge, q.

$$|\vec{F}_Q| = k\frac{qq_t}{r^2}$$

- Write the general definition for the electric field intensity.

$$\vec{E} = \frac{\vec{F}_Q}{q_t}$$

- Substitute the expression for force into the above equation and simplify.

$$|\vec{E}| = \frac{k\frac{q\cancel{q_t}}{r^2}}{\cancel{q_t}}$$

$$|\vec{E}| = k\frac{q}{r^2}$$

ELECTRIC FIELD INTENSITY NEAR A POINT CHARGE

The electric field intensity a distance away from a point charge is the product of Coulomb's constant and the charge, divided by the square of the distance from the charge. The direction of the field is radially outward from a positive point charge and radially inward toward a negative point charge.

$$|\vec{E}| = k\frac{q}{r^2}$$

Quantity	Symbol	SI unit
electric field intensity	$\vec{E}$	$\frac{\text{N}}{\text{C}}$ (newtons per coulomb)
Coulomb's constant	k	$\frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$ (newton · metres squared per coulomb squared)
source charge	q	C (coulombs)
distance	r	m (metres)

Unit Analysis

$$\frac{\text{newton} \cdot \text{metre}^2}{\text{coulomb}^2} \cdot \frac{\text{coulomb}}{\text{metre}^2} = \frac{\text{newton}}{\text{coulomb}}$$

$$\frac{\text{N} \cdot \cancel{\text{m}^2}}{\text{C}^{\cancel{2}}} \cdot \frac{\cancel{\text{C}}}{\cancel{\text{m}^2}} = \frac{\text{N}}{\text{C}}$$

Note: This equation applies only to the field surrounding an isolated point charge.

SAMPLE PROBLEMS

Field Intensity near a Charged Sphere

1. **What is the electric field intensity at a point 30.0 cm from the centre of a small sphere that has a positive charge of 2.0×10^{-6} C?**

Conceptualize the Problem

- At any point outside of a *charged sphere*, the *electric field* is the same as it would be if the charge was *concentrated at a point* at the *centre* of the sphere.
- The *electric field* is related to the source *charge* and *distance*.
- The *direction* of the field is the direction in which a *positive charge would move* if it was placed at that point in the field.

Identify the Goal

The electric field intensity, $\vec{E}$

Identify the Variables and Constants

Known	**Implied**	**Unknown**
$q = +2.0 \times 10^{-6}$ C	$k = 9.0 \times 10^9 \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}$	$\vec{E}$
$r = 0.30$ m		

Develop a Strategy

Find the field intensity by using the equation for the special case of the field near a point charge.

$$|\vec{E}| = k\frac{q}{r^2}$$

Substitute the numerical values for charge and distance and solve.

$$|\vec{E}| = \left(9.0 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.0 \times 10^{-6}\ \text{C}}{(0.30\ \text{m})^2}\right)$$

The direction is radially outward from the positive charge.

$$|\vec{E}| = 2.0 \times 10^5 \frac{\text{N}}{\text{C}}$$

The electric field intensity is 2.0×10^5 N/C in a direction pointing directly away from the source charge.

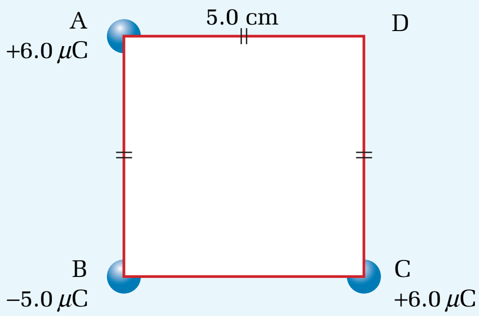
Validate the Solution

Close to a charge of "average" magnitude, the field is expected to be quite strong. Check that the units cancel to give N/C.

$$\frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{C}^2}}\cdot\frac{\cancel{\text{C}}}{\cancel{\text{m}^2}} = \frac{\text{N}}{\text{C}}$$

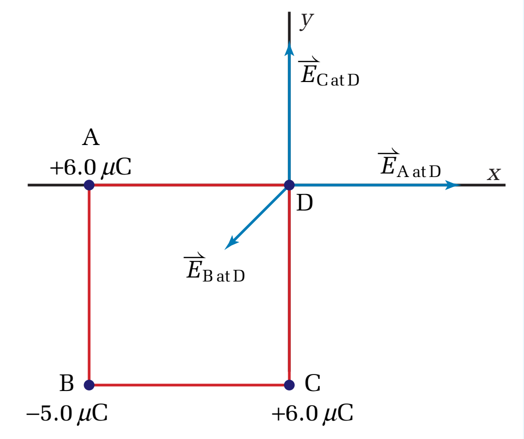
---

2. Three charges, A (+6.0 μC), B (–5.0 μC), and C (+6.0 μC), are located at the corners of a square with sides that are 5.0 cm long. What is the electric field intensity at point D?



Conceptualize the Problem

- Since field intensities are vectors they must also be *added vectorally*.
- The magnitude of the field vectors can be determined individually.
- Draw a *vector diagram* showing the *field intensity* vectors at point D and then superimpose an *x-y coordinate* system on the drawing, with the *origin* at point D.



Identify the Goal

The resultant electric field intensity, $\vec{E}$, at point D

Identify the Variables and Constants

Known	**Implied**	**Unknown**
$d_{AB} = d_{BC} = 5.0$ cm	$k = 9.0 \times 10^9 \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}$	$\vec{E}_D$
$q_A = +6.0\ \mu$C		d_{BD}
$q_B = -5.0\ \mu$C		
$q_c = +6.0\ \mu$C		

continued ►

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Develop a Strategy

Calculate the diagonal of the square by using the Pythagorean theorem.

Since the result is a distance, the negative root has no meaning. Use the positive root.

$$d_{BC}^2 = (5.0\text{ cm})^2 + (5.0\text{ cm})^2$$
$$d_{BC}^2 = 50.0\text{ cm}^2$$
$$d_{BC} = \pm\sqrt{50.0\text{ cm}^2}$$
$$d_{BC} = \pm 7.07\text{ cm}$$
$$d_{BC} = 7.07\text{ cm}$$

Calculate the magnitude of the electric field intensity of each of the given charges at point D, using the equation for the special case of the field intensity near a point charge.

$$|\vec{E}| = k\frac{q}{r^2}$$
$$|\vec{E}_{\text{A at D}}| = \left(9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{6.0 \times 10^{-6}\text{ C}}{(0.050\text{ m})^2}\right)$$
$$|\vec{E}_{\text{A at D}}| = 2.16 \times 10^7\ \frac{\text{N}}{\text{C}}$$
$$|\vec{E}_{\text{B at D}}| = \left(9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{5.0 \times 10^{-6}\text{ C}}{(0.0707\text{ m})^2}\right)$$
$$|\vec{E}_{\text{B at D}}| = 9.00 \times 10^6\ \frac{\text{N}}{\text{C}}$$
$$|\vec{E}_{\text{C at D}}| = \left(9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{6.0 \times 10^{-6}\text{ C}}{(0.050\text{ m})^2}\right)$$
$$|\vec{E}_{\text{C at D}}| = 2.16 \times 10^7\ \frac{\text{N}}{\text{C}}$$

Use the method of components to find the resultant electric field vector.

The angle between the x-axis and the vector for the field at point D due to charge B is 45°, because it points along the diagonal of a square.

***x*-components**

$$E_{(\text{A at D})x} = 2.16 \times 10^7\ \frac{\text{N}}{\text{C}}$$
$$E_{(\text{B at D})x} = -\left(9.00 \times 10^6\ \frac{\text{N}}{\text{C}}\right)\cos 45^\circ$$
$$E_{(\text{B at D})x} = -6.36 \times 10^6\ \frac{\text{N}}{\text{C}}$$
$$E_{(\text{C at D})x} = 0$$
$$E_{(\text{net})x} = 1.524 \times 10^7\ \frac{\text{N}}{\text{C}}$$

***y*-components**

$$E_{(\text{A at D})y} = 0$$
$$E_{(\text{B at D})y} = -\left(9.00 \times 10^6\ \frac{\text{N}}{\text{C}}\right)\sin 45^\circ$$
$$E_{(\text{B at D})y} = -6.36 \times 10^6\ \frac{\text{N}}{\text{C}}$$
$$E_{(\text{C at D})y} = 2.16 \times 10^7\ \frac{\text{N}}{\text{C}}$$
$$E_{(\text{net})y} = 1.524 \times 10^7\ \frac{\text{N}}{\text{C}}$$

Use the Pythagorean theorem to find the magnitude of the resultant vector.

$$|\vec{E}_{(\text{net})}|^2 = \left(E_{(\text{net})x}\right)^2 + \left(E_{(\text{net})y}\right)^2$$
$$|\vec{E}_{(\text{net})}|^2 = \left(1.524 \times 10^7\ \frac{\text{N}}{\text{C}}\right)^2 + \left(1.524 \times 10^7\ \frac{\text{N}}{\text{C}}\right)^2$$
$$|\vec{E}_{(\text{net})}|^2 = 4.6452 \times 10^{14}\left(\frac{\text{N}}{\text{C}}\right)^2$$
$$|\vec{E}_{(\text{net})}| = 2.1553 \times 10^7\ \frac{\text{N}}{\text{C}}$$
$$|\vec{E}_{(\text{net})}| \cong 2.2 \times 10^7\ \frac{\text{N}}{\text{C}}$$

Use the definition of the tangent function to find the direction of the electric field vector at point D.

$$\tan\theta = \frac{1.524 \times 10^7\ \frac{\text{N}}{\text{C}}}{1.524 \times 10^7\ \frac{\text{N}}{\text{C}}}$$
$$\tan\theta = 1.00$$
$$\tan\theta = \tan^{-1} 1.00$$
$$\tan\theta = 45^\circ$$

The electric field intensity at point D is 2.2×10^7 N/C at an angle of 45° counterclockwise from the positive x-axis.

Validate the Solution

Since two positive charges and one negative charge of similar magnitudes are creating the field, you would expect that the net field would be similar in magnitude to those created by the individual charges. The angle is 45° as predicted.

PRACTICE PROBLEMS

20. Calculate the electric field intensity at a point 18.0 cm from the centre of a small conducting sphere that has a charge of $-2.8\ \mu$C.

21. The electric field intensity at a point 0.20 m away from a point charge is 2.8×10^6 N/C, directed toward the charge. What is the magnitude and sign of the charge?

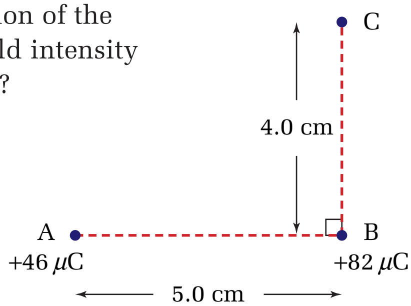
22. The electric field intensity at a point, P, near a spherical charge of 4.6×10^{-5} C, is 4.0×10^6 N/C. How far is point P from the centre of the charge?

23. How many electrons must be removed from a spherical conductor with a radius of 4.60 cm in order to make the electric field intensity just outside its surface 3.95×10^3 N/C?

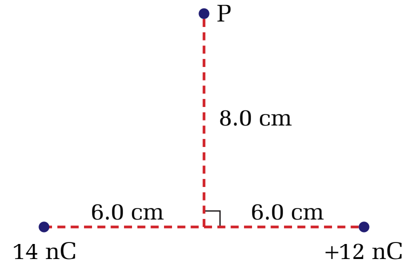
24. What is the electric field intensity at a point 15.2 cm from the centre of a sphere charged uniformly at $-3.8\ \mu$C?

25. A charge of $+7.4\ \mu$C establishes an electric field intensity at point M of 1.04×10^7 N/C. How far is point M from the centre of the charge?

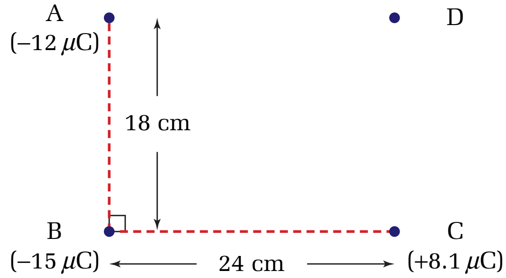
26. In the diagram, A and B represent small spherical charges of $+46\ \mu$C and $+82\ \mu$C, respectively. What is the magnitude and direction of the electric field intensity at point C ?



27. Determine the magnitude and direction of the electric field intensity at point P in the diagram.



28. The diagram shows three small charges at three corners of a rectangle. Calculate the magnitude and direction of the electric field intensity at the fourth corner, D.



29. Two point charges of $-40.0\ \mu$C and $+50.0\ \mu$C are placed 12.0 cm apart in air. What is the electric field intensity at a point midway between them?

30. Points A and B are 13.0 cm apart. A charge of $+8.0\ \mu$C is placed at A and another charge of $+5.0\ \mu$C is placed at B. Point P is located 5.0 cm from A and 12.0 cm from B. What is the magnitude and direction of the electric field intensity at P?

The approach taken above for electric fields can also be applied to gravitational fields. The following steps develop an expression for the gravitational field intensity near a point source. As stated previously, the field at any point outside of a spherical mass is the same as it would be if the mass was concentrated at a point at the centre of the sphere.

- Write the equation for the general definition of gravitational field intensity. $\vec{g} = \frac{\vec{F}_g}{m}$
- Write the general equation for the gravitational force between two masses. Let m_1 be the source of a gravitational field and m_2 be any mass, m, in that field. $|\vec{F}_g| = G\frac{m_s m}{r^2}$
- Substitute the expression for the force of gravity into the general expression for gravitational field intensity.

$$|\vec{g}| = \frac{G\frac{m_s \cancel{m}}{r^2}}{\cancel{m}}$$

$$|\vec{g}| = G\frac{m_s}{r^2}$$

GRAVITATIONAL FIELD INTENSITY NEAR A POINT MASS

The gravitational field intensity at a point a distance r from the centre of an object is the product of the universal gravitation constant and mass, divided by the square of the distance from the centre of the object. The direction of the gravitational field intensity is toward the centre of the object creating the field.

$$|\vec{g}| = G\frac{m_s}{r^2}$$

Quantity	**Symbol**	**SI unit**
gravitational field intensity	$\vec{g}$	$\frac{N}{kg}$ (newtons per kilogram)
universal gravitation constant	G	$\frac{N \cdot m^2}{kg^2}$ (newton · metres squared per kilogram squared)
mass of source of field	m_s	kg (kilograms)
distance from centre of source	r	m (metres)

Unit Analysis

$$\left(\frac{\text{newton} \cdot \text{metre}^2}{\text{kilogram}^2}\right)\left(\frac{\text{kilogram}}{\text{metre}^2}\right) = \left(\frac{\text{newton}}{\text{kilogram}}\right)$$

$$\frac{N \cdot \cancel{m^2}}{kg^{\cancel{2}}} \times \frac{\cancel{kg}}{\cancel{m^2}} = \frac{N}{kg}$$

SAMPLE PROBLEM

Field Intensity near Earth

Calculate the gravitational field intensity at a height of 300.0 km from Earth's surface.

Conceptualize the Problem

- Since the point in question is *outside* of the *sphere* of Earth, the gravitational field there is the same as it would be if Earth's mass was concentrated at *a point at Earth's centre*. Therefore, the equation for the *gravitational field intensity* near a *point mass* applies.

Identify the Goal

The gravitational field intensity, $\vec{g}$

Identify the Variables and Constants

Known	Implied	Unknown
$h = 300.0\ \text{km}$	$G = 6.67 \times 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	$\vec{g}$
	$r_\text{E} = 6.38 \times 10^{6}\ \text{m}$	
	$m_\text{E} = 5.98 \times 10^{24}\ \text{kg}$	

Develop a Strategy

Convert the height above Earth's surface into SI units and calculate the distance, r, from the centre of Earth.

$$h = 300.0\ \text{km} = 3.000 \times 10^{5}\ \text{m}$$
$$r = 3.000 \times 10^{5}\ \text{m} + 6.38 \times 10^{6}\ \text{m}$$
$$r = 6.68 \times 10^{6}\ \text{m}$$

Use the equation for the gravitational field intensity near a point source.

$$|\vec{g}| = G\frac{m_\text{s}}{r^2}$$

Substitute numerical values and solve.

$$|\vec{g}| = \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})}{(6.68 \times 10^{6}\ \text{m})^2}$$
$$|\vec{g}| = 8.9387\ \frac{\text{N}}{\text{kg}}$$
$$|\vec{g}| \cong 8.94\ \frac{\text{N}}{\text{kg}}$$

The gravitational field intensity 300.0 km from the surface of Earth is 8.94 N/kg.

Validate the Solution

You would expect the gravitational field intensity to be less than 9.81 N/kg at a great distance from Earth's surface.

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PRACTICE PROBLEMS

31. What is the gravitational field intensity at a distance of 8.4×10^{7} m from the centre of Earth?

32. If the gravitational field intensity at the surface of Saturn is 26.0 N/kg and its mass is 5.67×10^{26} kg, what is its radius?

33. What is the acceleration due to gravity on the surface of Venus? ($m_{\text{Venus}} = 4.83 \times 10^{24}$ kg; $r_{\text{Venus}} = 6.31 \times 10^{6}$ m)

34. An astronaut drops a 3.60 kg object onto the surface of a planet. It takes 2.60 s to fall 1.86 m to the ground. If the planet is known to have a radius of 8.40×10^{6} m, what is its mass?

35. What is the gravitational field intensity at a distance of 2.0 m from the centre of a spherical metal ball of mass 3.0 kg? (Calculate only the field due to the ball, not to Earth.)

36. Calculate the gravitational field intensity at a height of 560.0 m above the surface of the planet Venus. (See problem 33 for data.)

37. The planet Neptune has a gravitational field intensity of 10.3 N/kg at a height of 1.00×10^{6} m above its surface. If the radius of Neptune is 2.48×10^{7} m, what is its mass?

Field Lines

Electric Field Lines

You have learned that an electric field at a particular point can be represented by a vector arrow with a length that corresponds to the magnitude of the field intensity at a given point. The direction of the vector arrow indicates the direction of the electric field at that point.

If you wanted to visualize the entire field around an electric charge, however, you would need to draw a set of these vector arrows at many points in the space around the charge. This process would be very tedious and complicated, so an idea originally used by Michael Faraday has been adapted. Using this method, the vectors are replaced by a series of lines that follow the path that a tiny point charge would take if it was free to move in the electric field. These lines are called **electric field lines**. In the vicinity of a positive charge, such field lines would radiate straight out, just as a positive test charge would be pushed straight out.

The field lines are constructed so that, at every point on the line, the direction of the field is tangent to the line. The strength of the field is represented by the density of the lines. The farther apart these lines are, the weaker the field is. Figure 7.9 shows the electric field lines that represent the electric field in various charge arrangements.

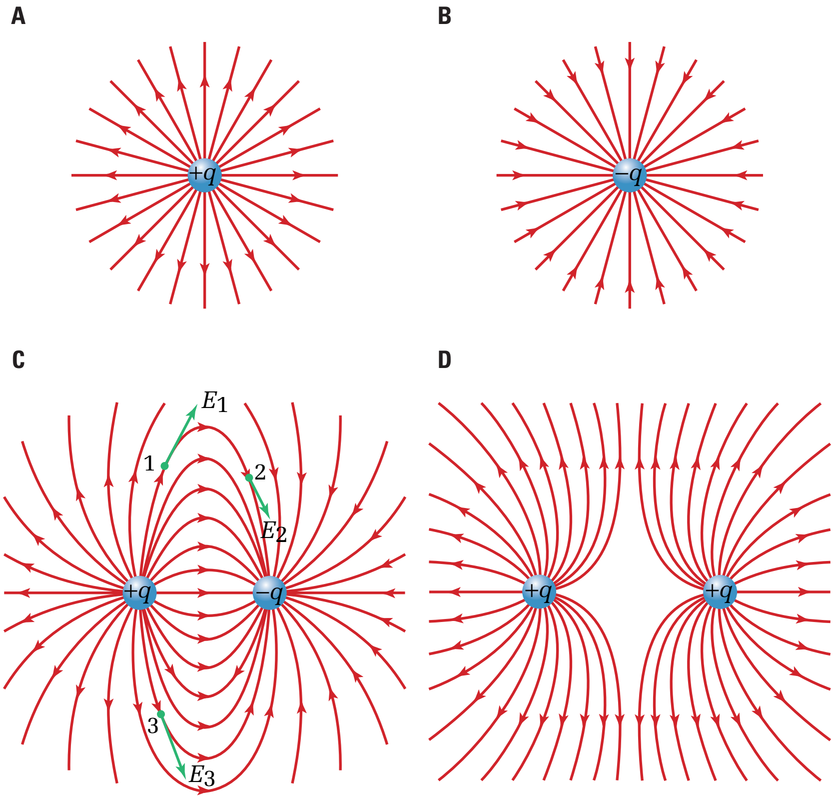


Figure 7.9 **(A)** The electric field lines from positive charge $+q$ are directed radially outward. **(B)** The electric field lines are directed radially inward toward negative point charge $-q$. **(C)** The electric field lines of an electric dipole are curved, and extend from the positive to the negative charge. At any point, such as 1, 2, or 3, the field created by the dipole is tangent to the line through the point. **(D)** The electric field lines for two identical positive point charges are shown. If both of the charges were negative, the directions of the lines would be reversed.

Note that when more than one electric source charge is present, the electric field vector at a point is the vector sum of the electric field attributable to each source charge separately. Since the field lines are often curved, this vector will be tangent to the field line at that point.

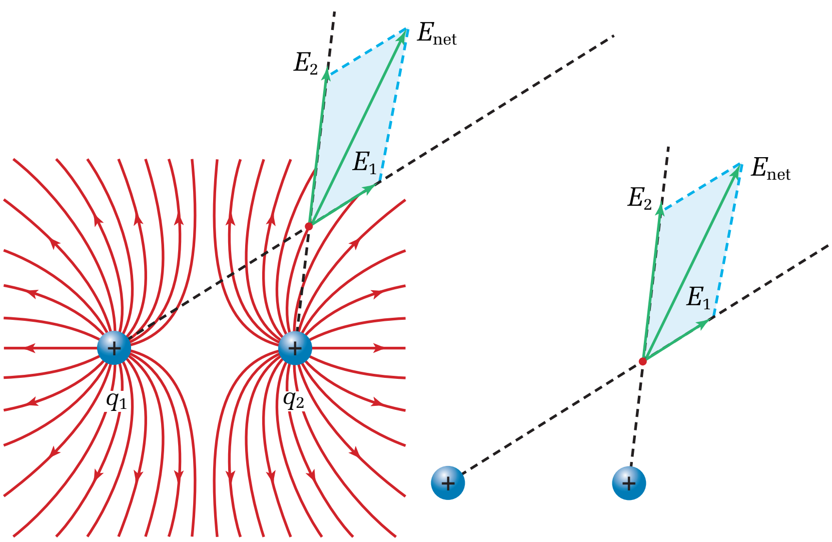


Figure 7.10 The electric field at a point near two positive charges

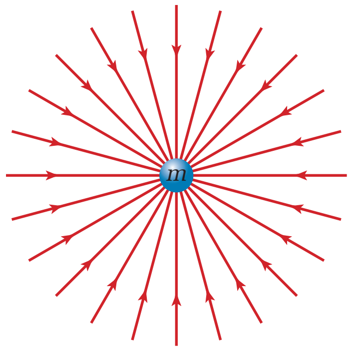


Figure 7.11 The gravitational field lines are directed radially inward toward a mass, *m*.

Gravitational Field Lines

Since the force of gravity is always attractive, the shape of **gravitational field lines** will resemble the electric field lines associated with a negative charge. Gravitational field lines will always point toward the centre of a spherical mass and arrive perpendicular to the surface.

Conceptual Problems

- Can there be a gravitational field diagram similar to the electric field in Figure 7.9 (D)? Explain.
- Sketch the gravitational field lines due to the two identical masses shown in the diagram here.

Magnetic Field Lines

Since there are no isolated magnetic poles (magnetic monopoles), the **magnetic field lines** have to be drawn so that they are associated with both poles of the magnet (magnetic dipole). The direction of the magnetic field at a particular location is defined as the direction in which the N-pole of a compass would point when placed at that location. The magnetic field lines leave the N-pole of a magnet, enter the S-pole, and continue to form a closed loop inside the magnet. The number of magnetic field lines, called the "magnetic flux," passing through a particular unit area is directly proportional to the magnetic field intensity. Consequently, flux lines are more concentrated at the poles of a magnet, where the magnetic field is greatest.

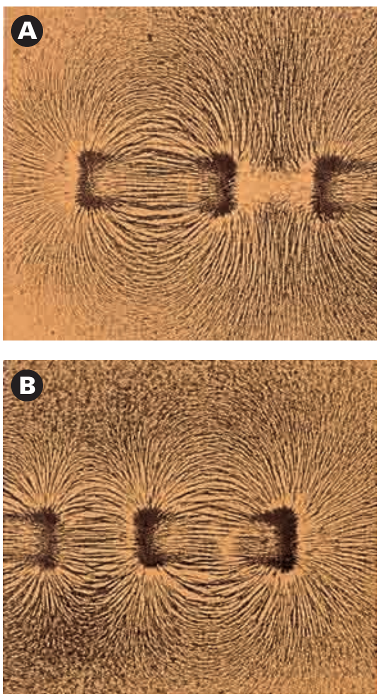


Figure 7.13 The field lines for **(A)** like poles and **(B)** unlike poles

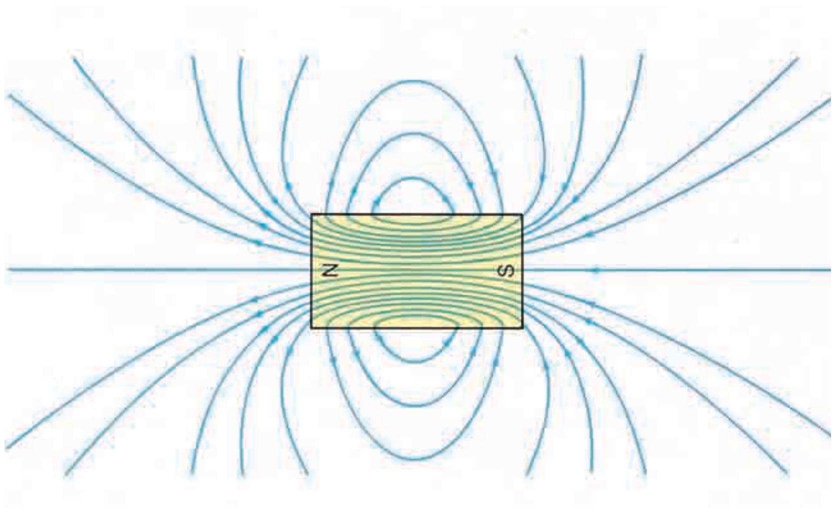


Figure 7.12 The magnetic field lines are closed loops leaving the N-pole of the magnet and entering the S-pole.

Conceptual Problems

- How does the magnetic dipole pattern compare with the electric field pattern of two opposite charges (an electric dipole)?
- What electrostatic evidence suggests that a water molecule is an electric dipole?
- What happens if you place a small bar magnet in a uniform magnetic field?
- What happens if you place a water molecule in a uniform electric field?

7.2 Section Review

1. **I** Place a strong bar magnet flat on a semi-rough surface, with the N-pole to the right. Place another bar magnet to the right of the first, but with its like N-pole to the left, suspended directly over the other N-pole. Adjust the top magnet until it balances. Now slide a piece of paper over the first magnet to hide it. Gently tap the suspended N-pole to start it vibrating vertically in space. What do your observations suggest about magnetic fields?
2. **K/U** What is the general definition for the electric field intensity at a distance r from a point charge q?
3. **I** Why is it not considered useful to define magnetic field intensity in the same way in which you defined the electric field intensity in question 2?
4. **C** Explain how you might calculate the gravitational field intensity at the various points along the path of a communication satellite orbiting Earth.
5. **K/U** In the vicinity of several point charges, how is the direction of the electric field intensity vector calculated?
6. **C** List four characteristics of electric field lines.

UNIT PROJECT PREP

A magnet held close to a refrigerator door is pulled toward the door. A ball rolls off a tabletop and is pulled toward the ground. Your hair sticks out in all directions after you remove a warm woollen cap. Each of these examples involve action at a distance. Forces are exerted without apparent contact.

- How does the use of fields help to explain action at a distance?
- Do descriptions of electric fields relate to descriptions of gravitational fields?
- Does an understanding of one type of field help with questions about another?

7.3 Fields and Potential Energy

SECTION EXPECTATIONS

- Define and describe the concepts and units related to electric and gravitational fields.
- Apply the concept of electric potential energy and compare the characteristics of electric potential energy with those of gravitational potential energy.

KEY TERMS

- electric potential difference
- equipotential surface

As a thundercloud billows, rising ice crystals collide with falling hailstones. The hail strips electrons from the rising ice and the top of the cloud becomes predominantly positive, while the bottom is mostly negative. Negative charges in the lower cloud repel negative charges on the ground, inducing a positive region, or "shadow," on Earth below. Electric fields build and a spark ignites a cloud-to-ground lightning flash through a potential difference of hundreds of millions of volts.

The lightning bolt featured in Figure 7.14 dramatically demonstrates that when a charge is placed in an electric field, it *will* move. The potential to move implies the existence of stored energy. In this chapter, you will focus on the energy stored in the gravitational and electric fields.

Figure 7.14 Tremendous amounts of electric energy are "stored" in the electric fields created by the separation of charge between thunderclouds and the ground. This energy is often released in the "explosion" of a lightning bolt.

Potential Energy

In Chapter 6, Energy and Motion in Space, you derived an equation for the gravitational potential energy of one mass due to the presence of a central mass. You started the derivation by determining the amount of work that you would have to do on the first mass to move it from a distance r_1 to a distance r_2 from a central mass. Then you learned that physicists have agreed on a reference position that is assigned a value of zero gravitational potential energy. That distance is infinitely far from the central mass. In this application, an infinite distance means so far away that the magnitude of the force of gravity is negligible.

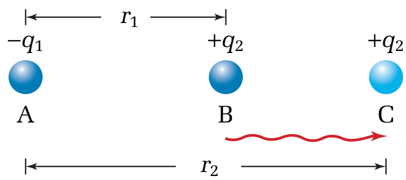


Figure 7.15 By doing work on charge q_2, you give it potential energy.

Physicists take the same approach in developing the concept of electric potential energy of a charge q_1 in the vicinity of another charge q_2 as shown in Figure 7.15. The change in electric potential energy of charge q_1 due to the presence of q_2, in moving q_1 from r_1 to r_2, is the work that you would have to do on the charge in moving it. In Figure 7.16, note the similarities in the equations for the force of gravity and the Coulomb force as well as the curves for force versus position.

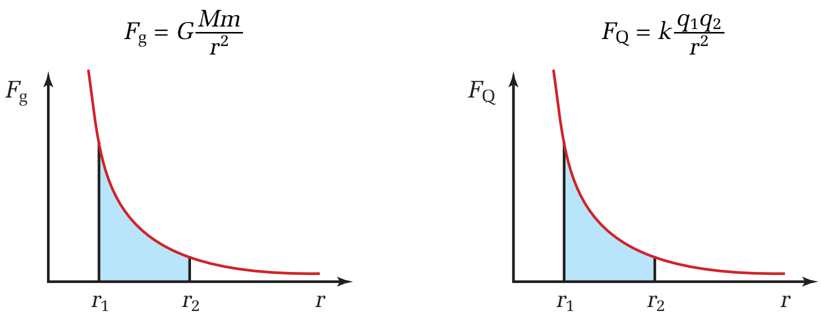


Figure 7.16 The Coulomb force and the force of gravity both follow inverse square relationships, so the curves of force versus position have exactly the same form.

Since the two equations and the two curves have identical mathematical forms, the result of the derivation of the change in the electric potential energy in moving a charge will be mathematically identical to the form of the change in the gravitational potential energy in moving a mass from position r_1 to position r_2.

$$\Delta E_g = \frac{GMm}{r_1} - \frac{GMm}{r_2} \qquad \Delta E_Q = \frac{kq_1q_2}{r_1} - \frac{kq_1q_2}{r_2}$$

The choice of a reference position for electric potential energy is the same as that for gravitational potential energy — an infinite distance — so far apart that the force between the two charges is negligible. Therefore, the equations for potential energy have the same mathematical form, with one small difference: There is no negative sign in the equation for the electric potential energy.

$$E_g = -\frac{GMm}{r} \qquad E_Q = \frac{kq_1q_2}{r}$$

The negative sign is absent from the equation for electric potential energy, because the energy might be negative or positive, depending on the sign of the charges. If the charges have opposite signs, the Coulomb force between them is attractive. Consequently, if one charge moves from infinity to a distance r from the second charge, it does work and therefore has less potential energy. Less than zero is negative. If the charges have the same sign, you must do work on one charge to move it from infinity to a distance r from the second charge, and therefore it has positive potential energy. If you include the sign of the charges when using the equation for electric potential energy, the final sign will tell you whether the potential is positive or negative.

- Two positive charges

  $$E_Q = \frac{k(q_1)(q_2)}{r}$$
  $$E_Q > 0$$

  Both q_1 and q_2 are positive, so the charges have positive potential energy when they are a distance r apart.

- Two negative charges

  $$E_Q = \frac{k(q_1)(q_2)}{r}$$
  $$E_Q > 0$$

  Both q_1 and q_2 are negative, so the charges have positive potential energy when they are a distance r apart.

- A positive and a negative charge.

  $$E_Q = \frac{k(q_1)(q_2)}{r}$$
  $$E_Q < 0$$

  The product q_1q_2 is negative, so the charges have negative potential energy when they are a distance r apart.

A second difference between electric potential energy and gravitational potential energy is that the two interacting charges might be similar in magnitude. Therefore, either charge could be considered the stationary or central charge, or the "movable" charge. You could therefore consider the two charges to be a system, and refer to the electric potential energy of the system that results from the proximity of the two charges.

SAMPLE PROBLEM

Electric Potential Energy

What is the electric potential energy stored between charges of +8.0 μC and +5.0 μC that are separated by 20.0 cm?

Conceptualize the Problem

- Two *charges* are close together and therefore they exert a *force* on each other.

- *Work* must be done on or to the charges in order to bring them close to each other.
- Since work was done on or by a charge, it has *electric potential energy*.

Identify the Goal

The electric potential energy, E_Q, stored between the charges

Identify the Variables and Constants

Known

$q_1 = 8.0 \times 10^{-6}$ C
$q_2 = 5.0 \times 10^{-6}$ C
$r = 0.200$ m

Implied

$k = 9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$

Unknown

E_Q

Develop a Strategy

Write the equation for electric potential energy between two charges.

$$E_Q = k\frac{q_1 q_2}{r}$$

Substitute numerical values and solve.

$$E_Q = \frac{\left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(+8.0 \times 10^{-6}\ \text{C})(+5.0 \times 10^{-6}\ \text{C})}{0.200\ \text{m}}$$

$$E_Q = +1.8\ \text{J}$$

The electric potential energy stored in the field between the charges is +1.8 J.

Validate the Solution

Magnitudes seem to be consistent. The units cancel to give J: $\frac{\text{N} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{C}^2}} \cdot \frac{\cancel{\text{C}} \cdot \cancel{\text{C}}}{\cancel{\text{m}}} = \text{N} \cdot \text{m} = \text{J}$. The sign is positive, indicating that the electric potential energy is positive. A positive sign is correct for like charges, because work was done *on* the charges to put them close each other.

PRACTICE PROBLEMS

38. Find the electric potential energy stored between charges of +2.6 μC and −3.2 μC placed 1.60 m apart.

39. Two identical charges of +2.0 μC are placed 10.0 cm apart in a vacuum. If they are released, what will be the final kinetic energy of each charged object (assuming that no other objects or fields interfere)?

40. How far apart must two charges of $+4.2 \times 10^{-4}$ C and -2.7×10^{-4} C be placed in order to have an electric potential energy with a magnitude of 2.0 J?

41. Two charges of equal magnitude, separated by a distance of 82.2 cm, have an electric potential energy of 2.64×10^2 J. What are the signs and magnitudes of the two charges?

CAREERS IN PHYSICS

Seeing Inside Storms

Blizzards can cause traffic accidents. Hurricanes can cause flooding. Tornadoes can destroy houses. Often, advance warning of these and other severe storms helps prevent deaths and reduce damage. For example, radio announcements can warn motorists to stay off roads, and municipal authorities can prepare to deal with possible flooding.

Giving advance warning is part of Dr. Paul Joe's work. Dr. Joe, a radar scientist and cloud physicist, is based at Environment Canada's radar site in King City, north of Toronto. Radar — short for *ra*dio *d*etection *a*nd *r*anging — involves transmitting pulses of electromagnetic waves from an antenna. When objects such as snowflakes or raindrops interrupt these pulses, part of their electromagnetic energy is reflected back. A receiver picks up the reflections, converting them into a visible form and indicating a storm's location and intensity.

Dr. Paul Joe, radar scientist and cloud physicist

Conventional radar cannot detect a storm's internal motions, however. This is why, in recent years, Environment Canada has been improving its radar sites across the country by adding Doppler capability. This improved radar technology applies the Doppler effect: If an object is moving toward the radar, the frequency of its reflected energy is increased from the frequency of the energy that the radar is transmitting. If an object is moving away from the radar, the frequency of its reflected energy is decreased.

"This is the same effect we notice with a subway train," Dr. Joe explains. "As it approaches, we hear a higher-pitched sound than when it leaves."

On Dr. Joe's radar screen, the frequency shifts are visualized using colours. In general, blue means an object is approaching; red means it is receding. But it's not that simple. Doppler images are complex and difficult for conventional weather forecasters to interpret, and Dr. Joe is working on ways to make them simpler. He also specializes in nowcasting — forecasting weather for the near future; for example, within an hour. As part of the 2000 Olympics, he went to Sydney, Australia, to join other scientists in demonstrating nowcasting technologies.

"I have it great," says Dr. Joe. "I love using what I've learned in mathematics, physics, and meteorology to decipher what Mother Nature is telling us and warning people about what she might do. Using the radar network, I can be everywhere chasing storms and seeing inside them in cyberspace."

Going Further

Dr. Joe's field, known in general as meteorology, includes radar science, cloud physics, climatology, and hydrometeorology. Research one of these fields and prepare a two-page report for presentation to the class.

WEB LINK

www.mcgrawhill.ca/links/physics12

The Canadian Hurricane Centre site maintained on the Internet by Environment Canada has a wide variety of information about hurricanes. Just go to the above Internet site and click on **Web Links**.

Electric Potential Difference

In previous physics courses, you learned that **electric potential difference** is the difference in the electric potential energy of a unit charge between two points in a circuit. You can broaden this definition to include any type of electric field, not just a field that is confined to an electric conductor. This concept allows you to describe the condition of a point in an electric field, relative to a reference point, without placing a charge at that point.

You have just derived an equation for the electric potential energy of a point charge, relative to infinity, a distance r from another point charge that can be considered as having created the field. For this case, you can find the electric potential difference between that point and infinity by considering the charge q_1 as the charge creating an electric field and q_2 as a unit charge.

- The definition of electric potential difference between a point and the reference point is $$V = \frac{E_Q}{q_2}$$
- Substitute the expression for the difference in electric potential energy of charge q_2 between the reference at infinity and the distance r from the charge q_1 due to the presence of q_1. $$V = \frac{\frac{kq_1q_2}{r}}{q_2}$$
- Since only one q, the charge creating the field, remains in the expression, there is no need for a subscript. $$V = \frac{kq}{r}$$

ELECTRIC POTENTIAL DIFFERENCE DUE TO A POINT CHARGE

The electric potential difference, a scalar, between any point in the field surrounding a point charge and the reference point at infinity charge is the product of Coulomb's constant and the electric charge divided by the distance from the centre of the charge to the point.

$$V = k\frac{q}{r}$$

Quantity	Symbol	SI unit
electric potential difference	V	V (volts)
Coulomb's constant	k	$\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$ (newton metres squared per coulombs squared)
electric charge	q	C (coulombs)
distance	r	m (metres)

Unit Analysis

$$\frac{\text{N}\cdot\text{m}^{\cancel{2}}\cdot\cancel{\text{C}}}{\text{C}^{\cancel{2}}\cdot\cancel{\text{m}}} = \frac{\text{N}\cdot\text{m}}{\text{C}} = \frac{\text{J}}{\text{C}} = \text{V}$$

PHYSICS FILE

Physicists often use the phrase, potential at a point, when they are referring to the potential difference between that point and the reference point an infinite distance away. It is not incorrect to use the phrase as long as you understand its meaning.

Problems involving electric potential difference can be extended, as can those involving electric field, to situations in which several source charges create an electric field. Since electric potential is a scalar quantity, the electric potential difference created by each individual charge is first calculated, being careful to use the correct sign, and then these scalar quantities are added algebraically.

You can go one step further and describe the electric potential difference between two points, P_1 and P_2, within a field. To avoid confusion, this quantity is symbolized ΔV and the relationship is written as follows.

$$\Delta V = V_2 - V_1$$

Always keep in mind that V_1 and V_2 represent the electric potential difference between point 1 and infinity, and point 2 and infinity — a location so far away that the field is negligible. The following sample problems will help you to clarify these concepts in your mind.

SAMPLE PROBLEMS

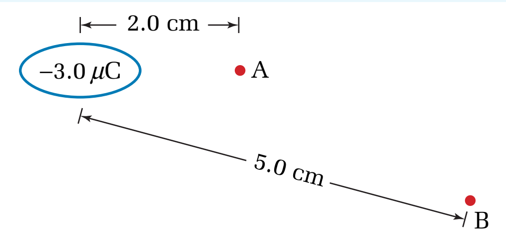
Calculations Involving Electric Potential Difference

1. A small sphere with a charge of −3.0 μC creates an electric field.

(a) Calculate the electric potential difference at point A, located 2.0 cm from the source charge, and at point B, located 5.0 cm from the same source charge.

(b) What is the potential difference between A and B?

(c) Which point is at the higher potential?



Conceptualize the Problem

- A *charged sphere* creates an *electric field*.
- At *any point* in the field, you can describe an *electric potential difference* between *that point* and a location an *infinite distance* away.
- Electric *potential difference* is a *scalar* quantity and depends only on the distance from the source charge and not the direction.
- The *potential difference* between *two points* is the *algebraic* difference between the individual potential differences of the points.

Identify the Goal

The electric potential difference, V, at each point
The electric potential difference, ΔV, between the two points
The point at a higher potential

Identify the Variables and Constants

Known
$q = -3.0 \times 10^{-6}$ C
$d_A = 2.0 \times 10^{-2}$ m
$d_B = 5.0 \times 10^{-2}$ m

Implied
$k = 9.0 \times 10^9 \frac{N \cdot m^2}{C^2}$

Unknown
V_A
V_B

Develop a Strategy

Use the equation for the electric potential difference at a point a distance r from a point charge.

$$V_A = k\frac{q}{d_A}$$

Substitute numerical values and solve.

$$V_A = \left(9.0 \times 10^9 \frac{N \cdot m}{C^2}\right)\left(\frac{-3.0 \times 10^{-6}\ C}{2.0 \times 10^{-2}\ m}\right)$$

$$V_A = -1.35 \times 10^6\ V$$

$$V_A \cong -1.4 \times 10^6\ V$$

$$V_B = k\frac{q}{d_B}$$

$$V_B = \left(9.0 \times 10^9 \frac{N \cdot m}{C^2}\right)\left(\frac{-3.0 \times 10^{-6}\ C}{5.0 \times 10^{-2}\ m}\right)$$

$$V_B = -5.4 \times 10^5\ V$$

(a) The electric potential difference is -1.4×10^6 V at point A, and -5.4×10^5 V at point B.

Use algebraic subtraction to determine the potential difference between the two points.

$$\Delta V = V_B - V_A$$

$$\Delta V = (-5.4 \times 10^5\ V) - (-1.35 \times 10^6\ V)$$

$$\Delta V = 8.1 \times 10^5\ V$$

(b) The electric potential difference, ΔV, between points A and B is 8.1×10^5 V.

Analyze the algebraic result and validate by considering the path of a positive test charge.

Algebraically, since $(V_B - V_A) > 0$, V_B is at the higher potential.

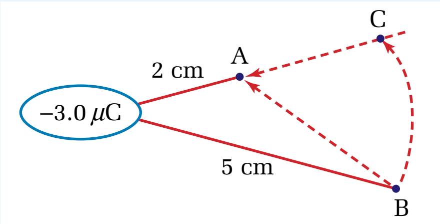
A positive test charge placed at point A would have to be dragged against the electric forces to get it to point B, which again places point B at the higher potential.

(c) Point B is at the higher potential.

Validate the Solution

The more distant point has a smaller magnitude potential, but its negative sign makes it a higher value. The analysis with a positive test charge validates the statement of higher potential.

Note: The answers were obtained in this sample problem by taking into account whether the two points were on the same radial line. The diagram shows two possible paths a test charge could take in moving from B to A. If the test charge followed the path BCA, no work would be done on it from B to C, because the force would be perpendicular to the path.



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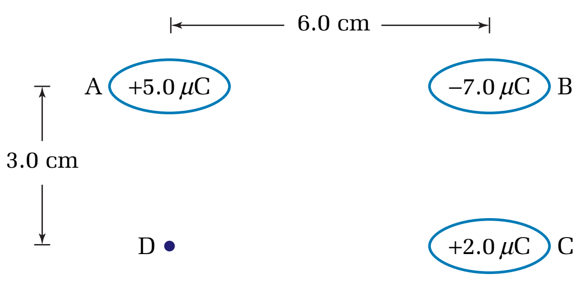
The only segment of the path where work is done, and therefore the electric potential energy changed, is from C to A, parallel to the direction of the force acting.

2. The diagram shows three charges, A (+5.0 μC), B (−7.0 μC), and C (+2.0 μC), placed at three corners of a rectangle. Point D is the fourth corner. What is the electric potential difference at point D?



Conceptualize the Problem

- There is an *electric potential difference* at point D, due to each of the separate charges.
- The separate potential values can be calculated and then *added algebraically*.

Identify the Goal

The electric potential difference, V, at point D

Identify the Variables and Constants

Known	Implied	Unknown
$q_A = 5.0\ \mu C$	$k = 9.0 \times 10^9\ \frac{N \cdot m^2}{C^2}$	$V_{at\,D}$
$q_B = -7.0\ \mu C$	$d_{CD} = 6.0$ cm	
$q_C = 2.0\ \mu C$		
$d_{AB} = 6.0$ cm		
$d_{AD} = 3.0$ cm		

Develop a Strategy

Calculate d_{BD}, using the Pythagorean theorem. Choose the positive value as a measure of the real distance.

$$d_{BD}^2 = (6.0\text{ cm})^2 + (3.0\text{ cm})^2$$
$$d_{BD}^2 = 45\text{ cm}^2$$
$$d_{BD} = \pm\sqrt{45\text{ cm}^2}$$
$$d_{BD} = \pm 6.7\text{ cm}$$

Calculate the contribution of each charge to the potential difference at point D independently.

$$V_{A\,at\,D} = \left(9.0 \times 10^9\ \frac{N \cdot m^2}{C^2}\right)\left(\frac{+5.0 \times 10^{-6}\ C}{0.030\ m}\right) = 1.5 \times 10^6\ V$$
$$V_{B\,at\,D} = \left(9.0 \times 10^9\ \frac{N \cdot m^2}{C^2}\right)\left(\frac{-7.0 \times 10^{-6}\ C}{0.067\ m}\right) = -9.4 \times 10^5\ V$$
$$V_{C\,at\,D} = \left(9.0 \times 10^9\ \frac{N \cdot m^2}{C^2}\right)\left(\frac{+2.0 \times 10^{-6}\ C}{0.060\ m}\right) = 3.0 \times 10^5\ V$$

Calculate the net potential difference at point D by adding the separate potential differences algebraically.

$$V_{at\,D} = (1.5 \times 10^6\ V) + (-9.4 \times 10^5\ V) + (3.0 \times 10^5\ V)$$
$$V_{at\,D} = 8.6 \times 10^5\ V$$

The electric potential difference at point D is 8.6×10^5 V.

Validate the Solution

The electric potential difference contributed by A is expected to be stronger, due to its closer proximity and average charge.

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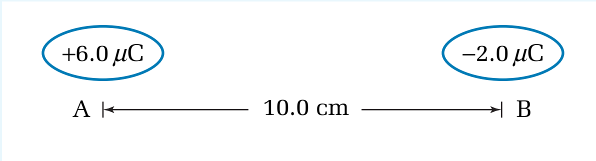
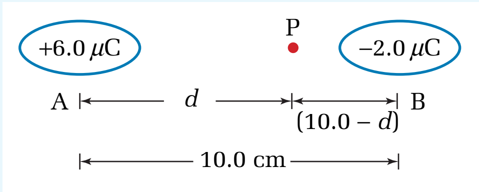
3. **A charge of +6.0 μC at point A is separated 10.0 cm from a charge of −2.0 μC at point B. At what locations on the line that passes through the two charges will the total electric potential be zero?**



Conceptualize the Problem



- The *total electric potential* due to the combination of charges is the *algebraic sum* of the electric potential due to *each point* alone.
- Draw a diagram and assess the likely position.
- Let the points be designated a distance d to the right of point A, and set the absolute magnitudes of the potential equal to each other. This allows for two algebraic scenarios.

Identify the Goal

The location of the point of zero total electric potential

Identify the Variables and Constants

Known

$q_A = +6.00 \times 10^{-6}$ C

$q_B = -2.00 \times 10^{-6}$ C

$d_{AB} = 10.0 \times 10^{-2}$ m

Implied

$k = 9.0 \times 10^9 \dfrac{\text{N} \cdot \text{m}^2}{\text{C}^2}$

Unknown

d at zero total electric potential

Develop a Strategy

For the potentials to cancel algebraically, the point cannot be to the left of point A, which would be closer to the larger positive charge and could not be balanced by the potential of the negative charge. That leaves two locations: one between points A and B, and one to the right of point B, where the smaller distance to the negative charge balances the smaller value of that charge.

$$|V_{\text{due to A}}| = |V_{\text{due to B}}|$$

Scenario 1

$$k\frac{q_A}{d} = k\frac{q_B}{(0.10 - d)}$$

$$q_A(0.10 - d) = q_B(d)$$

$$0.10q_A - q_A d = q_B d$$

$$0.10q_A = d(q_A + q_B)$$

$$d = \frac{0.10q_A}{q_A + q_B}$$

$$d = \frac{(0.10\ \text{m})(6.0\ \mu\text{C})}{6.0\ \mu\text{C} + (-2.0\ \mu\text{C})}$$

$$d = \frac{0.60\ \text{m} \cdot \cancel{\mu\text{C}}}{4.0\ \cancel{\mu\text{C}}}$$

$$d = 0.15\ \text{m}$$

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Scenario 2

$$k\frac{q_A}{d} = -k\frac{q_B}{(0.10 - d)}$$

$$q_A(0.10 - d) = -q_B(d)$$

$$0.10q_A - q_A d = -q_B d$$

$$0.10q_A = d(q_A - q_B)$$

$$d = \frac{0.10q_A}{q_A - q_B}$$

$$d = \frac{(0.10\ \text{m})(6.0\ \mu\text{C})}{6.0\ \mu\text{C} - (-2.0\ \mu\text{C})}$$

$$d = \frac{0.60\ \text{m}\cdot\mu\text{C}}{8.0\ \mu\text{C}}$$

$$d = 0.075\ \text{m}$$

The points of zero potential are 7.5 cm to the right of point A and 5.0 cm to the right of point B. (Note: 15 cm to the right of A is the same as 5 cm to the right of B.)

Validate the Solution

The electric potentials due to point A at the two points are

$$(9.0 \times 10^9)\left(\frac{+6.0 \times 10^{-6}}{0.075}\right) = +7.2 \times 10^5\ \text{V and } (9.0 \times 10^9)\left(\frac{+6.0 \times 10^{-6}}{0.15}\right) = +3.6 \times 10^5\ \text{V}$$

The electric potentials due to point B at the two points are

$$(9.0 \times 10^9)\left(\frac{-2.0 \times 10^{-6}}{0.025}\right) = -7.2 \times 10^5\ \text{V and } (9.0 \times 10^9)\left(\frac{-2.0 \times 10^{-6}}{0.050}\right) = -3.6 \times 10^5\ \text{V}$$

In both locations, the potentials due to points A and B add algebraically to zero.

PRACTICE PROBLEMS

42. Find the electric field due to a point charge of 4.2×10^{-7} C at a point 2.8 cm from the charge.

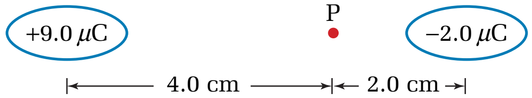
43. How far from a positive point source of 8.2 C will the electric potential difference be 5.0 V? (**Note:** 8.2 C is a very large charge!)

44. The electric potential difference due to a point charge is 4.8 V at a distance of 4.2 cm from the charge. What will be the electric potential energy of the system if a second charge of +6.0 μC is placed at that location?

45. The electric potential difference at a distance of 15 mm from a point charge is –2.8 V. What is the magnitude and sign of the charge?

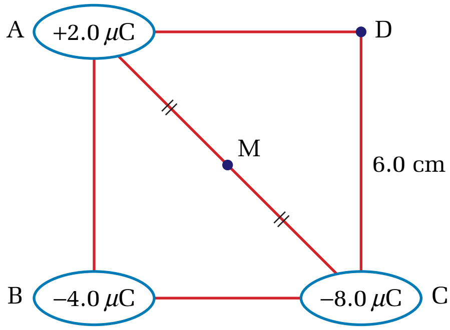
46. Point charges of +8.0 μC and –5.0 μC, respectively, are placed 10.0 cm apart in a vacuum. At what location along the line through them will the electric potential difference be zero?

47. What is the potential difference at point P situated between the charges +9.0 μC and –2.0 μC, as shown in the diagram.

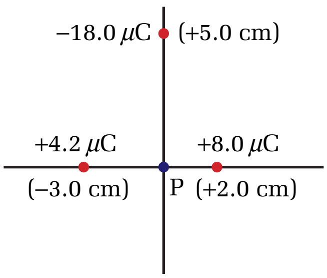


48. Point X has an electric potential difference of +4.8 V and point Y has a potential difference of –3.2 V. What is the electric potential difference, ΔV, between them?

49. Charges of +2.0 μC, –4.0 μC, and –8.0 μC are placed at three vertices of a square, as shown in the diagram. Calculate the electric potential difference at M, the midpoint of the diagonal AC.



50. The diagram shows three small charges located on the axes of a Cartesian coordinate system. Calculate the potential difference difference at point P.



51. Two charges are placed at the corners of a square. One charge, +4.0 μC, is fixed to one corner and another, –6.0 μC, is fixed to the opposite corner. What charge would need to be placed at the intersection of the diagonals of the square in order to make the potential difference zero at each of the two unoccupied corners?

52. Point A has an electric potential difference of +6.0 V. When a charge of 2.0 C is moved from point B to point A, 8.0 J of work are done on the charge. What was the electric potential difference of point B?

53. The potential difference between points X and Y is 12.0 V. If a charge of 1.0 C is released from the point of higher potential and allowed to move freely to the point of lower potential, how many joules of kinetic energy will it have?

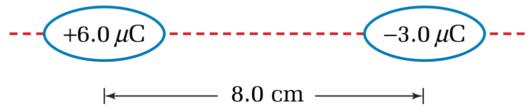
54. Identical charges of +2.0 μC are placed at the four vertices of a square of sides 10.0 cm. What is the potential difference between the point at the intersection of the diagonals and the midpoint of one of the sides of the square?

55. **(a)** If 6.2×10^{-4} J of work are required to move a charge of 3.2 nC (one nanocoulomb = 10^{-9} coulombs) from point B to point A in an electric field, what is the potential difference between A and B?

(b) How much work would have been required to move a 6.4 nC charge instead?

(c) Which point is at the higher electric potential? Explain.

56. Two different charges are placed 8.0 cm apart, as shown in the diagram. Calculate the location of the two positions along a line joining the two charges, where the electric potential is zero.

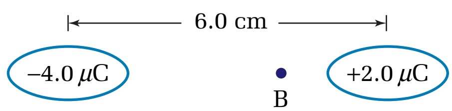


57. A charge of +8.2 nC is 10.0 cm to the left of a charge of –8.2 nC. Calculate the locations of three points, all of which are at zero electric potential.

58. A charge of –6.0 μC is located at the origin of a set of Cartesian coordinates. A charge of +8.0 μC. is 8.0 cm above it. What are the coordinates of the points at which the potential is zero?

59. A charge of +4.0 μC is 8.0 cm to the left of a point that has zero potential. Calculate three possible values for the magnitude and location of a second charge causing the potential to be zero.

60. Calculate the location of point B in the diagram below so that its electric potential is zero.



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Conceptual Problem

- In practice problem 56, do you think there could be locations (other than along a line joining the two charges) where the electric potential difference could be the same, but not zero? Explain.

Equipotential Surfaces

The quantities of gravitational potential energy, electric potential energy, and electric potential difference are all scalar quantities. Although it is rarely used, there is also a quantity called "gravitational potential difference," which is defined as gravitational potential energy per unit mass. It is expressed mathematically as $V_g = \frac{E_g}{m} = -\frac{GM}{r}$. Since these are scalar quantities, the direction from the charge or mass that is creating the field does not affect the values. If you connected all of the points that are equidistant from a point mass or an isolated point charge, they would have the same potential difference and they would be creating a spherical surface. Such a surface, illustrated in Figure 7.17, is called an **equipotential surface**.

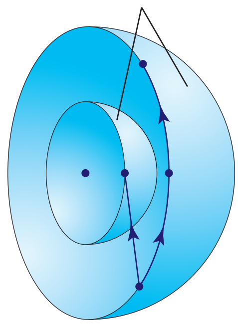


Figure 7.17 The spherical shells could represent equipotential surfaces either for a gravitational field around a point mass (or spherical mass) or for an electric field around an isolated point charge. In cross section, the equipotential spherical surfaces appear as concentric circles.

You will recall that the work done per unit charge in moving that charge from a potential V_1 to a potential V_2 is $\frac{W}{q} = V_2 - V_1$. Since, on an equipotential surface, $V_1 = V_2$, the work done must be zero. In other words, no work is required to move a charge or mass around on an equipotential surface, and the electric or gravitational force does no work on the charge or mass. Consequently, a field line must have no component along the equipotential surface. An equipotential surface must be perpendicular to the direction of the field lines at all points. Figure 7.18 shows the electric field lines and equipotential surfaces for pairs of point charges.

GEOGRAPHY LINK

The equipotential lines around a system of charges could be compared to the contour lines on topographical maps. Since these contour lines represent identical heights above sea level, they also represent points that have the same gravitational energy per unit mass, and so are equipotential lines.

$$E_g = mgh$$
$$\frac{E_g}{m} = gh$$
$$V_g \propto h$$

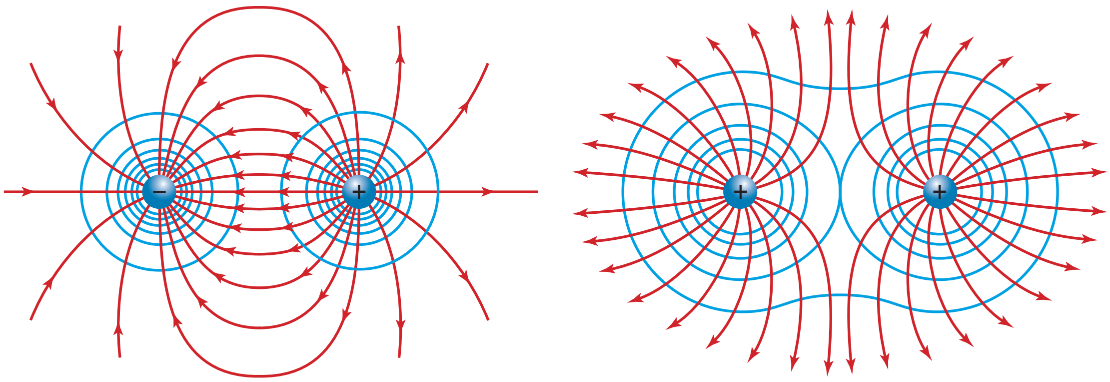


Figure 7.18 The field lines for these electric dipoles are shown in red and the cross section of the equipotential surfaces are in blue. Notice that field lines are always perpendicular to equipotential surfaces.

Conceptual Problem

- Could the barometric lines on a weather map be considered to be equipotential lines?

7.3 Section Review

1. K/U What are the differences in the data required to calculate the gravitational potential energy of a system and the electric potential energy?
2. K/U How does the amount of work done relate to the electric potential difference between two points in an electric field?
3. MC Research and briefly report on the use of electric potential differences in medical diagnostic techniques such as electrocardiograms.
4. MC Research and report on the role played by electric potential differences in the transmission of signals in the human nervous system.
5. C Can an equipotential surface in the vicinity of two like charges have a potential of zero? Explain the reason for your answer.
6. I Investigate Internet sites that use computer programs to draw the electric field lines near a variety of charge systems. Prepare a portfolio of various patterns.
7. K/U How could you draw in the equipotential surfaces associated with the patterns obtained in question 6?
8. I
   - **(a)** Why do you think atomic physicists tend to speak of the electrons in atoms as having "binding energy"?
   - **(b)** Investigate the use of the term "potential well" to describe the energy state of atoms.

CHAPTER 7 Review

REFLECTING ON CHAPTER 7

- The gravitational force and the Coulomb force both follow inverse square laws.
  $$F_g = G\frac{m_1 m_2}{r^2} \qquad F_Q = k\frac{q_1 q_2}{r^2}$$
- The equations for gravitational force and Coulomb force were developed for point masses and point charges. However, if the masses or charges are perfect spheres, the laws apply at any point outside of the spheres.
- Since magnetic monopoles do not exist or have never been detected, magnetic forces cannot be described in the same form as gravitational and electrostatic forces. However, they appear to follow an inverse square relationship.
- Because charges, masses, and magnets do not have to be in contact to exert forces on each other, early physicists classified their interactions as action-at-a-distance forces.
- Michael Faraday developed the concept of a field in which masses, charges, and magnets influence the space around themselves in the form of a field. When a second mass, charge, or magnet is placed in the field created by the first, the field exerts a force on the object.
- The strength of an electric field on a point P is described as the electricity field intensity and is mathematically expressed as "force per unit charge," $\vec{E} = \frac{\vec{F}_Q}{q_t}$.
- The direction of an electric field at any point is the direction that a *positive* charge would move if it was placed at that point.
- The strength of a gravitational field at any point P is called the gravitational field intensity and is mathematically expressed as "force per unit mass," $\vec{g} = \frac{\vec{F}_g}{m_t}$.
- The direction of a gravitational field is always toward the mass creating the field.
- For the special case of a point charge q creating the field, the electric field intensity at a point P, a distance r from the charge, is given by $|\vec{E}| = k\frac{q}{r^2}$.
- For the special case of a point mass m creating the field, the gravitational field intensity at a point P, a distance r from the mass, is given by $\vec{g} = G\frac{m}{r^2}$.
- To find the electric field intensity in the vicinity of several point charges, find the field intensity due to each charge alone and then add them vectorially.
- Field lines are used to describe a field over a large area or volume. Field lines are drawn so that the intensity of the field is proportional to the density of the lines. The direction of a field at any point is the tangent to the field line at that point.
- A charge placed in an electric field or a mass in a gravitational field has potential energy.
- Potential energy of any type is not absolute, but relative to an arbitrary reference position or condition. The reference position for gravitational or electric potential energy in a field created by a point source is often chosen to be at an infinite distance from the point source.
- The electric potential energy of two point charges a distance r apart is given by
  $$E_Q = k\frac{q_1 q_2}{r}$$
- The gravitational potential energy of two masses a distance r apart is given by
  $$E_g = -G\frac{m_1 m_2}{r}$$
- Electric potential difference is defined as the potential energy per unit charge and is expressed mathematically as $V = \frac{E_Q}{q}$.
- For the special case of the electric potential difference in an electric field created by a point charge q, the electric potential difference is $V = k\frac{q}{r}$. This is the potential difference between a point the distance r from the charge and an infinite distance from the charge.

- Since work must be done on a charge to give it potential energy, the change in the potential between two points is the amount of work done on a unit charge that was moved between those two points, or $\Delta V = \frac{W}{q}$.
- In any field, there will be many points that have the same potential. When all of the points that are at the same potential are connected, an equipotential surface is formed.
- No work is done when a charge or mass moves over an equipotential surface.

Knowledge/Understanding

1. In your own words, define
   (a) electric charge
   (b) Coulomb's law
   (c) field
2. The field of an unknown charge is first mapped with a 1.0×10^{-8} C test charge, then repeated with a 2.0×10^{-8} C test charge.
   (a) Would the same forces be measured with the two test charges? Explain your answer.
   (b) Would the same fields be determined using the two test charges? Explain your answer.
3. Both positive and negative charges produce electric fields. Which direction, toward or away from itself, does the field point for each charge?
4. What is the difference between electric field intensity, electric potential difference, and electric potential energy?
5. What determines the magnitude and direction of an electric field at a particular point away from a source charge?
6. Is electric field strength a scalar quantity or a vector quantity? Is electric potential difference a scalar or vector quantity?
7. If the gravitational potential energy for an object at height h above the ground is given by $mg\Delta h$, what is the gravitational potential difference (similar in nature to the electric potential difference) between the two levels? What are the units of gravitational potential difference?
8. Units of electric field strength can be given in N/C or volts per metre, V/m. Show that these units are equivalent.

Inquiry

9. Consider a charge of +2.0 μC placed at the origin of an x–y-coordinate system and a charge of –4.0 μC placed 40.0 cm to the right. Where must a third charge be placed — between the charges, to the left of the origin, or beyond the second charge — to experience a net force of zero? Argue your case qualitatively without working out a solution. Consider both positive and negative charges.
10. (a) In a room, gravity exerts a downward pull on a ball held by a string. Sketch the gravitational field in the room.
    (b) Suppose a room has a floor that is uniformly charged and positive and a ceiling that carries an equal amount of negative charge. Neglecting gravity, how will a small, positively charged sphere held by a string behave? Sketch the electric field in the room.
    (c) Comment on any similarities and differences between the above situations.

Communication

11. (a) Sketch the electric field lines for a positive charge and a negative charge that are very far apart.
    (b) Show how the field lines change if the two charges are then brought close together.
12. Sketch the field lines for two point charges, 2Q and –Q, that are close together.
13. Explain why electric field lines never cross.
14. What is the gravitational field intensity at the centre of Earth?

Making Connections

15. Develop a feeling for the unit of the coulomb by examining some everyday situations. How much charge do you discharge by touching a doorknob after walking on a wool rug? How much charge does a comb accumulate when combing dry hair? How much charge does a lightning bolt discharge? What is the smallest charge that can be measured in the laboratory? The largest charge?
16. Make a list of the magnitudes of some electric fields found in everyday life, such as in household electric wiring, in radio waves, in the atmosphere, in sunlight, in a lightning bolt, and so on. Where can you find the weakest and greatest electric fields?
17. In November 2001, NASA launched the Gravity Recovery and Climate Experiment, or GRACE, involving a pair of satellites designed to monitor tiny variations in Earth's gravitational field. The two satellites follow the same orbit, one 220 km ahead of the other. As both satellites are in free fall, regions of slightly stronger gravity will affect the lead satellite first. By accurately measuring the changes in the distance between the satellites with microwaves, GRACE will be able to detect minute fluctuations in the gravitational field. Research the goals and preliminary findings of GRACE. In particular, examine how both ocean studies and meteorological studies will benefit from GRACE.

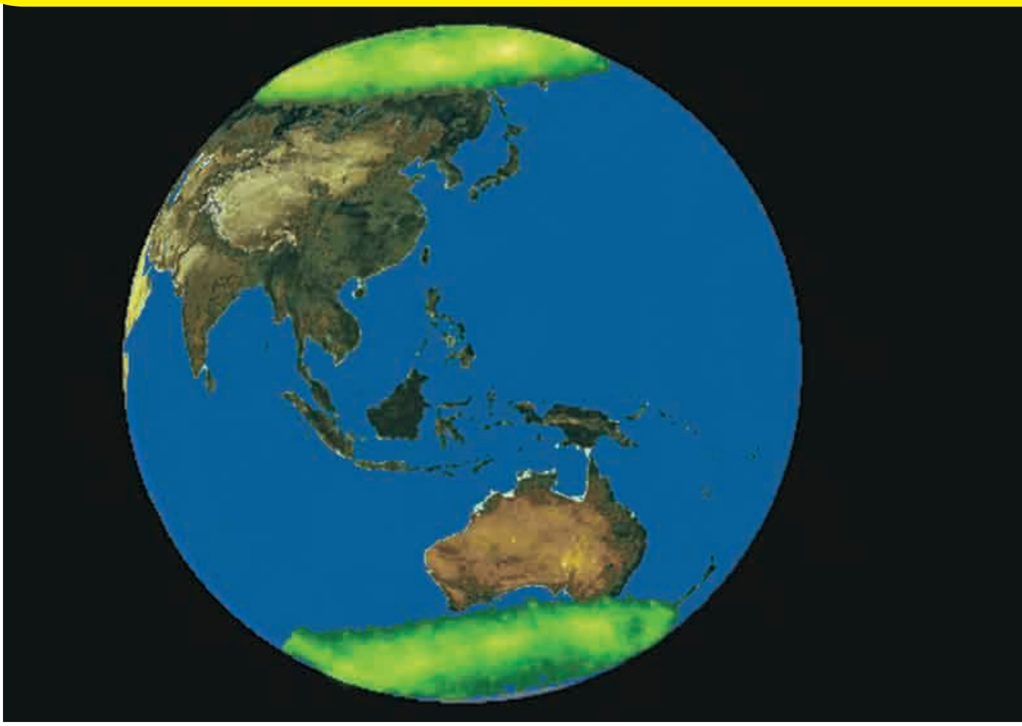
Problems for Understanding

18. What is the force of repulsion between two equal charges, each of 1 C, that are separated by a distance of 1 km?
19. Calculate the force between two free electrons separated by 0.10 nm.
20. The force of attraction between two charged Ping-Pong™ balls is 2.8×10^{-4} N. If the charges are +8.0 nC and –12.0 nC, how far apart are their centres?
21. Three point charges, A (+2.0 μC), B (+4.0 μC), and C (–6.0 μC), sit consecutively in a line. If A and B are separated by 1.0 m and B and C are separated by 1.0 m, what is the net force on each charge?
22. Three charges sit on the vertices of an equilateral triangle, the sides of which are 30.0 cm long. If the charges are A = +4.0 μC, B = +5.0 μC and C = +6.0 μC (clockwise from the top vertex), find the force on each charge.
23. In the Bohr model of the hydrogen atom, an electron orbits a proton at a radius of approximately 5.3×10^{-11} m. Compare the gravitational and the electrostatic forces between the proton and the electron.
24. Suppose the attractive force between Earth and the Moon, keeping the Moon in its orbit, was not gravitational but was, in fact, a Coulombic attraction. Predict the magnitude of the possible charges on Earth and the Moon that would cause an identical force of attraction.
25. What is the ratio of the electric force to the gravitational force between two electrons?
26. Calculate the charge (sign and magnitude) on a 0.30 g pith ball if it is supported in space by a downward field of 5.2×10^{-5} N/C.
27. A 3.0 g Ping Pong™ ball is suspended from a thread 35 cm long. When a comb is brought to the same height, the Ping Pong™ ball is repelled and the thread makes an angle of 10.0° with the vertical. What is the electric force exerted on the Ping Pong™ ball?
28. The gravitational field intensity at a height of 150 km (1.50×10^2 km) above the surface of Uranus is 8.71 N/kg. The radius of Uranus is 2.56×10^7 m.
    - **(a)** Calculate the mass of Uranus.
    - **(b)** Calculate the gravitational field intensity at the surface of Uranus.
    - **(c)** How much would a 100 kg (1.00×10^2 kg) person weigh on the surface of Uranus?
29. If a planet, P, has twice the mass of Earth and three times the radius of Earth, how would the gravitational field intensity at its surface compare to that of Earth?

30. The Bohr model of the hydrogen atom consists of an electron ($q_e = -e$) travelling in a circular orbit of radius 5.29×10^{-11} m around a proton ($q_p = +e$). The attraction between the two gives the electron the centripetal force required to stay in orbit. Calculate the
   (a) force between the two particles
   (b) speed of the electron
   (c) electric field the electron experiences
   (d) electric potential difference the electron experiences
31. What mass should an electron have if the gravitational and electric forces between two electrons were equal in magnitude? How many times greater than the accepted value of the electron mass is this?
32. A charge, $q_1 = +4$ nC, experiences a force of 3×10^{-5} N to the east when placed in an electric field. If the charge is replaced by another, $q_2 = -12$ nC, what will be the magnitude and direction of the force on the charge at that position?
33. If the electric potential energy between two charges of 1.5 μC and 6.0 μC is 0.16 J, what is their separation?
34. Two electric charges are located on a coordinate system as follows: $q_1 = +35$ μC at the origin (0,0) and $q_2 = -25$ μC at the point (3,0), where the coordinates are in units of metres. What is the electric field at the point (1,2)?
35. **(a)** What is the change in electric potential energy of a charge of –15 nC that moves in an electric field from an equipotential of +4 V to an equipotential of +9 V?
   (b) Does the charge gain energy or lose energy?
36. To move a charge of +180 nC from a position where the electric potential difference is +24 V to another position where the potential difference is +8 V, how much work must be done?
37. For breakfast, you toast two slices of bread. The toaster uses 31 000 J of energy, drawn from a 110 volt wall outlet. How much charge flows through the toaster?
38. A spherical Van de Graaff generator terminal (capable of building up a high voltage) has a radius of 15 cm.
   (a) Calculate the potential at the surface if the total charge on the terminal is 75 nC.
   (b) If you touch the generator with a hollow steel ball of radius 6.5 cm, are the spheres "equipotential" while in contact?
   (c) Calculate the charge on each sphere when they are separated.
39. Two identical charges, $q_1 = q_2 = 6.0$ μC, are separated by 1.0 m.
   (a) Calculate the electric field and electric potential difference at point P, midway between them.
   (b) Replace one of the charges with a charge of the same magnitude but opposite sign and repeat the calculation in (a).
   (c) Discuss your solutions.
40. Points X and Y are 30.0 mm and 58 mm away from a charge of +8.0 μC.
   (a) How much work must be done in moving a +2.0 μC charge from point Y to point X?
   (b) What is the potential difference between points X and Y?
   (c) Which point is at the higher potential?
41. Points R and S are 5.9 cm and 9.6 cm away from a charge of +6.8 μC.
   (a) What is the potential difference between the points R and S?
   (b) Which point is at the higher potential?

CHAPTER 8 Fields and Their Applications

CHAPTER CONTENTS



PREREQUISITE CONCEPTS AND SKILLS

- Newton's law of universal gravitation
- Electric potential difference
- Magnetic fields
- Moving charges in magnetic fields

The photograph above is the first image ever obtained of auroras at both the North Pole and the South Pole at the same time — a reminder that Earth's magnetic field protects all living organisms from frequent bombardment by high-energy, charged particles in the solar wind.

When the onslaught of charged particles enters Earth's magnetic field at an angle with the field, they curve away from Earth's surface. Many of the particles become trapped in the magnetic field and follow a helical path, circling back and forth in the field for long periods of time. These ions form the ionosphere. Only at the magnetic poles do the charged particles enter Earth's magnetic field parallel to the field lines and, therefore, are not diverted from their path. As these particles collide with oxygen and nitrogen molecules in the atmosphere, they excite the molecules, which then emit light as they return to their ground state.

Electric, magnetic, and gravitational fields exert a great influence on the structures in the universe. In this chapter, you will study how scientists and engineers are able to construct and manipulate some fields for practical purposes. The study of the behaviour of electric and magnetic fields has led to great progress in our understanding of the electromagnetic field and its enormous significance in, for example, telecommuncations.

MULTI LAB

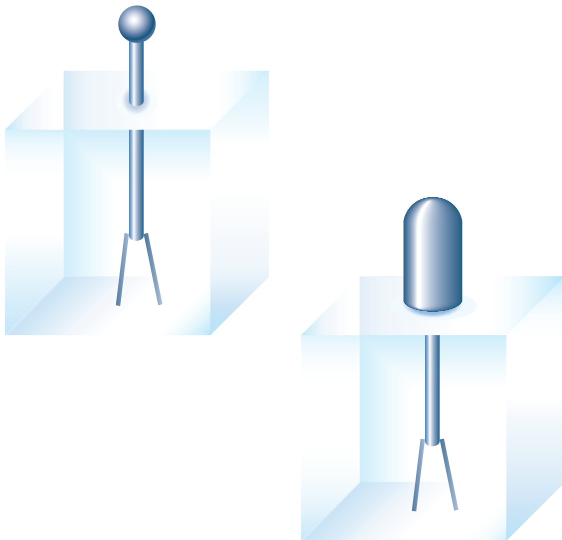
Electric Fields

TARGET SKILLS

- Predicting
- Performing and recording
- Analyzing and interpreting

Cover It Up

Rub an ebonite rod with fur. Bring the rod close to the cap of a metal leaf electroscope, then remove the rod. Sit a small inverted metal can over the cap of the electroscope and repeat the experiment.



Analyze and Conclude

1. What is the reason for the difference in the results of the two experiments?
2. Suggest an explanation of the role of the metal can.

Swinging Pith Ball

Support two aluminum squares (about 10 cm square) in grooved wooden blocks and place them 3.0 cm apart. Charge a pith ball with an ebonite rod rubbed with fur and suspend the pith ball at roughly the midpoint between the plates. Now, ask your teacher to connect a Van de Graaff generator or other charging device across the plates, using alligator leads. After the plates have been charged, disconnect the alligator leads. Predict how changing the separation of the plates will affect the pith ball. Predict how changing the length of time of charging by the generator will affect the pith ball. Test your predictions. When moving the plates, do not touch the plates themselves. Touch only the wooden supports.

CAUTION Care must be exercised in the use of charge generators. Serious heart or nerve injury could occur through contact with large potential differences, depending on the resulting current.

Analyze and Conclude

1. What relationship did you observe between the separation of the plates and the behaviour of the pith ball?
2. What relationship did you observe between the time of charging and the behaviour of the pith ball?
3. Propose an explanation for the behaviour of the pith ball under the changing conditions.

8.1 Field Structure

SECTION EXPECTATIONS

- Analyze and illustrate the electric field produced by various charge arrangements and two oppositely charged parallel plates.
- Describe and explain the electric field that exists inside and on the surface of a charged conductor.
- Analyze and explain the properties of electric fields.

KEY TERMS

- charge density
- gradient
- potential gradient
- Stokes' Law
- Millikan's oil-drop experiment

WEB LINK

www.mcgrawhill.ca/links/physics12

If you would like to experiment with creating charge distributions and generating field lines, go to the above Internet site and click on **Web Links**.

In Chapter 7, Fields and Forces, you learned about electric fields and studied a few special cases of fields, such as the electric field around a single point charge and the combination of two point charges, either like or unlike. Much more complex fields exist, however, both natural and generated in the laboratory. For example, Figure 8.1 shows areas of equal potential around the human heart.

In this section, you will be studying the electric field and the corresponding field line patterns of a number of different-shaped, charged conductors. Regardless of how many individual charges are included in the configuration, the electric field vector at any point can be determined by calculating the sum of electric field vectors contributed by each charge influencing the field. For some configurations, however, this method would become very tedious and time-consuming, so physicists have developed techniques for a few special cases of fields. In addition, computer programs have been developed that can generate the field lines for different arrangements of charges. The user can create the distribution of charges and the computer will generate the associated electric field lines.

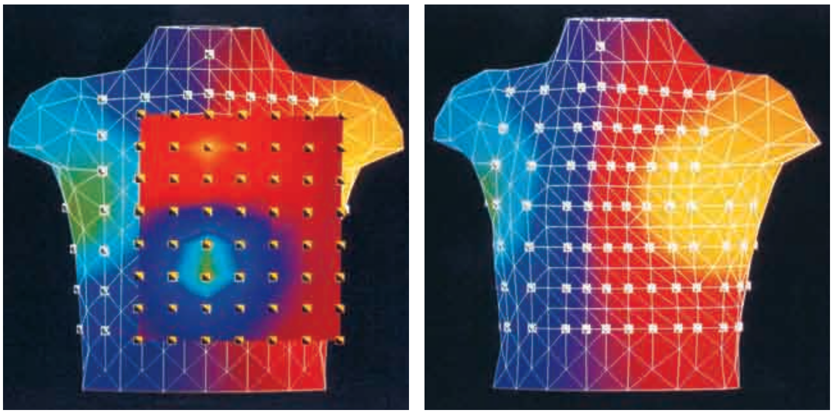


Figure 8.1 Electric and magnetic fields are a very real part of life. The photo on the right shows areas of equal electric potential difference, while the photo on the left shows areas of equal magnetic field intensity and direction around the human heart in varying shades of colour. The electric activities of the heart can provide a physician with important information about the health of a patient's heart.

Properties of an Electric Field Near a Conductor

Until now, you have been considering fields in the region of point charges. As you will see in the following Quick Lab, you can create some unusual fields with point charges. In real situations, however, charged conductors take on a variety of shapes, but the same basic concepts about fields that apply to point charges also apply to conductors of all shapes. In fact, you can think of a conductor as a very large number of point charges lined up very close together. One important concept to remember when working with conductors is that electric field lines enter and leave a conductor perpendicular to the surface. Figure 8.2 shows why field lines cannot contact a conductor at any angle other than 90˚.

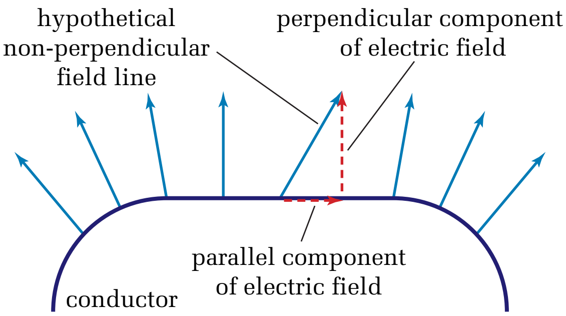


Figure 8.2 To indicate that an electric field line leaves a conductor at an angle implies that there is a component of the electric field that is parallel to the surface of the conductor. If this was the case, charges in the conductor would move until they had redistributed themselves in the conductor in a way that would change the field until there was no longer a parallel component.

QUICK LAB

Charge Arrays

TARGET SKILLS
- Predicting
- Analyzing and interpreting

In this Quick Lab, you will extend your knowledge into new and more complex charge arrangements. You will predict electric field lines and equipotential lines for several charge arrangements and then check your predictions.

For each of the following charge arrangements (arrays) located on the Cartesian coordinate plane, predict and sketch electric field line patterns and some equipotential lines. Use different colours for the field lines and for the equipotential lines.

(a) +1.0 C at (0,0)

(b) +1.0 C at (0,0) and an identical +1.0 C at (4,0)

(c) +1.0 C at (0,0) and –1.0 C at (4,0)

(d) +2.0 C at (0,0) and –1.0 C at (4,0)

(e) +3.0 C at (0,0) and –1.0 C at (4,0)

(f) +1.0 C at (0,0), +1.0 C at (4,0), and +1.0 C at (2,–4)

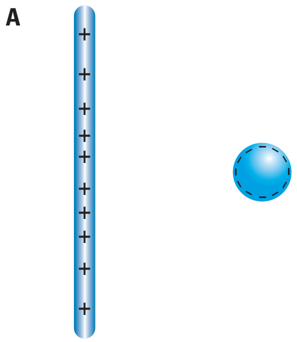
(g) +1.0 C at (0,0), +1.0 C at (4,0), and –1.0 C at (2,–4)

(h) +1.0 C at (0,0), +1.0 C at (4,0), –1.0 C at (4,–4), and +1.0 C at (0,–4)

Visit one of the Internet sites suggested by the Web Link on the previous page and simulate the charge arrays listed above. Observe the actual electric field lines and equipotentials that would be generated.

Analyze and Conclude

1. How well did your predicted patterns correspond to those generated by the simulation program?
2. How could you actually verify one specific value of the electric field intensity?



Consider the charged conductors in Figure 8.3 (A). Before you start to draw electric field lines, count the number of unit charges. Notice that the sphere has more negative charges than the plate has positive charges. So, more field lines will be ending on the sphere than leaving the plate. When you start to draw field lines, decide on the number entering and leaving each conductor so that the number of lines is proportional to the amount of charge on the conductor. Then, draw the beginning and end of each line perpendicular to the surface of the conductors, as shown in Figure 8.3 (B). Next, smoothly connect the lines so that those leaving the positive plate enter at adjacent lines ending on the negative sphere. The remaining lines on the sphere will spread out, but will not contact the positive plate. Finally, you can draw equipotential lines that are perpendicular to the electric field lines, as shown in Figure 8.3 (D).

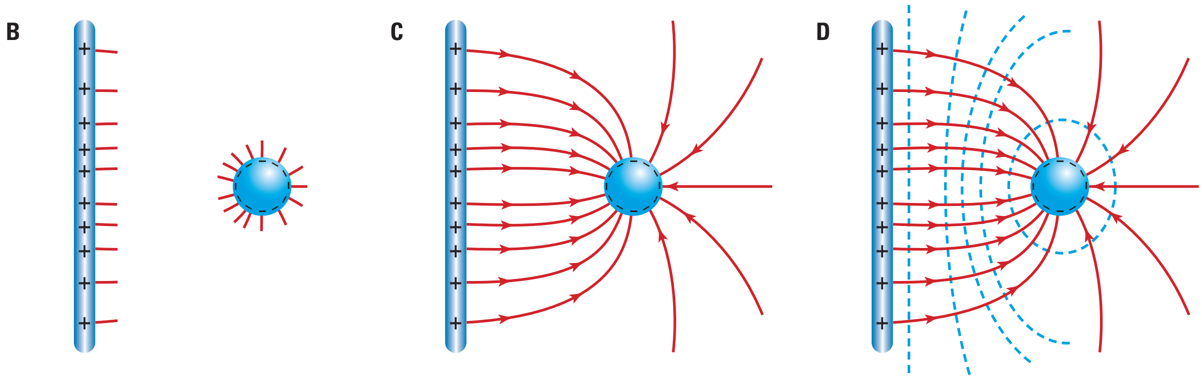
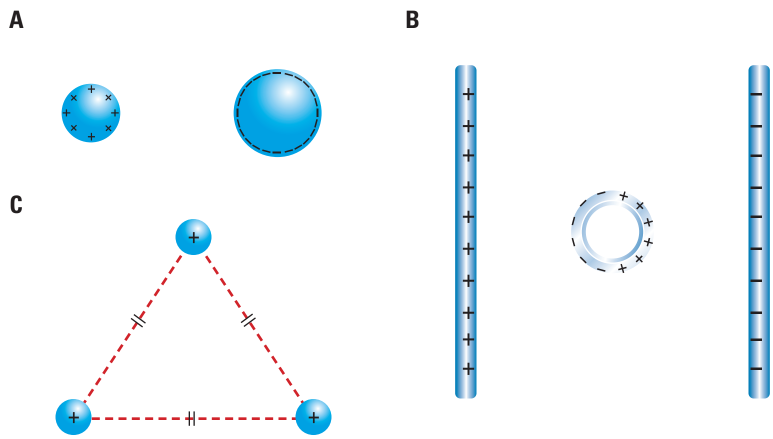


Figure 8.3 The positive plate attracts the negative charge on the sphere so that it is more dense on the side near the plate and less dense on the side away from the plate.

Conceptual Problem

- Copy each of the following diagrams (do not write on the diagrams in your textbook), showing various-shaped conductors, and draw in a representative sample of electric field lines and equipotential lines. Note that a uniform charge distribution has been assumed for each object except the cylinder in (B).



Parallel Plates

Charged parallel plates are a convenient way to create an electric field and therefore warrant in-depth examination. When two large, oppositely charged parallel plates are placed close together, the electric field between them is uniform, except for a certain spreading or "fringing" of the field at the edges of the plates, as shown in Figure 8.4.

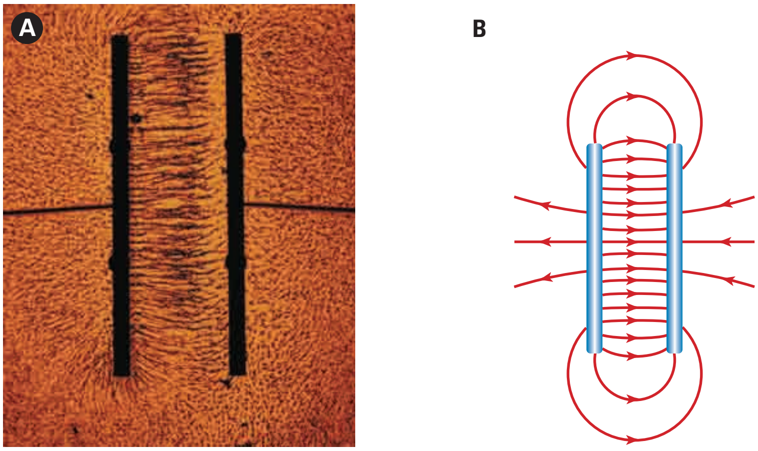


Figure 8.4 **(A)** When grass seeds are placed in an electric field between two parallel plates, they line up to reveal the shape of the electric field. **(B)** Using part (A) of this illustration as a model, a schematic diagram of an electric field is drawn.

The plates are too large to act like point charges, but the fact that the total charge on each plate is the sum of a large number of individual charges provides a way to explain the uniform field between the plates, as illustrated in Figure 8.5.

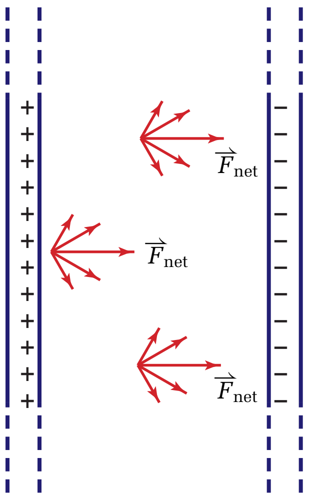


Figure 8.5 By symmetry, all of the force vectors add to produce a net force directly to the right.

A positive test charge placed at any point between the plates would experience a force from every positive charge on the left plate and every negative charge on the right plate. The magnitude of each of these forces would be determined by Coulomb's law, and the direction of each force would be along the line joining the test charge to each charge on the plates.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of charges and fields, go to your Electronic Learning Partner.

PHYSICS FILE

It is important to remember that the parallel plates are mounted on insulators, isolated from any circuit. If they were charged by a battery, the battery was disconnected, so the same amount of charge would remain on the plates. If instead the plates remained connected to the battery and, for example, the area of the plates or the distance between them was changed, the battery would then adjust the charge on the plates and the field would change as well. Parallel plates connected within a circuit are called "capacitors" and their operation is beyond the scope of this course.

The net (resultant) force on the test charge would then be determined by the vector sum of all of the forces acting on it. Since the system is perfectly symmetrical, for every upward force, there would be a force of equal magnitude pointing down. The net force on the test charge would be a constant vector perpendicular to the plates, regardless of its location between the plates. The resulting field between two parallel plates can be summarized as follows.

- The electric field intensity is uniform at all points between the parallel plates, independent of position.
- The magnitude of the electric field intensity at any point between the plates is proportional to the **charge density** on the plates or, mathematically, $|E_Q| \propto \sigma$, where $\sigma = q/A$ (charge density = charge per unit area).
- The electric field intensity in the region outside the plates is very low (close to zero), except for the fringe effects at the edges of the plates.

SAMPLE PROBLEM

Parallel Plates

An identical pair of metal plates is mounted parallel on insulating stands 20 cm apart and equal amounts of opposite charges are placed on the plates. The electric field intensity at the midpoint between the plates is 400 N/C.

(a) What is the electric field intensity at a point 5.0 cm from the positive plate?

(b) If the same amount of charge was placed on plates that have twice the area and are 20 cm apart, what would be the electric field intensity at the point 5.0 cm from the positive plate?

(c) What would be the electric field intensity of the original plates if the distance of separation of the plates was doubled?

Conceptualize the Problem

- The *electric field* between isolated parallel plates is *uniform*.
- The *electric field* between isolated parallel plates depends on the *charge density* on the plates.

Identify the Goal

The magnitude of the electric field, $|E_Q|$, under three different conditions

Identify the Variables and Constants

Known	Unknown
$|E_{Q(\text{initial})}| = 400$ N/C	$|E_{Q(\text{final})}|$

Develop a Strategy

The magnitude of the electric field intensity is uniform between parallel plates, so it will be the same at every point.

$$|E_Q| = 400\ \frac{\text{N}}{\text{C}}$$

(a) The magnitude of the electric field intensity is 400 N/C at a point 5 cm from the positive plate.

The magnitude of the electric field intensity is inversely proportional to the area of the plates.

$$|E_{Q_1}| \propto \frac{q_1}{A_1} \quad \text{and} \quad |E_{Q_2}| \propto \frac{q_2}{A_2}$$

Divide.

$$\frac{|E_{Q_2}|}{|E_{Q_1}|} = \frac{\frac{q_2}{A_2}}{\frac{q_1}{A_1}}$$

Substitute.

$$q_2 = q_1 \quad \text{and} \quad A_2 = 2A_1$$

$$\frac{|E_{Q_2}|}{|E_{Q_1}|} = \frac{\frac{\cancel{q_1}}{2\cancel{A_1}}}{\frac{\cancel{q_1}}{\cancel{A_1}}}$$

$$|E_{Q_2}| = \frac{|E_{Q_1}|}{2}$$

$$|E_{Q_2}| = \frac{400\frac{\text{N}}{\text{C}}}{2}$$

$$|E_{Q_2}| = 200\ \frac{\text{N}}{\text{C}}$$

(b) The magnitude of the electric field intensity is 200 N/C when the area is doubled.

(c) The magnitude of the electric field intensity is 400 N/C (unchanged) when the distance is doubled, because electric field intensity is independent of the distance of separation.

Validate the Solution

Only the charge density affects the field intensity between the plates. Therefore, changing the area of the plates and consequently reducing the charge density is the only change that will affect the value of the field intensity.

PRACTICE PROBLEMS

1. A pair of metal plates, mounted 1.0 cm apart on insulators, is charged oppositely. A test charge of +2.0 μC placed at the midpoint, M, between the plates experiences a force of 6.0×10^{-4} N[W].

(a) What is the electric field intensity at M?

(b) What is the electric field intensity at a point 2.0 mm from the negative plate?

(c) What is the electric field intensity at a point 1.0 mm from the positive plate?

(d) What are two possible ways in which you could double the strength of the electric field?

2. The electric field intensity at the midpoint, M, between two oppositely charged (isolated) parallel plates, 12.0 mm apart, is 5.0×10^3 N/C[E].

(a) What is the electric field intensity at a point 3.0 mm from the negative plate?

(b) If the plate separation is changed to 6.0 mm and the area of the plates is changed, the electric field intensity is found to be 2.0×10^4 N/C[E]. What was the change made to the area of the plates?

continued ▶

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3. A test charge of +5.0 μC experiences a force of 2.0×10^3 N[S] when placed at the midpoint of two oppositely charged parallel plates. Assuming that the plates are electrically isolated and have a distance of separation of 8.0 mm, what will be the force experienced by a different charge of −2.0 μC, located 2.0 mm from the negative plate?

Parallel Plates and Potential Difference

In Chapter 7, you learned that the potential difference between two points in an electric field is the work required to move a unit charge from one point to the other. What generalizations can you make about potential difference between two parallel plates?

Consider a test charge, q, placed against the negative plate of a pair of parallel plates. You can derive an expression for the potential difference between the plates by considering the work done on a test charge when moving it from the negative plate to the positive plate.

- Write the equation for the amount of work you would have to do to move the charge a displacement, Δd. To eliminate the $\cos\theta$, work only with the component of displacement that is parallel to the force and therefore to the electric field.

$$W = F\Delta d\cos\theta$$
$$W = F\Delta d \text{ (parallel to field)}$$

- Write the expression for the force on a charge in an electric field.

$$\vec{F} = q\vec{E}_Q$$

- Substitute the expression for force into the equation for work. (**Note:** The vector notation and absolute value symbol will be used with the electric field intensity to avoid confusion with electric potential energy.)

$$W = q|\vec{E}_Q|\Delta d$$

- Divide both sides of the equation by q.

$$\frac{W}{q} = |\vec{E}_Q|\Delta d$$

- The definition of electric potential difference is work per unit charge.

$$\Delta V = \frac{W}{q}$$
$$\Delta V = |\vec{E}_Q|\Delta d$$

- Another useful equation results when you divide both sides of the equation by displacement.

$$|\vec{E}_Q| = \frac{\Delta V}{\Delta d}$$

ELECTRIC FIELD AND POTENTIAL DIFFERENCE

The magnitude of the electric field intensity in the region between two points in a uniform electric field is the quotient of the electric potential difference between the points and the component of the displacement between the points that is parallel to the field.

$$\left|\vec{E}_Q\right| = \frac{\Delta V}{\Delta d}$$

Quantity	Symbol	SI unit
electric field intensity	$\vec{E}_Q$	$\frac{N}{C}$ (newtons per coulomb)
electric potential difference	ΔV	V (volts)
component of displacement between points, parallel to field	$\Delta \vec{d}$	m (metres)

Unit Analysis

$$\frac{V}{m} = \frac{J/C}{m} = \frac{N \cdot \cancel{m}}{C \cdot \cancel{m}} = \frac{N}{C}$$

Note: When doing a unit analysis, it is very useful to remember that a volt per metre is equivalent to a newton per coulomb.

Potential Gradient

In general, a gradient is similar to a rate. While a rate is a change in some quantity relative to a time interval, a **gradient** is a change in some quantity relative to a change in position, or displacement; therefore, the expression $\frac{\Delta V}{\Delta d}$ is known as the **potential gradient**.

As you move from one plate to the other, the electric potential difference changes linearly, since $\Delta V = \left|\vec{E}\right|\Delta d$ and $\vec{E}$ is constant. So, if the potential difference across the plates is 12 V, the potential difference at a point one third of the distance from the negative plate will be 4.0 V. The potential difference is higher close to the positive plate. In other words, the potential difference increases in a direction opposite to the direction of the electric field.

Physicists commonly refer to the "potential at a point" in an electric field. As you know, there are no absolute potentials, only potential differences. Therefore, the phrase "potential at a point" means the potential difference between that point and a reference point. In the case of parallel plates, the reference point is always the negatively charged plate.

SAMPLE PROBLEM

Field and Potential

Two parallel plates 5.0 cm apart are oppositely charged. The electric potential difference across the plates is 80.0 V.

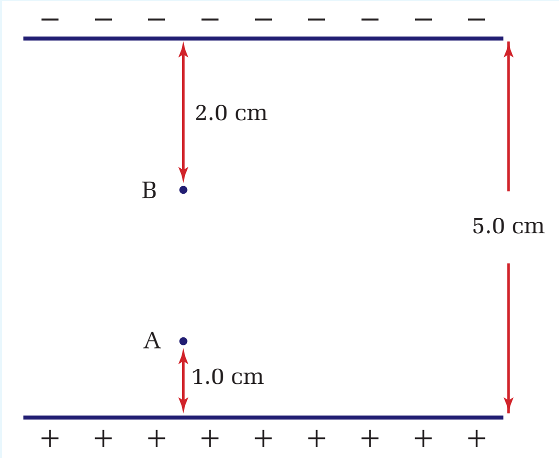
(a) What is the electric field intensity between the plates?

(b) What is the potential difference at point A?

(c) What is the potential difference at point B?

(d) What is the potential difference between points A and B?

(e) What force would be experienced by a small 2.0 μC charge placed at point A?



Conceptualize the Problem

- The *electric field* between *parallel plates* is *uniform*.
- Identify the lower plate as positive.
- The *electric field intensity* is related to the *potential difference* and the *distance of separation*.

Identify the Goal

The electric field intensity, $|\vec{E}_Q|$, between the plates

The potential difference at point A, V_A, and point B, V_B

The potential difference between points A and B, ΔV_{AB}

The electric force, $\vec{F}_Q$, on a charge placed at point A

Identify the Variables and Constants

Known	Unknown	
$\Delta d = 5.0$ cm	$\vec{E}_Q$	ΔV_{AB}
$\Delta V = 80.0$ V	V_A	$\vec{F}_Q$
Points A and B	V_B	
$q = 2.0\ \mu$C		

Develop a Strategy

The electric field is related to the potential difference and the distance of separation.

$$|\vec{E}_Q| = \frac{\Delta V}{\Delta d}$$

$$|\vec{E}_Q| = \frac{80.0\ \text{V}}{5.0 \times 10^{-2}\ \text{m}}$$

$$\vec{E}_Q = 1.6 \times 10^3\ \frac{\text{N}}{\text{C}}$$

directed from the positive to the negative plate

(a) The electric field intensity is 1.6×10^3 N/C away from the positive plate.

Use the equation that relates the electric potential difference to the electric field intensity. (**Note:** Point A is 4.0 cm from the negative plate.)

$$V_A = |\vec{E}_Q|\Delta d$$
$$V_A = \left(1.6 \times 10^3\ \frac{V}{\cancel{m}}\right)(0.040\ \cancel{m})$$
$$V_A = 64\ V$$

(b) The potential difference at point A is 64 V.

Use the equation that relates the electric potential difference to the electric field intensity.

$$V_B = |\vec{E}_Q|\Delta d$$
$$V_B = \left(1.6 \times 10^3\ \frac{V}{\cancel{m}}\right)(0.020\ \cancel{m})$$
$$V_B = 32\ V$$

(c) The potential difference at point B is 32 V.

Point A is at the higher potential, because it is closer to the positive plate.

$$\Delta V = V_A - V_B$$
$$\Delta V = 64\ V - 32\ V$$
$$\Delta V = 32\ V$$

(d) The potential difference between points A and B is 32 V.

The electric force is related to the field and charge.

$$\vec{F}_Q = q\vec{E}_Q$$
$$\vec{F}_Q = (2.0 \times 10^{-6}\ \cancel{C})\left(1.6 \times 10^3\ \frac{N}{\cancel{C}}\right)$$
$$\vec{F}_Q = 3.2 \times 10^{-3}\ N[\text{away from positive plate}]$$

(e) The force experienced by the small charge at point A is 3.2×10^{-3} N, away from the positive plate.

Validate the Solution

The values are reasonable in terms of the given data. The units are logical.

PRACTICE PROBLEMS

4. Calculate the electric field intensity between two parallel plates, 4.2 cm apart, which have a potential difference across them of 60.0 V.

5. The potential difference between two points 8.0 mm apart in the field between two parallel plates is 24 V.

(a) What is the electric field intensity between the plates?

(b) The plates themselves are 2.0 cm apart. What is the electric potential difference between them?

6. When an 80.0 V battery is connected across a pair of parallel plates, the electric field intensity between the plates is 360.0 N/C.

(a) What is the distance of separation of the plates?

(b) What force will be experienced by a charge of $-4.0\ \mu C$ placed at the midpoint between the plates?

(c) Calculate the force experienced by the charge in part (b) if it is located one quarter of the way from the positive plate.

7. What electric potential difference must be applied across two parallel metal plates 8.0 cm apart so that the electric field intensity between them will be 3.2×10^2 N/C?

8. The potential gradient between two parallel plates 2.0 cm apart is 2.0×10^3 V/m.

(a) What is the potential difference between the plates?

(b) What is the electric field intensity between the plates?

QUICK LAB

Quantum Pennies

TARGET SKILLS
- Hypothesizing
- Analyzing and interpreting

This Quick Lab will give you some insight into the approach that Robert Andrews Millikan (1868–1953) used to determine the charge on the electron. Instead of charge, however, you will determine the mass of a penny.

Your teacher has prepared a class set of small black film canisters that contain various numbers of identical pennies. You and your classmates will use an electronic balance to determine the mass of each film canister and its contents. Carry out this procedure and post the class results.

Analyze and Conclude

1. Draw a bar graph, with the canister number on the horizontal axis and the mass of that canister on the vertical axis.
2. Calculate and record the increments between each mass value and all other mass values.
3. Is there any minimum increment of which all other increments are a multiple?
4. What do you predict to be the mass of a penny? Check your prediction by direct measurement.

Apply and Extend

5. As a class, open the canisters and randomly add or remove pennies. Again, measure the mass, and repeat the analysis.

Millikan's Oil-Drop Experiment: Charge on the Electron

A very important series of experiments dependent on the uniform electric field between a pair of parallel plates was performed during the years 1909 to 1913 by Millikan. The results of these experiments, together with his contributions to research on the photoelectric effect (see Chapter 12), led to his Nobel Prize in Physics in 1923. Millikan was able not only to verify the existence of a fundamental electric charge — the electron — but also to provide the precise value of the charge carried by the electron. This had a tremendous impact on the further development of the theory of the structure of matter.

The experimental procedures used by Millikan were actually a modification of earlier techniques used by J.J. Thomson (1856–1940). A pair of parallel plates was very finely ground to smoothness and a tiny hole was drilled in the top plate. An atomizer was mounted above the plates and used to spray tiny droplets of oil into the region above the plates. These oil droplets acquired an electric charge as they were sprayed, presumably from friction. The whole apparatus was kept in a constant-temperature enclosure, and the region between the plates was illuminated with an arc lamp.

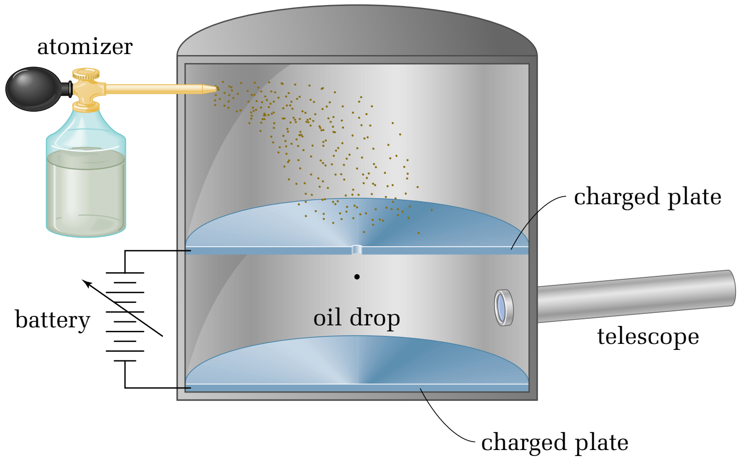


Figure 8.6 Not shown in this simplified cross-section of Millikan's apparatus is a source of X rays that could ionize the air in the electric field.

A droplet that fell through the hole and into the region between the plates could be viewed through a short telescope. The droplet would very quickly acquire terminal velocity as it fell under the influence of the force of gravity and the resistance (viscosity) of the air. This terminal velocity, v_0, could be measured by timing the drop as it fell between the lines on a graduated eyepiece. (In Millikan's apparatus, the distance between the cross hairs was 0.010 cm). A potential difference (3000 to 8000 V) was then applied across the plates (separated 1.600 cm) by means of a variable battery.

Usually the oil drops had attained a negative charge, so the potential difference would be applied so that the top plate was made positive. In that way, a negatively charged oil drop could be made to reach an upward terminal velocity under the action of the applied electric field, its effective weight, and air friction. This second terminal velocity, v_1, was also recorded.

Millikan then directed X rays to the region between the plates. The X rays ionized the air molecules between the plates and caused the charge on the oil drop to change as it either gained or lost electrons. Again, the procedure of determining the two terminal velocities, one with and one without the applied electric field, could be repeated many times with differently charged oil drops.

Millikan observed that the terminal velocity of the charged oil drops, which depended on the charge itself, varied from trial to trial. Over a very large number of trials, however, the velocity values could be grouped into categories, all of which represented an integral multiple of the lowest observed value. This led him to conclude that the charge on the oil drops themselves could be quantified as integral multiples of one fundamental value. Millikan then applied a mathematical analysis to determine that value. The following is a simplified version of Millikan's analysis.

- When the oil drop travelled with a uniform velocity, the upward electric force was equal in magnitude to the downward gravitational force.

$$\vec{F}_Q = \vec{F}_g$$

- Write the expressions for the electric force in terms of the electric field intensity and for the gravitational force.

$$\vec{E}_Q = \frac{\vec{F}_Q}{q} \qquad \vec{F}_g = m\vec{g}$$

$$\vec{F}_Q = q\vec{E}_Q$$

- Substitute the expressions into the first equation.

$$q\vec{E}_Q = m\vec{g}$$

- Express the electric field intensity in terms of the potential difference across the plates.

$$E_Q = \frac{\Delta V}{\Delta d}$$

- Substitute the expression for the electric field intensity.

$$\frac{qV}{\Delta d} = mg$$

- Solve for the charge, q.

$$q = \frac{mg\Delta d}{V}$$

All of the variables in this equation for q could be measured easily, except the mass. Millikan then turned to the work of Sir George Gabriel Stokes (1819–1903), a British mathematician and physicist who had helped to develop the laws of hydrostatics. Based on a particle's rate of fall as it falls through a viscous medium, **Stokes' law** can be used to calculate the particle's mass.

Millikan measured the terminal velocity of each oil drop with the battery turned off, that is, the rate of fall under the influence of gravity and the resistance of the fluid (air) only. When he substituted the mass of each oil drop into the equation for the charge q, above, he found that the magnitude of the charge on each oil drop was always an integral multiple of a fundamental value. He assumed that this particular fundamental charge was actually the charge on the electron, and the multiple values arose from the oil drops having two, three, or more excess or deficit electrons.

The electronic charge computed from many trials of Millikan's method is found to be $e = 1.6065 \times 10^{-19}$ C, which agrees well with values determined by other methods. The currently accepted value for the charge on the electron is $e = 1.602 \times 10^{-19}$ C. Knowing the charge, e, on an electron, it has become common practice to express the charge on an object in terms of the number, n, of the excess or deficit of electrons on the object, or $q = ne$.

SAMPLE PROBLEM

Millikan Experiment

Two horizontal plates in a Millikan-like apparatus are placed 16.0 mm apart. An oil drop of mass 3.00×10^{-15} kg remains at rest between the plates when a potential difference of 420.0 V is applied across the plates, the upper plate being positive. Calculate the

(a) net charge on the oil drop

(b) sign of the charge on the oil drop

(c) number of excess or deficit electrons on the oil drop

Conceptualize the Problem

- The oil drop is held in place by its own *weight* (down) and the *electric force* (up).
- The *electric force* depends on the *electric field* value between the plates.

Identify the Goal

The magnitude of the charge, q, on the oil drop; its sign, ±; and electron number, n, of excess or deficit electrons

Identify the Variables and Constants

Known	Implied	Unknown
$d = 16.0$ mm	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	q
$m = 3.00 \times 10^{-15}$ kg	$e = 1.602 \times 10^{-19}$ C	n
$V = 420.0$ V		

Develop a Strategy

The electric force will be equal in magnitude to the force of gravity.

$$F_Q = F_g$$
$$qE_Q = mg$$
$$q\frac{V}{\Delta d} = mg$$
$$q = \frac{mg\Delta d}{V}$$
$$q = \frac{(3.00 \times 10^{-15}\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.60 \times 10^{-2}\ \text{m})}{420.0\ \text{V}}$$
$$q = 1.1211 \times 10^{-18}\ \text{C}$$
$$q \cong 1.12 \times 10^{-18}\ \text{C}$$

(a) The charge on the oil drop is -1.12×10^{-18} C.

The drop was suspended.

Since the electric force was in a direction opposite to the gravitational force, it had to be "up." The upper plate was positive, so the charge had to be negative.

(b) The net charge was negative.

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The net charge is related to the charge on the electron.

$$n = \frac{q}{e}$$

$$n = \frac{1.12 \times 10^{-18}\,\text{C}}{1.60 \times 10^{-19}\,\text{C}}$$

$$n = 7.00$$

(c) There is an excess of seven electrons on the oil drop, causing its net negative charge.

Validate the Solution

The values are consistent with the size of the oil drop, the plate separation, and the potential difference. The units in the calculation are consistent.

$$\frac{\cancel{\text{kg}} \cdot \frac{\text{N}}{\cancel{\text{kg}}} \cdot \text{m}}{\text{V}} = \frac{\text{N} \cdot \text{m}}{\frac{\text{J}}{\text{C}}} = \frac{\cancel{\text{J}}}{\frac{\cancel{\text{J}}}{\text{C}}} = \text{C}$$

PRACTICE PROBLEMS

9. Two large horizontal parallel plates are separated by 2.00 cm. An oil drop, mass 4.02×10^{-15} kg, is held balanced between the plates when a potential difference of 820.0 V is applied across the plates, with the upper plate being negative.

(a) What is the charge on the drop?

(b) What is the number of excess or deficit electrons on the oil drop?

10. A small latex sphere experiences an electric force of 3.6×10^{-14} N when suspended halfway between a pair of large metal plates, which are separated by 48.0 mm. There is just enough electric force to balance the force of gravity on the sphere.

(a) What is the mass of the sphere?

(b) What is the potential difference between the plates, given that the charge on the sphere is 4.8×10^{-19} C?

11. The density of the oil used to form droplets in the Millikan experiment is $9.20 \times 10^{2}\,\text{kg/m}^{3}$ and the radius of a typical oil droplet is 2.00 μm. When the horizontal plates are placed 18.0 mm apart, an oil drop, later determined to have an excess of three electrons, is held in equilibrium. What potential difference must have been applied across the plates?

UNIT PROJECT PREP

Understanding the costs and benefits of any issue often begins by (a) gathering useful facts and (b) identifying personal bias.

- Identify your bias. Do you believe that fundamental research is worthwhile if it does not have any obvious applications?
- Do you believe that research resulting in greater understanding will some day, perhaps decades later, be put to use in an application?
- How would you complete the statement "Knowledge for the sake of knowledge … "?

INVESTIGATION 8-A

Millikan's Oil-Drop Experiment

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting

In this investigation, you will demonstrate that electric charge exists as a quantized entity, using apparatus that allows you to apply a potential difference across parallel plates as you observe the movement of latex spheres.

Problem

Does charge exist in fundamental units and can you find evidence of differently charged objects?

Equipment

- Millikan apparatus for use with latex spheres
- supply of latex spheres
- stopwatch

CAUTION Be careful not to touch open terminals that are connected to a high potential difference.

Procedure

1. Follow the manufacturer's instructions to adjust and focus the light source and also to connect the plates to the source of potential difference. Your aim is to make repeated measurements of the velocity of a sphere under the action of gravity alone (v_0, down) and also under the action of both the gravitational force and the electric force (v_1, up).
2. Examine the position and function of the voltage switch. In the *off* position no electric field will be applied and the sphere will fall under the action of the force of gravity alone. In the *on* position a potential difference will be applied across the plates, with the top plate being positive, and the sphere will rise as the electric force is greater than the force of gravity.
3. Place the switch in the *off* position and squeeze some latex spheres into the region between the plates. (You may need to practise observing the spheres before you actually start timing them. They will appear as tiny illuminated dots.) Follow the manufacturer's instructions for determining direction. The telescope usually inverts the field of view, so the force of gravity is then "up," although some manufacturers have included an extra lens to compensate.
4. Using the voltage switch, clear the field of fast-moving dots. They carry a large charge and are hard to measure. Choose one of the slowly falling spheres and measure its time of travel as it falls, under the action of the force of gravity. Observe the motion for several grid marks in the field of view. (Remember it might be falling "up" in your apparatus.) Without losing the sphere, change the switch so that the sphere rises under the action of the electric field and again measure the time of travel over the grid marks. Before the sphere disappears from the field of view, place the switch in the *off* position and again measure the time of travel over the grid marks. You will need a laboratory partner to record the results so that you can keep your eye on the selected sphere.
5. Repeat your observations for a different sphere from a new batch, and continue making observations for at least 20 different spheres. (Alternate with your lab partner to allow your eyes to rest!)

Analyze and Conclude

1. Calculate the velocity of the spheres for every trial, using an arbitrary unit for distance. For example, if one sphere moved 8.0 gridlines in 3.1 seconds, record its velocity as

$$v = \frac{\Delta d}{\Delta t} = \frac{8.0\ \text{grid lines}}{3.1\ \text{s}} = 2.6\ \frac{\text{grid lines}}{\text{s}}.$$

2. Record the velocity of the sphere in two different ways: v_0 to represent the velocity of the sphere under the force of gravity alone, and v_1 to represent the velocity when

the electric force up is greater than the gravitational force on the sphere. Record your data in a table similar to the one below.

Sphere	v_0	v_1	$v_0 + v_1$

3. Complete the calculations for each column in the table.

4. Since the value of $v_0 + v_1$ represents the strength of the electric force alone, acting on the sphere, it can also be considered to represent the electric charges on the sphere. Draw a bar graph with the quantity $v_0 + v_1$ on the vertical axis and "Trial number" evenly distributed on the horizontal axis.

5. Does your bar graph offer any evidence that electric charge exists as an integral multiple of a fundamental charge? Are you able to state the number of fundamental charges that are excess or deficit on your spheres? Explain your reasoning.

8.1 Section Review

1. K/U
   (a) Draw the electric field pattern for a +4 μC charge and a −4 μC charge separated by 4.0 cm. Include four equipotential lines.
   (b) Repeat part (a) for a −16 μC and a −4 μC charge.

2. I If you have access to the Internet, use the sites listed in your Electronic Learning Partner to verify your answers.

3. K/U
   (a) List four properties of electric field lines.
   (b) List two properties of equipotential surfaces.

4. MC Research and report on the use of electric fields in technology and medicine (for example, laser printers, electrocardiograms).

5. C With your classmates, prepare a dramatic skit to simulate Millikan and his colleagues preparing and performing his oil-drop experiment.

6. K/U A pair of parallel plates is placed 2.4 cm apart and a potential difference of 800.0 V is connected across them.
   (a) What is the electric field intensity at the midpoint between the plates?
   (b) What is the electric potential difference at that point?
   (c) What is the electric field intensity at a point 1.0 cm from the positive plate?
   (d) What is the electric potential difference at that point?

7. I A pair of horizontal metal plates are situated in a vacuum and separated by a distance of 1.8 cm. What potential difference would need to be connected across the plates in order to hold a single electron suspended at rest between them?

8.2 Conductors and Fields

You will now examine how conductors are used in the transport of electric current and electromagnetic signals. An electric field can be established not only in the spatial region around point charges or in the air gap between parallel plates, but also in the metal conducting wires that enable electric current to be transmitted. Shielded coaxial wires can also be used as a "guide" to transport electromagnetic waves to a convenient location (such as your television receiver) with minimal loss of strength. You will learn more about electromagnetic waves in Unit 4, but for now, you can at least gain a qualitative idea of how they can be transported efficiently, with minimal loss of energy.

SECTION EXPECTATIONS

- Define and describe the concepts related to electric fields.
- Describe and explain the electric field that exists inside and on the surface of a charged conductor.
- Demonstrate how an understanding of electric fields can be applied to control the electric field around a conductor.

KEY TERM

- Faraday cage

Conducting Wires

In previous science courses, you worked with conducting wires and circuits. You learned that if you placed an electric potential difference across the ends of the conductor, a current would flow. You have just learned that an electric potential difference creates an electric field and that charges in an electric field experience electric forces. Now you can examine conductors in more detail.

In previous studies, you learned that the copper atoms have heavy positive nuclei and a cloud of negative electrons surrounding it. An isolated atom has electrons filling up the lower energy "shells," but there are also a few electrons outside of these complete shells. These outer electrons can move relatively easily if they are replaced with another electron from another copper atom. The electrons are then free to move through the metal, colliding randomly with the stationary positive nuclei.

If a battery is connected to the ends of a metal wire, it will create an electric field inside the wire and parallel to its axis. Consequently, the free electrons will move in a direction opposite to the direction of the field, as shown in Figure 8.7.

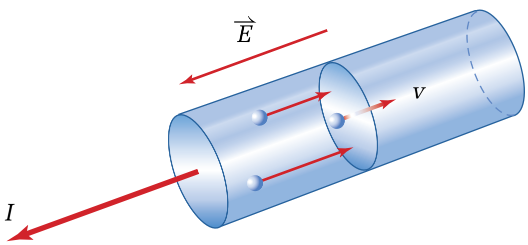


Figure 8.7 In a conductor, electrons move opposite to the direction of the electric field, because the direction of the field is defined as the direction in which a positive charge would move.

Hollow Conductors: Faraday's Ice-Pail Experiment

When you carried out the Cover It Up activity in the Multi-Lab at the beginning of this chapter, you probably noticed that when you placed the can over the sphere on the electroscope, it eliminated the effect that you originally observed when you brought the charged rod close to the electroscope sphere. You probably did not realize that you were performing an experiment very similar to one of the most famous experiments in the history of the study of electric fields — Michael Faraday's ice-pail experiment. Faraday devised this experiment to show that electric charge will reside only on the *outside* of a hollow conductor. The experiment is outlined schematically in Figure 8.8 and described in the steps that follow.

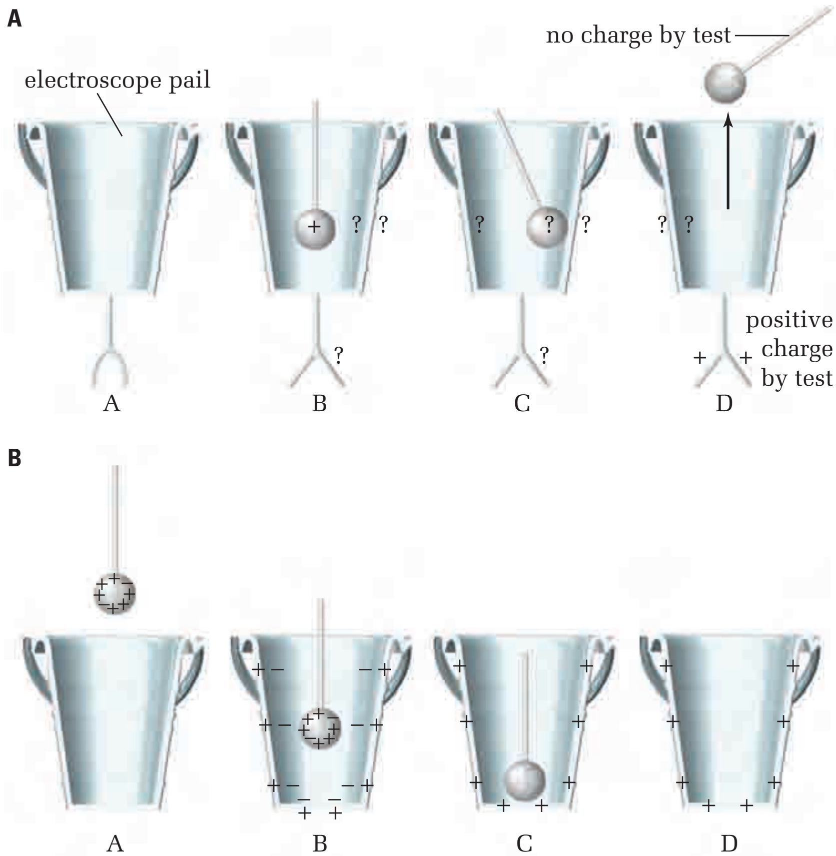


Figure 8.8 **(A)** The questions raised by Faraday's ice-pail experiment; **(B)** the answers

Faraday's Ice-Pail Experiment

- A hollow metal can (Faraday happened to use an ice pail), insulated from its surroundings, was connected to an uncharged electroscope.

- A positively charged metal ball was lowered into the pail by its insulated handle. The electroscope leaves diverged and stayed at a fixed divergence. When the metal ball was moved around inside the ice pail, the electroscope leaves stayed at a fixed angle of divergence.
- The metal ball was allowed to touch the inside of the ice pail. The angle of divergence of the leaves of the electroscope remained the same.
- The metal ball was then removed from the ice pail and the ball and the leaves were tested for charge. The ball was found to be uncharged, and the leaves were charged positively.

From his experiment, Faraday deduced the following.

- The positive ball had induced a negative charge on the inside wall of the pail and a positive charge on the outside wall.
- The induced charge was of the same magnitude as the charge on the ball, since the charges on the ball and the inside wall of the pail cancelled each other.
- The induced charges on the inside and outside walls of the pail were of equal magnitude, since the angle of divergence of the leaves did not change throughout the experiment.

Two general properties were illustrated by this experiment.

1. The formation of one charge is always accompanied by the formation of an equal, but opposite, charge.
2. The net charge in the interior of a hollow conductor is zero; all excess charge is found on the outside.

The latter property led to the general conclusion that an external electric field will not affect the inside of a hollow conductor. In fact, it will be shielded. Faraday pursued this with a further demonstration in which he built a very large metal cage, mounted on insulators, and then entered the cage to perform electrostatic experiments, while a very high electric field was generated all around him.

This electric screening was the basis of the Cover It Up activity in the Multi-Lab at the beginning of the chapter. If you charge an electroscope and then place a cage (or inverted can) over the sphere of the electroscope, you will shield it from external electric fields. If you bring a charged rod close to the cage, the leaves of the electroscope are unaffected. This principle has become a popular method for screening sensitive electric circuit elements by placing them in some form of metal cage. Today, anything that is used to shield a region from an external electric field is called a **Faraday cage**.

WEB LINK

www.mcgrawhill.ca/links/physics12

For more information about Michael Faraday, both scientific and personal, go to the above Internet site and click on **Web Links**.

QUICK LAB

Faraday Cage

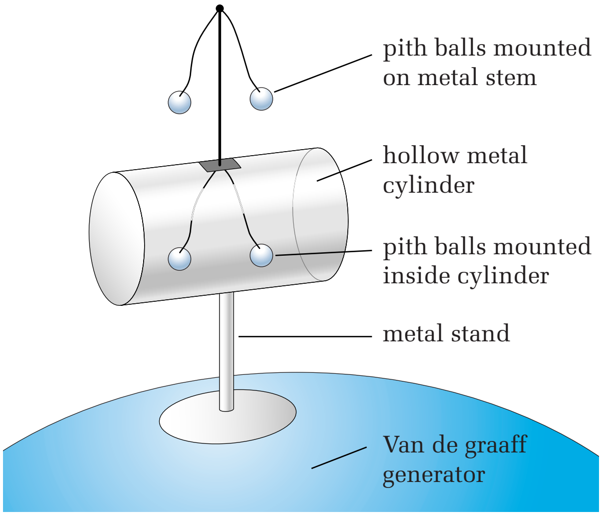
TARGET SKILLS
- Hypothesizing and predicting
- Analyzing and interpreting

CAUTION Van de Graaff generators generate very high potential differences that might cause harm to some individuals.

You can demonstrate the shielding effect of a hollow metal cylinder by using the apparatus shown in the diagram. Either use tape to attach the metal base of the hollow cylinder-pith ball apparatus to the top of the sphere on the Van de Graaff generator, or use an electric lead to connect the two. Turn on the generator and allow it to run for a few seconds.

Analyze and Conclude

1. How does the behaviour of the pith balls inside the hollow cylinder differ from the behaviour of the pith balls mounted outside the hollow cylinder?
2. What does this experiment demonstrate about the electric field inside a hollow conductor?



Coaxial Cable

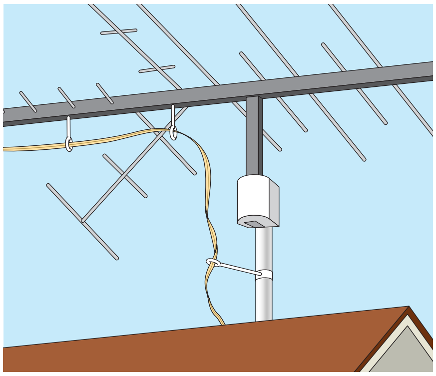
When electromagnetic waves, such as television signals, are transmitted to the home, either through or beyond the atmosphere, they are captured by a receiver (antenna) and then delivered to your television as an electric signal.

Early antenna cables consisted of a flat, twin-lead wire, with two braided wires (through which the signal was conducted) mounted in a flat, plastic insulating band. This type of wire has become less common, as it is very susceptible to interference from unwanted electromagnetic signals, such as those arising from sunspot activity, lightning storms, or even just local extraneous transmissions, such as those from power tools.

An improvement on the twin-lead wire was the shielded twin lead, in which the braided wires were each wrapped in foam insulation. The pair of wires was then wrapped in foil sheathing to provide shielding and then in an outer layer of plastic insulation.

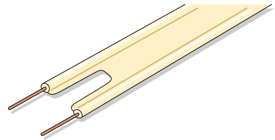
The most efficient and popular signal-conducting wire today is the coaxial cable, consisting of concentric rings: an inner conducting wire, sometimes stranded (stereo) but usually solid (television), a sheath of foam insulation, a second sheath of braided (or solid foil) conducting wire, and an outer sheath of plastic insulation. The actual mathematics and physics of the transport of a signal along a coaxial cable is quite complex, but for the purposes of this section, it is sufficient to say that the two wires transporting the signal are the inner core wire and the outer braided wire. The latter provides a form of Faraday shielding from external interference.

Conceptual Problems

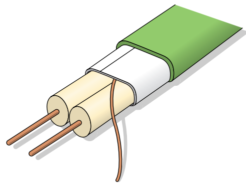


- When twin-lead wire is used to carry the television signal from the antenna to a television receiver, the directions require that the lead be twisted and not installed straight. What would be the purpose of this instruction?
- Why must the twin-lead wire be held away from the metal antenna mast, using insulating clamps?
- A homeowner knew some physics and decided to run a coaxial cable through the house inside the metal heating ducts.
  (a) What would be one advantage of this procedure?
  (b) State one disadvantage of this method.
- Before the advent of transistors, old superheterodyne “wireless” receivers used “radio valves” in the amplification circuit. Why were these valves often enclosed in metal cylinders?
- When transistors became the basic component of electric circuits, did they also need shielding? Research your answer.

A a flat twin-lead wire



B shielded twin-lead wire



C coaxial cable

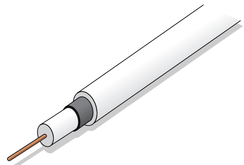


Figure 8.9 Common types of signal conducting wires.

8.2 Section Review

1. **K/U** What evidence supports the practice of enclosing electronic components in metal shells?
2. **MC** Some people who felt that TV antennas looked unsightly hid them in the attics of their houses. Discuss how the type of roof and siding material has relevance to this practice.
3. **MC** Some people feel that it is relatively safe to take shelter in a car during a lightning storm, because the rubber tires will provide insulation. However, a lightning strike that has travelled several kilometres is not going to be discouraged from jumping the last few centimetres. In what way does a car offer protection from lightning?

Levitation: How Does It Work?

Levitating an object should be easy. All you need is a repulsive force strong enough to counteract Earth's gravity. So why not use an electric charge or a magnet to create the repulsive force? Scientists have been thinking about this idea for years. In fact, the first proposal to use magnetism to levitate vehicles was made in 1912, just one year after the discovery of superconductivity.

Superconductors can conduct electricity with no resistance at all. In normal conductors, moving electrons collide with atoms, a process that resists the flow of current and causes the conductor to heat up. Superconductors can carry large currents without heating up, which means that they can be used to create powerful electromagnets. Once the current is introduced into the superconducting wire, it can flow indefinitely, without dissipation, because there is no resistance. So, over time, the created magnetic field will not lose strength.

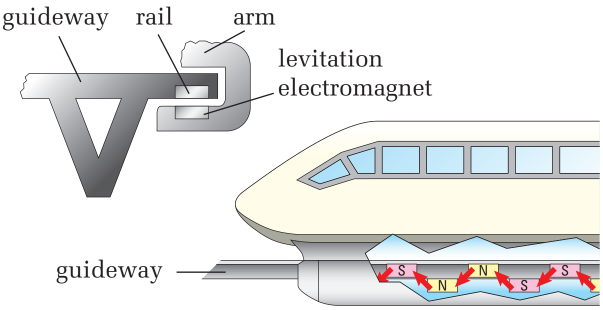
A Surprising Effect

What is the connection between superconductors and levitation? Superconductors have an additional surprising property when placed in a magnetic field. When a normal conductor is placed in a magnetic field, the magnetic field lines go right through the conductor — as if it was not there. When a superconductor is placed in a magnetic field, it expels the magnetic field from its interior and causes levitation. You can clearly see the levitation factor at work in a magnetically levitated (maglev) train — the train rides (or levitates) a few centimetres above the track (guideway).

Exciting Technology

Since the train rides above the guideway, what about propulsion? How does that happen? One design for a magnetically levitated train that is currently being built in Japan uses a push-pull system. Electromagnets are placed in the bottom of the train and along the track. The current is set so that the electromagnets along the track have opposite polarity to those on the train. It's possible, then, to have an unlike pole just ahead of each electromagnet on the train and a like pole just behind. The electromagnetic force pulls the unlike poles together and pushes the like poles apart.

After the train has moved forward enough to line up all of the electromagnets on the track and the train, the track electromagnets are briefly switched off. When they are switched on again, they have the opposite polarity, so that each electromagnet on the train is now pushed and pulled by those on the track. It's an intuitively basic design. The train's speed can be adjusted by timing the switches of the polarity of the track electromagnets. Slowing or braking the train is similarly accomplished.



Levitation electromagnets drawn up toward the rail in the guideway levitate Japan's magnetically levitated train.

Because the train rides a few centimetres above the track, there is no friction due to moving parts; this allows maglev trains to achieve much greater speeds than conventional trains. Test models in Japan and Germany have recorded speeds of 400 to 500 km/h. However, maglevs are expensive to build and operate. They have no wheels, so they cannot run on existing tracks and conventional trains cannot run on maglev tracks — so entirely new tracks must be built. Still, several countries are investigating how best to use this exciting new technology.

Levitation and Diamagnetic Materials

Levitation can also be achieved using certain types of materials, which are known as "diamagnetic" and are not superconductive. Diamagnetic materials are normally non-magnetic materials that become magnetized in a direction opposite to an applied magnetic field. Recently, physicists in Holland used a magnetic field of about 10 T to levitate a variety of seemingly non-magnetic materials, including a hazelnut, a strawberry, a drop of water, and a live frog. According to the researchers, the frog showed no ill effects from its adventure in levitation.

Frogs helped in levitation experiments and experienced no ill effects.

Then there's the other obvious question: Can people be levitated? In principle, the answer is yes, but in practice, the answer for now is no. Existing magnets are capable of levitating objects a few centimetres in diameter. Levitating a person would require an enormous electromagnet operating at 40 T, with about 1 GW of continuous power consumption. That's the same amount of power required to light 10 million 100 W light bulbs! Until more efficient ways can be found to make strong electromagnets, or people can be turned into frogs, people will be forced to walk with their feet on the ground.

Making Connections

1. Conduct a feasibility study for a maglev train to operate between two large cities.
2. Discuss the importance of space-based research and how diamagnetic levitation can be used to augment it.

8.3 Applications of Magnetic and Electric Fields

SECTION EXPECTATIONS

- Define and describe the concepts related to electric, gravitational, and magnetic fields.
- Predict the forces acting on a moving charge and on a current-carrying conductor in a uniform magnetic field.
- Determine the resulting motion of charged particles by collecting quantitative data from experiments or computer simulations.
- Describe instances where developments in technology resulted in advancement of scientific theories.

KEY TERMS

- particle accelerator
- mass spectrometer
- cyclotron
- synchrocyclotron
- betatron
- linear accelerator
- synchrotron

In previous science courses, you have probably read that the mass of an electron is 9.1094×10^{-31} kg and that the mass of a proton is 1.6726×10^{-27} kg. Did you ever wonder how it was possible for anyone to measure masses that small — especially to five significant digits — when there are no balances that can measure masses that small?

Atomic masses are determined by mass spectrometers, which are instruments that are based on the behaviour of moving charges in magnetic fields. The same principle causes motors to turn and prevents high-speed ions in the solar wind from bombarding Earth — except at the North and South Poles, as you read in the chapter introduction. In this section, you will learn more about moving charges in magnetic fields and many of the technologies based on this principle.

In Grade 11 physics, you were introduced to the force acting on a current-carrying conductor in a magnetic field and its application, the motor principle. The force acting on a conductor is actually due to the flow of charge through it and, in fact, the force acting on the charge is quite independent of the conductor through which the charge travels.

When a beam of charged particles is fired into a magnetic field, the following properties are observed.

- The beam will not be deflected if the direction of travel of the charges is parallel to the magnetic field.
- Maximum deflection occurs when the beam is aimed *perpendicular* to the direction of the magnetic field.
- The magnetic deflecting force is always perpendicular to *both* the direction of travel of the charge *and* the magnetic field.
- The magnitude of the magnetic deflecting force is directly proportional to the magnitude of the charge on each particle: $F_M \propto q$.
- There is no magnetic force on a stationary charge.
- The magnitude of the force is directly proportional to the speed of the charged particles: $F_M \propto v$.
- The magnitude of the force is directly proportional to the magnetic field intensity: $F_M \propto B$.
- The magnitude of the force depends on the sine of the angle between the direction of motion of the charge and the applied magnetic field: $F_M \propto \sin\theta$.

These proportional relationships can be summarized by one joint proportion statement.

$$F \propto qvB\sin\theta$$
$$F = kqvB\sin\theta$$

The definition of the unit for the magnetic field intensity, B, was chosen to make the value of the constant k equal to unity, so $F = qvB\sin\theta$. If you solve the equation for B, you can see the units that are equivalent to the unit for the magnetic field intensity.

$$B = \frac{F}{kqv\sin\theta}$$

The unit, one tesla (T), was chosen as the strength of the magnetic field. A charge of one coulomb, travelling with a speed of one metre per second perpendicular to the magnetic field ($\theta = 90^\circ$ and $\sin\theta = 1$) experiences a force of one newton. By substituting units into the equation for B and letting $k = 1$ and $\sin 90^\circ = 1$, you can find the equivalent of one tesla.

$$\text{tesla} = \frac{\text{newton}}{\text{coulomb}\frac{\text{metre}}{\text{second}}}$$
$$\text{T} = \frac{\text{N}}{\text{C}\frac{\text{m}}{\text{s}}} = \frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}$$

The direction of the magnetic force on the charge q follows a right-hand rule. If you arrange your right hand so that the fingers are pointing in the direction of the magnetic field, $\vec{B}$, and the thumb is pointing in the direction of motion of a *positively* charged particle, q, then the palm of the hand points in the direction of the magnetic force, $\vec{F}_M$, acting on the particle.

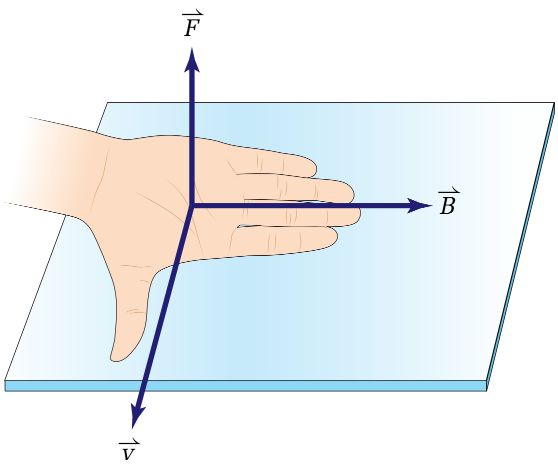


Figure 8.10 The direction of the magnetic force on the charge q follows a right-hand rule.

MATH LINK

You have probably noticed that the equation for the force on a moving charge in a magnetic field is the product of two vectors that yields another vector. In mathematics, this type of equation is called a "vector product" or "cross product" and is written $\vec{F} = q\vec{v} \times \vec{B}$. The magnitude of a vector product is equal to the product of the magnitudes of the two vectors times the sine of the angle between the vectors. The direction of the vector product is perpendicular to the plane defined by the vectors that are multiplied together.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner to enhance your understanding of magnetic fields.

FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

The magnitude of the magnetic force exerted on a moving charge is the product of the magnitudes of the charge, the velocity, the magnetic field intensity, and the sine of the angle between the velocity and magnetic field vectors.

$$F_{\mathrm{M}} = qvB\sin\theta$$

Quantity	Symbol	SI unit
magnetic force on a moving charged particle	F_{M}	N (newtons)
electric charge on the particle	q	C (coulombs)
magnitude of the velocity of the particle (speed)	v	$\frac{\mathrm{m}}{\mathrm{s}}$ (metres per second)
magnetic field intensity	B	T (teslas)
angle between the velocity vector and the magnetic field vector	θ	degree (The sine of an angle is a number and has no units.)

Unit Analysis

$$\text{newton} = \text{coulomb}\left(\frac{\text{metre}}{\text{second}}\right)\text{tesla}$$

$$\mathrm{N} = \mathrm{C}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)\mathrm{T} = \frac{\mathrm{C}\cdot\mathrm{m}\cdot\mathrm{T}}{\mathrm{s}} = \mathrm{N}$$

$\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.

Since one coulomb per second is defined as an ampere ($1\ \frac{\mathrm{C}}{\mathrm{s}} = 1$ A), the tesla is often defined as $T = \frac{\mathrm{N}}{\mathrm{A}\cdot\mathrm{m}}$.

Since the vectors $\vec{F}$, $\vec{v}$, and $\vec{B}$ are never in the same plane, physicists have accepted a convention for drawing magnetic fields. As shown in Figure 8.11, a magnetic field that is perpendicular to the plane of the page is drawn as crosses or dots. The crosses represent a field directed into the page and the dots represent a field coming out of the page.

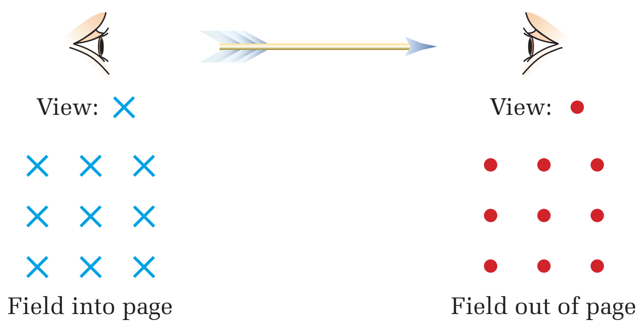


Figure 8.11 To remember the convention for drawing magnetic fields, think of the dot as the point of an arrow coming toward you. Think of the cross as the tail of the arrow going away from you.

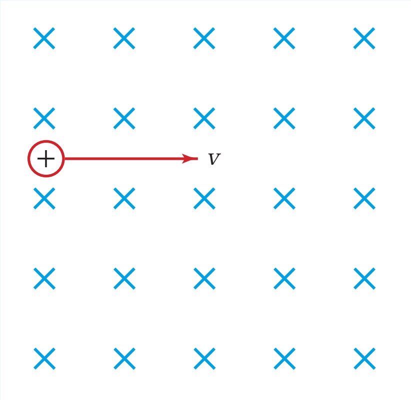
SAMPLE PROBLEM

Force on a Moving Charge

A particle carrying a charge of +2.50 μC enters a magnetic field travelling at 3.40×10^5 m/s to the right of the page. If a uniform magnetic field is pointing directly into the page and has a strength of 0.500 T, what is the magnitude and direction of the force acting on the charge as it just enters the magnetic field?

Conceptualize the Problem

- Make a sketch of the problem.
- The *charged particle* is *moving* through a *magnetic field*; therefore, it experiences a *force*.
- The *force* is always *perpendicular* to both the direction of the *velocity* and of the *magnetic field*.



Identify the Goal

The magnetic force, F_M, on the charged particle

Identify the Variables and Constants

Known	Implied	Unknown
$q = 2.50\,\mu\text{C}$	$\theta = 90^\circ$	F_M
$v = 3.40 \times 10^5\,\frac{\text{m}}{\text{s}}$		
$B = 0.500\,\text{T}$		

Develop a Strategy

Use the equation that relates the force on a charge in a magnetic field to the charge, velocity, and magnetic field intensity. Substitute numerical values and solve.

$$F_M = qvB\sin\theta$$

$$F_M = (2.50 \times 10^{-6}\,\text{C})\left(3.40 \times 10^5\,\frac{\text{m}}{\text{s}}\right)(0.500\,\text{T})(\sin 90^\circ)$$

$$F_M = 0.425\,\text{N}$$

Use the right-hand rule to determine the direction.

- Thumb represents travel of charge to the right.
- Fingers represent direction of magnetic field into the page.
- Palm represents direction of magnetic force on charge toward the top of the page.

The magnetic force on the moving charge is 4.25×10^{-1} N toward the top of the page.

Validate the Solution

A small charge combined with a high speed reasonably would produce the force calculated.

$$\cancel{\text{C}} \cdot \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \cdot \frac{\text{N} \cdot \cancel{\text{s}}}{\cancel{\text{C}} \cdot \cancel{\text{m}}} = \text{N}$$

continued ►

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PRACTICE PROBLEMS

12. An alpha particle, charge $+3.2 \times 10^{-19}$ C, enters a magnetic field of magnitude 0.18 T with a velocity of 2.4×10^6 m/s to the right. If the magnetic field is directed up out of the page, what is the magnitude and direction of the magnetic force on the alpha particle?

13. A proton is projected into a magnetic field of 0.5 T directed into the page. If the proton is travelling at 3.4×10^5 m/s in a direction [up 28° right], what is the magnitude and direction of the magnetic force on the proton?

14. An electron travelling at 6.00×10^5 m/s enters a magnetic field of 0.800 T. If the electron experiences a magnetic force of magnitude 3.84×10^{-14} N, what was the original direction of the electron's velocity relative to the magnetic field?

15. A particle having a mass of 0.200 g has a positive charge of magnitude 4.00×10^{-6} C. If the particle is fired horizontally at 5.0×10^4 m/s[E], what is the magnitude and direction of the magnetic field that will keep the particle moving in a horizontal direction as it passes through the field?

16. A +4.0 μC charge is projected along the positive *x*-axis with a speed of 3.0×10^5 m/s. If the charge experiences a force of 5.0×10^{-3} N in the direction of the negative *y*-axis, what must be the magnitude and direction of the magnetic field?

The magnetic force experienced by a charged particle moving freely through a perpendicular magnetic field can be compared to the force exerted on a current-carrying conductor that also is perpendicular to the magnetic field. The net force on a conductor of length l will be the total of the individual forces acting on each charge.

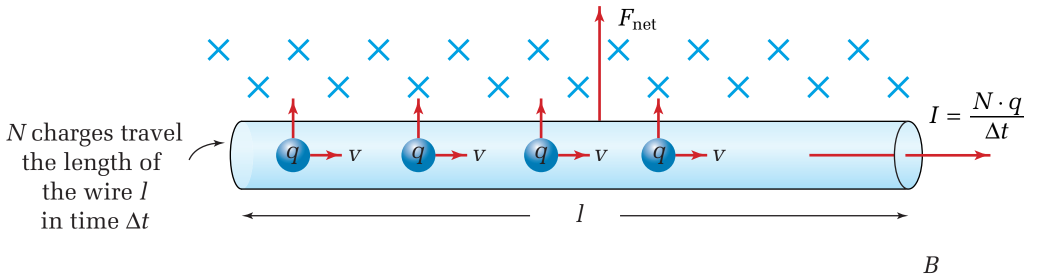


Figure 8.12 When the charges that are moving through a magnetic field are confined to a wire, the magnetic force appears to act on the wire.

If N charges, each of magnitude q, travel the distance equal to the length of the wire l in a time interval Δt, the velocity will be $l/\Delta t$. The net force will be as follows.

$$F_{net} = N \cdot qvB \sin \theta$$

$$F_{net} = N \cdot q \cdot \frac{l}{\Delta t} \cdot B \sin \theta$$

$$F_{net} = \left(\frac{N \cdot q}{\Delta t}\right) \cdot l \cdot B \sin \theta$$

$\frac{N \cdot q}{\Delta t}$ is the total charge per unit time, the current.

FORCE ON A CURRENT-CARRYING CONDUCTOR IN A MAGNETIC FIELD

The magnitude of force on a conductor carrying a current in a magnetic field is the product of the magnetic field intensity, the length of the conductor, the current in the conductor, and the sine of the angle that the electric current makes with the magnetic field vector.

$$F_M = IlB \sin \theta$$

Quantity	Symbol	SI unit
magnetic force on a current-carrying conductor	F_M	N (newtons)
electric current in the conductor	I	A (amperes)
length of the conductor	l	m (metres)
magnetic field intensity	B	T (teslas)
angle between the conductor and the magnetic field vector	θ	degree (The sine of an angle is a number and has no units.)

Unit Analysis

$$\mathrm{T \cdot A \cdot m} = \frac{\mathrm{N \cdot \cancel{s}}}{\mathrm{\cancel{C} \cdot \cancel{m}}} \cdot \frac{\mathrm{\cancel{C}}}{\mathrm{\cancel{s}}} \cdot \mathrm{\cancel{m}} = \mathrm{N}$$

Although many of the quantities in the equation for the magnetic force on a current-carrying wire are vectors, the equation can be used only to determine the magnitude of the force, so the vector notation has not been used. Directions must be determined by the relevant right-hand rules. $\vec{F}_M$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. The right-hand rule for the direction of the force is shown in Figure 8.13.

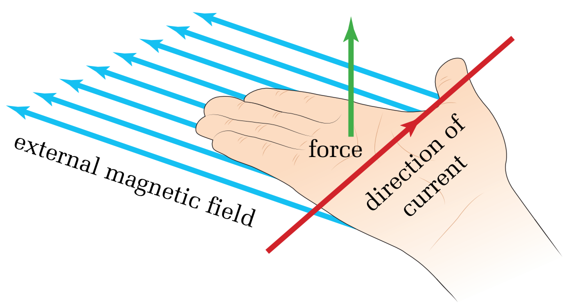


Figure 8.13 The thumb points in the direction of the current, the fingers point in the direction of the magnetic field vector, and the palm of the hand indicates the direction of the force on the conductor.

SAMPLE PROBLEM

Force on a Current-Carrying Conductor

A wire segment of length 40.0 cm, carrying a current of 12.0 A, crosses a magnetic field of 0.75 T[up] at an angle of [up 40° right]. What magnetic force is exerted on the wire?

Conceptualize the Problem

- *Charges* in the wire are *moving* through a *magnetic field*.
- *Moving charges* in a *magnetic field* experience a *force*.
- The *magnetic force* is related directly to the *magnetic field intensity*, the *electric current*, the *length* of the wire segment, and the *angle* between the wire and the magnetic field.

Identify the Goal

The magnetic force, F_M, on the wire segment

Identify the Variables and Constants

Known	Unknown
$l = 40.0\text{ cm}$	F_M
$I = 12.0\text{ A}$	
$B = 0.75\text{ T}$	
$\theta = 40°$ between B and I	

Develop a Strategy

Find the force using the relevant equation that relates force, magnetic field, current, and length of wire that is in the field.

$$F = IlB\sin\theta$$
$$F = (12.0\ A)(0.40\ m)(0.75\text{ T})(\sin 40°)$$
$$F = 2.3140\text{ N}$$
$$F \cong 2.3\text{ N}$$

Determine the direction using the right-hand rule; only the [right] component of the current, perpendicular to the magnetic field direction, contributes to the magnetic force.

- Thumb of right hand points right
- Fingers point up toward top of page

The force will be out of the page, according to the right-hand rule.

- Palm will be facing up out of the page

The force of the magnetic field on the conductor is 2.3 N[out of the page].

Validate the Solution

The force seems to be consistent with the magnetic field and current values. The direction is consistent with the right-hand rule.

$$\text{T}\cdot\text{A}\cdot\text{m} = \frac{\text{N}}{\cancel{\text{A}}\cdot\cancel{\text{m}}}\cdot\cancel{\text{A}}\cdot\cancel{\text{m}} = \text{N}$$

PRACTICE PROBLEMS

17. A wire 82.0 m long runs perpendicular to a magnetic field of strength 0.20 T. If a current of 18 A flows in the wire, what is the magnitude of the force of the magnetic field on the wire?

18. A wire 65 cm long carries a current of 20.0 A, running east through a uniform magnetic field. If the wire experiences a force of 1.2 N[N], what is the magnitude and direction of the magnetic field?

19. A segment of conducting wire runs perpendicular to a magnetic field of 2.2×10^{-2} T. When the wire carries a current of 15 A, it experiences a force of 0.60 N. What is the length of the wire segment?

20. **(a)** What current would need to flow east along the equator through a wire 5.0 m long, which weighs 0.20 N, if the magnetic field of Earth is to hold the wire up against the force of gravity? (Assume that Earth's horizontal magnetic field intensity at this location is 6.2×10^{-5} T.)

(b) Discuss the practicality of this result.

Circular Motion Caused by a Magnetic Field

When a charge enters a magnetic field at right angles, the resulting magnetic force on the particle is perpendicular to both the velocity vector and the magnetic field vector. Consequently, there is no component of the force in the direction of motion and the speed will not change. As the charge is deflected by the force, it still remains perpendicular to the magnetic field. This means that it will always experience a constant magnitude of force *perpendicular* to its motion. This is the standard requirement for circular motion at constant speed. The magnetic force is providing the centripetal force on the particle.

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv^{\cancel{2}}}{q\cancel{v}B}$$

$$r = \frac{mv}{qB}$$

Motion Due to Both Electric and Magnetic Fields

You have now studied ways in which electric and magnetic fields can exert forces on a charged particle. The following are examples in which both types of field affect the motion of a particle.

Simple Particle Accelerator

A simple **particle accelerator** consists of a particle source, a pair of parallel plates, and an accelerating potential difference. The particle source can be simply a spark gap that causes the surrounding gas molecule to become ionized, that is, separate into positive and negative particles. These "ions" then enter the region

COURSE CHALLENGE

Field Energy

A book falls from your desk, a movie plays on a television screen, and a homing pigeon can find its way home, all because of the energy within a field. Refer to page 604 of this textbook for suggestions relating field energy to your Course Challenge.

between the parallel plates and are accelerated by the potential difference between the plates. A hole in the opposite plate allows the particles to continue into the region beyond the plates. For this reason the apparatus is sometimes called a "particle gun," or in the case of electrons, an "electron gun." As a result, the kinetic energy of the emerging particles can be expressed in terms of the work done on them between the parallel plates: $\frac{1}{2}mv^2 = qV$.

Velocity Selector

A velocity selector is a device quite often associated with the parallel plate particle accelerator. A beam of particles having different velocities, as a result of carrying different charges, is "filtered" so that only those particles with the same velocity continue. The apparatus consists of a crossed (perpendicular) electric and magnetic field. A positively charged particle, for example, would experience an upward force due to the magnetic field and a downward force due to the electric field. If the two forces are equal, the particle will travel straight through the velocity selector.

You can determine the velocity of particles that will pass directly through the velocity selector by taking the following steps.

- Set the electric and magnetic forces equal to each other. $F_M = F_Q$
- Write the expressions for the two forces. $F_M = qvB$, $F_Q = qE$
- Substitute the expressions for the values of the forces into the first equation. $qvB = qE$
- Solve for the velocity.

$$v = \frac{qE}{qB}$$

$$v = \frac{E}{B}$$

Only charged particles with a velocity that matches the ratio of the electric field intensity to the magnetic field intensity will continue to travel in a straight line. Particles with other speeds will be deflected up or down and absorbed by the surrounding material.

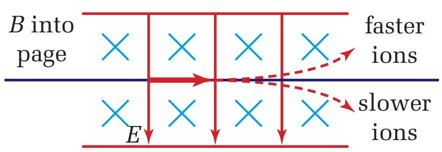


Figure 8.14 Only charges having one specific velocity will travel in a straight line. All others will be diverted up or down.

Mass Spectrometer

The **mass spectrometer** is an instrument that can separate particles of different mass and, in fact, measure that mass. The first stage of a mass spectrometer is a velocity selector. Then, ions of the selected speed enter a magnetic field in a direction perpendicular to the field. While in the magnetic field, the ions experience a magnetic force that is always perpendicular to the direction of their motion.

You will recognize this type of force as a centripetal force. You can see how the mass spectrometer separates particles of different masses by analyzing the following steps.

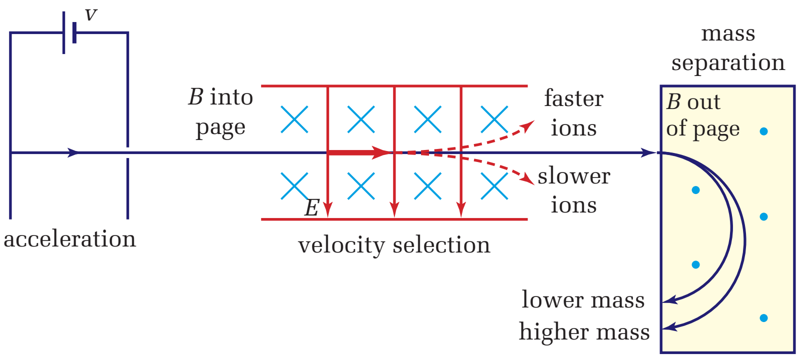


Figure 8.15 The path of positive ions through the acceleration, velocity selection, and mass separation process

- The magnetic field supplies the centripetal force. $F_C = F_M$
- Substitute the expressions for centripetal and magnetic forces. $\frac{mv^2}{r} = qvB$
- Solve for *m*. $m = \frac{rq\cancel{v}B}{v^{\cancel{2}}}$

$$m = \frac{rqB}{v}$$

The velocity of the particles is known because it was selected before the particles entered the magnetic field. The charge is known due to the method of creating ions that entered the velocity selector. The instrument measures the radius of the circular path. The only unknown quantity is the mass.

By observing the radius for particles of known charge, the mass can be determined. This is particularly useful for determining the relative proportions of "isotopes," atoms that have the same number of protons but different numbers of neutrons.

PHYSICS FILE

Portable mass spectrometers are used at airports and in other areas where security is a priority, in an attempt to detect particles associated with materials used in manufacturing explosives.

SAMPLE PROBLEM

Mass Spectrometer

A positive ion, having a charge of 3.20×10^{-19} C, enters at the extreme left of the parallel plate assembly associated with the velocity selector and mass spectrometer shown in Figure 8.15.

(a) If the potential difference across the simple accelerator is 1.20×10^3 V, what is the kinetic energy of the particle as it leaves through the hole in the right plate?

continued ▶

continued from previous page

(b) The parallel plates of the velocity selector are separated by 12.0 mm and have an electric potential difference across them of 360.0 V. If a magnetic field of strength 0.100 T is applied at right angles to the electric field, what is the speed of the particles that will be "selected" to pass on to the mass spectrometer?

(c) When these particles then enter the mass spectrometer, which shares a magnetic field with the velocity selector, the radius of the resulting circular path followed by the particles is 6.26 cm. What is the mass of the charged particles?

(d) What is the nature of the particles?

Conceptualize the Problem

- When the *charged* particles enter the *electric field*, the field does *work* on the particles, giving them *kinetic energy*.
- When the *moving* particles pass through the crossed *electric* and *magnetic* fields, only those of *one specific velocity* pass through undeflected.
- When the selected particles enter the *magnetic field*, the magnetic force provides a *centripetal force*.

Identify the Goal

(a) The kinetic energy, E_k, of the particle

(b) The speed, v, of the particles that will be "selected"

(c) The mass, m, of the charged particles

(d) The nature of the particles

Identify the Variables and Constants

Known		Unknown
$q = 3.20 \times 10^{-19}\,\text{C}$	$\Delta d = 12.0\ \text{mm}$	E_k
$V_1 = 1.20 \times 10^3\,\text{V}$	$B = 0.100\ \text{T}$	v
$V_S = 360.0\,\text{V}$	$r = 6.26\ \text{cm}$	m

Develop a Strategy

The energy of a charged particle is related to the accelerating potential difference.

$$E_k = qV$$

$$E_k = (3.20 \times 10^{-19}\ \text{C})(1.2 \times 10^3\ \text{V})$$

$$E_k = 3.84 \times 10^{-16}\ \text{J}$$

(a) The kinetic energy of the particle was 3.84×10^{-16} J.

The selected velocity is related to the electric and magnetic fields. The electric field is related to the potential difference and the distance of separation of the plates.

$$E = \frac{V}{\Delta d} = \frac{360.0\ \text{V}}{1.20 \times 10^{-2}\ \text{m}} = 3.00 \times 10^4\ \frac{\text{N}}{\text{C}}$$

$$v = \frac{E}{B} = \frac{3.00 \times 10^4\ \frac{\text{N}}{\text{C}}}{0.100\ \text{T}}$$

$$v = 3.00 \times 10^5\ \text{m/s}$$

(b) The speed of the particles was 3.00×10^5 m/s.

The mass of the particle is related to the charge, magnetic field, radius of path, and speed.

$$m = \frac{qBr}{v}$$

$$m = \frac{(3.20 \times 10^{-19}\ \text{C})(0.100\ \text{T})(6.26 \times 10^{-2}\ \text{m})}{3.00 \times 10^5 \frac{\text{m}}{\text{s}}}$$

$$m = 6.68 \times 10^{-27}\ \text{kg}$$

(c) The mass of the particles was 6.68×10^{-27} kg.

The charge is two times the charge on a proton. The mass is four times the mass of a proton. The particle seems to be an alpha particle, which is the positive nucleus of a helium atom.

(d) The particles seem to be alpha particles.

Validate the Solution.

The mass is what you would expect for a small atom. The units cancel to give kg, which is correct.

$$\frac{\text{C} \cdot \text{T} \cdot \text{m}}{\frac{\text{m}}{\text{s}}} = \cancel{\text{C}} \cdot \frac{\text{N} \cdot \text{s}}{\cancel{\text{C}} \cdot \cancel{\text{m}}} \cdot \cancel{\text{m}} \cdot \frac{\text{s}}{\text{m}} = \frac{\text{N} \cdot \text{s}^2}{\text{m}} = \frac{\text{kg} \cdot \cancel{\text{m}}}{\cancel{\text{s}^2}} \cdot \frac{\cancel{\text{s}^2}}{\cancel{\text{m}}} = \text{kg}$$

PRACTICE PROBLEMS

21. A proton is accelerated across parallel plates, through a potential difference of 180.0 V. Calculate

(a) the final kinetic energy of the proton

(b) the final velocity of the proton, assuming its mass is 1.67×10^{-27} kg

22. A particle of mass 1.2 g and charge +3.0 μC is held suspended against the force of gravity between a parallel pair of plates that are 15.0 mm apart.

(a) In which direction does the electric field vector point?

(b) What is the magnitude of the electric potential difference connected across the plates?

23. An isotope of hydrogen having a proton and a neutron in its nucleus is ionized and the resulting positive ion (deuteron) travels in a circular path of radius 36.0 cm in a perpendicular magnetic field of strength 0.80 T.

(a) Calculate the speed of the deuteron.

(b) What was the accelerating potential that gave the deuteron this speed?

24. An electron of mass 9.11×10^{-31} kg travels perpendicularly through a magnetic field of strength 6.8×10^{-5} T at a speed of 3.4×10^5 m/s. What is the radius of the path of the electron?

25. What is the speed of a beam of electrons if in passing through a 0.80 T magnetic field they remain undeflected, due to a balancing electric field of 5.4×10^3 N/C?

26. An isotope of hydrogen passes, without deflection, through a velocity selector that has an electric field of 2.40×10^5 N/C and a magnetic field of 0.400 T. It then enters a mass spectrometer that has an applied magnetic field of 0.494 T and consequently describes a circular path with a radius of 3.80 cm.

(a) What is the mass of the particle?

(b) Which isotope of hydrogen is it?

INVESTIGATION 8-B

Measuring a Magnetic Field

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting
- Communicating results

In this investigation, you will use a current balance to determine the strength of the magnetic field at the central axis of a solenoid.

Problem

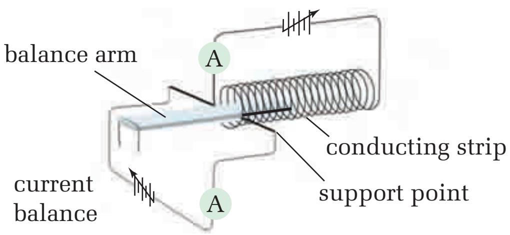
How can you measure magnetic field intensity with a current balance?

Equipment

- current balance and solenoid
- 2 variable power supplies (12 V DC)
- 2 DC ammeters
- electronic balance
- scissors
- string

Procedure

1. Set up the current balance-solenoid apparatus, as shown in the diagram.



2. With the power off, adjust the balance arm so that it is horizontal.
3. Turn on the power to the coil and balance arm. Adjust the polarity so that the conducting arm inside the solenoid is forced downward.

CAUTION The current in both the arm and solenoid can create enough heat to cause a burn.

4. Set and record the current in the solenoid to the upper range of its values. Set and record the current through the balance arm to the high end of its range, forcing down the balance arm inside the solenoid.
5. Loop a length of string over the outside end of the balance arm and, using scissors, adjust its length until the balance arm is horizontal.
6. Without changing any settings, turn off the current to both sources. Determine the mass of the string.
7. Keeping the solenoid current constant, repeat the experiment five more times, using a smaller balance current. Record the value of the balance current and the mass of the string each time.
8. Carefully measure
   - **(a)** the lengths of the solenoid and the current arm
   - **(b)** the number of turns in the solenoid
   - **(c)** the distance of the suspension point of the current balance to each of its ends (lever arms)

Analyze and Conclude

1. For each trial, use the mass of the string and the principle of levers to calculate the force acting down on the current arm. Record your data.
2. Draw a graph with the force acting on the current arm versus current in the current arm.
3. Describe the relationship between I and F when the magnetic field is kept constant?
4. Measure the slope of your graph. Use your data to determine the magnetic field, B, inside the solenoid.

Apply and Extend

5. Using your data and the equation below, calculate the strength of the magnetic field.

$$B = \mu_0 \frac{N \cdot I_S}{l}$$

$\mu_0 = 1.257 \times 10^{-6}\ \mathrm{T \cdot m/A}$, N is the number of turns in the solenoid, l is the length of the solenoid, and I_S the current flowing in the solenoid wire.

6. How did your two values for the magnetic field in the solenoid compare? What might cause them to differ?

Particle Accelerators

In the early part of the twentieth century, the development of the theory of the structure of the atom and its nucleus depended to a large degree either on the spontaneous disintegration of radioactive nuclei or on observations made when the products of those spontaneous disintegrations were directed at other nuclei. The particles emitted during natural disintegrations, however, such as the α-particles used by Rutherford in his experiments, provided only limited opportunity to observe nuclear reactions during bombardment. The particles were limited in energy and were emitted randomly in all directions, so they were difficult to harness in sufficient quantities to provide reliable results.

To overcome the difficulties of availability and reliability, particle accelerators were developed that were capable of emitting high-speed, subatomic-sized particles (protons, electrons) in sufficient numbers. Particle accelerators today are capable of accelerating charged particles to energies close to one million million electron volts, or 1000 GeV. This in turn has allowed physicists to investigate the fundamental composition of matter even more deeply, with the result that more and more fundamental particles are known to exist and complex models of the structure of matter have been developed. You will learn more about these models in Unit 5.

The Cockcroft-Walton Proton Accelerator

The first particle accelerator for use in nuclear research was built in 1932 by J.D. Cockcroft and E.T.S. Walton, students of Ernest Rutherford at the Cambridge Laboratory in England. In this accelerator, protons were introduced into the top of an evacuated glass tube and accelerated by using a potential difference between electrically charged metal cylinders. Since it is not possible to maintain a potential difference much more than 200 000 V between electrodes in an evacuated tube, Cockcroft and Walton used special multi-stage accelerator tubes, with each stage powered by a unique charging circuit. The protons accelerated by this arrangement approached energies of 1 MeV.

At the bottom of the glass tube, they placed a lithium target and consequently observed the first nuclear transformation caused by artificially accelerated particles. The bombardment of the lithium atoms with protons resulted in the formation of helium nuclei. For their work, Cockcroft and Walton were awarded the Nobel Prize in Physics in 1951.

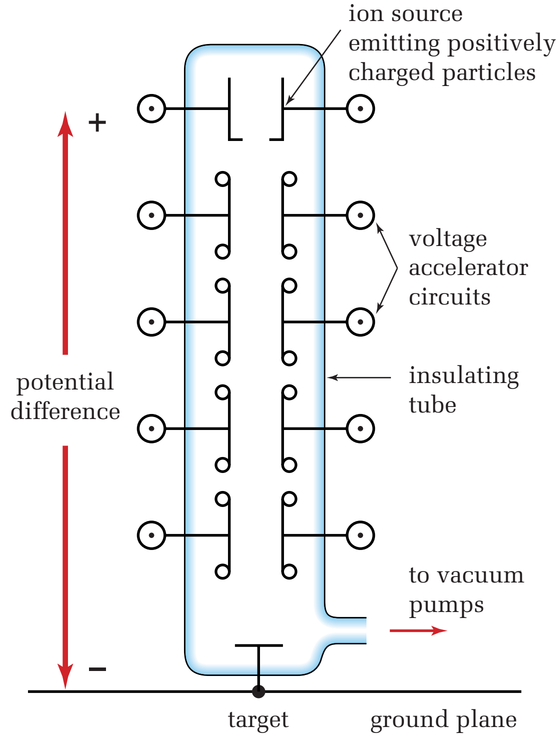


Figure 8.16 One type of multi-stage accelerator tube

The Cyclotron

To avoid the problems associated with very high voltages, Ernest O. Lawrence and his colleagues at the University of California at Berkeley designed an accelerator based on a circular path that subjected the charged particles to a large number of small increases in potential. This was achieved by the use of a pair of evacuated hollow semicircular chambers (called "dees," because they are shaped like the letter D). The charged particles are injected into the chambers at the centre. This device is called a **cyclotron**.

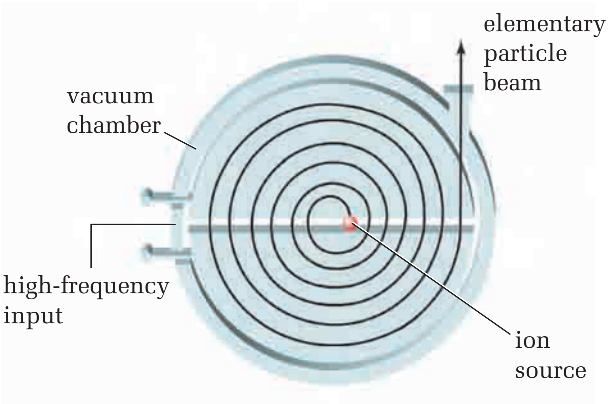


Figure 8.17 A cyclotron

The dees are positioned between the poles of an electromagnet that provides a uniform magnetic field perpendicular to the path of the charged particle inside the chamber, thus causing its circular motion. A potential difference is applied between the two chambers, so that as the charged particle crosses from one chamber to the next, it will be accelerated by the potential difference. The particle will speed up and, as a result, the radius of its path will increase. In order for the particle to speed up when it crosses the gap between the dees again, the direction of the potential difference must be reversed. This alternating potential difference is kept in phase with the frequency of orbit of the charged particle so that it will always speed up when it crosses the gap between the chambers. Consequently the particle will spiral outward until it reaches the outer edge of the dee, where a magnetic field is applied to deflect the particle out through a gate and onto a target. The first cyclotron built in 1931 produced ions of energy 80 keV, but by the latter part of that decade, energies of 30 MeV were quite common.

As you will learn in Unit 5, when particles reach speeds close to the speed of light, relativistic effects become prominent. In the case of the cyclotron, the mass of the particle increases to such an extent that it becomes necessary to synchronize the alternating potential difference with the time of travel of the particle.

The Synchrocyclotron

In the **synchrocyclotron**, an adaptation of the cyclotron, the frequency of the accelerating electric field, applied between the dees, is adjusted to allow for the relativistic increase in mass of the particles. Since the change in frequency required takes approximately 10 ms, the ions are delivered in small bursts, rather than continuously. This results in the intensity of the ion beam being lower than the conventional cyclotron. This is compensated for by using larger magnets, although cost then becomes a limiting factor.

The Betatron

The principle of the cyclotron has been adapted to allow for the acceleration of electrons. Since electrons were historically called "beta particles," the accelerator is called a **betatron**. Instead of allowing the electrons to spiral outward, a magnetic field applied along the central axis of an evacuated doughnut is uniformly increased. This increasing magnetic field induces an electric field that causes the electron to speed up but retain the same radius, inside the doughnut.

The Linear Accelerator (LINAC)

New **linear accelerators** differ from earlier machines, such as the Cockcroft-Walton accelerator, in that they use electric fields alternating at radio frequencies to accelerate the particles, rather than high voltages.

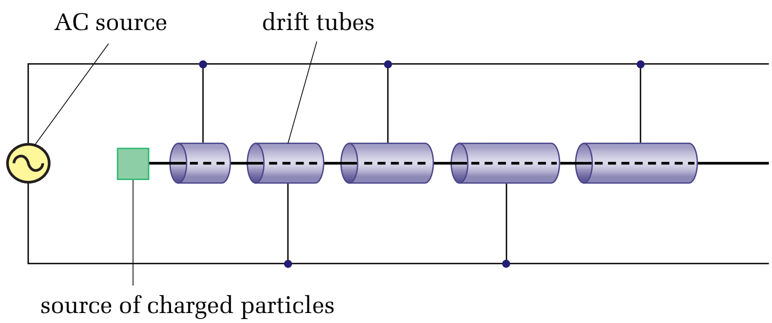


Figure 8.18 Schematic of a linear accelerator

The acceleration tube consists of many individual drift tubes that are charged alternately positive and negative. When a positive particle enters the tube, if the first drift tube is negative, it will attract the particle. Inside the tube, there is no electric field, so the particle "drifts" through at constant speed. If the electric field is reversed as the particle leaves the first tube, it will accelerate toward the second drift tube and enter it at a higher speed. This second tube is longer and the particle will leave it just as the potential reverses and it will be attracted to the third drift tube. Hence, the particle is accelerated between a long series of drift tubes. The Stanford Linear Accelerator Centre linear accelerator is

3.2 km long, contains 240 drift tubes, and is designed to accelerate electrons to energies above 20 GeV.

Synchrotron

A very efficient way to accelerate protons is to combine the features of the cyclotron and the linear accelerator. Such a device is the **synchrotron**.

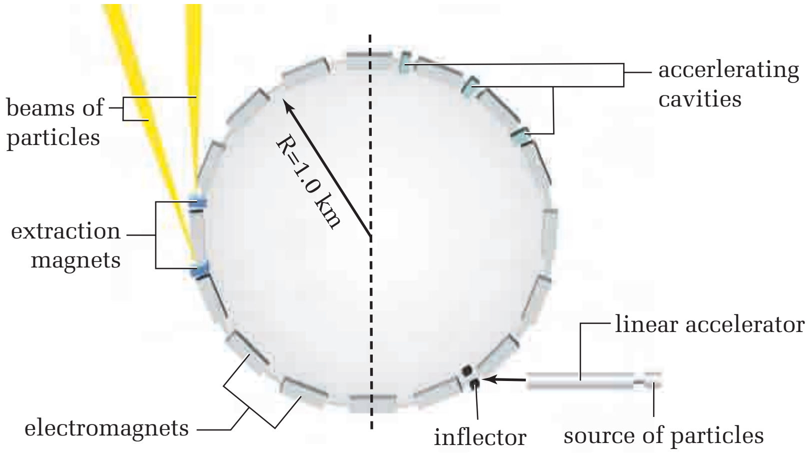


Figure 8.19 A synchrotron

Since a magnetic field is required only to maintain the circular orbit, rather than use one large central magnet, a series of ring magnets surrounding a doughnut-shaped vacuum tank is used, making the synchrotron much more economical. At repeated locations along the circular path, high-frequency accelerating cavities (much like short linear accelerators) are inserted to accelerate the protons. This combined technique produces protons of enormous energy that can in turn be directed at other targets and the resulting fundamental particles can be investigated. In 1954, Lawrence, the designer of the cyclotron, developed a synchrotron that produced protons with energies in the range of 6.2 billion electron volts. It was therefore called the "bevatron." (Today, it is identified as 6.2 GeV.) These protons were in turn used to discover the antiproton.

Other renowned synchrotron installations include the 1.0 TeV Tevatron at Fermilab (the Fermi National Accelerator Laboratory) in Illinois and the 400 GeV at CERN (European Council for Nuclear Research) near Geneva, Switzerland.

The Tokamak Fusion Test Reactor

An international group, International Thermonuclear Experimental Reactor (ITER), which includes Canada, is attempting to develop efficient nuclear fusion reactors, in which two isotopes of hydrogen (deuterium and tritium) collide with such high energy that they "fuse" to produce a helium nucleus, and at the same time release enormous amounts of energy. Fusion can occur only at temperatures equivalent to the centre of stars, about 10^8 °C. At these temperatures, the fusion reactants actually break down into individual positive nuclei and negative electrons. This ionized gas is called a "plasma." It is because these ions are charged that it has been found both possible, and necessary, to confine them within a toroidal (doughnut-shaped) magnetic bottle, since no material bottle can exist at such high temperatures for its containment.

WEB LINK

www.mcgrawhill.ca/links/physics12

For an award-winning photograph taken inside a Tokamak reactor, go to the above Internet site and click on **Web Links**.

Figure 8.20 The Tokamak fusion test reactor

This magnetic confinement seems to have the greatest potential and its popular design is based on the Tokamak system, developed in the former U.S.S.R. ("Tokamak" is an acronym for the Russian translation of "toroidal magnetic chamber.")

8.3 Section Review

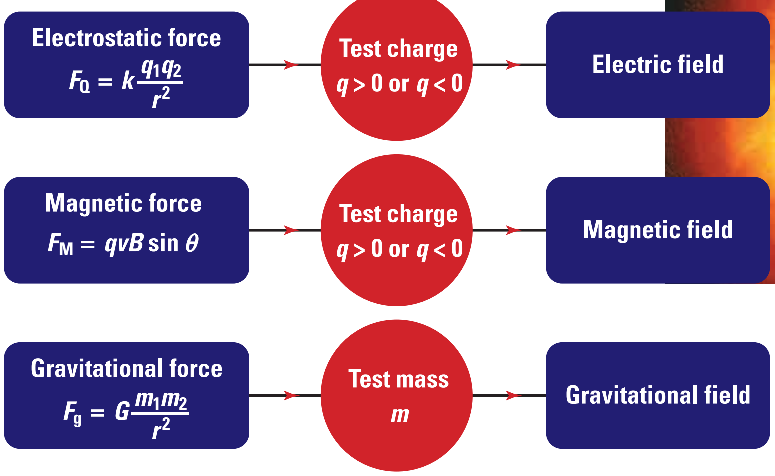
1. **K/U**
   - **(a)** Under what conditions will a charged particle be subject to the maximum possible deflecting force when entering a magnetic field?
   - **(b)** Under what conditions would the deflection be minimal?
2. **K/U** In what way is the force acting on a conductor carrying a current in a magnetic field similar to the deflecting forces described in question 1?
3. **C** Prepare a report or other presentation describing the many applications of the deflection of a charge by a magnetic field. Give a detailed account of the social significance of one of these applications.
4. **K/U** Explain how a particle accelerator and velocity selector complement the operation of a mass spectrometer.
5. **MC** In 2001, Canada and Japan were competing for the right to build a Tokamak-style fusion reactor. Canada's plan is to locate the reactor in Clarington, Ontario, adjacent to the Darlington nuclear plant. Research Canada's bid and make a presentation in which you
   - **(a)** outline the reasons Canada's ITER team had for wanting to build the reactor
   - **(b)** explain why that particular location was chosen
   - **(c)** give your own opinions on the merit of the plan

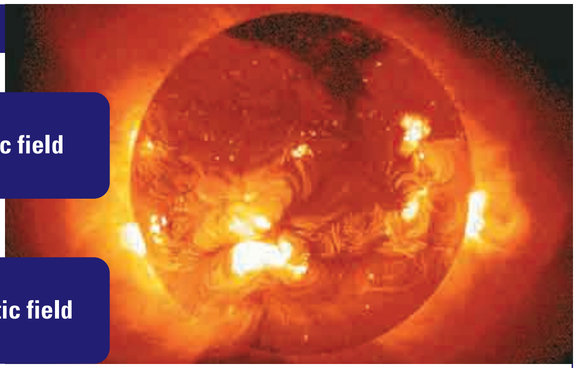
CHAPTER 8 Review

REFLECTING ON CHAPTER 8

- The electric field pattern for a collection of charges can be generated by considering the field vectors due to each individual charge.
- Electric field lines leave a positive charge and/or enter a negative charge and are always perpendicular to the surface of a conductor.
- The number of field lines is proportional to the magnitude of the net charge.
- Equipotential surfaces are always perpendicular to the electric field lines
- The electric field is uniform between two oppositely charged parallel plates placed close together.
- The magnitude of the electric field between parallel plates is proportional to the charge density on the plates.
- The magnitude of the electric field intensity between parallel plates is given by the equation $\vec{E} = \frac{\Delta V}{\Delta d}$.
- The potential gradient describes the linear change in electric potential difference at positions between the plates.
- Electric fields can be used to transfer kinetic energy to charged particles, so that $qV = \frac{1}{2}mv^2$.
- An electric field can also be used to balance the force of gravity on a charged particle.
- The electron volt is an alternative unit for the energy of a charged particle.
- The charge on the electron was determined by Robert Millikan by measuring the terminal velocities of oil drops placed between a parallel plate apparatus.
- The drift velocity of the electrons in a conductor carrying a current is very slow, compared with the speed of the current itself.
- The Faraday ice-pail experiment demonstrated that the net charge inside a hollow conductor is zero; all charge resides on the outer surface.
- Faraday shielding is a useful way of preventing external electromagnetic interference in circuit components.
- The magnetic force on a charged particle travelling in a magnetic field is $F_M = qvB \sin\theta$ and its direction is described by a right-hand rule.

Concept Organizer





Solar flares are a result of the build up and then release of magnetic energy. Electrons, protons and nuclei are accelerated into the solar atmosphere. An amount of energy equivalent to millions of 100 Mt bombs is released.

- The magnetic force on a conductor carrying a current in a magnetic field is $F_M = IlB \sin \theta$ and its direction is described by a right-hand rule.
- A magnetic field can be used to cause the circular motion of a charged particle, so that $qvB = \frac{mv^2}{r}$.
- The velocity of a charged particle can be determined by electric and magnetic fields.
- The motion of a charged particle under the action of electric and/or magnetic fields forms the basis for applications such as cyclotrons, synchrocyclotrons, and mass spectrometers, among others.
- The containment of a plasma in a Tokamak fusion reactor is achieved through magnetic fields.

Knowledge/Understanding

1. Answer the following questions about Millikan's oil-drop experiment.
   (a) Describe the main features of the experiment.
   (b) What were the results of the experiment and their significance?
   (c) Draw free-body diagrams of an oil drop that is between two horizontal, parallel electrically charged plates under three conditions: the oil drop is stationary, the oil drop is falling toward the bottom plate, the oil drop is drifting upward.
   (d) What was the effect of Millikan's use of X rays in his experiment?
   (e) Explain why the plates in the experiment need to be horizontal.
2. Imagine that you are probing the field around a charge of unknown magnitude and sign. At a distance r from the unknown charge, you place a test charge of q_1. You then substitute q_1 with a second test charge, q_2, that has twice the charge of $q_1(q_2 = 2q_1)$.
   (a) Compare the forces that would act on the two test charges.
   (b) Compare the electric field that would affect the two test charges.
3. State mathematically and describe in words the definition of a tesla.
4. Can the magnetic force change the energy of a moving charged particle? That is, can the magnetic force do work on the particle?
5. (a) What is the function of the alternating potential difference in a cyclotron? The potential difference is applied across which two parts of the cyclotron? Why does the potential difference have to alternate in polarity?
   (b) What is the function of the magnetic field in a cyclotron? Is the magnetic field constant or alternating in direction?
6. Mass spectrometers are used to determine the masses of positively charged atoms and molecules.
   (a) Draw a concept map of the physics principles on which mass spectrometers were developed.
   (b) Explain the function of a velocity selector when it is used in conjunction with a mass spectrometer.
7. What types of studies were conducted to probe atomic structure prior to the development of particle accelerators? What were the limitations of such studies for developing an understanding of the micro-world?
8. Contrast the designs of particle accelerators that accelerate particles linearly and those that accelerate particles in circular paths. What are the advantages and disadvantages of each design?

Inquiry

9. Research and make a model of one type of particle accelerator that is being used currently. Your model should include all critical components and show their relationship with each other. Write a report to describe the physics involved in the accelerator's operation.

10. Suppose a cyclotron that normally accelerates protons is now to be used with alpha particles. What changes will have to be made to maintain synchronism?
11. Write a proposal for a new experimental facility to study the structure of the atom. Evaluate different particle accelerators and make a case for why you want to use a particular design. Include a cost analysis in your proposal. What are the most expensive components?

Communication

12. Use the rules of electric field line formation to explain why the lines around a negatively charged sphere are uniformly spaced and directed radially inward.
13. Outline, using vector diagrams, why the electric field at any point between two parallel plates is uniform and independent of the distance between the plates.
14. Consider a large, positively charged sphere. Two positively charged objects, A and B, are the same distance away from the sphere. Object A has a charge three times as large as that of object B. Which property will be the same for the two objects, the electric potential energy or the electric potential difference? Which property will be three times as large for object A compared to object B, the electric potential energy or the electric potential difference?
15. A proton passes through a magnetic field without being deflected. What can be said about the direction of the magnetic field in the region? Draw a sketch to illustrate your reasoning.
16. An electron is moving vertically upward when it encounters a magnetic field directed to the west. In what direction is the force on the electron?
17. Consider two parallel current-carrying wires. If the currents are in the same direction, will the force between the wires be attractive or repulsive? If the currents are in opposite directions, will the force between the wires be attractive or repulsive? Draw sketches to illustrate your answers.
18. A simple particle accelerator consists of three components. Make a sketch that identifies each component and its function. Why must ions be used instead of neutral particles?

Making Connections

19. A television uses a cathode ray tube to direct a beam of electrons toward a screen.
    - (a) Draw a schematic diagram of a television picture tube as seen from the side and explain how electric and magnetic fields are used to accelerate and deflect the electrons.
    - (b) Although electrons do not orbit in the magnetic field of a television cathode ray tube, their trajectory does follow a definable circular arc. On your diagram, label where this circular arc is located and explain how the radius of the arc can be used to determine the size of the picture tube.
20. The origins of naturally occurring magnetic fields are still poorly understood. Outline theories that explain the origin of Earth's magnetic field, the Sun's magnetic field, and the magnetic field of the Milky Way galaxy. Explain how these theories can be tested.
21. Although the cause of Earth's magnetic field is uncertain, it is known to be unstable. Analysis of rock strata in Earth's crust suggests Earth's magnetic field has reversed itself several times over the past five million years. How is this analysis done? What is the current thinking on why this occurs?

Problems for Understanding

22. The electric field intensity between two large, charged parallel plates is 400 N/C. If the plates are 5.0 cm apart, what is the electric potential difference between them?
23. Two parallel charged metal plates are separated by 8.0 cm. Identify four points along a line between the plates, A, B, C, and D, located at the following distances from the negatively charged plate: 0.0 cm, 2.0 cm, 4.0 cm, and 6.0 cm. The electric potential difference at point B, V_B, is measured to be 40.0 V.

(a) What is the electric potential difference across the plates?
(b) What is the electric potential difference at points A, C, and D?
(c) What is the potential difference between points A and B, B and C, and A and D?
(d) What is the electric field strength between the plates?
(e) A 1.0 μC test charge is placed first at point B, then at point C. What force does it experience at each point?
(f) Repeat (e) above for a 2.0 μC test charge.

24. In a Millikan oil-drop experiment, an oil drop of unknown charge is suspended motionless when the electric field is 3500 N/C. If the upper plate is positive and the drop weighs 2.8×10^{-15} N, determine **(a)** the charge on the oil drop and **(b)** the number of excess or deficit electrons on the oil drop.

25. A pith ball has a charge of –5.0 nC. How many excess electrons are on the pith ball?

26. A 10.5 cm wire carries a current of 5.0 A. What is the magnitude of the magnetic force acting on the wire if the wire is perpendicular to a uniform magnetic field of 1.2 T?

27. A small body moving perpendicular to a magnetic field of 0.25 T carries a charge of 6.5 μC. If it experiences a sideways force of 0.52 N, how fast is it travelling?

28. Consider a horizontal, straight 2.0 m wire carrying a 22 A current that runs from west to east. If the wire is in Earth's magnetic field, which points north with a magnitude of 4.0×10^{-5} T, calculate
(a) the magnetic force on the wire
(b) the maximum mass of the wire that would be supported by Earth's magnetic field

29. A velocity selector consists of an electric field of 20 000 V/m (2.0×10^4 V/m) perpendicular to a magnetic field of magnitude 0.040 T. A beam of ions, having passed through a velocity selector, is passed into a mass spectrometer that has the same magnetic field. Under these conditions, the radii of curvature of the path of singly charged lithium ions is found to be 78 cm. Calculate the mass of the lithium ions.

30. The period of a charged particle's circular orbit in a uniform magnetic field can be calculated from the radius of its orbit and its tangential velocity. Interestingly, both the period and its inverse, the frequency, are independent of the particle's speed and the radius of its orbit. Consider two electrons moving perpendicular to a 0.40 T magnetic field. One has a speed of 1.0×10^7 m/s and the other has a speed of 2.0×10^7 m/s.
(a) Calculate the radii of the orbits of the two electrons.
(b) Calculate their periods.
(c) Calculate their frequencies.
(d) Comment on the above results.

31. Suppose an electron and a proton are each injected perpendicularly into a uniform magnetic field with equal kinetic energies.
(a) Compare the periods of their orbits.
(b) Compare the radii of their orbits.

32. Charged particles from the Sun can be trapped by the magnetic field that surrounds Earth. If the particles enter the atmosphere, they can excite atoms in the air, resulting in the phenomenon of auroras. Consider a proton with a speed of 1.2×10^7 m/s that approaches Earth perpendicular to Earth's magnetic field. It is trapped and spirals down a magnetic field line.
(a) If the magnetic field strength at the altitude where the proton is captured is 2.0×10^{-5} T, calculate the frequency and radius of curvature of the proton's orbital motion.
(b) Repeat (a) for a proton that comes in at half the speed of the first proton.

33. An electron moves with a velocity of 5.0×10^6 m/s in a horizontal plane perpendicular to a horizontal magnetic field. It experiences a magnetic force that just balances the gravitational force on the electron.
(a) Calculate the strength of the magnetic field.
(b) If the electron is travelling north, what is the magnetic field direction?

UNIT 3 ISSUE ANALYSIS

Costs and Benefits of Physics Research

Background

Throughout history, societies have expended tremendous amounts of money and other resources on the accumulation of scientific knowledge. Often, at the time of expenditure, the direct value in monetary or other terms was not readily evident, so the debate always arises as to whether the costs of scientific research and related high-tech applications outweigh their benefits.

This unit contained an overview of a number of different particle accelerators. In some cases, the device, such as a mass spectrometer, is used to identify the elements contained in a substance. In other cases, such as the Conseil Européen pour la Recherche Nucléaire and Fermi National Accelerator Laboratory accelerators, the device accelerates charged particles to a speed at which not only do their own properties change, but their collisions with other particles create the formation of yet new and different particles. The high-energy collisions made possible by particle accelerators lead to new understandings of the structure of matter.

History has shown that the more society learns about the structure and behaviour of matter, the more this knowledge can be used to improve society's standard of living. On the other hand, the costs of such endeavours are not all monetary. Often related to research and development are side effects that affect the environment and the health and freedoms of a society.

Challenge

Build a class consensus on the costs and benefits of continuing public support for using particle accelerators in research and development in particular, and for physics research in general.

The U.S. Department of Energy's Fermi National Accelerator Laboratory.

Plan and Present

A. As a class, research and compile a list of particle accelerators that are currently in use, either for pure scientific research or for a particular technological application. Identify a select list of accelerators for further study. Divide the class into groups, assigning one accelerator to each group. Each group is to write a report on the accelerator's function, its associated costs, and its potential benefits. While developing this report, in preparation for the class debate described below, class members should decide on which side of the debate they want to participate.

B. Set up a class debate on the costs and benefits to society of the public funding of research using particle accelerators in particular, and on physics research in general.

ASSESSMENT

After you complete this project

- **assess the clarity of your report in explaining the costs and benefits of particle accelerators for research and development**
- **assess the success of your team in convincing the audience of your perspective during the debate**
- **assess the ability of the class to come to a consensus on a rational position on the costs and benefits of physics research**

Action Plan

1. Establish an evaluation method by preparing
   - a class rubric for evaluating individual group reports
   - a rubric for evaluating the class debate
2. Establish groups and then
   - as a class, brainstorm and conduct preliminary research into the types of particle accelerators currently in use
   - establish small working groups to investigate a representative number of particle accelerators
3. For the assigned accelerator, each small group will gather data on
   - the location and size of the accelerator
   - the physics principles about which the accelerator is designed to further knowledge
   - the monetary cost of building and operating the accelerator
   - the source of its funding
   - the type of particle accelerated
   - the final energy of the particle
   - the type of research that can be accomplished only by using these high-energy particles
   - any monetary return (profit) from applications of the accelerator
   - possible future (direct or indirect) benefits derived from the knowledge gained as a result of the research made possible by use of the accelerator
   - the environmental and societal impact of the use of the accelerator or of the knowledge gained
4. Prepare a report that summarizes the information gathered by the group.
5. Delegate responsibilities for publishing the report.
6. Prepare for the debate by
   - setting up two class teams that will debate on the costs/benefits of physics research; the debating teams will include both debaters and technical advisers from each of the small working groups
   - selecting a neutral person to act as moderator

   Each debating team will
   - analyze the small groups' reports to determine the costs and benefits of accelerators
   - assign roles for the debate (e.g., organizing and compiling material, preparing notes, developing arguments, serving as debaters)
   - rehearse the debate
7. Publish and present the small group reports.
8. Conduct a class debate.

Evaluate

1. Small group publications and class debate: Use the rubric prepared in step 1 of the Action Plan to evaluate the publications and class debate.
2. After the debate, through class discussion, attempt to establish a class position on the costs and benefits of the public funding of particle accelerators, and of science funding in general.

UNIT 3 Review

Knowledge/Understanding

Multiple Choice

In your notebook, choose the most correct answer for each of the following questions. Outline your reasons for your choice.

1. Two parallel oppositely charged metal plates have an electric field between them. The magnitude is
   (a) greatest near the positive plate
   (b) greatest near the negative plate
   (c) zero
   (d) uniform throughout the region
2. The magnitude of the electric field at a point in space is equal to the
   (a) force a charge of 1 C would experience there
   (b) force a negative charge would experience there
   (c) force a positive charge would experience there
   (d) potential difference there
   (e) electric charge there
3. The force on a proton in an electric field of 100 N/C (1.0×10^2 N/C) is
   (a) 1.6×10^{-17} N
   (b) 1.6×10^{-19} N
   (c) 1.6×10^{-21} N
   (d) 6.2×10^{20} N
4. Magnetic fields do not interact with
   (a) stationary permanent magnets
   (b) moving permanent magnets
   (c) stationary electric charges
   (d) moving electric charges
   (e) none of the above
5. An electron moves horizontally to the east through a magnetic field that is downward. The force on the electron is toward the
   (a) N **(c)** E
   (b) S **(d)** W
6. A current is flowing west along a power line. Neglecting Earth's magnetic field, the direction of the magnetic field above it is
   (a) N **(c)** E
   (b) S **(d)** W
7. The electric and magnetic forces in a velocity selector are directed
   (a) at 90° to each other
   (b) parallel to each other, in the same direction
   (c) opposite to each other

Short Answer

8. Do electric field lines point in the direction of increasing or decreasing electric potential?
9. Why do electric field lines come out of positive charges and enter negative charges?
10. What similarities and differences are there between electric potential energy and gravitational potential energy?
11. In a 10 000 V power line, how many units of energy is carried by each unit of charge making up the current?
12. How is the principle of superposition used in problems of determining the field value due to multiple charges?
13. Explain why there is no parallel component to the electric field on the surface of conductors.
14. **(a)** The direction of motion of a positively charged particle, the direction of the magnetic field, and the direction of the magnetic force on the particle are mutually perpendicular. Draw a sketch of this situation and describe the right-hand rule that models the relationship among these directions.
    (b) The direction of a current in a conductor, the direction of the magnetic field, and the direction of the force on the conductor are mutually perpendicular. Draw a sketch of this situation and describe the right-hand rule that models the relationship among these directions.
15. **(a)** Describe the characteristics of the force required to create and maintain circular motion at constant speed.
    (b) Discuss examples that illustrate how each of the following fields can provide such a force on an object or charged particle and cause circular motion: gravitational field, electric field, and magnetic field.

16. Why is it more difficult to provide a simple equation for the strength of a magnetic force than it is for the strength of a gravitational force, the universal law of gravitation, or the strength of an electrostatic force, Coulomb's law?

17. Consider an electric field around an irregularly shaped, positively charged object. Draw a sketch of this situation by placing the charged object at the origin of a Cartesian coordinate system. Make labelled drawings to illustrate your written answers to the following questions.
    (a) In which direction will the field push a small positive test charge?
    (b) Where does the positive test charge have its greatest electric potential energy?
    (c) In which direction will the field push a small negative charge?
    (d) Where does the negative charge have the greatest magnitude of its electric potential energy?

18. **(a)** Describe the main features of coaxial cable.
    (b) Explain why coaxial cables were designed to replace flat, twin-lead wire.

19. Explain whether it is possible to determine the charge and mass of a charged particle by separate electric or magnetic forces, that is, individually and not simultaneously.

Inquiry

20. Describe an experiment in which you could determine whether the charges on a proton and electron were the same in magnitude.

21. Devise an experiment that verifies Coulomb's law. Show that the electric force should be proportional to the product of the charges and show that the electric force should be proportional to the inverse square of the distance.

22. You place a neutral object between a pair of parallel charged plates. Will it experience a net force? Will it rotate?

23. The following table shows some results that Millikan obtained during his oil-drop experiment. In this trial, the distance over which the oil drop was measured (the distance between the cross hairs in the eyepiece) was always 1.0220 cm. The second column shows the time of travel under the action of gravity alone, and the third column shows the time for an oil drop to rise when the electric field was turned on.
    (a) Calculate the velocity that corresponds to each trial.
    (b) Group the common velocities.
    (c) Analyze the velocities in a manner similar to Millikan's and show the evidence for a fundamental charge.

Trial	Fall (seconds)	Rise (seconds)
1	51.13	30.55
2	51.25	21.86
3	51.19	50.72
4	51.32	148.63
5	51.53	147.46
6	51.69	50.29
7	51.55	50.25
8	51.54	50.39
9	51.98	49.70
10	51.64	146.41

24. Two identical pith balls, mass 1.26 g, have a charge of +4.00 nC. One ball (A) is attached to the end of a light rod made of insulating material; the other (B) is suspended from a fixed point by an insulated thread 80.0 cm long. When ball A is held at various horizontal distances from B, the angle between the thread and the vertical is measured. Determine whether the results support Coulomb's law.

Horizontal distance between A and B (cm)	Angular displacement of thread
0.50	25.0°
1.00	6.65°
1.50	2.97°
1.80	2.06°
2.10	1.51°
2.50	1.07°

Communication

25. The two statements "like poles repel" and "unlike poles attract" are throwbacks to the action-at-a-distance theory, in that they imply the two poles interact with each other directly. Rewrite these two statements to reflect a field theory perspective.

26. Use the concepts of the electric field and electric field lines to convince someone that like charges should repel each other.

27. Explain how it would be possible to measure Coulomb's constant.

28. Contrast the concepts of potential difference and difference of potential energy.

29. Use Newton's law of universal gravitation to explain why Earth is round.

30. Determine the direction of the unknown vector for each of the following situations. Consider north as the top of the page and sketch the directions of the magnetic field lines, the direction of the charged particle and the force that acts on it.

(a) an electron moving east, experiencing a force directed into the page

(b) a proton moving north in a magnetic field directed west

(c) an electron moving in a magnetic field directed into the page, experiencing a force to the south

(d) a proton experiencing a force to the east, moving north

(e) an electron, experiencing no force, moving in a magnetic field directed east

(f) a proton experiencing a force to the south as it travels west

31. Describe the significance to twentieth-century physics of the Millikan oil-drop experiment.

32. Consider a stream of protons moving parallel to a stream of electrons. Is the electric force between the streams attractive or repulsive? Is the magnetic force between the streams attractive or repulsive? What factor(s) determine which force will dominate?

33. **(a)** Sketch the electric field between two parallel charged plates. Label the orientation of the charges on the plates. Show the trajectory of a positive charge sent into the field in a direction perpendicular to the field. In which direction is the electric force on the particle? Is work done on the particle as it passes between the plates?

(b) Sketch the magnetic field between the north pole of one magnet and the south pole of a different magnet. Both are set up in such a way that the field will be uniform. Show the trajectory of a positive charge sent into the field in a direction perpendicular to the field. In which direction is the magnetic force on the particle? Is work done on the particle as it passes through the field?

34. A current runs from west to east in a horizontal wire. If Earth's magnetic field points due north at this location, what is the direction of the force on the current?

35. Explain how a current balance can be used to measure the intensity of the magnetic field along the axis of a solenoid.

36. Explain how a velocity selector is able to filter a beam of particles of different velocities so that only particles with the same velocity continue in a mass spectrometer.

Making Connections

37. **(a)** Use science journals, your library, and/or the Internet to determine how auroras are formed.

(b) Discuss the phenomenon in terms of electric, gravitational, and magnetic fields.

(c) The photograph opening Chapter 8 shows the aurora borealis and the aurora australis occurring simultaneously. Explain whether you think this is a unique occurrence or one that will recur.

38. The torsion balance played an essential role in Coulomb's work. Research the history of the use of the torsion balance in physics. How is a torsion pendulum different?

39. Research and report on how the concept of the field has evolved. Discuss Faraday's and Maxwell's contributions. Also, discuss the role of Einstein's general theory of relativity in our present view of gravitational fields.

40. Albert Einstein spent the last several years of his life trying to devise a unified field theory that would show that gravity and the electric and magnetic forces were different aspects of the same phenomenon. He did not succeed. In the 1960s, it was shown that electric and magnetic forces and the weak nuclear force are different aspects of the same force: the unified electroweak force. To date, no one has linked gravity or the strong nuclear force with the unified electroweak force. Research the unification of forces and explain why the problem is so difficult to solve.

41. The Sun's magnetic field is responsible for sunspots, small regions on the surface of the Sun that are cooler and have a much higher magnetic field concentration than their surroundings. The Sun's magnetic field is also responsible for producing solar flares and other solar activity. Prepare a report that summarizes the latest research on the Sun's magnetic field and the types of solar phenomena that are being examined. Incorporate into your report the findings provided by the orbiting solar satellite, the Solar and Heliospheric Observatory (SOHO).

42. Research the principle behind the defibrilator and the steps that have been made to ensure its presence on all major aircraft.

43. Research the structure of an electrostatic air cleaner and discuss the function of the charging electrode and the grid.

44. In what way is electrostatic force used in the electroplating process in automobile manufacturing?

45. Research and explain the part played by the electric field in
(a) the xerographic process
(b) laser printers
(c) inkjet printers

46. Prepare a cost-benefit analysis of the use of the electric car.

47. "Electron guns" are used in television sets to propel electrons toward the screen. What techniques are then used to deflect the electron beam and "paint" a picture?

48. Discuss the role of electric potential difference in the following medical diagnostic techniques.
(a) electroencephalography
(b) electroretinography

Problems for Understanding

49. What is the total charge on 1.0 g of electrons?

50. What is the magnitude of the electric force between a proton and electron in a hydrogen atom if they are 52.9 pm apart?

51. A nucleus of argon has a charge of +18 e and a nucleus of krypton has a charge of +36 e, where e is the elementary charge, 1.60×10^{-19} C. If they are 8.0 nm apart, what force does one exert on the other?

52. Two small ball bearings sit 0.75 m apart on a table and carry identical charges. If each ball bearing experiences a force of 3.0 N, how large is the charge on each?

53. How many electrons must be removed from an isolated conducting sphere 12 cm in diameter to produce an electric field of intensity 1.5×10^{-3} N/C just outside its surface?

54. Two identical charges exert a force of 50.0 N[repulsion] on each other. Calculate the new force if

(a) one of the charges is changed to the exact opposite
(b) instead, the distance between the charges is tripled
(c) instead, one charge is doubled in magnitude and the other is reduced to one third of its magnitude
(d) all of the above changes are made

55. Two identical pith balls, each with a mass of 0.50 g, carry identical charges and are suspended from the same point by two threads of the same length, 25.0 cm. In their equilibrium position, the angle between the two threads at their suspension point is 60˚. What are the charges on the balls?

56. Suppose you wanted to replace the gravitational force that holds the Moon in orbit around Earth by an equivalent electric force. Let the Moon have a net negative charge of $-q$ and Earth have a net positive charge of $+10\ q$. What value of q do you require to give the same magnitude force as gravity?

57. Earth carries a net charge of -4.3×10^5 C. When the force due to this charge acts on objects above Earth's surface, it behaves as though the charge was located at Earth's centre. How much charge would you have to place on a 1.0 g mass in order for the electric and gravitational forces on it to balance?

58. Suppose you want to bring two protons close enough together that the electric force between them will equal the weight of either at Earth's surface. How close must they be?

59. Calculate the repulsive force between two 60 kg people, 1.0 m apart, if each person were to have 1% more electrons than protons. (Assume for simplicity that a neutral human body has equal numbers of protons and neutrons.)

60. What will be the net force, considering both gravitational and electrostatic forces, between a deuterium ion and a tritium ion placed 5.0 cm apart?

61. What must be the charge on a pith ball of mass 3.2 g for it to remain suspended in space when placed in an electric field of 2.8×10^3 N/C[up]?

62. (a) Calculate the repulsive Coulomb force between two protons separated by 5×10^{-15} m in an atomic nucleus.
(b) How is it possible that such a force does not cause the nucleus to fly apart?

63. The electric potential difference between two large, charged parallel plates is 50 V. The plates are 2.5 cm apart. What is the electric field between them?

64. How many electrons make up a charge of 1.0 μC?

65. A 2.0 pC charge is located at point A on an imaginary spherical surface which is centred on a 4.0 μC point charge 2.8 cm away. How much work is required to move the 2.0 pC charge to the following two points?
(a) to point B, which is located on the same spherical surface an arc length 3.0 cm away
(b) to point C, which is located radially outward from A on another imaginary spherical surface of radius 4.2 cm
(c) What name could be used to describe these spherical surfaces?

66. Two horizontal plates used in an oil-drop experiment are 12 mm apart, with the upper plate being negative. An oil drop, with a mass of 6.53×10^{-14} kg, is suspended between the plates. The electric potential difference is 1.6×10^4 V. Calculate the
(a) total charge on the oil drop
(b) number of excess or deficit electrons on the oil drop
(c) electric potential difference required to suspend the oil drop if an electron is knocked off it by an X ray

67. A current of 2.0 A runs through a wire segment of 3.5 cm. If the wire is perpendicular to a uniform magnetic field and feels a magnetic force of 7.0×10^{-3} N, what is the magnitude of the magnetic field?

68. A small body of unknown charge, travelling 6.1×10^5 m/s, enters a 0.40 T magnetic field directed perpendicular to its motion.
(a) If the particle experiences a force of 9.0×10^{-4} N, what is the magnitude of the charge?

(b) If the object is sent into the magnetic field so that its velocity makes an angle of 30.0° with the magnetic field, by how much will the magnetic force be reduced?

69. Consider a proton that is travelling northward with a velocity of 5.8×10^6 m/s in a particle accelerator. It enters an east-directed magnetic field of 0.25 T.

(a) Calculate the magnetic force acting on the proton.

(b) What is the magnitude and direction of its acceleration?

70. A proton travelling at 2×10^7 m/s horizontally enters a magnetic field of strength 2.4×10^{-1} T, which is directed vertically downward. Calculate the consequent radius of orbit of the proton.

71. Prove that the radius of orbit of a particle in a mass spectrometer is equal to p/qB, where p is its momentum.

72. **(a)** An electron is fired into a 0.20 T magnetic field at right angles to the field. What will be its period if it goes into a circular orbit?

(b) If the electron is moving at 1.0×10^7 m/s, what is the radius of its orbit?

73. You want to create a beam of charged particles that have a speed of 1.5×10^6 m/s. You use a crossed electric and magnetic field and choose a magnet with a strength of 2.2×10^{-4} T. What must be the magnitude of the electric field?

74. A charged particle that is sent into a magnetic field at an angle will follow a helical path, the characteristics of which can be calculated from the particle's velocity parallel and perpendicular to the field. Consider a magnetic field of strength 0.26 T directed toward the east. A proton with a speed of 6.5×10^6 m/s is shot into the magnetic field in the direction [E30.0°N].

(a) Calculate the proton's velocity in the directions parallel and perpendicular to the magnetic field.

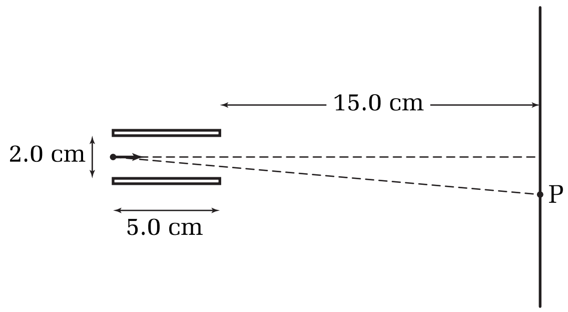
(b) Calculate the radius of the proton's orbit as it spirals around the magnetic field. (Hint: Which component of the velocity contributes to this motion?)

(c) How long will it take the proton to complete a singular circular orbit?

(d) During the time that it takes the proton to complete one orbit, how far will the proton travel toward the east? (Hint: Which component of the proton's velocity contributes to this motion?)

(e) Sketch the proton's path as seen from the side and as seen looking west into the magnetic field.

75. The diagram shows an electron entering the region between the plates of a cathode ray tube (the basic structure of a television tube). The electron has an initial velocity of 2.7×10^7 m/s horizontally and enters at the exact mid-axis of the plates. The electric field intensity between the plates is 2.80×10^4 N/C upward. How far below the axis of the plates will the electron strike the screen at point P?



COURSE CHALLENGE

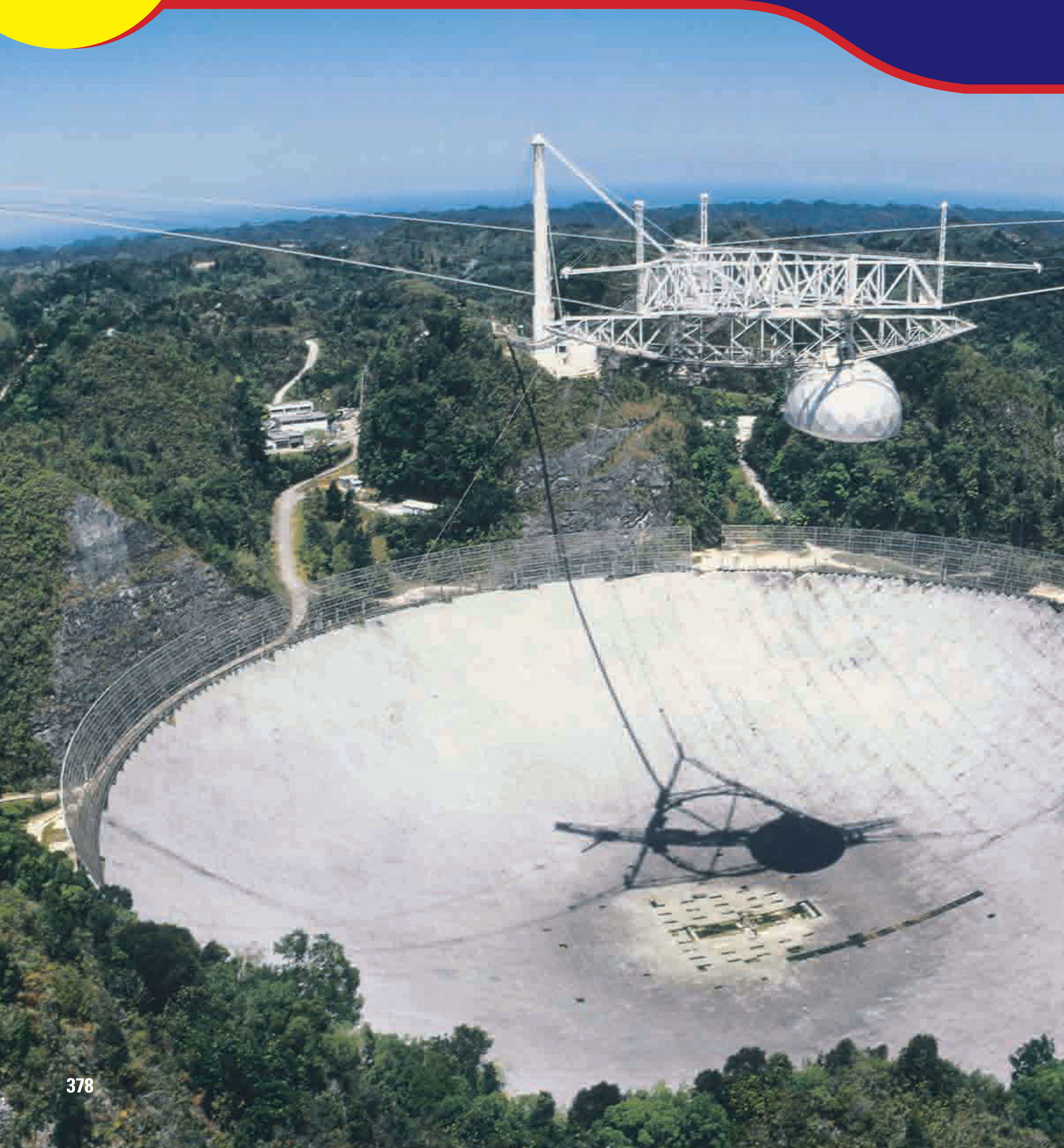
Scanning Technologies: Today and Tomorrow

Plan for your end-of-course project by considering the following.

- Are you able to incorporate electric, gravitational, and magnetic fields into your analysis?
- Consider time and equipment requirements that might arise as you design project-related investigations.
- Examine the information that you have gathered to this point. Produce a detailed plan, including a time line, to guide you as you continue gathering information.

UNIT 4

The Wave Nature of Light



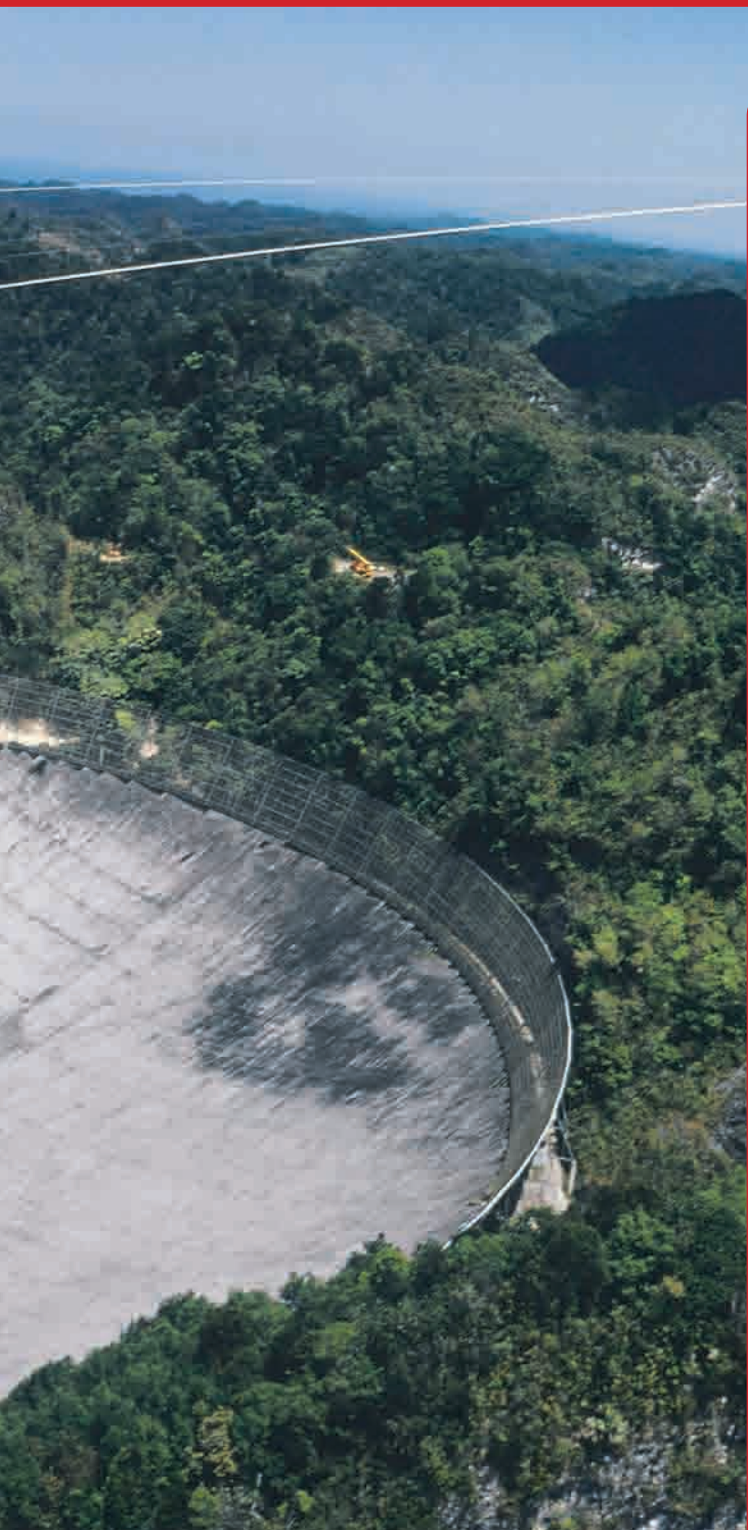
OVERALL EXPECTATIONS

DEMONSTRATE an understanding of the wave model of electromagnetic radiation.

PERFORM experiments relating to the wave model of light and applications of electromagnetic radiation.

ANALYZE light phenomena and explain how the wave model provides a basis for technological devices.

UNIT CONTENTS



Is there life beyond Earth? It seems inconceivable that life has formed only on this planet, yet there is no direct evidence that there is life outside our own solar system. If civilizations exist in space, might they be discovered by electromagnetic radiation monitoring from here on Earth?

The Search for Extraterrestrial Intelligence (SETI) program, a range of research projects dedicated to the search for intelligent life beyond Earth, is investigating this possibility. Using the world's largest radio telescope, located in Arecibo, Puerto Rico (shown in the photograph), the sky is scanned around the clock for non-natural electromagnetic signals. SETI research projects attempt to answer questions, such as: How many stars might have planets? And of those planets, how many have environments that could support life?

Developing an understanding of electromagnetic radiation has provided modern civilization with a powerful communication tool. This unit will introduce the theoretical framework that predicted the existence of electromagnetic waves, how these waves (including light) are produced and detected, their properties, and some applications in modern society.

UNIT PROJECT PREP

Refer to pages 454–455. In this unit project, you will have the opportunity to build and test an FM transmitter.

- How will an understanding of a wave model for electromagnetic radiation help you to understand FM transmission?
- What properties of electromagnetic waves will be easiest to verify using your transmitter?

CHAPTER 9

Wave Properties of Light

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Physical properties of waves
- Reflection and refraction of waves
- Superposition of waves
- Using a ripple tank

Peering through a telescope, you can see the "Red Planet," Mars, and, off in the distance, Jupiter's stripes. Earth looks like a blue marble and the gas giant Neptune appears to be crystal blue. This composite photograph reveals a richness of knowledge transmitted in the form of light that reaches Earth from the expanse of space.

What are the properties of light that allow it to travel millions of kilometres through deep space from the Sun, to the other planets, and back to our telescope, carrying information in the form of colour and intensity. Careful visual observation of solar system objects yields a great deal of knowledge.

Galileo used a telescope that today would be considered primitive to discover four of Jupiter's moons. His discovery solidified in his mind that Copernicus' concept of a Sun-centred solar system was correct, even though such a concept clashed with the scientific and religious theories of his time.

Less than 50 years later, a new debate raged, not about the solar system, but about the very nature of light, which streams from the Sun, illuminates Earth, and seems to light up a room instantly. The new debate struggled to compare light to something more common to everyday experience, attempting to classify this elusive form of energy as either a wave or a particle.

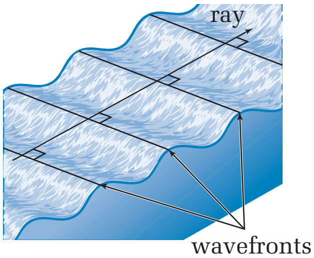
In this chapter, you will learn about the attempts to formulate and verify a model for light. You will discover that the techniques that established the wave model for light also led to some practical applications and research tools.

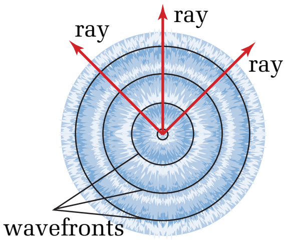
INVESTIGATION 9-A

Properties of Waves

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting
- Identifying variables

For light to be classified as a wave, it must exhibit specific properties of waves. In this investigation, you will analyze an important property of water waves that must also be true of light — if light is, in fact, a wave.





Problem

Investigate how waves behave when they

(a) encounter a small barrier

(b) pass through a narrow slit

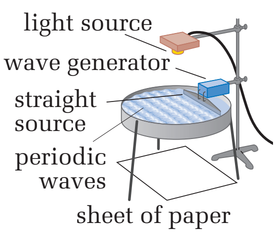
Equipment

- ripple tank
- wave generator
- 2 solid barriers (less than half the width of the tank)
- wooden dowel

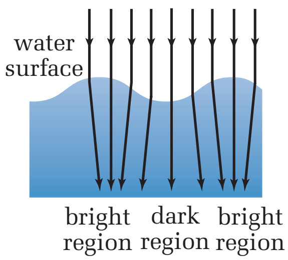
CAUTION Care must be taken with any electrical equipment near ripple tanks. Firmly attach lights and wave generators to the tank or lab bench, and keep all electrical wiring away from the water.

Procedure

1. Assemble the ripple tank, light source, and wave generator as shown in the diagram. Add water and carefully level the tank so that the depth of the water is approximately 1.5 cm at all points in the ripple tank.



A Wave tank set-up



B Water as a lens

2. Align the straight-wave generator so that parallel wavefronts travel perpendicularly from the dowel. Vary the frequency of the generator to find a wavelength that produces the clearest image on the paper below the tank. Use the light and dark regions cast on the paper to view the wave properties during the investigation.
3. Place a solid barrier in the tank that is about half the width of the tank. Send straight waves at the barrier and observe their behaviour. Sketch the appearance of the waves as they pass the edge of the barrier.
4. Vary the wavelength of the incident waves. Draw cases that exhibit maximum and minimum spreading around the edge of the barrier.
5. Place two solid barriers in the tank, leaving a narrow slit between them. Send straight waves toward the narrow opening and observe the nature of the waves that pass through it.
6. Systematically vary the width of the opening and then the wavelength to determine a general relationship between the amount of the spreading of the waves, wavelength, and the size of the opening.

Analyze and Conclude

1. Describe what happens to waves when they pass the edge of a solid barrier. Is the effect altered as wavelength is changed? If so, how?
2. Describe what happens when waves pass through a narrow opening between two solid barriers. What relationship between the wavelength and the width of the opening appears to be the most significant?

Apply and Extend

3. In your experience, does light exhibit any of the properties of waves that you have just studied? Provide examples.

9.1 Two Models for Light

SECTION EXPECTATION

- Define and explain the units and concepts related to the wave nature of light.

KEY TERMS

- dispersion
- diffraction
- Huygens' principle
- superposition of waves
- constructive interference
- destructive interference
- nodal point

You flip a switch as you walk into a room, flooding the room with light that instantly reaches every corner. Objects in the path of the light generate shadows, and yet the light reaches far enough under your bed to illuminate an old shirt. What allows light to seemingly be everywhere instantaneously, be blocked by objects, and yet be able to reflect and bounce into tiny nooks and crannies?

Physicists have been trying to develop a complete model of light for centuries. Is light best modelled as a particle or as a wave? These competing models for light originated in the late 1600s, proposed by physicists who were attempting to describe the propagation of light.

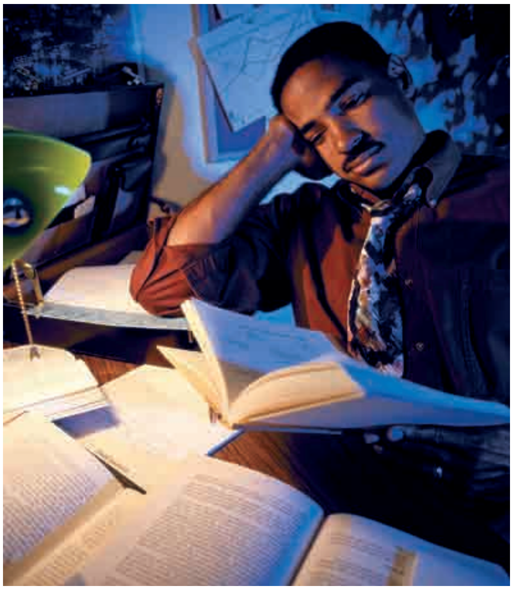


Figure 9.1 Even though the light casts shadows, you can still clearly see objects in those shadows. A little light seems to reach everywhere.

The basic properties of light that were understood in the 1600s (and that any acceptable model must be able to explain) were straight-line propagation, reflection, refraction, dispersion, and the ability of light to travel undisturbed across millions of kilometres of space. Which model — particle or wave — could best explain and predict these properties?

Newton's Corpuscular Model

Although scientists and philosophers had been hypothesizing about the nature of light for centuries, Sir Isaac Newton (1642–1727) was the first to formulate a detailed, systematic model of light. He published his "corpuscular" theory of light in 1704. Newton's proposed corpuscles were particles with exceedingly small masses that travelled in straight lines through space, penetrated some media, and bounced off other solid surfaces. Newton could explain refraction (the bending of light when it travels from one medium to another) if the speed of the particles increased when entering a more-dense medium. Although speeding up in a dense medium does not seem logical, Newton explained it by proposing that an attractive interaction existed between the light particles and the medium.

Conceptual Problem

- Use the conservation of momentum to show that when a particle such as a billiard ball collides with a solid wall, it follows the law of reflection, which states that the angle of reflection is equal to the angle of incidence.

The **dispersion** of light, which is the separation of light into the colours of the spectrum when passing through a prism, had been observed by scientists before Newton proposed his theory of light. Newton himself had demonstrated that white light was actually a composite of all of the colours of the rainbow by showing that the colours could be combined by a second prism and produce white light.

Figure 9.2 Using a refracting prism, white light can be separated into a spectrum of colour.

According to Newton's corpuscular model, each colour had a different mass. Violet light was refracted the most and therefore must have the least amount of mass, making it easiest to divert from its original path. Blue light was more massive than violet and therefore refracted less. Following this argument, Newton assumed that red light particles were the most massive of all of the visible colours.

Another question that Newton's corpuscular theory was able to answer was: What occupies the space between Earth and the Sun? If light was, in fact, small particles of insignificant mass, the particles would be able to travel millions of kilometres to Earth from the Sun. Possibly, the most important reason that Newton did not consider the wave model for light was the apparent lack of

ELECTRONIC LEARNING PARTNER

Go to your Electronic Learning Partner to enhance your understanding of diffraction.

diffraction — the spreading of a wave after encountering a barrier. Italian scientist Francesco Grimaldi (1606–1680) provided evidence that light does undergo diffraction by demonstrating that light passing through a small opening in a barrier produced a spot of light on a distant screen that was larger than strict ray diagrams predicted. The edges of the bright spot also appeared fuzzy: The region of light faded into dark, rather than being crisply divided into two regions. Newton and the proponents of the corpuscular theory of light discounted the effects, citing that the amount of diffraction seemed to be too small to be of consequence.

Newton was not entirely convinced of the correctness of his own corpuscular model for light and was surprised that some of his proponents approved of it so strongly. Nevertheless, until stronger evidence of wave-like properties was obtained, Newton would not accept a wave model for light.

Huygen's Wave Model

Christiaan Huygens (1629–1695) refined and expanded the wave model of light, originally proposed by Robert Hooke (1635–1703). Hooke rejected a particle model partly because two beams of light can pass through each other without scattering each other, as particles do. One problem with the wave model was the ability of light to travel through the apparently empty space of the universe.

During early discussions about the nature of light, scientists knew that mechanical waves required a medium through which to propagate. Various properties of the medium would undergo periodic changes from a maximum, to an equilibrium, to a minimum, and back through the cycle again. For example, in a water wave, the particles of water actually move between a maximum and minimum height. In sound waves, the pressure of the medium increases and decreases. What medium could be carrying light energy? Since no medium was known to exist throughout space, scientists proposed that an as yet undetected medium called "ether" existed to carry light waves.

Huygens developed a priniciple that is still helpful in analyzing and predicting the behaviour of waves. He compared the propagation of light to the travelling disturbance observed when a pebble is dropped into a pond of still water. The disturbance, or wave pulse, moves outward in concentric circles from the pebble's point of impact. Huygens realized that the waves travelling outward from the centre continue to travel even after the pebble has struck the pond's bottom. The waves are effectively travelling without a source.

Extending this example, he postulated that disturbances existing at each point on a wavefront could be a source for disturbances along the wavefront an instant later. Figure 9.3 illustrates Huygens' thinking for straight and circular waves.

PHYSICS FILE

Robert Hooke contributed to science in several fields, from cellular biology to the physics of light. Hooke developed the wave theory of light, while Newton proposed the corpuscular or particle theory of light. Hooke and Newton had argued before, because Hooke had developed an early version of Newton's gravitation equation and felt that Newton had not given him enough credit. Both men realized, however, that more evidence would be necessary before either of their models would be accepted.

Huygens' principle states: Every point on an advancing wavefront can be considered to be a source of secondary waves called "wavelets." The new position of the wavefront is the envelope of the wavelets emitted from all points of the wavefront in its previous position.

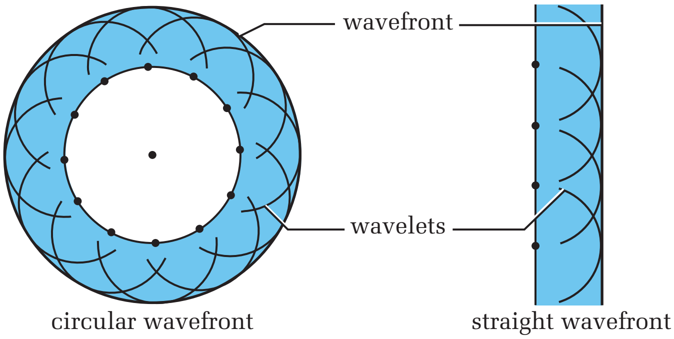
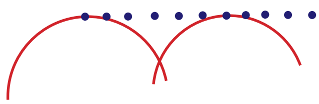


Figure 9.3 Each point of a wavefront can be considered to be a source of a secondary wave, called a "wavelet."

Conceptual Problem

- Carefully mark a small dot every 0.5 cm over a distance of 8.0 cm on a blank sheet of white paper. Beginning with the first dot, use a 25 cent coin to draw semicircles on every fourth dot. Ensure that each semicircle arc is drawn with the leading edge always closest to the top of the page, intersecting a single dot as shown. Draw more arcs, one for each dot.
  (a) Does the leading edge of the sum of the arcs form a more complete wavefront?
  (b) Would an infinite number of wavelets form a continuous wavefront? What is happening behind the wavefront to cause a single wavefront to form?



Either by applying Huygens' principle or by observing visible waves such as water waves, you can show that waves propagate in straight lines while moving unobstructed through a single medium, and that they reflect off solid or opaque barriers. Huygens also showed that if the velocity of light decreases when it passes from a less-dense to a more-dense medium, it will bend or refract in such a way that the angle of refraction is smaller than the angle of incidence. A slight difference in the speed of the various colours of light in a given medium could also explain dispersion.

Huygens' wave model accurately predicted the behaviour of light as strongly or even more strongly than did Newton's model in terms of rectilinear propagation, reflection, refraction, and dispersion. At that time, however, there were no tests or observations that could eliminate either model, so Newton's stature in the scientific community (gained for his many and varied contributions, including the laws of motion) resulted in his winning the approval of other scientists for his less eloquent corpuscular model. What type of experiment would be necessary in order to accept or reject one of the models? What would reveal the greatest contrast between the properties of waves and particles?

Superposition of Waves

PHYSICS FILE

The superposition of waves principle holds true for linear waves. Linear waves are characterized by small amplitudes. Interestingly, the superposition principle does not apply to non-linear waves, characterized by large amplitudes. This textbook does not deal with non-linear waves.

What happens when two particles or two waves attempt to occupy the same point in space at the same time? Obviously, as two particles approach the same point, they will collide and move in a way that will obey the law of conservation of momentum. Two waves, however, *can* occupy the same space at the same time. When two waves pass through one location in the medium, the medium will oscillate in a way that resembles the sum of the effects from both waves. Waves that reach the same point simultaneously interfere with each other in terms of the displacement of the medium.

This result, called the **superposition of waves**, is formally stated as: When two or more waves propagate through the same location in a medium, the resultant displacement of the medium will be the algebraic sum of the displacements caused by each wave. Each wave behaves as though the other did not exist and, once past the area of interest, proceeds unchanged.

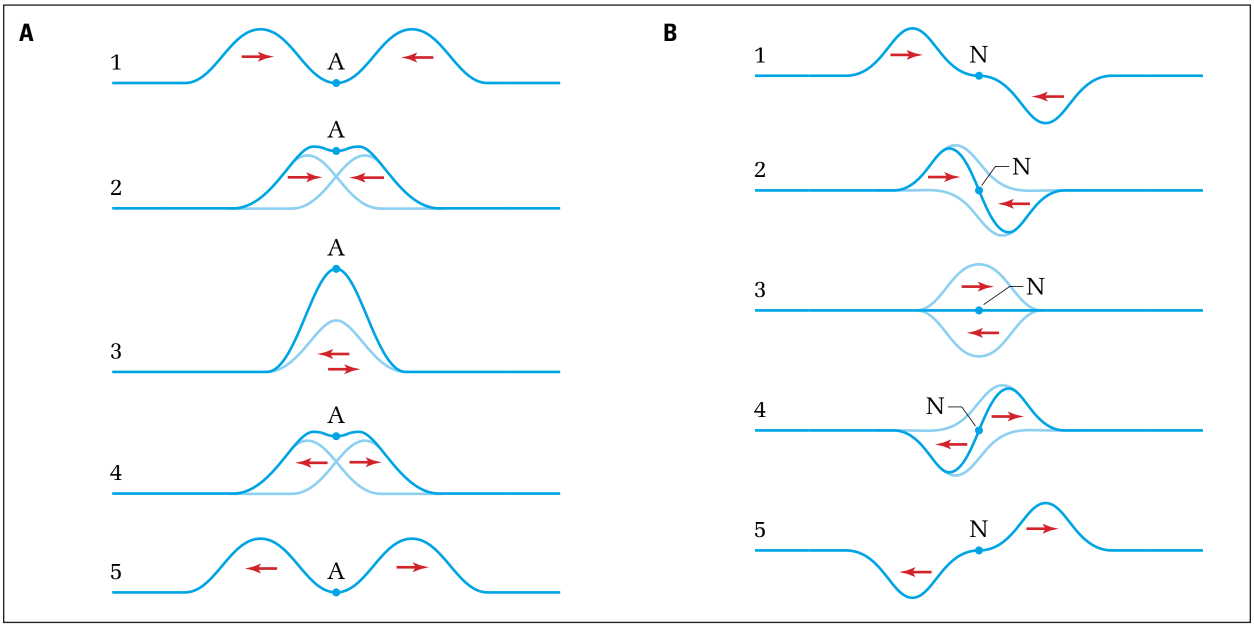


Figure 9.4 **(A)** Constructive interference results in a wave pulse that is larger than either individual pulse. **(B)** Destructive interference results in a wave pulse that is smaller than the larger of the component waves. Two identical but inverted pulses will yield a momentary amplitude of zero.

The resultant displacement of the medium caused by the superposition of two waves is unlike either of the individual waves that added together to form it. Figure 9.4 (A) illustrates two pulses travelling toward each other. The darkened wave at point A is the result of **constructive interference** of the two pulses. Constructive interference results in a wave with larger amplitude than any individual wave. Perfectly constructive interference occurs when waves are completely in phase with each other. Figure 9.4 (B) illustrates **destructive interference**. In this case, the resultant wave amplitude is smaller than the largest component wave. A **nodal point** exists in the medium when two waves with identical but inverted amplitudes exist simultaneously. The resultant displacement at a nodal point is zero.

The most definitive test of a phenomenon that could classify it as a wave is to show that it undergoes interference. In the next section, you will learn how to observe and verify that light exhibits interference and therefore must behave like a wave.

9.1 Section Review

1. MC Sound, a form of energy, can be modelled by using two distinctly different approaches.
   - **(a)** Describe the propagation of sound energy through air by discussing the motion of individual particles. Include possible mathematical equations that might apply.
   - **(b)** Describe the propagation of sound energy through air by discussing waves. Include possible mathematical equations that might apply.
   - **(c)** Does one method provide a more easily understood explanation?
   - **(d)** Does one method provide a more simple mathematical model?
2. K/U Describe Huygens' concept of wavelets.
3. **(a)** K/U Draw a series of Huygens wavelets so that they produce a circular wavefront.
   - **(b)** Describe how the wavelets that form the wavefront apparently vanish behind it.
4. **(a)** MC Do you believe that some current accepted scientific models or theories might be held in high regard because of the stature of the scientists who proposed them?
   - **(b)** Suggest one possible model or theory that is currently accepted by a majority of people that you feel might be significantly inaccurate. Explain.

UNIT PROJECT PREP

An FM transmitter produces a carrier wave with a specific frequency. A fixed frequency wave of any type is produced by periodic motion of a source.

- Hypothesize about what might experience periodic motion in the creation of FM radio waves.
- How is an understanding of frequency important in the construction of a radio transmitter?
- Apply Huygens' wave model to various waves with which you are familiar as you study this unit. Can his model predict the behaviour of all waves?

CANADIANS IN PHYSICS

The Light Fantastic

Even as a young girl, Dr. Geraldine Kenney-Wallace knew that she wanted to be a scientist. She preferred playing with crystals, minerals, and fossils rather than toys and was fascinated by electric motors, trains, and radios. In school, art, math and science were her favourite classes.

After high school, she worked as a summer research student at the Clarendon Physics Laboratories, part of England's Oxford University. Although lasers were then new, Dr. Kenney-Wallace's work at Clarendon brought her into daily contact with them. There were hints that lasers could be used in new and exciting applications to investigate atoms, molecules and semiconductors in particular.

Dr. Kenney-Wallace continued her research while working on her bachelor's degree in London. She then came to Canada for graduate work. While teaching in Toronto, she was able to indulge her passion for lasers, physics and chemistry by establishing the first University ultrafast laser laboratory in Canada.

Dr. Geraldine Kenney-Wallace

A pulsed laser works by pumping certain kinds of crystals or gases with so much energy that the electrons are pushed to a very high quantum level. Spontaneous emission occurs. In a laser cavity, multiple reflections between the end mirrors trigger *stimulated* emission so that the electrons all simultaneously drop down to a lower energy level, releasing their stored energy as photons. Then, the process begins building up in the cavity again. While they are recharging, however, lasers cannot generate any light output.

Lasers are used to investigate chemical reactions by bouncing the laser photons off the atoms and molecules. What happens, though, if the reaction takes place faster than the time it takes for the laser to recharge? Researchers realized that conventional lasers could not be used to study these fast reactions. They needed a laser that fired and recharged very quickly — an ultrafast laser. Dr. Kenney-Wallace was in the forefront of the design and application of these new ultra-fast lasers.

Dr. Kenney-Wallace's interests and talents extended beyond the frontiers of scientific research. In addition to her scientific research, she devoted part of her career to consultation on business and public policy issues, usually related to the areas dearest to her: Research and Development, and science and education.

Her ongoing work has earned her international recognition. She has been quoted in the House of Commons, interviewed by Canada's national media, and has given dozens of professional seminars throughout the world. She has 13 honorary degrees, is the former president of McMaster University, a Fellow of the Royal Society of Canada, and the first woman to hold the Chair of the Science Council of Canada. For the past few years, she has been working in England, helping to set up a number of virtual universities. She is not only at the forefront of laser research, but also at the cutting edge of e-learning education. Dr. Kenney-Wallace is living proof that, at least intellectually, you can have it all!

Going Further

1. One of Dr. Geraldine Kenney-Wallace's great strengths is her multidisciplinary background. She has interests in both chemistry and physics and also in business. Discuss other combinations of fields that might be helpful to a career in science or how a science background can help you in another career. For example, might a science background help you in business? How? Alternatively, can you think of how a knowledge of economics, for example, might be of help to a chemist? How about the combination of biology and law?
2. What do you find most interesting about Dr. Kenney-Wallace's career? Why? Report on this to your class.

9.2 Interference and the Wave Model for Light

SECTION EXPECTATIONS

- Describe the concepts related to diffraction and interference.
- Describe interference of light in qualitative and quantitative terms.
- Collect and interpret experimental data in support of a scientific model.
- Identify interference patterns produced by light.
- Describe experimental evidence supporting the wave model of light.

KEY TERMS

- coherent
- fringe
- Fraunhofer diffraction
- Fresnel diffraction

A siren pierces the serenity of a quiet evening. Although you are unable to see the emergency vehicle generating the noise, you are certainly able to hear it. The sound is able to bend around corners, pass through doorways, and eventually reach your ears. The ability of sound energy to bend around corners and spread around barriers is not only a property of sound, but is also a property of all waves.

Figure 9.5 You can usually hear a siren long before you see the emergency vehicle, because sound can "bend" around corners.

A Definitive Experiment

Bending around corners — a form of diffraction — is a property of all waves. However, scientists studying light at the time of Newton and Huygens were not able to detect any significant diffraction of light. To determine with confidence whether light behaved like a wave or a particle, scientists needed a carefully planned experiment that could clearly show evidence or lack of evidence of interference of light. To visualize the type of experiment that would be definitive, observe the pattern of water waves in Figure 9.6, which results from two point sources creating a periodic disturbance.

Figure 9.6 Nodal lines, resulting from total destructive interference, are clearly visible, radiating outward from between the two sources.

INVESTIGATION 9-B

Diffraction of Sound

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

Diffraction is readily observed using mechanical waves. In this activity, you will study some variables associated with diffraction of sound waves.

Problem

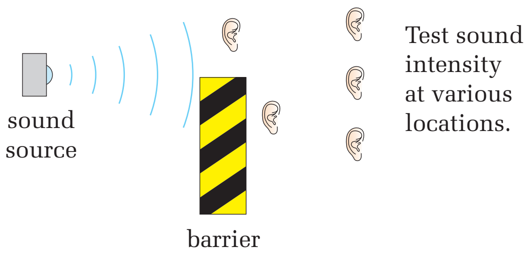
Identify some variables associated with diffraction of sound waves.

Equipment

- audio frequency generator
- speaker

Procedure

1. Using the wave model for sound, predict **(a)** how sound intensity will vary (e.g., sharply, gradually) behind the edge of a solid barrier and **(b)** how changes in wavelength will influence the results from part (a).
2. Use an audio frequency generator connected to a single speaker to act as a sound source. Recall that the frequency of sound will cause the wavelength to vary, according to the wave equation $v = f\lambda$. Use a relatively sound-proof barrier, such as a wall with a wide door, to test your predictions. A door opening into a large open space, as shown in the diagram, will reduce the amount of reflection from walls and will therefore yield the best results. Select and maintain a single, relatively low intensity (volume) to reduce effects produced by reflection of sound off nearby objects.



3. Carefully analyze how the intensity of the source varies at different locations behind the barrier, as shown in the diagram. Select an appropriate method to illustrate how the sound intensity varied.
4. Experiment to see how wavelength affects the amount of diffraction.

Analyze and Conclude

1. Were your predictions about the diffraction of sound accurate? Explain.
2. Describe and illustrate how the sound intensity varied at different locations behind the edge of the barrier.
3. Does varying the wavelength of the source affect the amount of sound wave diffraction? Explain and provide evidence.
4. Do your results validate the wave model of sound? Explain.
5. Suggest why only one speaker is used in this activity. Include the principle of superposition of waves in your answer.

In Figure 9.6, you can see lines emanating from the sources that show constructive interference — standing waves — and destructive interference — no movement of the water. If light behaves like a wave, a similar experiment should reveal bright areas — constructive interference — and dark areas — destructive interference — on a screen. Figure 9.7 shows how interference resulting from two light sources would create light and dark regions.

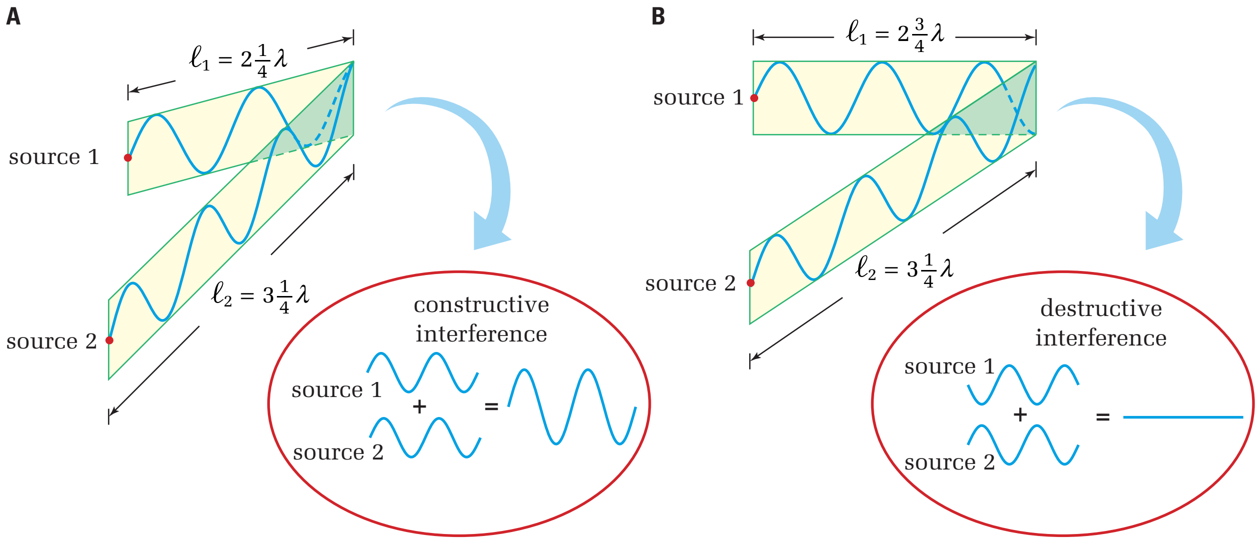


Figure 9.7 **(A)** Waves from each source with a path difference of whole-number multiples of wavelength interfere constructively. **(B)** Waves from each source with a path difference of multiples of one half wavelength interfere destructively.

Examination of Figure 9.7 reveals two important features that must be designed into the experiment. First, the sources must produce coherent waves. **Coherent** sources produce waves of the same frequency and in phase with each other. Second, the distance between the sources must be of the order of magnitude of the wavelength of the waves. If the sources are placed too far apart, the light and dark areas on a screen will be too close together to be observed. (As a rule of thumb, the sources must be no farther than 10 wavelengths apart.)

These conditions were exceedingly difficult for scientists to create in the 1700s. Physicists could produce light of one frequency by passing it through a prism but, before the invention of the laser, coherent light sources did not exist. The phases of light emanating from a source were random. Therefore, constructive or destructive interference would occur in a random way and the effects would be an average of light and dark, so that they appeared to be uniform. In addition, since physicists did not even know whether light behaved like a particle or a wave, they had no way of knowing what the wavelength might be. It took nearly 100 years after Newton and Huygens presented their models of light for the debate to be resolved.

ELECTRONIC LEARNING PARTNER

Go to your Electronic Learning Partner to enhance your understanding of interference.

Young's Double-Slit Experiment

Thomas Young (1773–1829) devised an ingenious experiment, as illustrated in Figure 9.8, that produced an interference pattern with light. Using one monochromatic light source, Young allowed the light to fall onto an opaque material with a single, narrow slit. According to Huygens' principle, this slit acted as a new source. The light passing through the single slit spread as it travelled to a second opaque barrier. The second barrier had two narrow slits placed very closely together.

In part (B) of Figure 9.8, you can see that two parts of the same wavefront from the single slit reach the double slits at the same time. Since these two parts of the same wavefront behave as new sources at the double slits, the light leaving the double slits is essentially coherent. Young experimented with this set-up for more than two years before he realized that the double slits had to be so close together that they almost appeared to be one slit to the unaided eye. The light that passed through the double-slit barrier fell on a nearby screen, producing the historic pattern of light and dark lines caused by the interference of light waves. Young's results catapulted the wave model for light into centre stage, where it would remain unchallenged for more than 100 years.

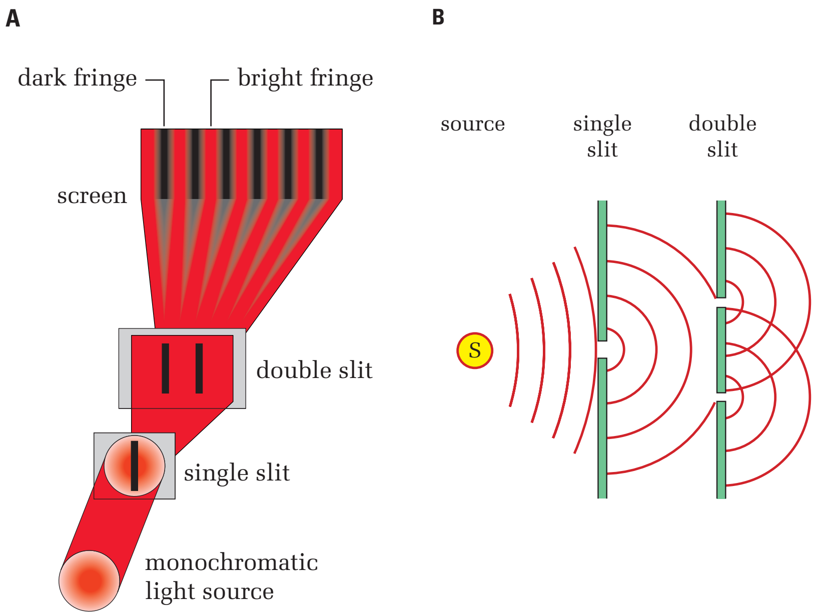


Figure 9.8 **(A)** Young's experiment used a single incandescent bulb and two narrow slits to produce coherent sources. He successfully showed that light could form an interference pattern similar to those produced with mechanical waves. **(B)** The wavefronts emanating from the double slits resemble the water waves generated by two point sources.

Young was successful where others had failed for several reasons.

- He used a monochromatic (single wavelength) light source.
- The double slits acted as two sources and were spaced much more closely together than was possible if two separate light sources were used.
- The light passing through the initial single slit acted as a point source. When a wavefront from the point source reached the double slits, two parts of the same wavefront became new sources for the double slits and were therefore coherent.

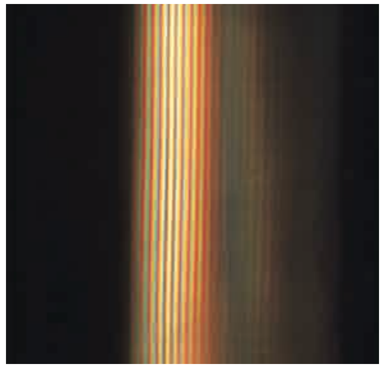


Figure 9.9 Photograph of an interference pattern from Young's experiment (notice how the intensity reduces toward the edges)

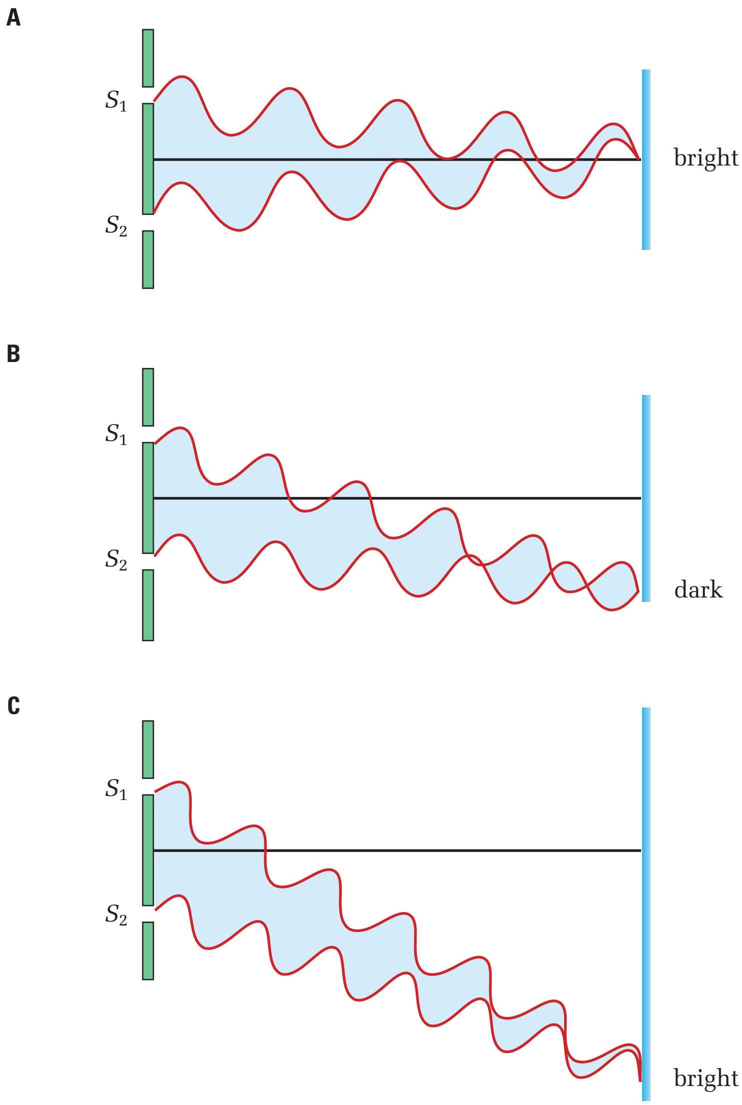


Figure 9.10 Light and dark fringes result from interfering waves.

To understand the mathematical analysis of the pattern produced by Young's double-slit experiment, examine Figure 9.11. In part (A), you see coherent light waves entering the two slits and passing through. Light leaves the slits in all directions, but you can study one direction at a time. Since the distance between the slits and the screen (labelled x) is approximately a million times larger than the distance between the slits, you can assume that the waves leaving the slits parallel to each other will hit the screen at the same point. Part (B) is drawn to the scale of the slit-to-screen distance and therefore the two parallel rays appear as one line.

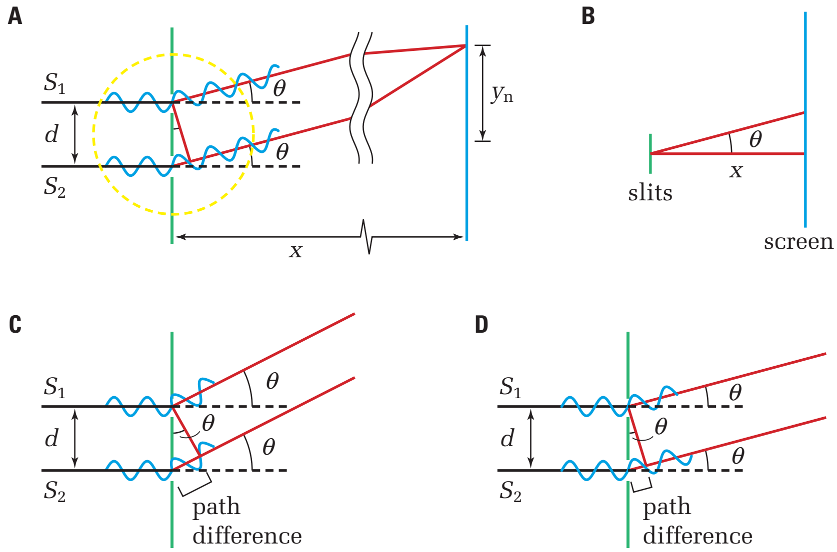


Figure 9.11 The path difference between slits that light travels to reach the screen is given by $d \sin \theta$.

Parts (C) and (D) of Figure 9.11 illustrate two special cases — constructive interference and destructive interference. Inspection of the right triangle in part (C) shows that the hypotenuse is the distance, d, between the two slits. One side is formed by a line drawn from slit 1 that is perpendicular to the ray leaving slit 2. The third side of the triangle is the distance that ray 2 must travel farther than ray 1 to reach the screen. When this path difference, PD, is exactly one wavelength, the two waves continue from the slit in phase and therefore experience constructive interference.

When the two rays reach the screen, they will produce a bright spot on the screen, called a bright **fringe**. Using trigonometry, you can see that the path difference is equal to $d \sin \theta$. In fact, if the path difference is any integer number of full wavelengths, the waves will remain in phase and will create a bright fringe on the screen. Notice that, from the geometry of the apparatus, the angle θ, formed by the slit separation and the perpendicular line between the light rays, is the same as the angle between the

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and click on **Web Links** for an in-depth activity about the interference effects of light.

horizontal line going to the screen and the direction of the rays going toward the screen. The result of this analysis can be expressed mathematically as shown in the following box.

CONSTRUCTIVE INTERFERENCE

A bright fringe will appear on a screen when an integer number of wavelengths of light is equal to the product of the slit separation and the sine of the angle between the slit separation and the line perpendicular to the light rays leaving the slits.

$$n\lambda = d\sin\theta \qquad \text{where } n = 0, 1, 2, 3, \ldots\ldots$$

Quantity	Symbol	SI unit
integer number of full wavelengths	n	none
wavelength of light	λ	m (metres)
distance between slits	d	m (metres)
angle between slit separation and line perpendicular to light rays	θ	unitless (degrees are not a unit)

Unit Analysis

metre = metre   m = m

Inspection of part (D) of Figure 9.11 shows that when the path difference is a half wavelength, the light waves that leave the slits are out of phase and experience destructive interference. When the waves reach the screen, they will cancel each other and the screen will be dark. Between the bright and dark fringes, the screen will appear to be shaded. A complete analysis shows that when the path difference is exactly half a wavelength more than any number of full wavelengths, the waves will destructively interfere and a dark spot or dark fringe will appear on the screen. This condition can be described mathematically as

$$\left(n - \frac{1}{2}\right)\lambda = d\sin\theta \qquad \text{where } n = 1, 2, 3, \ldots$$

In a typical experiment, you would not be able to measure the path difference or the angle θ. Instead, you would measure the distance y_{n} between the central bright fringe and another bright fringe of your choice. You could then determine the angle θ by applying trigonometry to part (B) of Figure 9.11, which gives

$$\tan\theta = \frac{y_{\text{n}}}{x}$$

In this expression, the variable n has the same meaning as it does in the previous relationships. When $n = 1$, the path difference is one full wavelength and y_1 describes the distance to the first bright fringe.

For very small angles, you can make an approximation that combines the two relationships above, as shown below.

- For very small angles, the sine of an angle is approximately equal to the tangent of the angle. $\sin\theta \cong \tan\theta$
- Using this approximation, you can write the expression for the wavelength, as shown. $n\lambda \cong d\tan\theta$
- Substitute the expression for $\tan\theta$ and substitute into the equation for the wavelength. $\tan\theta = \frac{y_n}{x}$ $n\lambda \cong d\frac{y_n}{x}$

You can set n equal to 1 by using the distance between adjacent fringes and obtain the relationship shown in the following box.

APPROXIMATION OF THE WAVELENGTH OF LIGHT

The wavelength of light is approximately equal to the product of the distance between fringes and the distance between slits, divided by the slit-to-screen distance.

$$\lambda \cong \frac{\Delta yd}{x}$$

Quantity	Symbol	SI unit
wavelength	λ	m (metres)
distance separating adjacent fringes	Δy	m (metres)
distance between slits	d	m (metres)
distance from source to screen	x	m (metres)

Note 1: The distance between nodal line centres is identical to the distance between bright fringe centres. Therefore, this relationship applies equally to dark fringes (nodal lines) and bright fringes.

Note 2: This relationship is based on an approximation. Use it only for very small angles.

INVESTIGATION 9-C

Young's Double-Slit Experiment

TARGET SKILLS

- Predicting
- Identifying variables
- Communicating results

Young's ingenious double-slit experiment is readily duplicated with only simple equipment. In this investigation, you will reproduce results similar to those that Young produced — the results that convinced the scientific community that light was a wave.

Problem

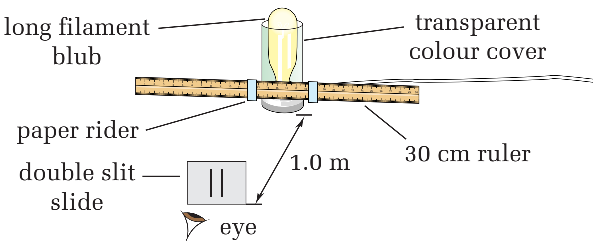
Is it possible to produce an interference pattern using light?

Prediction

- Make a prediction about the requirements of the experimental design that will be necessary to produce an interference pattern based on the wavelength of light and the nature of incandescent light sources.
- Make a second prediction about the nature of an interference pattern produced with short wavelength light (such as blue or green) compared to longer wavelength light (such as yellow or red). Which wavelength will allow you to make the most accurate measurements? Explain your prediction in detail.

Equipment

- long filament light source
- magnifying glass
- double-slit slides
- transparent colour light covers
- metre stick
- ruler with fractions of a millimetre markings
- 30 cm ruler



Procedure

1. Using a magnifying glass and finely ruled ruler, measure and record the centre-to-centre width of the slit separation.
2. Cut two paper riders for the 30 cm ruler to mark the width of the observed interference pattern.
3. Place a transparent colour cover over the portion of the light bulb with the straightest filament.
4. Place the 30 cm ruler with paper riders in front of the bulb. With your eye exactly 1 m from the bulb, observe the filament through the double slits.
5. Count the number of bright *or* dark fringes that you are able to clearly distinguish. Use the paper riders to mark off the edges of the observed fringes.
6. Repeat the experiment, varying the slit width and the wavelength of light.

Analyze and Conclude

1. Describe the effect on the observed interference pattern of **(a)** altering the slit width and **(b)** altering the wavelength of light.
2. Use your data to determine the wavelength of light used for each trial. How well did your calculated wavelength compare to expected values?
3. Was it easier to obtain data on one wavelength than on the others? If so, was your original prediction accurate? Explain.

Apply and Extend

CAUTION *Do not* look directly into the laser.

4. If time permits, use a helium-neon laser to verify the double-slit equation. Place a double-slit slide of known width in front of the laser beam. Observe the interference pattern on a screen a known distance from the laser. Calculate the wavelength of laser light. Determine the percentage deviation of the calculated value and your experimental value.

Single-Slit Diffraction

Wavelets originating at two separate but closely spaced sources produce clear interference patterns, as observed in Young's experiment. It is also possible to obtain an interference pattern from a single slit.

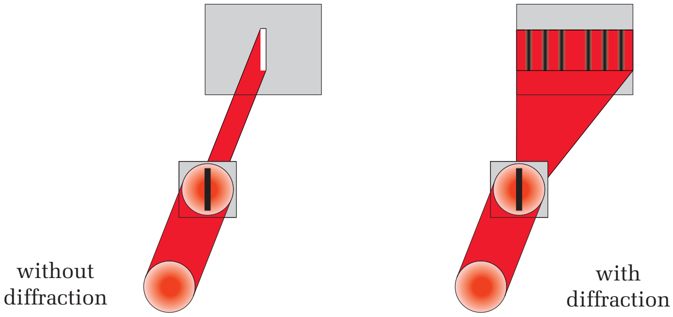


Figure 9.12 If diffraction did not occur, only a thin sliver of light would appear on the screen. In fact, an interference pattern results from the diffraction of incident light through a single slit.

The diagrams in Figure 9.13 illustrate two types of single-slit diffraction that might occur. The **Fraunhofer diffraction** pattern results when the parallel rays of light (straight or planar wavefronts) are incident on the slit. Fraunhofer diffraction produces clear interference patterns that are readily analyzed when a converging lens is used to bring the parallel rays into focus. When the incident light rays are not parallel, **Fresnel diffraction** occurs. Analysis of Fresnel diffraction requires mathematical processes that are beyond the scope of this course. However, analysis of both Fraunhofer and Fresnel diffraction is based on Huygens' principle.

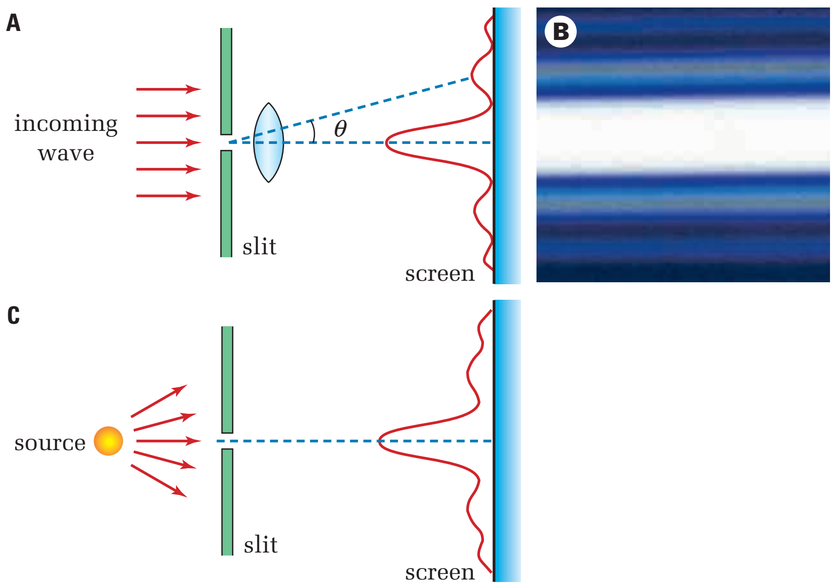


Figure 9.13 **(A)** Fraunhofer diffraction pattern created from parallel rays striking a single slit. **(B)** Photograph of Fraunhofer diffraction. Note the double-wide central maximum and the reduction of intensity of subsequent fringes. **(C)** Fresnel diffraction patterns are created when incident rays falling onto a single slit are not parallel.

To apply Huygens' principle to single-slit diffraction, imagine many point sources across the single slit. Wavelets produced by each source will interfere with each other, generating an interference pattern on a screen. Wavelets passing directly through the slit interfere constructively, producing a bright central fringe.

To understand how the destructive interference occurs, visualize the slit as two halves. Each of the infinite number of wavelets — represented and simplified by numbers 1 to 5 — are in phase as they pass through the opening. The wavelets leaving the slit at an angle θ will no longer be in phase. For example, wavelet 3, originating at the middle of the slit, will travel farther than wavelet 1 by a path difference of $\frac{1}{2}\lambda$. The wavelet just below wavelet 1 will be exactly $\frac{1}{2}\lambda$ ahead of the wavelet just below wavelet 3. In this way, all wavelets in the top half of the slit will interfere destructively with wavelets from the lower half of the slit. When the wavelets reach the screen, they will interfere destructively, as shown in Figure 9.14.

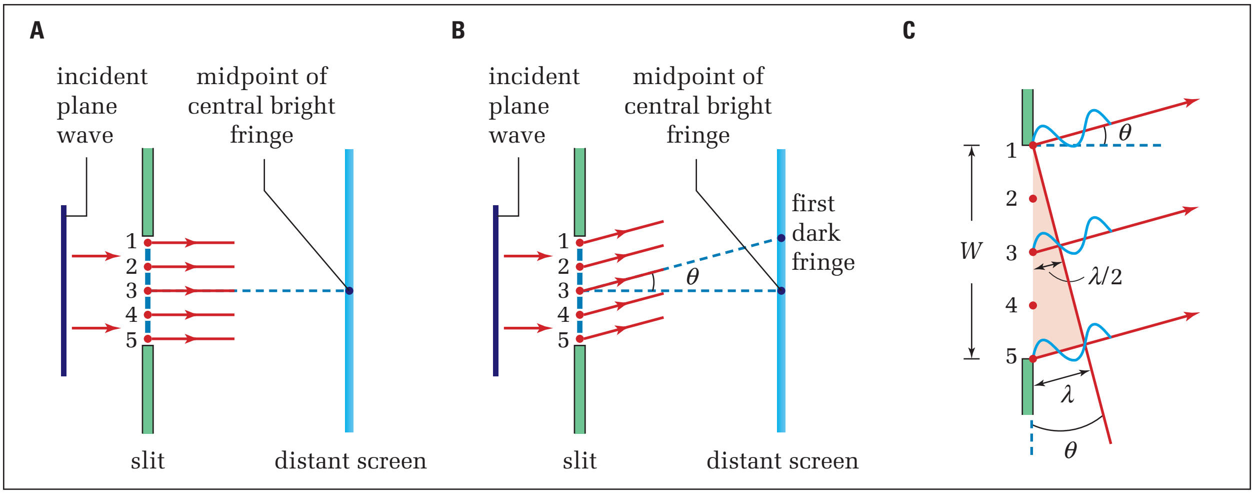


Figure 9.14 **(A)** The plane wave incident on the single slit is shown as five sources for Huygens' wavelets. The wavelets interfere constructively at the central midpoint, generating a large, bright central maximum. **(B)** The first nodal line (dark fringe) occurs when the Huygens' wavelets from each source interfere destructively. **(C)** Destructive interference requires a path difference of $\frac{1}{2}\lambda$.

The second dark fringe measured at an angle of θ is shown in Figure 9.15. Again, consider the single slit as two separate halves. This second-order dark fringe results because the path difference travelled between wavelets 1 and 2 is exactly $\frac{1}{2}\lambda$. The wavelets just under wavelet 1 and wavelet 2 will also strike the distant screen exactly $\frac{1}{2}\lambda$ apart. This process repeats for the entire top half of the slit. The wavelets in the bottom half of the slit interfere in the same way as do those in the top half. The net result is a dark fringe. Between the dark fringes, wavelets interfere constructively, forming bright fringes.

The process of destructive interference occurs repeatedly for angles that produce a path difference that is an integral multiple of the wavelength of light.

$$\sin\theta = \frac{m\lambda}{W} \qquad (m = \pm1, \pm2, \pm3 \ldots)$$

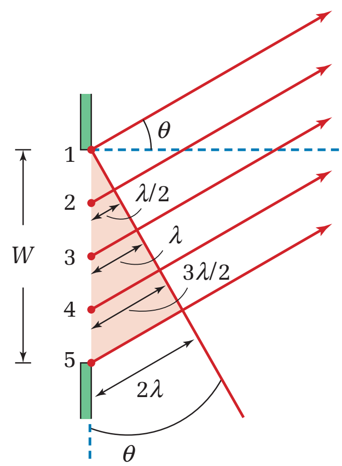


Figure 9.15 The second nodal line occurs when the path difference of every wavelet from the top half of the slit interferes with each wavelet from the bottom half.

The distance to the screen is much larger than the slit width or the separation of the dark fringes. Therefore, the perpendicular distance to the screen, L, is approximately the same length as the distance from the slit to the dark fringes, L_1, L_2, L_3, ..., as shown in Figure 9.16.

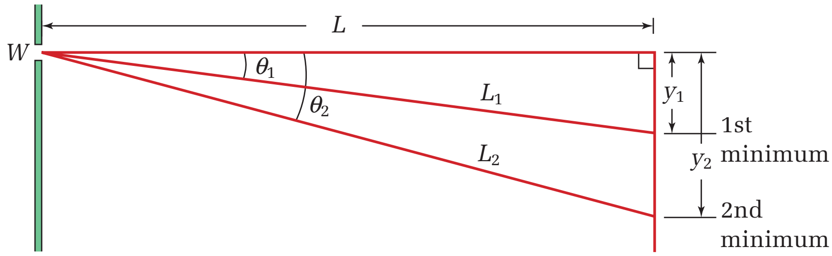


Figure 9.16 Slit width, W, is much smaller than the distance to the screen, L, allowing for the approximation that $L_1 \approx L_2 \approx L_n \approx L$.

Because $L >> y$, $\sin\theta = m\lambda/W$ $(m = \pm1, \pm2, \pm3\ldots)$ can be approximated as

$$\tan\theta = m\lambda/W \qquad (m = \pm1, \pm2, \pm3\ldots)$$

$$y_m/L = m\lambda/W \qquad (m = \pm1, \pm2, \pm3\ldots)$$

$$y_m = m\lambda L/W \qquad (m = \pm1, \pm2, \pm3\ldots)$$

SINGLE-SLIT INTERFERENCE

Dark fringes will exist on a distinct screen at regular, whole-numbered intervals

$$y_m = \frac{m\lambda L}{W} \qquad (m = \pm1, \pm2, \pm3\ldots) \quad \text{Destructive}$$

$$y_m = \frac{(m + \frac{1}{2})\lambda L}{W} \qquad (m = \pm1, \pm2, \pm3\ldots) \quad \text{Constructive}$$

Quantity	**Symbol**	**SI unit**
distance to fringe from the central bisector	y_m	m (metres)
distance to screen	L	m (metres)
fringe order number $(\pm1, \pm2, \pm3\ldots)$	m	unitless
width of slit	W	m (metres)

Notice that the intensity of the bright fringes drops off dramatically in higher-order fringes. Intensity is a result of the amount of light energy striking a unit area per second. Recall that only wavelets interacting with the slit's edge are diffracted. This generates a double-wide central bright fringe, created by the light travelling directly through to the screen, unaffected by the slit's edge, plus the constructively interfering diffracted wavelets. The intensity steadily decreases as wavelets begin to destructively interfere, due to subtle changes in path difference.

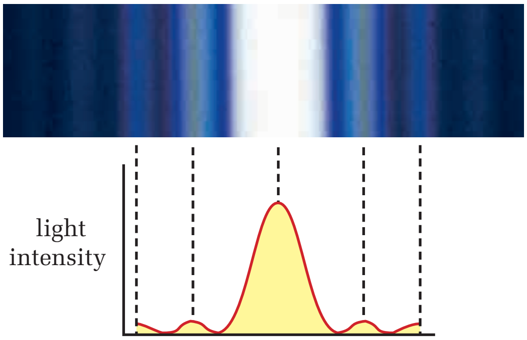


Figure 9.17 Bright and twice as wide, the central maximum appears much more intense than subsequent fringes.

SAMPLE PROBLEM

Single-Slit Diffraction

Viewing a 645 nm red light through a narrow slit cut into a piece of paper yields a series of bright and dark fringes. You estimate that five dark fringes appear in a space of 1.0 mm. If the paper is 32 cm from your eye, calculate the width of the slit.

Conceptualize the Problem

- *Light* passing through a very *narrow slit* will be *diffracted*, causing an interference pattern to be visible.
- *Dark fringes* result from *destructive interference*.
- The distance, y_1 , to the first dark fringe can be calculated by first determining the space between fringes, Δy.

Identify the Goal

The width, W, of the single slit

Identify the Variables and Constants

Known	Unknown
$m = 1$	Δy
$\lambda = 645$ m	y_m for $m = 1$
$L = 0.32$ m	W

Develop a Strategy

Determine the fringe spacing. Recall that there is always one less space between the fringes than there are fringes.

5 fringes in 1.0 mm = $4\Delta y$
$\Delta y = 0.25$ mm

continued ►

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The distance between dark fringes, Δy, is also the distance from the central bisector to the first dark fringe.
Slit width can be determined by using the single-slit interference equation for dark fringes.

$y_1 = 0.25$ mm, $m = 1$

$$y_m = \frac{m\lambda L}{W}$$

$$W = \frac{m\lambda L}{y_1}$$

$$W = \frac{1(645 \times 10^{-9}\ \text{m})(0.32\ \cancel{\text{m}})}{0.25 \times 10^{-3}\ \cancel{\text{m}}}$$

$$W = 8.256 \times 10^{-4}\ \text{m}$$

$$W \cong 8.3 \times 10^{-4}\ \text{m}$$

The slit is 0.8 mm wide.

Validate the Solution

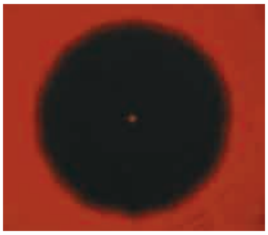
Interference fringes resulting from a single slit cut into a piece of paper would be very difficult to observe. The number, 5, must be taken as a number containing only one significant digit. The final answer is therefore provided to only one significant digit. A width of 0.8 mm is reasonable.

PRACTICE PROBLEMS

1. Determine the distance that the third bright fringe would lie from the central bisector in a single-slit diffraction pattern generated with 542 nm light incident on a 1.2×10^{-4} m slit falling onto a screen 68 cm away.
2. A special effects creator wants to generate an interference pattern on a screen 6.8 m away from a single slit. She uses 445 nm light and hopes to get the second dark fringe exactly 48 cm from the middle of the central bright maximum. What width of slit does she require?
3. Predict whether violet light ($\lambda = 404$ nm) or red light ($\lambda = 702$ nm) will have a wider central maximum when used to generate a single-slit diffraction pattern. Calculate the difference if the light is incident on a 6.9×10^{-5} m wide slit falling onto a screen 85 cm away.

9.2 Section Review

1. K/U Diffraction is defined as the spreading of light that passes by the edge of an opaque barrier. Explain why the term "spreading" is used rather than "bending."
2. I Suggest a property of light that posed the greatest difficulty for physicists attempting to observe the diffraction of light.
3. C This photograph is the result of illuminating a penny with a single point source of light. Describe what must be occurring in order to form the observed image both in the centre and around the edges.



4. K/U Describe what is meant by the term "coherent sources."
5. MC Why did the success of Young's experiment convince physicists of the time that light was some type of wave?
6. K/U Double-slit interference patterns form with equal spacing between light and dark fringes. Single-slit diffraction generates an interference pattern containing a central bright fringe that is twice as wide as any other. Explain these differing results.

9.3 Examples and Applications of Interference Effects

Not only did Young's double-slit experiment demonstrate the wave nature of light, it also paved the way for applications of interference and explained many phenomena that had been observed but not understood. If fact, Newton himself had observed some effects of interference of light, but he did not know that interference caused these effects.

SECTION EXPECTATIONS

- Describe how new technology resulted in the advancement of scientific theory.
- Outline the scientific understanding made possible through technological devices.
- Analyze thin films using diffraction, refraction, and wave interference.

KEY TERMS

- diffraction grating
- line spectrum (emission spectrum)
- resolving power
- Rayleigh criterion

Diffraction Gratings

Hold a compact disc in sunlight and a rainbow of colours will appear. Observe an Indigo snake moving through bright light and the full spectrum of colours will shimmer across its scales. The colours are separated from white light by diffraction from hundreds, even thousands, of tiny parallel ridges.

Figure 9.18 A compact disc has a thin transparent coating over a shiny metallic disk. What is the source of the rainbow colours?

If two slits are good, are 2000 slits better? For many applications, 2000 slits are definitely better. Such a device, called a **diffraction grating**, can create very fine, bright fringes that are separated by large dark fringes. A typical diffraction grating has several thousand slits or lines per centimetre. For example, a grating with 2000 lines/cm would have a slit spacing $d = (1/2000)\text{ cm} = 5 \times 10^{-4}\text{ cm}$. Diffraction gratings might be transmission gratings (light passes through the slits) or reflective gratings, in which light is reflected by smooth lines separated by non-reflective surfaces.

The principle on which a diffraction grating is based is the same as that of a double slit. The diffraction grating simply has thousands of pairs of double slits that all work together. As shown in Figure 9.19, constructive interference occurs when the distance travelled by a light ray from one slit is longer than that of the adjacent slit by an integral multiple of the wavelength of light.

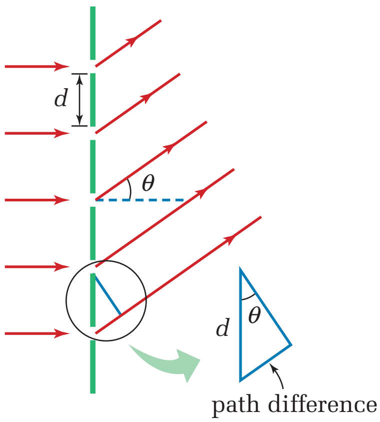


Figure 9.19 At very precise angles, the path difference travelled by waves passing through any pair of slits in the entire diffraction grating is an integer multiple of the wavelength of the light.

When $m = 0$ and the path lengths of all of the rays are the same, the rays go directly through the grating, creating a central bright fringe. The next bright fringe above or below the central fringe is called the "first-order fringe." The naming continues with second order, third order, and up to the last visible fringe.

DIFFRACTION GRATING BRIGHT FRINGES

Bright fringes will strike the screen when the path is an integer number of wavelengths of light.

$$m\lambda = d \sin \theta \qquad (m = 0, 1, 2, \ldots)$$

Quantity	Symbol	SI unit
wavelength of light	λ	m (metres)
integer number of wavelengths (0, 1, 2, …)	m	none
distance between slit centres	d	m (metres)
angle from horizontal to the ray resulting from constructive intererence	θ	unitless (degree is not a unit)

Note: Both transmission and reflection diffraction gratings can be modelled with this relationship.

The advantage of a diffraction grating over a double slit is the amount of destructive interference between the peaks of constructive interference. At the precise angles given by the equation $m\lambda = d \sin \theta$, waves from every slit interfere constructively with each other. However, at any other angle, waves from some combination of slits interfere destructively with each other. Figure 9.20 shows a typical double-slit pattern and compares it with a pattern obtained with five slits. With thousands of slits, the peaks become fine vertical lines and the space between is flat.

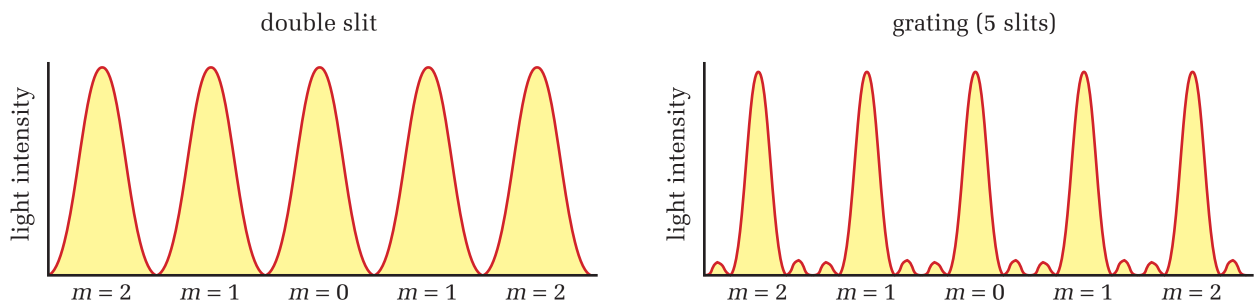


Figure 9.20 Destructive interference from distant fringes generates very narrow, bright fringes, compared to double-slit interference patterns.

For a given diffraction grating with constant slit separation, the angle that results in constructive interference depends on the wavelength of the light. Since different colours have different wavelengths, colours are separated when light passes through a grating, as shown by the rainbow of colours in Figure 9.21 (A). Figure 9.21 (B) shows what you would see if two colours passed through a diffraction grating together. This property of diffraction gratings makes them very useful in several types of instruments.

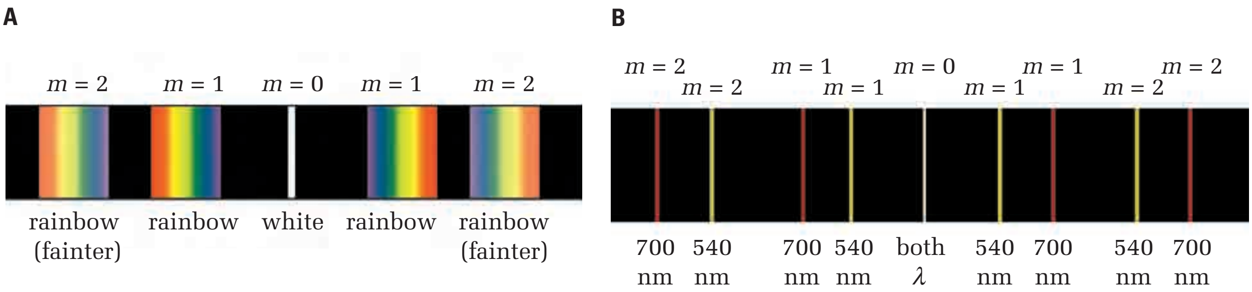


Figure 9.21 **(A)** A diffraction grating will separate white light into a rainbow of colours, because different wavelengths will be diffracted by different amounts. Higher-order fringes are more spread out. **(B)** This theoretical result would occur if a two-wavelength (700 nm, 540 nm) source was viewed through a spectroscope.

A spectroscope uses a diffraction grating to separate light into very narrow bands of specific colours (wavelengths) that you can then analyze. For example, you can identify the atoms or molecules in a gas discharge tube. When a gas is heated or has an electric discharge passed through it, it will emit light at very specific wavelengths. The set of wavelengths emitted by a pure substance is called the substance's **line spectrum** or **emission spectrum**. Figure 9.22 shows the line spectrum of several common substances.

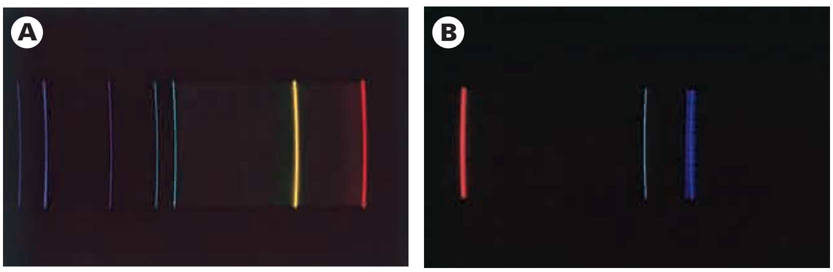


Figure 9.22 Line or emission spectrum of pure **(A)** helium, **(B)** atomic hydrogen, and **(C)** cadium as viewed through a spectroscope.

ELECTRONIC LEARNING PARTNER

Your Electronic Learning Partner contains an excellent reference source of emission and absorption spectra for every element in the periodic table.

Spectroscopes can also analyze absorption spectra. For example, the core of the Sun emits a continuous spectrum. However, atoms and molecules in the Sun's outer atmosphere absorb specific wavelengths, causing the Sun's spectrum to have several narrow black lines. Atoms and molecules also absorb light at the same wavelengths at which they emit it. Therefore, by identifying the wavelengths of light that have been absorbed by the Sun's outer atmosphere, physicists are able to identify the atoms that are present there. Careful analysis of the Sun's absorption spectrum reveals that at least two thirds of all elements present on Earth are present in the Sun. In fact, this technique is used to identify the composition of stars throughout our galaxy.

An instrument called a "spectrophotometer" is used in chemistry and biochemistry laboratories to identify and measure compounds in solutions. A spectrophotometer has a diffraction grating that separates white light into all wavelengths. You can select a specific wavelength and send it through a sample of a solution. The spectrophotometer then measures the amount of light of the wavelength that is absorbed by the sample and you can then calculate the concentration of the compound in the solution.

Interference of Thin Films

Soap bubbles always shimmer, with colour flowing across their surface. This interference phenomenon is a result of light reflecting off both surfaces of a thin film. To understand what happens when light strikes a thin film, review the process of reflection that is illustrated in Figure 9.23. In the upper left-hand photograph, a wave is moving through a "slow" medium (a heavy spring) toward an interface with a "fast" medium (a lightweight spring). On reaching the interface, both the reflected and the transmitted wave remain on the same side of the spring. In the lower left-hand photograph, a wave is moving from the right within the "fast" medium to the left toward the "slow" medium. When the wave reaches the interface, the transmitted wave remains on the same side of the medium, but the reflected wave has undergone a phase change or inversion and is on the opposite side of the medium. This change of phase or inversion that occurs when a wave reflects off an interface with a "slower" medium is a property of all waves.

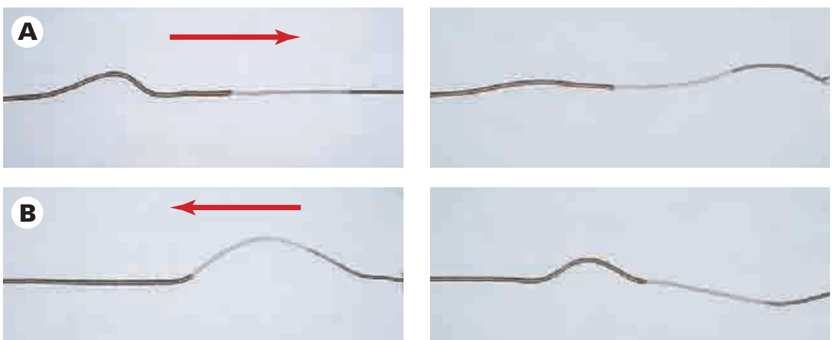


Figure 9.23 **(A)** At a slow-to-fast interface between two media, the transmitted and reflected pulses are on the same side of the spring. **(B)** At a fast-to-slow interface between two media, the transmitted pulse is on the same side of the spring, but the reflected pulse is inverted.

Very Thin Films: Destructive Interference ($t << \lambda$)

Destructive interference occurs when the film thickness, t, is much less than the wavelength, λ, of incident light, $t << \lambda$. Light travelling through air toward the very thin film of a soap bubble will behave in the same way that the wave in the spring behaves. Light incident on the surface of the soap bubble will be partially reflected and partially transmitted. In Figure 9.24, since the reflected wave (1) encounters the surface of a more-dense and therefore "slower" medium, it undergoes a phase change. When the transmitted light reaches the far surface of the soap film, it reflects off the interface with air, a "faster" medium, and therefore the reflected wave does not undergo a phase change, as indicated in Figure 9.24.

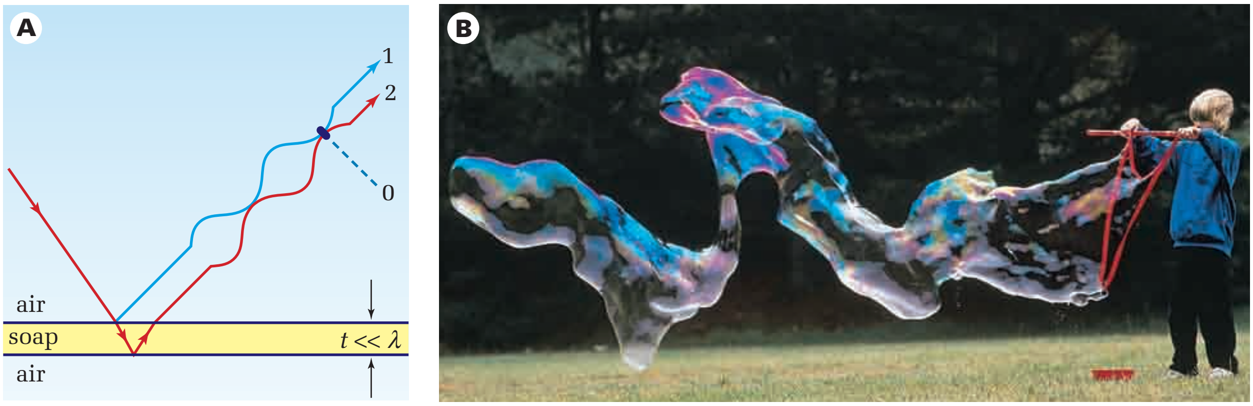


Figure 9.24 **(A)** Phase inversion causes destructive interference in very thin film when the film thickness, t, is much less than the wavelength of light, λ. **(B)** A soap bubble varies slightly in thickness, causing the destructive interference of varying colours.

In Figure 9.24, the thickness of the soap film is exaggerated relative to the wavelength of the light. In reality, the distance that wave 2 travels farther than wave 1 is negligible. The result is that when the inverted wave 1 rejoins reflected wave 2, the two waves are out of phase and undergo destructive interference. In the case of a soap bubble, the thickness varies as material flows throughout the film. The small fluctuations in thickness determine which wavelengths of light interfere destructively. The remaining wavelengths provide the colour that your eye sees.

Thin Films: Constructive Interference ($t = \lambda/4$)

Thin films that have a thickness of approximately a quarter of the incident wavelength of light cause constructive interference (see Figure 9.25). Once again, wave 1 reflects off an interface with a more-dense (slower) medium and therefore undergoes a phase change or inversion. In this case, the thickness of the film is significant. Wave 2 travels a half wavelength farther than wave 1. As a result, the two waves rejoin each other in phase and undergo constructive interference.

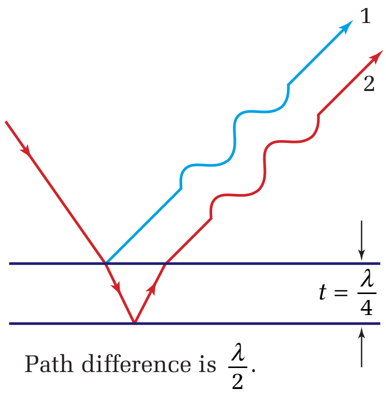


Figure 9.25 A total path difference of $\lambda/2$ combined with a phase inversion of one wave results in constructive interference.

Thin Films: Destructive Interference ($t = \lambda/2$)

A thin film with a thickness of one half of the wavelength of the incident light will cause destructive interference. Once again, wave 1 experiences a phase inversion at the top (fast-to-slow) reflecting surface. Wave 2 does not experience a phase inversion at the bottom (slow-to-fast) boundary, but does travel farther by a distance equal to one wavelength. Transmitted waves do not experience a phase shift; therefore, wave 1 and wave 2 proceed exactly out of phase and interfere destructively.

Although thin films are often just an attractive novelty, they can be useful. Eyeglass manufacturers make use of destructive interference caused by thin films to prevent reflection from lenses. By applying a coating of a carefully designed thickness to the outer surface of eyeglass lenses, specific wavelengths of reflected light can be cancelled.

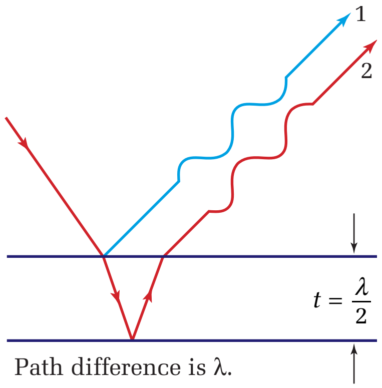


Figure 9.26 A total path difference of one wavelength, λ, combined with a phase inversion of one wave results in destructive interference.

Digital Videodiscs

A digital videodisc (DVD) is composed of several layers of plastic on a reflective aluminum disc. The distinctive gold colour of a DVD results from a semi-reflecting layer of gold used to separate each data layer. Data is stored in the plastic layers as series of pits or bumps that follow a spiral path from close to the centre of the disc to the outer edge. On the reflective aluminum side of a DVD, they are pits, but on the side from which the laser reads information, they are actually "bumps." Each bump has a thickness of exactly 120 nm, which is one fourth of a wavelength of the 640 nm laser light that reads the bumps.

DVD players read the information by shining laser light on the edge of the bumps and detecting the reflections. Part of the laser beam falls on the flat surface of the disc, while the other part strikes the bumps. Since the bumps are one fourth of a wavelength, the part of the beam striking the bump travels a distance that is half a wavelength shorter than the part that remains on the flat surface. When both parts of the reflected beam rejoin each other, destructive interference occurs in a way that is similar to thin film interference. The detector converts the differences in the amount of reflected laser light into the images and sound that you observe on your DVD player.

If it was possible to lift the long spiral of data-storing bumps off a DVD and stretch it into a straight line, the line would extend more than 48 km. A double-sided DVD can store up to seven times more information than a compact disc — approximately 15.9 GB. The increased data storage results from a combination of more efficient techniques to digitize the data and tighter data "bump" spacing, thanks to the use of shorter wavelength laser light. Each lap of the spiral is spaced exactly 740 nm from the previous one.

The series of tracks behaves like a diffraction grating, producing the varying colour patterns that you see when light reflects from the surface of a DVD.

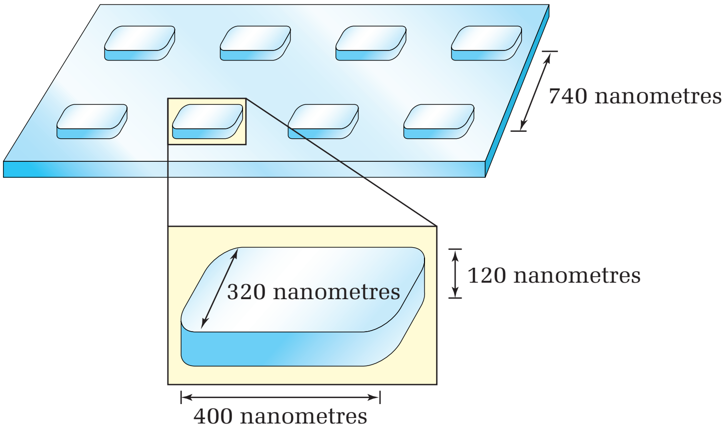


Figure 9.27 Data are stored on DVDs in the form of tiny "bumps" that are less than the wavelength of light.

Resolving Power

A light travels toward you from the distance. Is the object a motorcycle with a single headlight or a car with two separated headlights? As the object approaches, the single light slowly grows into an oval shape and then finally into two individual and distinct headlights of a car. The ability of an optical instrument, such as the human eye or a microscope, to distinguish two objects is called the **resolving power** of the instrument. An eagle has eyesight that is much better at resolving objects than is the human eye. A microscope uses lenses with resolving powers that are even greater.

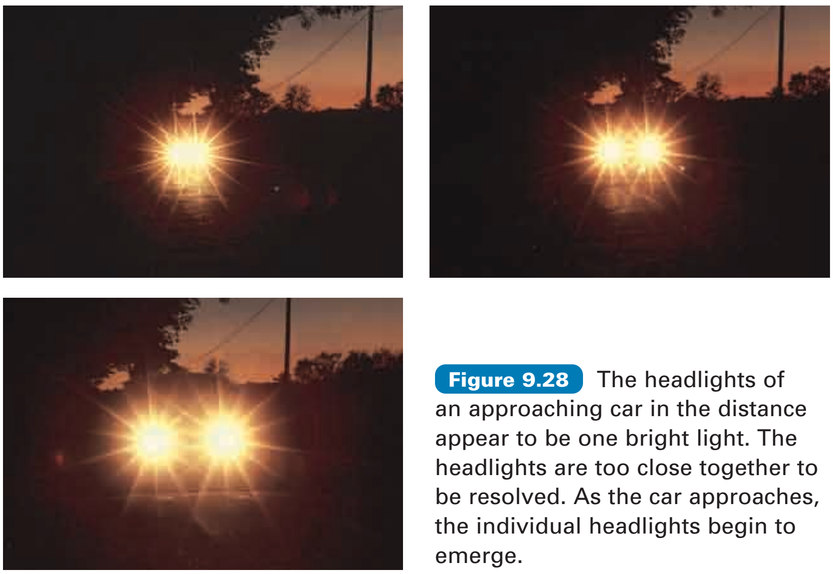


Figure 9.28 The headlights of an approaching car in the distance appear to be one bright light. The headlights are too close together to be resolved. As the car approaches, the individual headlights begin to emerge.

Light travelling through a small opening or aperture is diffracted. To distinguish between two objects through a single aperture (such as the pupil of an eye), the central bright fringes of the two sources must not overlap. Although many factors might affect the ability of an instrument to resolve two objects, the fundamental factors are the size of the aperture, the distance between the two objects, and the distance between the aperture and the objects. Lord Rayleigh (John William Strutt: 1842–1919) suggested a criterion that is still practical for use today. The **Rayleigh criterion** states that "Two points are just resolved when the first dark fringe in the diffraction pattern falls directly on the central bright fringe in the diffraction pattern of the other." Figure 9.29 illustrates the Rayleigh criterion.

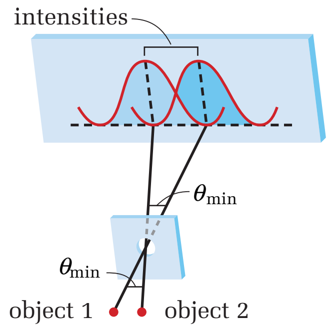


Figure 9.29 The ability to resolve two objects occurs when the first dark fringe just falls on the other object's first bright fringe. This is known as the Rayleigh criterion.

Recall that for a single slit, the first dark fringe occurs when $\lambda = W \sin \theta$. Since the Rayleigh criterion states that the central fringe of the second object must be no closer to the first object than its first dark fringe, you can use the same equation to describe the spatial relationship between the two objects. Since resolution depends on both the distance between the objects and their distance from the aperture, it is convenient to combine those distances and express them as the angle between rays coming from each object to the aperture. This is exactly the angle θ in the equation above. Therefore, the Rayleigh criterion for a single slit aperture is as follows.

$$\sin \theta = \frac{\lambda}{W}$$

For very small angles, the sine of the angle is numerically almost the same as the angle itself expressed in radians. Since the angles describing resolving power are always very small, you can express the Rayleigh criterion as shown below.

$$\theta = \frac{\lambda}{W}$$

Most optical instruments use circular apertures, rather than rectangular ones. Experimental evidence shows that the minimum angle that a circular aperture is just able to resolve is as follows.

$$\theta = \frac{1.22\lambda}{D}$$

D is the diameter of the aperture and, once again, the angle θ is expressed in radians.

RESOLVING POWER

In order to resolve two objects, the minimum angle between rays from the two objects passing through a rectangular aperture is the quotient of the wavelength and the width of the aperture. For a circular aperture, the minimum angle is the quotient of 1.22 times the wavelength and the diameter of the aperture.

$$\theta_{min} = \frac{\lambda}{W}$$ rectangular slit aperture

$$\theta_{min} = \frac{1.22\lambda}{D}$$ circular aperture

Quantity	Symbol	SI unit
minimum angle for resolution	θ	unitless (radian is not a unit)
wavelength of light	λ	m (metres)
width of rectangular aperture	W	m (metres)
diameter of circular aperture	D	m (metres)

Note: The angle measure must be provided in radians.

$360^\circ = 2\pi$ rad or 1 rad $= 57.3^\circ$

SAMPLE PROBLEM

Resolving Power

A skydiver is falling toward the ground. How close to the ground will she have to be before she is able to distinguish two yellow baseballs lying 25.0 cm apart, reflecting 625 nm light in air? Her pupil diameter is 3.35 mm. Assume that the speed of light inside the human eye is 2.21×10^8 m/s.

Conceptualize the Problem

- The human pupil is a circular opening; therefore, the circular aperture equation for resolving power applies.
- The wavelength of light will be different inside the material of the eye because the speed is less than it would be in air. The reduced speed must be used to calculate the wavelength of the light in the eye.

Identify the Goal

The maximum height, h, at which the skydiver can resolve the two objects that are 0.250 cm apart.

Identify the Variables and Constants

Known	Unknown
$s = 25.0$ cm	λ_{eye}
$\lambda_{air} = 625$ nm	θ_{min} (radians)
$D = 3.35$ mm	
$v_{eye} = 2.21 \times 10^8 \frac{m}{s}$	

continued ►

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Develop a Strategy

- Determine the wavelength of the light inside her eye. Use the wave equation and the fact that the frequency of a wave does not change when it passes from one medium into another.

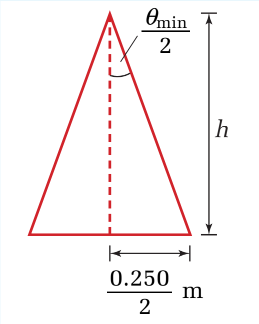
$$v = f\lambda$$
$$f = \frac{v}{\lambda}$$
$$f = \frac{v_{air}}{\lambda_{air}} = \frac{v_{eye}}{\lambda_{eye}}$$
$$\lambda_{eye} = \frac{\lambda_{air} v_{eye}}{v_{air}}$$
$$\lambda_{eye} = \frac{(625\ \text{nm})\left(2.21 \times 10^8\ \frac{\text{m}}{\text{s}}\right)}{3.00 \times 10^8\ \frac{\text{m}}{\text{s}}}$$
$$\lambda_{eye} = 460\ \text{nm}$$

- Determine the minimum angle for resolution, using the Raleigh criterion.

$$\theta_{min} = \frac{1.22\lambda}{D}$$
$$\theta_{min} = \frac{(1.22)(460 \times 10^{-9}\ \text{m})}{3.35 \times 10^{-3}\ \text{m}}$$
$$\theta_{min} = 1.6767 \times 10^{-4}\ \text{rad}$$

- Divide the isosceles triangle in half, as shown, and use the tangent function to calculate the height. (Hint: Remember that the equation gives the angle in radians. Be sure that your calculator is set to radians while performing your calculations.)

$$\tan\left(\frac{\theta_{min}}{2}\right) = \frac{\frac{0.250\ \text{m}}{2}}{h}$$
$$h = \frac{0.125\ \text{m}}{8.3837 \times 10^{-5}}$$
$$h = 1.491 \times 10^3\ \text{m}$$
$$h \cong 1.49\ \text{km}$$



If only resolving power is considered, she will just be able to distinguish the baseballs when she is 1.49 km from the ground.

Validate the Solution

Resolving objects separated by 25 cm could not possibly be accomplished from a distance of 1.5 km. Most likely, she would have to be much closer, because of the effects caused by the high-speed descent. Owls have pupils that are as much as 10 times larger than human pupils. How would such an adaptation be helpful?

PRACTICE PROBLEMS

4. (a) Commercial satellites are able to resolve objects separated by only 1.0 m. If these satellites orbit Earth at an altitude of 650 km, determine the size of the satellites' circular imaging aperture. Use 455 nm light for the light in the lenses of the satellites.

(b) Describe why the value from part (a) is a theoretical best-case result. What other effects would play a role in a satellite's ability to resolve objects on the surface of Earth?

5. Calculate the resolving power of a microscope with a 1.30 cm aperture using 540 nm light. The index of refraction of the lens slows the light inside the glass to 1.98×10^8 m/s.

6. You are about to open a new business and need to select a colour scheme for your backlighted sign. You want people to be able to see your sign clearly from a highway some distance away. Assuming that brightness is not a problem for either colour, should you use blue or red lettering? Develop an answer, using resolving power arguments. Include numerical examples.

Concept Organizer

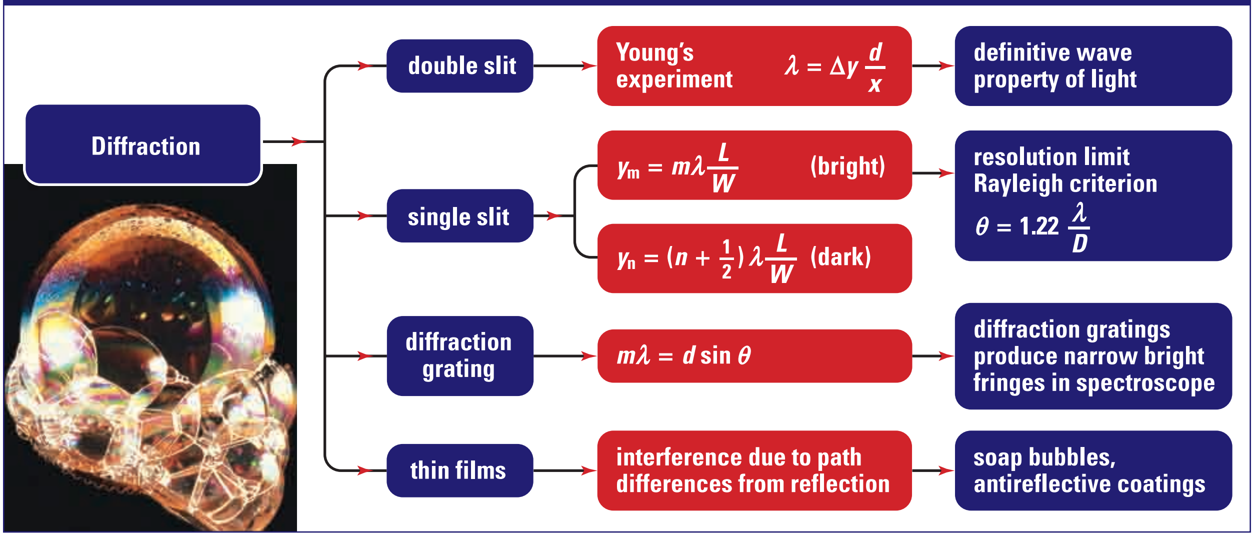


Figure 9.30 Diffraction

9.3 Section Review

1. K/U Why does a diffraction grating produce much narrower bright fringes than a double-slit interference pattern?

2. I Describe why an astronomer would pass light from a distant star through a diffraction grating? Provide the name of the instrument used and possible facts that could be learned from its use.

3. K/U Classify each of the following examples as one of **(a)** very thin film ($t << \lambda$) or **(b)** thin film ($t = \lambda/4$) or ($t = \lambda/2$) interference.
   - oil floating on water
   - antireflective coating on a television screen

4. K/U Describe how DVD technology makes use of thin film interference and partial reflection-partial refraction.

5. MC Looking through a terrestrial telescope, you see a single light shining from a distant farmhouse. The single image is the result of light streaming from two identical windows placed side by side. Describe two possible steps you could take so that the lighted windows would appear individually.

6. K/U Why is resolving power often expressed as an angle?

New Views of Earth's Surface

It's a dark and stormy night — perfect conditions to study minute changes in Earth's crust. Not from Earth's surface, but from radar satellites hundreds of kilometres away in space. Optical satellite images are used to observe weather patterns, but the data used to generate radar images contain more information than is displayed in the optical images. This additional information can be exploited to provide precision measurements of Earth's surface.

Radar satellites have two distinct advantages over optical satellites. First, they operate at longer wavelengths, which allows them to penetrate clouds. Second, they provide their own illumination instead of relying on reflected sunlight, so they can obtain images day or night and, more importantly, coherent radiation can be used. In coherent radiation, the individual waves are all emitted in step or in "phase" with one another.

The Importance of Phase Information

This phase information provides the basis for satellite radar interferometry. When the satellite views a patch of Earth's surface at an angle, the distance from the target point on the surface to each of the satellite's two antennae will be different. Coherent radiation emitted by the satellite will be received in a particular phase by the first antenna and in a different phase by the second antenna because of the different path lengths. Effectively, the phase information is like a stopwatch that indicates how long the wave has been travelling. Since the wave travels at the speed of light, this time is easily converted into a distance. The phase difference between the waves in the received signals can be used to reconstruct the height of a point on Earth's surface. A couple of snapshots taken in seconds, or even in repeat satellite passes, can provide precise topographical detail that would take geologists and surveyors years to match.

If positions and heights can be accurately measured, then by spacing observations over time, changes in Earth's surface can be detected. In two radar images taken from the same altitude but at different times, each of the corresponding picture elements, (pixels) on the two images should have the same phase. If the ground has moved toward or away from the satellite in the time between the images, this would be detected as a phase difference in the pixels. This is easily seen by constructing an "interferogram."

Displaying the Phase Information

As the name suggests, the process of constructing an interferogram involves allowing light waves to interfere with each other. When any two waves combine, they can reinforce each other, cancel each other, or do something in between, depending on the relative phases. Suppose you represent places on the resultant image where two corresponding pixels (from images of the same area taken at different times) reinforce each other with red pixels, and places where they cancel each other with blue pixels. Cases in between these extremes can be represented by colours in the spectrum between red and blue. A cycle from red to blue would then indicate a displacement on the ground equivalent to half of a wavelength. A typical interferogram will show several complete colour cycles, or fringes, because the phase between any two pixels can differ by any number of whole wavelengths (1, 2, 3,…). By counting these fringes, the total displacement can be determined.

Radar Satellites in Operation Today

Although the four radar satellites presently in operation — the Canadian RADARSAT, the European ERS-1 and ERS-2, and the Japanese JERS-1 — orbit at an altitude of several hundred kilometres, radar interferometry allows them to monitor changes in Earth's surface on the scale of half of a radar wavelength, approximately 2 to 4 cm, or smaller. The method was first applied in 1992 to examine the deformation of Earth's crust after an earthquake in Landers, California. Although the maximum displacement of the fault was 6 m, researchers were able to detect a tiny slip of 7 mm on a fault located 100 km away from where the quake had struck. Since then, applications of the method have grown rapidly.

Examples of present uses of radar interferometry include the examination of a variety of geophysical phenomena. By studying the after-effects of earthquakes worldwide, critical information can be gained that someday could be used to predict earthquakes. The subsidence of surface land due to extraction of coal or oil can be monitored. The advance and recession of glaciers and ice flow velocities can be routinely studied to improve hydrological models and assess global climate change.

Several recent studies have applied radar interferometry to volcanoes. As magma fills or drains chambers under the volcano's surface, subtle deformation of the volcano, not detected by other methods, is revealed in interferograms. Detecting uplift and swelling of volcanoes could in some cases provide early warning of an eruption.

With some promising results so far, radar interferometry is destined to become an important tool for analyzing our ever-changing Earth.

Making Connections

1. Compare and contrast the information gained from radar interferometry and from land surveying.
2. Research the limitations involved in using radar interferometry.
3. The Canadian RADARSAT has just completed an important survey of Antarctica. Report on some of the findings.

CHAPTER 9 Review

REFLECTING ON CHAPTER 9

- Mechanical waves are disturbances that transfer energy from one location to another through a medium. All waves, under appropriate conditions, are known to exhibit rectilinear propagation, reflection, refraction, partial reflection and partial refraction, and diffraction.
- Light energy reaches Earth after travelling through the void of outer space. Light, under appropriate conditions exhibits rectilinear propagation, reflection, refraction, partial reflection and partial refraction, and diffraction.
- Waves interfere with one another according to the superposition of waves, which states: When two or more waves propagate through the same location in a medium, the resultant displacement of the medium will be the algebraic sum of the displacements caused by each individual wave. When two or more waves propagate through the same location in a medium, the waves behave as though the other waves did not exist.
- Huygens' principle models light as a wave that results from the superposition of an infinite number of wavelets. The principle states: Every point on an advancing wavefront can be considered as a source of secondary waves called "wavelets." The new position of the wavefront is the envelope of the wavelets emitted from all points of the wavefront in its previous position.
- Interference is a property exhibited by waves.
- Young's double-slit experiment demonstrated that light experiences interference and forms diffraction patterns. Young's experiment can be used to determine the wavelength of a specific colour of light by the relationship $\lambda \cong \frac{\Delta y d}{x}$.
- Light passing through a single slit experiences interference and forms diffraction patterns. Single-slit interference forms distinctive patterns, according to the relationship $y_{\text{m}} \cong \frac{m\lambda L}{W}$ for dark fringes and $y_{\text{m}} \cong \frac{(m + \frac{1}{2})\lambda L}{W}$ for bright fringes.
- A diffraction grating, composed of several equally spaced slits, produces diffraction patterns with more distinct bright and dark fringes. Diffraction from each of the slits increases the degree of constructive and destructive interference.
- Spectrometers make use of the diffraction of light, splitting the incident light into fine bands of colour. The resulting spectrum is used to identify the atomic composition of the light source. Astronomers use absorption line spectra to determine the composition of stars.
- The Rayleigh criterion states that "Two points are just resolved when the first dark fringe in the diffraction pattern falls directly on the central bright fringe in the diffraction pattern of the other." Experimental evidence shows that the minimum angle that a circular aperture is just able to resolve is given by $\theta_{\text{min}} = \frac{1.22\lambda}{D}$.

Knowledge/Understanding

1. Distinguish between dispersion and diffraction.
2. What happens to the energy of light waves in which destructive interference leads to dark lines in an interference pattern?
3. How did Thomas Young's experiment support the wave model of light?
4. An interference maximum is produced on a screen by two portions of a beam originally from the same source. If the light travelled entirely in air, what can be said about the path difference of the two beams?
5. The same formula is used for the positions of light maxima produced by two slits as for a grating with a large number of finely ruled slits or lines. What is the justification for creating and using many-lined gratings?
6. (a) What is the difference between the first-order and the second-order spectra produced by a grating?
   (b) Which is wider?
   (c) Does a prism produce spectra of different orders?
7. Why is it important that monochromatic light be used in slit experiments?
8. How does a thin film, such as a soap bubble or gasoline on water, create an interference pattern?

Inquiry

9. Devise a simple experiment to demonstrate the interference between sound waves from two sources.
10. Sketch the diffraction pattern produced by parallel wavefronts incident on a very wide slit. How does the pattern change as the slit size decreases?
11. Describe simple experiments to determine the following.
    (a) the resolving power of a small, backyard telescope
    (b) the wavelength of a source of monochromatic light
    (c) the separation between the rulings in a diffraction grating
12. Photographers often use small apertures to maximize depth of focus and image sharpness. However, at smaller apertures, diffraction effects become more significant. If you have access to a single-lens reflex camera, try to evaluate at which aperture diffraction effects become problematic in a particular lens. Also, try to determine the resolution of the lens. How does it compare to the manufacturer's stated value?
13. Suppose you have a source that emits light of two discrete wavelengths, one red and one blue. Assume for the sake of simplicity that each colour is emitted with the same intensity. Imagine allowing the light to pass through a diffraction grating onto a screen. Draw the appearance of the resulting line spectrum.
14. Single-slit diffraction affects the interference pattern of a double slit. Consider a double slit with slits that are 0.130 mm wide and spaced 0.390 mm apart, centre to centre.
    (a) Which orders of the double-slit pattern will be washed out by the minima of the single-slit pattern?
    (b) Sketch the interference pattern and demonstrate the above solution by superimposing the single-slit pattern on the double-slit pattern.
15. An ingenious physics student wants to remove the amount of glare reflecting from her computer screen. Describe, with the aid of a diagram, how she could make use of her knowledge of thin films to accomplish her task.

Communication

16. Explain whether a beam of light can be made increasingly narrow by passing it through narrower and narrower slits.
17. A friend who has never taken a physics course asks why light that passes through a slit produces a series of bright and dark fringes. How would you explain this phenomenon?

18. Discuss how we know that the wavelength of visible light must be very much less than a centimetre.

19. Both sound and light waves diffract on passing through an open doorway. Why does a sound wave diffract much more than a light wave? In other words, why can you hear around corners, but not see around corners?

20. Atoms have diameters of about 10^{-10} m. Visible light wavelengths are about 5×10^{-7} m. Can visible light be used to "see" an atom? Explain why or why not.

21. Suppose that, in a double-slit experiment, monochromatic blue light used to illuminate the slits was replaced by monochromatic red light. Discuss whether the fringes would be more closely or widely spaced.

22. Explain the source of colour seen on the surface of compact discs.

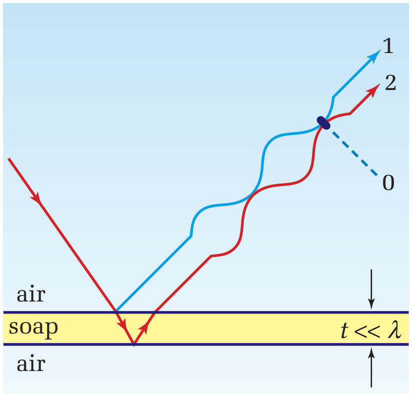
23. Discuss why interference fringes are not visible from thick films. (Hint: What is the effect of rays incident on thin films and thick films?)

24. The illustration depicts interference caused by very thin films.

(a) Describe the type of interference depicted by the illustration.

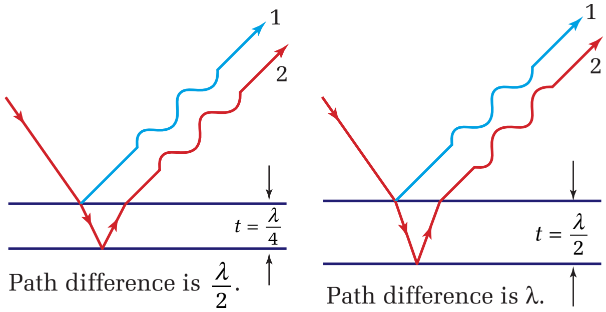
(b) Use wave model arguments to explain how the interference pattern from part (a) is caused.

(c) Would it be possible to coat an object with a very thin film to make it invisible in white light? Explain.



25. **(a)** Analyze carefully the illustrations following this question. Write a description tracing the path of the incident light as it encounters each medium interface. Describe what is happening to the wave in each case as the wave is **(i)** reflected and **(ii)** transmitted.

(b) Compare the resulting interference patterns from each illustration. What is the fundamental difference between the path that the light takes in each case?



Making Connections

26. Bats use echolocation to detect and locate their prey — insects. Why do they use ultrasonic vibrations for echolocation rather than audible sound?

27. CD and DVD players both utilize the effects of interference to retrieve digital information. Explain how this is done.

28. Explain how one observer's blue sky could be related to another observer's view of a red sunset.

29. Many butterflies have coloured wings due to pigmentation. In some, however, such as the Morpho butterfly, the colours do not result from pigmentation and, when the wing is viewed from different angles, the colours change. Explain how these colours are produced.

30. By studying the spectrum of a star (for example, the Sun), many physical properties can be determined in addition to the chemical composition. In fact, besides the telescope, the spectroscope is probably an astronomer's most useful tool. Write an essay discussing the information obtained about stars from spectra, and the impact of the spectroscope on modern astronomy.

Problems for Understanding

31. In a ripple tank, two point sources that are 4.0 cm apart generate identical waves that interfere. The frequency of the waves is 10.0 Hz. A point on the second nodal line is located 15 cm from one source and 18 cm from the other. Calculate
(a) the wavelength of the waves
(b) the speed of the waves

32. Blue light is incident on two slits separated by 1.8×10^{-5} m. A first-order line appears 21.1 mm from the central bright line on a screen, 0.80 m from the slits. What is the wavelength of the blue light?

33. A sodium-vapour lamp illuminates, with monochromatic yellow light, two narrow slits that are 1.00 mm apart. If the viewing screen is 1.00 m from the slits and the distance from the central bright line to the next bright line is 0.589 mm, what is the wavelength of the light?

34. Under ordinary illumination conditions, the pupils of a person's eye are 3.0 mm in diameter and vision is generally clearest at 25 cm. Assuming the eye is limited only by diffraction, what is its resolving power? (Choose 550 nm, in the middle of the visible spectrum, for your calculation.)

35. Assuming that the eye is limited only by diffraction, how far away from your eye could you place two light sources that are 50.0 cm apart and still see them as distinct?

36. The Canada-France-Hawaii telescope has a concave mirror that is 3.6 m in diameter. If the telescope was limited only by diffraction, how many metres apart must two features on the Moon's surface be in order to be resolved by this telescope? Take the Earth-Moon distance as 385 000 km and use 550 nm for the wavelength of the light.

37. A diffraction grating with 2000 slits per cm is used to measure the wavelengths emitted by hydrogen gas. If two lines are found in the first order at angles $\theta_1 = 9.72 \times 10^{-2}$ rad and $\theta_2 = 1.32 \times 10^{-1}$ rad, what are the wavelengths of these lines?

38. The range of visible light is approximately from 4.0×10^{-7} m (violet light) to 7.0×10^{-7} m (red light).
(a) What is the angular width of the first-order spectrum (from violet to red) produced by a grating ruled with 8000 lines per cm?
(b) Will this angular width increase or decrease if the grating is replaced by one ruled with 4000 lines per cm?

39. Show that there will be yellow light but no red light in the third-order spectrum produced by a diffraction grating ruled 530 lines per mm.

40. Suppose a grating is used to examine two spectral lines. What is the ratio of the wavelengths of the lines if the second-order image of one line coincides with the third-order image of the other line?

41. **(a)** What is the largest order image of green light, 540 nm, that can be viewed with a diffraction grating ruled 4000 lines per cm?
(b) At what angle does that order appear?

42. Red light is incident normally onto a diffraction grating ruled with 4000 lines per cm, and the second-order image is diffracted 33.0° from the normal. What is the wavelength of the light?

43. Suppose you shine a light on a soap bubble that is 2.50×10^{-7} m thick. What colour will be missing from the light reflected from the soap bubble? Assume the speed of light in water is 2.25×10^{8} m/s.

CHAPTER 10

Electromagnetic Waves

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Ampère's law
- Coulomb's law
- Faraday's law
- Vibrations and waves

A camera and some rolls of film packed safely inside your favourite sweatshirt appear clearly on a monitor as your suitcase passes through the airport's security X-ray system. Various materials absorb the low-energy X rays differently, and sophisticated software analyzes the varying intensities of the X-ray signals that are transmitted. The software then interprets these signals and converts them into a colour picture for the security guard to see.

This entire airport security process is based on the production, transmission, and reception of electromagnetic radiation. The intensity of the radiation can be finely adjusted and focussed, producing clear pictures of the contents of opaque containers such as luggage without damaging even sensitive camera film.

Electromagnetic radiation and its applications provide much more than just airport security. Global communications, radar, digital videodisc players, and television remote controls also use electromagnetic radiation. This chapter explores how physicists attempt to understand electromagnetic radiation by using a wave model. You will gain a better understanding of how electromagnetic radiation is produced, how varied forms of it behave, including light, and how its properties are utilized in various communication and medical applications.

MULTI LAB

Properties of Electromagnetic Waves

TARGET SKILLS
- Analyzing and interpreting
- Identifying variables
- Communicating results

Transmission of Ultraviolet Radiation

In this lab, you will analyze the transmission of ultraviolet (UV) radiation through various substances.

Tonic water, which contains quinine, emits a blue glow when exposed to UV radiation, while pure water does not.

Fill one clear, plastic cup with tonic water and one with pure water. Shine UV radiation onto the tops of both filled glasses. From the side, observe the top centimetre of the tonic water and the pure water. Place a dark cloth behind the cups to add contrast. Record the amount and depth of the blue glow. Repeat this procedure with transparent materials such as glass, plastic, and cellulose acetate placed over the cups.

Analyze and Conclude

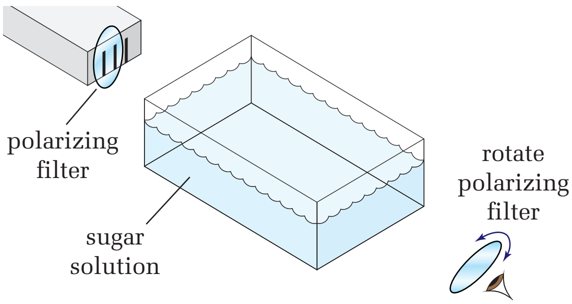
1. List the materials that you tested in the order of their effectiveness in absorbing UV radiation, starting with the material that was most effective.
2. Suggest why both the time of day and time of year affect the amount of dangerous UV radiation reaching the surface of Earth.

Apply and Extend

3. If time permits, devise an experiment to test the ability of various sunblock strengths to absorb UV radiation. (Hint: Smear the sunblock over a medium that is transparent to UV radiation.)
4. Increased amounts of UV radiation reaching Earth's surface could pose a risk to wildlife. Devise a simple experiment to verify if UV radiation is able to penetrate the surface of lakes and rivers.

Polarizing Light with Sugar

Fill a transparent, rectangular container with a supersaturated sugar solution. Using a ray box, shine three rays of light through the solution. Ensure that two rays of light pass through a polarizing filter before passing through the sugar solution. Carefully observe the rays of light through a second polarizing filter. Slowly rotate the filter closest to your eye while observing the rays of light passing through the solution.



Analyze and Conclude

1. Describe what you observed for **(a)** the two rays that passed through the first polarizing filter before entering the sugar solution and **(b)** the ray that did not pass through the first polarizing filter.
2. **(a)** Formulate an hypothesis that could explain your observations.

   (b) Is your hypothesis based on a wave or particle model of light?

10.1 The Nature of Electromagnetic Waves

SECTION EXPECTATIONS

- Describe how electromagnetic radiation is produced.
- Analyze the transmission of electromagnetic radiation.
- Define and explain the concepts related to the wave nature of electromagnetic radiation.
- Explain the underlying principle of polarizing filters.
- Identify experimental evidence for the polarization of light.
- Describe how electromagnetic radiation, as a form of energy, interacts with matter.

KEY TERMS

- Maxwell's equations
- electromagnetic wave
- electric permittivity
- magnetic permeability
- plane polarized
- photoelastic

While Huygens, Young, and others were studying the properties of light, physicists in another sector of the scientific community were exploring electric and magnetic fields. They were not yet aware of the connections among these fields of study. While a student at Cambridge, a young Scotsman, James Clerk Maxwell (1831–1879), became interested in the work of Lord Kelvin (William Thomson: 1824–1907) and Michael Faraday (1791–1867) in electric and magnetic fields and lines of force. Soon after Maxwell graduated in 1854, he gathered all of the fundamental information and publications that he could find in the fields of electricity and magnetism. After carrying out a thorough study and detailed mathematical analysis, Maxwell synthesized the work into four fundamental equations that are now known as **Maxwell's equations**. These equations form the foundation of classical electromagnetic field theory in the same way that Newton's laws form the foundation of classical mechanics.

Maxwell's Equations

Maxwell did not create the equations; he adapted and expanded mathematical descriptions of electric and magnetic fields that had been developed by others. The mathematical form of Maxwell's equations is well beyond the scope of this course, but the qualitative concepts and the implications of his equations are quite logical.

Maxwell's first two equations are based on concepts and equations developed by Carl Friedrich Gauss (1777–1855) and called "Gauss's law for electric fields" and "Gauss's law for magnetic fields" — concepts with which you are already familiar. Simply stated, Maxwell's first equation (Gauss's law for electric fields), illustrated in Figure 10.1 (A), states that for any imaginary closed surface, the number of electric field lines exiting the surface is proportional to the amount of charge enclosed inside the surface. Note that a field line entering the surface will cancel a line emerging from the surface. Fundamentally, this equation is based on the concept that electric field lines start on positive charges and end on negative charges.

Maxwell's second equation (Gauss's law for magnetic fields), illustrated in Figure 10.1 (B), states that for any imaginary closed surface, the number of magnetic field lines exiting the surface is zero. This equation simply describes the concept that magnetic field lines form closed loops and do not begin or end. If a field line enters a closed surface, it will eventually leave the surface. On first considering these equations, they do not appear to carry much significance. However, Maxwell and others after him were able to use the mathematical equations to make very significant predictions. For example, Maxwell showed that accelerating charges radiated energy in the form of electromagnetic waves.

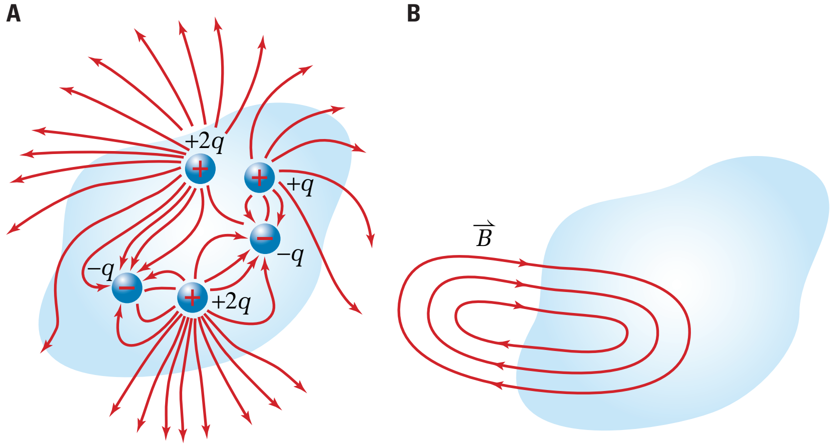


Figure 10.1 **(A)** The number of electric field lines leaving any imaginary closed surface — also called a Gaussian surface — is proportional to the amount of charge enclosed within the surface. **(B)** The number of magnetic field lines entering any imaginary closed surface is equal to the number of magnetic field lines leaving the surface.

Maxwell based his third equation on Faraday's discovery of the generator effect. You will probably recall from previous physics courses that when you move a magnet through a coil of wire, the changing magnetic field induces a current to flow in the coil, as shown in Figure 10.2 (A).

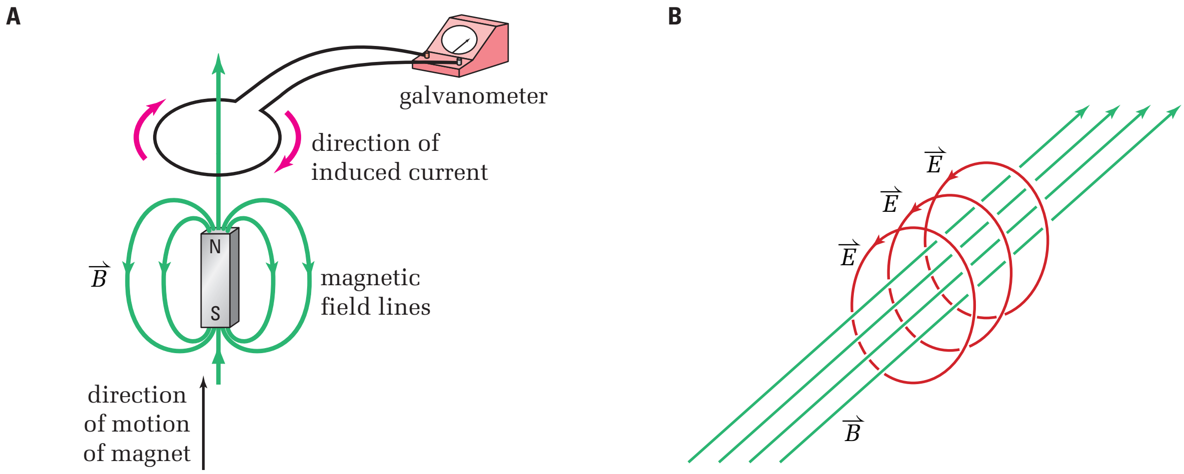


Figure 10.2 The moving magnet causes the magnetic field in and around the coil of wire to change. Since the changing magnetic field causes a current to flow in the wire, it must be generating an electric field in the region of the conductor.

HISTORY LINK

When Gauss was a child in primary school, the teacher punished the class for misbehaving by telling them to add all of the numbers from 1 to 100. The teacher noticed that, while all of the other students were writing vigorously, Gauss was staring out of the window. Then he wrote down a number. Gauss was the only student in the class who had the right answer. When the teacher asked him how he solved the problem, Gauss explained, "When I added 1 and 100, I got 101. When I added 2 plus 99, the answer was again 101. There are 50 of those combinations so the answer had to be 5050." Gauss rapidly became known for his mathematical abilities.

Maxwell expanded the concept to describe the phenomenon even when there was no coil present. To understand how he was able to make the generalization, ask yourself a few questions.

Q: What could cause the charges in the coil to move, making a current?

A: The charges must experience a force to start them moving and to overcome the frictional forces in the wire to keep the charges moving.

Q: If no visible source of a force is present, what might be providing the force?

A: An electric field exerts a force on charges that are placed in it, and if they are able to move, they will move.

These questions and answers lead to the conclusion that an electric field must exist around a changing magnetic field. If a coil is placed in the region, a current will flow. Maxwell's third equation states, in mathematical form, that a changing magnetic field induces an electric field, which is always perpendicular to the magnetic field, as illustrated in Figure 10.2 (B).

Maxwell's fourth equation is based on an observation made by Hans Christian Oersted (1777–1851) that André-Marie Ampère (1775–1836) developed into a law. Oersted observed that a current passing through a conductor produces a magnetic field around the conductor. Once again, Maxwell generalized the phenomenon to include the situation in which no wire was present. Ask yourself some more questions.

Q: What condition must exist in order for a current to flow in a wire?

A: Regardless of its origin, an electric field must exist in the wire in order for a current to flow.

Q: Can the presence of an electric field produce a magnetic field?

A: Since the current produced a magnetic field around the wire, it is probable that it was the electric field driving the current that actually produced the magnetic field.

Through mathematical derivations, Maxwell showed in his fourth equation that a changing electric field generates a magnetic field.

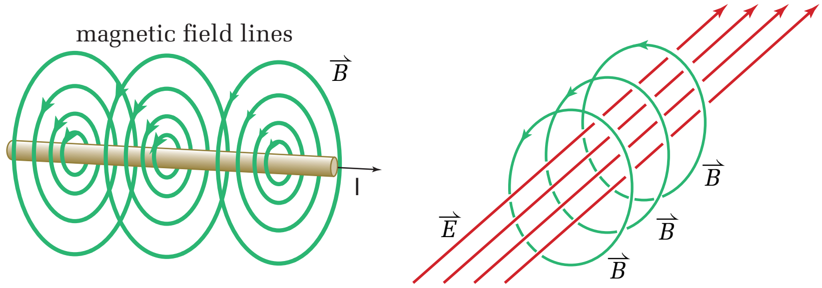


Figure 10.3 Maxwell showed mathematically that a changing electric field that exists in the absence of a conductor will produce a magnetic field around it, in the same way that a current in a conductor will produce a magnetic field.

Electromagnetic Waves

Maxwell's four equations and his excellent mathematical skills gave him exceptional tools for making predictions about electromagnetism. By applying his third and fourth equations, Maxwell was able to predict the existence of electromagnetic waves, as well as many of their properties. Think about what happens when you combine the two concepts — a changing electric field produces a magnetic field, and a changing magnetic field produces an electric field. Imagine that you generate a changing electric field. Initially, there is no magnetic field, so when the changing electric field produces a magnetic field, it has to be changing from zero intensity up to some maximum intensity. This changing magnetic field would then induce an electric field that, of course, would be changing. You have just predicted the existence of an **electromagnetic wave**.

Recall that, by applying his first equation, Maxwell showed that an accelerating charge can radiate energy. The energy that leaves the accelerating charge will be stored in the electric and magnetic fields that radiate through space. A good way to visualize this process is to envision an antenna in which electrons are oscillating up and down, as illustrated in Figure 10.4.

ELECTRONIC LEARNING PARTNER

Refer to your Electronic Learning Partner for a graphic representation of an electromagnetic wave.

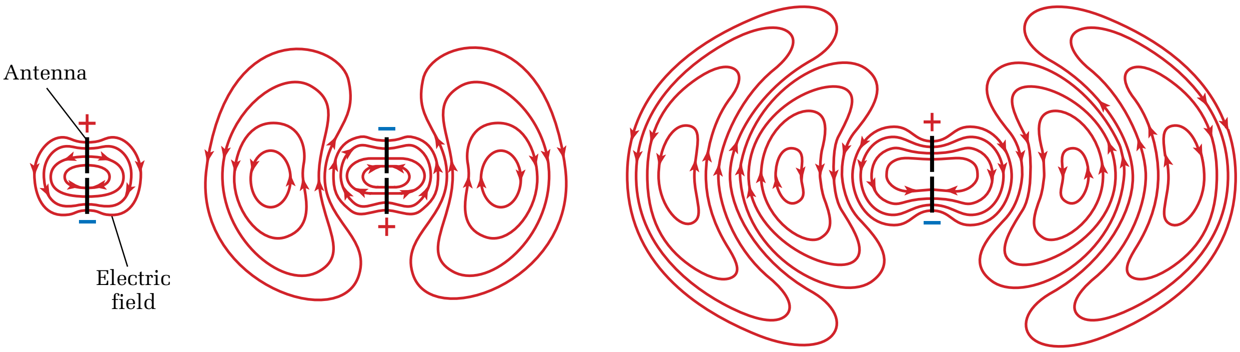


Figure 10.4 Although it is an oversimplification of the concept, it might help you to visualize the formation of an electromagnetic wave if you imagine that when the charges in the antenna reverse direction, the electric field pinches off the antenna.

In the first step, as shown in Figure 10.4, the motion of electrons made the bottom of the antenna negative, leaving the top positive. The separation of charge produced an electric field. As the electrons continued to oscillate, the antenna reversed its polarity, producing another field, with the direction of the field lines reversed from the first. Finally, the electrons moved again, producing a third field. Keep in mind that these events occur in three-dimensional space. Try to visualize each set of loops as forming a doughnut shape, coming out of the page and behind the page.

Although the magnetic field lines are not included in Figure 10.4, the changing electric fields have produced magnetic fields in which the direction of the field lines is always perpendicular to the electric field lines. To visualize these lines without making the image too complex, only one loop of electric field lines is drawn

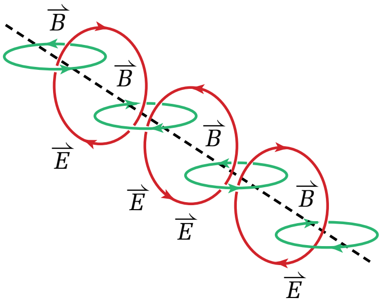


Figure 10.5 Every changing electric field generates a changing magnetic field, and every changing magnetic field generates a changing electric field. The electric and magnetic fields are always perpendicular to each other.

for each step. In Figure 10.5, you can see that the magnetic field lines loop into adjacent electric field lines.

The field lines in the illustrations provided so far show the direction of the electric and magnetic fields, but not their intensity. You can use a diagram, however, to estimate the relative strengths of the fields at any point in space. For example, start with the diagram in Figure 10.4 and draw a horizontal line from the centre of the antenna to the right, as shown in Figure 10.6 (A). As you move to the right of centre, the direction of the field is down. At the point where the two sets of loops meet, there are many field lines, so the field is at its greatest intensity. Farther to the right, the intensity decreases until, at the centre of the loops, it is zero. The field then changes direction and becomes stronger. As the wave propagates out into space, this pattern repeats itself over and over.

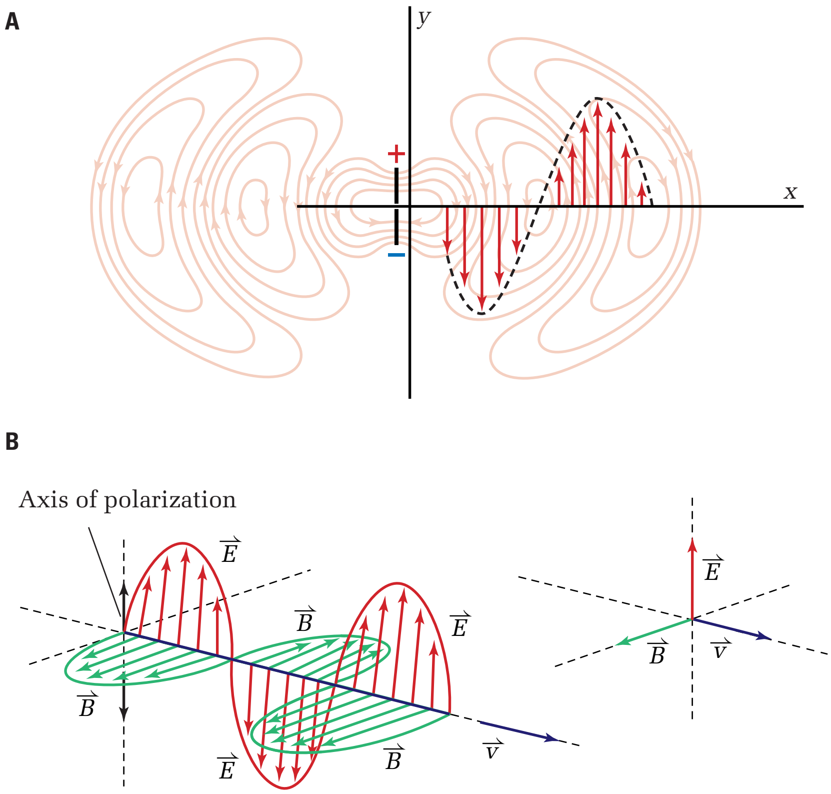


Figure 10.6 The length of each of the red arrows represents the intensity of the electric field at the point at which the base of the arrow meets the *x*-axis. The length of each of the green arrows represents the intensity of the magnetic field at the point at which the base of the arrow meets the *x*-axis.

Part (B) of Figure 10.6 duplicates the electric field lines in part (A) and adds vectors that show the magnetic field intensity. This figure shows the typical diagram for electromagnetic waves. No material objects are moving. The entities that are represented by the waves are the strengths of the electric and magnetic fields. An electromagnetic wave is a transverse wave in which electric and magnetic fields are oscillating in directions that are perpendicular to each other and perpendicular to the direction of propagation of the wave.

Experimental Evidence for Electromagnetic Waves

Although Maxwell correctly predicted the existence of electromagnetic waves and many of their properties, such as speed and the ability to reflect, refract, and undergo interference, he never saw any experimental evidence of their existence. It was not until eight years after Maxwell's death that Heinrich Hertz (1857–1894) demonstrated in his laboratory the existence of electromagnetic waves.

Hertz placed two spherical electrodes close to each other and connected them, through conductors, to the ends of an induction coil that provided short bursts of high voltage. When the voltage between the two electrodes was large enough, the air between them ionized, allowing a spark to jump from one electrode to the other. The momentary spark was evidence of electrons moving between the electrodes. The acceleration of the charges between the electrodes radiated electromagnetic energy away from the source, as predicted by Maxwell's equations.

As illustrated in Figure 10.7, Hertz used a single conducting loop with a second spark gap as a receiver. He verified the creation of electromagnetic waves by observing sparks produced in the receiver. The electromagnetic waves that Hertz produced were in the range of what is now called "radio waves."

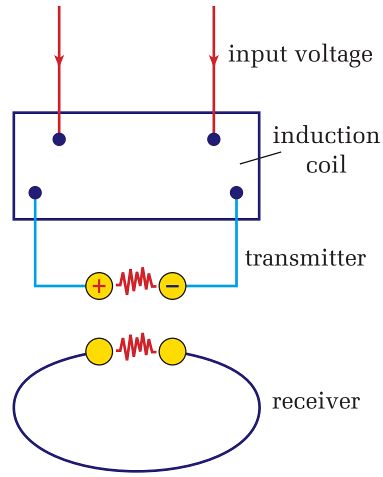


Figure 10.7 Sparks produced at the transmitter by voltage surges generate electromagnetic waves that travel to the receiver, causing a second spark to flash.

The Speed of Electromagnetic Waves in a Vacuum

As Hertz continued to study the properties of electromagnetic waves, he used the properties of interference and reflection to determine the speed of these waves.

Hertz set up a standing wave interference pattern, as illustrated in Figure 10.8. A wave of a known frequency emitted from the source and reflected back on itself, setting up a standing wave pattern. Hertz was able to detect the location of nodal points in the pattern by using a receiving antenna. Using the nodal point locations, he could determine the wavelength.

Hertz then calculated the speed of the wave, using the wave equation $v = f\lambda$. His calculated value for the speed of electromagnetic waves came very close to values of the speed of light that had been estimated and measured by several physicists in the middle 1800s.

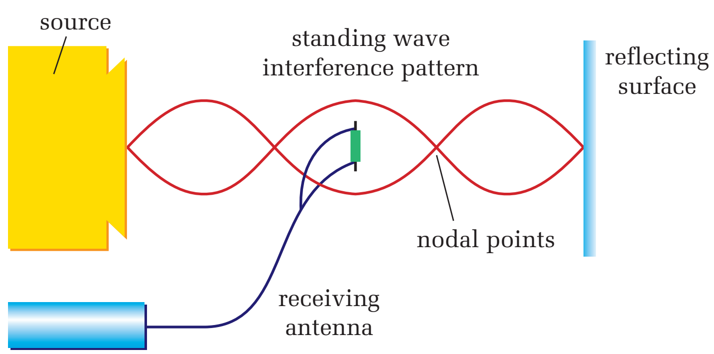


Figure 10.8 When the receiver was at an antinode, as shown here, it detected a strong signal. When it was moved to a node, it detected nothing.

In 1905, Albert A. Michelson (1852–1931) made the most accurate measurement of the speed of light of any that had made previously. In fact, it was extremely close to the value of modern measurements made with lasers. Michelson perfected a method developed by Jean Foucault (1819–1868) and illustrated in Figure 10.9. Michelson set up an apparatus on Mount Wilson in California and positioned a mirror 35 km away. A light source reflected off one side of an eight-sided mirror, then off the distant mirror, and finally off the viewing mirror. The rate of rotation (up to 32 000 times per minute) of the eight-sided mirror had to be precise for the reflection to be seen. By determining the exact rotation rate that gave a reflection and combining that with the total distance that the light travelled, he calculated the speed of light to be 2.997×10^8 m/s.

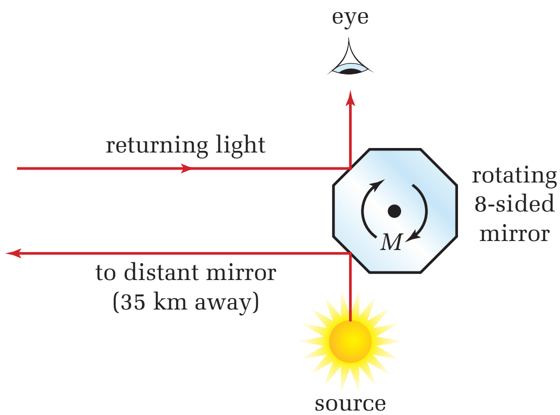


Figure 10.9 Until the laser was developed, Michelson's measurements of the speed of light using an apparatus similar to this were the best measurements of the speed of light that were available.

Maxwell's equations also provided a method for calculating the theoretical speed of electromagnetic waves. The equations include the speed as well as two constants that depend on the way in which the medium through which the waves are travelling affects electric and magnetic fields. The **electric permittivity** (ε) is a measure of the ability of a medium to resist the formation of an electric field within the medium. The constant is directly related to the Coulomb constant in Coulomb's law. The second constant, called the **magnetic permeability** (μ), is a measure of the ability of the medium to become magnetized. When electric and magnetic fields exist in a vacuum, often called "free space," the constants are written with subscript zeros: ε_0 and μ_0. Their values are known to be as follows.

$$\varepsilon_0 = 8.854\ 187\ 82 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2} \quad \text{and} \quad \mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$$

Maxwell's equations show that the speed of electromagnetic waves in a vacuum or free space should be given by the equation in the following box.

PHYSICS FILE

Coulomb's law is sometimes written $F_Q = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$, where $\frac{1}{4\pi\varepsilon} = k$.

QUICK LAB

Producing Electromagnetic Waves

TARGET SKILLS
- Predicting
- Identifying variables
- Analyzing and interpreting

Predict whether the visible sparks between a Van de Graaff generator (set up by your teacher) and a grounded object will produce electromagnetic radiation other than light. Use a portable radio to test for electromagnetic radiation with wavelengths similar to radio waves. Clearly tune in an AM radio station before generating the sparks. Predict how the portable radio will react if the sparks generate radio waves. Generate spark discharges and observe. Repeat for an FM station. Test how the distance between the spark source and the receiver (the radio) affects observed results. If the radio has a movable antenna, test different orientations of the antenna to find out if one orientation has any greater effect than the others.

Analyze and Conclude

1. What theoretical basis exists to suggest that the sparks will produce electromagnetic radiation in the form of both light and radio waves?
2. Does the presence of small electric sparks suggest the acceleration of charged particles? Explain.
3. **(a)** Describe what happened to the portable radio when sparks were produced.
   (b) How do your results verify the production of electromagnetic radiation?
4. In terms of frequency and wavelength, how are AM and FM radio signals different?

SPEED OF ELECTROMAGNETIC RADIATION

The speed of all electromagnetic radiation is the inverse of the square root of the product of the electric permittivity of free space and the magnetic permeability of free space.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Quantity	Symbol	SI unit
speed of light	c	$\frac{\text{m}}{\text{s}}$ (metres per second)
permeability of free space	μ_0	$\frac{\text{N}}{\text{A}^2}$ (newtons per ampere squared)
permittivity of free space	ε_0	$\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$ (coulombs squared per newton metre squared)

Unit Analysis

$$\frac{1}{\sqrt{\left(\frac{\text{N}}{\text{A}^2}\right)\left(\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}} = \frac{1}{\sqrt{\left(\frac{\cancel{\text{N}}}{\frac{\cancel{\text{C}^2}}{\text{s}^2}}\right)\left(\frac{\cancel{\text{C}^2}}{\cancel{\text{N}}\cdot\text{m}^2}\right)}} = \frac{1}{\sqrt{\frac{\text{s}^2}{\text{m}^2}}} = \frac{\text{m}}{\text{s}}$$

Note: The symbol c, by definition, represents the speed of light in a vacuum. Since all electromagnetic waves travel at the same speed in a vacuum and light is an electromagnetic wave, it is appropriate to use c for electromagnetic waves in general.

SAMPLE PROBLEM

Speed of Electromagnetic Waves

Use the solution to Maxwell's equations for the velocity of light in free space to determine a numerical value for the speed.

Conceptualize the Problem

- Maxwell's theory predicts the velocity of light to have a theoretical speed, given by $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$.
- Free space has a constant value for electric field permittivity.
- Free space has a constant value for magnetic field permeability.

Identify the Goal

The numerical value for the theoretical speed of electromagnetic waves, including light

Identify the Variables and Constants

Known

$\mu_0 = 4\pi \times 10^{-7}\ \dfrac{\text{N}}{\text{A}^2}$

$\varepsilon_0 = 8.854\ 187\ 82 \times 10^{-12}\ \dfrac{\text{C}^2}{\text{N} \cdot \text{m}^2}$

Unknown

c

Develop a Strategy

Use Maxwell's theoretical speed equation for free space.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Substitute in the values and compute the result.

$$c = \frac{1}{\sqrt{\left(4\pi \times 10^{-7}\ \frac{\text{N}}{\text{A}^2}\right)\left(8.854\ 187\ 82 \times 10^{-12}\ \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)}}$$

$$c = 299\ 792\ 458\ \frac{\text{m}}{\text{s}}$$

The numerical value for the theoretical speed of electromagnetic waves, including light, is 299 792 458 m/s.

Validate the Solution

The speed should be exceptionally fast, which it is. As shown on the previous page, the units cancel to give m/s which is correct for speed.

PRACTICE PROBLEMS

1. News media often conduct live interviews from locations halfway around the world. There is obviously a time-lag between when a signal is sent and when it is received.
   (a) Calculate how long the time-lag should be for a signal sent from locations on Earth separated by 2.00×10^4 km.
   (b) Suggest reasons why the actual time-lag differs from the value in (a).
2. What is the speed of light in water if, in water, $\varepsilon = 7.10 \times 10^{-10}\ \text{C}^2/\text{N} \cdot \text{m}^2$ and $\mu = 2.77 \times 10^{-8}\ \text{N/A}^2$?

Since light and electromagnetic waves all exhibit the properties of reflection, refraction, and interference, and have identical theoretical and experimental speeds in a vacuum, there is no doubt that light is no more than a form of electromagnetic waves that is detected by the human eye.

Polarization of Electromagnetic Waves

Polarized sunglasses eliminate the glare of reflected light from the highway and the hood of a car, while other sunglasses do not. What is unique about polarized lenses? The answer is based on a specific property of electromagnetic radiation including light. Evidence for this property was first reported by Danish scientist Erasmus Bartholinus (1625–1692) in 1669. Although he could not explain what he saw, Bartholinus observed that a single ray of light separated into two distinct rays while passing through a piece of naturally occurring calcite crystal. Figure 10.10 illustrates how a light entering a crystal from only one source (a single hole) splits while travelling through the crystal. Check it out yourself in the Quick Lab that follows.

Figure 10.10 A single ray is split into two as it passes through the calcite crystal.

QUICK LAB: Calcite Crystals

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting
- Communicating results

Obtain a piece of calcite crystal and a piece of cardboard. Poke a small hole in the centre of the cardboard with the tip of a pencil. Place the calcite crystal tightly against the cardboard, with the hole at the crystal's centre. Hold the cardboard-crystal apparatus in front of a light source, with the crystal on the side facing you. Observe the light passing through the small hole into the crystal. Repeat the procedure, placing a single polarizing filter on the back of the cardboard, over the hole. Rotate the filter while viewing the light passing into the crystal.

Analyze and Conclude

1. How many dots of light were visible exiting the crystal when the light source was viewed without the polarizing filter?
2. Describe what happened to the light passing through the crystal when the polarizing filter was being rotated.
3. What might cause a ray of light to change path?
   (a) What would happen if the electric field of an electromagnetic wave oriented vertically was able to pass through a substance at a different speed than if it was oriented horizontally?
   (b) Could your results suggest that calcite crystals have a different refractive index for light, depending on the alignment of the electric field of the wave? Explain.

To understand the principles behind polarized lenses and the splitting of a ray of light by calcite crystals, you first need to grasp the concept of polarization. As you know, electromagnetic waves are transverse waves in which both the electric and magnetic fields are perpendicular to the direction of propagation of the wave. However, the electric field might be pointing in any direction within a plane that is perpendicular to the direction of propagation, as illustrated on the left side of Figure 10.11. Make a mental note that, when examining illustrations such this, only the electric field vector is drawn, so a magnetic field exists perpendicular to the electric field.

Polarizing filters, developed in the 1920s, have the ability to selectively absorb all but one orientation of the electric fields in electromagnetic waves, as shown in Figure 10.11. After light or any electromagnetic wave has passed through such a filter, all of the electric fields lie in one plane and the wave is said to be **plane polarized**. The following Quick Lab will help you to understand polarization.

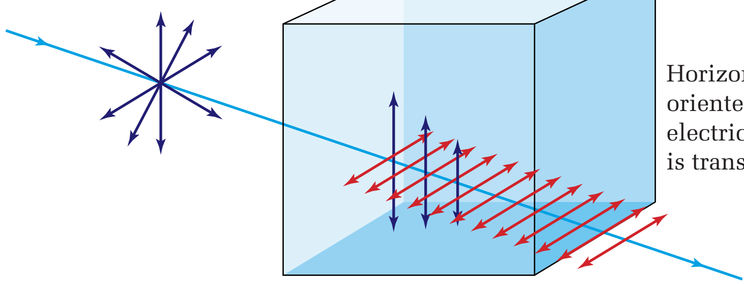


Figure 10.11 Natural light has waves with electric vectors pointing in all possible directions perpendicular to the direction of propagation of the wave. Polarizing filters absorb the energy of the waves that have electric fields in all but one orientation.

QUICK LAB

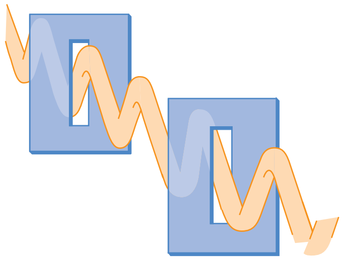
Polarization

TARGET SKILLS

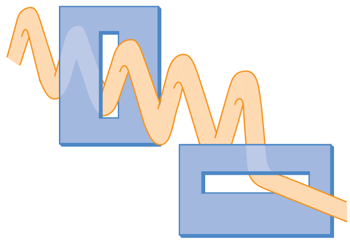
- Performing and recording
- Analyzing and interpreting
- Communicating results

Part A: Modelling Polarization with Rope

A



B

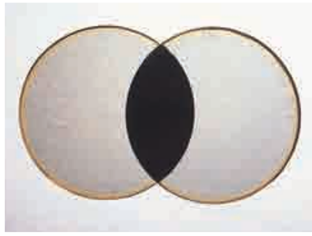


Working in groups of three, practise generating randomly polarized pulses in a length of rope held at both ends. Only one person in each group should generate the pulses. Have the third person insert a board with a horizontal slit cut into it. Observe how the pulses change after passing through the slit in the board. Rotate the board so that the slit is oriented vertically. Again, observe how the pulse changes after passing through the board. Repeat the process again, this time inserting a second board. Observe the pulses when the slits in the boards are both aligned (a) vertically and (b) at 90° to each other.

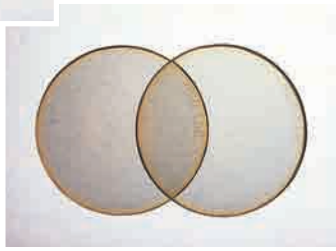
Analyze and Conclude

1. Explain the meaning of “randomly polarized pulses.”
2. How do the pulses change as they pass through the **(a)** horizontally and **(b)** vertically oriented slit in the board?
3. What happens to the energy contained in pulses that are not aligned with the slit in the board?
4. Could this experiment be repeated using a spring and longitudinal pulses?

Part B: Polarization of Light



A polarizing filters with axes perpendicular



B polarizing filters with axes parallel

1. Obtain two polarizing filters. Design a simple procedure using both filters to determine whether light can be polarized.
2. Design a simple procedure to determine if reflected light (such as sunlight reflecting off a desktop) is polarized.
3. Observe the sky, preferably on a day with a bright blue sky and some fluffy white clouds. Rotate a single polarizing filter while viewing the sky to determine if the blue light from the sky is polarized.

Analyze and Conclude

1. Hypothesize what is occurring when two polarizing filters are aligned so that they **(a)** allow light through and **(b)** block all of the light.
2. Did you find any evidence for the polarization of reflected light? Explain.
3. **(a)** Describe how the image of the blue sky and white clouds changes as the polarizing filter is rotated.

   (b) Based on your observations, determine whether the blue light of the sky polarized.

You now know how polarized lenses affect light, but how do they exclusively absorb glare from the light that is reflected from a road surface or the hood of a car? When light strikes a surface such as a street or pool of water, the electric fields that are perpendicular to that surface are absorbed and the parallel or horizontal electric fields are reflected. Therefore, reflected light is polarized.

As illustrated in Figure 10.12, polarized lenses in sunglasses are oriented so that they allow only vertical electric fields to pass through and thus absorb most of the horizontally polarized reflected light. Figure 10.13 shows photos taken with and without a polarizing filter. The fish beneath the water's surface is clearly visible when the bright glare, consisting mainly of horizontally polarized light, is removed.

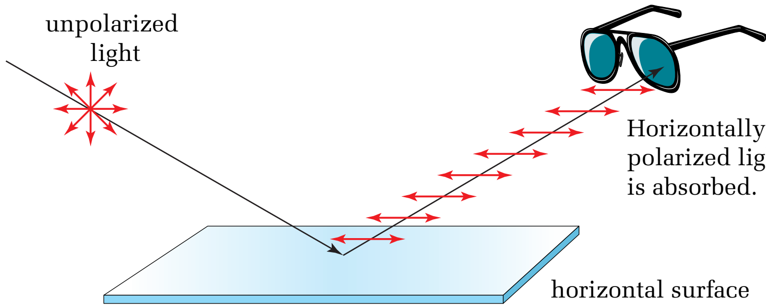


Figure 10.12 Since glare is caused by reflected light that is horizontally polarized, sunglasses with polarized lenses can eliminate glare by allowing only vertically polarized light to pass through.

Figure 10.13 Bright sunlight reflecting off the surface of water creates a lot of glare, preventing you from seeing objects below the surface. Polarized filters allow you to clearly see the fish swimming in this pond.

How can the phenomenon of polarization explain the ability of calcite crystals to split a beam of light into two beams? Again, ask yourself some questions.

Q: What happens to light when it passes from one medium, such as air, into another medium, such as a calcite crystal?

A: Light appears to bend or refract, because the speed of light is different in the two different media.

Q: What determines the extent of bending or refraction of the light?

A: The ratio of the indices of refraction of the two media determines the angle of refraction of light. The speed of light in a medium determines its index of refraction.

Q: How can a single crystal refract a single beam of light at two different angles?

A: The crystal must have two different indices of refraction for different properties of light.

Q: How can a beam of light have different properties?

A: Natural light has electric fields pointing in different directions.

Crystals, in general, are very orderly structures. The compounds in calcite are uniquely oriented so that the speed of light polarized in one direction is different than the speed of the light polarized perpendicular to the first. As a result, a beam of light is split into two beams, because light polarized in different planes refracts to different extents. Substances such as calcite, which have a different refractive indices depending on the polarization of the light, is said to be doubly refractive.

Certain materials, such as Lucite™, exhibit double refractive properties when under mechanical stress. The stress causes molecules in the material to align and behave similar to the orderly compounds in calcite crystals. Such materials are said to be **photoelastic**. Figure 10.14 demonstrates stress patterns that become visible in the Lucite™ when it is placed between polarizing and analyzing filters.

Mechanical stress changes the refractive index of Lucite™. As the level of stress varies in a sample, so does the amount of refraction. The plane of polarization of incident plane polarized light will be rotated it travels. A changing refractive index also means that the speed of propagation will be different for each electric field orientation. Light reaching an analyzer — a second polarizing filter — will be polarized in a different plane and form a pattern highlighting the stresses in the sample.

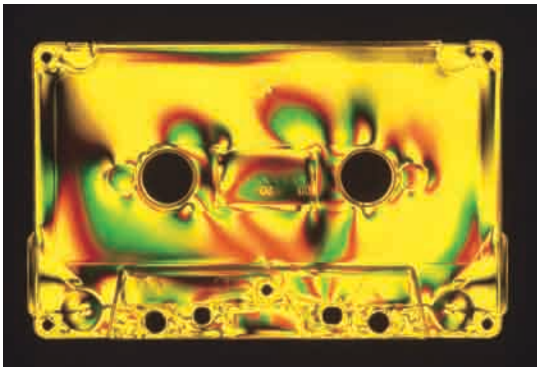


Figure 10.14 Lucite™ sandwiched between polarizing and analyzing filters yields an interference pattern showing the distribution of mechanical stress.

It is possible to produce reflective photoelastic coatings that are painted onto solid objects, allowing engineers to monitor mechanical stress and potential areas of failure. This technique is used to analyze materials for otherwise undetectable cracks and flaws.

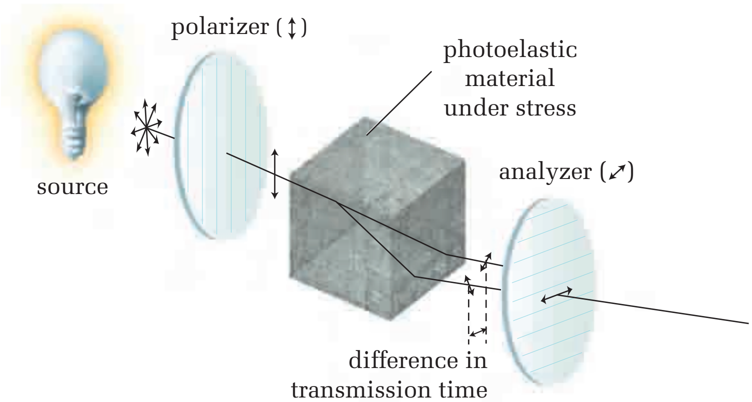


Figure 10.15 The refractive indices of doubly refractive photoelastic material varies under mechanical stress, producing interference patterns used to detect flaws.

Reflection and Absorption of Electromagnetic Waves

Light reflecting from a mirror is a common experience. So is the presence of satellite dishes used for satellite television. The satellite dishes are often a grey colour and are not nearly as smooth to the touch as a mirror. It might seem strange that the satellite dishes are not made of shiny, highly reflective material to help capture and reflect the radio waves for the receiver. In fact, although the grey coloured dishes are not highly reflective to light, they are highly reflective to radio waves. The amount of energy that is reflected depends on the wavelength of the incident wave and the material it is striking.

Light reflects off a mirror. Is the wavelength of light smaller or larger than the atoms that make up a mirror? The atoms need to be much smaller than the wavelength of light; otherwise, the mirror surface would appear bumpy. A tiny scratch in the mirror is easily visible, because it is much larger than the wavelength of light. Theoretically, if atoms were larger than the wavelength of light, it would be impossible to make a mirror that acted as a good reflector.

Conceptual Problem

- Would a mirror designed to reflect longer wavelength infrared radiation need to be smoother than a mirror designed to reflect shorter wavelength ultraviolet radiation?

Radio telescopes work by reflecting radio waves to a central receiver. The radio waves have wavelengths in the order of several metres. Therefore, the reflecting dishes can be constructed of conducting material, such as metallic fencing. To the long wavelengths, the fencing would act as a smooth surface. In general, the shorter the wavelength, the smoother the reflecting surface must be.

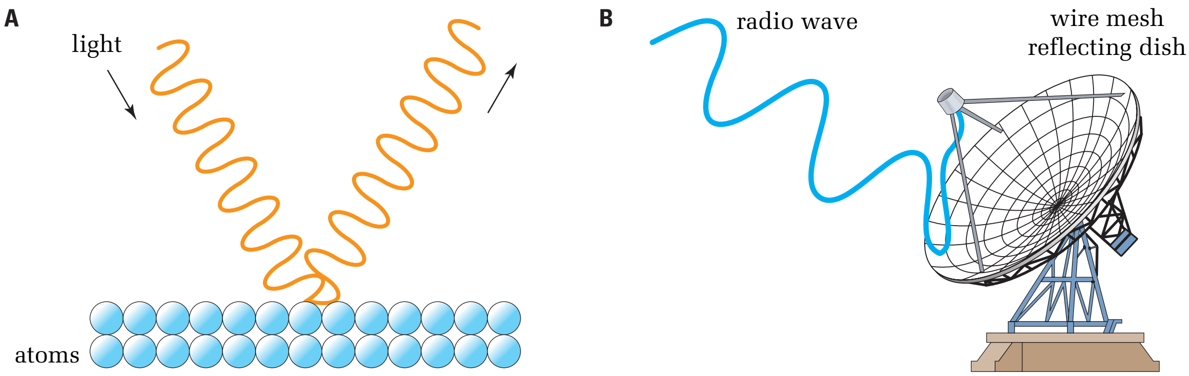


Figure 10.16 A mirror is a smooth reflector for light in the same way that wire mesh is a smooth reflector for long wavelength radio waves.

10.1 Section Review

1. K/U Qualitatively explain the meaning of Maxwell's third and fourth equations.

2. K/U What did Maxwell predict would be necessary to generate an electromagnetic wave?

3. K/U How did Hertz verify Maxwell's theory of electromagnetic waves?

4. K/U What is relationship in space between the electric and magnetic fields in an electromagnetic wave?

5. C Describe the process that allows an electromagnetic wave to exist as it radiates away from the source that created it.

6. C Describe the meaning of the terms "electric permittivity" and "magnetic permeability."

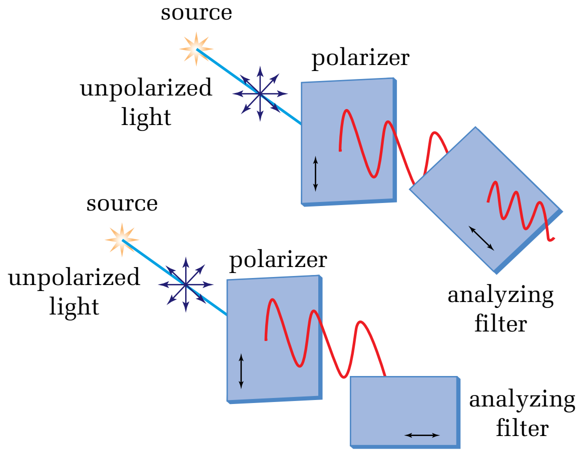
7. C Describe the apparatus that Michelson used to measure the speed of light.

8. C Describe one mechanism by which light is polarized in nature.

9. MC Assume that you are wearing polarized sunglasses while driving a car. You come to a traffic light and stop behind another car. You see that the rear window of the car ahead has a distinct pattern of light and dark areas. Explain.

10. C Define the term "photoelastic." Explain how light interacts with a photoelastic material.

11. C As illustrated in the diagram, describe the process involved as light travels from the source to the analyzing filter.



UNIT PROJECT PREP

Radio frequencies of the electromagnetic spectrum spread information around the globe at the speed of light.

- Investigate the relationship between a signal's wavelength and the length of transmitting and receiving antennas.
- Is there a relationship between the frequency of a radio signal and the range over which it might be received?
- If you could select the frequency at which your transmitter will operate, what frequency would you chose? Explain.

10.2 The Electromagnetic Spectrum

SECTION EXPECTATIONS

- Define and explain the concepts and units related to the electromagnetic spectrum.
- Describe technological applications of the electromagnetic spectrum.
- Describe and explain the design and operation of technologies related to the electromagnetic spectrum.
- Describe the development of new technologies resulting from revision of scientific theories.

KEY TERMS

- electromagnetic spectrum
- triangulation

When Hertz designed and carried out his experiments, his only intention was to test Maxwell's theories of electromagnetism. He had no idea that his success in generating and detecting electromagnetic waves would influence technology and the daily lives of the average citizen. Today, with electromagnetic radiation, people talk on cellphones, watch television that is receiving signals from satellites, and diagnose and treat disease.

There is literally no limit to the possible wavelengths and, consequently, the frequencies that an electromagnetic wave could have. Wavelengths of electromagnetic waves as long as hundreds of kilometres (10^3 Hz) to less than 10^{-13} m (3×10^{21} Hz) have been generated or detected. The **electromagnetic spectrum** has arbitrarily been divided into seven categories, based in some cases on historical situations or by their method of generation. In fact, some of the categories overlap. The following is a summary of the generation and applications of these seven categories of electromagnetic waves.

Radio Waves

By far the broadest electromagnetic wave category comprises radio waves, ranging from the longest possible wavelength or lowest frequency to about a 0.3 m wavelength or a frequency of 10^9 Hz. Radio wave frequencies are easily generated by oscillating electric circuits. They are broadcast by antennas made of electric conductors in which charges oscillate, as illustrated in Figure 10.4 on page 425. Radio waves are divided into subcategories by governments to restrict the use of certain ranges of waves to specific purposes.

Extremely low-frequency communication — 3 to 3000 Hz — is reserved for military and navigational purposes. Submarine-to-shore communications use the lowest of these frequencies when deeply submerged. Electromagnetic signals travelling through salt-water are absorbed, making communication with a deeply submerged vessel difficult. The very low frequencies are better able to penetrate the salt-water than are higher frequencies. However, such low frequencies have extremely long wavelengths that require very long antennas. In order to use these frequencies, submarines drag behind the ship a cable that can be as long as several hundred metres, to act as an antenna. Above ground, the transmission antenna consists of more than 140 km of suspended cabling.

Amplitude modulated (AM) radio — 535 to 1700 kHz — was the first widely used type of radio communication in the early part of the twentieth century. The range of frequencies that constitute the AM band was chosen arbitrarily, based primarily on the ability of the technology to generate the signals.

An AM radio station is assigned a specific "carrier" frequency on which to transmit signals. The information is carried by increasing and decreasing (modulating) the amplitude of the wave. For example, if the information is in the form of a voice or music, a microphone converts the sound waves into electric signals that are then combined or mixed with the carrier wave, as shown in Figure 10.17. A radio receiver picks up the signals by tuning an oscillating circuit in the instrument to the same frequency as the carrier frequency and then amplifies that wave. The electronic circuitry filters out the carrier wave and the "envelope" wave drives a speaker, converting the electric signal back into sound.

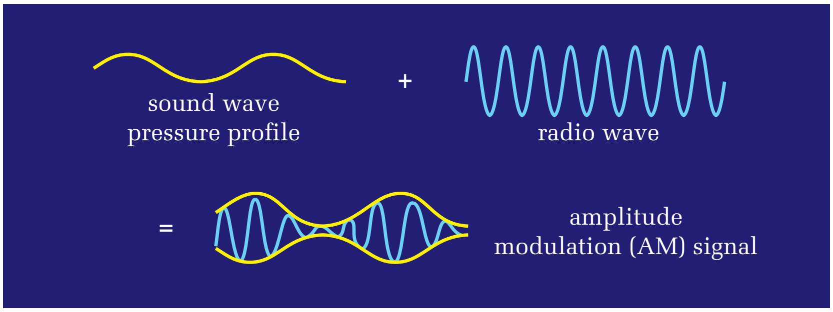


Figure 10.17 The radio wave in this figure is the carrier wave that is broadcast by the radio station. The term "modulation" refers to the changes in the amplitude of the carrier wave to match the sound wave, which contains information such as music, spoken words, and special effects.

The greatest problem with AM reception is that many machines and instruments emit random electromagnetic waves over a broad range of radio frequencies. For example, electric motors, automobile ignitions, and lightning bolts emit random "noise" signals that add to the amplitude of many AM waves. On a nearby AM radio, the signals will be picked up as static.

Short–wave radio is a range of frequencies — 5.9 MHz to 26.1 MHz — reserved for individual communication. Before satellite communications and cellular telephones, many people, called "ham operators," built their own transmitters and receivers and communicated with other ham operators around the world. Novice operators were licensed to transmit only Morse code, but they usually advanced quite rapidly and obtained licences to transmit voice. Occasionally, ham operators were the only people listening for signals when a boat or downed airplane was sending SOS calls. Ham operators were responsible for saving many lives.

Citizens' band (CB) radio frequencies — 29.96 MHz to 27.41 MHz — are reserved for communication between individuals over very short distances. The range of frequencies is divided into 40 separate channels. Because licences restrict the power of CB radios, they cannot transmit over long distances, so many people can use

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Your calculator might be a radio wave transmitter. Turn your AM radio dial to a very low frequency and make sure that it is between stations so that there is very little sound. Turn on your electronic calculator, hold it very close to the radio, and press various calculator buttons. You might be able to play a tune on your radio.

the same band at the same time, because the ranges do not overlap. Before the advent of cellphones and other more sophisticated wireless communication systems, CB and walkie-talkie communication provided a link between homes, businesses, and people travelling in vehicles. CB radio almost created a subculture and a language among truck drivers and other long-distance travellers, who used CB radios to pass the time.

Cordless telephones use a range of frequencies between 40 and 50 MHz. The range of a cordless telephone is designed for use in a home, and is shared with garage door openers and home security systems close to 40 MHz and baby monitors close to 49 MHz. The possibility of receiving a telephone conversion from a cordless telephone over a baby monitor exists, although it is unlikely, due to the very low power of both a telephone and a baby monitor. Some cordless telephones are also designed to operate close to 900 MHz.

Television channels 2 through 6 are broadcast in the 54 to 88 MHz frequency range, which lies just below FM radio. Channels 7 through 13 are broadcast over a frequency band between 174 to 220 MHz, which lies just above FM radio. These TV signals are broadcast signals that can be received only with a TV aerial. Cable and satellite signals are quite different. The method of transmission and reception are essentially the same as radio. The television picture is transmitted as an AM signal and the sound is transmitted as an FM signal.

Wildlife tracking collars use some of the same frequencies that are used by television. Understanding complex ecological interactions sometimes involves tracking wild animal populations over long distances. Canada has become a world leader in the design and manufacturing of wildlife tagging and tracking technology.

The animal in Figure 10.18 is a cheetah, a member of an endangered species. It is wearing a tracking collar that emits an electromagnetic signal of a specific frequency, allowing researchers to follow the animal's day-to-day movements. Similar systems have been developed for various climates and conditions, including cold Arctic climates and underwater environments.

Figure 10.18 Wildlife researchers track individual animals belonging to endangered species to learn about their behaviour, in an attempt to find ways to prevent extinction of their species.

In the past, animals were tagged with identification bands. If the same animal was captured again, migration patterns could be deduced. The process required tagging a very large sample of animals and data accumulation was slow, since the recapture of tagged animals could not be guaranteed. Microchip tags on the tracking collars have dramatically increased wildlife tracking research capabilities, providing sample and log data such as ambient temperature, light, and underwater depth, as well as transmitting a tracking signal. Similar, although less sophisticated, tags are used by pet owners to identify their animals. Pet identification tags store information about the pet and its owners. The tag is inserted under the animal's skin, where it will remain for life. Placing a receiving antenna near the tag retrieves the data.

Frequency modulated (FM) radio transmits frequencies from 88 MHz to 108 MHz. FM radio was introduced in the 1940s to improve the sound quality by reducing the static that is common on AM radio. FM radio eliminates static, because a change in the amplitude of the wave has no effect on the signal. An FM carrier wave has a single frequency, as does the AM carrier. However, as shown in Figure 10.19, information is carried in the form of slight increases and decreases in the frequency of the carrier. FM frequencies are absorbed more easily by the atmosphere than are AM frequencies, so the range of an FM station is shorter than an AM station.

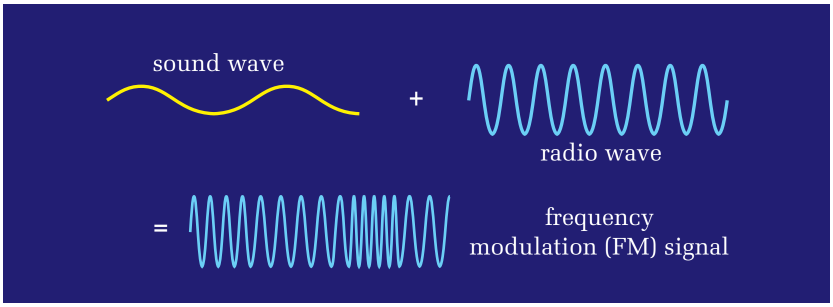


Figure 10.19 The amplitude of a carrier wave from an FM radio station never changes. In fact, if external noise from a nearby machine increases the amplitude, the radio receiver crops off the waves, accepting only a constant amplitude. The sound signal modulates, or varies, the frequency of the carrier wave.

The transmission of all radio wave frequencies is called a "line of sight" transmission, because radio waves are absorbed by the ground. Radio waves do of course penetrate walls and objects that are not extremely thick and dense, but cannot penetrate large amounts of matter. Sometimes, however, you might pick up a radio station at a distance greater than line of sight. The explanation for this phenomenon is the ability of the ionosphere to reflect radio waves, as shown in Figure 10.20. The ionosphere is a layer of charged atoms and molecules in the upper atmosphere that is created by high-energy radiation from the Sun stripping electrons from the gases. At night, the altitude of the ionosphere increases and allows signals to travel farther than during the day. You might have noticed that you can pick up radio stations at night that you cannot receive during the day.

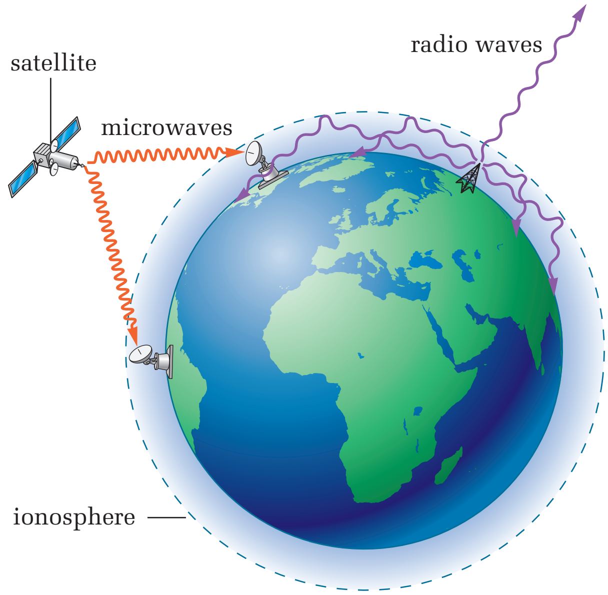


Figure 10.20 The longer wavelength radio waves reflect from the ionosphere, and most of the waves return to Earth. Although radio waves cannot be used for satellite communications, they can travel farther around Earth's surface than can the shorter microwaves. In contrast, shorter microwaves are unaffected by the ionosphere and can therefore be used in satellite communications.

The ionosphere is most effective in reflecting short-wave radio signals. Multiple reflections of short-wave radio signals between the ground and the ionosphere allow signals to travel over incredibly long distances. Ham operators are often able to communicate with others halfway around the world.

Magnetic resonance imaging (MRI) is a unique application of radio waves that is used to diagnose certain types of illnesses and injuries. MRI provides incredible detail in the study of nerves, muscles, ligaments, bones, and other body tissues by using electromagnetic signals to create image "slices" of the human body.

The largest component of an MRI system is a powerful magnet, with a tube called the "bore" running horizontally through the magnet. The patient slides on a special table into the bore. The magnetic field interacts with the nucleus of atoms of hydrogen, because these nuclei behave like tiny magnets that align themselves in the field. A pulse of radio waves is absorbed by the hydrogen, causing the hydrogen atoms to "flip" and become aligned against the external magnetic field. When the electromagnetic pulse stops, the atoms relax back to their original alignment and release absorbed energy in the form of electromagnetic waves. Sensors detect the emitted waves and send signals to a computer system that converts the electrical signals into a digital image that can be put on film.

Microwaves

Microwaves ranging from 1.0×10^{10} Hz to 3.0×10^{11} Hz have such high frequencies that there is no electronic circuitry capable of oscillating this fast. Research into the development of a device able to generate microwaves was stimulated by the development of radar. Physicist Henry Boot and biophysicist John T. Randall, both British scientists, invented an electron tube called the "resonant-cavity magnetron" that could produce microwaves. Similar tubes, called Klystron™ tubes, are the two main devices that generate microwaves today. The first application of microwave was, of course, radar, which has revolutionized safety in aviation as well as detecting weather data around the world.

Soon after the technology to generate microwaves was developed, the number of applications grew rapidly. Microwave ovens generate microwaves that have a frequency of 2450 MHz, which is close to the natural frequency of vibration of water molecules. As a result, these microwaves are efficiently absorbed by the water molecules in food, causing a dramatic increase in temperature, which cooks the food.

Microwaves have revolutionized communications for one fundamental reason — in contrast to radio waves, microwaves penetrate the ionosphere, making them useful for space-based communication. Any location, anywhere on Earth, can be reached by satellite communication.

The Global Positioning System (GPS), for example, makes it possible to determine your location and altitude anywhere on Earth through the use of geostationary satellites. Global positioning satellites, originally part of the military infrastructure, provide businesses, rescue workers, and outdoor enthusiasts with instantaneous position and tracking data. The global positioning satellite network consists of 24 geostationary satellites that are in constant communication with each other and with several ground stations.

Triangulation is the basis of the GPS. Figure 10.21 demonstrates how you could locate your exact position if you knew how far you were from three points. For example, if you knew you were exactly 65 km from Toronto, you could be anywhere in a circle with a 65 km radius around Toronto. If you also knew that you were 194 km from Windsor, you could now determine your

PHYSICS FILE

Invented by the British and shared with the U.S. military while the World War II Battle of Britain raged in 1940, the resonant-cavity magnetron was described as being capable of generating "ten kilowatts of power at ten centimetres, roughly a thousand times the output of the best U.S. [vacuum] tube on the same wavelength." From this realization flowed numerous developments, including gun-laying radar, radar-bombing systems, and air-intercept radar, as well as the first blind-landing system. As most veterans of the "Rad Lab" came to believe: "The atomic bomb only ended the war. Radar won it."

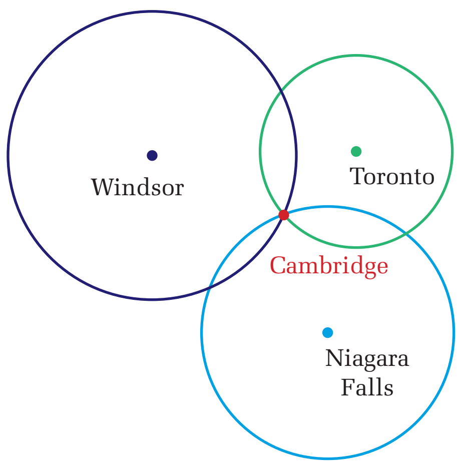


Figure 10.21 Circles generated from three known distances intersect at only one point.

location as being one of two sites where the two circles intersect. Knowing a third measure, such as that you were 79 km from Niagara Falls, you could pin-point your exact location. All three circles intersect at only one point, revealing your location to be downtown Cambridge. Global positioning technology takes this process one step further, using four spheres instead of three circles. This allows a location to be determined in three-dimensional space, including altitude.

GPS satellites continually send radio signals between each other and to Earth. The network of 24 satellites ensures that no matter where you are on Earth, at least four satellites will have a direct line of sight to your position. Hand-held GPS receivers measure the amount of time required for a microwave signal to travel from each satellite. The receivers are then able to calculate the distance from each satellite, knowing the speed of the signal ($c = 3.00 \times 10^8$ m/s). GPS hand-held systems are effective because they are an inexpensive and accurate method to determine the time the signal took to travel from the satellite to the receiver.

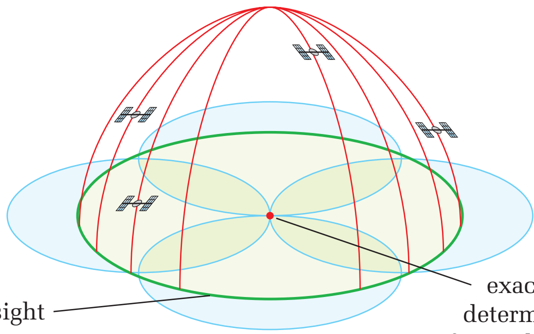


Figure 10.22 GPS receivers determine exact location by computing a single point at which imaginary spheres from each satellite intersect.

GPS technology has been made possible only through the merging of several branches of science and engineering. For example, complex mathematical models are used to calculate the speed of electromagnetic signals through our continually changing atmosphere and ionosphere. Atomic clocks on board each satellite are a product of research in atomic physics. Aerospace and rocketry advances rely on chemistry and physics.

Infrared Radiation

Infrared radiation with frequencies from 3.0×10^{11} Hz to 3.85×10^{14} Hz lies between microwaves and visible light. Infrared radiation was accidentally discovered in 1800 by Sir William Herschel (1738–1822). He was separating the colours of the visible spectrum and measuring the ability of different colours to heat the objects that were absorbing the light. He was very surprised when he placed his thermometer just beside the red light, where no visible light was falling, and discovered that the thermometer showed the greatest increase in temperature. He rightly concluded that there was some form of invisible radiant energy just beyond red light.

Any warm object, including your body, emits infrared radiation. The natural frequency of vibration and rotation of many different types of molecules lies in the infrared region. For this reason, these molecules can efficiently absorb and emit infrared radiation.

The ability to detect infrared radiation has led to several varied applications, from electronic night-scopes, which convert the heat of an animal or person into a visible image, to satellite imaging able to "see" through clouds to gather information related to the health of vegetation or the hot spots of a forest fire.

> **COURSE CHALLENGE**
>
> **How Far Can It Go?**
>
> Scientific discoveries breed new applications with capabilities once unimagined. Page 604 of this text provides suggestions for you to consider for your *Course Challenge*.

Visible Light

Visible light encompassing frequencies between about 3.85×10^{14} Hz and 7.7×10^{14} Hz is defined as light, simply because the human eye is sensitive to electromagnetic waves within this range. In some applications, it is more common to refer to the wavelength than the frequency, so you might see the range of light waves reported as encompassing wavelengths between 400 and 700 nm.

Light technologies are too numerous to mention. You probably know more about light than any other range of the electromagnetic spectrum, because entire units in your previous science courses were based on the properties of light. Visible light is emitted from all very hot objects, due to excited electrons in molecules dropping down to lower energy levels and emitting light energy.

Ultraviolet Radiation

Ultraviolet (UV) radiation, with frequencies between 7.7×10^{14} Hz and 2.4×10^{16} Hz, has the ability to knock valence electrons free from their neutral atoms — a process known as "ionization." When electrons drop from very high energy levels in atoms to much lower levels, UV radiation is emitted. Ionization is responsible for creating the ionosphere around the globe. UV radiation activates the synthesis of vitamin D in the skin, which is very important to your health. However, large quantities of UV radiation can cause skin cancer and cataracts. Ultraviolet wavelengths are used extensively in radio astronomy.

UV radiation was discovered just one year after infrared radiation was discovered and also by "accident." J. Ritter was studying the ability of light to turn silver chloride black by releasing metallic silver. He discovered that when silver chloride was placed just beyond the violet light in a spectrum of sunlight created by a prism, it was blackened even more efficiently than when exposed to the blue or violet light.

X Rays

X rays with frequencies between 2.4×10^{16} Hz and 5.0×10^{19} Hz have great penetrating power and are very effective in ionizing atoms and molecules. X rays can be produced when electrons in outer shells of an atom fall down to a very low, empty level. Commercial generators produce X rays by directing very high energy electrons that have been accelerated by a high voltage onto a solid metal surface inside a vacuum tube as shown in Figure 10.23. When the electrons are abruptly stopped by the target electrode, X rays are emitted. Such X ray generators are used extensively in medical applications and in industry for material inspection.

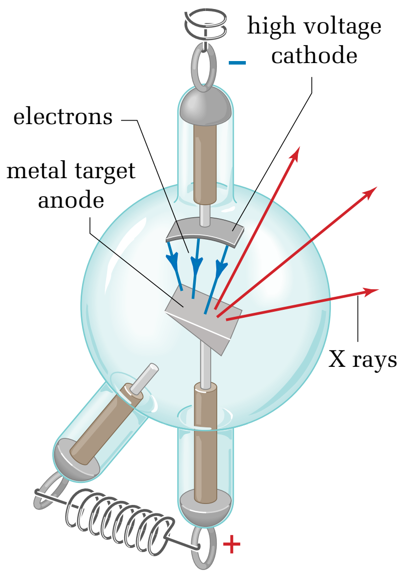


Figure 10.23 An X-ray tube must be evacuated so that the high-energy electrons are not scattered by gas molecules. The X rays produced when the electrons collide with the target and are suddenly stopped are sometimes called "Bremsstrahlung," which means "braking radiation."

MISCONCEPTION

Cosmic Rays Are *Not* Rays!

Cosmic rays are not rays at all, but rather are high-energy particles ejected from stars, including the Sun, during solar flares. These particles, although they travel at very high speeds, do not travel at the speed of electromagnetic radiation. A gamma ray emitted by the Sun will arrive at Earth in approximately 8 min, whereas high-energy particles referred to as "cosmic rays" can take between several hours to several days to travel from the Sun to Earth.

X-ray images of the Sun can yield important clues about solar flares and other changes on the Sun that can affect space weather.

Gamma Rays

Gamma rays are the highest frequency, naturally occurring electromagnetic waves, with frequencies ranging from 2.4×10^{18} Hz to 2.4×10^{21} Hz. Gamma rays are distinguished from X rays only by their source: Whereas X rays are produced by the acceleration and action of electrons, gamma rays are produced by the nuclei of certain atoms. Just as electrons in atoms can become excited by the absorption of energy, the nucleus of an atom can also be excited. An atom with an excited nucleus is said to be "radioactive." When an excited or radioactive nucleus releases energy to drop down to a more stable state, it releases gamma rays.

Due to their great penetrating and ionizing power, gamma rays are sometimes used to destroy malignant tumors deep inside the body. Radioactive atoms are also used extensively in research to track and identify specific elements. Gamma ray images of our universe provide information on the life and death of stars and on other violent processes in the universe. Gamma rays are extremely energetic and can be very harmful to life.

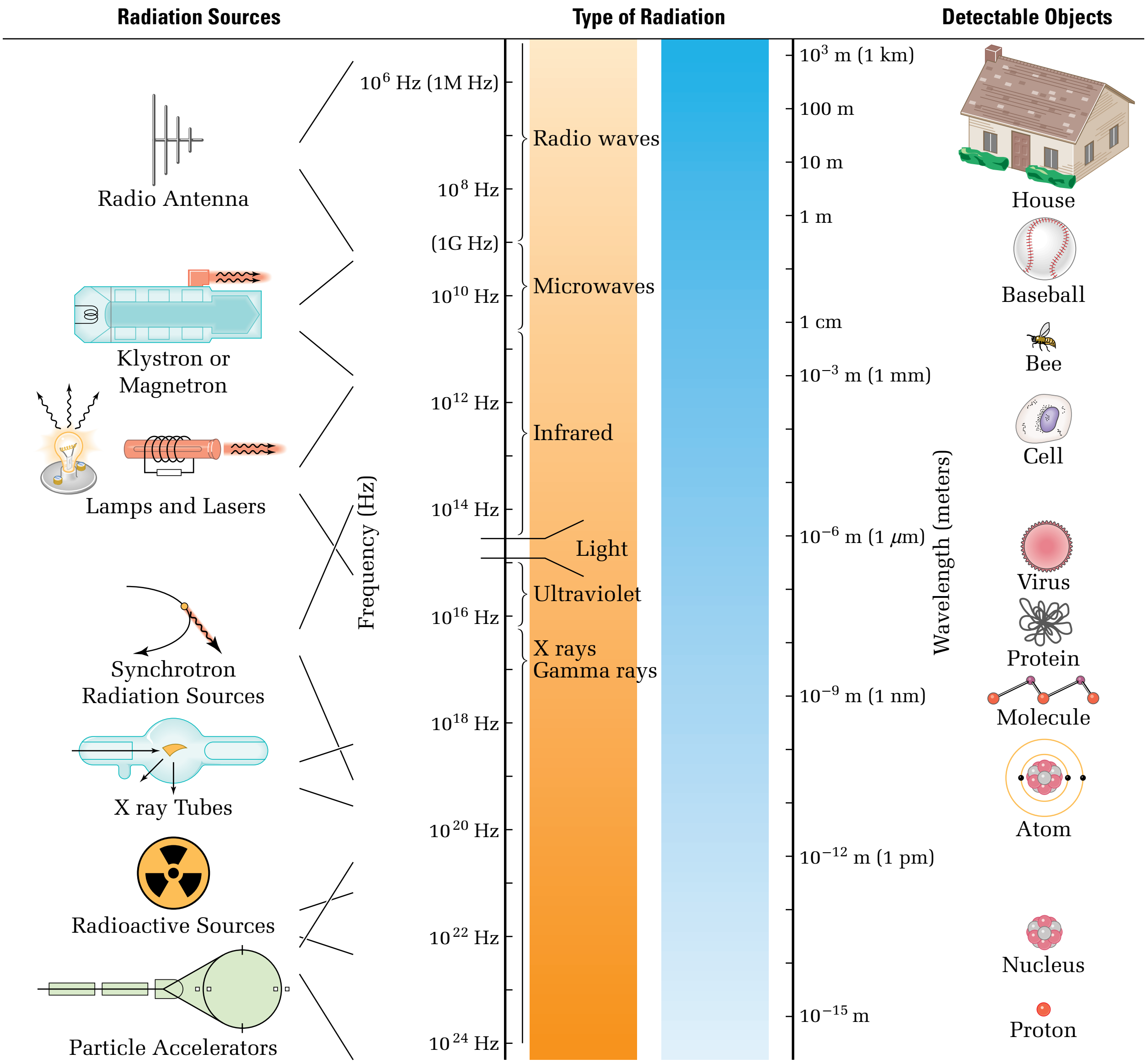


Figure 10.24 The electromagnetic spectrum is continuous throughout a range of frequencies covering more than 18 orders of magnitude (powers of 10). The subdivisions are artificial, but give scientists a way to quickly communicate the range of electromagnetic waves and the general properties of the waves of interest.

Conceptual Problems

- Describe which frequency range is best for long-distance communication. Explain.
- Suggest why gamma rays penetrate farther into matter than UV, despite the generalization that longer wavelengths have greater penetration power.

Electromagnetic Waves and the Wave Equation

The wave equation that you learned while studying mechanical waves, $v = f\lambda$, also applies to electromagnetic waves. Since electromagnetic waves always travel with the same speed in a vacuum, the wave equation can be expressed in terms of c instead of v. Since the speed of electromagnetic waves in air is almost identical to their speed in a vacuum, this equation can be used for air as well as for a vacuum or free space.

ELECTROMAGNETIC WAVE EQUATION

The speed of electromagnetic waves is the product of their frequency and wavelength.

$$c = f\lambda$$

Quantity	Symbol	SI unit
speed of electromagnetic radiation in a vacuum	$c = 3.00 \times 10^8$ m/s	$\frac{\text{m}}{\text{s}}$ (metres per second)
frequency	f	Hz (hertz or $\frac{1}{\text{s}}$)
wavelength	λ	m (metres)

Unit Analysis

$$(\text{hertz})(\text{metre}) = \left(\frac{1}{\text{s}}\right)(\text{m}) = \frac{\text{m}}{\text{s}}$$

Note: The velocity of all electromagnetic radiation in a vacuum is denoted as c. However, electromagnetic radiation travelling through air is slowed only slightly, and therefore the value of c is often used to approximate velocity values in air as well.

SAMPLE PROBLEM

Radio Waves

An FM radio station broadcasts at a frequency of 2.3×10^8 Hz. Determine the wavelength of the FM radio waves from this station.

Conceptualize the Problem

- *Radio wave* transmission is a form of *electromagnetic* radiation.
- The *speed* of electromagnetic radiation in air is approximated as c.
- *Electromagnetic waves* can be described by using the *wave equation*.

Identify the Goal

The wavelength of 2.3×10^8 Hz electromagnetic waves

Identify the Variables and Constants

Known	Implied	Unknown
$f = 2.3 \times 10^8$ Hz	$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$	λ

Develop a Strategy

Use the wave equation.

$$c = f\lambda$$

Manipulate the equation, solving for wavelength.

$$\lambda = \frac{c}{f}$$

Substitute and solve.

$$\lambda = \frac{3.00 \times 10^8}{2.3 \times 10^8 \text{ Hz}}$$

$$\lambda = 1.3043 \frac{\frac{\text{m}}{\text{s}}}{\text{s}^{-1}}$$

$$\lambda \cong 1.3 \text{ m}$$

The wavelength of 2.3×10^8 Hz electromagnetic radiation is 1.3 m.

Validate the Solution

Both the speed and the frequency were in the order of 10^8, which would suggest that the answer should be near unity, which it is.

PRACTICE PROBLEMS

3. (a) Determine the wavelength of an AM radio signal with a frequency of 6.40×10^6 Hz.
   (b) Suggest why AM radio transmitting antennas are hundreds of metres tall.
4. Microwave oven doors have metallic screens embedded in them. Light is able to pass through these screens, but the microwaves are not. Assume that the microwave radiation is in the order of 10^{10} Hz and the light in the order of 10^{14} Hz.
   (a) Calculate the wavelengths of both the microwave radiation and visible radiation.
   (b) Suggest why a metallic screen is used in microwave oven doors.

10.2 Section Review

1. K/U What is the basis for naming the categories within radio waves?
2. C Explain the difference between frequency modulation (FM) and amplitude modulation (AM).
3. K/U Why does AM radio exhibit much more static than FM radio?
4. K/U Why are microwaves used for satellite communications rather than radio waves?
5. K/U How do microwaves "cook" food?
6. K/U Describe the difference between ionizing and non-ionizing electromagnetic radiation.
7. K/U In many spectra, you will see an overlap of X rays and gamma rays. What distinguishes X rays from gamma rays?
8. C Explain how GPS can help you to locate your position.
9. MC How has the application of radio tracking collars impacted wildlife research?
10. MC Magnetic resonance imaging (MRI) is not always a safe option for some patients. Suggest possible reasons that might make an MRI scan unsafe for a patient. Support your suggestions with Internet research.

UNIT PROJECT PREP

Your FM transmitter will transform your words into electromagnetic radiation to be received and heard on a typical portable radio.

- How are electromagnetic waves produced?
- Investigate the factors that determine the quality of a transmitting antenna.
- Think about the differences and similarities between the electromagnetic signal Hertz first sent and received to those used today.

CHAPTER 10 Review

REFLECTING ON CHAPTER 10

- Maxwell unified concepts from electricity and magnetism into a new field called "electromagnetism." He presented his ideas in the form of four equations.
- Maxwell's equations show that a changing electric field generates a magnetic field and a changing magnetic field generates an electric field.
- Hertz produced electromagnetic waves in the laboratory, verifying Maxwell's predictions.
- Electromagnetic waves are produced when charges are accelerated. An electromagnetic wave consists of an oscillating electric field and magnetic field at right angles to each other that propagate in a direction that is perpendicular to both fields.
- Electromagnetic waves travel with a speed of $c = 3.00 \times 10^8$ m/s through empty space.
- Electromagnetic radiation exhibits wave behaviour, undergoing diffraction and forming interference patterns.
- Electromagnetic radiation can be polarized. The electric field of plane polarized light oscillates in only one plane.
- Since the magnetic field in an electromagnetic wave is always perpendicular to the electric field, when the electric field is polarized, the magnetic field must also be polarized.
- Electromagnetic radiation can have frequencies ranging from below 1 Hz to above 10^{22} Hz, called the "electromagnetic spectrum."
- Light is one narrow section of the electromagnetic spectrum. Colour is identified by frequency or wavelength.

Colour	Wavelength (nm)
violet	400 – 450
blue	450 – 500
green	500 – 570
yellow	570 – 590
orange	590 – 610
red	610 – 750

- Radio waves are produced by oscillating charge in an antenna.
- Microwaves can be produced by Klystron™ and magnetron tubes.
- Infrared radiation and light can be produced when high-energy electrons in very hot objects drop to lower energy levels.
- Ultraviolet radiation can be produced when electrons in excited atoms drop to lower energy levels.
- X rays are produced by rapidly stopping very energetic electrons that have been accelerated by a large potential difference.
- Gamma rays are emitted by unstable nuclei when they return to a more stable state.
- A deeper understanding of electromagnetic radiation has led to several applications, including television and radio broadcasts, global positioning systems, wildlife tracking systems, and magnetic resonance imaging.

Knowledge/Understanding

1. A magnetic field in an electromagnetic wave travelling south oscillates in an east-west plane. What is the direction of the electric field vector in this wave?
2. Describe how an antenna works for transmitting and receiving radiation.
3. If you could see the electric fields in light, how would the electric fields appear if you were looking straight toward a light source?
4. How can unpolarized light be transformed into polarized light?
5. **(a)** What is the cause of glare?
   (b) How do Polaroid sunglasses reduce glare?

6. What happens if sunglasses polarized to allow vertical vibrations through are turned 90°?
7. Sketch an electromagnetic wave and label the appropriate parts.
8. Television antennas that receive broadcast stations (not cable or satellite) have the conductors oriented horizontally. What does this imply about the way signals are broadcast by the stations?
9. Why cannot a radio station transmit microwaves?
10. Why do you not see interference effects from light entering a room from two different windows?
11. Does the speed of an electromagnetic wave depend on either the frequency or wavelength?
12. Is light a longitudinal or transverse wave? How do you know?
13. Is it possible to get a sunburn through a closed window?

Inquiry

14. Before cable and satellite television were available, most people had indoor antennas called "rabbit ears" that they could manually move around to get the best reception. Often, when someone would be touching the antenna while moving it, the reception would be good. When the person walked away from the antenna, the reception became poor again. Suggest a possible reason for this phenomenon.
15. Devise one or more situations involving an electron, a proton, or a neutron in constant motion, accelerated motion, or at rest, to produce the following.
    - **(a)** an electric field only
    - **(b)** both electric and magnetic fields
    - **(c)** an electromagnetic wave
    - **(d)** none of these
16. Suppose two pairs of identical polarizing sunglasses are placed in front of each other. Explain clearly your answers to the following.
    - **(a)** What would you observe through them?
    - **(b)** If one pair is rotated 90° in relation to the other, what would you observe?
    - **(c)** If a third pair, oriented randomly, is inserted between the two pairs, what would you observe?
17. All objects, including human beings, emit electromagnetic radiation according to their temperatures. Through thought experiments, predict whether hotter objects would emit longer or shorter wavelength radiation than cooler objects.

Communication

18. Make sketches to demonstrate that a mirror's surface appears to be smooth if its atoms are smaller than the wavelength of light and that it would appear to be bumpy if its atoms were larger than a wavelength of light.
19. Find the approximate frequency and wavelength of the waves associated with the following.
    - **(a)** your favourite AM radio station
    - **(b)** your favourite FM radio station
    - **(c)** a microwave oven
    - **(d)** a conventional oven
    - **(e)** green light
    - **(f)** dental X rays
20. How can you test the light of the blue sky to determine its direction of polarization?
21. Describe how you can determine whether your sunglasses are polarizing material or tinted glass.
22. Explain why a flashlight using old batteries gives off reddish light, while light from a flashlight using new batteries is white.
23. A doctor shows a patient an X ray of a fractured bone. Explain how the image is produced. What kinds of materials can and cannot X rays penetrate?
24. At night, you can often pick up more distant radio stations than in the daytime. Explain why this is so.

Making Connections

25. Ultraviolet rays, X rays, and gamma rays can be very harmful to living things. What is unique about these forms of electromagnetic waves that could cause damage to living cells?

26. (a) Discuss methods to measure the speed of a race car and the speed of a bullet.
(b) What are the largest sources of error?
(c) What difficulties do you encounter if you try to apply these methods to measure the speed of light?

27. Some species of snakes, called "pit vipers," have sensors that can detect infrared radiation. What do you think is the function of these sensors?

28. Reflectors left on the Moon's surface by the Apollo astronauts can be used to accurately measure the Earth-Moon distance with lasers. To achieve an accuracy of 10 m, what must be the accuracy of the timing device?

29. Investigate the relationship between coloured light and coloured cloth. Make different coloured lights (with coloured glass or transparent plastic film). Explain why the colours of coloured cloths change as they are viewed under different-coloured light sources.

30. Suppose your eyes were sensitive to radio waves instead of visible light.
(a) What size of radio dish would you need on your face to achieve the same resolution?
(b) What things would look bright?
(c) What things would look faint?

31. You see a lightning flash and simultaneously hear static on an AM radio. Explain why.

32. (a) Why does an ordinary glass dish become hot in a conventional oven but not in a microwave oven?
(b) Why should metal not be used in a microwave oven?
(c) Microwave ovens often have "dead spots" where food does not cook properly. Why might this occur?

33. Research the basic components required for a radio transmitter and receiver. Describe how a signal is transmitted and received.

34. Film used for modern medical and dental X rays is far more sensitive to X rays than film that was used when these forms of X rays were first developed. Why do you think it was important to increase the sensitivity of the film?

Problems for Understanding

35. If a gamma ray has a frequency of 1.21×10^{21} Hz, what is its wavelength?

36. Radio waves 300 m long have been observed on Earth from deep space. What is the frequency of these waves?

37. The announcer on an FM radio station in Toronto identifies the station as "The Edge 102.1," where the number 102.1 is the frequency in some units. What is the wavelength and frequency of the waves emitted by the radio station?

38. A 100 kW (1.00×10^5 W) radio station emits electromagnetic waves uniformly in all directions.
(a) How much energy per second crosses a 1.0 m^2 area receiver that is 100.0 m from the transmitting antenna? (Hint: The surface area of a sphere is $4\pi r^2$.)
(b) Repeat the above calculation for a distance of 10.0 km from the antenna.
(c) If you double the distance between the transmitter and the receiver, by what factor will the energy per second crossing the area decrease?

39. A light-minute is the distance light travels in one minute. Calculate how many light minutes the Sun is from Earth.

40. Airplanes have radar altimeters that bounce radio waves off the ground and measure the round-trip travel time. If the measured time is 75 μs, what is the airplane's altitude?

41. If you make an intercontinental telephone call, your voice is transformed into electromagnetic waves and routed via a satellite in geosynchronous orbit at an altitude of 36 000 km. About how long does it take before your voice is heard at the other end?

42. You charge a comb by running it through your hair.
(a) If you then shake the comb up and down, are you producing electromagnetic waves?
(b) With what frequency would you have to shake the comb to produce visible light?

UNIT 4 PROJECT

Constructing Your Own FM Transmitter

Background

Radio waves, a form of electromagnetic radiation, play a major role in communication technology, including radio and television program transmission, cellphone service, wireless computer connections, and satellite operations. In this activity, you will construct an FM transmitter that is capable of broadcasting your voice to any nearby portable radio. You will then use the transmitter to investigate the variables that affect it.

Challenge

Construct and test an FM transmitter, using the kit provided. To accomplish this task, you will need to learn how to identify and then solder circuit components.

Design and conduct experiments to investigate relationships between the signals emitted by your transmitter and predictions made by the wave model for electromagnetic radiation.

Materials

- FM transmitter kit
- portable radio
- soldering pencil
- solder
- small wire cutters
- small screw driver
- battery
- wet sponge

Safety Precautions

- Ensure that all electrical equipment is properly grounded.
- Be extremely careful when working with a soldering pencil. The heated iron tip can cause serious burns in an instant.
- Ensure that you are wearing eye protection. Solder might splatter when you are cleaning the soldering pencil with the wet sponge.
- Avoid inhaling fumes generated by the solder. Work in a well-ventilated area.

Design Criteria

A. As a class, develop assessment criteria to address the operation of each transmitter. You might want to include some or all of the following categories.
  - ability to transmit your voice from the transmitter to a portable radio
  - range of reliable signal transmission
  - construction quality of transmitter
  - manufacture of a peaking circuit to test transmitter; a peaking circuit is a very simple circuit that allows you to tune your transmitter to its maximum output by simply measuring the potential difference across an element in the peaking circuit
  - technical statistics, such as maximum output voltage radiated by your transmitter, detected using a peaking circuit

B. Develop experiment procedures that will allow you to test predictions made by the wave model for electromagnetic radiation. You might want to design and conduct experiments to investigate the relationship between
  - aerial length and transmitting frequency
  - range of the transmitter and the amount of input energy
  - amount of signal absorption and the amount of input energy
  - environmental factors and reflection or interference effects

  As a class, develop criteria to assess both the experiment design and the validity of the obtained results.

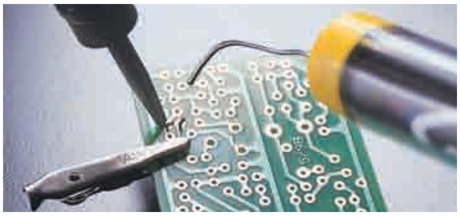
After you complete this project

- assess the development of your technical skills during the construction phase
- assess the quality of your transmitter based on established criteria
- assess the ability of your experiment design to test specified theoretical predictions

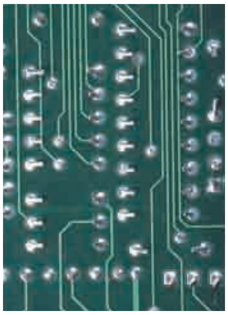
Action Plan

Part 1: Build the Transmitter

1. Investigate the proper soldering technique, referring to the Internet or other resources. Before you open your transmitter kit, practise soldering on an old circuit board and components. Place a hot, clean soldering pencil against both the conducting surface of the circuit board and the component for 1 or 2 s. Carefully dab and remove the solder at the point where the pencil is touching both the component and the circuit board. The solder must come into contact only with the component (for example, the resistor or capacitor) and the metal conducting surface of the board. Adjacent soldered connections must not touch. Always test each soldered joint by gently pushing on the component. The component should not move. Always clean excess solder from the iron by using a wet sponge between soldering attempts. Attach an alligator clip to the board to radiate away excess heat.



2. Open the FM transmitter kit. Using the kit's instructions, identify each component and its proper location on the circuit board. Begin construction by carefully soldering the shortest components first.
3. Once each component is in place, inspect each soldered joint. Ensure that they are secure and that each soldered joint is free from contact with adjacent joints.
4. Test your transmitter using a portable radio. (This might involve the construction and use of a simple peaking circuit.)

A  B 

(A) Robotically soldered circuit board. **(B)** Improperly soldered connections that connect two different components.

5. Test your transmitter based on the criteria decided on by your class.

Part 2: Testing the Wave Theory

6. Design and conduct investigations relating to criteria decided on by your class. Ensure that your experiment design clearly identifies the conditions that you will control and the variables that you are testing. Record your theoretical predictions, based on the wave model, before conducting the experiment. To ensure that the appropriate safety measures are being taken, check with your teacher before conducting any experiments,.
7. Prepare a report of your findings.

Evaluate

1. Assess the success of your FM transmitter, based on your class's previously determined criteria. Compare your results with your classmates, taking careful note of differences in the construction of the device.
2. Assess your experiment design and the results you generated, based on the class's previously determined criteria. Recommend ideas for further experimentation.

UNIT 4 Review

Knowledge/Understanding

Multiple Choice

In your notebook, write the letter of the best answer for each of the following questions. Outline your reasons for your choice.

1. Two objects are just able to be resolved when the
   (a) central maximum of one falls on the first minimum of the other
   (b) central maxima of the objects do not overlap
   (c) angle separating them is greater than the wavelength of light
   (d) distance from the objects to your eyes is sufficiently reduced
2. Digital videodiscs (DVDs) use
   (a) thin-film principles
   (b) interference effects
   (c) shorter laser light than compact disc players
   (d) All of the above.
3. An electron falling through a potential difference generates
   (a) a magnetic field only
   (b) an electric field only
   (c) stationary electric and magnetic fields
   (d) no fields
   (e) an electromagnetic wave
4. Electromagnetic waves propagate in a direction
   (a) parallel to the oscillation of the electric field
   (b) parallel to the oscillation of the magnetic field
   (c) perpendicular to the oscillations of both the magnetic and electric fields
   (d) independent of the oscillations of either the magnetic or electric field
5. Which of the following phenomena leads to the interpretation that electromagnetic radiation is a transverse wave?
   (a) diffraction
   (b) partial reflection, partial refraction
   (c) linear propagation
   (d) polarization
6. Which of the following is *not* a result of the superposition of waves?
   (a) destructive interference
   (b) diffraction
   (c) refraction
   (d) constructive interference
7. Electromagnetic waves differ from mechanical waves because
   (a) they undergo diffraction
   (b) they do not require a medium in which to travel
   (c) they are transverse waves
   (d) their speed is determined by the medium through which they are travelling
8. Which of the following statements is *not* true about the properties of electromagnetic waves?
   (a) X rays are emitted by unstable nuclei of atoms.
   (b) Extremely low frequency radio waves penetrate salt-water better than higher frequencies.
   (c) Ultraviolet light was discovered by accident.
   (d) Satellites that detect infrared radiation can see through clouds.
9. Maxwell's first law, known as Gauss's law,
   (a) relates a changing magnetic field and the induced *emf*
   (b) relates electric field lines to the charges that create them
   (c) relates magnetic field lines to the charges that create them
   (d) predicts the existence of electromagnetic waves

Short Answer

10. Explain why you do not see interference effects from light entering a room from two different windows.
11. Explain the effect of turning polarized sunglasses through an angle of 90°. What happens if sunglasses polarized to allow vertical vibrations through are turned 90°?
12. The equation, $\lambda \cong \frac{\Delta y d}{x}$, relates the wavelength of light to the distance between slits in a diffraction grating, the distance from the grating to a screen, and the distance between fringes on the screen. What approximation was made in deriving this relationship and under what conditions is the approximation valid?

13. What does the Maxwell's 4th equation predict?

14. Do research on "cosmic rays" anthem. Compare the differences between cosmic rays and electromagnetic radiation.

15. **(a)** In single-slit diffraction, how does the width of the central maximum compare to the width of the other maxima?
    (b) How does the width of the maxima produced by a diffraction grating compare to those produced by a double slit?

16. Describe the necessary conditions for two light waves incident at a single location to produce a dark fringe.

17. How is it possible to determine how thick a coating should be on a pair of glasses to reduce the amount of reflection? How thick should the coating be?

18. **(a)** Define electromagnetic radiation.
    (b) What evidence is there that light is electromagnetic radiation?
    (c) What evidence is there that sound is not electromagnetic radiation?

19. Consider an electromagnetic wave propagating in the positive x-direction. At a time, t_0, the electric field points in the positive y-direction. In what direction does the magnetic field point at this time? Sketch the electromagnetic wave. In what directions will the electric and magnetic fields point half a period later?

Inquiry

20. How do magnetic resonance imaging systems make use of hydrogen atoms?

21. Two friends are hired by a telemarketing firm for the summer. Each friend has a desk that is separated from the other workers by only a thin half-wall. One friend notices a continuous humming sound when she is sitting at her desk. The other notices that there is no humming noise audible at his desk. Some investigation finds that the company has two speakers placed at one end of the working floor, 8.0 m apart. A continuous, low-frequency hum is generated to mask conversations from nearby desks. The two students conduct a survey and find that several people do not hear the humming noise at their desks. For each of these people, they measure the distance from each speaker to the desk. The table below is the data they collected.

Person	Distance from speaker A (m)	Distance from speaker B (m)
1	14	8
2	10	4
3	12	10
4	10	12
5	6	12

The friends also sample the air temperature and find that it is always 23°C.

(a) Provide an explanation, based on the characteristics of waves, for why some workers will hear the hum while others will not.
(b) Use the data provided in the table to determine the frequency of the low-frequency hum.
(c) The company is unhappy to learn that five people are unable to hear the low-frequency hum intended to mask nearby conversations. Suggest how the company could mask conversations more effectively.

22. You notice that a telephone pole casts a clear shadow of the light from a distant source. Why is there no such effect for the sound of a distant car horn?

23. Design and make a simple model of a laser. Identify and explain the function of the principal components. Discuss why a laser beam is so narrow.

24. Suppose white light is used in a Young's experiment. Describe the characteristics of the resulting fringe pattern and sketch it.

25. Challenge: Many experiments and optical instruments exploit the property of rectilinear propagation of light by reflecting a light beam many times for a desired effect. In this challenge, use your knowledge of physics to

control a television or VCR from outside a room or around a corner. Construct several $10 \text{ cm} \times 10 \text{ cm}$ reflector cards by covering the cards with aluminum foil and see how many times you can reflect a remote control beam and still turn on the television or VCR. Test long distances (out in the hall, down another hall) as well.

26. Triangulation is a basic geometrical method that has been used since the time of the ancient Greeks. Surveyors use it to determine the distance to an object by sighting it from two different positions a known distance apart. With two angles and a side, or a side and two angles, the dimensions of the triangle can be determined. How could you use the method to determine the altitude of a global positioning system satellite or the distance to a planet or nearby star? What effect will a 1% error in the measured angle have on the calculated distance in each case?

Communication

27. Although the wavelengths of optical radiation are very small, they can be measured with high accuracy. Explain how this is possible.

28. You see a lightning flash and simultaneously hear static on an AM radio. Explain how these occurrences are related.

29. Explain why an optical telescope must have a smooth surface, while the surface of a radio telescope is not as highly machined.

30. The wave model of light predicted that the speed of light would slow down when travelling from one medium into another of greater optical density. Use Huygens' principle to demonstrate this prediction. Include a diagram.

31. Make a list of the characteristics of light that a model should explain: rectilinear propagation, reflection, refraction, partial reflection, partial refraction, dispersion, and diffraction. Briefly discuss how the wave model of light and the particle model of light explain these phenomena.

32. Compare the collision between two oppositely directed particles with the collision between two oppositely directed water waves. What are the similarities and differences between these interactions?

33. **(a)** Test light diffraction with the shadow of your hand. How can you cast the sharpest and fuzziest shadows?
(b) Describe other examples of light diffraction.

34. **(a)** With a sketch that shows individual waves of light, demonstrate how Young used diffraction to create a two-point light source that was exactly in phase.
(b) Show how these two sources interfered to produce a series of light and dark bands on a screen.

35. Due to the popularity of Newton's particle model of light, Young's work on light interference was received with scepticism by British scientists. Explain how the evidence of wave behaviour that you observed in ripple tanks models the results of Young's experiment and his conclusion that light behaves like a wave.

36. Select one of the following statements about the competing models of light and develop an argument to refute it.
(a) Both models found support because neither model could adequately describe every observed property of light.
(b) Both models found support because each model adequately described every observed property of light at the time.
(c) Newton's model for light found support primarily because of his fame and respected stature.

37. **(a)** Explain how the definition of "one metre" was redefined in 1961.
(b) In 1983, the metre was redefined again to be the distance light travels in 1/299 752 458 s. Why do you think this was done?

38. Thin films, such as soap bubbles or gasoline on water, often have a multicoloured appearance that sometimes changes while you are watcing. Explain the multicoloured appearance of these films and why their appearance changes with time.

Making Connections

39. In the seventeenth century, it was not known whether light travelled instantaneously or with finite speed.

(a) Early in that century, Galileo attempted to measure the speed of light by stationing one helper one kilometre away and timing how long it took a pulse of light to travel the distance. Explain why that attempt was unsuccessful.

(b) Ole Roemer made the first successful attempt to measure the speed of light around 1675. He made a long series of observations to accurately determine the period of one of Jupiter's moons, Io, around Jupiter. When he later used this information to predict when Io would be eclipsed by Jupiter, he found that his predictions were too early or too late, compared to the observations, depending on Earth's position in its orbit. Investigate Roemer's method and clearly explain, with the aid of a diagram, how he was able to successfully measure the speed of light. What factor(s) limited the accuracy of Roemer's method?

(c) An elegant and much more accurate measurement of the speed of light was made by Albert Michelson in 1880. He used a rotating octagonal mirror and sent a beam of light to a stationary mirror on a mountaintop 35 km away. Sketch Michelson's experiment set-up. He refined this experiment over many years. Discuss how he was able to measure the speed of light so accurately.

40. Cochlear implants are sometimes used to assist the hearing of deaf people. Their operation relies on the broadcasting and receiving of electromagnetic waves and the ultimate stimulation of the auditory nerve. If the auditory nerve is intact, the deaf person can learn to recognize sounds. Sketch the components of a cochlear implant and describe how it works.

41. In an interferometer, light following different paths is allowed to interfere. By measuring the interference fringes, the different path lengths can be precisely determined. Gravitational wave detectors use interferometers to search for ripples in the fabric of space and time. These ripples were predicted by Einstein's theory of general relativity and are thought to be produced by collisions of two black holes or the collapse of massive stars in supernova explosions. Research the operations of the Laser Interferometer Gravity-Wave Observatory and the proposed Laser Interferometer Space Antenna or other gravitational wave observatories. Why are the interferometers used in these observatories so long? What sensitivity do the scientists hope to achieve? What are the goals of these projects? How will detection of gravitational waves change our view of the universe?

42. Investigate and explain in detail why glass is transparent to visible light, but opaque to ultraviolet and infrared light.

43. For more than a century, photographic plates and film have been used to record light in various detectors. Now these are being replaced by devices that record light digitally, such as charged coupled devices. Contrast these two technologies in terms of sensitivity (the ability to detect faint objects) and resolution.

44. **(a)** Investigate different methods of using polarized glasses to produce 3-D films. How does the IMAX technology (see Chapter 2, The *Big* Motion Picture) differ from virtual reality technology?

(b) Some viewers complain of feeling nauseated during 3-D films. Why does this occur? How can it be avoided or minimized?

45. Astronomy is described as an observational science, not an experimental science such as physics, biology, and chemistry. Stars cannot be reproduced in the lab, so starlight and light from other sources must be studied as thoroughly as possible. Research and write an essay on how astronomers maximize the information obtained from light through the use of detectors that measure light's intensity, wavelength, direction, and polarization. Include a discussion of how astronomy has benefited from the space age by the launching of satellite observatories that observe wavelength ranges blocked by the atmosphere. What kinds of astrophysical phenomena are best studied in each wavelength range of the electromagnetic spectrum?

46. In the Search for Extraterrestrial Intelligence (SETI) program, signals in the radio wavelength region of the electromagnetic spectrum are being examined for unusual patterns that might indicate an intelligent (instead of natural) source.
  (a) Why has the radio wavelength region of the spectrum been chosen?
  (b) Are there frequencies within the radio region for which a detection might be more likely than others?
  (c) What might an intelligent signal look like?
  (d) The SETI@home program asks participants to use their home computers to scan data currently being obtained from the Arecibo radio telescope. What progress has been made in SETI's search to date?

47. The coherence of a laser beam allows it to be broken up into extremely short pulses called "bits." These bits allow information to be stored in digital form. Investigate the role of the laser in the storage, transmission, and retrieval of information in various media. Evaluate the efficiency of this process and discuss how consumers might expect it to evolve in the future. Summarize your findings in a report.

48. What is the limit of resolution of an optical microscope? How does this affect the types of things that can be studied with this instrument? To study finer detail, scanning electron microscopes are used (see Chapter 12, Quantum Mechanics and the Atom). What is the limit of resolution for these instruments?

49. Different physical processes are responsible for producing electromagnetic radiation of different wavelengths. For each wavelength region of the electromagnetic spectrum (radio, infrared, visible, ultraviolet, X ray, gamma ray), identify at least one physical process that produces radiation in that wavelength range.

Problems for Understanding

50. A detector tuned to microwave wavelengths registers 2500 wave crests in 1.0 μs. What is the wavelength, frequency, and period of the incoming wave?

51. If an electromagnetic wave has a period of 4.8 μs, what is its frequency and wavelength?

52. Calculate the wavelength of a 10^{21} Hz gamma ray.

53. How many cycles of a 5.5×10^{-9} m ultraviolet wave are registered in 1.0 s?

54. What is the colour of light that has a frequency of 7.0×10^{14} Hz?

55. The most efficient antennas have a size of half the wavelength of the radiation they are emitting. How long should an antenna be to broadcast at 980 kHz?

56. How long should a microwave antenna be for use on a frequency of 4400 MHz?

57. A light-year is the distance light travels in one year. How far is this in metres? (There are 365.25 days in one year.)

58. A concert in Halifax is simultaneously broadcast to Vancouver on FM radio. Determine whether people listening in Vancouver, approximately 5.0×10^3 km away, will hear the music just before or just after someone sitting in the back of the 82 m concert hall in Halifax. Assume that the speed of sound in the concert hall is 342 m/s.

59. Yellow light is incident on a single slit 0.0315 mm wide. On a screen 70.0 cm away, a dark band appears 13.0 mm from the centre of the bright central band. Calculate the wavelength of the light.

60. **(a)** The beam of a helium-neon laser ($\lambda = 632.8$ nm) is incident on a slit of width 0.085 mm. A screen is placed 95.0 cm away from the slit. How far from the central band is the first dark band?

(b) If the slit was two times wider, would the first dark band be closer or farther from the central band?

61. If a spectrum is to have no second order for any visible wavelength, how many lines per cm must the grating have?

62. A certain beetle has wings with a series of bands across them. When 600 nm light is incident normally and reflects off the wings, the wings appear to be bright when viewed at an angle of 49°. How far apart are the lines in the bands?

63. Radio astronomers utilize interferometry to build large arrays of radio dishes and thereby achieve much greater resolution. In fact, when the signals from two small dishes 1 km apart are properly combined, the two dishes have the same resolving power as one giant dish that is 1 km across.

(a) The Very Large Array, in New Mexico, has a maximum dish separation of 36 km. What resolution can be obtained at a wavelength of 6.0 cm?

(b) New interferometers are proposed, which would stretch across entire continents. What is the resolution of a very long baseline interferometer at this wavelength if the dishes are separated by 3600 km?

(c) What minimum separation between two radio sources at the centre of the Milky Way galaxy, 24 000 light-years away, could this interferometer distinguish? Compare this to the radius of Pluto's orbit, 5.9×10^{12} m.

(d) Would the resolution of these telescopes become better or worse if they operated at shorter radio wavelengths?

64. What size of orbiting optical space telescope would you require to measure the diameter of a star that has the same diameter as the Sun and is 10.0 light-years away? (Use a wavelength of 550 nm, and take the Sun's diameter as 1.40×10^9 m.)

65. A diffraction grating has $5000 (5.000 \times 10^3)$ lines per cm. Monochromatic light with a wavelength of 486 nm is incident normally on the grating. At what angles from the normal will the first, second, and fourth order maxima exit the grating? Do the angles increase linearly with the order or the maxima? Explain.

66. Cherenkov radiation is light emitted by a particle moving through a medium with a speed greater than the speed of light in the medium. (**Note:** The speed of the particle is not greater than the speed of light in a vacuum.) Consider a beam of electrons passing through water with an index of refraction of 1.33. If Cherenkov light is emitted, what is the minimum speed of the electrons?

67. If the first order maximum of He-Ne light ($\lambda = 632.8$ nm) exits a diffraction grating at an angle of 40.7°, at what angle will the first order maximum of violet light with a wavelength of 418 nm exit?

COURSE CHALLENGE

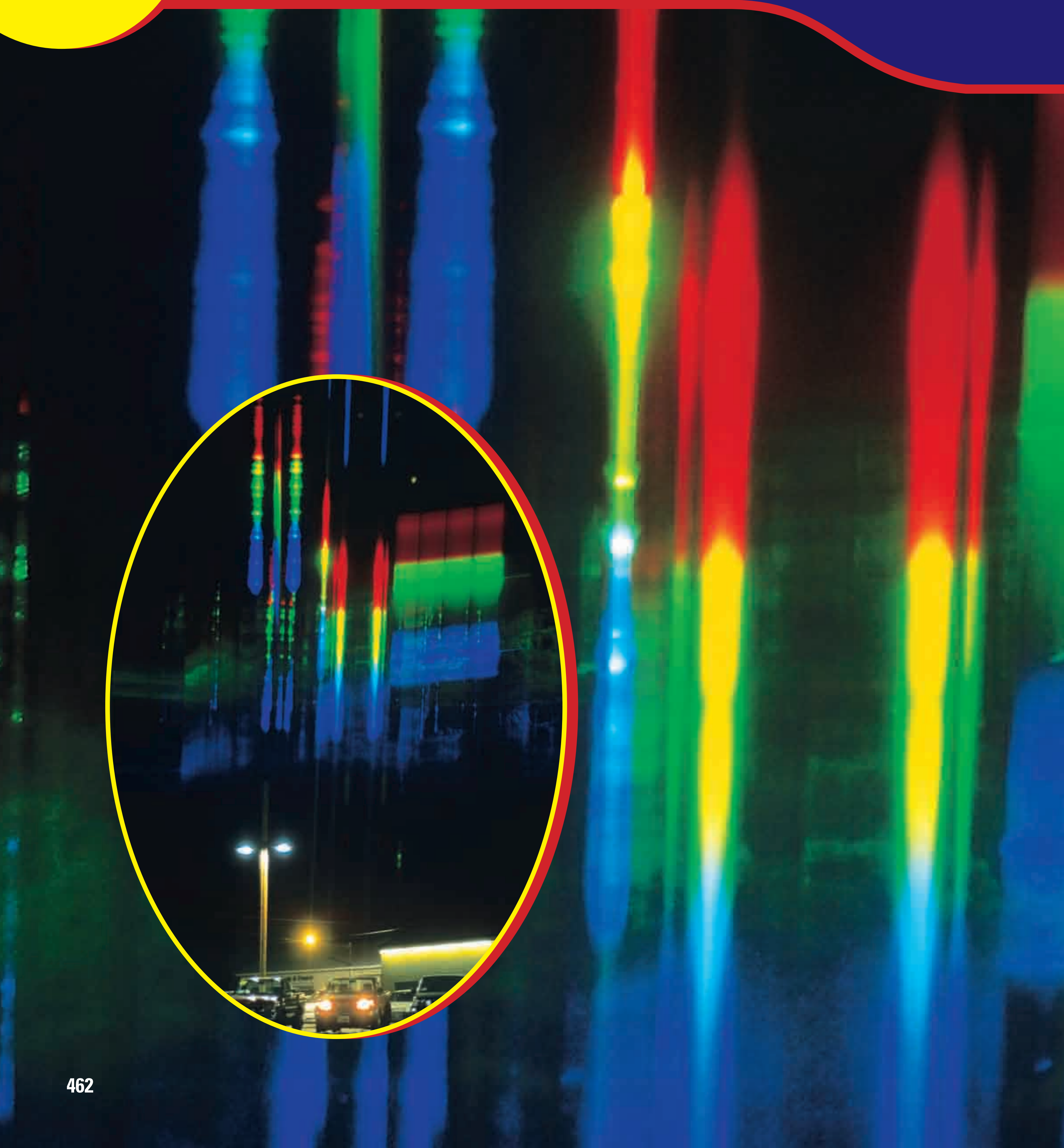
Scanning Technologies: Today and Tomorrow

Continue to plan for your end-of-course project by considering the following.

- How do the wave properties of electromagnetic radiation relate to your project?
- Are you able to incorporate newly learned skills from this unit into your project?
- Analyze the information contained in your research portfolio to identify knowledge or skills gaps that should be filled during the last unit of the course.

UNIT 5

Matter-Energy Interface



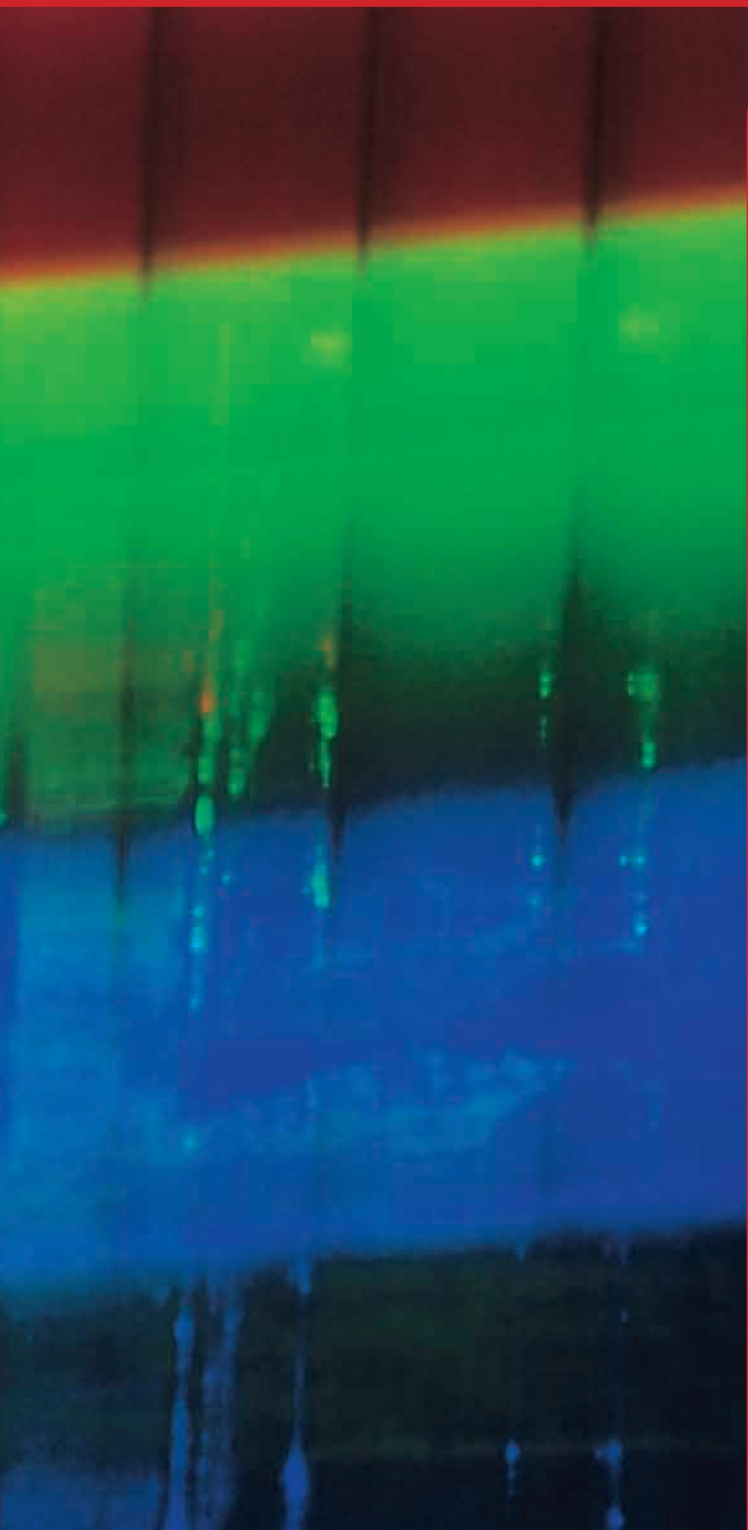
OVERALL EXPECTATIONS

DEMONSTRATE an understanding of the basic concepts of Einstein's special theory of relativity and of the development of models of matter that involve an interface between matter and energy, based on classical and early quantum mechanics.

INTERPRET data to support scientific models of matter and conduct thought experiments as a way of exploring abstract scientific ideas.

DESCRIBE how the introduction of new conceptual models and theories can influence and change scientific thought and lead to the development of new technologies.

UNIT CONTENTS



The turn of the twentieth century was a time of excitement and turmoil in science. In 1888, Heinrich Hertz demonstrated the existence of radio waves and then, in 1895, Wilhelm Conrad Röntgen discovered X rays. The following year, Antoine Henri Becquerel discovered radioactivity and, a year later, J.J. Thomson discovered the electron. Then, Philipp Lenard observed the photoelectric effect in which light ejected electrons from metals.

Along with these discoveries came a number of puzzles. How could radioactive substances emit radiation without any apparent source of energy? Why could only certain colours of light eject electrons from metals? For nearly 50 years, spectroscopists wondered why each element gave off a unique spectrum of light. Since light crossed the vacuum of space between Earth and the stars, physicists assumed that a substance called "luminiferous ether" must exist to carry light waves. However, all attempts to detect Earth's motion through it failed.

In this unit, not only will you learn more about the discoveries of some of the most outstanding scientists who ever lived, but also you will learn the answers to some of the questions that baffled them.

UNIT PROJECT PREP

Refer to pages 590–591 before beginning this unit. In this unit project, you will examine the parallels between scientists and their theories with societal pressures and realities.

- Revisit the time line on page xiv and try to remember some significant societal events that took place during the years represented.
- How closely tied do you think scientific research is to societal pressures?

CHAPTER 11

Special Theory of Relativity

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Interaction of electric and magnetic fields
- vector addition
- frames of reference
- relative velocity

The name of Albert Einstein has towered over the field of physics during the twentieth century and on into the twenty-first century. In the space of a few years, Einstein not only changed the world's way of thinking about electromagnetic radiation such as light, he also radically changed the commonly accepted picture of the universe with his two relativity theories — the special theory of relativity and the general theory of relativity.

The special theory of relativity, which you will be studying in this chapter, was not well received at first. The idea that fundamental measurements such as time, distance, and mass depended on the relative motion of the observer seemed absurd to many. In fact, Einstein was awarded the 1921 Nobel Prize for physics for his development of the concept of photons and the resulting explanation of the photoelectric effect, not for his theories of relativity.

With the advent of high-energy physics, however, Einstein's theory of special relativity became essential to the understanding of the behaviours of all high-speed subatomic particles.

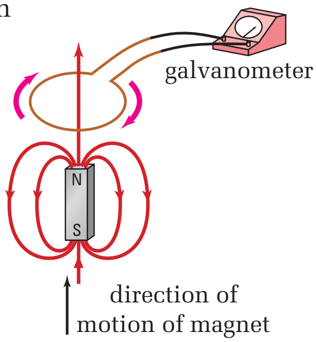
QUICK LAB

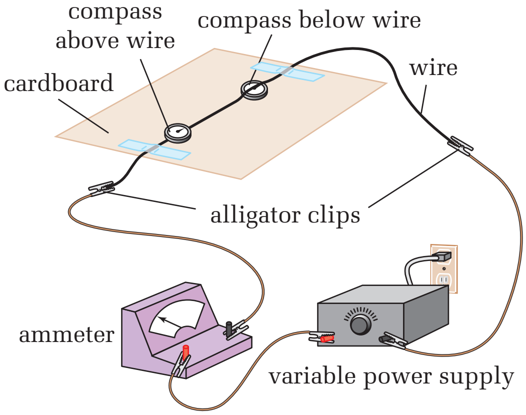
Generating Electromagnetic Fields

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting

One of the problems that led to Einstein's special theory of relativity came from an analysis of the way in which electric and magnetic fields spread out through space as electromagnetic waves. In previous science courses, you have studied various characteristics of magnetic and electric fields; now, examine carefully the two diagrams and try to answer the questions that follow each of them. Discuss the answers with your classmates. Then, carry out the activity that follows these questions.

- Under what condition does a magnetic field generate an electric current?
- What determines the magnitude of the current?
- What determines the direction of the current?
- How do you know that an electric field must have been generated across the coil?

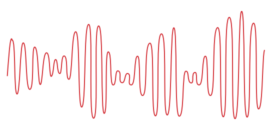




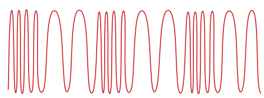
- What evidence is there that an electric current can generate a magnetic field?
- What affects the strength of the magnetic field?
- What affects the direction of the magnetic field?
- What kind of field is needed to produce an electric current?

Lab

Obtain a radio with a movable antenna. Turn on the radio and set it in the AM range. ("AM" refers to amplitude modulation, a process in which a signal is impressed on a radio carrier frequency by varying its amplitude, as shown in the diagram.) Turn on an induction coil and allow an arc (or spark) to pass between the points of the electrodes. Listen for the effect on the radio. Find the antenna orientation for which the effect is (a) greatest and (b) least. Repeat these steps with the radio tuned to an FM station. ("FM" stands for frequency modulation, in which the signal is impressed on the carrier wave through variations in its frequency, as shown.)



AM signal



FM signal

Analyze and Conclude

1. What evidence is there that radio waves (electromagnetic radiation) are travelling from the arc to the radio?
2. Is there any relationship between the orientation of the arc and the orientation of the antenna for maximum and minimum effects?
3. If there is a relationship in question 2, what does that indicate about the nature of the waves produced from the arc?
4. **(a)** What difference do you notice with the FM station? **(b)** Try to explain this difference.

11.1 Troubles with the Speed of Light

SECTION EXPECTATIONS

- State Einstein's two postulates for the special theory of relativity.
- Conduct thought experiments as a way of developing an abstract understanding of the physical world.
- Outline the historical development of scientific views and models of matter and energy.

KEY TERMS

- interferometer
- Lorentz-Fitzgerald contraction

Toward the end of the nineteenth century, many scientists felt that they were close to a complete understanding of the physical world. Newton's laws described motion. Maxwell's laws described radiant energy. The chemists were learning more and more about the behaviour of atoms. No one realized that their fundamental concepts of space, time, matter, and energy were seriously limited.

The Michelson-Morley Experiment

The first indication of a difficulty came from a critical experiment performed in 1881 by Albert Michelson (1852–1931), using an **interferometer**, an instrument he had devised for measuring wavelengths of light. In this experiment, he unsuccessfully attempted to detect the motion of Earth through the luminiferous ether, the substance that was then believed to be the medium through which light waves could travel through space. The apparent failure of Michelson's first experiment to find any such motion prompted many physicists to drop the ether concept.

Later, in 1887, Michelson and Edward Williams Morley (1838–1923) performed a refined version of the experiment, using an improved version of the interferometer. They reasoned that if light behaved like a sound wave or a wave on water, if you moved toward an oncoming beam of light, it would seem to approach you at a higher speed than if you were moving away from it. These different speeds would affect the interference pattern in the interferometer. By comparing interference patterns for light beams travelling perpendicular to each other, Michelson and Morley hoped to detect and measure the speed with which Earth passed through the ether.

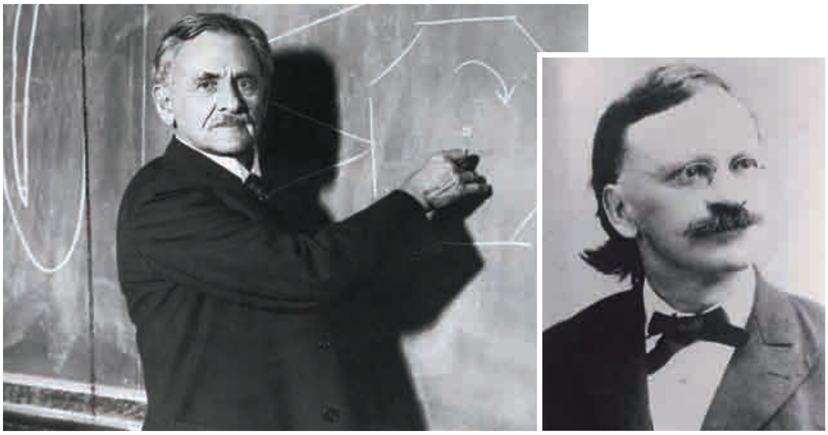


Figure 11.1 Albert Michelson (left) and Edward Morley used Michelson's interferometer (see Figure 11.2) to conduct an experiment that later became the foundation of Einstein's special theory of relativity.

To understand the basis of this experiment, consider the following scenario involving relative velocities. Two identical boats, X and Y, are about to travel in a stream. Boat Y will go straight across the stream and straight back. Boat X will travel the same distance downstream and then return to its starting point. Which boat will make the trip in the shortest time? Examine Figure 11.3 and then follow the steps below to determine the time required for boat Y to travel across the stream and back.

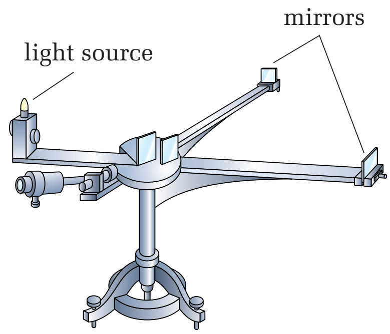


Figure 11.2 Michelson's first interferometer was designed to determine wavelengths of light. It should also be able to determine whether light travelling in directions perpendicular to each other travelled at different speeds.

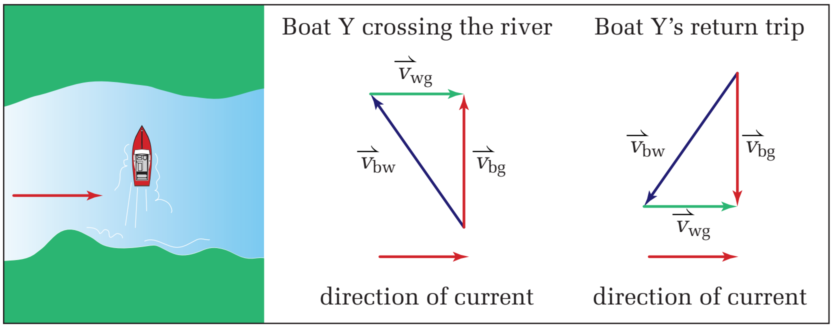


Figure 11.3 Since boat Y must go directly across the stream, the driver must angle the boat upstream while crossing either way perpendicular to the current.

- Define the symbols.

$\vec{v}_{bw}$: velocity of the boat relative to the water
$\vec{v}_{wg}$: velocity of the water relative to the ground
$\vec{v}_{bg}$: velocity of the boat relative to the ground
L : distance travelled along each leg of the trip
Δt : total time for the trip

- Write the definition for velocity and solve it for the time interval.

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$
$$\Delta t = \frac{\Delta \vec{d}}{\vec{v}}$$

- Use vector addition to find the magnitude of the velocity of the boat relative to the ground. Notice in Figure 11.3 that this velocity is the same for both legs of the trip.

$$(v_{bw})^2 = (v_{bg})^2 + (v_{wg})^2$$
$$(v_{bg})^2 = (v_{bw})^2 - (v_{wg})^2$$
$$v_{bg} = \sqrt{(v_{bw})^2 - (v_{wg})^2}$$

- Substitute the total length of the trip ($2L$) and the magnitude of the velocity into the expression for the time interval to find the time required for boat Y to make the round trip.

$$\Delta t_Y = \frac{2L}{\sqrt{(v_{bw})^2 - (v_{wg})^2}}$$

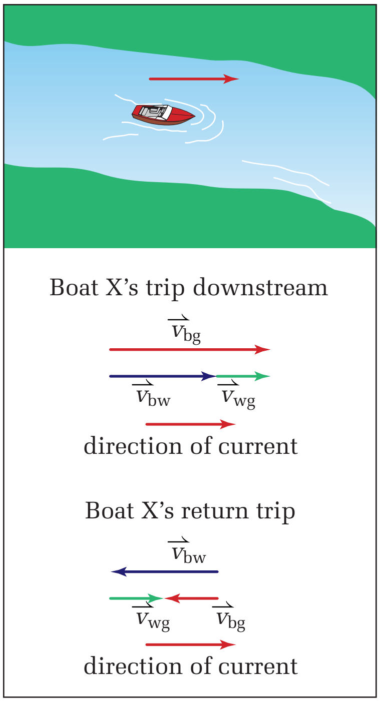


Figure 11.4 Boat X travels with the current when it is going downstream and against the current on its return trip.

Study Figure 11.4 to determine the velocities of boat X as it makes its trip downstream and back. Then, follow the steps below that determine the time for boat X to make the trip.

- Since the direction of the velocities of boat X and of the stream are in one dimension, the magnitudes can be added algebraically.

  Trip downstream: $v_{bg} = v_{bw} + v_{wg}$
  Trip upstream: $v_{bg} = v_{bw} - v_{wg}$

- Use the equation for the time interval in terms of displacement and velocity to write the time interval for boat X to travel downstream.

$$\Delta t_{down} = \frac{L}{v_{bw} + v_{wg}}$$

- Write the time interval for boat X to travel back upstream.

$$\Delta t_{up} = \frac{L}{v_{bw} - v_{wg}}$$

- To find the total time for boat X to make the round trip, add the time intervals for the two directions.

$$\Delta t_X = \frac{L}{v_{bw} + v_{wg}} + \frac{L}{v_{bw} - v_{wg}}$$

- Find a common denominator and simplify.

$$\Delta t_X = \frac{L(v_{bw} - v_{wg}) + L(v_{bw} + v_{wg})}{(v_{bw} + v_{wg})(v_{bw} - v_{wg})}$$

$$\Delta t_X = \frac{Lv_{bw} - Lv_{wg} + Lv_{bw} + Lv_{wg}}{(v_{bw})^2 - (v_{wg})^2}$$

- The time required for boat X to travel downstream and return is

$$\Delta t_X = \frac{2Lv_{bw}}{(v_{bw})^2 - (v_{wg})^2}$$

So, did boat Y or boat X complete the trip more quickly? You can find this out by dividing Δt_X by Δt_Y.

- Divide Δt_X by Δt_Y.

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{\dfrac{2v_{bw}L}{(v_{bw})^2 - (v_{wg})^2}}{\dfrac{2L}{\sqrt{(v_{bw})^2 - (v_{wg})^2}}}$$

- Simplify.

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{v_{bw}}{\sqrt{(v_{bw})^2 - (v_{wg})^2}}$$

- Divide the numerator and denominator by v_{bw} and simplify.

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{1}{\sqrt{1 - \dfrac{(v_{wg})^2}{(v_{bw})^2}}}$$

Since the denominator is less than one, the ratio is greater than one; thus, Δt_X is greater than Δt_Y — boat Y was faster.

MATH LINK

Normally, taking a square root results in both positive and negative roots. However, since both time intervals were measured forward from a common starting point, they must both be positive, so the ratio must also be positive.

In the Michelson-Morley experiment, the speed of light through the luminiferous ether, usually represented by c, is equivalent to the speed of a boat through water. The speed of the water relative to the ground is equivalent to the speed of the ether relative to Earth. Because motion is relative, it is also the speed of Earth relative to the ether. If this speed is represented by v, the time ratio can be written as

$$\frac{\Delta t_X}{\Delta t_Y} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This means that light that is moving back and forth parallel to the motion of Earth should take longer to complete the trip than light that is moving back and forth perpendicular to the motion of Earth.

Conceptual Problems

- As you study the Michelson-Morley experiment, you will find similarities to this problem. Keep your answers in mind as you read further. Suppose two identical boats can travel at 5.0 m/s relative to the water. A river is flowing at 3.0 m/s. Boat Y travels 1.00×10^2 m straight across the river and then the same distance back. Boat X travels 1.00×10^2 m upstream and then returns the same distance.
  - **(a)** Which boat makes the trip in the shortest time?
  - **(b)** How much sooner does it arrive than the other boat?
- Imagine that both of the two identical boats in the previous problem headed out from the same point at the same time. The river flows due east. Boat Y travelled 1.00×10^2 m[NW] relative to its starting point on shore and then returned straight to its starting point. Boat X travelled 1.00×10^2 m[NE] relative to its starting point on shore and then returned straight to its starting point. Which boat will make the trip in the shortest time? Hint: Sketch the vector diagrams for each case. You might not have to do any calculations.

Michelson's Interferometer

In Michelson's interferometer, a light beam is split into two beams as it passes through the beam splitter, such as a half-silvered mirror. Beam X continues straight on, while beam Y reflects at right angles to its original path. The beams reflect from mirrors and recombine as they once again pass through the beam splitter. Since the two beams do not travel precisely the same distance before they recombine, they interfere with each other as they head toward the telescope. This combination produces an interference pattern that can be observed with the telescope. Anything that

LANGUAGE LINK

The "ether" to which the text refers is not the chemical form of ether. It stems from the Latin word *aether* and was thought to be a highly rarefied medium through which light and other electromagnetic waves travelled. The word "ethereal" comes from this concept.

changes the time of travel of the two beams, such as moving the adjustable mirror even a small distance, produces obvious changes in the interference pattern.

When Michelson and Morley used the interferometer, they assumed that if beam X was parallel to the direction in which the planet was travelling, then that beam would take longer to reach the telescope than beam Y. This would produce a certain interference pattern. However, if the apparatus was rotated through 90˚, beam Y would lag behind. During the rotation, the interference pattern should change as the arrival time for each beam changed. Their hope was to measure this change and use it to measure the speed of Earth through the ether. The relationship between the motion of Earth and two perpendicular interferometer positions is shown in Figure 11.6.

It was an elegant experiment, and yet it seemed to be a disaster. The interference pattern refused to change. This lack of change, or nul result, greatly discouraged the two experimenters and was a source of puzzlement for other physicists. Could it be that Earth really did not move at all relative to the ether? This did not make sense, since Earth obviously orbited the Sun. Did Earth drag the luminiferous ether along with it? This did not seem likely, since that would affect the appearance of stars as seen from Earth.

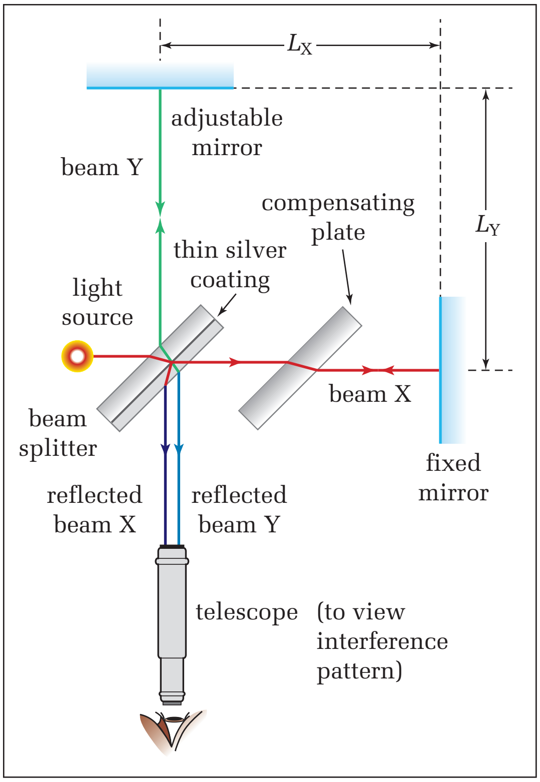


Figure 11.5 If the two beams (X and Y) are not in phase, they will interfere with each other, producing a pattern that can be seen in the telescope.

One guess, which in a sense paved the way for the relativity answer, was that objects that moved through the ether were compressed, just as a spring could be compressed if it was pushed lengthwise through oil. This contraction would cause a shortening of lengths in the direction of motion, thus reducing the time required for the light to make the round trip. In this way, both light beam X and light beam Y would always arrive at the telescope at the same time. This hypothesis was known as the **Lorentz-Fitzgerald contraction**.

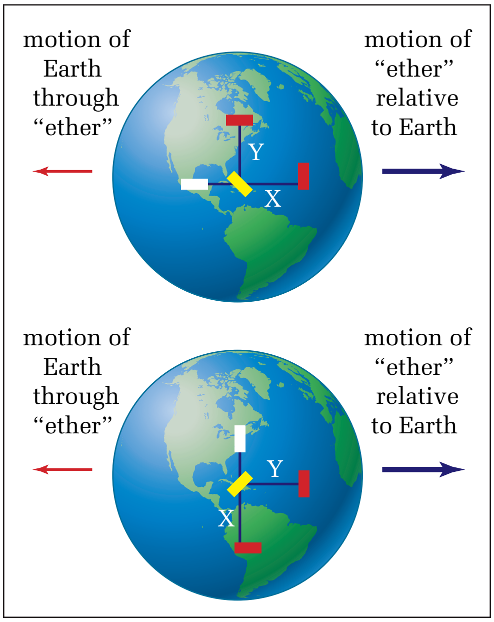


Figure 11.6 Arrival times for the light beams were expected to change when the interferometer was rotated by 90˚, but this did not happen.

The Theoretical Speed of Light

The strange results of the Michelson-Morley experiment remained a mystery for nearly two decades. Then, in 1905, the explanation came with Albert Einstein's publication of his special theory of relativity. He had developed this theory while considering the propagation of electromagnetic waves, as described by James Clerk Maxwell. Maxwell's equations showed how electromagnetic waves would spread out from accelerated charges.

In the early 1870s, Maxwell realized that a changing magnetic field could induce a changing electric field and that the changing electric field could in turn induce a changing magnetic field. Most importantly, he realized that these mutually inducing fields could spread out through space with a speed given by $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, where c represents the speed at which the fields spread out through space (the speed of light in a vacuum), ε_0 represents the electric permittivity of free space ($\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$), and μ_0 represents the magnetic permeability of free space ($\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$).

This formula yields a speed for electromagnetic radiation through space of 3.00×10^8 m/s. This was a major triumph in the field of theoretical physics, since it predicted the speed of light in terms of basic properties involving the behaviour of electric and magnetic fields in space. In addition, there was now no necessity for assuming the existence of luminiferous ether — magnetic and electric fields can exist in space without such a medium.

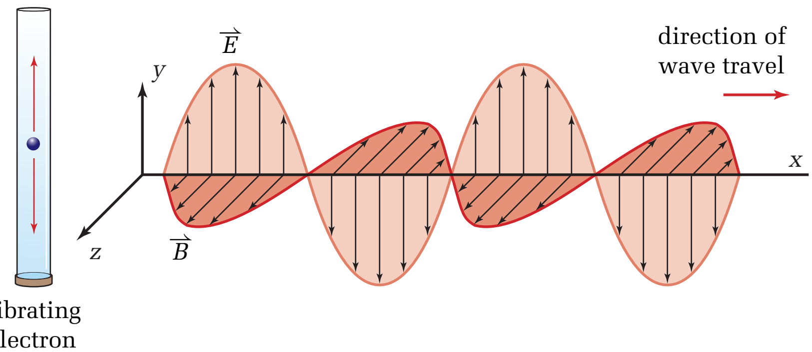


Figure 11.7 In this diagram, $\vec{E}$ represents the electric field, while $\vec{B}$ represents the magnetic field.

Einstein, however, was puzzled by an apparent inconsistency in this equation. It did not indicate any particular frame of reference. The laws of physics are expected to be valid in any inertial frame of reference. However, quantities such as speed and velocity could appear to be different from different frames of reference. For example, a race car can be seen to travel at a high speed relative to spectators in the stands. However, it might have zero velocity relative to another race car.

PHYSICS FILE

Toward the end of Michelson's life, Einstein praised him publicly for his ground-breaking experiments, which provided the first experimental confirmation for the special theory of relativity.

PHYSICS FILE

Electric permittivity is related to the Coulomb constant (k): $\varepsilon_0 = \frac{1}{4\pi k}$. Magnetic permeability comes from the expression for the strength of the magnetic field in the vicinity of a current-carrying conductor. The equation for the magnetic field, $\vec{B}$, is $\vec{B} = \frac{\mu_0 I}{2\pi r}$, where I is the current in the wire in amperes and r is the radial distance from the wire.

Apparently, there was no specified frame of reference for the speed of light in Maxwell's equation. This implied that the speed of light (and, in fact, of all members of the electromagnetic spectrum) through a vacuum should be seen as being the same in any inertial frame of reference. Einstein realized that this was indeed the case and announced his special theory of relativity.

Figure 11.8 In any race, relative velocity is all that counts.

The Special Theory of Relativity

Einstein based his special theory of relativity on two postulates.

1. All physical laws must be equally valid in all inertial frames of reference.
2. The speed of light through a vacuum will be measured to be the same in all inertial frames of reference.

The first statement had been accepted since the time of Galileo and Newton. The second one was a radical departure from the common understanding of the basics of physics, so it took scientists a long time to accept it. Eventually it was accepted, though, and the special theory of relativity is now considered to be one of the principal scientific triumphs of the twentieth century.

11.1 Section Review

1. **K/U** Make sketches of the velocity vectors identical to those in Figures 11.3 and 11.4 on pages 467 and 468. Label the vectors as though they represented the velocities of light through the ether, the ether relative to Earth, and light relative to Earth.
2. **MC** Show that the units used in Maxwell's equation for the speed of light simplify to metres per second. Note that $1\ T = 1\ N/A \cdot m$.
3. **I** In the interferometer shown in Figure 11.5, how far in wavelengths would the adjustable mirror have to move so that the interference pattern would return to its initial appearance? Hint: Review the interference relationships found in Chapter 9.
4. **K/U** What caused physicists to assume that space was filled with a medium that they called the "luminiferous ether"?
5. **MC**
   - **(a)** State the two basic postulates of the special theory of relativity.
   - **(b)** Explain why the constancy of the speed of a light beam, as seen from different inertial frames of reference, seems to be wrong. Try to use commonplace examples to make your point.

11.2 The Basics of the Special Theory of Relativity

Einstein's special theory of relativity changed our fundamental understanding of distance, time, and mass. He used his famous thought experiments to illustrate these new concepts. This section contains several thought experiments similar to the ones Einstein used.

SECTION EXPECTATIONS

- Describe Einstein's thought experiments relating to the constancy of the speed of light in all inertial frames of reference, time dilation, and length contraction.

KEY TERMS

- simultaneity
- time dilation
- proper time
- dilated time
- length contraction
- proper length
- relativistic speeds
- gamma

Thought Experiment 1: Simultaneity

Imagine that you are sitting high on a hill on Canada Day and you can see two different celebrations going on in the distance. You are startled when two sets of fireworks ignite at exactly the same time — one off to your left and the other far to your right. About 100 m behind you, a car is travelling along a highway at 95 km/h. Do the passengers in the car see the fireworks igniting simultaneously or do they think that one set ignited before the other? Your immediate reaction is probably, "Of course they saw the fireworks igniting simultaneously — they were simultaneous!"

According to Einstein's special theory of relativity, however, the answer is not quite so simple. To restate the question more precisely, are two events that are simultaneous for an observer in one inertial reference frame simultaneous for observers in all inertial reference frames? The answer is no. The constancy of the speed of light creates problems with the **simultaneity** of events, as the situation in Figure 11.9 illustrates.

In Figure 11.9 (A), observers A and B are seated equidistant from a light source (S). The light source flashes. Since the light must travel an equal distance to both observers, they would say that they received the flash at exactly the same time, that the arrival of the flash was simultaneous for both of them.

Now imagine that these two observers are actually sitting on a railway flatcar that is moving to the right with velocity $\vec{v}$ relative to the ground and to observer C in Figure 11.9 (B). Observer C makes two observations.

1. B is moving away from the point from which the light was emitted.
2. A is moving toward the point from which the light was emitted.

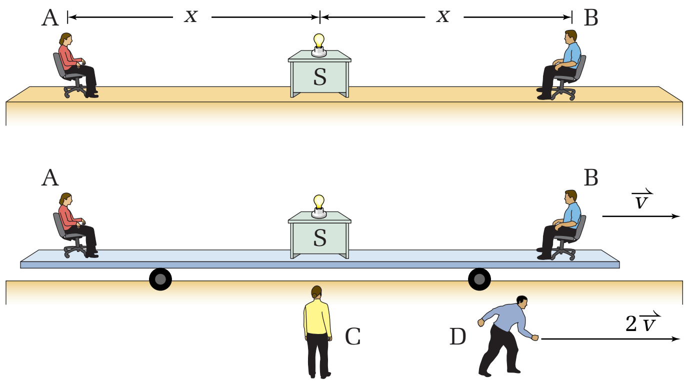


Figure 11.9 Do events that appear to be simultaneous to observers A and B also appear to be simultaneous to observers C and D?

Observer C concludes that it takes longer for light to reach B than it does to reach A. Thus, according to observer C, observer A received the flash first and B received it second. The arrivals are not simultaneous in C's frame of reference, and yet it is an inertial reference just as much as is the frame of reference of the flatcar.

In the frame of reference for observer D, who is moving to the right with a velocity of $2\vec{v}$, the flatcar is moving toward the left with a velocity of $\vec{v}$. Now, it is A who is moving away from the point from which the flash was emitted and B is moving toward that emission point. The light would take longer to reach A, so the light would arrive at observer B first.

As you can see from this example, the whole concept of simultaneity, of past, present, and future, is fuzzy in relativity. What is a future event in one frame of reference becomes a past event in another. This is due entirely to the fact that the speed of light is the same in all inertial frames of reference, regardless of their relative velocities.

Thought Experiment 2: Time Dilation

Imagine yourself back on the hilltop, watching fireworks. You look at your watch at the moment that the fireworks ignite and it says 11:23 P.M. What do the watches of the passengers in the car read? If they saw the fireworks ignite at different times, their watches cannot possibly agree with yours.

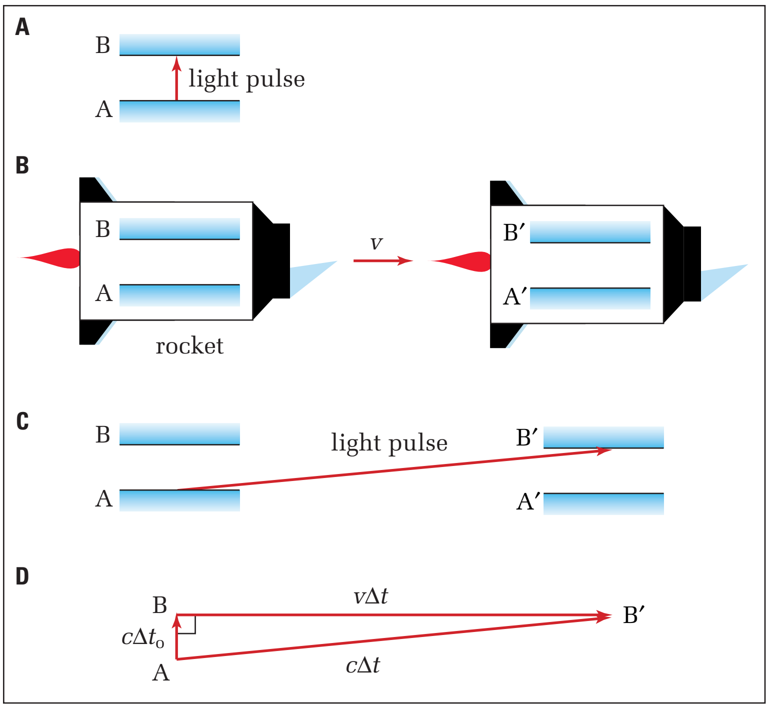


Figure 11.10 If the speed of light is the same to all observers, then light takes longer to travel from A to B′ than it does to travel from A to B.

The constancy of the speed of light creates problems with time intervals. The term **time dilation** applies to situations in which time intervals appear different to observers in different inertial frames of reference. To understand the implications of this constant speed of light for time measurement, assume that an experimenter has devised a light clock. In it, a pulse of light reflects back and forth between two mirrors, A and B. The time that it takes for the pulse to travel between the mirrors is the basic tick of this clock. Figure 11.10 (A) shows such a "tick."

Now, picture this clock in a spacecraft that is speeding past Earth. An observer in the spacecraft sees the light as reflecting back and forth as it was before, so the basic tick of the clock has not changed. However, an observer on Earth would see that the mirrors moved while the light pulse travelled from A to B, as shown in Figure 11.10 (B). Since the pulse actually has to travel from A to B′, it must take longer, as indicated in Figure 11.10 (C). The tick of the clock therefore takes longer to occur in the Earth frame of reference than in the spacecraft observer's frame of reference. In fact, if the spacecraft observer was wearing a watch, the Earth observer would say that the watch was counting out the seconds too slowly. The spacecraft observer, however, would say that the watch and the light clock were working properly.

The relationship between times as measured in the spacecraft and on Earth can be deduced from Figure 11.10 (D). Assume that

- c is the speed of light, which is the same for all observers
- Δt is the time that the Earth observer says it takes for the pulse to travel between the mirrors
- Δt_o is the time that the spacecraft observer says it takes for the pulse to travel between the mirrors

The distance from A to B would be $c\Delta t_o$. The distance travelled by the spacecraft would be $v\Delta t$, since this involves a distance, speed, and time observed by the Earth observer.

The Earth observer claims that the light pulse actually travelled a distance of $c\Delta t$. These distances represent the lengths of the sides of a right-angled triangle, as seen in Figure 11.10 (D). Notice how similar this result is to the arrival-time equation in the boat X-boat Y scenario on pages 467 and 468.

- Apply the Pythagorean theorem and expand.

$$(c\Delta t)^2 = (c\Delta t_o)^2 + (v\Delta t)^2$$
$$c^2\Delta t^2 = c^2\Delta t_o^2 + v^2\Delta t^2$$

- Solve for $c^2\Delta t_o^2$.

$$c^2\Delta t_o^2 = c^2\Delta t^2 - v^2\Delta t^2$$

- Factor out a Δt^2.

$$c^2\Delta t_o^2 = \Delta t^2(c^2 - v^2)$$

- Divide by c^2.

$$\Delta t_o^2 = \frac{\Delta t^2(c^2 - v^2)}{c^2}$$

- Simplify, then take the square root of both sides of the equation.

$$\Delta t_o^2 = \Delta t^2\left(1 - \frac{v^2}{c^2}\right)$$
$$\Delta t_o = \Delta t\sqrt{1 - \frac{v^2}{c^2}}$$

- Solve for Δt.

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In any question involving relativistic times, it is important to carefully identify the times.

- Δt_o is the time as measured by a person at rest relative to the object or the event. It is called the **proper time**. You could think of it as the "rest time," although this term is not generally used. Another way to picture it is as the "one-point" time, the time for an observer who sees the clock as staying at only one point.

ELECTRONIC LEARNING PARTNER

Relativity

Experiment with near-light speeds and time dilation by using your Electronic Learning Partner.

MATH LINK

Note that the negative square root has no meaning in this situation. Both times will be seen as positive. In addition, v must be less than c. If it was greater than c, the denominator would become the square root of a negative number. Although such a square root can be expressed using complex numbers, it is not expected that a time measurement would involve anything other than the set of real numbers.

- Δt is the expanded or **dilated time**. Since the denominator $\sqrt{1 - \frac{v^2}{c^2}}$ is less than one, Δt is *always* greater than Δt_o. It can also be thought of as the "two-point" time, the time as measured by an observer who sees the clock as moving between two points.

DILATED TIME

The dilated time is the quotient of the proper time and the expression: square root of one minus the velocity of the moving reference frame squared divided by the speed of light squared.

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Quantity	Symbol	SI unit
dilated time	Δt	s (seconds)
proper time	Δt_o	s (seconds)
velocity of the moving reference frame	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
speed of light	c	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$\text{seconds} = \frac{\text{seconds}}{\sqrt{1 - \frac{\left(\frac{\text{metres}}{\text{second}}\right)^2}{\left(\frac{\text{metres}}{\text{seconds}}\right)^2}}} = \text{seconds} \qquad \text{s} = \frac{\text{s}}{\sqrt{1 - \frac{\left(\frac{\text{m}}{\text{s}}\right)^2}{\left(\frac{\text{m}}{\text{s}}\right)^2}}} = \text{s}$$

SAMPLE PROBLEM

Relative Times

A rocket speeds past an asteroid at 0.800 *c*. If an observer in the rocket sees 10.0 s pass on her watch, how long would that time interval be as seen by an observer on the asteroid?

Conceptualize the Problem

- *Proper time*, Δt_o, and *dilated time*, Δt, are not the same. *Time intervals* appear to be *shorter* to the observer who is *moving* at a velocity close to the speed of light.
- Proper time, Δt_o, and dilated time, Δt, are related by the *speed of light*, *c*.

Identify the Goal

The amount of time, Δt, that passes for the observer on the asteroid while 10.0 s passes for the observer on the rocket

PROBLEM TIP

Since $\frac{v^2}{c^2}$ is a ratio, the speeds can have any units as long as they are the same for both the numerator and the denominator. It is often useful to express *v* in terms of *c*.

Identify the Variables and Constants

Known	Implied	Unknown
$v_{rocket} = 0.800\ c$	$c = 3.00 \times 10^8\ \frac{\text{m}}{\text{s}}$	Δt
$\Delta t_o = 10.0\ \text{s}$		

Develop a Strategy

Select the equation that relates dilated time to proper time.

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Substitute into the equation.

$$\Delta t = \frac{10.0\ \text{s}}{\sqrt{1 - \frac{(0.800\ c)^2}{c^2}}}$$

Solve.

$$\Delta t = \frac{10.0\ \text{s}}{0.600}$$

$$\Delta t = 16.67\ \text{s}$$

$$\Delta t \cong 16.7\ \text{s}$$

The time as seen by an observer on the asteroid would be 16.7 s.

Validate the Solution

The dilated time is expected to be longer than the proper time, and it is.

PRACTICE PROBLEMS

1. A tau (τ) particle has a lifetime measured at rest in the laboratory of 1.5×10^{-13} s. If it is accelerated to 0.950 c, what will be its lifetime as measured in **(a)** the laboratory frame of reference, and **(b)** the τ particle's frame of reference?
2. A rocket passes by Earth at a speed of 0.300 c. If a person on the rocket takes 245 s to drink a cup of coffee, according to his watch, how long would that same event take according to an observer on Earth?
3. A kaon particle (κ) has a lifetime at rest in a laboratory of 1.2×10^{-8} s. At what speed must it travel to have its lifetime measured as 3.6×10^{-8} s?

Thought Experiment 3: Length Contraction

Imagine the following situation. Captain Quick is a comic book hero who can run at nearly the speed of light. In her hand, she is carrying a flare with a lit fuse set to explode in 1.50 μs (1.50×10^{-6} s). The flare must be placed into its bracket before this happens. The distance (L) between the flare and the bracket is 402 m.

PHYSICS FILE

As you will discover in Chapter 13, The Nucleus and Elementary Particles, many subatomic particles come into existence and decay into some other particles in very short periods of time. The tau and kaon particles are examples of these subatomic particles.

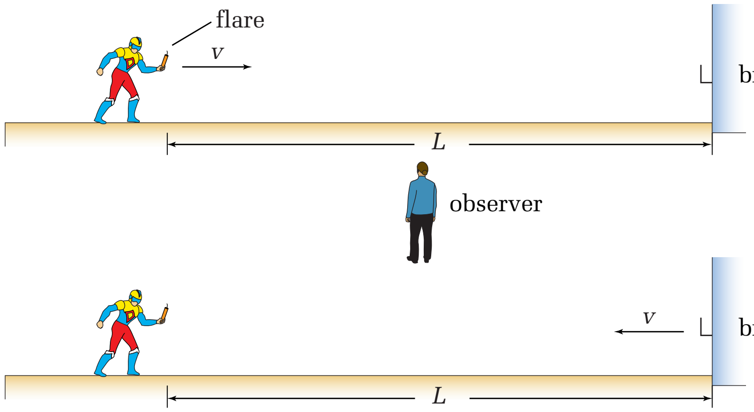


Figure 11.11 A race against time

- Captain Quick runs at $\frac{2}{3}c$ (2.00×10^8 m/s) and arrives at the bracket in time. According to classical mechanics, this would not be possible because it should take 2.01 μs as shown on the right.

$$\Delta t = \frac{L}{v}$$
$$\Delta t = \frac{402\ \text{m}}{2.00 \times 10^8\ \frac{\text{m}}{\text{s}}}$$
$$\Delta t = 2.01 \times 10^{-6}\ \text{s or } 2.01\ \mu\text{s}$$

- However, to an observer in the stationary frame of reference, the time for the fuse to burn will be dilated in relation to his own frame of reference. It will take 2.01 μs for the fuse to burn and therefore, Captain Quick will reach the bracket in time.

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$\Delta t = \frac{1.50 \times 10^{-6}\ \text{s}}{\sqrt{1 - \frac{\left(\frac{2}{3}c\right)^2}{c^2}}}$$
$$\Delta t = \frac{1.50 \times 10^{-6}\ \text{s}}{0.7454}$$
$$\Delta t = 2.01 \times 10^{-6}\ \text{s}$$

- Since Captain Quick and the fuse are in the same frame of reference, however, Captain Quick should observe the fuse burning in 1.50 μs. How did she make it in time? Then she realized that the only way she could have arrived in time was if the *distance* to the bracket in her moving frame of reference was less than the 402 m in the stationary frame. The distance must have been *multiplied* by the same factor by which the time was *divided* in the observer's frame of reference.

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$
$$L = (402\ \text{m})\sqrt{1 - \frac{\left(\frac{2}{3}c\right)^2}{c^2}}$$
$$L = (402\ \text{m})(0.7454)$$
$$L = 300\ \text{m}$$

- If the distance was smaller, then Captain Quick could make it to the bracket before the fuse burned out.

$$\Delta t = \frac{L}{v}$$

$$\Delta t = \frac{300\ \text{m}}{2.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}}$$

$$\Delta t = 1.50 \times 10^{-6}\ \text{s or } 1.50\ \mu\text{s}$$

This thought experiment illustrates that two ideas go hand in hand. If two observers are moving relative to each other, then a time dilation from one observer's point of view will be balanced by a corresponding **length contraction** from the other observer's point of view.

In the box below, L_o represents the **proper length**, which is the length as measured by an observer at rest relative to the object or event and L is the contracted length seen by the moving observer.

LENGTH CONTRACTION

The contracted length is the product of the proper length and the expression, square root of one minus the velocity of the moving reference frame squared divided by the speed of light squared.

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

Quantity	Symbol	SI unit
contracted length	L	m (metres)
proper length	L_o	m (metres)
velocity of the moving reference frame	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
speed of light	c	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$\text{metres} = \frac{\text{metres}}{\sqrt{1 - \frac{\left(\frac{\text{metres}}{\text{second}}\right)^2}{\left(\frac{\text{metres}}{\text{seconds}}\right)^2}}} = \text{metres}$$

$$\text{m} = \frac{\text{m}}{\sqrt{1 - \frac{\left(\frac{\text{m}}{\text{s}}\right)^2}{\left(\frac{\text{m}}{\text{s}}\right)^2}}} = \text{m}$$

Note: Length contraction applies *only* to lengths measured *parallel* to the direction of the velocity. Lengths measured perpendicular to the velocity are not affected.

This thought experiment seems to yield strange results that go against common experience. However, the results explain a phenomenon involving a tiny particle called the "mu meson" (or muon). This particle has a lifetime of 2.2×10^{-6} s and is formed about 1.0×10^4 m above the surface of Earth, speeding downward at about 0.998 c. At that speed (according to classical mechanics), it should travel only about 660 m before decaying into other particles, but it is observed in great numbers at Earth's surface. The relativistic explanation is that the muon's lifetime as measured by Earth-based observers has been dilated as follows.

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t = \frac{2.2 \times 10^{-6}\ \text{s}}{\sqrt{1 - \frac{(0.998\,c)^2}{c^2}}}$$

$$\Delta t = \frac{2.2 \times 10^{-6}\ \text{s}}{0.0632}$$

$$\Delta t = 3.5 \times 10^{-5}\ \text{s}$$

The distance travelled becomes

$$\Delta d = v\Delta t$$

$$\Delta d = (0.998)\left(3.00 \times 10^8\ \frac{\text{m}}{\text{s}}\right)(3.5 \times 10^{-5}\ \text{s})$$

$$\Delta d = 1.0 \times 10^4\ \text{m}$$

At that speed, the muon's lifetime is so expanded (according to the observers on Earth) that the particle can reach the surface. On the other hand, the muon sees its own lifetime as unchanged, and from its frame of reference, Earth's surface is rushing toward it at 0.998 c. The distance it sees to Earth's surface is given by

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

$$L = (1.0 \times 10^4\ \text{m})\sqrt{1 - \frac{(0.998\,c)^2}{c^2}}$$

$$L = (1.0 \times 10^4\ \text{m})(0.0632)$$

$$L = 632\ \text{m}$$

This reduced distance would take a shorter time, given by

$$\Delta t = \frac{\Delta x}{v}$$

$$\Delta t = \frac{632\ \text{m}}{(0.998)\left(3.0 \times 10^8\ \frac{\text{m}}{\text{s}}\right)}$$

$$\Delta t = 2.1 \times 10^{-6}\ \text{s}$$

The muon therefore can reach Earth's surface before decaying.

Which Is Correct?

The physicist standing on the surface of Earth claims that the lifetime of the muon is 3.5×10^{-5} s and its height above Earth's surface is 1.0×10^4 m. From the muon's point of view, however, its lifetime is 2.2×10^{-6} s and its height is 632 m. Which is correct?

Both statements are correct. The value of any measurement is tied to the frame of reference in which that measurement is taken. Going from one inertial frame of reference to another will involve differences in the measurement of lengths and times. Normally, these differences are too small to be observed, but as relative speeds approach the speed of light, these differences become quite apparent.

Gamma Saves Time

When solving problems involving **relativistic speeds** (speeds approaching the speed of light), you will often need to calculate the value of $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$. Physicists have assigned the symbol **gamma** (γ) to this value, or $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$. Using the γ notation, the length and time equations become $\Delta t = \gamma \Delta t_o$ and $L = \frac{L_o}{\gamma}$.

SAMPLE PROBLEM

Relativistic Lengths

A spacecraft passes Earth at a speed of 2.00×10^8 m/s. If observers on Earth measure the length of the spacecraft to be 554 m, how long would it be according to its passengers?

Conceptualize the Problem

- *Length* appears to be shorter, or *contracted*, to the *observer* who is *moving* relative to the object being measured.
- The amount of *length contraction* that occurs is determined by the *relative speeds* of the reference frames of the two observers.

Identify the Goal

The length of the spacecraft, L_o, as seen by its passengers

Identify the Variables and Constants

Known	Implied	Unknown
$v = 2.00 \times 10^8 \frac{\text{m}}{\text{s}}$	$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$	L_o
$L = 554$ m		

continued ▶

continued from previous page

Develop a Strategy

Calculate gamma.

$$\gamma = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\gamma = \sqrt{1 - \frac{(2.00 \times 10^8\ \frac{\text{m}}{\text{s}})^2}{(3.00 \times 10^8\ \frac{\text{m}}{\text{s}})^2}}$$

$$\gamma = 1.342$$

Use the equation that describes length contraction.

$$L = \frac{L_o}{\gamma}$$

$$L_o = L\gamma$$

Solve.

$$L_o = (554\ \text{m})(1.342)$$

$$L_o = 743.2\ \text{m}$$

$$L_o \cong 743\ \text{m}$$

The length of the spacecraft as seen by its passengers is 743 m.

Validate the Solution

The proper length is expected to be longer than the contracted length, and it is.

PRACTICE PROBLEMS

4. An asteroid has a long axis of 725 km. A rocket passes by parallel to the long axis at a speed of 0.250 c. What will be the length of the long axis as measured by observers in the rocket?

5. An electron is moving at 0.95 c parallel to a metre stick. How long will the metre stick be in the electron's frame of reference?

6. A spacecraft passes a spherical space station. Observers in the spacecraft see the station's minimum diameter as 265 m and the maximum diameter as 325 m.

(a) How fast is the spacecraft travelling relative to the space station?

(b) Why does the station not look like a sphere to the observers in the spacecraft?

PHYSICS FILE

Einstein's equations allow a particle to travel faster than light if it was already travelling faster than light when it was created. For such particles (called "tachyons"), the speed of light represents the slowest speed limit. Although the equations say that tachyons can exist, there is no evidence that they do. In fact, no one knows how they would interact with normal matter.

The Universal Speed Limit

Calculation of expanded times and contracted lengths involve the expression $\sqrt{1 - \frac{v^2}{c^2}}$. Since times and lengths are measurements, they must be represented by real numbers, so the value under the square root must be a *positive* real number. For this to be true, $\frac{v^2}{c^2} < 1$. This implies that $v < c$. If v approaches c, the value of gamma approaches infinity. Consider what happens to Δt when v approaches c in $\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$. The denominator approaches zero.

Division of a non-zero real number by zero is undefined so an object's speed must be less than the speed of light.

This speed limit applies only to material objects. Obviously, light can travel at the speed of light. Also, once a light pulse has been slowed down by passing into a medium such as water, objects can travel faster through that medium than can the pulse. The blue glow (called "Cerenkov radiation") emanating from water in which radioactive material is being stored is created by high-speed electrons (beta particles) that are travelling through the water faster than the speed of light through water. This phenomenon is sometimes compared to sonic boom, in which particles (in the form of a jet airplane) are travelling faster than the speed of sound in air.

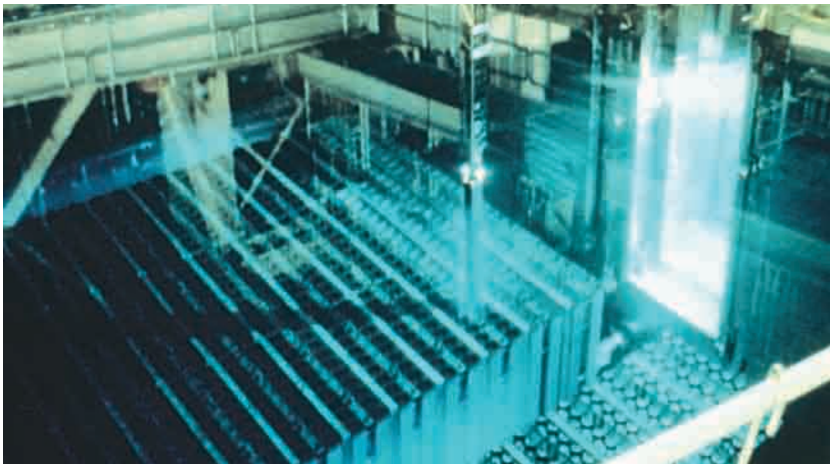


Figure 11.12 The blue glow from this storage pool in a nuclear generating station comes from particles that are travelling through the water faster than the speed of light through water.

11.2 Section Review

1. K/U
   (a) Explain what is meant by an inertial frame of reference.
   (b) Would a rotating merry-go-round be an inertial frame of reference? Give reasons for your answer.

2. K/U Explain the meaning of the terms "proper length" and "proper time."

3. I An arrow and a pipe have exactly the same length when lying side by side on a table. The arrow is then fired at a relativistic speed through the pipe, which is still lying on the table. Determine whether there is a frame of reference in which the arrow can
   (a) be completely inside the pipe with extra pipe at each end
   (b) overhang the pipe at each end

   Give reasons for your answers.

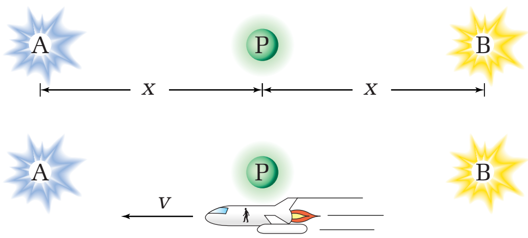
4. K/U Explain the meaning of the terms "length contraction" and "time dilation."

5. C Explain why the results of the Michelson-Morley experiment were so important.

6. **K/U**

(a) In the diagram, two stars (A and B) are equidistant from a planet (P) and are at rest relative to that planet. They both explode into novas at the same time, according to an observer on the planet. From the point of view of passengers in a rocket ship travelling past at relativistic speeds, however, which star went nova first? Give reasons for your answer.

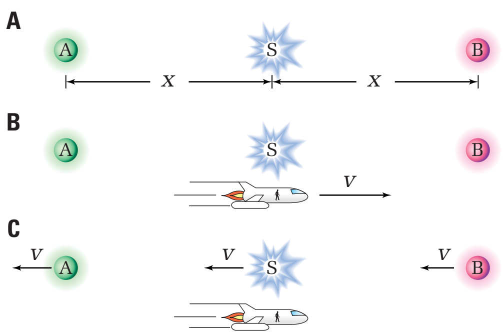
(b) Where could the observer stand on the planet in order to see both stars at the same time?



7. **K/U** Part (A) of the diagram shows a star (S) located at the midpoint between two planets (A) and (B), which are at rest relative to the star. The star explodes into a supernova.

(a) In the frame of reference of the planets, which planet saw the supernova first? Give reasons for your answer.

(b) A spacecraft is passing by as shown in part (B). In its frame of reference, the star and planets are moving as shown in part (C). In the spacecraft frame of reference, which planet saw the supernova first? Give reasons for your answer.



8. **I**

(a) Imagine that you are riding along on a motorcycle at 22 m/s and throw a ball ahead of you with a speed of 35 m/s. What will be the speed of that ball relative to the ground?

(b) If the velocity of the motorcycle relative to the ground is v_{mg}, the velocity of the ball relative to the motorcycle is v_{bm}, and the velocity of the ball relative to the ground is v_{bg}, state the vector equation for calculating the velocity of the ball relative to the ground.

(c) Apply this formula to a situation in which the motorcycle is travelling at 0.60 c and the ball is thrown forward with a speed of 0.80 c. What is the speed of the ball relative to the ground? What is wrong with this answer?

(d) In the special theory of relativity, the formula for adding these velocities is

$$v_{bg} = \frac{v_{bm} + v_{mg}}{1 + \frac{v_{bm} \cdot v_{mg}}{c^2}}$$

- What does this formula predict for the answer to (c)?
- What does this formula predict for the answer to (a)?
- Imagine that you are travelling in your car at a speed of 0.60 c and you shine a light beam ahead of you that travels away from you at a speed of c. According to this formula, what would be the speed of that light beam relative to the ground?

UNIT PROJECT PREP

How would the general public have received the new information in Einstein's special theory of relativity?

- Do you believe that at the turn of the twentieth century society had more or less faith in science than people do today? Why or why not?
- Dramatic events often steer thinking into new directions. Do you believe that Einstein was affected by any one particular event as he developed his theories?
- Are you able to link recent societal events with current changes in the direction of scientific research?

CAREERS IN PHYSICS

Not Even the Sky's the Limit!

When a signal leaves a satellite or interplanetary space probe, a special code is embedded in it to give it a time-stamp. When the signal is picked up on Earth, that time-stamp is compared to a terrestrial clock. Subtracting the two gives the travel time between the satellite or space probe and the ground station. Since the signal travels at the speed of light, all you should need to do then is multiply the time by *c* to determine the distance — but it's not that simple.

Gravity is part of the problem. According to Einstein's general theory of relativity, clocks run more slowly in a gravitational field. The stronger the field is, the slower the clocks run. Clocks on board spacecraft and satellites run slightly faster in interplanetary space than they do near Earth. These timing differences result in distance measurement differences between what is observed from a ground station and from a spacecraft. The situation becomes even more complicated as the spacecraft dips into and out of the gravitational fields of planets that it encounters on its voyage.

A second problem results from the relative velocity between the spacecraft and the ground station. Einstein's special theory of relativity, discussed in detail in this chapter, describes how time intervals and distance measurements vary between inertial frames of reference that are in motion relative to each other. This relative velocity is continually changing as a result of the gravity of the Sun and planets and due to Earth's orbital and rotational velocities. Relativistic corrections — numerical adjustments based on the theory of relativity — are an ongoing challenge in spacecraft instrument design.

There are "a whole suite of careers that utilize these things," Steve Lichten, manager of the Tracking Systems and Applications Section of NASA's Jet Propulsion Laboratory, says of relativity. Einsteinian physics is no longer the sole property of university researchers. Commercial satellite manufacturers must have an understanding of relativity in order for their products to work.

Theoreticians, engineers, and computer scientists must work together to help a spacecraft communicate with its ground station, so the companies that manufacture spacecraft and commercial satellites are always on the lookout for people with the necessary knowledge. Generally, an advanced graduate degree in engineering, physics, or mathematics is preferred, although a bachelor's degree in science with a demonstrated understanding of the concepts and techniques involved will go a long way.

So brush up your math skills and keep doing those thought experiments. Some day, they might take you to the stars!

Going Further

1. Describe some examples of satellites that require extremely precise distance and time measurements. Explain why such precision is necessary for those satellites.
2. Many companies that manufacture satellites or equipment for use on satellites (including space stations) offer summer internship programs for interested students. Find out if any of these companies are located near you and call them. You might be able to get a head start on a great career!
3. Research the space probes, such as the one shown in the photograph, that are currently active. Explain why precise knowledge of time intervals and distances is of extreme importance to the operation of space probes.

WEB LINK

www.mcgrawhill.ca/links/physics12

For information about the NASA Jet Propulsion Laboratory's past, current, and planned space missions, go to the above Internet site and click on **Web Links**.

11.3 Mass and Energy

SECTION EXPECTATIONS

- Conduct thought experiments as a way of developing an abstract understanding of the physical world as it relates to mass increase when an object approaches the speed of light.
- Apply quantitatively the laws of conservation of mass and energy, using Einstein's mass-energy equivalence.

KEY TERMS

- rest mass
- relativistic mass
- total energy
- rest energy

In the last section, you read a discussion based on mathematical equations that explained why no object with mass can travel at or above the speed of light. The discussion probably left you wondering why. If, for example, a spacecraft is travelling at 0.999 c, what would prevent it from burning more fuel, exerting more reaction force, and increasing its speed up to c?

The fact that no amount of extra force will provide that last change in velocity is explained when you discover that the mass of the spacecraft is also increasing. Einstein showed that, just as time dilates and length contracts when an object approaches the speed of light, its mass increases according to the equation $m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$. In the equation, m is the mass as measured by an observer who sees the object moving with speed v, and m_o is the mass as measured by an observer at rest relative to the object. The mass, m, is sometimes called the **relativistic mass** and m_o is known as the **rest mass**.

As the speed of the object increases, the value of the denominator $\sqrt{1 - \frac{v^2}{c^2}}$ decreases. As v approaches c, the denominator approaches zero and the mass increases enormously. If v could equal c, the mass would become $\frac{m_o}{0}$, which is undefined. The speed of an object, measured from any inertial frame of reference, therefore must be less than the speed of light through space.

RELATIVISTIC MASS

Relativistic mass is the quotient of rest mass and gamma.

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Quantity	Symbol	SI unit
relativistic mass	m	kg (kilograms)
rest mass	m_o	kg (kilograms)
speed of the mass relative to observer	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
speed of light	c	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$\text{kilograms} = \frac{\text{kilograms}}{\sqrt{1 - \frac{\left(\frac{\text{metres}}{\text{second}}\right)^2}{\left(\frac{\text{metres}}{\text{seconds}}\right)^2}}} = \text{kilograms} \qquad \text{kg} = \frac{\text{kg}}{\sqrt{1 - \frac{\left(\frac{\text{m}}{\text{s}}\right)^2}{\left(\frac{\text{m}}{\text{s}}\right)^2}}} = \text{kg}$$

SAMPLE PROBLEM

Relativistic Masses

An electron has a rest mass of 9.11×10^{-31} kg. In a detector, it behaves as if it has a mass of 12.55×10^{-31} kg. How fast is that electron moving relative to the detector?

Conceptualize the Problem

- The *mass* of the object appears to be *much greater* to an *observer* in a frame of reference that is *moving at relativistic speeds* than it does to an observer in the *frame of reference of the object*.
- The amount of the *increase in mass* is determined by the ratio of the *object's speed* and the *speed of light*.

Identify the Goal

Determine the speed, v, of the electron relative to the detector

Identify the Variables and Constants

Known	Implied	Unknown
$m_o = 9.11 \times 10^{-31}$ kg $m = 12.55 \times 10^{-31}$ kg	$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$	v

Develop a Strategy

Use the equation that relates the relativistic mass, rest mass, and speed.

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Solve the equation for speed.

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{m_o}{m}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{m_o}{m}\right)^2$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{m_o}{m}\right)^2$$

$$v^2 = c^2\left(1 - \left(\frac{m_o}{m}\right)^2\right)$$

$$v = c\sqrt{1 - \left(\frac{m_o}{m}\right)^2}$$

Substitute numerical values and solve.

$$v = \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)\sqrt{1 - \left(\frac{9.11 \times 10^{-31} \text{ kg}}{12.55 \times 10^{-31} \text{ kg}}\right)^2}$$

$$v = \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)\sqrt{1 - 0.52692}$$

$$v = \pm\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)(0.68780)$$

$$v = \pm 2.0634 \times 10^8 \frac{\text{m}}{\text{s}}$$

$$v \cong \pm 2.06 \times 10^8 \frac{\text{m}}{\text{s}}$$

PROBLEM TIP

In questions involving masses, the masses form a ratio. It does not matter, therefore, what the actual mass units are, as long as they are the same for both the rest mass (m_o) and the moving (relativistic) mass (m).

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Since the problem is asking only for relative speed, the negative root has no meaning. Choose the positive value.

The speed of the electron relative to the detector is $2.06 \times 10^8 \frac{\text{m}}{\text{s}}$ or $0.688c$.

Validate the Solution

Since there is an appreciable mass increase, the object must be moving at a relativistic speed.

PRACTICE PROBLEMS

7. A speck of dust in space has a rest mass of 463 μg. If it is approaching Earth with a relative speed of $0.100c$, what will be its mass as measured in the Earth frame of reference? Remember, in questions involving masses, the masses form a ratio, so it does not matter what the actual mass units are, as long as they are the same for both the rest mass and the moving, or relativistic, mass.

8. A neutron is measured to have a mass of 1.71×10^{-27} kg when travelling at 6.00×10^7 m/s. Determine its rest mass.

9. How fast should a particle be travelling relative to an experimenter in order to have a measured mass that is 20.00 times its rest mass?

Where Is the Energy?

At the start of this section, you examined the relativistic effects that occur when a spacecraft is approaching the speed of light. The conclusion was that its increasing mass must prevent it from accelerating up to the speed of light. However, while the thrusters on the spacecraft are firing, force is being exerted over a displacement, indicating that work was being done on the spacecraft. You know that, at non-relativistic speeds, the work would increase the spacecraft's kinetic energy. At relativistic speeds, however, the speed and thus the kinetic energy increase can only be very small. What, then, is happening to the energy that the work is transferring to the spacecraft?

Einstein deduced that the increased mass represented the increased energy. He expressed it in the formula $E_k = mc^2 - m_o c^2$ or $E_k = (\Delta m)c^2$. As before, m is the mass of the particle travelling at speed v, and m_o is its rest mass. The expression mc^2 is known as the **total energy** of the particle, while $m_o c^2$ is the **rest energy** of the particle. Rearranging the previous equation leads to $mc^2 = m_o c^2 + E_k$. The total energy of the particle equals the rest energy of the particle plus its kinetic energy.

TOTAL ENERGY

The total energy (relativistic mass times the square of the speed of light) of an object is the sum of the rest energy (rest mass times the square of the speed of light) and its kinetic energy.

$$mc^2 = m_o c^2 + E_k$$

Quantity	Symbol	SI unit
relativistic mass	m	kg (kilograms)
rest mass	m_o	kg (kilograms)
speed of light	c	$\frac{m}{s}$ (metres per second)
kinetic energy	E_k	J (joules)

Unit Analysis

$$\text{kilogram}\left(\frac{\text{metres}}{\text{second}}\right)^2 = \text{kilogram}\left(\frac{\text{metres}}{\text{second}}\right)^2 + \text{joule} = \text{joule}$$

$$\text{kg}\left(\frac{\text{m}}{\text{s}}\right)^2 = \text{kg}\left(\frac{\text{m}}{\text{s}}\right)^2 + \text{J} = \text{J}$$

No wonder physicists had difficulty accepting Einstein's theory! In these equations, he is saying that mass and energy are basically the same thing and that the conversion factor relating them is c^2, the square of the speed of light. At the time that Einstein published his work, such changes in mass could not be measured. Eventually, with the advent of high-energy physics, these measurements have become possible.

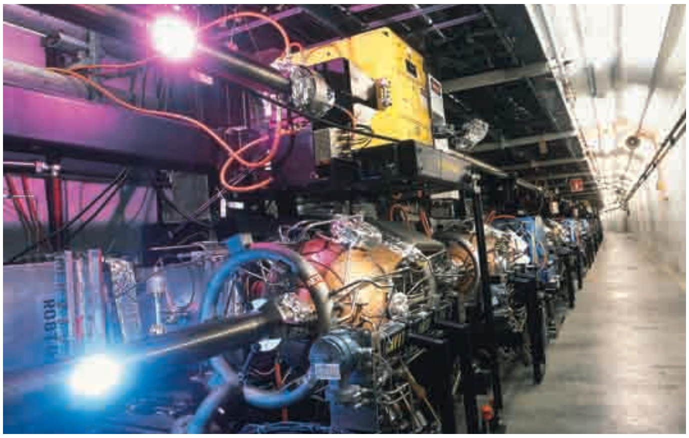

Figure 11.13 In particle accelerators, such as this one at the Stanford Linear Accelerator Center in California, particles are accelerated to speeds very close to the speed of light. Their masses are measured and are found to agree with Einstein's prediction.

SAMPLE PROBLEMS

Kinetic Energy in a Rocket and in a Test Tube

1. A rocket car with a mass of 2.00×10^3 kg is accelerated to 1.00×10^8 m/s. Calculate its kinetic energy

(a) using the classical or general equation for kinetic energy

(b) using the relativistic equation for kinetic energy

Conceptualize the Problem

- The *classical* equation for *kinetic energy* is directly related to the object's *mass* and the *square of its velocity*.

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- The *relativistic equation* for *kinetic energy* takes into account the concept that the *object's mass changes* with its *velocity* and accounts for this *relativistic mass*.

Identify the Goal

The kinetic energy, E_k, of the rocket car, using the classical expression for kinetic energy and then the relativistic expression for kinetic energy

Identify the Variables and Constants

Known	Implied	Unknown
$m_o = 2.00 \times 10^3$ kg	$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$	m
$v = 1.00 \times 10^8 \frac{\text{m}}{\text{s}}$		E_k

Develop a Strategy

Select the classical equation for kinetic energy. Substitute into the equation. Solve.

(a) $E_k = \frac{1}{2}mv^2$

$$E_k = \frac{1}{2}(2.00 \times 10^3\ \text{kg})\left(1.00 \times 10^8\ \frac{\text{m}}{\text{s}}\right)^2$$

$$E_k = 1.00 \times 10^{19}\ \text{J}$$

Calculate gamma.

(b) $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$

$$\gamma = \sqrt{1 - \frac{(1.00 \times 10^8\ \frac{\text{m}}{\text{s}})^2}{(3.00 \times 10^8\ \frac{\text{m}}{\text{s}})^2}}$$

$$\gamma = 1.061$$

Select the relativistic equation for E_k. Rearrange in terms of gamma.

$$E_k = mc^2 - m_oc^2$$

$$E_k = c^2\left(m_o\gamma - m_o\right)$$

$$E_k = m_oc^2\left(\gamma - 1\right)$$

Substitute into the equation.

$$E_k = \left(2.00 \times 10^3\ \text{kg}\right)\left(3.00 \times 10^8\ \frac{\text{m}}{\text{s}}\right)^2\left(1.061 - 1\right)$$

Solve the equation.

$$E_k = 1.09 \times 10^{19}\ \text{J}$$

(a) The classical expression for kinetic energy yields 1.00×10^{19} J.

(b) The relativistic expression for kinetic energy yields 1.09×10^{19} J.

Validate the Solution

Since gamma is not far from unity, a speed of 1.00×10^8 m/s does not provide a high degree of relativistic difference, so the two kinetic energies should not be too far apart.

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2. A certain chemical reaction requires 13.8 J of thermal energy.

(a) What mass gain does this represent?

(b) Why would the chemist still believe in the law of conservation of mass?

Conceptualize the Problem

- *Thermal energy* is the *kinetic energy* of *molecules*.

- Einstein's equation for *relativistic kinetic energy*, which represents the difference between the *total energy* and the *rest energy*, applies to the motion of molecules as well as to rockets.
- If *thermal energy* seems to *disappear* during a chemical reaction, it must have been *converted into mass*.

Identify the Goal

The mass gain, Δm, during an absorption of 13.8 J of energy

Identify the Variables and Constants

Known	Implied	Unknown
$E_k = 13.8$ J	$c = 3.00 \times 10^8 \frac{m}{s}$	Δm

Develop a Strategy

Select the equation linking kinetic energy and mass.

$$E_k = \Delta mc^2$$

Rearrange to give the mass change.

$$\Delta m = \frac{E_k}{c^2}$$

Solve.

$$\Delta m = \frac{13.8 \text{ J}}{\left(3.00 \times 10^8 \frac{m}{s}\right)^2}$$

$$\Delta m = 1.533 \times 10^{-16} \text{ kg}$$

$$\Delta m \cong 1.53 \times 10^{-16} \text{ kg}$$

(a) The gain in mass is 1.53×10^{-16} kg.

(b) This mass change is too small for a chemist to measure with a balance, so the total mass of the products would appear to be the same as the total mass of the reactants.

Validate the Solution

The mass change at non-relativistic speeds should be extremely small.
Note: The source of the energy that is released during a chemical reaction is a loss of mass.

PRACTICE PROBLEMS

10. A physicist measures the mass of a speeding proton as being 2.20×10^{-27} kg. If its rest mass is 1.68×10^{-27} kg, how much kinetic energy does the proton possess?

11. A neutron has a rest mass of 1.68×10^{-27} kg. How much kinetic energy would it possess if it was travelling at $0.800c$?

12. How fast must a neutron be travelling relative to a detector in order to have a measured kinetic energy that is equal to its rest energy? Express your answer to two significant digits.

13. How much energy would be required to produce a kaon particle (κ) at rest with a rest mass of 8.79×10^{-28} kg?

14. If an electron and a positron (antielectron), each with a rest mass of 9.11×10^{-31} kg, met and annihilated each other, how much radiant energy would be produced? (In such a reaction involving matter and antimatter, the mass is completely converted into energy in the form of gamma rays.) Assume that the particles were barely moving before the reaction.

15. If the mass loss during a nuclear reaction is 14 μg, how much energy is released?

16. The Sun radiates away energy at the rate of 3.9×10^{26} W. At what rate is it losing mass due to this radiation?

Relativistic and Classical Kinetic Energy

It might seem odd that there are two apparently different equations for kinetic energy.

- At relativistic speeds, $E_k = mc^2 - m_o c^2$.
- At low (classical) speeds, $E_k = \frac{1}{2}mv^2$.

These equations are not as different as they appear, however. The first equation expands as follows.

$$E_k = mc^2 - m_o c^2$$
$$E_k = c^2(m - m_o)$$
$$E_k = c^2\left(\frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} - m_o\right)$$
$$E_k = m_o c^2\left[\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} - 1\right]$$

This expression can be simplified by using an advanced mathematical approximation that reduces to the following when $v \ll c$.

$$E_k = c^2 m_o\left[1 - (-\tfrac{1}{2})\frac{v^2}{c^2} - 1\right]$$
$$E_k = \frac{1}{2}m_o v^2$$

The relativistic expression for kinetic energy therefore becomes the classical expression for kinetic energy at normal speeds.

You have now examined the basics of the special theory of relativity and have seen how the measurement of time, length, and mass depends on the inertial frame of reference of the observer. You have also seen that mass and energy are equivalent, that matter could be considered as a condensed form of energy. What happens, though, when the frame of reference is not inertial? Such considerations are the subject of Einstein's general theory of relativity, which deals with gravitation and curved space — concepts that are beyond the scope of this course.

11.3 Section Review

1. K/U What do the terms "total energy" and "rest energy" mean?
2. K/U What term represents the lowest possible mass for an object?
3. C Using the equations involved in relativity, give two reasons why the speed of light is an unattainable speed for any material object.
4. C Imagine that the speed of light was about 400 m/s. Describe three effects that would be seen in everyday life due to relativistic effects.
5. I How might an experimenter demonstrate that high-speed (relativistic) particles have greater mass than when they are travelling at a slower speed? Assume that the experimenter has some way of measuring the speeds of these particles.
6. MC
   - **(a)** What must be true about the masses of the reactants and products for a combustion reaction? Why?
   - **(b)** Why would a chemist never notice the effect in part (a)?

CHAPTER 11 Review

REFLECTING ON CHAPTER 11

- The Michelson-Morley experiment indicated that the speed of light was the same for observers in any inertial frame of reference.
- The special theory of relativity is based on two postulates.
  1. All physical laws hold true in any inertial frame of reference.
  2. The speed of light is the same for observers in any inertial frame of reference.
- The special theory of relativity predicts that events that are simultaneous for observers in one inertial frame of reference are not necessarily simultaneous for observers in a different inertial frame of reference.
- The special theory of relativity predicts that if you are observing events in an inertial frame of reference that is moving rapidly relative to you, times in that observed frame of reference will appear to slow down. This is known as "time dilation." The effect is expressed in the formula
$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$
- The special theory of relativity predicts that if you are observing objects in an inertial frame of reference that is moving rapidly relative to you, lengths in that observed frame of reference in the direction of the motion will appear to be shorter. This is known as "length contraction." The effect is expressed in the formula
$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$
- The special theory of relativity predicts that objects moving at a high rate of speed relative to a given inertial frame of reference will have greater mass than when they are at rest in that frame of reference. This is known as "mass increase." The effect is expressed in the formula
$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$
- The previous equations are shortened by the use of the quantity called "gamma."
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
Using gamma, the equations become
$$\Delta t = \gamma \Delta t_o$$
$$L = \frac{L_o}{\gamma}$$
$$m = m_o\gamma$$
- Relativistic kinetic energy is given by
$$E_k = mc^2 - m_oc^2$$
$$\text{or } E_k = (\Delta m)c^2$$
- The speed of light through a vacuum is the fastest speed possible. Objects with mass must travel slower than this speed. Massless objects such as photons must travel at this speed.
- mc^2 represents the total energy of the particle.

Knowledge/Understanding

1. Briefly describe the Michelson-Morley experiment.
   (a) What were the predicted results?
   (b) What did Michelson and Morley actually observe?
   (c) Why did the observed results cause a complete rethinking of their basic postulates?
2. **(a)** In the Michelson-Morley experiment, what was the purpose of rotating the apparatus 90°?
   (b) What were the results and implications of this procedural step?
3. While riding in a streetcar in Bern, Switzerland, Einstein realized that moving clocks might not run at the same rate as

stationary clocks. He looked at a clock on a tower and realized that if the streetcar moved away from the clock at the speed of light, it would appear to him as if the clock had stopped. Describe a thought experiment to illustrate his thinking.

4. Explain, using examples, why seemingly simultaneous events might occur at different times and different places, depending on your frame of reference.
5. The speed of light in water is 2.25×10^8 m/s. Using Einstein's thinking, explain whether it is possible for a particle to travel through water at a speed greater than 2.25×10^8 m/s.
6. Explain how the behaviour of muons is used as evidence for the concepts of time dilation and length contraction.
7. Why is it postulated that electrons, protons, and other forms of matter can never travel at the speed of light?
8. A photon can be considered as a particle with a specific energy that travels at the speed of light.
   - **(a)** According to the special theory of relativity, what is the rest mass of the photon?
   - **(b)** If a photon has energy, does that mean it also has momentum?
9. Explain how the equation $E = mc^2$ is consistent with the law of conservation of energy.

Inquiry

10. Examine the question of when relativistic effects become important by plotting graphs of $1/\gamma$ versus velocity and γ versus velocity for speeds of 0 to $1.0c$.
    - **(a)** At what speed would an observer experience a 1.0% time dilation effect or a 1.0% length contraction effect?
    - **(b)** Repeat part (a) for 10%, 50%, 90%, and 99.99%.
11. Explain the first postulate of the theory of special relativity by describing how the laws of classical physics hold in an inertial frame of reference, but do not hold in a non-inertial frame of reference.
12. Describe a thought experiment to consider the effect on your everyday life if the speed of light was 3.0×10^2 m/s, rather than 3.0×10^8 m/s. Assume appropriate rates of speed and consider how much younger you would be if you flew from Toronto to Vancouver and back than if you stayed home. If the distance from Toronto to Vancouver is 2.8×10^3 km, measured at a walking pace, what distance will you have covered from the airplane's frame of reference? Assume that the airplane is flying at approximately 800 km/h.
13. Estimate the number of lights in the city of Toronto or Ottawa. Make reasonable assumptions. Suppose all of the light energy used in the city in 1 h in the evening could be captured and put into a box. Approximately how much heavier would the box become?

Communication

14. Sketch the appearance of a baseball as it flies past an observer at low speeds and at speeds that approach the speed of light.
15. A friend states that, according to Einstein, "Everything is relative." Disprove this popular statement by making a list of quantities that according to special relativity are **(a)** relative, that is, their value depends on the frame of reference, and **(b)** invariant, that is, their value is the same for all inertial observers.
16. Your lab partner is trying to convince you that a spaceship, which can travel at 0.9 c, can fit into a garage shorter than the spaceship's actual length. He suggests that if the spaceship is backed into the garage at full speed, it will undergo length contraction and thus fit into the garage. Explain to him the flaws in his thinking.

Making Connections

17. Until recently, the neutrino was thought to be massless and, therefore, to travel at the speed of light. Evidence from the Sudbury Neutrino Observatory (see the Physics Magazine in Chapter 13, The Nucleus and Elementary

Particles), published in June 2001, suggests that the neutrino has a tiny mass. Research the latest developments.

(a) What is the neutrino's mass now considered to be?

(b) Why has it been so difficult to measure?

(c) If the premise that neutrinos have mass is accepted, what are the implications in relation to setting an upper limit on their speed?

Problems for Understanding

18. How fast must a spaceship be moving for you to measure its length to be half its rest length?

19. You are speeding along in your sports car when your friend passes you on a relativistic motorcycle at $0.60c$. You see your own car as being 4.0 m long and your friend's motorcycle as being 1.5 m long. You also notice that your friend's watch indicates that 8.5×10^{-8} s elapsed as she passed you. (It is a very large watch!)

(a) How long is your car as seen by your friend?

(b) How long is the motorcycle as seen by your friend?

(c) How much time passed on your watch while 8.5×10^{-8} s passed on your friend's watch?

20. A proton has a rest mass in a laboratory of 1.67×10^{-27} kg.

(a) What would its mass be relative to the laboratory if it was accelerated up to a speed of $0.75c$?

(b) While the experimenter was determining the proton's mass in (a), what would be the proton's mass in its own frame of reference?

21. Create a graph showing the observed mass of an object that has a 1.0 kg rest mass as its speed goes from rest to $0.99c$.

22. If a clock in an airplane is found to slow down by 5 parts in 10^{13}, (i.e., $\Delta t/\Delta t_{\text{o}} = 1.0 + 5.0 \times 10^{-13}$), at what speed is the airplane travelling? (**Hint:** You might need to use an expansion for γ.)

23. A spaceship travelling at 0.9 c fires a beam of light straight ahead.

(a) How fast would the crew on the spaceship measure the light beam's speed to be?

(b) How fast would a stationary observer on a spacewalk measure the light beam's speed to be?

(c) How fast would the crew on another spaceship travelling parallel to the first at the same speed of $0.9c$ measure the light beam's speed to be?

24. A pion is an unstable elementary particle that has a lifetime of 1.8×10^{-8} s. Assume that a beam of pions produced in a lab has a velocity of $0.95c$.

(a) By what factor is the pions' lifetime increased?

(b) What will be their measured lifetimes?

(c) How far will they travel in this time?

25. What is the mass of an electron travelling at two thirds of the speed of light? Compare this to its rest mass.

26. What is the kinetic energy of an electron with the following speeds?

(a) $0.0010c$; **(b)** $0.10c$; **(c)** $0.50c$; **(d)** $0.99c$

(e) For which, if any, of these speeds, can you use the non-relativistic expression $\frac{1}{2}mv^2$ and have an error of less than 10%?

27. If an object has a mass that is 1.0% larger than its rest mass, how fast must it be moving?

28. How many 100 W light bulbs could be powered for one year by the direct conversion of 1 g of matter into energy?

29. An electron is accelerated from rest through a potential difference of 2.2 MV, so that it acquires an energy of 2.2 MeV. Calculate its mass, the ratio of its mass to its rest mass, and its speed.

30. An object at rest explodes into two fragments, each of which has a rest mass of 0.50 g.

(a) If the fragments move apart at speeds of $0.70c$ relative to the original object, what is the rest mass of the original object?

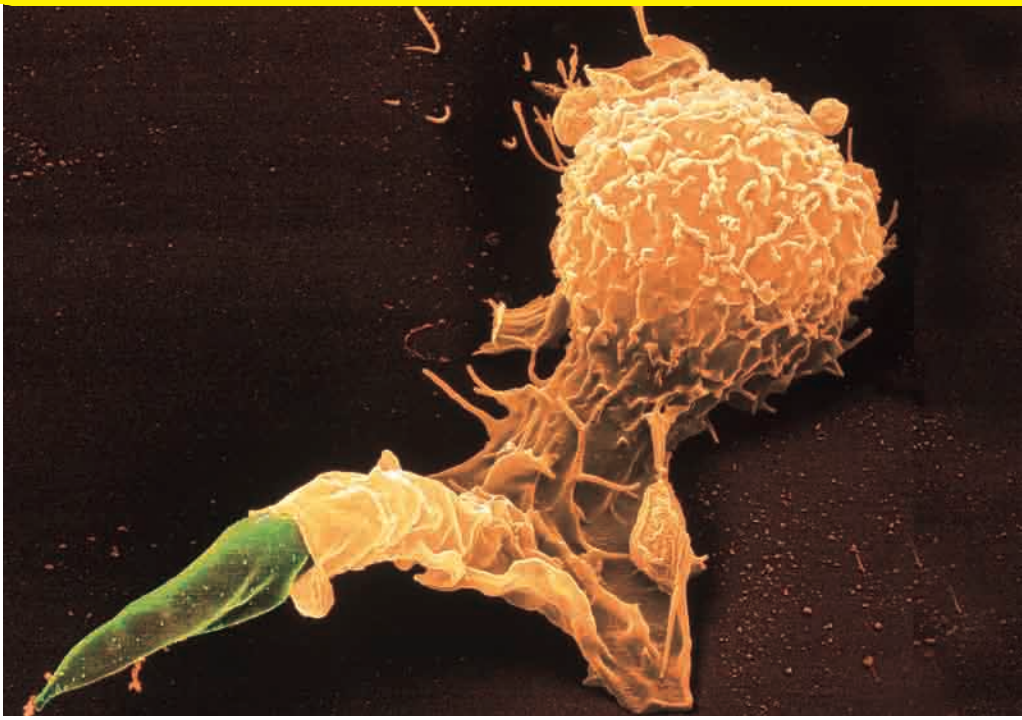
(b) How much of the object's original mass became kinetic energy of the fragments in the explosion?

CHAPTER 12 Quantum Mechanics and the Atom

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- J.J. Thomson's discovery of the electron
- Rutherford's scattering experiment



In the photograph above, a white blood cell is engulfing and destroying a parasite. This process, called "phagocytosis," is one way in which your immune system protects you from disease. The image of the white blood cell was formed not by light waves, but by electrons. In previous courses, you learned in detail how light waves form images. You discovered that the wave properties of light made image formation possible. It would seem logical then, that in order for electrons to form images, they must behave like waves.

The idea that electrons, and all forms of matter, have wavelike properties was one of the concepts that shook the world of physics in the early 1900s. This discovery, along with the observation that light behaves like particles, helped form the basis of quantum theory — a theory that has permanently changed scientists' perception of the physical world. The early observations and concepts seemed so theoretical and distant from the everyday world that it was difficult to see any potential impact on the daily lives of non-scientists. However, out of quantum theory grew such technologies as electron microscopes, lasers, semiconductor electronics, light meters, and many other practical tools. In this chapter, you will follow, step by step, how and why quantum theory developed and how it influenced scientists' concept of the atom.

INVESTIGATION 12-A

Discharging an Electroscope

TARGET SKILLS
- Hypothesizing
- Performing and recording
- Analyzing and interpreting

In this investigation, you will use an electroscope to analyze the interaction between ultraviolet light and a zinc plate.

Problem

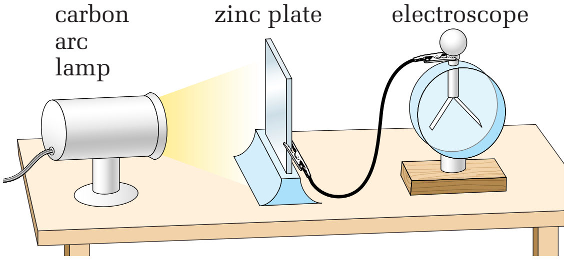
How can you discharge an electroscope when it is isolated from any source of electric grounding?

Equipment

- metal leaf electroscope
- carbon arc lamp (or source of intense ultraviolet light)
- insulating stand
- conducting wire with alligator clips
- zinc plate
- ebonite rod
- glass rod
- emery paper
- fur
- silk

Procedure

1. Polish the zinc plate with the emery paper until the plate shines.
2. Assemble the apparatus as shown in the diagram, leaving the lamp turned off. Ensure that the shiny side of the zinc plate faces the lamp.



3. Rub the ebonite rod with the fur to give the rod a negative charge.
4. Touch the ebonite rod to the sphere of the electroscope. Record the appearance of the electroscope.
5. Observe and record any changes in the electroscope over a period of 2 to 3 min.
6. Turn on the carbon arc lamp and observe and record any changes in the electroscope over a 2 to 3 min period.

CAUTION When the carbon arc lamp is on, do *not* look directly at the light or any reflected light. Ultraviolet light could damage your eyes.

7. Turn the lamp off. Touch the sphere of the electroscope with your hand to fully discharge the leaves.
8. Rub the glass rod with the silk to give it a positive charge. Touch the rod to the sphere of the electroscope.
9. Turn on the carbon arc lamp and observe and record any changes in the electroscope over a period of 2 to 3 min.
10. Turn the lamp off and discharge the electroscope.

Analyze and Conclude

1. Describe the exact conditions under which the electroscope discharged. For example, did it discharge when it was carrying a net negative charge or net positive charge? Was the carbon arc lamp on or off when this occurred?
2. Describe the conditions under which the electroscope did not discharge.
3. What entity had to escape from the electroscope in order for it to discharge?
4. Formulate a hypothesis about a mechanism that would have allowed the entity in question 3 to escape.
5. As you study this chapter, compare your hypothesis with the explanation given by physicists.

12.1 The Particle Nature of Light

SECTION EXPECTATIONS

- Describe the photoelectric effect and outline the experimental evidence that supports a particle model of light.
- Describe how the development of the quantum theory has led to technological advances such as the light meter.

KEY TERMS

- classical physics
- blackbody
- ultraviolet catastrophe
- empirical equation
- quantized
- quantum
- photoelectric effect
- stopping potential
- photon
- work function
- threshold frequency
- electron volt

In Unit 4, The Wave Nature of Light, you studied light and electromagnetic radiation. You learned that Christiaan Huygens (1629–1695) revived the wave theory of light in 1678. In 1801, Thomas Young (1773–1829) demonstrated conclusively with his famous double-slit experiment that light consisted of waves.

For more than 200 years, physicists studied electromagnetism and accumulated evidence for the wave nature of light and all forms of electromagnetic radiation. In fact, in 1873, James Clerk Maxwell (1831–1879) published his *Treatise on Electricity and Magnetism*, in which he summarized in four equations everything that was known about electromagnetism and electromagnetic waves. Maxwell's equations form the basis of electromagnetism in much the same way that Newton's laws form the basis of mechanics.

These areas of study, Newtonian mechanics and electricity and magnetism, along with thermodynamics, constitute **classical physics**. By the late 1800s, classical physics was well established. Many years of experiments and observations supported the theories of Newton and Maxwell. However, the scientific community was about to be shaken by events to come with the turn of the century.

How could observations on something as seemingly simple as a blackbody expose a flaw in these well-established theories? What, exactly, is a blackbody?

Blackbody Radiation

Based on his studies on the emission and absorption spectra of gases, Gustav Kirchhoff (1824–1887) defined the properties of a blackbody. While working with Robert Bunsen (1811–1899), Kirchhoff observed that, when heated to incandescence, gases emit certain, characteristic frequencies of light. When white light shines through the gases, they absorb the same frequencies of light that they emit, so Kirchhoff proposed that all objects absorb the same frequencies of radiation that they emit. He further reasoned that since black objects absorb all frequencies of light, they should emit all frequencies when heated to incandescence. Thus, the term **blackbody** was defined as a "perfect radiator," a body that emits a complete spectrum of electromagnetic radiation.

Fortunately for experimenters, blackbodies are not difficult to simulate in the laboratory. Any cavity with the inner walls heated to a very high temperature and with a very small hole to allow radiation to escape (see Figure 12.1) will emit a spectrum of radiation nearly identical to that of a blackbody.

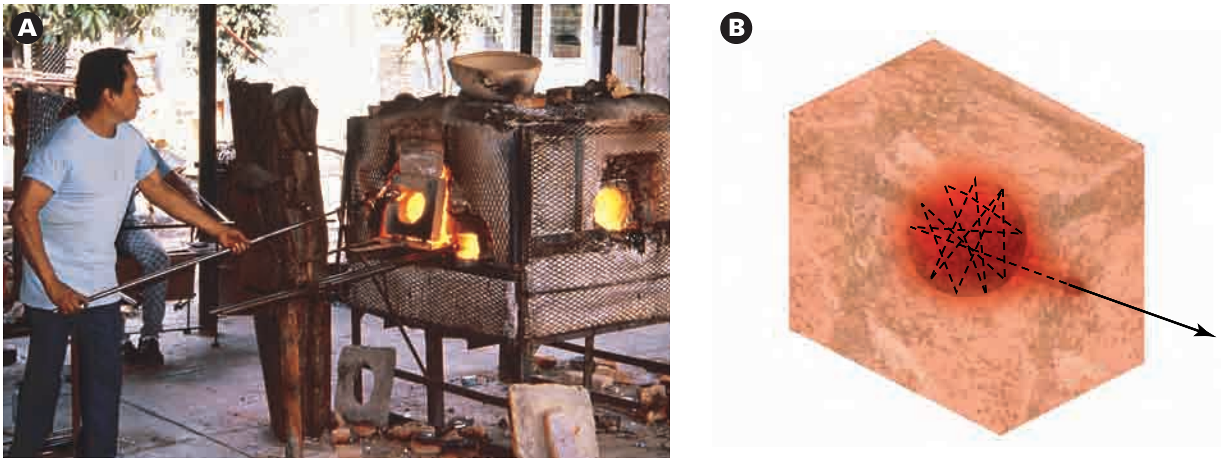


Figure 12.1 **(A)** When the temperature of a kiln surpasses 1000 K, the radiation is independent of the nature of the material in the kiln and depends only on the temperature. **(B)** A tiny hole in a very hot cavity “samples” the radiation that is being emitted and absorbed by the walls inside.

Figure 12.2 shows graphs of the blackbody radiation distribution at several different temperatures. The frequency of the radiation is plotted on the horizontal axis and the intensity of the radiation emitted at each frequency is plotted on the vertical axis. The area under the curve represents the total amount of energy emitted by a blackbody in a given time interval.

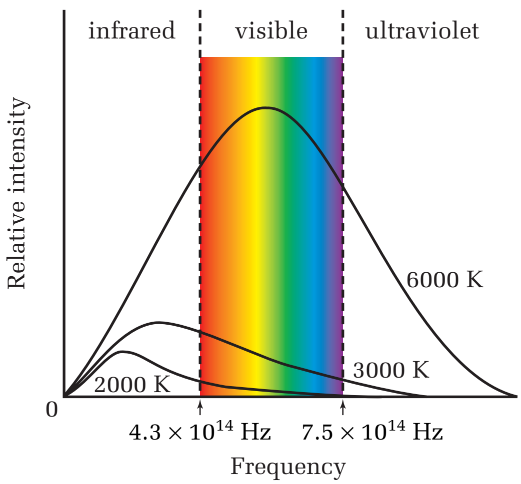


Figure 12.2 As the temperature of an incandescent body increases, the frequency that is emitted with the highest intensity (the peak of the curve) becomes higher.

Using data such as those in Figure 12.2, Kirchhoff was able to show that the power radiated by a blackbody depends on the blackbody’s temperature. He also showed that the intensity of the radiation was related to the frequency in a complex way and that the distribution of intensities was different at different temperatures. Kirchhoff was unable to find the exact form of the mathematical relationships, so he challenged the scientific community to do so.

Figure 12.3 **(A)** While an object such as a crowbar emits no visible radiation at room temperature, it is actually emitting infrared radiation. **(B)** When a stove coil reaches 600 K, it emits mostly invisible, infrared radiation. The radiation appears to be red, because it emits a little visible radiation in the red end of the spectrum. **(C)** At 2000 K, a light bulb filament looks white, because it emits all frequencies in the visible range.

According to electromagnetic theory, accelerating charges emit electromagnetic radiation. Maxwell's equations describe the nature of these oscillations and the associated radiation. A blackbody therefore must have vibrating, or oscillating, charges on the surface that are emitting (or absorbing) electromagnetic energy.

Josef Stefan (1835–1893) showed experimentally in 1879 that the power (energy per unit time) emitted by a blackbody is related to the fourth power of the temperature ($P \propto T^4$). In other words, if the temperature of a blackbody doubles, the power emitted will increase by 2^4, or 16 times. Five years later, Ludwig Boltzmann (1844–1906) used Maxwell's electromagnetic theory, as well as methods Boltzmann himself had developed for thermodynamics, to provide a theoretical basis for the fourth-power relationship.

The exact mathematical relationship between frequency and intensity of radiation emitted by a blackbody is much more complex than the relationship between temperature and power. Nevertheless, Lord Rayleigh (John William Strutt, 1842–1919) and Sir James Hopwood Jeans (1877–1946) attempted to apply the same principles that Boltzmann had used for the energy-temperature relationship. When they applied these theories to blackbody radiation, they obtained the upper curve shown in Figure 12.4. The lower curve represents experimental data for the same temperature.

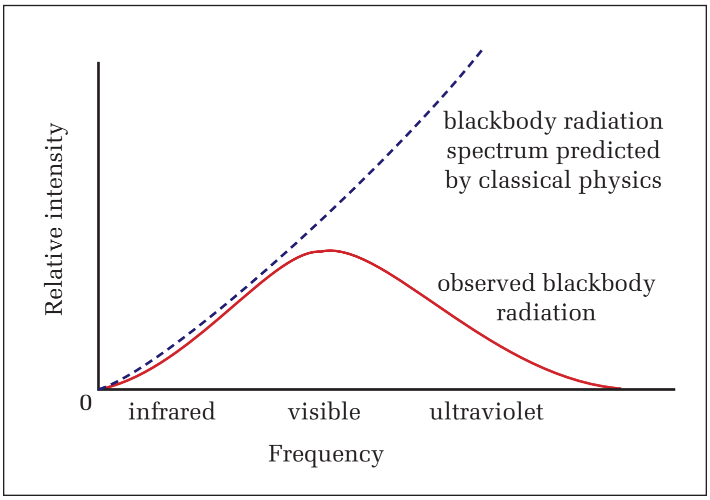


Figure 12.4 At low frequencies, predictions based on classical theory agree with observed data for the intensity of radiation from a blackbody. At high frequencies, however, theory and observation diverge quite drastically.

As you can see in Figure 12.4, classical theory applied to blackbody radiation agrees with observed data at low frequencies, but predicts that energy radiated from incandescent objects should continue to increase as the frequency increases. This discrepancy between theory and observation shocked the physicists of the day so much that they called it the **ultraviolet catastrophe**. How could a theory that had explained all of the data collected for 200 years fail to predict the emission spectrum of a blackbody? Little did they realize what was in store.

The Birth of Quantum Theory

Max Planck (1858–1947), a student of Kirchhoff, developed an empirical mathematical relationship between intensity and frequency of blackbody radiation. (An **empirical equation** is one that fits the observed data but is not based on any theory.) To develop the theory behind his empirical relationship, Planck turned to a statistical technique that Boltzmann had developed to solve certain thermodynamic problems. Planck had to make a "minor adjustment" to apply this method to energies of oscillators in the walls of a blackbody, however.

Boltzmann's statistical method required the use of discrete units, such as individual molecules of a gas. Although the energies of oscillators had always been considered to be continuous, for the sake of the mathematical method, Planck assigned discrete energy levels to the oscillators. He set the value of the allowed energies of the oscillators equal to a constant times the frequency, or $E = hf$, where h is the proportionality constant.

According to this hypothetical system, an oscillator could exist with an energy of zero or any integral multiple of hf, but not at energy levels in between, as illustrated in Figure 12.5. When the blackbody emitted radiation, it had to drop down one or more levels and emit a unit of energy equal to the difference between the allowed energy levels of the oscillator. A system such as this is said to be **quantized**, meaning that there is a minimum amount of energy, or a **quantum** of energy, that can be exchanged in any interaction.

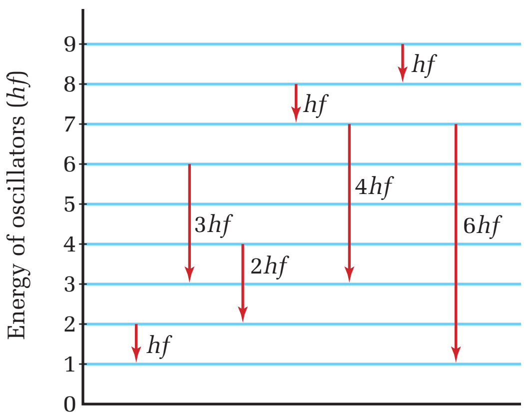


Figure 12.5 The "allowed" energy levels of the oscillators in the walls of a blackbody can be described as $E = nhf$, where n is any positive integer — 0, 1, 2, and up.

PHYSICS FILE

Shortly after Planck presented his paper on blackbody radiation, Einstein corrected one small error in the mathematics. He showed that the energy levels of the oscillators had to be $E = (n + \frac{1}{2})hf$. The addition of $\frac{1}{2}$ did not affect the *difference* between energy levels and thus did not change the prediction of the spectrum of blackbody radiation. However, it did show that the minimum possible energy of an oscillator is not zero, but $\frac{1}{2}hf$.

With discrete units of energy defined, Planck could now apply Boltzmann's statistical methods to his analysis of blackbodies. His plan was to develop an equation and then apply another mathematical technique that would allow the separation of energy levels to become smaller and smaller, until the energies were once again continuous. Planck developed the equation, but when he performed the mathematical operation to make oscillator energies continuous, the prediction reverted to the Rayleigh-Jeans curve. However, his equation fit the experimental data perfectly if the allowed energies of the oscillators remained discrete instead of continuous.

Planck was quite surprised, but he continued to analyze the equation. By matching his theoretical equation to experimental data, he was able to determine that the value of h, the proportionality constant, was approximately 6.55×10^{-34} J · s. Today, h is known as Planck's constant and its value is measured to be $6.626\,075\,5 \times 10^{-34}$ J · s.

With a theory in hand that could precisely predict the observed data for blackbody radiation, Planck presented his findings to the German Physical Society on December 14, 1900, and modern physics was born. Planck's revolutionary theory created quite a stir at the meeting, but the ideas were so new and radical that physicists — Planck included — could not readily accept them. More evidence would be needed before the scientific community would embrace the theory of the quantization of energy.

The Photoelectric Effect

The photoelectric effect, which would eventually confirm the theory of the quantization of energy, was discovered quite by accident. In 1887, Heinrich Hertz (1857–1894) was attempting to verify experimentally Maxwell's theories of electromagnetism. He assembled an electric circuit that generated an oscillating current, causing sparks to jump back and forth across a gap between electrodes, as illustrated in Figure 12.6. He showed that the sparks were generating electromagnetic waves by placing, on the far side of the room, a small coil or wire with a tiny gap. When the "transmitter" generated sparks, he observed that sparks were also forming in the gap of the "receiver" coil on the far side of the room. The electromagnetic energy had been transmitted across the room.

Hertz was able to show that these electromagnetic waves travelled with the speed of light and could be reflected and refracted, verifying Maxwell's theories. Ironically, however, Hertz made an observation that set the stage for experiments that would support the particle nature of electromagnetic radiation — the sparks were enhanced when the metal electrodes were exposed to ultraviolet light. At the time of Hertz's experiments, this phenomenon was difficult to explain.

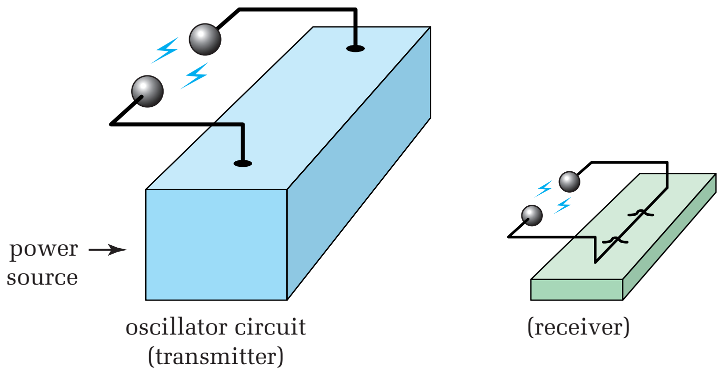


Figure 12.6 The electromagnetic waves that Hertz generated with his spark gap were in the frequency range that is now called "radio waves." Although Hertz's only intention was to verify Maxwell's theories, his experiments led to invention of the wireless telegraph, radio, television, microwave communications, and radar.

It was not until 10 years after Hertz carried out his experiments that Joseph John Thomson (1856–1940) discovered the electron. With this new knowledge, physicists suggested that the ultraviolet light had ejected electrons from Hertz's metal electrodes, thus creating a "conducting path" for the sparks to follow. The ejection of electrons by ultraviolet light became known as the **photoelectric effect**.

Early Photoelectric Effect Experiments

In 1902, physicist Philipp Lenard (1862–1947) performed more detailed experiments on the photoelectric effect. He designed an apparatus like the one shown in Figure 12.7. Electrodes are sealed in an evacuated glass tube that has a quartz window. (Ultraviolet light will not penetrate glass.) A very sensitive galvanometer detects any current passing through the circuit. Notice that the variable power supply can be connected so that it can make either electrode positive or negative.

To determine whether photoelectrons were, in fact, ejected from the "emitter," Lenard made the emitter negative and the collector positive. When he exposed the emitter to ultraviolet light, the galvanometer registered a current. The ultraviolet light had ejected electrons, which were attracted to the collector and then passed through the circuit. When Lenard increased the intensity of the ultraviolet light, he observed an increase in the current.

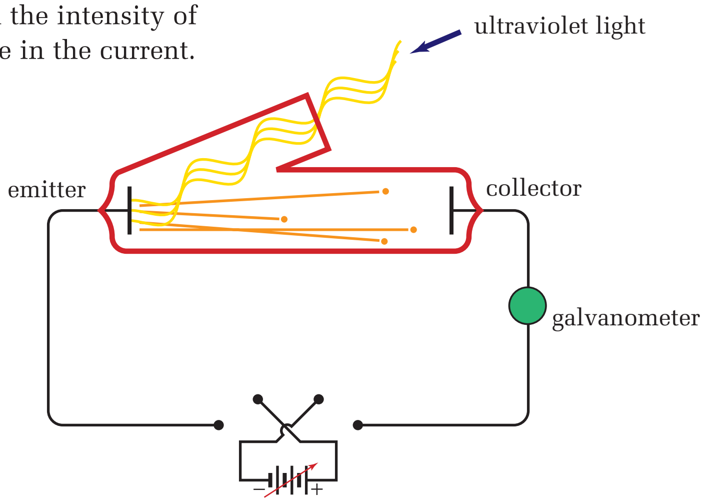


Figure 12.7 These glass tubes for experiments on the photoelectric effect had to be sealed in a vacuum so that the electrons would not collide with molecules of gas.

WEB LINK

www.mcgrawhill.ca/links/physics12

Many of the physicists who contributed to the development of quantum theory won the Nobel Prize in Physics. Find out who they were and learn more about their contributions to modern physics by going to the above Internet site and clicking on **Web Links**.

To learn more about the relative kinetic energies of photoelectrons, Lenard reversed the polarity of the power supply so that the electric field between the electrodes would oppose the motion of the photoelectrons. Starting each experiment with a very small potential difference opposing the motion of the electrons, he gradually increased the voltage and observed the effect on the current. The photoelectrons would leave the emitter with kinetic energy. He theorized that if the kinetic energy was great enough to overcome the potential difference between the plates, the electron would strike the collector. Any electrons that reached the collector would pass through the circuit, registering a current in the galvanometer. Electrons that did not have enough kinetic energy to overcome the potential difference would be forced back to the emitter.

Lenard discovered that as he increased the potential difference, the current gradually decreased until it finally stopped flowing entirely. The opposing potential had turned back even the most energetic electrons. The potential difference that stopped all photoelectrons is now called the **stopping potential**. Lenard's data indicated that ultraviolet light with a constant intensity ejected electrons with a variety of energies but that there was always a maximum kinetic energy.

In a critical study, Lenard used a prism to direct narrow ranges of frequencies of light onto the emitter. He observed that the stopping potential for higher frequencies of light was greater than it was for lower frequencies. This result means that, regardless of its intensity, light of higher frequency ejects electrons with greater kinetic energies than does light with lower frequencies. Once again, a greater *intensity* of any given frequency of light increased only the flowing current, or *number* of electrons, and had no effect on the electrons' stopping potential and, thus, no effect on their maximum kinetic energy. In summary, Lenard's investigations demonstrated the following.

- When the intensity of the light striking the emitter increases, the number of electrons ejected increases.
- The maximum kinetic energy of the electrons ejected from the metal emitter is determined *only* by the frequency of the light and is not affected by its intensity.

Lenard's first result is in agreement with the classical wave theory of light: As the intensity of the light increases, the amount of energy absorbed by the surface per unit time increases, so the number of photoelectrons should increase. However, classical theory also predicts that the kinetic energy of the photoelectrons should increase with an increase in the intensity of the light. Lenard's second finding — that the kinetic energy of the photoelectrons is determined *only* by the frequency of the light — cannot be explained by the classical wave theory of light.

Einstein and the Photoelectric Effect

Just a few years after Planck's quantum theory raised questions about the nature of electromagnetic radiation, the photoelectric effect raised even more questions. After publication, Planck's theory had been, for the most part, neglected. Now, however, because of the new evidence pointing to a flaw in the wave theory of light, Planck's ideas were revisited.

It was Albert Einstein (1879–1955) who saw the link between Planck's quantum of energy and the photoelectric effect. In 1905, Einstein published a paper in which he proposed that light must not only be emitted as quanta, or packets of energy, but it must also be absorbed as quanta. By treating light as quanta or **photons**, as they were named later, Einstein could explain Lenard's results for the photoelectric effect.

Einstein suggested that Planck's unit of energy, $E = hf$, is the energy of a photon. He proposed that when a photon strikes a metal surface, all of its energy is absorbed by one electron in one event. Since the energy of a photon is related to the frequency of the light, a photon with a higher frequency would have more energy to give an electron than would a photon with a lower frequency. This concept immediately explains why the maximum kinetic energy of photoelectrons depends only on frequency. Increasing the intensity of light of a given frequency increases only the *number* of photons and has no effect on the energy of a single photon.

Since the kinetic energy of the photoelectrons varied, some of the energy of the photons was being converted into a form of energy other than kinetic. Einstein proposed that some energy must be used to overcome the attractive forces that hold the electron onto the surface of the metal. Since some electrons are buried "deeper" in the metal, a larger amount of energy is needed to eject them from the surface. These electrons leave the emitter with less kinetic energy. The electrons with maximum kinetic energy must be the most loosely bound. Einstein gave the name **work function** (W) to this minimum amount of energy necessary to remove an electron from the metal surface. He predicted that the value would depend on the type of metal. The following mathematical expression describes the division of photon energy into the work function of the metal and the kinetic energy of the photoelectron.

$$hf = W + E_{k(max)}$$

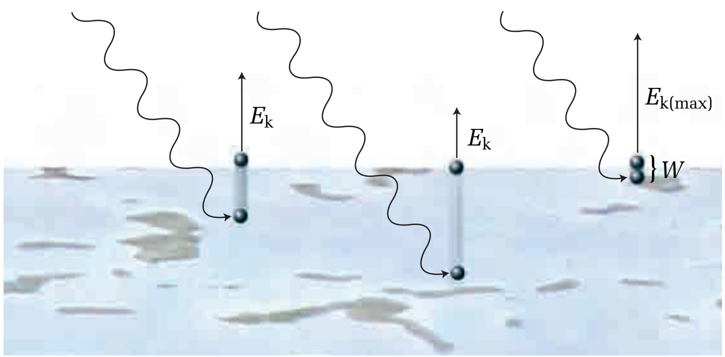


Figure 12.8 The energy of the photon must first extract the electron from the metal surface. The remainder of the energy becomes the kinetic energy of the electron.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of photoelectric effect, go to your Electronic Learning Partner.

PHYSICS FILE

Albert Einstein never actually carried out any laboratory experiments. He was a genius, however, at interpreting and explaining the results of others. In addition, the technology needed to test many of his theories did not exist until many years after he published them. Einstein was truly a theoretical physicist.

While Einstein's explanation could account for all of the observations of the photoelectric effect, very few physicists, including Planck, accepted Einstein's arguments regarding the quantum (or particle) nature of light. It was very difficult to put aside the 200 years of observations that supported the wave theory. Unfortunately, when Einstein wrote his paper on the photoelectric effect, the charge on the electron was not yet known, so there was no way to prove him right or wrong.

Millikan and the Photoelectric Effect

By 1916, Robert Millikan (1868–1953) had established that the magnitude of the charge on an electron was 1.60×10^{-19} C. With this "ammunition" in hand, Millikan set out to prove that Einstein's assumptions regarding the quantum nature of light were incorrect. Like others, Millikan felt that the evidence for the wave nature of light was overwhelming.

Millikan improved on Lenard's design and built photoelectric tubes with emitters composed of various metals. For each metal, he measured the stopping potential for a variety of frequencies. Using his experimentally determined value for the charge on an electron, he calculated the values for the maximum kinetic energy for each frequency, using the familiar relation $E = qV$. In this application, E is the energy of a charge, q, that has fallen through a potential difference, V. In Millikan's case, E was the maximum kinetic energy of the photoelectrons and q was the charge on an electron. The equation becomes $E_{\mathrm{k(max)}} = eV_{\mathrm{stop}}$. Millikan then plotted graphs of kinetic energy versus frequency for each type of metal emitter.

To relate graphs of $E_{\mathrm{k(max)}}$ versus f to Einstein's equation, it is convenient to solve for $E_{\mathrm{k(max)}}$, resulting in the following equation.

$$E_{\mathrm{k(max)}} = hf - W$$

$E_{\mathrm{k(max)}}$ is the dependent variable, f is the independent variable, and h and W are constants for a given experiment. Notice that the equation has the form of the slope-intercept equation of a straight line.

$$y = mx + b$$

Comparing the equations, you can see that Planck's constant (h) is the slope (m), and the negative of the work function ($-W$) is the y-intercept (b).

When Millikan plotted his data, they resulted in straight lines, as shown in Figure 12.9. The slopes of the lines from all experiments were the same and were equal to Planck's constant. When Millikan extrapolated the lines to cross the vertical axis, the value gave the negative of the work function of the metal. Much to Millikan's disappointment, he had proven that Einstein's equations perfectly predicted all of his results. He begrudgingly had to concede that Einstein's assumptions about the quantum nature of light were probably correct.

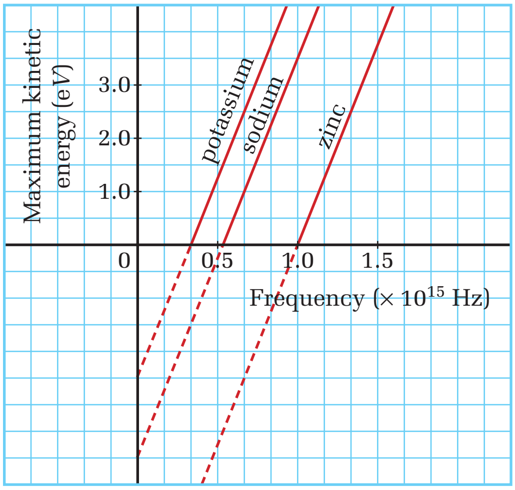


Figure 12.9 The graphs of Millikan's data were straight lines with equal slopes. The only differences were the points at which the extrapolated lines crossed the axes.

PHOTOELECTRIC EFFECT

The maximum kinetic energy of a photoelectron is the difference of the energy of the photon and the work function of the metal emitter.

$$E_{k(max)} = hf - W$$

Quantity	Symbol	SI unit
maximum kinetic energy of a photoelectron	$E_{k(max)}$	J (joules)
Planck's constant	h	J · s (joule · seconds)
frequency of electromagnetic radiation	f	Hz (hertz: equivalent to s^{-1})
work function of metal	W	J (joules)

Unit Analysis

(joule · second)(hertz) – joule = (J · s)(s^{-1}) = J

Another critical feature of a graph of maximum kinetic energy versus frequency is the point at which each line intersects the horizontal axis. On this axis, the maximum kinetic energy of the photoelectrons is zero. The frequency at this horizontal intercept is called **threshold frequency** (f_o), because it is the lowest frequency (smallest photon energy) that can eject a photoelectron from the metal. When photons with threshold frequency strike the emitter, they have just enough energy to raise the most loosely bound electrons to the surface of the emitter, but they have no energy left with which to give the photoelectrons kinetic energy. These photoelectrons are drawn back into the emitter.

Conceptual Problem

- At threshold frequency (f_o), the maximum kinetic energy of the photoelectrons is zero ($E_{k(max)} = 0$). Substitute these terms (f_o and 0) into Einstein's equation for the photoelectric effect and solve for the work function (W). Explain the meaning of the relationship that you found and how you can use it to find the work function of a metal.

You might have noticed that the unit for energy on the vertical axis in Figure 12.9 was symbolized as "eV," which in this case stands for "electron volt." The need for this new unit will become apparent when you start to use the photoelectric equation. You will find that the kinetic energy of even the most energetic electrons is an extremely small fraction of a joule. Since working with numbers such as 1.23×10^{-17} J becomes tedious and it is difficult to compare values, physicists working with subatomic particles developed the electron volt as the energy unit suitable for such particles and for photons. The **electron volt** is defined as the energy gained by one electron as it falls through the potential difference of one volt. The following calculation shows the relationship between electron volts and joules.

$$E = qV$$
$$1 \text{ eV} = (1 \text{ e})(1 \text{ V})$$
$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(1 \text{ V})$$
$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

Table 12.1 lists the work functions, in units of electron volts, of several common metals that have been studied as emitters in photoelectric experiments.

Like many other theoretical developments in physics, scientists soon found some practical applications for the photoelectric effect. The first light meters used the photoelectric effect to measure the intensity of light. Light meters have specialized metal emitters that are sensitive to visible light. When light strikes the metal, electrons are released and then collected by a positive electrode. The amount of current produced is proportional to the intensity of the light. The photon that physicists once had difficulty accepting is now almost a household word.

Table 12.1 Work Functions of Some Common Metals

Metal	Work function (eV)
aluminum	4.28
calcium	2.87
cesium	2.14
copper	4.65
iron	4.50
lead	4.25
lithium	2.90
nickel	5.15
platinum	5.65
potassium	2.30
tin	4.42
tungsten	4.55
zinc	4.33

12.1 Section Review

1. K/U Explain how a very hot oven can simulate a blackbody.

2. K/U Why was Planck's theory of blackbody radiation considered to be revolutionary?

3. C (a) Describe how the Hertz experiment, in which he used spark gaps to transmit and receive electromagnetic radiation, also provided early evidence for the photoelectric effect.
   (b) Name one modern technology that has its origin in the Hertz experiment. Briefly describe how it is related to this experiment.

4. C Describe how Einstein used Planck's concept of quanta of energy to explain the photoelectric effect.

5. K/U Define the terms "work function" and "threshold frequency."

6. C Describe how the quantum (photon) model for light better explains the photoelectric effect than does the classical wave theory.

7. MC What instruments have you used that rely on the photoelectric effect?

8. I Plot a graph of the following data from a photoelectric effect experiment and use the graph to determine Planck's constant, the threshold frequency, and the work function of the metal. Consult Table 12.1 and determine what metal was probably used as the target for electrons in the phototube.

Stopping potential (V)	Frequency of light (Hz)
0.91	9.0×10^{14}
1.62	10.7×10^{14}
2.35	12.4×10^{14}
3.50	15.0×10^{14}
4.21	16.5×10^{14}

12.2 Light Particles and Matter Waves

SECTION EXPECTATIONS

- Define and describe the concepts related to the understanding of matter waves.
- Describe how the development of quantum theory has led to technological advances such as the electron microscope.

KEY TERMS

- Compton effect
- de Broglie wavelength
- wave-particle duality

When Millikan's experimental results verified Einstein's interpretation of the photoelectric effect, the scientific community began to accept the particle nature of light. Physicists started to ask more questions about the extent to which particles of light, or photons, resembled particles of matter. U.S. physicist Arthur Compton (1892–1962) decided to study elastic collisions between photons and electrons. Would the law of conservation of momentum apply to such collisions? How could physicists determine the momentum (mv) of a particle that has no mass?

The Compton Effect

The ideal way to study collisions between particles is to start with free particles. Preferably, the only force acting on either particle at the moment of the collision is the impact of the other particle. However, electrons rarely exist free of atoms. So, Compton reasoned that if the photon's energy was significantly greater than the work function of the metal, the energy required to free an electron from the metal would be negligible when compared to the energy of the interaction. He needed a source of highly energetic photons.

About 30 years prior to Compton's work, Wilhelm Conrad Röntgen (1845–1923) discovered X rays and demonstrated that they are high-frequency electromagnetic waves. Thus, X-ray photons would have the amount of energy that Compton needed for his studies. In 1923, Compton carried out some very sophisticated experiments on collisions between X-ray photons and electrons. The phenomenon that he discovered is now known as the **Compton effect** and is illustrated in Figure 12.10. When a very high-energy X-ray photon collides with a "free" electron, it gives some of its energy to the electron and a lower-energy photon scatters off the electron.

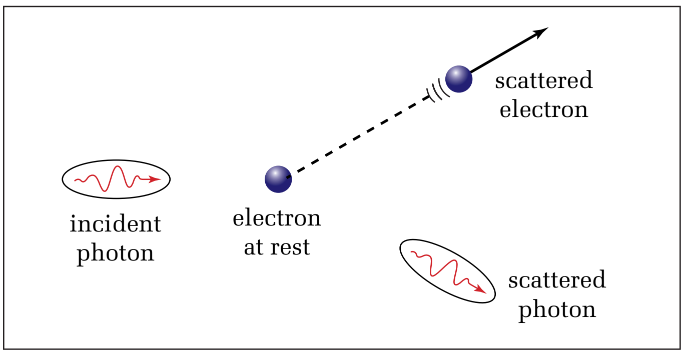


Figure 12.10 When a high-energy photon collides with a "free" electron, both energy and momentum are conserved.

You can describe mathematically the conservation of energy in a photon-electron collision as follows, where hf is the energy of the photon before the collision, hf' is the energy of the photon after the collision, and $\frac{1}{2}m_e v'^2$ is the kinetic energy of the electron after the collision.

$$hf = hf' + \frac{1}{2}m_e v'^2$$

Since the scattered photon has a lower energy, it must have a lower frequency and a longer wavelength than the original photon. Compton's measurements showed that the scattered photon had a lower frequency, and that kinetic energy gained by an electron in a collision with a photon was equal to the energy lost by the photon.

The more difficult task for Compton was finding a way to determine whether momentum had been conserved in the collision. The familiar expression for momentum, $p = mv$, contains the object's mass, but photons have no mass. So Compton turned to Einstein's now famous equation, $E = mc^2$, to find the mass equivalence of a photon. The following steps show how Compton used Einstein's relationship to derive an expression for the momentum of a photon. Since the goal is to find the magnitude of the momentum, vector notations are omitted.

- Write Einstein's equation that describes the energy equivalent of mass. $$E = mc^2$$
- Divide both sides of the equation by c^2 to solve for mass. $$m = \frac{E}{c^2}$$
- Write the equation for momentum. $$p = mv$$
- Substitute the energy equivalent of mass into the equation for momentum. $$p = \frac{E}{c^2}v$$
- Since the velocity of a photon is c, substitute c for v and simplify. $$P = \frac{E}{c^{\cancel{2}}}\cancel{c} = \frac{E}{c}$$
- Substitute the expression for the energy of a photon (hf) for E in the equation for momentum. $$p = \frac{hf}{c}$$
- The momentum of a photon is usually expressed in terms of wavelength, rather than frequency. Use the equation for the velocity of a wave to find the expression for f in terms of v. Note that the velocity of a light wave is c. $$f\lambda = v$$ $$f\lambda = c$$ $$f = \frac{c}{\lambda}$$
- Substitute the expression for frequency into the momentum equation and simplify. $$p = \frac{h\frac{\cancel{c}}{\lambda}}{\cancel{c}}$$ $$p = \frac{h}{\lambda}$$

When Compton calculated the momentum of a photon using $p = \frac{h}{\lambda}$, he was able to show that momentum is conserved in collisions between photons and electrons. These collisions obey all of the laws for collisions between two masses. The line between matter and energy was becoming more and more faint.

MOMENTUM OF A PHOTON

The momentum of a photon is the quotient of Planck's constant and the wavelength of the photon.

$$p = \frac{h}{\lambda}$$

Quantity	Symbol	SI unit
momentum	p	$\frac{\text{kg} \cdot \text{m}}{\text{s}}$ (kilogram metres per seconds)
Planck's constant	h	$\text{J} \cdot \text{s}$ (joule seconds)
wavelength	λ	m (metres)

Unit Analysis

$$\frac{\text{kilogram} \cdot \text{metre}}{\text{second}} = \frac{\text{joule} \cdot \text{second}}{\text{metre}}$$

$$\frac{\text{kg} \cdot \text{m}}{\text{s}} = \frac{\text{J} \cdot \text{s}}{\text{m}} = \frac{\frac{\text{kg} \cdot \text{m}^{\cancel{2}}}{\text{s}^{\cancel{2}}} \cdot \cancel{\text{s}}}{\cancel{\text{m}}} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

The following problem will help you to develop a feeling for the amount of momentum that is carried by photons.

SAMPLE PROBLEM

Momentum of a Photon

Calculate the momentum of a photon of light that has a frequency of 5.09×10^{14} Hz.

Conceptualize the Problem

- The *momentum* of a *photon* is related to its *wavelength*.
- A photon's *wavelength* is related to its *frequency* and the speed of light.

Identify the Goal

The momentum, p, of the photon

Identify the Variables and Constants

Known	Implied	Unknown
$f = 5.09 \times 10^{14}$ Hz	$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$	λ
		p

Develop a Strategy

Find the wavelength by using the equation for the speed of waves and the value for the speed of light.

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{5.09 \times 10^{14}\ \text{s}^{-1}}$$

$$\lambda = 5.8939 \times 10^{-7}\ \text{m}$$

Use the equation that relates the momentum of a photon to its wavelength.

$$p = \frac{h}{\lambda}$$

$$p = \frac{6.63 \times 10^{-34}\ \mathrm{J \cdot s}}{5.8939 \times 10^{-7}\ \mathrm{m}}$$

$$p = 1.1249 \times 10^{-27}\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

$$p \cong 1.12 \times 10^{-27}\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

The momentum of a photon with a frequency of 5.09×10^{14} Hz is $1.12 \times 10^{-27}\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$.

Validate the Solution

You would expect the momentum of a photon to be exceedingly small, and it is. Check to see if the units cancel to give $\frac{\mathrm{kg \cdot m}}{\mathrm{s}}$.

$$\frac{\mathrm{J \cdot s}}{\mathrm{m}} = \frac{\frac{\mathrm{kg \cdot m^{\cancel{2}}}}{\mathrm{s^{\cancel{2}}}} \cdot \cancel{\mathrm{s}}}{\cancel{\mathrm{m}}} = \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

PRACTICE PROBLEMS

1. Find the momentum of a photon with a wavelength of 1.55 m (radio wave).
2. Find the momentum of a gamma ray photon with a frequency of 4.27×10^{20} Hz.
3. What would be the wavelength of a photon that had the same momentum as a neutron travelling at 8.26×10^{7} m/s?
4. How many photons with a wavelength of 5.89×10^{-7} m would it take to equal the momentum of a 5.00 g Ping-Pong™ ball moving at 8.25 m/s?
5. What would be the frequency of a photon with a momentum of $2.45 \times 10^{-32}\ \mathrm{kg \cdot m/s}$? In what part of the electromagnetic spectrum would this photon be?

Matter Waves

By the 1920s, physicists had accepted the quantum theory of light and continued to refine the concepts. Once again, however, the scientific community was startled by the revolutionary theory proposed by a young French graduate student, who was studying at the Sorbonne. As part of his doctoral dissertation, Louis de Broglie (1892–1987) proposed that not only do light waves behave as particles, but also that particulate matter has wave properties.

De Broglie's professors at the Sorbonne thought that the concept was rather bizarre, so they sent the manuscript to Einstein and asked for his response to the proposal. Einstein read the dissertation with excitement and strongly supported de Broglie's proposal. De Broglie was promptly granted his Ph.D., and six years later, he was honoured with the Nobel Award in Physics for his theory of matter waves. The following steps lead to what is now called the **de Broglie wavelength** of matter waves.

- Write Compton's equation for the momentum of a photon. $p = \frac{h}{\lambda}$
- Solve the equation for wavelength. $\lambda = \frac{h}{p}$
- Substitute the value for the momentum of a particle for *p*. $\lambda = \frac{h}{mv}$

DE BROGLIE WAVELENGTH OF MATTER WAVES

The de Broglie wavelength of matter waves is the quotient of Planck's constant and the momentum of the mass.

$$\lambda = \frac{h}{mv}$$

Quantity	Symbol	SI unit
wavelength (of a matter wave)	λ	m (metres)
Planck's constant	h	J · s (joule seconds)
mass	m	kg (kilograms)
velocity	v	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$\frac{\text{joule} \cdot \text{second}}{\text{kilogram}\frac{\text{metre}}{\text{second}}} = \frac{\text{J} \cdot \text{s}}{\text{kg}\frac{\text{m}}{\text{s}}} = \frac{\text{J} \cdot \text{s}}{\text{kg}} \cdot \frac{\text{s}}{\text{m}} = \frac{\frac{\cancel{\text{kg}} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}\cancel{\text{s}^2}}{\cancel{\text{kg}} \cdot \cancel{\text{m}}} = \text{m}$$

Note: Since wavelength is a scalar quantity, vector notations are not used for velocity.

SAMPLE PROBLEM

Matter Waves

Calculate the wavelength of an electron moving with a velocity of 6.39×10^6 m/s.

Conceptualize the Problem

- *Moving particles* have *wave* properties.
- The *wavelength* of particle waves depends on *Planck's constant* and the *momentum* of the particle.

Identify the Goal

The wavelength, λ, of the electron

Identify the Variables and Constants

Known	Implied	Unknown
$v = 6.39 \times 10^6\ \frac{m}{s}$	$h = 6.63 \times 10^{-34}\ J \cdot s$	λ
	$m_e = 9.11 \times 10^{-31}\ kg$	

Develop a Strategy

Use the equation for the de Broglie wavelength.

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}\ J \cdot s}{(9.11 \times 10^{-31}\ kg)(6.39 \times 10^6\ \frac{m}{s})}$$

$$\lambda = 1.1389 \times 10^{-10}\ m$$

$$\lambda \cong 1.14 \times 10^{-10}\ m$$

The de Broglie wavelength of an electron travelling at 6.39×10^6 m/s is 1.14×10^{-10} m.

Validate the Solution

Since Planck's constant is in the numerator, you would expect that the value would be very small. Check the units to ensure that the final answer has the unit of metres.

$$\frac{J \cdot s}{kg \frac{m}{s}} = \frac{\frac{\cancel{kg} \cdot m^{\cancel{2}}}{s^{\cancel{2}}} \cdot \cancel{s}}{\cancel{kg}} \cdot \frac{\cancel{s}}{\cancel{m}} = m$$

PRACTICE PROBLEMS

6. Calculate the wavelength of a proton that is moving at 3.79×10^6 m/s.
7. Calculate the wavelength of an alpha particle that is moving at 1.28×10^7 m/s.
8. What is the wavelength of a 5.00 g Ping-Pong™ ball moving at 12.7 m/s?
9. Find the wavelength of a jet airplane with a mass of 1.12×10^5 kg that is cruising at 891 km/h.
10. Calculate the wavelength of a beta particle (electron) that has an energy of 4.35×10^4 eV.
11. What is the speed of an electron that has a wavelength of 3.32×10^{-10} m?

To verify de Broglie's hypothesis that particles have wavelike properties, an experimenter would need to show that electrons exhibit interference. A technique such as Young's double-slit experiment would be ideal. This technique is not feasible for particles such as electrons, however, because the electrons have wavelengths in the range of 10^{-10} m. It simply is not possible to mechanically cut slits this small and close together. Fortunately, a new technique for observing interference of waves with very small wavelengths had recently been devised.

PHYSICS FILE

As you know, in 1897, J.J. Thomson provided solid evidence for the existence of the electron, a subatomic particle that is contained in all atoms. Ironically, just 30 years later, his son George P. Thomson demonstrated that electrons behave like waves.

During the 10 years prior to de Broglie's proposal, physicists Max von Laue (1879–1960) and Sir Lawrence Bragg (1890–1971) were developing the theory and technique for diffraction of X rays by crystals. The spacing between atoms in crystals is in the same order of magnitude as both the wavelength of X rays and electrons, about 10^{-10} m. As illustrated in Figure 12.11, when X rays scatter from the atoms in a crystal, they form diffraction patterns in much the same way that light forms diffraction patterns when it passes through a double slit or a diffraction grating. If electrons have wave properties, then the same crystals that diffract X rays should diffract electrons and create a pattern.

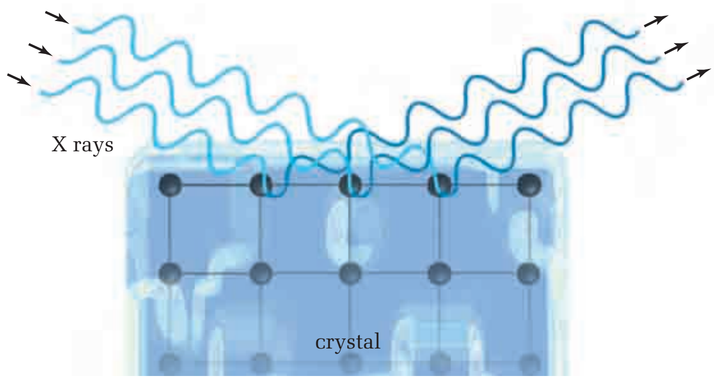


Figure 12.11 X rays scattered from regularly spaced atoms in a crystal will remain in phase only at certain scattering angles.

Within three years after de Broglie published his theory of matter waves, Clinton J. Davisson (1881–1958) and Lester H. Germer (1896–1971) of the United States and, working separately, George P. Thomson (1892–1975) of England carried out electron diffraction experiments. Both teams obtained patterns very similar to those formed by X rays. The wave nature of electrons was confirmed. In the years since, physicists have produced diffraction patterns with neutrons and other subatomic particles. Figure 12.12 shows diffraction patterns from aluminum foil formed by a beam of (A) X rays and (B) electrons.

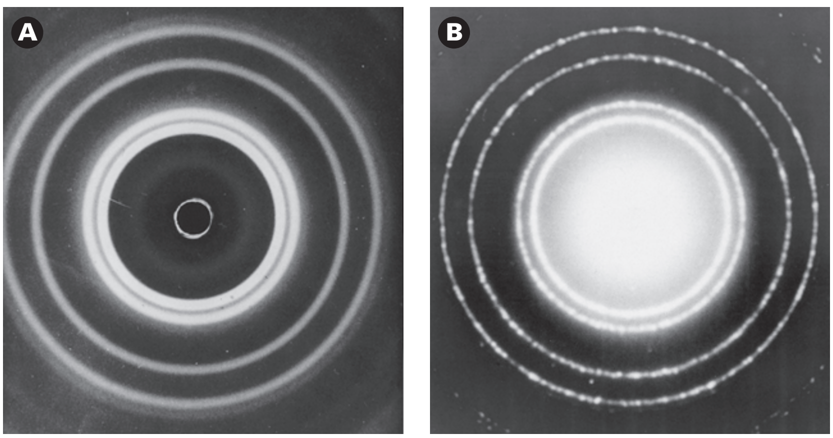


Figure 12.12 These patterns were created by diffraction of **(A)** X rays and **(B)** electrons by aluminum foil. Diffraction occurs as a result of the interference of waves. The similarity of these patterns verifies that electrons behave like waves.

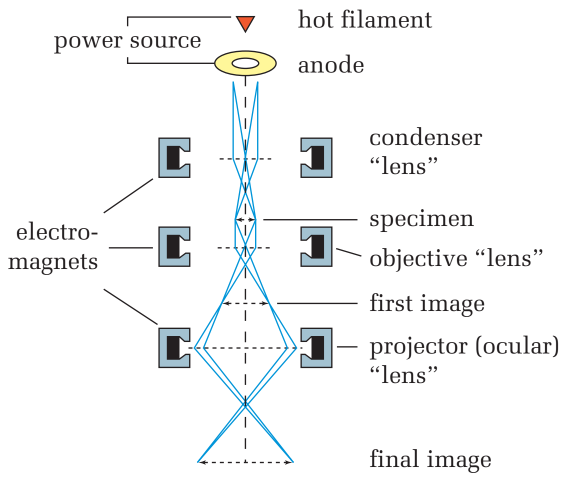
CANADIANS IN PHYSICS

University of Toronto Graduate Students Make History

Although many research groups around the world were attempting to design and build electron microscopes in the 1930s, the first high-resolution electron microscope that was practical and therefore became the prototype for the first commercial instrument was designed, built, and tested by two graduate students at the University of Toronto. James Hillier and Albert Prebus are shown in the photograph with the electron microscope that they built in 1938. Hillier continued to perfect and use the electron microscope while he completed his Ph.D. degree. In 1940, Hillier joined the staff of the Radio Corporation of America (RCA) in Camden, New Jersey, where he continued to improve the electron microscope. In 1969, Hillier became the executive vice president in charge of research and engineering for RCA. In this position, he was responsible for all of the research, development, and engineering programs.

The race to build electron microscopes was based on Davisson and Germer's verification of the wave properties of electrons. Electron microscopes have much greater resolving power than light microscopes, due to their very short wavelengths. Resolving power is the ability to distinguish two or more objects as separate entities, rather than as one large object. If the distance between two objects is much less than the wavelength, a microscope "sees" them as one particle, rather than as two. You can magnify the image to any size, but all that you will see is one large, blurred object. Since the shortest wavelength of visible light is about 400 nm and electrons can have wavelengths of 0.005 nm, electron microscopes could theoretically have a resolving power more than 10 000 times greater than light microscopes. In practice, however, electron microscopes have resolving powers about 1000 times greater than light microscopes.

The diagram shows the typical design of a transmission electron microscope. The barrel of the microscope must be evacuated, because electrons would be scattered by molecules in the air. Electrons would not penetrate glass lenses, of course, so focussing is accomplished by magnetic fields created by electromagnets. These magnetic "lenses" do not have to be moved or changed, because their focal lengths can be changed simply by adjusting the magnetic field strength of the electromagnets. Since electrons cannot penetrate glass, the extremely thin electron microscope specimens are placed on a wire mesh so that the electrons can penetrate the areas between the tiny wires.



The photograph at the beginning of this chapter was produced by a scanning electron microscope. These instruments function on a very different principle than do transmission electron microscopes. A very tiny beam of electrons sweeps back and forth across the specimen, and electrons that bounce back up from the sample are detected. Scanning electron microscopes were first developed in 1942, but they were not commercially available until 1965.

WEB LINK

www.mcgrawhill.ca/links/physics12

For more information about Hillier and Prebus and the history of the electron microscope, including diagrams and photographs, go to the above Internet site and click on **Web Links**.

The Wave-Particle Duality

Within 30 years after Planck presented his revolutionary theory to the German Physical Society, physicists had come to accept the particle nature of light and the wave nature of subatomic particles. They did not, however, forsake Maxwellian electromagnetism or Newtonian mechanics. Newton's concepts have made it possible for astronauts to travel to the Moon and back and to put satellites into orbit. Maxwell's electromagnetism permits engineers to develop the technology to send microwaves to and from these satellites. Physicists accept the dual nature of radiant energy that propagates through space as waves and interacts with matter as particles or discrete packets of energy.

Matter also has a dual nature, but only the subatomic particles have a small enough mass, and thus a large enough wavelength, to exhibit their wave nature. In 1924, Albert Einstein wrote, "There are therefore now two theories of light, both indispensable, and — as one must admit today despite twenty years of tremendous effort on the part of theoretical physicists — without any logical connection." Some physicists hope that in the future we will have a clearer picture of matter waves and quanta of energy. For now, we accept the **wave-particle duality:** Both matter and electromagnetic energy exhibit some properties of waves and some properties of particles.

12.2 Section Review

1. **C** Explain how Compton determined the momentum of a photon — a particle that has no mass.
2. **C** Describe the Compton effect.
3. **K/U** What was the most important result of Compton's experiments with the collisions between photons and electrons?
4. **K/U** Compton was able to ignore the work function of the metal in which the electrons were embedded in his momentum calculations. How was he able to justify this?
5. **C** Describe the reasoning that de Broglie used to come up with the idea that matter might have wave properties.
6. **C** When you walk through a doorway, you represent a particle having momentum and, therefore, having a wavelength. Why is it improbable that you will be "diffracted" as you pass through the doorway?
7. **K/U** Attempts to demonstrate the existence of de Broglie matter waves by using a beam of electrons incident on Young's double-slit apparatus proved unsuccessful. Give one possible explanation.
8. **C** Explain the technique that Davisson, Germer, and George P. Thomson used to verify the wave nature of electrons.
9. **MC** Research the production of X rays and prepare a display poster. In your display, include a diagram of the general structure of the X-ray tube, an explanation of how electrons cause the production of X rays, and an indication of the societal importance of the technology.

12.3 The Bohr Atom and Beyond

The new discoveries in quantum theory revealed phenomena that can be observed on the scale of subatomic particles, but are undetectable on a larger scale. These discoveries gave physicists the tools they needed to probe the structure of atoms in much more detail than ever before. The refinement of atomic theory grew side by side with the development of quantum theory.

SECTION EXPECTATIONS

- Describe and explain the Bohr model of the hydrogen atom.
- Collect and interpret experimental data in support of Bohr's model of the atom.
- Outline the historical development of scientific models from Bohr's model of the hydrogen atom to present-day theories of atomic structure.
- Describe how the development of quantum theory has led to technological advances such as lasers.

KEY TERMS

- nuclear model
- Balmer series
- Rydberg constant
- Bohr radius
- principal quantum number
- Zeeman effect
- Schrödinger wave equation
- wave function
- orbital
- orbital quantum number
- magnetic quantum number
- spin quantum number
- Pauli exclusion principle
- ground state

Atomic Theory before Bohr

As you have learned in previous science courses, the first significant theory of the atom was proposed by John Dalton (1766–1844) in 1808. Dalton's model could be called the "billiard ball model" because he pictured atoms as solid, indivisible spheres. According to Dalton's model, atoms of each element are identical to each other in mass and all other properties, while atoms of one element differed from atoms of each other element. Dalton's model could explain most of what was known about the chemistry of atoms and molecules for nearly a hundred years.

Figure 12.13 Dalton proposed that atoms were the smallest particles that make up matter and that they were indestructible. With his model, Dalton could predict most of what was known about chemistry at the time.

The Dalton model of the atom was replaced when J.J. Thomson established in 1897 that the atom was divisible. He discovered that the "cathode rays" in gas discharge tubes (see Figure 12.14) were negatively charged particles with a mass nearly 2000 times smaller than a hydrogen atom, the smallest known atom. These negatively charged particles, later named "electrons," appeared to have come off the metal atoms in one of the electrodes in the gas discharge tubes. Based on this new information, Thomson developed another model of the atom, which consisted of a positively charged sphere with the negatively charged electrons imbedded in it, as illustrated in Figure 12.15.

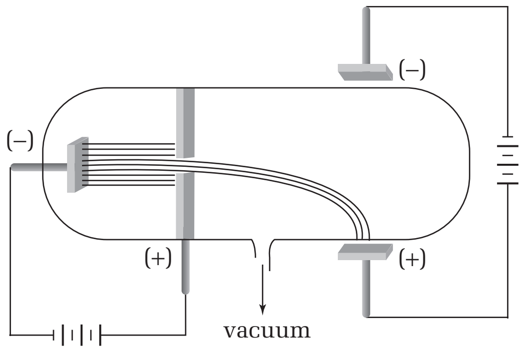


Figure 12.14 Metal electrodes were sealed in a glass tube that had been evacuated of all but a trace of a gas. A potential difference was created between the two electrodes. "Cathode rays" emanated from the negative electrode and a few passed through a hole in the positive electrode. Thomson showed that these "cathode rays" carried a negative charge by placing another set of electrodes outside the tube. The positive plate attracted the "rays."

Even as Thomson was developing his model of the atom, Ernest Rutherford (1871–1937) was beginning a series of experiments that would lead to replacement of Thomson's model. Rutherford was born and educated in New Zealand. In 1895, he went to England to continue his studies in the laboratory of J.J. Thomson. While there, he became interested in radioactivity and characterized the "rays" emitted by uranium, naming them "alpha rays" and "beta rays." He discovered that alpha rays were actually positively charged particles.

In 1898, Rutherford accepted a position in physics at McGill University in Montréal, where he continued his studies of alpha particles and published 80 scientific papers. Nine years later, Rutherford returned to England, where he accepted a position at the University of Manchester.

While in Manchester, Rutherford and his research assistant Hans Geiger (1882–1945) designed an apparatus (see Figure 12.16) to study the bombardment of very thin gold foils by highly energetic alpha particles. If Thomson's model of the atom was correct, the alpha particles would pass straight through, with little or no deflection. In their preliminary observations, most of the alpha particles did, in fact, pass straight through the gold foil. However, in a matter of days, Geiger excitedly went to Rutherford with the news that they had observed some alpha particles scatter at an angle greater than 90°. Rutherford's famous response was, "It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15 inch shell at a piece of tissue paper and it come back and hit you!" The observations were consistent: Approximately 1 in every 20 000 alpha particles was deflected more than 90°. These results could not be explained by Thomson's model of the atom.

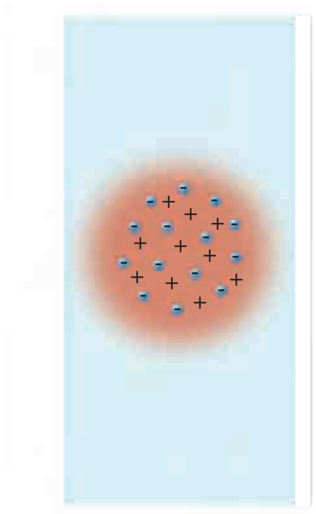


Figure 12.15 Thomson named his model the "plum pudding model" because it resembled a pudding with raisins distributed throughout.

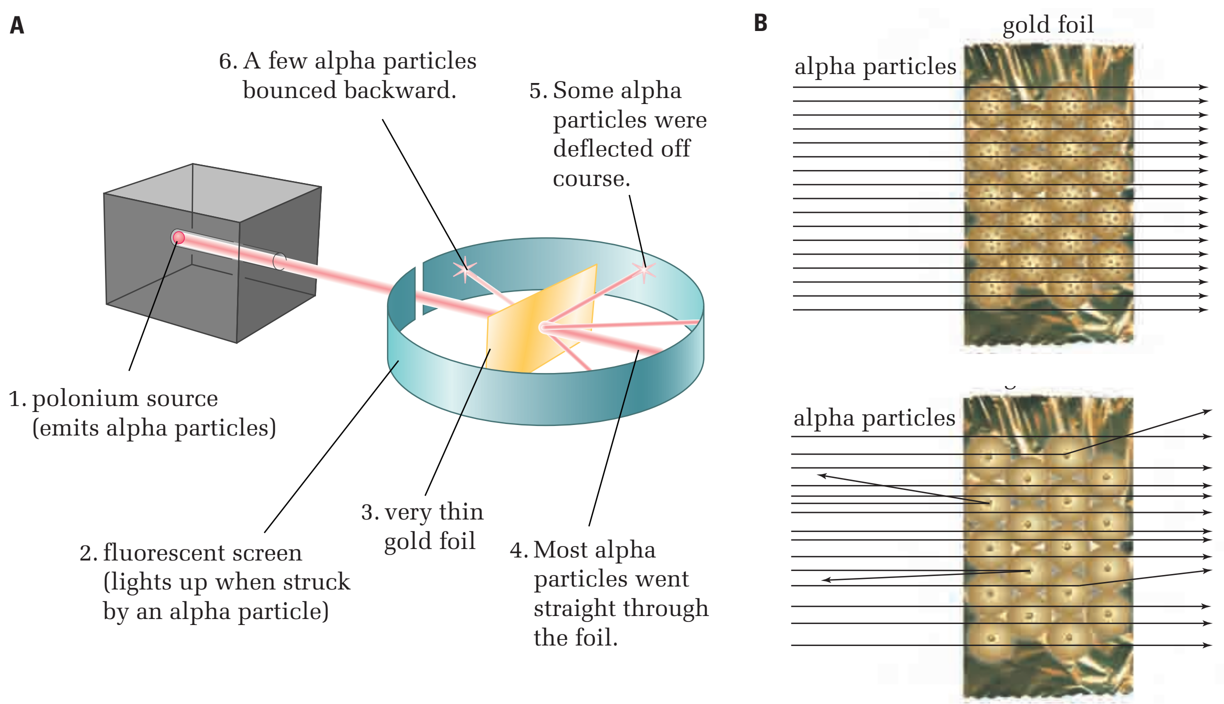


Figure 12.16 **(A)** A fine beam of alpha particles was directed at a very thin gold foil. The circular screen around the foil was coated with zinc sulfide, which emitted a flash of light when hit by an alpha particle. **(B)** If positive and negative charges were equally distributed throughout the foil, they would have little effect on the direction of the alpha particles. **(C)** If all of the positive charge in each atom was concentrated in a very tiny point, it would create a large electric field close to the point. The field would deflect alpha particles that are moving directly toward or very close to the tiny area where the positive charge is located.

What force could possibly be strong enough to repel such a highly energetic alpha particle? Rutherford searched his mind and performed many calculations. He concluded that the only force great enough to repel the alpha particles would be an extremely strong electrostatic field. The only way that a field this strong could exist was if all of the positive charge was confined in an extremely small space at the centre of the atom. Thus, Rutherford proposed his **nuclear model** of the atom. All of the positive charge and nearly all of the mass of an atom is concentrated in a very small area at the centre of the atom, while the negatively charged electrons circulate around this "nucleus," somewhat like planets around the Sun, as illustrated in Figure 12.17.

In the following Quick Lab, you will apply some of the same concepts that Rutherford used to estimate the size of the atomic nucleus.

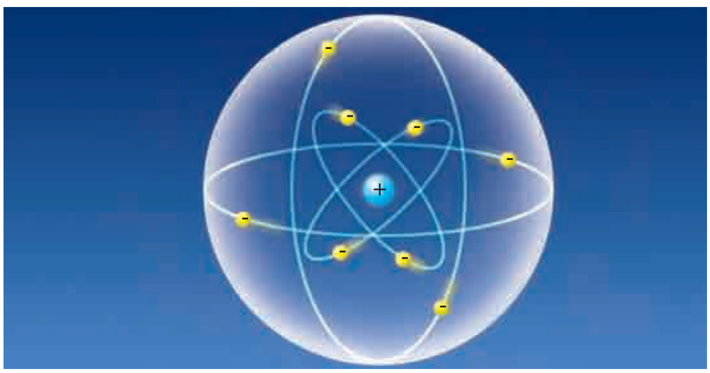


Figure 12.17 Rutherford's nuclear model resembles a solar system in which the positively charged nucleus could be likened to the Sun and the electrons are like planets orbiting the Sun.

QUICK LAB

Estimating the Size of the Nucleus

TARGET SKILLS
- Hypothesizing
- Analyzing and interpreting

Method 1

At the time that Rutherford was performing his experiments, physicists knew that the diameter of the entire atom was about 10^{-10} m.

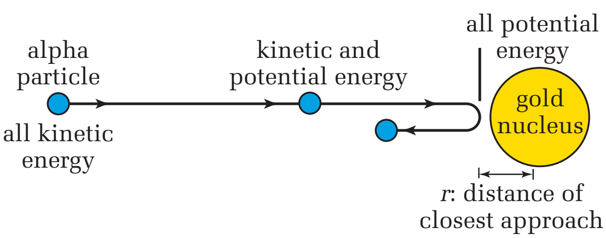
- Calculate the cross-sectional area of the atom, assuming that its diameter is 10^{-10} m.

Rutherford's students observed that about 1 in every 20 000 alpha particles scattered backward from the foil. If they were all headed toward the atom but only 1 in 20 000 was headed directly toward the nucleus, what must be the cross-sectional area of the nucleus?

- Calculate the cross-sectional area of the nucleus based on the above information.
- Using your cross-sectional area of the nucleus, calculate the diameter of the nucleus.

Method 2

Make a second estimate of the size of the nucleus based on the conservation of mechanical energy of the alpha particle. As shown in the diagram, at a large distance from the nucleus, the energy of the alpha particle is all kinetic energy. As it approaches the nucleus, its kinetic energy is converted into electric potential energy. When all of its kinetic energy is converted into electric potential energy, the alpha particle will stop. At this point, called the "distance of closest approach," the repulsive Coulomb forces will drive the alpha particle directly backward. If the alpha particle penetrated the nucleus, it would be trapped and would not scatter backward.



- The equation below states that the kinetic energy of the alpha particle at a large distance from the nucleus is equal to the electric potential energy of the alpha particle at the distance of closest approach. Substitute into the equality the mathematical expressions for kinetic energy and electric potential energy between two point charges a distance, r, apart.

$$E_k \text{ (very far from nucleus)} = E_Q \text{ (distance of closest approach)}$$

- The mass of an alpha particle is about 6.6×10^{-27} kg and those that Rutherford used had an initial velocity of 1.5×10^7 m/s. Calculate the kinetic energy of the alpha particle.
- An alpha particle has 2 positive charges and a gold nucleus has 79 positive charges. Using the magnitude of one elementary charge (1.6×10^{-19} C), calculate the magnitude of the charges needed for the determination of the electric potential energy.
- Substitute all of the known values into the equation above. You will find that r is the only unknown variable. Solve the equation for r, the distance of closest approach.

Analyze and Conclude

1. Comment on the validity of each of the two methods. What types of errors might affect the results?
2. How well do your two methods agree?
3. The accepted size of an average nucleus is in the order of magnitude of 10^{-14} m. How well do your calculations agree with the accepted value?

The Bohr Model of the Atom

Rutherford's model of the atom was based on solid experimental data, but it had one nagging problem that he did not address. According to classical electromagnetism, an accelerating charge should radiate electromagnetic waves and lose energy. If electrons are orbiting around a nucleus, then they are accelerating and they should be radiating electromagnetic waves. If the electrons lost energy through radiation, they would spiral into the nucleus. According to Rutherford's model, electrons remain permanently in orbit.

Niels Henrik David Bohr (1885–1962) addressed the problem of electrons that do not obey classical electromagnetic theory. Bohr was born and educated in Denmark, and in 1912, went to study in Rutherford's laboratory in Manchester. (Rutherford said of Bohr, "This young Dane is the most intelligent chap I've ever met.") Convinced that Rutherford was on the right track with the nuclear atom, Bohr returned home to Copenhagen, where he continued his search for an explanation for the inconsistency of the nuclear atom with classical theory.

Bohr was very aware of the recent publications of Planck and Einstein on blackbody radiation and the photoelectric effect, and that these phenomena did not appear to obey the laws of classical physics. He realized that some phenomena that are unobservable on the macroscopic level become apparent on the level of the atom. Thus, he did not hesitate to propose characteristics for the atom that appeared to contradict classical laws.

Bohr had another, very significant piece of evidence available to him — atomic spectra. When Kirchhoff defined blackbodies, he was studying very low-pressure gases in gas discharge tubes. Kirchhoff discovered that when gases of individual elements were sealed in gas discharge tubes and bombarded with "cathode rays," each element produced a unique spectrum of light. The spectrum of hydrogen is shown in Figure 12.18.

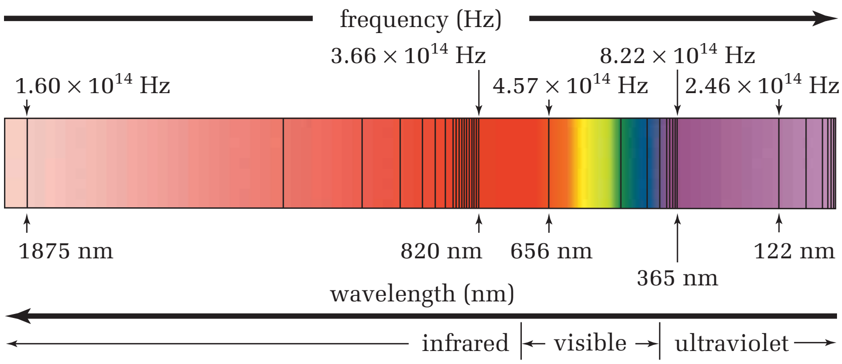


Figure 12.18 When bombarded by high-energy electrons, hydrogen atoms emit a very precise set of frequencies of electromagnetic radiation, extending from the infrared region, through the visible region, and well into the ultraviolet region of the spectrum.

Since emission spectra did not have an immediately obvious pattern, Bohr thought them too complex to be useful. However, a friend who had studied spectroscopy directed Bohr to a pattern that had been determined in 1885 by Swiss secondary school teacher Johann Jakob Balmer (1825–1898). Balmer had studied the visible range of the hydrogen spectrum and found an empirical expression that could produce the wavelength of any line in that region of the spectrum. Balmer's formula is given below. Remember that empirical equations are developed from experimental data and are not associated with any theory. Balmer could not explain why his formula had the form that it did. He could demonstrate only that it worked.

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right], \quad \text{where } n = 3, 4, 5, \ldots \text{ and } R = 1.097\,373\,15 \times 10^7\ \text{m}^{-1}$$

The spectral lines of hydrogen that lie in the visible range are now known as the **Balmer series**. As spectroscopists developed methods to observe lines in the infrared and ultraviolet regions of the spectrum, they found more series of lines. Swedish physicist Johannes Robert Rydberg (1854–1919) modified Balmer's formula, as shown below, to incorporate all possible lines in the hydrogen spectrum. The constant R is known as the **Rydberg constant**.

$$\frac{1}{\lambda} = R\left[\frac{1}{m^2} - \frac{1}{n^2}\right], \quad \text{where } m \text{ and } n \text{ are integers; } 1, 2, 3, 4, \ldots \text{ and } n > m$$

Bohr Postulates

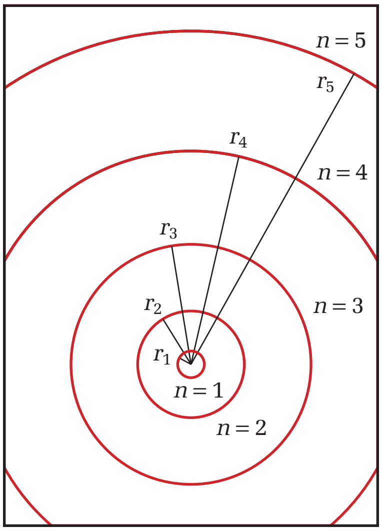


Figure 12.19 According to Bohr's model of the atom, electrons can exist in specific, allowed energy levels and can jump from one level to another by absorbing or emitting energy.

When Bohr saw these mathematical patterns, he said, "As soon as I saw Balmer's formula, the whole thing was immediately clear to me." Bohr was ready to develop his model of the atom. Bohr's model, illustrated in Figure 12.19, was based on the following postulates.

- Electrons exist in circular orbits, much like planetary orbits. However, the central force that holds them in orbit is the electrostatic force between the positive nucleus and the negative charge on the electrons, rather than a gravitational force.
- Electrons can exist only in a series of "allowed" orbits. Electrons, much like planets, have different amounts of total energy (kinetic plus potential) in each orbit, so these orbits can also be described as "energy levels." Since only certain orbits are allowed, then only certain energy levels are allowed, meaning that the energy of electrons in atoms is quantized.
- Contrary to classical theory, while an electron remains in one orbit, it does not radiate energy.
- Electrons can "jump" between orbits, or energy levels, by absorbing or emitting an amount of energy that is equal to the *difference* in the energy levels.

Electrons can "jump" to higher energy levels by absorbing thermal energy (collision with an energetic atom or molecule), by bombardment with an energetic electron (as in gas discharge tubes), or by absorbing photons of radiant energy with energies that exactly match the difference in energy levels of the electrons in the atom. Likewise, electrons can "drop" to a lower energy level by emitting a photon that has an energy equal to the *difference* between the energy levels. Since the energy of a photon is directly related to the frequency of the electromagnetic waves, you could express this relationship between energy levels and photons as follows.

$$|E_f - E_i| = hf$$

E_f is the energy of the final energy level, E_i is the energy of the initial energy level, and hf is the photon energy. This concept gives meaning to the "2" and the "n" in Balmer's formula, because the "2" represents the second energy level and "n" is any energy level above the second one. In Rydberg's more general formula, m is the final energy level, which can be any level. Likewise, n is the initial level and must therefore be higher than the final level.

To find the exact energies of these "allowed energy levels," Bohr had to determine exactly what property of the electron was quantized. An important clue comes from the units of Planck's constant — joule · seconds. First, simplify joules to base units.

$$\mathrm{J \cdot s = N \cdot m \cdot s = \frac{kg \cdot m}{s^{\cancel{2}}} \cdot m \cdot \cancel{s} = kg \cdot \frac{m}{s} \cdot m}$$

The final units, $\mathrm{kg \cdot \frac{m}{s} \cdot m}$, are the units for the quantities of mass, speed, and distance, or *mvd*. At one time in physics, this combination was called "action." In fact, Planck called his constant, h, the "quantum of action." If you apply these quantities to the electron in an orbit of radius r, you will get $m_e v_n 2\pi r$, where $2\pi r$ is the distance that the electron travels during one orbit around the nucleus. If this value is quantized, you would have the following.

$$2\pi m_e v_n r_n = nh$$

You might recognize the expression $m_e v_n r_n$ as the angular momentum of the electron in the n^{th} orbit. Following a similar logic, Bohr proposed that the angular momentum was quantized and then tested that hypothesis. The angular momentum of the n^{th} orbit can be written as follows.

$$m_e v_n r_n = n\frac{h}{2\pi}$$

You can test the theory by using the equation for the quantized angular momentum to find allowed radii and allowed energies of electrons. Then, you can compare these differences between energy levels to Balmer's formula and the Rydberg constant. Since you have two unknown quantities, r and v, you will need more relationships to find values for either r or v in terms of known

PHYSICS FILE

Johann Balmer's life was a contrast to that of most other contributors to the theory of the atom. Balmer taught mathematics in a secondary school for girls and lectured at the University of Basel in Switzerland. He published only two scientific papers in his career, one when he was 60 years old and one when he was 72. Balmer died 15 years before Niels Bohr provided an explanation for Balmer's now famous formula for the emission spectrum of hydrogen.

constants. Because Bohr based his concept on circular orbits, you can use the fact that the electrostatic force between the electron and the nucleus provides the centripetal force that keeps the electron in a circular orbit. The following steps will lead you through the procedure.

Deriving the Bohr Radius

- Write Coulomb's law. $$F = k\frac{q_1q_2}{r^2}$$
- Let Z be the number of positive charges in the nucleus. Therefore, Ze is the charge of the nucleus. The charge on an electron is, of course, e. Let r_n be the radius of the n^{th} orbit. Substitute these values into Coulomb's law. $$F = k\frac{Ze^2}{{r_n}^2}$$
- Set the coulomb force equal to the centripetal force. $$k\frac{Ze^2}{r_n^2} = \frac{m_e v_n^2}{r_n}$$
- Multiply both sides by r_n^2. $$kZe^2 = m_e v_n^2 r_n$$
- Divide both sides by $m_e v_n^2$. $$r_n = \frac{kZe^2}{m_e v_n^2}$$
- Write Bohr's condition for quantization of angular momentum. $$m_e v_n r_n = n\frac{h}{2\pi}$$
- Solve for v_n. $$v_n = \frac{nh}{2\pi m_e r_n}$$
- Substitute this expression for v_n into the equation for r_n. $$r_n = \frac{kZe^2}{m_e\left(\frac{nh}{2\pi m_e r_n}\right)^2}$$
- Start the simplification by inverting the fraction in the denominator in brackets and then multiplying by the inverted fraction. $$r_n = \frac{kZe^2}{m_e} \cdot \frac{4\pi^2 m_e^2 r_n^2}{n^2h^2}$$
- Divide both sides of the equation by r_n. $$1 = \left(\frac{4\pi^2 kZe^2 m_e^{\not 2}}{\not m_e n^2h^2}\right)r_n$$
- Invert and multiply by the expression in brackets. $$r_n = \frac{n^2h^2}{4\pi^2 kZe^2 m_e}$$

The expression, $\frac{h}{2\pi}$, occurs so frequently in quantum theory that the symbol $\hbar$ is often used in place of $\frac{h}{2\pi}$. The final expression is usually written as follows.

$$r_n = n^2\frac{\hbar^2}{m_e kZe^2}$$

For the first allowed radius of the electron in a hydrogen atom, $Z = 1$ and $n = 1$. All of the other values in the equation are constants and if you substitute them into the equation and simplify, you will obtain $r_1 = 0.052\ 917\ 7$ nm. This value is known as the **Bohr radius**.

Deriving Allowed Energy Levels

You can use the equation for the radius of the n^{th} orbit of an electron to find the energy for an electron in the n^{th} energy level in an atom as shown in the following steps.

- Write the expression for the total energy (kinetic plus potential) of a charge a distance, r, from another charge.

$$E = \frac{1}{2}mv^2 - k\frac{q_1q_2}{r}$$

- Substitute in the values for an electron at a distance, r_n, from a nucleus.

$$E_n = \frac{1}{2}m_e v_n^2 - k\frac{Ze^2}{r_n}$$

- To eliminate the variable, v, from the equation, go back to the expression you wrote when you set the Coulomb force equal to the centripetal force.

$$k\frac{Ze^2}{r_n^2} = \frac{m_e v^2}{r_n}$$

- Multiply both sides of the expression by $\frac{r_n}{2}$ and simplify.

$$\left(k\frac{Ze^2}{r_n^2}\right)\left(\frac{r_n}{2}\right) = \left(\frac{m_e v^2}{r_n}\right)\left(\frac{r_n}{2}\right)$$

$$\frac{kZe^2}{2r_n} = \frac{1}{2}m_e v_n^2$$

- Substitute this value found in the last step for kinetic energy, $\frac{1}{2}m_e v_n^2$, in the second equation and then simplify.

$$E_n = \frac{kZe^2}{2r_n} - k\frac{Ze^2}{r_n}$$

$$E_n = -\frac{kZe^2}{2r_n}$$

- Substitute the value for r_n into the expression for energy.

$$E_n = -\frac{kZe^2}{2\left(n^2\frac{\hbar^2}{m_e kZe^2}\right)}$$

- To simplify, invert the fraction in the denominator and multiply.

$$E_n = -\frac{kZe^2}{2(n^2)} \cdot \frac{m_e kZe^2}{\hbar^2}$$

$$E_n = -\frac{k^2e^4m_e}{2\hbar^2} \cdot \frac{Z^2}{n^2}$$

Once again, you can write a general formula for the total energy of an electron in the n^{th} level of a hydrogen atom ($Z = 1$) by substituting the correct values for the constants. You will discover that $E_n = -\frac{13.6\ \text{eV}}{n^2}$. The integer, n, is now known as the **principal quantum number**.

Recalling Bohr's hypothesis that the difference in the energy levels would be the energies of the photons emitted from an atom, you can now use this formula to compare Bohr's model of the atom with the observed frequencies of the spectral lines for hydrogen atoms. For example, you should be able to calculate the frequency of the first line in the Balmer series by doing the following.

$$hf = E_3 - E_2$$

$$hf = \frac{-13.6\ \text{eV}}{3^2} - \left(-\frac{13.6\ \text{eV}}{2^2}\right)$$

$$hf = -1.511\ \text{eV} + 3.40\ \text{eV}$$

$$hf = 1.89\ \text{eV}$$

$$f = \left(\frac{1.89\ \cancel{\text{eV}}}{h}\right)\left(\frac{1.6 \times 10^{-19}\ \text{J}}{\cancel{\text{eV}}}\right)$$

$$f = \frac{3.0222 \times 10^{-19}\ \cancel{\text{J}}}{6.63 \times 10^{-34}\ \cancel{\text{J}} \cdot \text{s}}$$

$$f = 4.56 \times 10^{14}\ \text{Hz}$$

This value is in excellent agreement with the observed frequency of the first line in the Balmer series. If you performed similar calculations for the other lines in the Balmer series, you would find the same excellent agreement with observations.

Spectroscopists continued to find series of lines that were matched with electrons falling from higher levels of the hydrogen atom down into the first five energy levels. These series are named and illustrated in Figure 12.20.

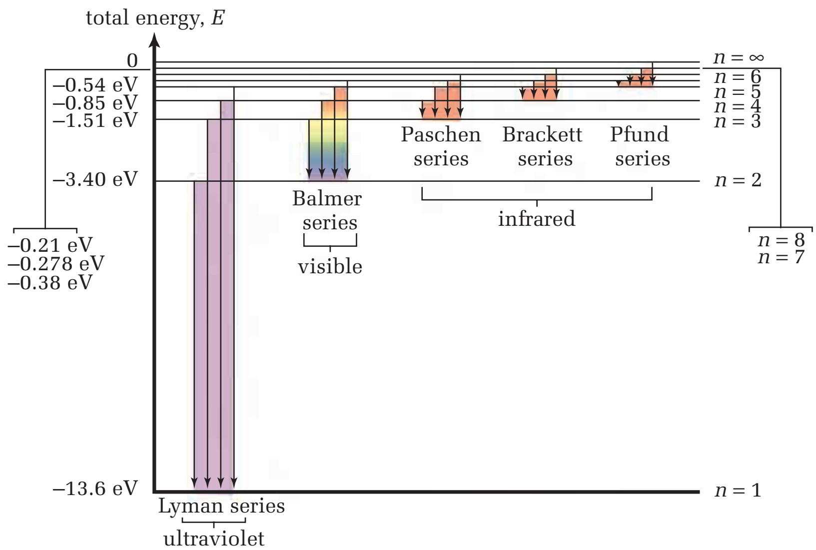


Figure 12.20 Photons from transitions that *end* at the same energy level have energies (and therefore frequencies) that are relatively close together. When inspecting a particular range of frequencies emitted by an element, therefore, an observer would find a set of spectral lines quite close together. Each set of lines is named after the person who observed and described them.

You could perform calculations such as the sample calculation of the frequency of the first line in the Balmer series for any combination of energy levels and find agreement with the corresponding line in the hydrogen spectrum. Bohr's model of the atom was thoroughly tested and was found to be in agreement with most of the data available at the time.

Conceptual Problems

- Start with the expression $hf = |E_f - E_i|$, then substitute the equation for the energy of the n^{th} level of an electron into E_f and E_i into the first expression. Finally, use the relationship $c = f\lambda$ to derive the following expression.

$$\frac{1}{\lambda} = \left| \frac{2\pi^2 k^2 e^4 m_e Z^2}{h^3 c} \left[\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2} \right] \right|$$

- The equation above would be identical to the Rydberg equation if the combination of constants $\frac{2\pi^2 k^2 e^4 m_e}{h^3 c}$ was equal to the Rydberg constant for hydrogen atoms ($Z = 1$). Calculate the value of the constants and compare your answer with the Rydberg constant. What does this result tell you about Bohr's model of the atom?

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Enhance your understanding of the Bohr atom, modelled as a wave or as a particle, by referring to your Electronic Learning Partner.

The Quantum Mechanical Atom

Bohr's model of the atom very successfully explained many of the confusing properties of the atom — it marked a monumental first step into the quantum nature of the atom. Nevertheless, the model was incomplete. For example, a very precise examination of the spectrum of hydrogen showed that what had at first appeared to be individual lines in the spectrum were actually several lines that were extremely close together. As illustrated in Figure 12.21, this "fine structure," as it is sometimes called, could best be explained if one or more energy levels was broken up into several very closely spaced energy levels.

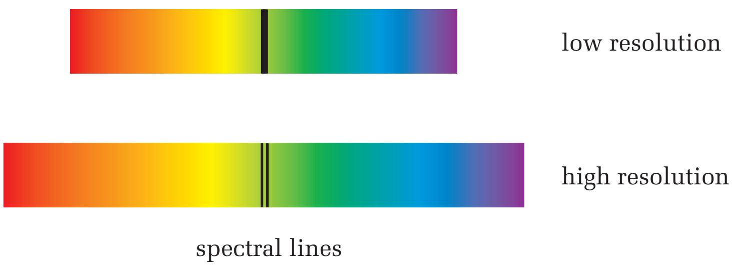

Figure 12.21 Very close examination of the lines in the hydrogen spectrum showed that some of the lines were made up of several fine lines that were very close together.

Another feature of emission spectra that the Bohr atom could not explain was observed in 1896 by Dutch physicist Pieter Zeeman (1865–1943). He placed a sodium flame in a strong magnetic field and then examined the emission spectrum of the flame with a very fine diffraction grating. He observed that the magnetic field caused certain spectral lines to "split" — what had been one line in the spectrum became two or more lines when a magnetic

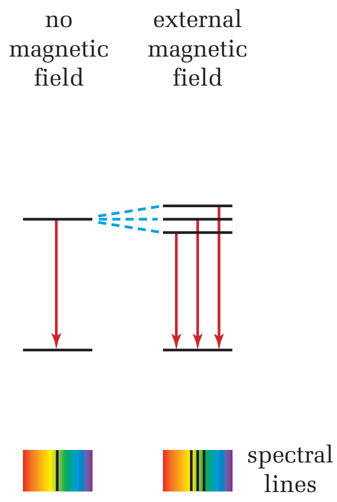

Figure 12.22 When a sample is placed in a magnetic field, some individual spectral lines become a set of closely spaced lines.

field was present. This phenomenon, illustrated in Figure 12.22, is now called the **Zeeman effect**.

Several physicists attempted with some success to modify the Bohr model to account for the fine structure and the Zeeman effect. The greatest success, however, came from an entirely different approach to modelling the atom: De Broglie's concept of matter waves paved the way to the new quantum mechanics or, as it is often called, "wave mechanics."

When de Broglie proposed his hypothesis about matter waves (about 10 years after Bohr had developed his model of the atom), he applied the ideas to the Bohr model. De Broglie suggested that when electrons were moving in circular orbits around the nucleus, the associated "pilot waves," as de Broglie named them, must form standing waves. Otherwise, destructive interference would eliminate the waves. To form a standing wave on a circular path, the length of the path would have to be an integral number of wavelengths, as shown in Figure 12.23. The procedure that follows the illustration will guide you through the first few steps of de Broglie's method for determining the radius of the orbit of the electron matter waves around the nucleus.

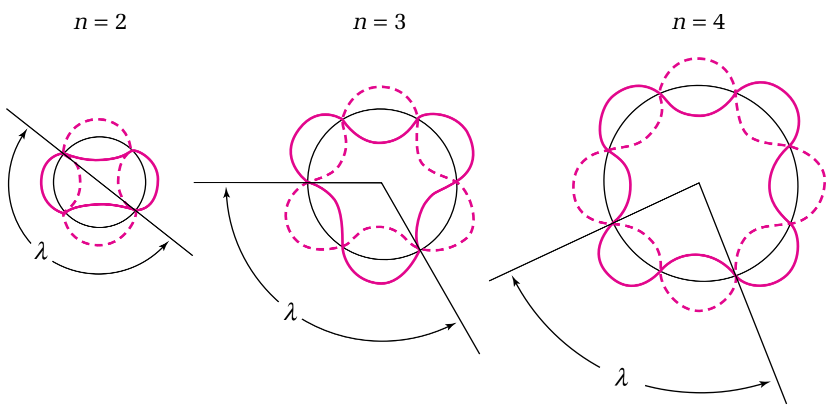

Figure 12.23 The number of wavelengths of matter waves that lie on the radius of an electron orbit is equal to the value of n for that energy level. No other wavelengths are allowed because they would interfere destructively with themselves.

- Write the formula for the circumference of a circle and set it equal to any integer (n) times the wavelength. $2\pi r_n = n\lambda$, where $n = 1, 2, 3, \ldots$
- Write de Broglie's formula for the wavelength of a matter wave. $\lambda = \dfrac{h}{mv}$
- Substitute de Broglie's wavelength of an electron into the first equation. $2\pi r_n = n\dfrac{h}{m_e v_n}$
- Divide both sides of the equation by 2π. $r_n = \dfrac{nh}{2\pi m_e v_n}$
- Multiply both sides of the equation by $m_e v_n$ $m_e v_n r_n = \dfrac{nh}{2\pi}$

Notice that the last equation is the same as Bohr's expression for the quantization of angular momentum. From this point on, the derivation of the equation for the radius of allowed orbits would be exactly the same as Bohr's derivation. Using two entirely different approaches to the quantization of electron orbits, Bohr's and de Broglie's results were identical.

In 1925, Viennese physicist Erwin Schrödinger (1887–1961) read de Broglie's thesis with fascination. Within a matter of weeks, Schrödinger had developed a very complex mathematical equation that can be solved to produce detailed information about matter waves and the atom. The now-famous equation, called the **Schrödinger wave equation**, forms the foundation of quantum mechanics. When you insert data describing the potential energy of an electron or electrons in an atom into the wave equation and solve the equation, you obtain mathematical expressions called "wave functions." These **wave functions**, represented by the Greek letter ψ (psi), provide information about the allowed orbits and energy levels of electrons in the atom.

Wave functions account for most of the details of the hydrogen spectra that the original Bohr model could not explain. However, Schrödinger's wave functions could not predict one small, magnetic "splitting" of energy levels. British physicist Paul Adrien Maurice Dirac (1902–1984) realized that electrons travelling in the lower orbits in an atom would be travelling at excessively high speeds, high enough to exhibit relativistic effects. In 1928, Dirac modified Schrödinger's equation to account for relativistic effects. The equation could then account for all observed properties of electrons in atomic orbits. In addition, it predicted many phenomena that had not yet been discovered when the equation was developed.

You are probably wondering, "What are wave functions and what do the amplitude and velocity of a matter wave describe?" Many physicists in the early 1900s asked the same question.

Wave functions do not describe such properties as the changing pressure of air in a sound wave or the changing electric field strength in an electromagnetic wave. In fact, wave functions cannot describe any real property, because they contain the imaginary number i ($i = \sqrt{-1}$, which does not exist). You must carry out a mathematical operation on the wave functions to eliminate the imaginary number in order to describe anything real about the atom.

The result of this operation, symbolized $\psi^*\psi$, represents the probability that the electron will occupy a certain position in the atom at a certain time. You could call the wave function a "wave of probability." You can no longer think of the electron as a solid particle that is moving in a specific path around the nucleus of an atom, but rather must try to envision a cloud such as the one shown in Figure 12.24 (A) and interpret the density of the cloud as the probability that the electron is in that location. These "regions

COURSE CHALLENGE

Waves and Particles

The process of science continues to discover truths about the nature of our universe. How far have we really come? How well are we able to describe our world? Refer to page 605 for ideas to help you incorporate philosophical debate into your *Course Challenge*.

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Solutions to Dirac's modification of the Schrödinger wave equation predicted twice as many particles as were known to exist in the systems for which the equation was defined. Dirac realized that one stage in the solution contained a square root that yielded both positive and negative values. For example, $\sqrt{16y^4} = \pm 4y^2$. You have probably solved problems involving projectile motion or some other form of motion and found both positive and negative values for time. You simply said that a negative time had no meaning and you chose the positive value. Dirac tried this approach, but it changed the final results. Dirac's original results seemed erroneous because they predicted the existence of antiparticles, which had not yet been discovered. Soon after, antiparticles were observed experimentally by other scientists. You will learn about antiparticles in Chapter 13, The Nucleus and Elementary Particles.

in space" occupied by an electron are often called **orbitals**. If the orbital of the electron is pictured as solid in appearance, as shown in Figure 12.24 (B), it means that there is a 95% probability that the electron is within the enclosed space.

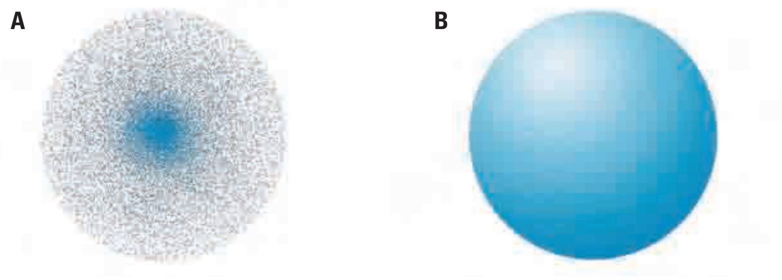


Figure 12.24 When you plot $\psi^*\psi$, you obtain orbitals such as these. **(A)** Orbitals drawn in this manner show the probability of finding the electron. **(B)** Often orbitals are drawn with solid outlines. The probability that the electron is within the enclosed space is 95%.

You might also wonder if the Bohr model was wrong and should be discarded. The answer to that question is a resounding no. The wave functions — that is, the solutions to the Schrödinger wave equation — give the same energy levels and the same principal quantum number (n) that the Bohr model gave. Also, the distance from the nucleus for which the probability of finding the electron is greatest is exactly the same as the Bohr radius. These results show that the general features of the Bohr model are correct and that it is a very useful model for general properties of the atom. The wave equation is necessary only in the finer details of structure.

ELECTRONIC LEARNING PARTNER

Your Electronic Learning Partner contains an excellent reference source of emission and absorption spectra for every element in the periodic table.

Quantum Numbers

The wave functions obtained from Schrödinger's wave equation include two more quantum numbers in addition to the principal quantum number, n. Dirac's relativistic modification of the Schrödinger equation adds another quantum number, making a total of four quantum numbers that specify the characteristics of each electron in an atom. Each quantum number represents one property of the electron that is quantized.

The principal quantum number, n, represents exactly the same property of the atom in both the Bohr model and the Schrödinger model and specifies the energy level of the electron. The value of n can be any positive integer: 1, 2, 3, 4, … . These energy levels are sometimes referred to as "shells."

The **orbital quantum number**, ℓ, specifies the shape of the orbital. The value of ℓ can be any non-negative integer less than n. For example, when $n = 1$, $\ell = 0$. When $n = 2$, ℓ can be 0 or 1. In chemistry, orbitals with different values of ℓ (0, 1, 2, 3, ...) are assigned the letters *s*, *p*, *d*, *f*, … . Figure 12.25 shows the shapes of orbitals for the first three values of ℓ.

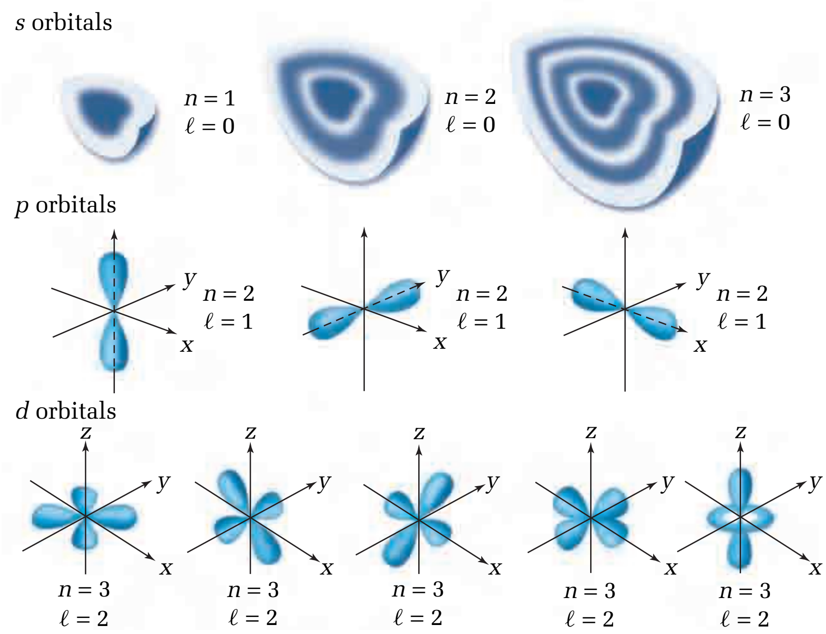


Figure 12.25 Orbitals for which $\ell = 0$ (*s* orbitals) are always spherical. When $\ell = 1$ (*p* orbitals), each orbital has two lobes. Four of the $\ell = 2$ (*d*) orbitals have four lobes and the fifth $\ell = 2$ orbital has two lobes plus a disk.

The orbital quantum number is sometimes called the "angular momentum quantum number," because it determines the angular momentum of the electron. If an electron was to move along a curved path, it would have angular momentum. Although it is not accurate to think of the electron as a tiny, solid piece of matter orbiting around the nucleus, some properties of the electron clouds of orbitals with ℓ greater than zero give angular momentum to the electron cloud. Electrons that have the same value of n but have different values of ℓ possess slightly different energies. As illustrated in Figure 12.26, these closely spaced energy levels account for the fine structure in an emission spectrum of the element.

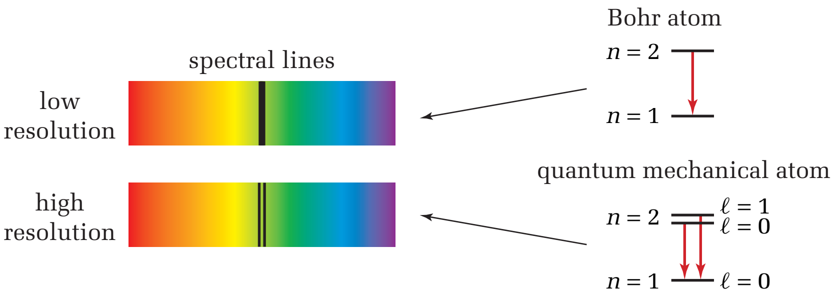


Figure 12.26 For any energy level (shell) for which $n > 0$, there is more than one value of the orbital quantum number, ℓ. The orbitals for each value of ℓ have slightly different energies. These closely spaced energies account for the fine structure, that is, the presence of more than one spectral line very close together.

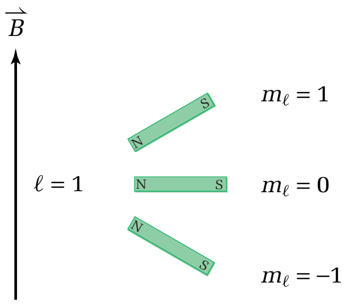


Figure 12.27 In the absence of an external magnetic field, the ℓ orbitals can take any random orientation in space. When a sample is placed in a magnetic field, the ℓ orbitals take on specific orientations in relation to the external field.

The **magnetic quantum number**, m_ℓ, determines the orientation of the orbitals when the atom is placed in an external magnetic field. To develop a sense of what this quantum number means, it is once again helpful, although not entirely accurate, to think of the electron in its cloud as an electric current flowing around the nucleus. As you know, a current flowing in a loop creates a magnetic field. The magnetic quantum number determines how this internal field is oriented if the atom is placed in an external magnetic field. In Figure 12.27, the electron's magnetic field is represented by a small bar magnet with different orientations in an external magnetic field.

The **spin quantum number**, m_s, results from the relativistic form of the wave equation. The term "spin" is used because the effect is the same as it would be if the electron was a spherical charged object that was spinning. A spinning charge creates its own magnetic field in much the same way that a circular current does. The value of m_s can be only $+\frac{1}{2}$ or $-\frac{1}{2}$. Similar to the magnetic quantum number, the spin quantum number has an effect on the energy of the electron only when the atom is placed in an external magnetic field. The two orientations in the external magnetic field are often called "spin up" and "spin down." Figure 12.28 illustrates the two possible orientations of the electron spin and its effect on the electron's energy and spectrum in an external magnetic field.

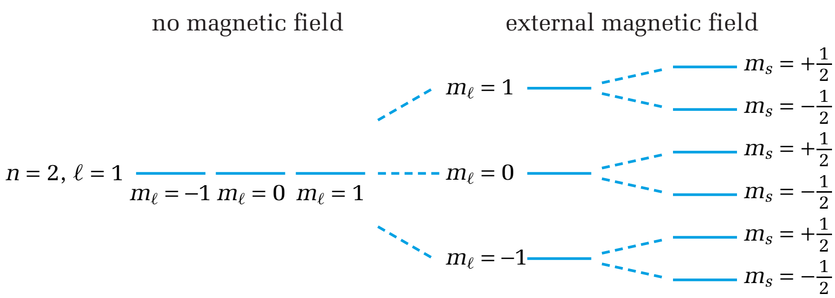
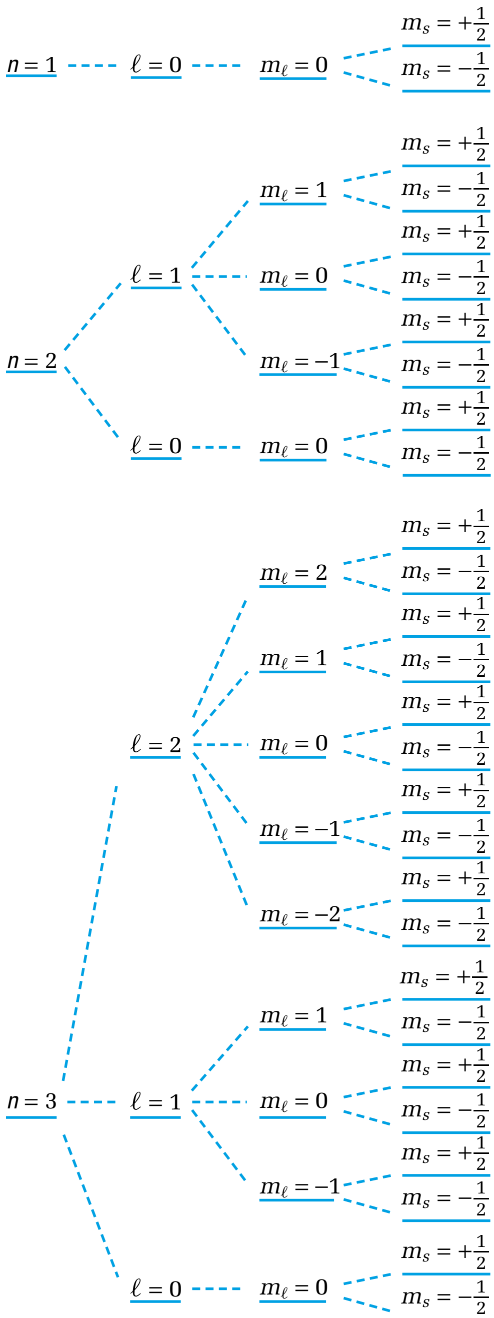


Figure 12.28 The electron spin can assume any orientation in the absence of an external magnetic field, but can take only two orientations when placed in a magnetic field — spin up or spin down.

These four quantum numbers and the associated wave functions can explain and predict essentially all of the observed characteristics of atoms. Two questions might arise, however: If almost all of the mass of an atom is confined to a very tiny nucleus and electrons, with very little mass, are in "clouds" that are enormous compared to the nucleus, why does matter seem so "solid"? Why cannot atoms be compressed into much smaller volumes?

Austrian physicist Wolfgang Pauli (1900–1958) answered those questions in 1925. According to the **Pauli exclusion principle**, *no two electrons in the same atom can occupy the same state.* An easier way of saying the same thing is that *no two electrons in the same atom can have the same four quantum numbers.* Electron clouds of atoms cannot overlap.

The Pauli exclusion principle also tells us how many electrons can fit into each energy level of an atom. The tree diagrams in Figure 12.29 show how many electrons can fit into the first three energy levels, $n = 1$, $n = 2$, and $n = 3$.



State				Total number of states
n	ℓ	m_ℓ	m_s	
1	0	0	$+\frac{1}{2}$	2
1	0	0	$-\frac{1}{2}$	
2	1	1	$+\frac{1}{2}$	
2	1	1	$-\frac{1}{2}$	
2	1	0	$+\frac{1}{2}$	
2	1	0	$-\frac{1}{2}$	8
2	1	−1	$+\frac{1}{2}$	
2	1	−1	$-\frac{1}{2}$	
2	0	0	$+\frac{1}{2}$	
2	0	0	$-\frac{1}{2}$	
3	2	2	$+\frac{1}{2}$	
3	2	2	$-\frac{1}{2}$	
3	2	1	$+\frac{1}{2}$	
3	2	1	$-\frac{1}{2}$	
3	2	0	$+\frac{1}{2}$	
3	2	0	$-\frac{1}{2}$	
3	2	−1	$+\frac{1}{2}$	
3	2	−1	$-\frac{1}{2}$	
3	2	−2	$+\frac{1}{2}$	18
3	2	−2	$-\frac{1}{2}$	
3	1	1	$+\frac{1}{2}$	
3	1	1	$-\frac{1}{2}$	
3	1	0	$+\frac{1}{2}$	
3	1	0	$-\frac{1}{2}$	
3	1	−1	$+\frac{1}{2}$	
3	1	−1	$-\frac{1}{2}$	
3	0	0	$+\frac{1}{2}$	
3	0	0	$-\frac{1}{2}$	

Figure 12.29 These tree diagrams show the energy levels, both in the absence and the presence of an external magnetic field, for the first three values of the principal quantum number, *n*.

Conceptual Problem

- Study Figure 12.29 and then draw a tree diagram for the next energy level, $n = 4$. How many electrons will fit into the fourth energy level?

Hydrogen has only one proton in the nucleus, and thus one electron in an orbital. When a hydrogen atom is not excited, the electron is in the $n = 1$, $\ell = 0$ energy level. However, the atom can absorb energy and become excited and the electron can "jump" up to any allowed orbital. All elements other than hydrogen have more than one electron. When atoms are not excited, the electrons are in the lowest possible energy levels that do not conflict with the Pauli exclusion principle. Figure 12.30 gives examples of three different elements with their electrons in the lowest possible energy levels. This condition is called the **ground state** of the atom. Similar to the electron in hydrogen, the electrons of other elements can absorb energy and rise to higher energy levels.

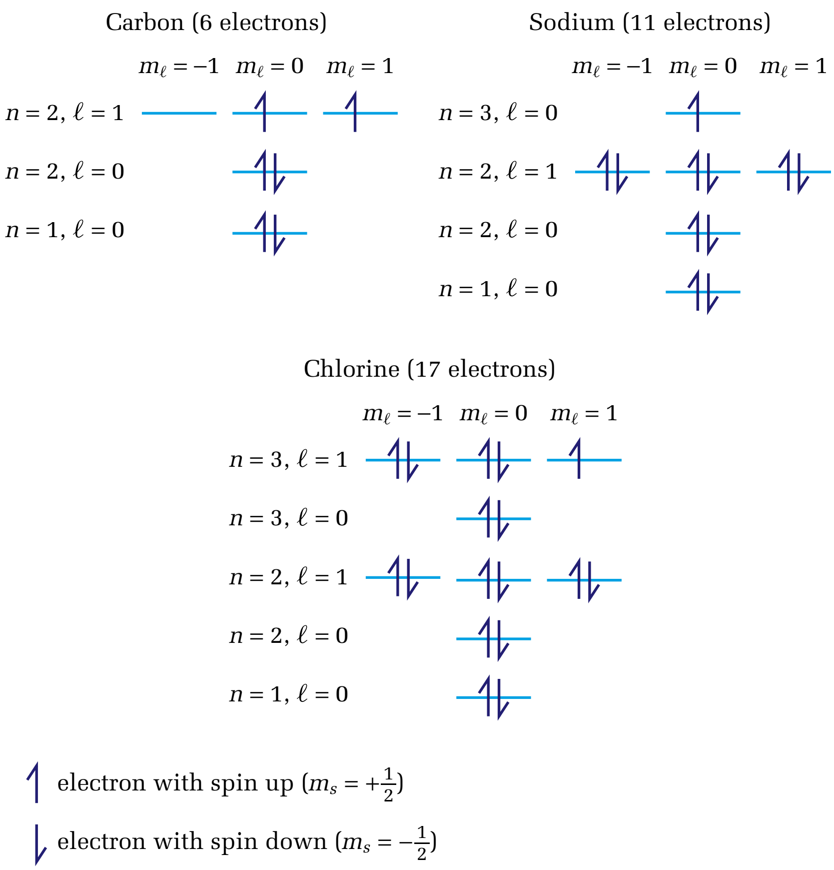


Figure 12.30 Electrons "fill" the energy levels from the lowest upward until there are as many electrons in orbitals as there are protons in the nucleus.

INVESTIGATION 12-B

Identifying Elements by Their Emission Spectra

TARGET SKILLS
- Predicting
- Performing and recording
- Analyzing and interpreting
- Communicating results

The emission spectra of atomic hydrogen gas obtained using gas discharge tubes provided Bohr with critical information that helped him to develop his model of the atom. These spectra also gave him experimental data with which to compare predictions based on his model. In this investigation, you will identify gases from observation of their emission spectra.

Problem

Identify gases from observation of their emission spectra.

Equipment

- hand-held spectroscope
- lighted incandescent bulb
- gas discharge tubes

Procedure

1. Practise using the spectroscope by observing a small incandescent light bulb. Point the slit of the spectroscope toward the bulb and move the spectroscope until you can clearly see the spectrum.
2. Record the appearance of the spectrum from the incandescent bulb.
3. Several numbered gas discharge tubes will be assembled and ready to view. Observe each tube with the spectroscope.

CAUTION A very high voltage is required to operate the gas discharge tubes. Do not come into contact with the source while viewing the tubes.

4. Make a sketch of each spectrum. Draw the relative distances between the lines as accurately as possible. Label each of the lines in each sketch with colour and wavelength to two significant figures.
5. Observe a fluorescent bulb with the spectroscope.
6. Record the appearance of the spectrum from the fluorescent bulb.

Analyze and Conclude

1. In a phrase, describe the spectrum of the incandescent bulb. Explain why the incandescent bulb emits the type of spectrum that you described.
2. Your teacher will provide you with spectra of a variety of types of gases. Compare your sketches with the spectra and attempt to identify each gas in the discharge tubes.
3. Compare your observations of the fluorescent bulb with the spectra from both the incandescent bulb and the gas discharge tubes. Which type of spectrum does the spectrum from the fluorescent bulb most resemble?
4. A fluorescent bulb is a type of gas discharge tube. However, the emissions of the gas are absorbed by a coating on the inside of the bulb and the atoms in the coating are excited and emit light. Based on this description, explain the features of the spectrum of the fluorescent bulb.
5. Is it possible to identify the gas in the fluorescent bulb? Explain why or why not.

Apply and Extend

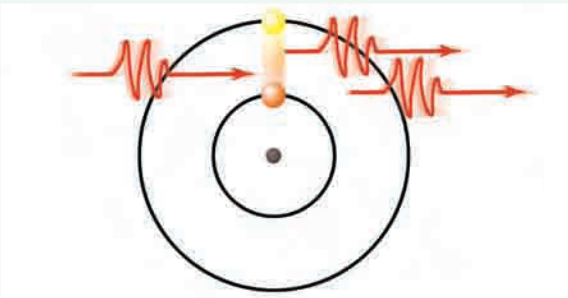
6. Select one of the central lines in the spectrum of atomic hydrogen. Predict which transition (from which energy level to which energy level) created this line.
7. Check your prediction by using Balmer's formula to calculate the wavelength that the transition would have caused. Compare the calculated wavelength with the wavelength of the spectral line that you selected.

PHYSICS & TECHNOLOGY

TARGET SKILLS
- Hypothesizing
- Analyzing and interpreting

Atoms and Lasers

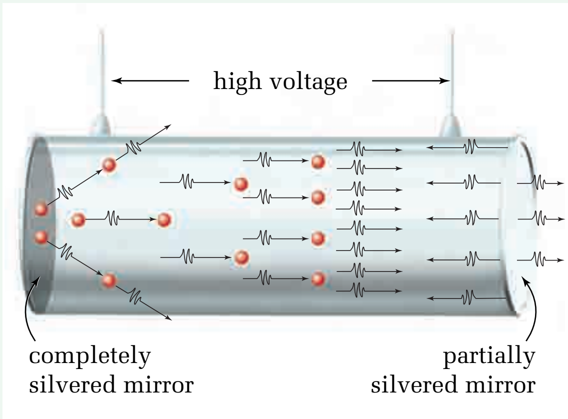
A thorough understanding of the energy levels of electrons in atoms and of transitions between these states was necessary before anyone could even imagine that a laser could be developed. Another critical property of electrons that was necessary in order to develop lasers was predicted by Einstein in 1917 — the stimulation of emission of a photon. As shown in the diagram, if an electron is in an excited state (that is, in a higher energy level), a photon with an energy level equal to the difference in allowed energy levels will stimulate the electron to drop to the lower energy level and emit another identical photon. In addition, the two photons are perfectly in phase.



If more electrons exist in the excited state than in the ground state, it is more probable that a photon will stimulate an emission instead of being absorbed. Two conditions are necessary in order to create and maintain this condition. Normally, most electrons are in the ground state at room temperature, so a stimulus is needed to excite the electrons. This stimulus can be provided by a high voltage that will accelerate free electrons, and then collisions with atoms will excite their electrons. The process is called "optical pumping."

If the excited electrons spend a longer than normal time in the excited state, stimulated emission will be more probable than spontaneous emission. This condition is met by selecting atoms of elements that have specific energy levels called the "metastable state." Electrons remain in metastable states for about 10^{-3} s, rather than the normal 10^{-8} s.

A typical gas laser tube is shown in the diagram. A high voltage excites the electrons in the gas, maintaining more atoms in an excited state than the ground state. As some photons are emitted spontaneously, they stimulate the emission of other photons. The ends of the laser tube are silvered to reflect the photons. This reflection causes more photons to stimulate the emission of a very large number of photons. Any photons that are not travelling parallel to the sides of the tube exit the tube and do not contribute to the beam. One end of the tube is only partially reflecting, and a fraction of the photons escape. These escaping photons have the same wavelength and frequency and are all in phase, creating a beam of what is called "coherent light."



Analyze

1. The word "laser" is an acronym for "light amplification by stimulated emission of radiation." Explain the significance of each term in the name.

2. Laser beams remain small and do not spread out, as does light from other sources. Based on the unique characteristics of laser light, try to explain why the beams do not spread out.

3. List as many applications of laser as you can.

The years from 1900 to 1930 were exciting ones in physics. No longer could physicists speak of waves and particles as separate entities — the boundary between the two became blurred. The long-standing Dalton model of the atom gave way to the Thomson model, which was soon usurped by the Rutherford model and, soon thereafter, by the Bohr model. Eventually, all models that represented electrons as discrete particles yielded to the quantum mechanical model described by the Schrödinger wave equation.

Today, the wave equation is still considered to be the most acceptable model. In fact, physicists have been able to show that the wave equation can give information about the nucleus and particles that was not known to exist when Schrödinger presented his equation. In the next chapter, you will learn about properties of the nucleus and particles that exist for time intervals as small as 10^{-20} s.

12.3 Section Review

1. C Discuss the similarities and differences between Dalton's model of the atom and J.J. Thomson's model of the atom.
2. K/U What surprising observation did Rutherford and Geiger make that motivated Rutherford to define a totally new model of the atom?
3. K/U In what way did Rutherford's nuclear model of the atom conflict with classical theory?
4. C Explain how experimentally observed spectra of atomic hydrogen helped Bohr develop his model of the atom.
5. K/U According to Bohr's model of the atom, what property of electrons in atoms must be quantized?
6. K/U List the four postulates on which Bohr based his model of the atom.
7. C Explain how Coulomb's law played a role in the determination of the Bohr radius.
8. C Describe the two features of the emission spectrum of atomic hydrogen that revealed a flaw in Bohr's model of the atom.
9. K/U How did Dirac improve Schrödinger's wave equation?
10. K/U What is a wave function and what type of information does a wave function provide about atoms?
11. K/U List and define the four quantum numbers.
12. C Balmer's work on the spectrum of hydrogen helped Bohr to modify Rutherford's model of the atom. Explain how he did this.
13. K/U Write down Rydberg's modification of Balmer's formula and define the terms.
14. K/U What can cause an electron in the Bohr model to "jump" to a higher energy level?
15. K/U Explain the term "principal quantum number."

UNIT PROJECT PREP

Inquisitive minds following unexpected results often lead to advances in our scientific understanding of the universe.

- Do you believe, and can you support, the idea that unexpected experimental results have contributed more to scientific discovery than any other means?
- Which theory, special relativity or quantum mechanics, was received with more skepticism by the general public of the time? Suggest reasons.

CHAPTER 12 Review

REFLECTING ON CHAPTER 12

- Oscillators on the surface of a blackbody can oscillate only with specific frequencies. When they emit electromagnetic radiation, they drop from one allowed frequency to a lower allowed frequency.
- The photoelectric effect demonstrated that electromagnetic energy can be absorbed only in discrete quanta of energy. Electromagnetic energy travels like a wave, but interacts with matter like a particle.
- When a photon ejects an electron from a metal surface, the maximum kinetic energy of the electron can be calculated from the equation $E_{k(max)} = hf - W$, where W is the work function of the metal.
- The Compton effect shows that both energy and momentum are conserved when a quantum of light energy, or a photon, collides with a free electron.
- The energy of a photon is $E = hf$.
- The momentum of a photon is $p = \frac{h}{\lambda}$.
- The diffraction of electrons by crystals demonstrated that electrons have wave properties. The wavelength of a particle of matter is $\lambda = \frac{h}{mv}$.
- Physicists accept the dual properties of matter and electromagnetic energy. Electromagnetic energy behaves like particles and particles of matter have wave properties. These concepts are called the "wave-particle duality."
- Dalton believed that atoms were the smallest, indivisible particles in nature. J.J. Thomson demonstrated that electrons could be removed from atoms and, therefore, that atoms were made up of smaller particles.
- By observing the scattering of alpha particles by a thin gold foil, Rutherford demonstrated that the positive charge in an atom must be condensed into an extremely small area at the centre of the atom.
- Bohr proposed that electrons in atoms could exist only in specific allowed energy levels. Electrons in these energy levels are in orbits with specific allowed radii.
- The energies of the photons in the observed spectra of atomic hydrogen have amounts of energy that are exactly equal to the difference in Bohr's allowed energy levels. This fact supports Bohr's concept that electrons can drop from a high energy level to a lower level by emitting a photon.
- Detailed inspection of emission spectra of gases showed that some of the spectral lines are actually made up of two or more lines that are very close together. Also, when placed in an external magnetic field, some single spectral lines split into two or more lines. These data show that Bohr's model of the atom is incomplete.
- Schrödinger's wave equation forms the foundation of quantum mechanics, or wave mechanics. Solutions to the wave equation, called "wave functions," provide information about the properties of electrons in an atom. The operation, $\psi^*\psi$ on the wave function gives the probability that an electron will be found at a specific point in space.
- Dirac modified Schrödinger's wave equation to account for relativistic effects of electrons in atoms travelling close to the speed of light. Wave functions obtained by solving this wave equation contain four quantum numbers. Each quantum number describes one property of electrons that is quantized.
- The Pauli exclusion principle states that no two electrons in the same atom can have the same four quantum numbers.

Knowledge/Understanding

1. Describe how a negatively charged electroscope can be used to provide evidence for the photoelectric effect.
2. (a) Describe the properties of a blackbody and explain how it is simulated in the laboratory.
   (b) How did the actual radiation spectrum emitted by a heated blackbody differ from the predictions of the classical wave theory?
3. The ultraviolet catastrophe was considered to be a flaw in the explanation of the blackbody emission spectra by the classical wave theory. In what way was it unexplained?
4. (a) The results of Lenard's photoelectric experiment partly correlated with the classical wave theory of light. Explain how it agreed.
   (b) In what way did Lenard's results differ from the predictions of the classical wave theory of light?
5. (a) Einstein saw a connection between the photoelectric effect and the Planck proposal that energy be quantized. Explain how Einstein developed an equation to describe the photoelectric effect.
   (b) Einstein's photoelectric equation is actually another example of conservation of energy. Explain how this applies.
6. A lithium surface in a photoelectric cell will emit electrons when the incident light is blue. Platinum, however, requires ultraviolet light to eject electrons from its surface.
   (a) Which of the two metals has a larger value for its work function? Explain your answer.
   (b) Which of the two metals has a higher threshold frequency? Explain your answer.
7. (a) How did a knowledge of the charge on an electron make it possible to calculate the numerical values of the kinetic energies of electrons emitted from a metal surface?
   (b) How did the data from Millikan's photoelectric experiments support Einstein's theory of the photoelectric effect?
8. (a) Explain the sequence by which Compton derived an expression for the momentum of a photon, considering that it has no mass.
   (b) In what way does a photon change "colour" after it has collided with an electron. Is "colour" always a suitable term to use?
9. Based on his premise regarding the momentum of a photon, Compton showed that momentum was conserved in collisions between photons and electrons. As a result, what can be concluded from this experiment?
10. (a) Explain the sequence that de Broglie used in taking Compton's expression for the momentum of a light photon and proposing that particles of matter have a corresponding matter wave and wavelength.
    (b) How can the matter wavelength of a particle be increased to make it more easily detectable?
    (c) Compare (through calculations) the de Broglie wavelength of an electron of mass 9.11×10^{-31} kg, travelling at 3.60 km/h, with that of a hockey puck of mass 0.15 kg, travelling at the same speed.
11. (a) What property of electromagnetic radiation represented a flaw in the Rutherford model of the atom?
    (b) Balmer's equation represents an "empirical expression." What is the significance of this term?
12. In what way did Rydberg modify Balmer's equation?
13. (a) Describe the key features of the Bohr model of the atom, and indicate how this model contradicts classical theory.
    (b) When electrons occupy a higher energy level, what are they likely to do? What options do they have?
14. (a) Write the equation linking the energy of a photon emitted from the Bohr atom to the energy levels of the atom.
    (b) How does this manifest itself in the emission spectrum of an atom?
15. (a) According to Bohr, what property of the electron in its orbit is quantized?

(b) In general terms, explain how Bohr used the equations for Coulomb's law, circular motion, and angular momentum to determine the "Bohr radius."

16. (a) Describe how Bohr used the equations for kinetic energy, Coulomb's law, and the Bohr radius to determine the general formula for the total energy of an electron in a hydrogen atom.

(b) Explain how the Bohr equation for the total energy of an electron in orbit in a hydrogen atom relates to the observed emission spectrum.

17. (a) Show that the speed of an electron as it moves in an "allowed" orbit can be represented by the equation

$$v_{\mathrm{n}} = \frac{2\pi ke^2}{nh}$$

(b) Calculate the de Broglie wavelength associated with an electron in the first orbit of the Bohr atom.

18. Schrödinger responded to de Broglie's thesis by developing the Schrödinger wave equation. In what general way did this equation account for the discrepancies in the emission spectra?

19. (a) Do you feel Schrödinger's wave equation is just an abstract model, or is there a "real" significance?

(b) Discuss whether you believe that the Schrödinger wave functions made the Bohr model obsolete.

Inquiry

20. Schrödinger's wave equation was a famous contribution to what is now called "quantum mechanics." At your library or through the Internet, find and record the exact text of the equation and describe, in general terms, the mathematical operations it incorporated.

21. (a) Research and describe the principle of complementarity.

(b) Under what conditions does light tend to show its wave properties, and under what conditions does the particle (photon) nature of light predominate?

22. Design and sketch a simple door opener that will open electrically when a person passes through a light beam.

Communication

23. Explain how Lenard was able to determine the maximum kinetic energy of the electrons coming from the emitter of his photoelectric apparatus.

24. (a) Explain how Einstein was able to include the properties of different types of emitter metals in his photoelectric equation.

(b) Initially, very few physicists accepted Einstein's claim for the quantum nature of light. Why did this opposition exist?

25. Briefly outline the key features of the model of the atom proposed by John Dalton, J.J. Thomson, and Ernest Rutherford.

26. Based on the results of the scattering experiments, Rutherford was led to believe that the atom was mainly empty space with a small charged core. Explain why he deduced this.

27. Some science fiction writers use a large sail to enable a space vehicle to move through space. It is argued that sunlight will exert a pressure on the sail, causing it to move away from the Sun. Prepare a report and/or display in which you indicate

(a) whether the proposal has merit

(b) what type of surface should be used for the sail

28. "Wave-particle duality" is a term used to describe the dual properties of both light and particles in motion. Has this meant that Maxwell's equations for electromagnetic wave propagation and Newton's classical mechanics have been discarded? Discuss your opinion.

Making Connections

29. A light meter is used by a photographer to ensure correct exposure for photographs. If the photocell in the meter is to operate satisfactorily up to the red light wavelength of 650 nm, what should be the work function of the emitter material?

30. Some television picture tubes emit electrons from the rear cathode and accelerate them forward through an electric potential difference of 15 000 V.
    (a) What is the de Broglie wavelength of the electron just before it hits the screen?
    (b) Discuss whether you think diffraction of the electron beam might pose a problem with the resulting picture.
31. Prepare a report on laser technology, including reference to the terms "spontaneous emission," "stimulated emission," "population inversion," and "metastable." In your report, include a reference to the medical applications of lasers.

Problems for Understanding

32. (a) The work function for a nickel surface is 5.15 eV. What is the minimum frequency of the radiation that will just eject an electron from the surface?
    (b) What is the general name given to this minimum frequency?
33. (a) The longest wavelength of light that will just eject electrons from a particular surface is 428.7 nm. What is the work function of this surface?
    (b) Use Table 12.1 to identify the material used in the surface.
34. When ultraviolet radiation was used to eject electrons from a lead surface, the maximum kinetic energy of the electrons emitted was 2.0 eV. What was the frequency of the radiation used?
35. The electrons emitted from a surface illuminated by light of wavelength 460 nm have a maximum speed of 4.2×10^5 m/s. Given that an electron has a mass of 9.11×10^{-31} kg, calculate the work function (in eV) of the surface material.
36. Assume that a particular 40.0 W light bulb emits only monochromatic light of wavelength 582 nm. If the light bulb is 5.0% efficient in converting electric energy into light, how many photons per second leave the light bulb?
37. (a) Calculate the momentum of a photon of light with a wavelength of 560 nm.
    (b) Calculate the momentum of the photons of light with a frequency of 6.0×10^{14} Hz.
    (c) A photon has an energy of 186 eV. What is its momentum?
38. An electron is moving at a speed of 4.2×10^5 m/s. What is the frequency of a photon that has an identical momentum?
39. What is the momentum of a microwave photon if the average wavelength of the microwaves is approximately 12 cm?
40. A particle has a de Broglie wavelength of 6.8×10^{-14} m. Calculate the mass of the particle if it is travelling at a speed of 1.4×10^6 m/s.
41. (a) Calculate the wavelength of a 4.0 eV photon.
    (b) What is the de Broglie wavelength of a 4.0 eV electron?
    (c) What is the momentum of an electron if its de Broglie wavelength is 1.4×10^{-10} m?
42. (a) Calculate the radius of the third orbit of an electron in the hydrogen atom.
    (b) What is the energy level of the electron in the above orbit?
43. Calculate the wavelength of the second line in the Balmer series.
44. A photon of light is absorbed by a hydrogen atom in which the electron is already in the second energy level. The electron is lifted to the fifth energy level.
    (a) What was the frequency of the absorbed photon?
    (b) What was its wavelength?
    (c) What is the total energy of the electron in the fifth energy level?
    (d) Calculate the radius of the orbit representing the fifth energy orbit.
    (e) If the electron subsequently returns to the first energy level in one "jump," calculate the wavelength of the corresponding photon to be emitted.

CHAPTER 13 The Nucleus and Elementary Particles

CHAPTER CONTENTS

PREREQUISITE CONCEPTS AND SKILLS

- Electric force and Coulomb's law
- Equivalence of mass and energy
- Conservation of mass-energy
- Potential and kinetic energy

Twentieth-century physics was ruled to a large extent by a quest for ultra-high energies. It might seem strange that such energies are required in order to investigate the tiniest, most subtle particles of matter. However, probing inside the nucleus and then inside the particles of that nucleus requires instruments capable of accelerating electrons and protons into the mega-electron volt (10^6 eV) and giga-electron volt (10^9 eV) ranges of kinetic energy.

Such instruments, called "particle accelerators," are located in university and government laboratories around the world. The above photograph shows an accelerator used at Conseil Européen pour la Recherche Nucléaire (CERN) located near Geneva, Switzerland. Along with other high-energy physics laboratories, CERN searches for new particles formed during energetic collisions between subatomic particles.

This chapter begins with the structure and properties of the nucleus and then examines the field of elementary particles, such as the ones created at CERN.

MULTI LAB

Radioactive Decay

TARGET SKILLS
- Performing and recording
- Analyzing and interpreting

Penetrating Ability of Radioactive Emissions

Obtain an end-window Geiger counter, sources of beta and gamma radiation, and shielding materials such as sheets of lead and cardboard.

CAUTION Handle the sources using tongs.

Position the tube of the Geiger counter so that either source (beta or gamma) can be placed close to the end window and so that sheets of cardboard or lead can be placed between the source and the Geiger tube. Turn on the Geiger counter and slide the beta source under it. Note the reading on the counter. Insert a sheet of cardboard between the beta source and the tube and take the reading again. Continue adding sheets until the reading is close to zero. Determine the thickness of an individual sheet and calculate the thickness of cardboard between the tube and the beta source for each radiation reading.

Repeat the process with the gamma source. If the cardboard does not provide much change to the reading, add sheets of lead instead. Continue adding sheets until the reading is close to zero (or as close as you can get). Determine the thickness of an individual sheet and calculate the total thickness of the barrier.

Analyze and Conclude

1. Plot a graph of the radioactivity reading (y-axis) against the thickness of cardboard for the beta source.
2. Plot a graph of the radioactivity reading (y-axis) against the thickness of cardboard or lead for the gamma source.
3. Which type of radioactive emission is the more penetrating?
4. Try to determine from the graphs the thickness of material that would reduce the reading to half of the unshielded value.

Half-Life

Perform this investigation as a class activity. Start with each member of the class holding a coin heads-up. Count and record the number of heads. Next, everyone should flip their coins. Count and record the number of heads. One minute later, each person who got heads on the first flip should flip again. Those who got tails are "out." Again, count and record the number of heads. Repeat the process every minute until only one or two heads remain.

Analyze and Conclude

1. Draw a graph of the number of heads remaining versus the number of flips.
2. Draw a graph of the number of heads that changed to tails versus the number of flips.
3. What are the chances that a coin will change from head to tail during a flip?
4. Why would a minute be known as the "half-life" of the coin?
5. The number of changes during each flip represents the activity. How does the activity change as time passes?
6. If you start with 160 heads, how many flips should you expect it to take to reduce the number of heads to 5? Explain your reasoning.

13.1 Structure of the Nucleus

SECTION EXPECTATIONS

- Define and describe the concepts and units related to the present-day understanding of the nature of elementary particles (e.g., mass-energy equivalence).
- Apply quantitatively the laws of conservation of mass and energy, using Einstein's mass-energy equivalence.

KEY TERMS

- proton
- neutron
- nucleon
- chemical symbol
- atomic number
- atomic mass number
- nucleon number
- strong nuclear force
- nuclide
- isotope
- mass defect
- atomic mass unit

Excitement was high in the scientific community in the early 1900s when Ernest Rutherford (1871–1937) proposed his model of the nucleus and Niels Bohr (1885–1962) developed a model of the atom that explained the spectrum of hydrogen. As is the case with many scientific breakthroughs, however, the answers to a few questions gave rise to many more.

The most obvious question arose from the realization that a great amount of positive charge was concentrated in a very small space inside the nucleus. The strength of the Coulomb repulsive force between like charges had long been established. Two positive charges located as close together as they would have to be in Rutherford's model of the nucleus would exert a mutual repulsive force of about 50 N on each other. For such tiny particles, this is a tremendous force. There had to be another, as yet unidentified, attractive force that was strong enough to overcome the repulsive Coulomb force. What is the nature of the particles that make up the nucleus and what force holds them together?

Protons and Neutrons

Again, it was Rutherford who discovered — and eventually named — the proton. When he was bombarding nitrogen gas with alpha particles, Rutherford detected the emission of positively charged particles with the same properties as the hydrogen nucleus. As evidence accumulated, it became apparent to physicists that the **proton** was identical to the hydrogen nucleus and was the fundamental particle that carried a positive charge, equal in magnitude to the charge on the electron and with a mass 1836 times as great as the mass of an electron. The positive charge of all nuclei consisted of enough protons to account for the charge.

Using the principle on which the mass spectrometer is based (refer to Chapter 8, Fields and Their Applications), several physicists discovered that the mass of most nuclei was roughly twice the size of the number of protons that would account for the charge. Rutherford encouraged the young physicists in his laboratory to search for a neutrally charged particle that could account for the excess mass of the nucleus. Finally, in 1932, English physicist James Chadwick (1891–1974) discovered such a particle. That particle, now called the **neutron**, has a mass that is nearly the same as that of a proton. The proton, neutron, and electron now account for all of the mass and charge of the atom. Since protons and neutrons have many characteristics in common, other than the charge, physicists call them **nucleons**.

Table 13.1 Properties of Particles in the Atom

Particle	Mass (kg)	Charge (C)
proton	$1.672\,614 \times 10^{-27}$ kg	$+1.602 \times 10^{-19}$ C
neutron	$1.674\,920 \times 10^{-27}$ kg	0 C
electron	$9.109\,56 \times 10^{-31}$ kg	-1.602×10^{-19} C

Representing the Atom

As physicists and chemists learned more about the nucleus and atoms, they needed a way to symbolically describe them. The following symbol convention communicates much information about the particles in the atom.

$$^{A}_{Z}X$$

X is the **chemical symbol** for the element to which the atom belongs. For example, the symbol for carbon is C, while the symbol for krypton is Kr.

Z is the **atomic number**, which represents the number of protons in the nucleus and is also the charge of the nucleus.

A is the **atomic mass number**, the total number of protons and neutrons in the nucleus. Since the particles in the nucleus are called "nucleons," the atomic mass number is sometimes called the **nucleon number**.

If N represents the number of neutrons in a nucleus, then

$$A = Z + N$$

The atomic number (Z) also indicates the number of electrons in the neutral atom, since one electron must be in orbit outside the nucleus for each proton inside the nucleus. In addition, the manner in which atoms chemically interact with each other depends on the arrangement of their outer electrons. This is influenced in turn by the atomic number. Since all atoms of an element behave the same chemically, all atoms of a given element must have the same atomic number. For example, all carbon atoms have an atomic number of 6 and all uranium atoms have an atomic number of 92.

The Strong Nuclear Force

By the end of the 1930s, physicists were beginning to accumulate data about the elusive force that holds the nucleus together. They discovered that any two protons (p↔p), two neutrons (n↔n), or a proton and a neutron (p↔n) attract each other with the most potent force known to physicists — the **strong nuclear force**.

When two protons are about 2 fm (femtometres: 2×10^{-15} m) apart, the nuclear force is roughly 100 times stronger than the repulsive Coulomb force. However, at 3 fm of separation, the nuclear force is almost non-existent. Whereas the gravitational and electrostatic forces have an unlimited range — both follow a $1/r^2$ law — the nuclear force has an exceptionally short range, which is roughly the diameter of a nucleon. Therefore, inside the nucleus, the nuclear force acts only between adjacent nucleons. When the separation distance between nucleons decreases to about 0.5 fm, the nuclear force becomes repulsive. This repulsion possibly occurs because nucleons cannot overlap. Estimates of the radius of a nucleon range from 0.3 fm to 1 fm. Figure 13.1 shows a graph of an approximated net force between two protons, relative to their separation distance.

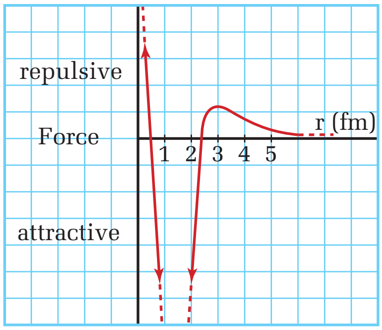


Figure 13.1 Assume that one proton is at the origin of the co-ordinate system and another proton approaches it. As the second proton approaches, it experiences a repulsive Coulomb force. If the second proton has enough energy to overcome the repulsion, it will reach a point where the net force is zero. Beyond that point, the strong nuclear force attracts it strongly.

Stability of the Nucleus

If the nuclear force is so strong, why cannot nucleons come together to form nuclei of ever-increasing size? The short range of the nuclear force accounts for this. When a nucleus contains more than approximately 20 nucleons, the nucleons on one side of the nucleus are so far from those on the opposite side that they no longer attract each other. However, the repulsive Coulomb force between protons is still very strong. Figure 13.2 shows the number of protons (Z) and neutrons (N) in all stable nuclei.

As you can see in Figure 13.2, the number of neutrons and protons is approximately equal up to a total of 40 nucleons. For example, oxygen has 8 protons and 8 neutrons and calcium has 20 protons and 20 neutrons $\left({}^{40}_{20}\text{Ca}\right)$. Beyond 40 nucleons, the ratio of neutrons to protons increases gradually up to the largest element. One form of uranium has 92 protons and 146 neutrons $\left({}^{238}_{92}\text{U}\right)$. This unbalanced ratio results in more nucleons experiencing the attractive force for each pair of protons experiencing the repulsive Coulomb forces. This combination appears to stabilize larger nuclei. Nuclei that do not lie in the range of stability will disintegrate.

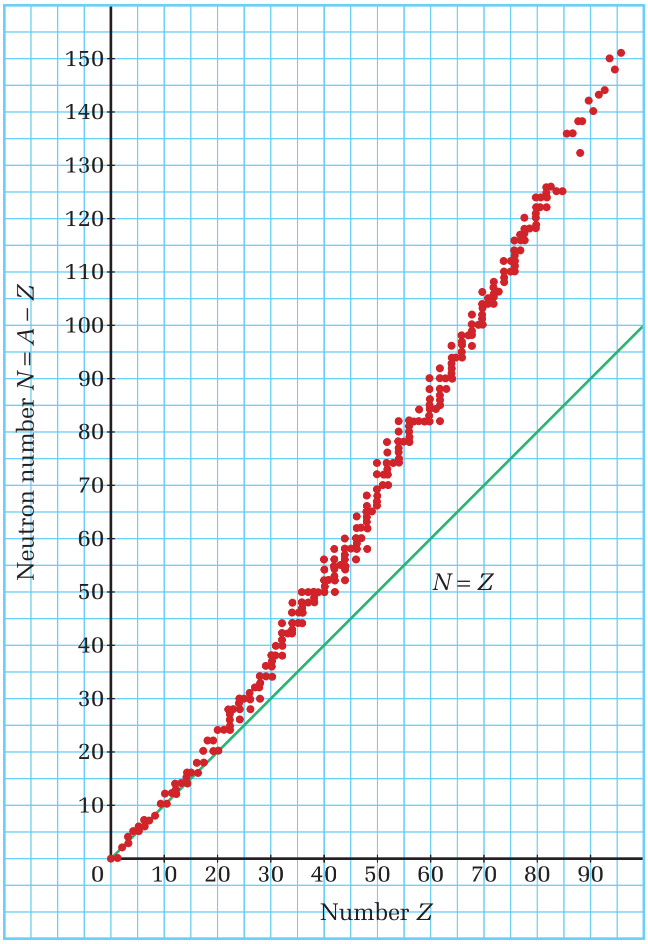


Figure 13.2 Each dot represents a stable nucleus, with the number of neutrons shown on the vertical axis and the number of protons on the horizontal axis.

Nuclides and Isotopes

Each dot in Figure 13.2 represents a unique stable nucleus with a different combination of protons and neutrons. Physicists call these unique combinations **nuclides**. The many columns of vertical dots indicate that several nuclides have the same number of protons. Since the number of protons determines the identity of the element, all of the nuclides in a vertical column are different forms of the same element, differing only in the number of neutrons. These sets of nuclides are called **isotopes**. For example, nitrogen, with 7 protons, might have 7 neutrons $\left({}^{14}_{7}\mathrm{N}\right)$ or 8 neutrons $\left({}^{15}_{7}\mathrm{N}\right)$. Figure 13.3 illustrates isotopes of hydrogen and helium.

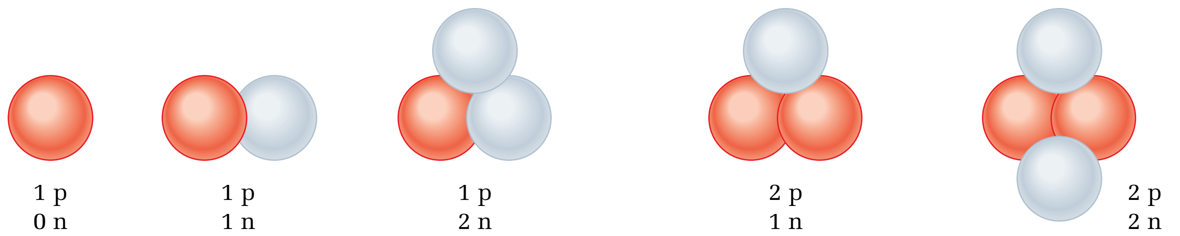


Figure 13.3 The isotopes of hydrogen are the only isotopes to which physicists have given different names: $^{2}_{1}$H is called "deuterium" and $^{3}_{1}$H is called "tritium." For most isotopes, physicists simply use the atomic mass number to describe the isotope: $^{3}_{2}$He is called "helium-3" and $^{4}_{2}$He is called "helium-4."

Nuclear Binding Energy and Mass Defect

When you consider the strength of the nuclear force, you realize that it would take a tremendous amount of energy to remove a nucleon from a nucleus. For the sake of comparison, recall that it takes 13.6 eV to ionize a hydrogen atom, which is the removal of the electron. To remove a neutron from $^{4}_{2}$He would require more than 20 million eV (20 MeV). The amount of energy required to separate all of the nucleons in a nucleus is called the binding energy of the nucleus. Figure 13.4 shows the average binding energy per nucleon plotted against atomic mass number A.

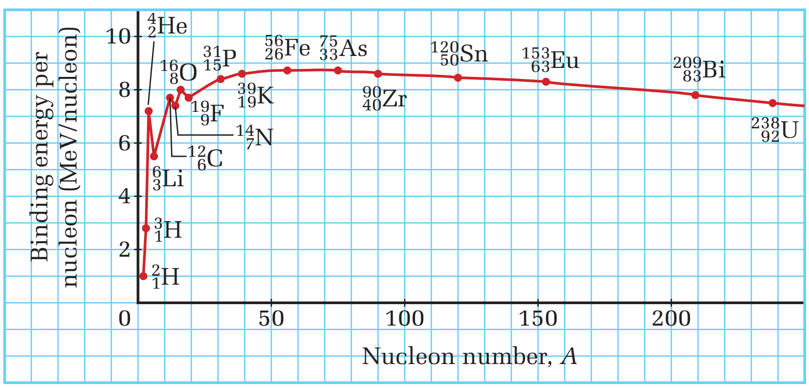


Figure 13.4 The average binding energy per nucleon is calculated by determining the total binding energy of the nucleus and dividing by the number of nuclides.

WEB LINK

www.mcgrawhill.ca/links/physics12

You can find many properties, including the mass, of all stable nuclides and many unstable nuclides in charts of the nuclides on the Internet. Just go to the above Internet site and click on **Web Links**.

Imagine that you were able to remove a neutron from $^{4}_{2}$He. What would happen to the 20 MeV of energy that you had to add in order to remove the neutron? The answer lies in Einstein's special theory of relativity: Energy is equivalent to mass. If you look up the masses of the nuclides, you would find that the mass of $^{4}_{2}$He is smaller than the sum of the masses of ${}^{3}_{2}\text{He}$ plus a neutron $\left({}^{1}_{0}\text{n}\right)$. The energy that was added to remove a neutron from ${}^{4}_{2}\text{He}$ became mass. This difference between the mass of a nuclide and the sum of the masses of its constituents is called the **mass defect**. Einstein's equation $E = \Delta mc^2$ allows you to calculate the energy equivalent of the mass defect, Δm.

When dealing with reactions involving atoms or nuclei, expressing masses in kilograms can be cumbersome. Consequently, physicists defined a new unit — the **atomic mass unit** (u). One atomic mass unit is defined as $\frac{1}{12}$ the mass of the most common isotope of carbon $\left({}^{12}_{6}\text{C}\right)$. This gives a value for the atomic mass unit of $1\ \text{u} = 1.6605 \times 10^{-27}$ kg. Table 13.2 lists the masses of the particles in atoms.

Table 13.2 Masses of Common Elementary Particles

Particle	Mass (kg)	Mass (u)
electron	$9.109\ 56 \times 10^{-31}$ kg	0.000 549 u
proton	$1.672\ 614 \times 10^{-27}$ kg	1.007 276 u
neutron	$1.674\ 920 \times 10^{-27}$ kg	1.008 665 u

SAMPLE PROBLEM

Calculate the Binding Energy of a Nucleus

Determine the binding energy in electron volts and joules for an iron nucleus of $\left({}^{56}_{26}\text{Fe}\right)$, given that the nuclear mass is 55.9206 u.

Conceptualize the Problem

- The *energy equivalent* of the *mass defect* is the *binding energy* for the nucleus.
- The *mass defect* is the *difference* of the mass of the *nucleus* and the *sum* of the masses of the *individual particles*.

Identify the Goal

The binding energy, E, of ${}^{56}_{26}\text{Fe}$

Identify the Variables and Constants

Known	Implied	Unknown
$m_{\text{nucleus}} = 55.9206$ u	$c = 2.998 \times 10^{8}\ \frac{\text{m}}{\text{s}}$	N
$A = 56$	$m_{\text{p}} = 1.007\ 276$ u	Δm
$Z = 26$	$m_{\text{n}} = 1.008\ 665$ u	E

continued ►

continued from previous page

Develop a Strategy

Calculate the number of neutrons.

$$N = A - Z$$
$$N = 56 - 26$$
$$N = 30$$

Determine the total mass of the separate nucleons by finding the masses of the protons and neutrons and adding them together.

$$m_{p(total)} = (26)(1.007\ 276\ \text{u})$$
$$m_{p(total)} = 26.189\ 176\ \text{u}$$

Find the mass defect by subtracting the mass of the nucleus from the total nucleon mass.

$$m_{n(total)} = (30)(1.008\ 665\ \text{u})$$
$$m_{n(total)} = 30.259\ 95\ \text{u}$$
$$m_{total} = 26.189\ 176\ \text{u} + 30.259\ 95\ \text{u}$$
$$m_{total} = 56.449\ 126\ \text{u}$$
$$\Delta m = 56.449\ 126\ \text{u} - 55.9206\ \text{u}$$
$$\Delta m = 0.528\ 526\ \text{u}$$

Convert this mass into kilograms.

$$\Delta m = (0.528\ 526\ \cancel{\text{u}})\left(1.6605 \times 10^{-27}\ \frac{\text{kg}}{\cancel{\text{u}}}\right)$$
$$\Delta m = 8.7762 \times 10^{-28}\ \text{kg}$$

Find the energy equivalent of the mass defect.

$$E = \Delta mc^2$$

Find the energy in electron volts.

$$\Delta E = \left(8.7762 \times 10^{-28}\ \text{kg}\right)\left(2.998 \times 10^{8}\ \frac{\text{m}}{\text{s}}\right)^2$$
$$\Delta E = 7.888 \times 10^{-11}\ \text{J}$$
$$\Delta E = \frac{7.888 \times 10^{-11}\ \cancel{\text{J}}}{1.602 \times 10^{-19}\ \frac{\cancel{\text{J}}}{\text{eV}}}$$
$$\Delta E = 4.9239 \times 10^{8}\ \text{eV}$$

The binding energy of the nucleus is 4.924×10^8 eV, or 7.888×10^{-11} J.

Validate the Solution

The binding energy of a nucleus should be extremely small.
You would expect the binding energy per nucleon to be about 8 MeV.

$$\frac{4.93 \times 10^8\ \text{eV}}{56} = 8.79 \times 10^6\ \text{eV} = 8.79\ \text{MeV}$$

PRACTICE PROBLEMS

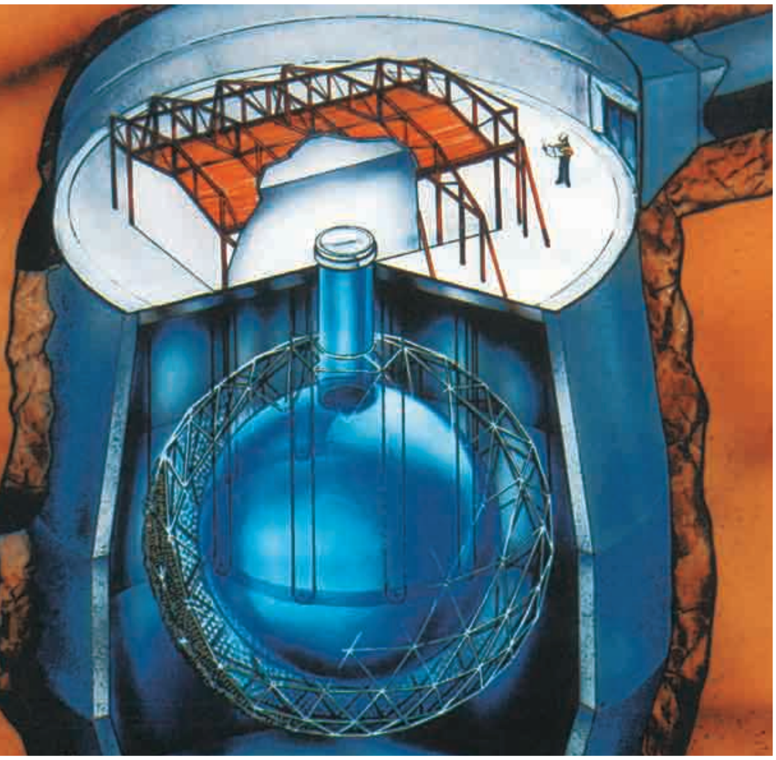
1. Determine the mass defect for ${}^{8}_{4}\text{Be}$ with a nuclear mass of 8.003 104 u.
2. Determine the binding energy for ${}^{3}_{2}\text{He}$ with a nuclear mass of 3.014 932 u.
3. Determine the binding energy for ${}^{235}_{92}\text{U}$ with a nuclear mass of 234.9934 u.

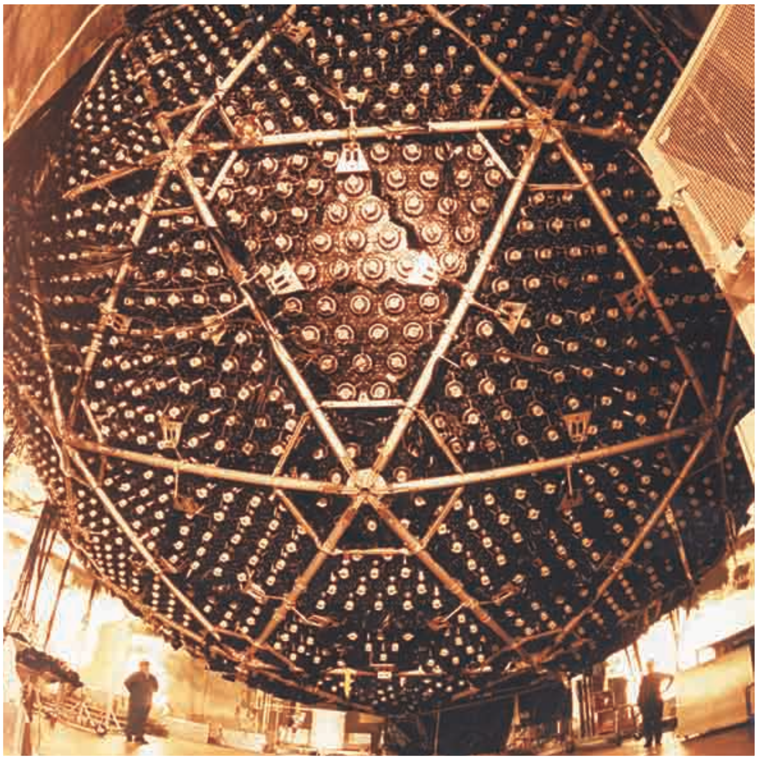
13.1 Section Review

1. K/U List the contribution of each of the following physicists to the study of the nucleus.
   (a) Ernest Rutherford
   (b) James Chadwick
2. C Why did physicists believe that a neutral particle must exist, even before the neutron was discovered?
3. K/U State the meaning of the following terms.
   (a) nucleon
   (b) atomic mass number
   (c) atomic number
4. K/U For the atom symbolized by ${}^{200}_{80}Hg$, state the number of
   (a) nucleons
   (b) protons
   (c) neutrons
   (d) electrons, if the atom is electrically neutral
   (e) electrons, if the atom is a doubly charged positive ion
5. K/U How did scientists know that some force other than the electromagnetic force had to exist within the nucleus?
6. C Describe the characteristics of the nuclear force.
7. C Describe the general trend of stable nuclei in relation to the proton number and neutron number.
8. C Explain the concept of binding energy.
9. C Define the term "mass defect" and explain how to determine it for a given nucleus.
10. MC The structure of the atom is often compared to a solar system, with the nucleus as the Sun and the electrons as orbiting planets. If you were going to use this analogy, which planet should you use to represent an electron, based on its comparative distance from the Sun?

Atom	Solar system
radius of nucleus ≈ 1×10^{-15} m	radius of Sun ≈ 6.96×10^{8} m
radius of typical electron orbit ≈ 1×10^{-10} m	radii of planetary orbits $r_{Mercury} \approx 6 \times 10^{10}$ m $r_{Earth} \approx 1.49 \times 10^{11}$ m $r_{Jupiter} \approx 8 \times 10^{11}$ m $r_{Pluto} \approx 6 \times 10^{12}$ m

Not Your Average Observatory





When you think of an observatory, you probably think of a large telescope on a remote mountaintop that collects light from stars and galaxies. You're not likely to think of 1000 t of heavy water at the bottom of a mine shaft, 2 km below Earth's surface. But that's exactly what you will find at the Sudbury Neutrino Observatory — also known as SNO — where scientists are trying to detect a wily, elusive particle called the "neutrino."

The existence of the neutrino was first predicted in 1931 by Wolfgang Pauli, when certain nuclear reactions appeared to be violating the laws of conservation of energy and momentum. Rather than modify or discard the law, Pauli suggested that an unseen, chargeless and probably massless particle was carrying away some of the energy and momentum. Italian physicist Enrico Fermi later named this mysterious particle the "neutrino," which means "little neutral one" in Italian.

Not Your Average Particle

Like the photon, the neutrino is produced in enormous quantities by nuclear reactions in the centres of stars, such as the Sun. They travel at close to the speed of light and carry away substantial amounts of energy from the star's hot core. Just as photons are collected by telescopes to analyze the processes that create them, neutrinos are observed in order to understand what's happening in the centres of stars. By counting neutrinos, physicists learn about the rate of fusion reactions in stellar cores. Although there are now known to be three types of neutrinos, stars produce the type known as the "electron-neutrino."

Unlike the photon, which interacts strongly with matter, the neutrino scarcely interacts with matter at all. Some 60 billion neutrinos pass unhindered through each square centimetre of your body each second. A beam of neutrinos could sail through a shield of lead 1 light-year (10 thousand billion kilometres) thick without being reflected or absorbed. This is why it took physicists almost 30 years after Pauli's prediction to verify the neutrino's existence.

Detecting a Neutrino

Detecting a neutrino is tricky, because it passes right through photographic film and electronic detectors, the devices that register photons. This is where heavy water is useful. Heavy water is made up of oxygen and deuterium, which is a hydrogen nucleus with an added neutron. It's about 10% heavier than "light" water, and its symbol is D_2O. Occasionally, in one of several possible reactions, neutrinos will transform a deuterium nucleus into a pair of protons and an electron. Neutrinos enter the tank with super-high energies and, because energy is conserved, the electron produced in the reaction will be jettisoned at speeds faster than the speed of light in water. It's like a high-speed crash, where even the debris of the collision flies out at high speed. As the energetic electron slows down in the water, it emits a flash of light, or a shock wave — the optical equivalent of a sonic boom. About 500 to 800 photons will be generated in the flash, with a total energy proportional to that of the incident neutrino.

Outside the tank of heavy water, several of the 10 thousand photomultiplier tubes will detect this tiny light flash. These photomultipliers comprise the main part of the detector. Together, they are about 200 000 times more sensitive than the human eye. This sensitivity is required because the flash of light is only as bright as the flash of a camera seen from the distance of the Moon! So, even though the neutrino is not detected directly, the product of its interaction is. Despite the 1000 t of heavy water, only about 10 neutrinos are detected per day.

One Mystery Solved

One longstanding problem that the Sudbury Neutrino Observatory investigated was the significant difference between the number of predicted neutrinos based on solar models and the number of neutrinos that were actually observed. Was the difference due to observation errors in the experiments and detectors, which were begun in the 1970s, or to errors in the scientific model calculations? Despite refinements in both over nearly 30 years, the difference persisted. Why?

The unexpected answer, which SNO helped provide in June 2001, was due to the neutrinos themselves. Because the Sun generates only electron-neutrinos, the earlier experiments were designed to detect only this type of neutrino, and not the other two types. Recent results from SNO, together with those of another detector in Japan, indicate that after neutrinos are generated in the Sun, they oscillate between the three types while they travel. Because of its ability to detect different types of neutrino reactions, SNO is sensitive to all three types of neutrinos, and it was able to show that earlier experiments simply missed the transformed neutrinos — but they were there all along.

For such transformations to take place, the neutrino must have a tiny mass, contrary to what was originally thought. SNO has measured this mass to be roughly 60 000 times less than that of an electron. What this new result means for elementary particle theory remains to be seen. How the neutrinos change type is also still not understood, but with continued monitoring of incoming neutrinos, SNO hopes to shed light on that too.

Making Connections

1. Could neutrinos enter the Sudbury Neutrino Observatory after passing through the far side of Earth, that is, on their way back out into space?
2. How do the laws of conservation of energy and momentum apply to reactions between particles?
3. How does the SNO differ from other neutrino observatories worldwide?

13.2 Radioactivity and Nuclear Reactions

SECTION EXPECTATIONS

- Define and describe the concepts and units related to radioactivity.
- Describe the principal forms of nuclear decay.
- Compare the properties of alpha particles, beta particles, and gamma rays in terms of mass, charge, speed, penetrating power, and ionizing ability.
- Compile, organize, and display data or simulations to determine and display the half-lives for radioactive decay of isotopes.

KEY TERMS

- radioactive material
- alpha particle
- beta particle
- gamma ray
- radioactive isotope (radioisotope)
- parent nucleus
- daughter nucleus
- transmutation
- ionizing radiation
- neutrino
- antineutrino
- positron
- half-life
- nuclear fission
- nuclear fusion

Observation of the effects of cathode ray tubes carried out by J.J. Thomson and others stimulated many other scientists to perform related studies in which a material was bombarded with "rays" of various types. When Wilhelm Conrad Röntgen (1845–1923) was using a cathode ray tube, he was surprised to see a fluorescent screen glowing on the far side of the room. Because he did not know the nature of these rays, he called them "X rays." French physicist Henri Becquerel (1852–1908) became curious about the emission of these X rays and wondered if luminescent materials, when exposed to light, might also emit X rays.

At first, Becquerel's experiment seemed to confirm his hypothesis. He wrapped photographic film to shield it from natural light and placed it under phosphorescent uranium salts. When he exposed the phosphorescent salts to sunlight, silhouettes of the crystals appeared when he developed the film. The salts appeared to absorb sunlight and reemit the energy as X rays that then passed through the film's wrapping. However, during a cloudy period, Becquerel stored the uranium salts and wrapped film in a drawer. When he later developed the film, he discovered that it had been exposed while in the drawer. This is the first recorded observation of the effects of radioactivity.

Radioactive Isotopes

Physicists discovered, studied, and used **radioactive materials** (materials that emit high-energy particles and rays) long before they learned the reason for these emissions. As you know, Rutherford discovered **alpha particles** (α) and used them in many of his famous experiments. He examined the nature of alpha particles by passing some through an evacuated glass tube and then performing a spectral analysis of the tube's contents. The trapped alpha particles displayed the characteristic spectrum of helium; alpha particles are simply helium nuclei.

Rutherford also discovered **beta particles**, and other scientists studied their charge-to-mass ratio and showed that beta (β) particles were identical to electrons. French physicist Paul Villard discovered that, in addition to beta particles, radium emitted another form of very penetrating radiation, which was given the name "gamma (γ) rays." **Gamma rays** are a very high-frequency electromagnetic wave. Figure 13.5 shows the separation of these radioactive emissions as they pass between oppositely charged plates.

In Section 13.1, you learned about the nuclei of atoms and about many of the characteristics that made them stable. Did you wonder what would happen to a nucleus if it was not stable? The answer is that it would disintegrate by emitting some form of radiation and transform into a more stable nucleus. Unstable nuclei are called **radioactive isotopes** (or "radioisotopes"). When a nucleus disintegrates or decays, the process obeys several conservation laws — conservation of mass-energy, conservation of momentum, conservation of nucleon number, and conservation of charge. The following subsections summarize the important characteristics of alpha, beta, and gamma radiation.

PHYSICS FILE

Pierre and Marie Curie once gave Henri Becquerel a sample of radium that they had prepared. When Becquerel carried the sample in his vest pocket, it burned his skin slightly. This observation triggered interest among physicians and eventually led to the use of radioactivity for medical purposes. Becquerel shared the 1903 Nobel Prize in Physics with the Curies.

Alpha Decay

When a radioactive isotope emits an alpha particle, it loses two protons and two neutrons. As a result, the atomic number (Z) decreases by two and the atomic mass number (A) decreases by four. Physicists describe this form of decay as shown below, where P represents the original nucleus or **parent nucleus** and D represents the resulting nucleus or **daughter nucleus**.

$$^{A}_{Z}\text{P} \rightarrow {}^{4}_{2}\text{He} + {}^{A-4}_{Z-2}\text{D}$$

Only very large nuclei emit alpha particles. One such reaction would be the alpha emission from radium-223 ($^{223}_{88}\text{Ra}$). To determine the identity of the daughter nucleus, write as much as you know about the reaction.

$$^{223}_{88}\text{Ra} \rightarrow {}^{4}_{2}\text{He} + {}^{219}_{86}?$$

Then look up the identity of an element with an atomic number of 86, and you will find that it is radon. The final equation becomes

$$^{223}_{88}\text{Ra} \rightarrow {}^{4}_{2}\text{He} + {}^{219}_{86}\text{Rn}$$

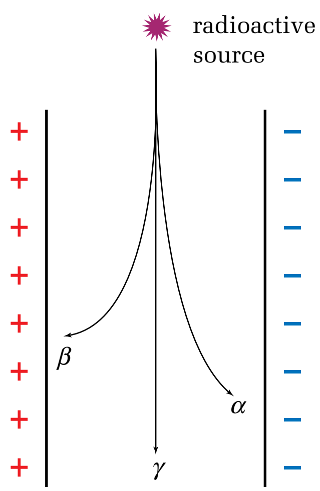


Figure 13.5 Positive alpha particles are attracted to the negative plate, while negatively charged beta particles are attracted to the positive plate. Gamma rays are not attracted to either plate, indicating that they do not carry a charge.

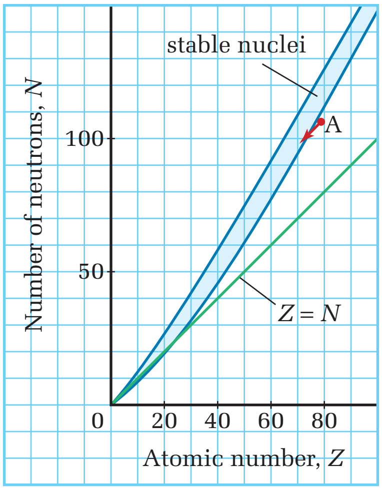


Figure 13.6 The emission of an alpha particle is represented here as a diagonal arrow going down and to the left. This process brings the tip of the arrow to a nucleus that has two fewer neutrons and two fewer protons than the nucleus at the tail of the arrow.

During this reaction, one element is converted into a different element. Such a change is called **transmutation**. Why would such a transmutation result in a more stable nucleus? You can find the answer by studying the simplified representation of stable nuclei in Figure 13.6. The point labelled "A" represents a nuclide that lies outside of the range of stability. The arrow shows the location of the daughter nucleus when the unstable parent loses two neutrons and two protons. As you can see, the daughter nucleus lies within the range of stability. In addition, the helium nucleus — alpha particle — is one of the most stable nuclei of all. Since you now have two nuclei that are more stable than the parent nucleus, the total binding energy increased. The mass defect becomes kinetic energy of the alpha particle and daughter nucleus. Typical alpha particle energies are between 4 MeV and 10 MeV.

Conceptual Problem

- Write the nuclear reaction for the alpha decay of the following nuclei.
  - **(a)** $^{222}_{86}\text{Rn}$
  - **(b)** $^{210}_{84}\text{Po}$
  - **(c)** $^{214}_{83}\text{Bi}$
  - **(d)** $^{230}_{90}\text{Th}$

Alpha particles do not penetrate materials very well. A thick sheet of paper or about 5 cm of air can stop an alpha particle. In stopping, it severely affects the atoms and molecules that are in its way. With the alpha particle's positive charge, relatively large mass, and very high speed (possibly close to 2×10^7 m/s), it gives some of the electrons in the atoms enough energy to break free, leaving a charged ion behind. For this reason, alpha particles are classified as **ionizing radiation**. These ions can disrupt biological molecules. Because of its low penetrating ability, alpha radiation is not usually harmful, unless the radioactive material is inhaled or ingested.

Beta Decay

When a radioactive isotope emits a beta particle, it appears to lose an electron from within the nucleus. However, electrons as such do not exist in the nucleus — a transformation of a nucleon had to take place to create the electron. In fact, in the process, a neutron becomes a proton, so the total nucleon number (A) remains the same, but the atomic number (Z) increases by one. You can write the general reaction for beta decay as follows, where $^{0}_{-1}\text{e}$ represents the beta particle, which is a high-energy electron. The superscript zero does not mean zero mass, because an electron has mass. The zero means that there are no nucleons.

$$^{A}_{Z}\text{P} \rightarrow {}^{0}_{-1}\text{e} + {}^{A}_{Z+1}\text{D}$$

Many common elements such as carbon have isotopes that are beta emitters.

$$^{14}_{6}\text{C} \rightarrow {}^{0}_{-1}\text{e} + {}^{14}_{7}?$$

When you look up the identity of an element with an atomic number of 7, you will find that it is nitrogen. The final equation becomes

$$^{14}_{6}\text{C} \rightarrow {}^{0}_{-1}\text{e} + {}^{14}_{7}\text{N}$$

When physicists were doing some of the original research on beta decay, they made some very puzzling observations. Linear momentum of the beta particle and daughter nucleus was not conserved. As well, they determined the spin of each particle and observed that angular momentum was not conserved. To add to the puzzle, the physicists calculated the mass defect and discovered that mass-energy was not conserved.

Some physicists were ready to accept that these subatomic particles did not follow the conservation laws. However, Wolfgang Pauli (1900–1958) proposed an explanation for these apparent violations of the fundamental laws of physics. He proposed the existence of an as yet unknown, undiscovered particle that would account for all of the missing momentum and energy. It was more than 25 years before this elusive particle, the **neutrino** (ν_e), was discovered.

In reality, the particle that is emitted with a beta particle is an **antineutrino**, a form of antimatter. The antineutrino has a very small or zero rest mass and so can travel at or near the speed of light. It accounts for all of the "missing pieces" of beta decay. The correct reaction for beta decay should be written as follows. The bar above the symbol ν_e for the neutrino indicates that it is an antiparticle.

$$^{A}_{Z}\text{P} \rightarrow {}^{0}_{-1}\text{e} + {}_{Z+1}{}^{A}\text{D} + \overline{\nu_e}$$

Physicists soon discovered a different form of beta decay — the emission of a "positive electron" that is, in fact, an antielectron. It has properties identical to those of electrons, except that it has a positive charge. The more common name for the antielectron is **positron**. Since the parent nucleus loses a positive charge but does not lose any nucleons, the value of A does not change, but Z decreases by one. A proton in the parent nucleus is transformed into a neutron. As you might suspect, the emission of a neutrino accompanies the positron. The reaction for positive electron or positron emission is written as follows.

$$^{A}_{Z}\text{P} \rightarrow {}^{0}_{+1}\text{e} + {}_{Z-1}{}^{A}\text{D} + \nu_e$$

You can understand why beta emission produces a more stable nucleus by examining Figure 13.7. The emission of an electron changes a neutron to a proton; in the chart, this is represented by an arrow going diagonally down to the right. Emission of a positron changes a proton into a neutron and the arrow in the chart goes diagonally upward and to the left.

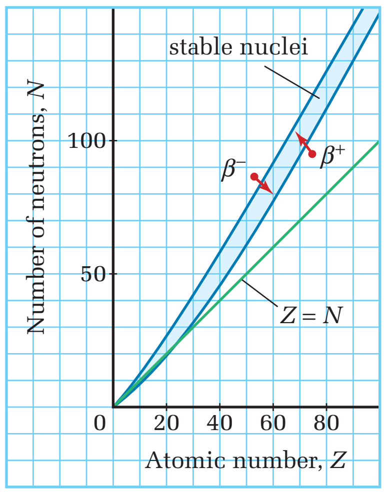
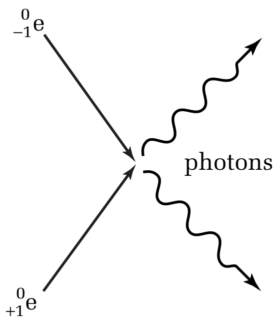


Figure 13.7 If a nucleus lies above the range of stability, it can transform into a more stable nucleus by beta emission. If it lies below the range of stability, it can transform into a more stable ion by emitting a positron. (Arrows are not drawn to scale.)

PHYSICS FILE

The positron is the antimatter particle for the electron. When they meet, they annihilate each other and release their mass-energy as a gamma photon.

$${}^{0}_{1}e + {}^{0}_{-1}e \rightarrow \gamma$$



The collision of a particle with its own antimatter particle results in the annihilation of the particles and the creation of two gamma ray photons.

Beta particles penetrate matter to a far greater extent than do alpha particles, mainly due to their much smaller mass, size, and charge. They can penetrate about 0.1 mm of lead or about 10 m of air. Although they can penetrate better than alpha particles, they are only about 5% to 10% as biologically destructive. Like alpha particles, they do their damage by ionizing atoms and molecules, and so are classified as ionizing radiation.

Conceptual Problems

- Free neutrons $\left({}^{1}_{0}n\right)$ decay by beta minus emission. Write the reaction.
- Free protons $\left({}^{1}_{1}p\right)$ can decay by beta plus emission. Write the reaction.
- Tritium, the isotope of hydrogen that consists of a proton and two neutrons, decays by beta minus emission. Write the reaction.
- Carbon-10 decays by positron emission. Write the reaction.
- Calcium-39 $\left({}^{39}_{20}Ca\right)$ decays into potassium-39 $\left({}^{39}_{19}K\right)$. Write the equation and identify the emitted particle.
- Plutonium-240 $\left({}^{240}_{94}Pu\right)$ decays into uranium-236 $\left({}^{236}_{92}U\right)$. Write the equation and identify the emitted particle.
- Lead-109 $\left({}^{109}_{46}Pb\right)$ decays into silver-109 $\left({}^{109}_{47}Ag\right)$. Write the equation and identify the emitted particle.
- Write the equation for the alpha decay of fermium-252 $\left({}^{252}_{100}Fm\right)$.
- Write the equation for the beta positive decay of vanadium-48 $\left({}^{48}_{23}V\right)$.
- Write the equation for the beta negative decay of gold-198 $\left({}^{198}_{79}Au\right)$.

Gamma Decay

When a nucleus decays by alpha or beta emission, the daughter nucleus is often left in an excited state. The nucleus then emits a gamma ray to drop down to its ground state. This process can be compared to an electron in an atom that is in a high-energy level. When it drops to its ground state, it emits a photon. However, a gamma ray photon has much more energy than a photon emitted by an atom. The decay process can be expressed as follows, where the star indicates that the nucleus is in an excited state.

$${}^{A}_{Z}P^{*} \rightarrow {}^{A}_{Z}P + {}^{0}_{0}\gamma$$

The following is an example of gamma decay:

$${}^{192}_{77}Ir^{*} \rightarrow {}^{192}_{77}Ir + {}^{0}_{0}\gamma.$$

Gamma radiation is the most penetrating of all. It can pass through about 10 cm of lead or about 2 km of air. The penetrating ability of gamma radiation is due to two factors. First, it carries no electric charge and therefore does not tend to disrupt electrons as it passes by. Second, its photon energy is far beyond any electron energy level in the atoms. Consequently, it cannot be absorbed through electron jumps between energy levels.

However, when gamma radiation is absorbed, it frees an electron from an atom, leaving behind a positive ion and producing an electron with the same range of kinetic energy as a beta particle — often called "secondary electron emission." For this reason, gamma radiation is found to be just as biologically damaging as beta radiation. As in the case of alpha and beta radiation, gamma is classified as ionizing radiation.

Decay Series

When a large nucleus decays by the emission of an alpha or beta particle, the daughter nucleus is more stable than the parent is; however, the daughter nucleus might still be unstable. Consequently, a nucleus can tumble through numerous transmutations before it reaches stability. Figure 13.8 shows one such decay sequence for uranium-238. Notice that the end product is lead-82, then go back to Figure 13.4 on page 550. You will find lead at the peak of the curve of binding energy per nucleon. Lead is one of the most stable nuclei of all of the elements.

Notice that during the progress of the transmutations the following occurs.

- An alpha decay decreases the atomic number by 2 and decreases the atomic mass number by 4.
- A beta negative decay increases the atomic number by 1, while leaving the atomic mass number unchanged.

Knowledge of decay sequences such as the one in Figure 13.8 gives scientists information about the history of materials that contain lead. For example, if a rock contains traces of lead-82, that isotope of lead probably came from the decay of uranium-238 that was trapped in crystals as molten rock solidified in the past. A geologist can determine the original amount of uranium-238 in the rock and compare it to the amount of uranium-238 that remains. Knowing the disintegration rate of the isotopes in the series, a geologist can determine the age of the rock. This method was used to determine that the Canadian Shield contains some of the most ancient rock in the world, aged close to 4 billion years.

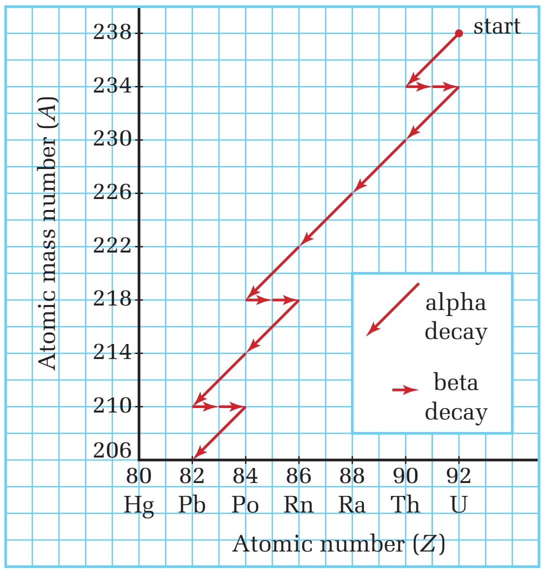


Figure 13.8 This series represents only one of several possible pathways of decay for uranium-238.

Rate of Radioactive Decay

You cannot predict exactly when a specific nucleus will disintegrate. You can only state the probability that it will disintegrate within a given time interval. Using probabilities might seem to be very imprecise, but if you have an exceedingly large number of atoms of the same isotope, you can state very precisely when half of them will have disintegrated. Physicists use the term **half-life**, symbolized by $T_{\frac{1}{2}}$, to describe the decay rate of radioactive isotopes. One half-life is the time during which the nucleus has a 50% probability of decaying. The half-life is also the time interval over which half of the nuclei in a large sample will disintegrate.

Imagine that you had a sample of polonium-218 $\left({}^{218}_{84}\text{Po}\right)$. It decays by alpha emission with a half-life of 3.0 min. If you started with 160.0 μg of the pure substance, it would decay as shown in Table 13.3.

Table 13.3 Decay of Polonium-218

Time (min)	Mass of Po-218 remaining (μg)
0	160.0
3.0	80.0
6.0	40.0
9.0	20.0
12.0	10.0
15.0	5.0

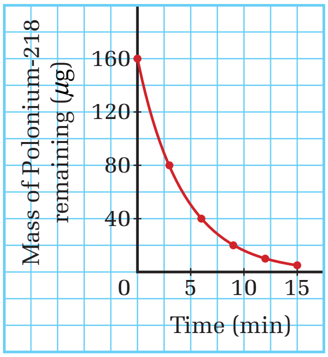


Figure 13.9 A graph of the decay of polonium and all other radioactive isotopes is an exponential curve.

From Figure 13.9 we can estimate the following.

- After 7.0 min, there should be about 32 μg of polonium-218 remaining.
- It would take about 13 min to reduce the mass of polonium-218 to 8.0 μg.

You can obtain more accurate values by using a mathematical equation that relates the mass of the isotope and time interval. You can derive such an equation as follows.

- Let N represent the amount of the original sample remaining after any given time interval.
- Let N_o represent the original amount in the sample; must be given in the same units as N.
- Let Δt represent the time interval, and $T_{\frac{1}{2}}$ represent the half-life.
- After 1 half-life, $N = \frac{1}{2}N_o$.
- After 2 half-lives, $N = \frac{1}{2}\left(\frac{1}{2}N_o\right) = \left(\frac{1}{2}\right)^2 N_o$.

ELECTRONIC LEARNING PARTNER

To enhance your understanding of radioactive decay and half-life, go to your Electronic Learning Partner.

- After 3 half-lives, $N = \frac{1}{2}\left(\frac{1}{2}\right)^2 N_o = \left(\frac{1}{2}\right)^3 N_o$.
- After 4 half-lives, $N = \frac{1}{2}\left(\frac{1}{2}\right)^3 N_o = \left(\frac{1}{2}\right)^4 N_o$.
- You can now see a pattern emerging and can state the general expression in which "n" is the number of half-lives. $N = \left(\frac{1}{2}\right)^n N_o$
- However, the number, n, of half-lives is equal to the time interval divided by the time for 1 half-life. $n = \frac{\Delta t}{T_{\frac{1}{2}}}$
- Substituting the value for n, you obtain the final equation. $N = \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}} N_o$

The amount of sample, N, can be expressed as the number of nuclei, the number of moles of the isotope, the mass in grams, the decay rate, or any measurement that describes an amount of a sample. The unit for decay rate in disintegrations per second is the becquerel, symbolized as Bq in honour of Henri Becquerel.

RADIOACTIVE DECAY

The amount of a sample remaining is one half to the exponent time interval divided by the half-life, all times the amount of the original sample.

$$N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

Quantity	Symbol	SI unit
amount of sample remaining	N	kilograms, moles, or Bq (might also be in number of atoms)
amount in original sample	N_o	kilograms, moles, or Bq (might also be in number of atoms)
elapsed time	Δt	s (often reported in min, days, years, etc.)
half life	$T_{\frac{1}{2}}$	s (often reported in min, days, years, etc.)

Unit Analysis

kilograms = kilograms   kg = kg

Note: The elapsed time and the half-life must be given in the same units so that they will cancel, making the exponent of one half a pure number. Also, the amount of the sample remaining and in the original sample at time zero must be given in the same units.

SAMPLE PROBLEM

Decay of Polonium-218

You have a 160.0 μg sample of polonium-218 that has a half-life of 3.0 min.

(a) How much will remain after 7.0 min?

(b) How long will it take to decrease the mass of the polonium-218 to 8.0 micrograms?

Conceptualize the Problem

- The *half-life* of a radioactive isotope determines the *amount* of a sample at any given *time*.

Identify the Goal

Amount of polonium-218 remaining after 7.0 min

Length of time required for the mass of the sample to decrease to 8.0 μg

> **PROBLEM TIPS**
>
> The data for amounts of a sample, time intervals, and half-lives in decay rate problems can be given in a variety of units. Always be sure that, in your calculations, the amounts of a sample, N and N_o, are in the same units and that the time interval and the half-life are in the same units.

Identify the Variables and Constants

Known	Unknown
$m_o = 160.0\ \mu g$	m (at 7.0 min)
$T_{\frac{1}{2}} = 3.0$ min	Δt (at 8.0 μg)
$\Delta t = 7.0$ min	
$m = 8.0\ \mu g$	

Develop a Strategy

Write the decay relationship

$$N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

Substitute and solve.

$$N = 160.0\ \mu g\left(\frac{1}{2}\right)^{\frac{7.0\ \text{min}}{3.0\ \text{min}}}$$

$$N = 160.0\ \mu g(0.198\ 425)$$

$$N = 31.748\ \mu g$$

$$N \cong 32\ \mu g$$

(a) The mass remaining after 7.0 min will be 32 μg.

Write the decay equation.

$$N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

Rearrange the equation to solve for the ratio N to N_o.

$$\frac{N}{N_o} = \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

Substitute numerical values.

$$\left(\frac{1}{2}\right)^{\frac{\Delta t}{3.0\ \text{min}}} = \frac{8.0\ \mu g}{160.0\ \mu g}$$

Solve by taking logarithms on both sides.

$$\log\left(\frac{1}{2}\right)^{\frac{\Delta t}{3.0\text{ min}}} = \log\frac{8.0}{160.0}$$

$$\frac{\Delta t}{3.0\text{ min}}\log\left(\frac{1}{2}\right) = \log 0.050$$

$$\Delta t = (3.0\text{ min})\frac{\log 0.050}{\log\left(\frac{1}{2}\right)}$$

$$\Delta t = (3.0\text{ min})\left(\frac{-1.301\ 003}{-0.301\ 03}\right)$$

$$\Delta t = 12.965\ 78\text{ min}$$

$$\Delta t \cong 13\text{ min}$$

(b) The time interval after which only 8.0 μg of polonium-218 will remain is 13 min.

Validate the Solution

These answers are the same as the answers estimated from the graph in Figure 13.9.

PRACTICE PROBLEMS

4. When a sample of lava solidified, it contained 27.4 mg of uranium-238, which has a half-life of 4.5×10^9 a (annum or year). If that lava sample was later found to contain only 18.3 mg of U-238, how many years had passed since the lava solidified?

5. Carbon-14 has a half-life of 5730 a. Every gram of living plant or animal tissue absorbs enough radioactive C-14 to provide an activity of 0.23 Bq. Once the plant or animal dies, no more C-14 is taken in. If ashes from a fire (equivalent to 1 g of tissue) have an activity of 0.15 Bq, how old are they? Assume that all of the radiation comes from the remaining C-14.

6. Radioactive iodine-128, with a half-life of 24.99 min, is sometimes used to treat thyroid problems. If 40.0 mg of I-128 is injected into a patient, how much will remain after 12.0 h?

Nuclear Reactions

When you were solving the problems above, you encountered radioactive isotopes that have half-lives of 3.0 min and 25 min. Did you wonder how any such isotopes could exist and why they had not decayed entirely? Most of the radioactive isotopes that are used in medicine and research are produced artificially. One of the first observations of artificial production of a radioisotope was accomplished by bombarding aluminum-27 with alpha particles as follows.

$$^{4}_{2}\text{He} + ^{27}_{13}\text{Al} \rightarrow ^{31}_{15}\text{P}^{*} \rightarrow ^{30}_{15}\text{P} + ^{1}_{0}\text{n}$$

The star on the phosphorus-31 indicates that it is very unstable and decays into phosphorus-30 and a neutron. Phosphorus-30 is a radioisotope that emits a positron. Today, many artificial isotopes are produced by bombarding stable isotopes with neutrons in nuclear reactors. For example, stable sodium-23 can absorb a neutron and become radioactive sodium-24.

Nuclear Fission

One of the most important reactions that is stimulated by absorbing a neutron is **nuclear fission**, the reaction in which a very large nucleus splits into two large nuclei plus two or more neutrons. The two most common isotopes that can undergo fission are $^{235}_{92}U$ and $^{239}_{94}Pu$. When a nucleus fissions, or splits, a tremendous amount of energy is released in the form of kinetic energy of the fission products — the resulting smaller nuclei. Since the kinetic energy of atoms and molecules is thermal energy, the temperature of the material rises dramatically. You can understand why such large amounts of energy are released by examining the graph of binding energy per nucleon in Figure 13.10.

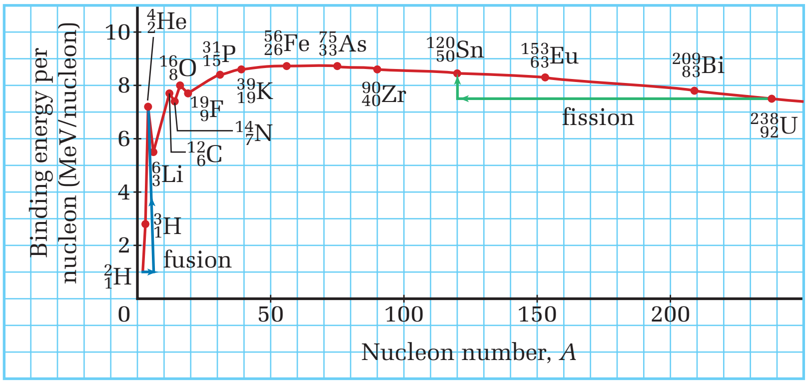


Figure 13.10 The binding energy of mid-range nuclei is greater than that of either very large or very small nuclei.

As you can see in Figure 13.10, when a large nucleus fissions, the smaller nuclei have much larger binding energies than the original nucleus did. Consequently, the sum of the masses of the fission products is much smaller than the mass of the original nucleus. This large mass defect yields the large amount of energy.

Nuclear fission is the reaction that occurs in nuclear reactors. The thermal energy that is released is then used to produce steam to drive electric generators. In this reaction, uranium-235 captures a slow neutron, producing a nucleus of uranium-236. This nucleus is quite unstable and will rapidly split apart. One possible result of this splitting or fission is shown in Figure 13.11. Notice that several neutrons are ejected during the fission. These neutrons can then cause further fissions, causing a chain reaction. However, the neutrons must be slowed down, or the uranium-235 nuclei cannot absorb them. In most Canadian reactors, heavy water is used, since neutrons are slowed down when they collide with the deuterium ($^{2}_{1}H$ or $^{2}_{1}D$) nuclei in the water. The reaction portrayed in

Figure 13.11, ${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3{}^{1}_{0}\text{n}$, is only one of a large number of possibilities. Many different fission products are formed.

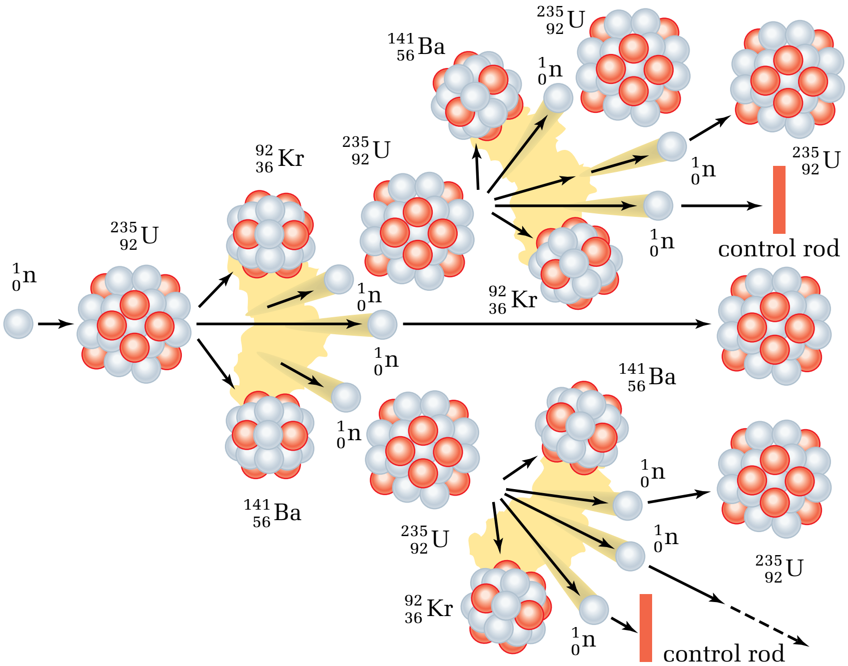


Figure 13.11 The neutrons given off by this nuclear fission reaction must be slowed down before they can be captured by other uranium-235 nuclei and produce further fission.

You have probably heard about the hazards of nuclear energy and the problems with the disposal of the products. Uranium-235 is an alpha emitter with a very long half-life, so it is not a serious danger itself. Alpha radiation is not very penetrating and the long half-life implies a low activity. The fission products cause the hazards. The reason becomes obvious when you examine Figure 13.12, which represents the stable nuclides. Fission products have about the same neutron-to-proton ratio as does the parent uranium nucleus. The fission products therefore lie far outside of the range of stability and so are highly radioactive.

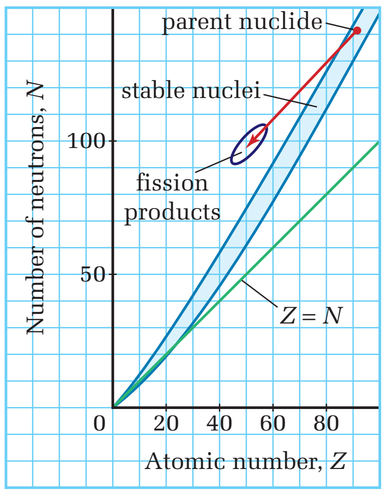


Figure 13.12 Fission products lie above the range of stability and are therefore mostly beta emitters.

Nuclear Fusion

Nuclear fusion is the opposite reaction to nuclear fission. In this process, small nuclei combine together to create larger nuclei. One such fusion reaction involves the combining of two isotopes of hydrogen, deuterium $({}^{2}_{1}\text{H})$ and tritium $({}^{3}_{1}\text{H})$. During the process, a neutron is released. The equation for the fusion reaction illustration in Figure 13.13 is ${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$.

PHYSICS FILE

Most stars generate energy through a process often called "hydrogen burning." This is not a good term, since no combustion is involved. Instead, single protons or hydrogen nuclei are fused together through a sequence of steps to form helium. Once the amount of hydrogen has diminished to the point where it no longer emits enough radiation to support the outer layers of the star against the inward pull of gravity, the star begins to collapse. This compression of the core causes its temperature to rise to the point at which helium begins to fuse. The star now swells up to become a Red Giant. This process of successive partial collapses and new fusion can continue until the core tries to fuse iron. At this point, the fusion reaction requires energy to continue, rather than releasing energy. The collapse is now catastrophic and the star blazes into a supernova.

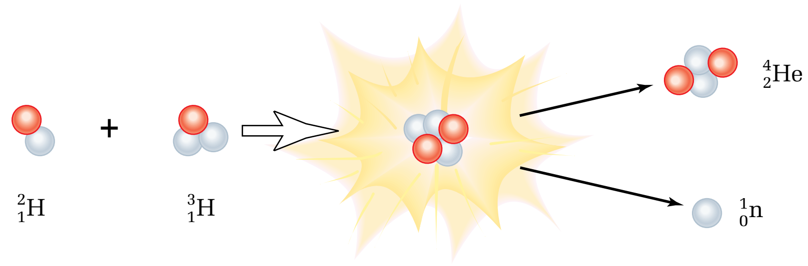


Figure 13.13 The helium nucleus has a much larger binding energy than either deuterium or tritium, so large amounts of energy are released in this nuclear fusion reaction.

Since nuclei repel each other due to their positive charges, they must be travelling at an extremely high speed for them to get close enough for the nuclear force to pull them together. An extremely high temperature can produce such speeds. As long as the product comes before iron in the periodic table, this reaction releases energy (is exothermic). After iron, the reaction requires an input of energy (is endothermic).

Fusion reactions occur in the cores of stars and in hydrogen bombs. Eventually it might be possible to control the fusion reaction so that it can be used to provide reasonably safe energy on Earth for many centuries to come. After all, the oceans contain vast amounts of hydrogen isotopes. Unfortunately, controlled fusion reactions have yet to provide a net output of energy.

SAMPLE PROBLEM

Energy from Nuclear Reactions

Determine the mass defect in the fission reaction given in the text and the amount of energy released due to each fission.

Data

Particle	Nuclear mass (u)
$^{235}_{92}U$	234.993
$^{1}_{0}n$	1.008
$^{141}_{56}Ba$	140.883
$^{92}_{36}Kr$	91.905

Conceptualize the Problem

- *Mass defect* is the *difference* between the total *mass* of the *reactants* and the total *mass* of the *fission products*.
- The *energy* released is the *energy equivalent* of the *mass defect*.

Identify the Goal

The mass defect, Δm, and the energy, E, released during each fission reaction

Identify the Variables and Constants

Known	Implied	Unknown
A, Z and m for all particles	$c = 2.998 \times 10^8 \frac{\text{m}}{\text{s}}$	Δm E

Develop a Strategy

Find the total mass of reactants.

$$m_{\text{neutron}} = 1.008\,665\ \text{u}$$
$$m({}^{235}_{92}\text{U}) = 234.993\ \text{u}$$
$$m_{\text{reactants}} = 1.008\,665\ \text{u} + 234.993\ \text{u}$$
$$m_{\text{reactants}} = 236.002\ \text{u}$$
$$m({}^{141}_{56}\text{Ba}) = 140.883\ \text{u}$$
$$m({}^{92}_{36}\text{Kr}) = 91.905\ \text{u}$$
$$m_{3\ \text{neutrons}} = 3 \times 1.008\,665\ \text{u}$$
$$m_{3\ \text{neutrons}} = 3.025\,995\ \text{u}$$

Find the total mass of the products.

$$m_{\text{products}} = 140.883\ \text{u} + 91.905\ \text{u} + 3.026\ \text{u}$$
$$m_{\text{products}} = 235.814\ \text{u}$$

Find the mass defect by subtraction.

$$\Delta m = 236.002\ \text{u} - 235.814\ \text{u}$$
$$\Delta m = 0.18767\ \text{u}$$

Convert the mass defect into kilograms.

$$\Delta m = (0.18767\ \cancel{\text{u}})\left(1.6605 \times 10^{-27}\ \frac{\text{kg}}{\cancel{\text{u}}}\right)$$
$$\Delta m = 3.1163 \times 10^{-28}\ \text{kg}$$

Convert the mass into energy, using $\Delta E = \Delta mc^2$.

$$\Delta E = \Delta mc^2$$
$$\Delta E = (3.1163 \times 10^{-28}\ \text{kg})\left(2.998 \times 10^8\ \frac{\text{m}}{\text{s}}\right)^2$$
$$\Delta E = 2.8009 \times 10^{-11}\ \text{J}$$

The mass defect is 0.1877 u or 3.116×10^{-28} kg. This is equivalent to an energy of 2.801×10^{-11} J.

Validate the Solution

The mass defect is positive, indicating an energy release.

PRACTICE PROBLEMS

7. Another possible fission reaction involving uranium-235 would proceed as follows.

$${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{90}_{38}\text{Sr} + {}^{135}_{54}\text{Xe} + 11\,{}^{1}_{0}\text{n}$$

Determine the mass loss and the amount of energy released in this reaction.

Particle	Mass (u)
${}^{1}_{0}\text{n}$	1.008 665
${}^{235}_{92}\text{U}$	234.993
${}^{90}_{38}\text{Sr}$	89.886
${}^{135}_{54}\text{Xe}$	134.879

continued ▶

continued from previous page

8. Determine the energy that would be released by the fusion of the nuclei of deuterium and tritium as indicated by the equation $^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$.

Particle	Mass (u)
$^{2}_{1}H$	2.013 553
$^{3}_{1}H$	3.015 500
$^{4}_{2}He$	4.001 506
$^{1}_{0}n$	1.008 665

9. In the Sun, four hydrogen nuclei are combined into a single helium nucleus by a series of reactions. The overall effect is given by the following equation.

$$4^{1}_{1}H \rightarrow ^{4}_{2}He + 2^{0}_{1}e$$

(a) Calculate the mass defect for the reaction and the energy produced by this fusion.

(b) If 4.00 g of helium contain 6.02×10^{23} nuclei, determine how much energy is released by the production of 1.00 g of helium.

Particle	Mass (u)
$^{1}_{1}H$	1.007 276
$^{4}_{2}He$	4.001 506
$^{0}_{1}e$	0.000 549

Detecting Radiation

Most people have heard of a Geiger counter, which is used to detect ionizing radiation; however, it is only one of a wide variety of instruments used for measuring radiation. Each is designed for a specific purpose, but they all function in the way that alpha, beta, and gamma radiation interacts with matter — by ionizing or exciting atoms or molecules in the object. Some possible interactions, summarized in Figure 13.14, are exposing film, ionizing atoms, or exciting atoms or molecules and causing them to fluoresce or phosphoresce.

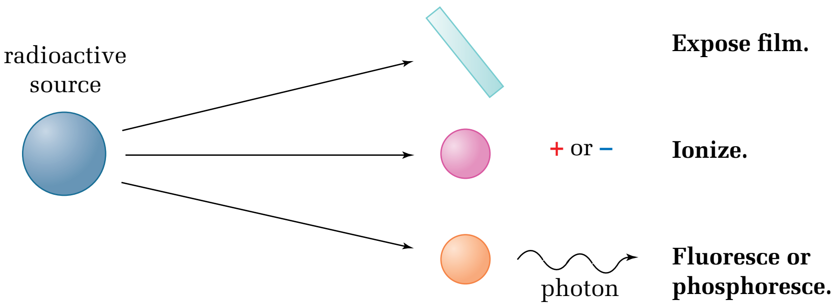


Figure 13.14 Radiation detectors commonly use one of three effects of radiation — the exposing of film, the ionization of matter, or the fluorescence of matter.

Radioactivity was discovered because it darkened film that was wrapped to protect it from light. For many years, people who worked in the nuclear industry or in laboratories where radioisotopes were used wore film badges. Technicians would develop the film, determine the degree of darkening, and calculate the amount of radiation to which the wearer of the badge had been exposed. Currently, many personnel badges contain lithium fluoride, a compound that enters an excited state when it absorbs energy from radiation. The material is thermoluminescent, meaning that when it becomes excited, it cannot return to the ground state unless it is heated. When heated, it emits light as it returns to its stable state. The technician collects the badges, puts the lithium fluoride in a device that heats it, and reads the amount of light emitted.

Geiger counters and other similar instruments detect the ions created by radiation as it passes through a probe that contains a gas at low pressure. When ionizing radiation passes through the gas, it ionizes some of the atoms in the gas. A high voltage between the wire and the cylinder accelerates the ions, giving them enough kinetic energy to collide with other gas molecules and ionize them. The process continues until an avalanche of electrons arrives at the central wire. The electronic circuitry registers the current pulse.

Geiger counters work well for low levels of radiation, but become saturated by higher levels. Ionization chambers are similar to Geiger counters, but they do not accelerate the ions formed by the radiation. They simply collect the primary ions formed by the radiation, which creates a current in the detector that is proportional to the amount of radiation present in the vicinity of the instrument. One such detector for high levels of radiation is called a "cutie pie."

Figure 13.15 This badge indicates the amount of radiation received by its wearer.

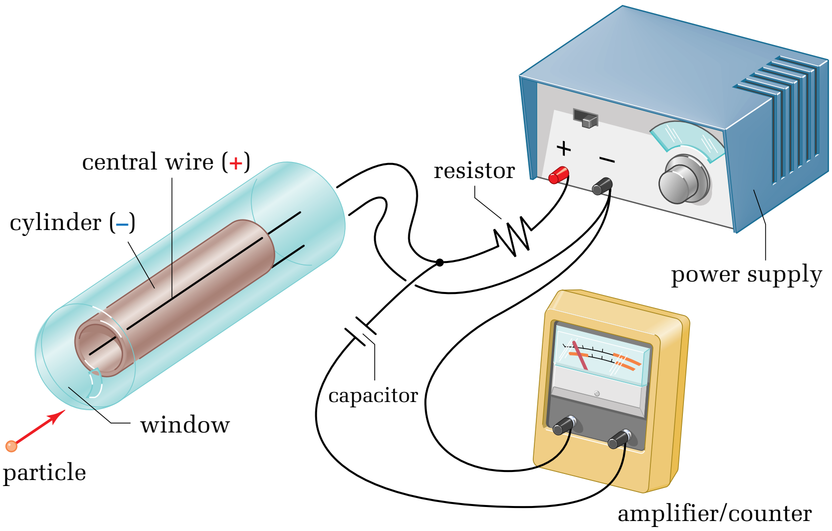


Figure 13.16 The passage of ionizing radiation through this tube creates an avalanche of electrons.

For accurately counting very small amounts of radioactivity, you would probably choose a scintillation counter. A crystal or a liquid consists of a material that, when excited by the absorption of radiation, will emit a pulse of light. Photomultiplier tubes that function on the principle of the photoelectric effect will detect the light and generate an electrical signal that is registered by electronic circuitry.

WEB LINK

www.mcgrawhill.ca/links/physics12

For in-depth information about radiation detection and protection, visit the TRIUMF Internet site. TRIUMF is Canada's national laboratory for particle and nuclear physics, located at the University of British Columbia in Vancouver. Just go to the above Internet site and click on **Web Links**.

INVESTIGATION 13-A

Half-Life of a Radioactive Isotope

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

In the second Multi-Lab at the start of this chapter (Half-Life), you investigated the concept of half-life by flipping coins. This investigation will allow you to actually determine the half-life of a radioactive isotope. A common type of generator for a half-life investigation contains cesium-137, which slowly decays into an excited nucleus, barium-137m. This in turn emits gamma radiation as it drops to its ground state, barium-137. The excited nucleus is leached from the system to provide a slightly radioactive solution.

Problem

The object of this investigation is to determine the half-life of barium-137m.

Equipment

- barium-137m
- Geiger counter
- small test tube and holder
- gloves
- tongs

CAUTION This investigation should be performed as a class demonstration. The experimenter should wear gloves and wash up at the end of the demonstration. All radioactive materials must be safely secured and locked up at the end of the demonstration.

Dispose of the barium solution according to WHMIS procedures.

Procedure

1. Prepare a table with the following headings: Time (min), Measured activity (Bq), Background radiation (Bq), and Net activity (Bq). **Note:** 1 Bq is one count per second.
2. Turn on the Geiger counter and place the tube of the counter close to the test tube holder. Measure an average value for the background radiation.
3. Prepare the barium solution and pour it into the test tube.
4. Take activity readings every half minute until the activity of the source is close to zero.
5. Subtract the background radiation from each reading in order to obtain the activity from the source.

Analyze and Conclude

1. Draw a graph of actual activity against time with the activity on the *y*-axis.
2. From the graph, determine the time interval during which the actual activity decreases by 50%. Repeat this determination in several regions of the graph. How constant is this time interval?
3. What is the half-life of this radioisotope?

Apply and Extend

4. From your graph, how would you determine the rate at which the activity is changing? Perform this determination at two different locations along the curve.
5. Brainstorm a number of possible uses for knowledge of the half-life of a radioisotope.

Applications of Radioactive Isotopes

Exposure to radiation can cause cancer, but it also can destroy cancerous tumours. How can radiation do both?

As you have learned, alpha, beta, and gamma radiation ionize atoms and molecules in their paths. In living cells, the resulting ions cause chemical reactions that can damage critical biological molecules. If that damage occurs in a few very precise regions of the genetic material, the result can be a mutation that destroys the cell's ability to control growth and cell division. Then, the cell divides over and over, out of control, and becomes a cancerous tumour.

On the other hand, if the amount of radiation is much higher, the damage to the molecules that maintain the cell functions will be too great, and the cell will die. If a few healthy cells die, they can usually be replaced, so little or no harm is caused to the individual. If cancerous cells die, the tumour could be destroyed and the person would be free of the cancer.

Great care must be taken when treating tumours with radiation, since healthy cells in the area are exposed to radiation and might themselves become cancerous. If the amount of irradiation is excessive (in a nuclear accident, for example) and the entire body is exposed, too many cells could die at the same time, seriously affecting the ability of the organs to function. Death would result.

Irradiating tumours with gamma radiation is sometimes the only feasible way to treat a tumour, however, and it can be very successful. Figure 13.17 shows one method of treating a tumour with radiation from the radioisotope cobalt-60. A thin beam of gamma rays is aimed at the tumour and then the unit rotates so that the beam is constantly aimed at the tumour. In this way the tumour is highly irradiated, while the surrounding tissue receives much less radiation.

Figure 13.17 Gamma radiation from cobalt-60 is used to destroy tumours.

COURSE CHALLENGE

"Seeing" with Radioisotopes

Techniques exist by which radioisotopes, injected into living bodies, will accumulate in infected areas or other diseased tissues. Observing the location of the radioisotopes provides critical information. Refer to page 605 for ideas to help you include these scanning techniques in your *Course Challenge*.

Radioactive Tracers

Because traces of radioactivity can be detected and identified, scientists can use very small quantities of radioactive substances to follow the chemical or physical activity of specific compounds. For example, iodine-131 is useful for investigating the heart and the thyroid gland. Phosphorus-32 accumulates in cancerous tumours, identifying their location. Technetium-99 portrays the structure of organs. Other applications include the following.

- Slight amounts of a radioisotope added to a fluid passing through an underground pipe allows technicians to locate leaks.
- Gamma radiation is used to sterilize food so that it will stay fresh longer.
- Exposing plants to radioactive carbon dioxide allows researchers to determine the long series of chemical reactions that convert carbon dioxide and water into glucose.
- Radioisotopes are a common tool in biochemistry research.

Smoke Detectors

Many smoke detectors contain a small amount of a radioisotope that emits alpha radiation. Because the gas in the detector is ionized, a current can pass through and be measured. When soot and ash particles in smoke enter the detector, they tend to collect these ions and neutralize them. The resulting drop in current triggers the smoke detector alarm.

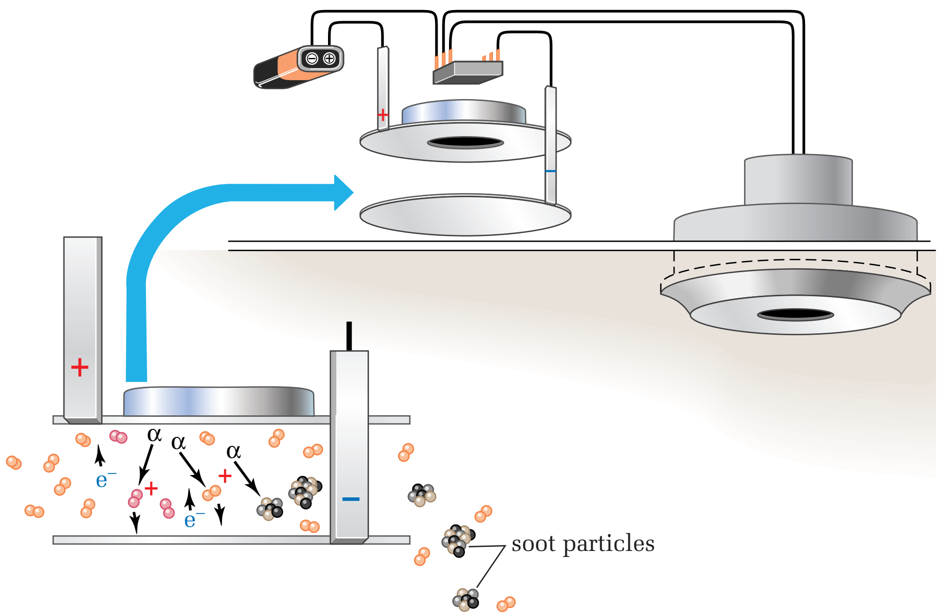


Figure 13.18 When alpha particles ionize molecules in the air, the positive ions are attracted to the negative electrode and the electrons are attracted to the positive electrode and a current passes through the circuit. Soot particles absorb and neutralizes some of the ions and the current decreases.

13.2 Section Review

1. K/U Explain why beta negative radiation tends to do less biological damage than an equal amount of alpha radiation.
2. K/U Which type of particle would you expect to penetrate best through lead, a beta positive particle (positron) or a beta negative particle? Give a reason for your choice.
3. K/U Why is gamma radiation much more penetrating than beta negative radiation?
4. K/U State the conservation laws used in writing nuclear reactions.
5. C Prepare a table for alpha radiation, beta negative radiation, and gamma radiation, comparing them with respect to mass, charge, relative penetrating ability, and relative biological damage.
6. C Draw a graph to illustrate the decay of carbon-14 in a wooden relic. Assume that the initial mass of the isotope in the wood was 240 mg.
7. C Draw a decay sequence similar to the one shown in Figure 13.8 on page 561. Begin with $\left({}^{255}_{101}\text{Md}\right)$. It emits four alpha particles in succession, then a beta negative particle, followed by two alpha particles and then a beta negative particle. Another alpha emission is followed by another beta emission. (There are more, but this is enough for this question.)
8. C Fission is a process in which a nucleus splits into two parts that are roughly half the size of the original nucleus. In fusion, two nuclei fuse, or combine, to form one nucleus. These reactions seem to be opposite to each other, yet they both release large amounts of energy. Explain why this is not really a contradiction. Use the graph of binding energy per nucleon versus atomic mass number in your explanation.
9. MC Give a possible reason why a smoke detector uses an alpha source rather than a beta or gamma emitter.
10. MC Suggest an equation to represent the transformation of nitrogen-14 into carbon-14.
11. I Research the use of radioisotopes for medical or non-medical purposes and prepare a poster to illustrate your findings.

UNIT PROJECT PREP

The eventual identification of radioactivity began as a "mysterious" laboratory result at the turn of the nineteenth century.

- While working on your unit project, have you found information about people who could be called "visionaries" because of their belief that radioactivity would eventually play a role in everyone's daily life?
- Can you identify emerging scientific discoveries that you believe will result in wide-ranging applications by the end of the twenty-first century, as radioactivity has over the past 100 years?

13.3 Elementary Particles

SECTION EXPECTATIONS

- Define and describe the concepts and units related to the present-day understanding of elementary particles.
- Analyze images of the trajectories of elementary particles to determine the mass-versus-charge ratio
- Describe the standard model of elementary particles in terms of the characteristic properties of quarks, leptons, and bosons.
- Identify the quarks that form familiar particles such as the proton and the neutron.

KEY TERMS

- lepton
- hadron
- quark
- standard model

By the 1930s, scientists believed that they knew the particles on which all matter was built — the proton, the neutron, and the electron. These were the "elementary" particles, in that nothing was more basic. Nature, however, was not that simple. The first hint of a greater complexity came with the missing energy and momentum in beta decay and Pauli's proposal of the neutrino.

Today, even the proposition that these particles are massless is being challenged. Several difficulties with our understanding of the nuclear reactions in the Sun would be cleared up if neutrinos actually had mass. Neutrinos are extremely tiny and neutral, so they barely interact with matter. Consequently, during the day, neutrinos rain down through us from the Sun. At night, they pour through the planet and stream upward through us.

In 1935, Japanese physicist, Hideki Yukawa (1907–1981) proposed that the strong nuclear force that bound the nuclei together was carried by a particle with a mass between that of the electron and the proton. This particle came to be known as a "meson," because of its intermediate mass. Eventually, it was discovered in 1947, when it was termed the "π meson" (or simply, the "pion"). In fact, though, these particles are not the ones that hold the nuclei together — they never interact with other particles or nuclei by means of the strong force. Elementary particle physics is one of the most active fields in theoretical physics.

The Search for New Particles

In Section 13.2, you learned that research into beta decay revealed the existence of neutrinos and positrons. Physicists discovered that positrons were, in fact, antielectrons. When a particle (positron) and its antiparticle (electron) interact, they annihilate each other and transform into two gamma rays. With the realization that the proton, neutron, and electron were not the only subatomic particles, physicists began to search in earnest for more elementary particles.

In Chapter 8, Fields and Their Applications, you learned how electric and magnetic fields are used in powerful instruments to accelerate protons and electrons to close to the speed of light. When these extremely high-energy particles collide with other particles and nuclei, new, very short-lived particles are produced. For example, when a very high-energy proton collides with another proton, a neutral particle called a "neutral pion" (π^0) is produced. The particle exists for only about 0.8×10^{-16} s, and then decays into two gamma rays.

You are probably wondering how physicists can study a particle that disappears within 10^{-16} s after it is produced. One instrument that physicists use is called a "cloud chamber," which was invented in 1894 by C.T.R. Wilson (1869–1959). A cloud chamber contains a cold, supersaturated gas. When any form of ionizing radiation passes through, the ions formed in the path of the particle form condensation nuclei and the cold gas liquefies, creating a visible droplet. If the cloud chamber is placed in a magnetic field, charged particles will follow curved paths. Physicists analyze photographs of the "tracks" in the cloud chamber and can determine their size and speed. In the following investigation, you will make observations using a cloud chamber.

During the 1930s and 1940s, several more particles were discovered, and physicists also realized that every elementary particle has an antiparticle. U.S. physicists S.H. Neddermeyer and C. D. Anderson discovered positive and negative muons (μ^+ and μ^-) that have a mass about 207 times that of an electron. Muons have a lifetime of 2.2×10^{-6} s and decay as shown below.

$$\mu^- \rightarrow e^- + \nu_\mu + \overline{\nu}_e$$
$$\mu^+ \rightarrow e^+ + \overline{\nu}_\mu + \nu_e$$

These discoveries also revealed the existence of muon neutrinos (ν_μ) and muon antineutrinos ($\overline{\nu}_\mu$). The subscript e must now be used to indicate electron neutrinos.

In the 1940s, two more pions were discovered, one having a positive charge and the other a negative charge. These pions have a lifetime of 2.6×10^{-8} s and decay as shown below.

$$\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$$
$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

Since these early discoveries, physicists have continued to identify many more extraordinary particles. Eventually, a pattern evolved and physicists were able to start classifying these elementary particles according to the types of forces through which they interact with other particles. For example, protons and neutrons interact through the strong nuclear force, whereas electrons do not experience the strong nuclear force. Particles might be affected by one or more of the four fundamental forces — gravitational, electromagnetic, strong nuclear, and weak nuclear forces.

Families of Particles

The smallest family of particles is the photon family, consisting only of the photon itself. Photons interact only through the electromagnetic force and interact only with charged particles. The photon is its own antiparticle.

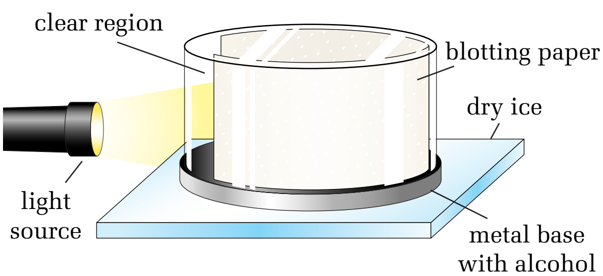
The **lepton** family of particles interacts by means of the weak nuclear force. Leptons can interact through the gravitational force,

INVESTIGATION 13-B

The Wilson Cloud Chamber

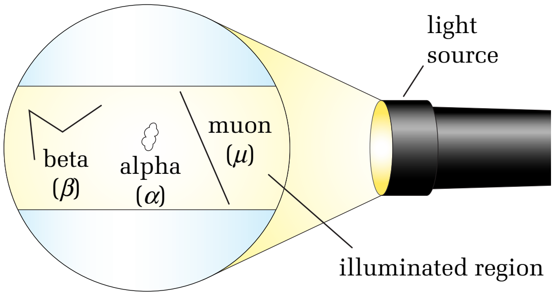
TARGET SKILLS
- Performing and recording
- Analyzing and interpreting

The cloud chamber that you will use consists of a short, transparent cylinder with a glass top and a metal base. The sides are lined with blotting paper, except for a section through which you will shine an intense light beam, as shown in the diagram. The light will make the liquid drops visible against the dark background.



The next diagram shows the general appearance of some tracks that you might see. Alpha radiation appears as a short (1 cm to 2 cm long), thick puff of white "cloud." Beta particles (high-speed electrons) appear as long, thin strands that bend gradually or zigzag from collisions with atoms.

Because the muon is much more massive than the beta particle, it appears as a thin, extremely straight strand that goes across the chamber. Many muons angle downward and are difficult to observe.



Since Earth is constantly bombarded by cosmic rays (which are really high-energy particles), you can nearly always observe tracks in a cloud chamber.

Equipment
- cloud chamber
- radioactive source (optional)
- light source
- alcohol
- dry ice

CAUTION Do not touch dry ice unless you are wearing thick gloves.

Procedure
1. Place the cloud chamber on the block of dry ice and pour in the alcohol to a depth of about 1 cm. Put on the glass cover and let the chamber stand for a few minutes, until the alcohol has a chance to reach equilibrium.
2. Working in groups of three or four, take turns watching the cloud chamber carefully for a total of at least 15 min. Make a sketch of every track that you see.
3. Obtain similar data from all of the groups that are performing the observations.
4. (Optional) If your cloud chamber has a small access hole in the side and if you have a small radioactive source on the end of a pin, insert it into the hole.
5. Make a sketch of the tracks that you observe emanating from the source.

Analyze and Conclude
1. Try to identify the tracks that you observed.
2. List the types of radiation observed in this investigation, from the most common to the least common.
3. What type of radioactive source did you use? Were the tracks consistent with the nature of the radiation emitted by the source? Explain.

and if they are charged, through the electromagnetic force, but are immune to the strong nuclear force. Once called the “beta decay interaction,” the weak nuclear force is involved in beta decay.

As you would probably expect, electrons and electron neutrinos are leptons. Muons and their neutrinos are also leptons. A more recently discovered particle, the tau (τ) particle and its neutrino, also fit into the lepton family. As previously stated, for every particle, there is an antiparticle. The antiparticles always have the same mass as the particle, and if the particle has a charge, the antiparticle has the opposite charge. When the particles are neutrally charged, the antiparticle is also neutral but opposite in some other property. In such cases, the antiparticles are denoted with a bar over the symbol. Leptons and their antiparticles appear to be true elementary particles. There is no indication that they consist of any more fundamental particles.

Particles of the **hadron** family interact through the strong and weak nuclear forces. Hadrons can also interact through the gravitational force, and if they are charged, through the electromagnetic force. The hadron family is the largest family and is subdivided into the groups, mesons and baryons. The common proton and neutron and their antiparticles are baryons, while pions are mesons. Pions were at one time called “pi mesons.” As larger and more powerful particle accelerators were built, more and more hadrons were discovered.

Table 13.4 summarizes the properties of most of the subatomic particles that have been discovered. However, the list of hadrons is incomplete and will certainly continue to grow as physicists continue their search. Since most of the particles are very short-lived and are eventually transformed back into energy, Table 13.4 reports the energy equivalent of the rest masses of the particles in units of MeV, rather than reporting in units of mass. If you calculated the energy equivalent of 1 u, you would find that it is about 931.5 MeV.

Quarks

As the number of hadrons that had been discovered grew, physicists became suspicious that hadrons might not really be elementary particles. Some physicists were studying the scattering of electrons off protons and neutrons and saw evidence that there were three “centres” of some type within the nucleons. At the same time, theoretical physicists Murray Gell-Mann (1929–) and George Zweig (1937–), working independently, proposed the existence of truly elementary particles that made up hadrons. Gell-Mann somewhat jokingly called these particles **quarks**, from a line in *Finnegan’s Wake* by James Joyce — “Three quarks for Muster Mark.” The name stuck. Today, physicists accept that quarks are the elementary particles of which all hadrons consist.

Table 13.4 Some Particles and Their Properties

Family	Particle	Particle Symbol	Antiparticle symbol	Rest energy (MeV)	Lifetime (s)
Photon	photon	γ	self*	0	stable
Lepton	electron	e^-	e^+	0.511	stable
	muon	μ^-	μ^+	105.7	2.2×10^{-6}
	tau	τ^-	τ^+	1784	10^{-13}
	electron neutrino	ν_e	$\overline{\nu}_e$	≈0	stable
	muon neutrino	ν_μ	$\overline{\nu}_\mu$	≈0	stable
	tau neutrino	ν_τ	$\overline{\nu}_\tau$	≈0	stable
Hadron					
Mesons	pion	π^+	π^-	139.6	2.6×10^{-8}
		π^0	self*	135.0	0.8×10^{-16}
	kaon	K^+	K^-	493.7	1.2×10^{-8}
		K^0_S	$\overline{K}^0_S$	497.7	0.9×10^{-10}
		K^0_L	$\overline{K}^0_L$	497.7	5.2×10^{-8}
	eta	η^0	self*	548.8	$<10^{-18}$
Baryons	proton	p	$\overline{p}$	938.8	stable
	neutron	n	$\overline{n}$	939.6	900
	lambda	Λ^0	$\overline{\Lambda}^0$	1116	2.6×10^{-10}
	sigma	Σ^+	$\overline{\Sigma}^-$	1189	0.8×10^{-10}
		Σ^0	$\overline{\Sigma}^0$	1192	6×10^{-20}
		Σ^-	$\overline{\Sigma}^+$	1197	1.5×10^{-10}
	omega	Ω^-	Ω^+	1672	0.8×10^{-10}

*The particle is its own antiparticle.

At the time that quarks were proposed, three quarks and their antiquarks could account for all known hadrons. Mesons consisted of two quarks, and baryons consisted of three quarks, given the names "up" (u), "down" (d), and "strange" (s). Uniquely, quarks have fractional charges of $+\frac{2}{3}e$, $-\frac{1}{3}e$, $-\frac{1}{3}e$, respectively, while the antiquarks have charges of the same size but opposite charge. Figure 13.19 gives examples of the quarks that make up the common neutron, proton, and positive and negative pions. Notice that the baryons consist of three quarks and the mesons consist of two.

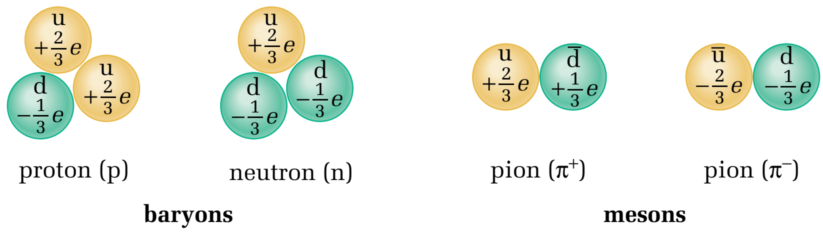


Figure 13.19 The combination of quarks in hadrons always results in a neutral charge or in a unit charge.

The quark model worked very well in explaining the properties of hadrons until about 1974, when more hadrons were discovered. Eventually, physicists discovered that six quarks were necessary in order to account for all of the newly discovered hadrons. The three new quarks were given the names "charmed" (c), "top" (t), and "bottom" (b), although some physicists, particularly in Europe, prefer to call the last two quarks, "truth" and "beauty." The quarks and some of their properties are summarized in Table 13.5.

Table 13.5 The Quarks

Quark name	Rest energy (GeV)	Quark		Antiquark	
		Symbol	Charge	Symbol	Charge
up	0.004	u	$+\frac{2}{3}e$	$\bar{u}$	$-\frac{2}{3}e$
down	0.008	d	$-\frac{1}{3}e$	$\bar{d}$	$+\frac{1}{3}e$
strange	0.15	s	$-\frac{1}{3}e$	$\bar{s}$	$+\frac{1}{3}e$
charm	1.5	c	$+\frac{2}{3}e$	$\bar{c}$	$-\frac{2}{3}e$
top (or truth)	176	t	$+\frac{2}{3}e$	$\bar{t}$	$-\frac{2}{3}e$
bottom (or beauty)	4.7	b	$-\frac{1}{3}e$	$\bar{b}$	$+\frac{1}{3}e$

As physicists collected more and more details about hadrons and their quarks, they discovered that quarks have more properties than just charge. A property that physicists call "colour" explains many of their observations, as well as placing the quark in agreement with the Pauli exclusion principle.

Exchange Particles

Physicists' current view of the structure of matter is summarized in Figure 13.20, but this summary does not present the complete picture. You have read many times about the four fundamental forces of nature, the properties of these forces, and how elementary particles are even categorized according to the forces that they experience. The question remains: How do these forces work? While studying elementary particles, physicists also discovered some basic information about the fundamental forces of nature. The **standard model** refers to the currently accepted mechanisms of the strong, weak, and electromagnetic forces. Physicists hope to bring the gravitational force into the model, but so far, it has been elusive.

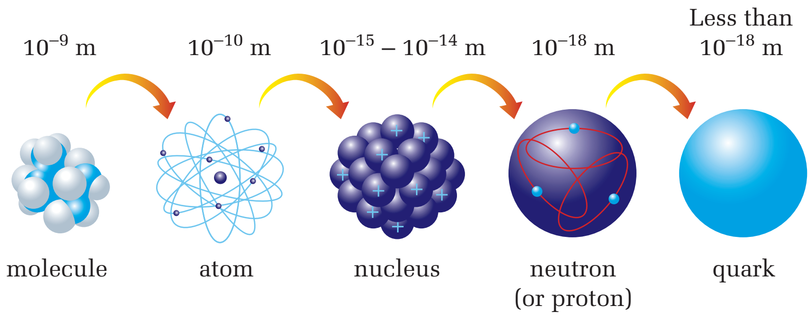


Figure 13.20 Scientists' view of the smallest indivisible piece of matter has changed greatly over the past century — going from Dalton's model of the atom to the current view of the quark.

Physicists have found particles that are exchanged by the elementary particles that account for the interactions between them. Some of the properties of these exchange particles are listed in Table 13.6.

When charged particles interact through the electromagnetic force, they exchange a photon. Because photons have no mass and travel at the speed of light, the range of the force is unlimited. In the opposite extreme, the weak nuclear force is mediated by bosons that have a large mass and such a short lifetime that the range of the interaction is extremely short.

Table 13.6 Force Carriers

Force	Name of Particle	Symbol	Mass (GeV)	Charge	Range (m)
electromagnetic	photon	γ	0	0	unlimited
weak nuclear	weak boson	W^+ W^- Z^o	80.2 80.2 91.2	$+e$ $-e$ 0	10^{-17}
strong nuclear	gluon	g	0	0	10^{-15}
gravitational	graviton*	G	0	0	unlimited

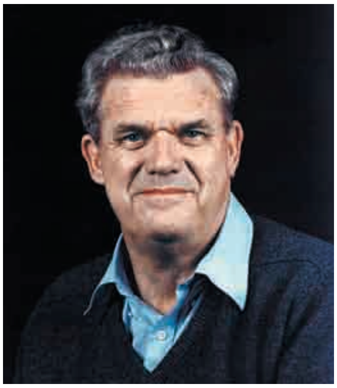
*The graviton has been proposed as a carrier of gravitational force. However, its existence has yet to be confirmed.

The exchange of gluons holds quarks together in hadrons. The theory is that when quarks exchange gluons, they change colour. Physicists have proposed the existence of a graviton as an exchange particle for the gravitational force and have determined some of the properties that such a particle would have to have. However, they have never observed any indication that gravitons exist. As you can see, the story is far from complete and there are many more challenges ahead for elementary particle physicists.

CANADIANS IN PHYSICS

"Not the Brightest Student" — But Wins Nobel Prize

There is no greater prize for a scientist than the Nobel Prize, and in 1990, Dr. Richard Taylor became the first Canadian to win the prestigious award in physics. He and two U.S. colleagues shared the award for proving the existence of quarks. The team used a powerful linear accelerator, operating at 21 GeV, to bombard protons and neutrons with electrons. They discovered that protons and neutrons, once thought to be indivisible, are made of these quarks, the existence of which had been theorized but never proven.



Dr. Richard Taylor
Courtesy Stanford Linear Accelerator Center

Born and raised in Medicine Hat, Alberta, Dr. Taylor was interested in experimental science from an early age, and this interest resulted in an accident that could have prematurely ended his science career. Several older boys showed him a formula for a better type of gunpowder than was available at that time. His attempt to follow the formula resulted in a powerful explosion that amputated three fingers of his left hand.

Dr. Taylor has said in interviews that he wasn't the brightest student in high school. "I did reasonably well in mathematics and science, thanks to some talented and dedicated teachers," he commented, "but I wasn't an outstanding student, although I did read quite a bit and high school mathematics came quite easily to me. You don't necessarily have to be a great student to do well later in life, although it is always important to work hard."

After completing his undergraduate work at the University of Alberta, Dr. Taylor was accepted into the graduate program at Stanford University in California, where he has spent much of his working life. "I found I had to work hard to keep up with my fellow students," said Dr. Taylor, "but learning physics was great fun in those surroundings."

Dr. Taylor stresses the importance of an inquiring mind. "It's fun to understand things and you should learn all you can. Reading gives you independence and a sense of freedom," he said, adding that he believes it is important to be educated in a broad range of subjects. Curiosity and a love of experimentation drive Dr. Taylor. While he has a great respect for theoretical physicists, calling them "smarter" than experimental physicists, he feels that "in experimental science, you can make contributions more easily."

Still a resident of California, Dr. Taylor works at the Stanford Linear Accelerator Center, also spending time in Europe at the HERA laboratories in Germany. After his prize-winning work to discover the quark, he is now interested in searching for gravitational waves and is involved with a new satellite experiment to detect high-energy gamma rays from sources in outer space.

Although at age 71 Dr. Taylor considers most of his scientific contributions to be behind him, much more work lies ahead in the field of particle physics. To the next generation of physicists, he says, "What the young people have to deal with is the fact that there are three generations of quarks. There are the quarks that everything we know of is made of, and then there are two more sets of quarks. The question is: Why are they there?" Dr. Taylor expects this question to occupy the physicists of tomorrow "for the next 50 years."

INVESTIGATION 13-C

Measuring the Mass-to-Charge Ratio for Electrons

TARGET SKILLS
- Identifying variables
- Performing and recording
- Conducting research

In this investigation, you will perform an experiment very similar to the one in which J.J. Thomson discovered and characterized the electron. You will accelerate electrons by means of a large potential difference and then deflect them in a cathode ray tube by means of a known magnetic field.

Problem

(1) Determine the speed of electrons that pass through a cathode ray tube and **(2)** measure the ratio of the mass to the charge for the electron.

Equipment

- DC power supply for heated cathode tubes
- Helmholtz coils
- DC power supply for Helmholtz coils
- ammeter
- Thomson deflection tube

CAUTION Avoid touching the high voltage connections.

A cathode ray tube emits a small amount of X rays, so stay in front of it very briefly.

Procedure

1. With all power supplies turned off, set the anode voltage to zero.
2. Connect the Thomson deflection tube to the power supply according to the instructions in the manual for the tube. Check that all connections are secure and correct.
3. Set the power supply for the Helmholtz coils to zero. Connect the ammeter in series with the power supply and the coils.
4. Measure the radius of the Helmholtz coils (or record the value provided with the coils).
5. Turn on the deflection tube power supply. Make sure that the filament voltage is set correctly, according the manual (probably 6.3 V).
6. Increase the anode voltage to 5000 V and observe the glowing trace of the cathode rays across the screen.
7. Gradually increase the voltage of the Helmholtz coils until the electron beam has been strongly deflected by the time the beam leaves the screen. Ensure that the maximum current for the coils is not exceeded. Record the value of the current.
8. Record the coordinates for two grid points along the trajectory of the beam.
9. Reduce all voltages to zero and turn off the power supplies.

Calculating the Radius of the Circular Trajectory

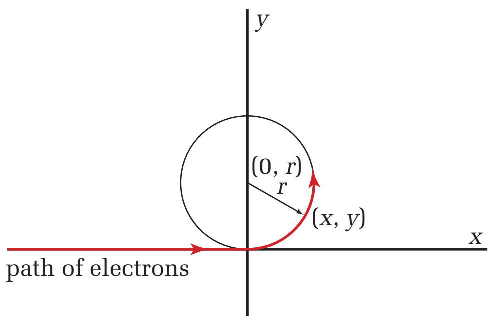
In general, the distance between any two points with known coordinates is given by

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

For the circular trajectory, the distance between any point on the circle and the centre is the radius, r.

Since the deflection begins when the beam passes throught the origin of the graph, the centre of the circle must be at $(0, r)$

Thus, $r = \sqrt{(x - 0)^2 + (y - r)^2}$ (3)



Analyze and Conclude

1. **(a)** State the coordinates for the two observed points on the electron beam's trajectory.

   (b) Write an expression for r based on the equation given above.

   (c) Calculate the value of r for each point and find the average. Use this average in your further calculations.

2. The magnetic field between the two Helmholtz coils is given by the equation

$$B = \frac{32\pi nI}{5\sqrt{5}(R_c)} \times 10^{-7}\ \text{T},$$

   where n is the number of turns in the coils (as indicated on the coils), I is the current in amperes, and R_c is the radius of the coils. Calculate the magnetic field (B) between the coils.

3. Use the equation $v = \frac{2V}{Br}$ to determine the speed of the electrons. Substitute the value for v into an expression, $\frac{m}{e} = \frac{2V}{v^2}$, to find the charge-to-mass ratio.

4. Review the information in Chapter 8, Fields and Their Applications, about the motion of charged particles moving through a magnetic field and derive the equations above.

13.3 Section Review

1. K/U How do physicists know of the existence of particles with lifetimes that are as short as 10^{-10} s, and how can they determine any properties of these particles?

2. K/U Why are the electrons in the lowest energy level of an atom not affected by the strong nuclear force?

3. K/U

   (a) State two ways in which leptons differ from hadrons.

   (b) In what ways are mesons similar to baryons?

   (c) How are mesons different from baryons?

4. MC Using quark notation, how could you represent **(a)** a negative pion and **(b)** an antiproton?

5. MC An antineutron must be neutral and have exactly the same mass as the neutron. What should its quark composition be?

6. I If neutrinos barely interact with matter, how can they be detected? Research the question and provide a diagram to explain the process.

CHAPTER 13 Review

REFLECTING ON CHAPTER 13

- The neutron was discovered by James Chadwick.
- The particles in the nucleus are called "nucleons" and consist of protons and neutrons. Their number is indicated as the atomic mass number (A).
- The number of protons in the nucleus is indicated by the atomic number (Z).
- In a neutral atom, the number of electrons orbiting the nucleus equals the number of protons in the nucleus.
- A common mass unit for atoms and nuclei is the atomic mass unit (u).

$$1\ \text{u} = 1.6605 \times 10^{-27}\ \text{kg}$$

- The mass defect is the difference between the separate total mass of the nucleons and the mass of the nucleus. It represents the binding energy for that nucleus.
- Nuclear fission is the splitting apart of a very large nucleus to produce two smaller nuclei plus several neutrons and energy.
- Nuclear fusion is the joining of two low-mass nuclei to form a larger nucleus.
- Henri Becquerel discovered radioactivity.
- Radioactivity consists of the emission of alpha particles (helium nuclei), beta negative particles (high-speed electrons), beta positive particles (high-speed positrons), and gamma rays (photons).
- Alpha, beta, and gamma radiation vary in their mass, charge, penetrating ability, and possible biological damage. The passage of any of these rays through matter leaves ions behind, so the radiation is called "ionizing radiation."
- Radiation can be detected by exposing film; causing ionization in matter by using, for example, the Geiger counter; and identifying the fluorescence or phosphorescence that radiation creates in some substances.
- Radioactivity has many uses, both medical and non-medical. For example, it is commonly used in smoke detectors.
- During any nuclear reaction the total atomic mass number (A) and the total atomic number (Z) remain unchanged.
- Transmutation is the conversion of one element into another.
- The rate of radioactive decay is indicated by the half-life of the radioisotope.
- Radioactive decay rates can be used to determine the age of ancient materials.
- The amount of a radioactive isotope remaining after a given time interval can be determined by using the following equation.

$$N = N_0\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

- A common unit in the field of radioactivity and radiation is the becquerel.
- Exposure to radiation can lead to various levels of sickness and, if severe enough, to death.
- Subatomic particles are grouped into three families — photons, leptons and hadrons. Hadrons consist of particles that are built from quarks.
- According to the standard model, forces are the result of the exchange of particles.
- The model of matter that involves particles as force carriers and the concept that all hadrons, such as protons and neutrons, are composed of quarks is known as the standard model.

Knowledge/Understanding

1. Use Einstein's theory to explain how the term "mass defect" refers to an amount of energy.
2. Outline the rationale for postulating the existence of a strong nuclear force as one of the fundamental forces of nature.
3. Compare the range of the field of influence of a strong nuclear force with that of an electromagnetic force when considering the effect of each on a proton near or in the nucleus of an atom.
4. Explain, with the aid of a series of sketches, the relative effects of an electromagnetic force and a strong nuclear force at several stages as a proton is propelled toward a nucleus in a fusion reaction.
5. Describe the characteristics of the three common forms of radioactivity.
6. Explain, based on our scientific understanding of radiation, why it is now useful to use the concept of a nucleon rather than a proton as a basic particle located in an atom's nucleus.
7. Explain why the daughter nuclei from fission reactions are likely to be radioactive.
8. Describe the concept of a *force carrier*. Outline how this concept is an explanatory device for outlining a scientific model in which mass and energy are simply different forms of the same phenomena.

Inquiry

9. The concept of antimatter has stimulated the imagination of many science fiction writers. Research and prepare a report of the scientific discoveries that led to the inclusion of antimatter particles in scientific models of matter.
10. Insight into nuclear structure can be gained by considering the binding energy per nucleon, $\Delta E/A$, for different elements. **(a)** Describe how the calculation of $\Delta E/A$ is used to indicate nucleons in a specific nucleus are tightly bound or loosely bound. **(b)** Calculate the binding energy, in both joules and MeV, for the following 12 elements: helium, carbon, neon, oxygen, chlorine, manganese, iron, cobalt, silver, gold, cesium, and uranium. In each case, divide the binding energy by the mass number, A.
    (c) Write the equation for a common fusion reaction. Locate the position of the initial nuclei, by their nucleon number, on the graph on page 550. Locate the position of the fused nuclei on the graph. Describe the effect of fusion on the binding-energy-per-nucleon ratio.
    (d) Write the equation for a common fission reaction. Locate on the graph on page 550 the position of the initial nuclei, by their nucleon number. Locate the position of the daughter nuclei on the graph. Describe the effect of fission on the binding energy per nucleon ratio.
    (e) Locate on the graph the range of nucleon numbers of those elements that are more likely to undergo fusion and the range of nucleon numbers for those that are more likely to undergo fission.
11. Suppose an experiment is designed to allow continuous observation of a single atom of a certain radioactive material. If the half-life is 1.5 h, can the observer predict when the atom will decay?
12. Use conservation laws to determine which of the following reactions are possible. Explain your reasoning in each case.
    (a) $p + p \rightarrow p + n + \pi^+$
    (b) $p + p \rightarrow p + p + n$
    (c) $p + p \rightarrow p + \pi^+$
    (d) $p + p \rightarrow p + p + \pi^0$

Communication

13. Explain why neutrons are said to make better "nuclear bullets" than either protons or electrons.
14. Use the concepts of fission, fusion, and binding energies to provide a scientific explanation of what limits the size a stable nucleus.

Making Connections

15. Explain why, to date, nuclear reactors have been constructed to use fission, but none have been constructed to use fusion.

16. Food and surgical supplies are sometimes sterilized by radiation. What are the advantages and disadvantages of using this procedure rather than sterilization by heating?

17. In 1989, two scientists at the University of Utah announced to the public that they had produced excess energy in a fusion-like experiment at room temperatures. The experiment was dubbed "cold fusion" and the scientists thought they had identified a new, cheap energy source. However, other experimenters failed to reproduce the results of this experiment, so even today, most of the scientific community does not consider cold fusion as a real possibility. Research this episode of physics history and use it to discuss the roles of peer review and reproducing results in scientific methodology.

18. Prepare a report on how radioactive tracers are used to either **(a)** follow the path of rainwater through groundwater reservoirs to lakes, streams, and wells or **(b)** map ocean currents.

19. The word "radiation" strikes fear into the hearts of many people. In fact, many would not live anywhere near a nuclear power station. Gather information about common concerns and misconceptions about "radiation" by interviewing people and generating a file of newspaper articles. Identify four or five of the common issues. Write a scientific perspective on each. Make a recommendation of the safety features that you consider essential for operating a nuclear power station in such a way that you would feel comfortable living within a one kilometre radius of it.

Problems for Understanding

20. Determine the number of protons, neutrons and electrons in **(a)** a doubly ionized calcium ion $^{40}_{20}Ca^{++}$ **(b)** an iron atom $^{56}_{26}Fe$ **(c)** a singly charged chlorine ion $^{35}_{17}Cl^{-}$.

21. Calculate the binding energy for **(a)** $^{12}_{6}C$ with a atomic mass of 12.000 000 u **(b)** $^{133}_{55}Cs$ with a atomic mass of 132.905 429 u.

22. Write the equation for the alpha decay of thorium: $^{230}_{90}Th$.

23. What fraction of the original number of nuclei in a sample are left after **(a)** two half-lives, **(b)** four half-lives, and **(c)** 12 half-lives?

24. **(a)** How much energy is released when radium-226 (nuclear mass 225.977 09 u) alpha decays and becomes radon-222 (nuclear mass 221.970 356 u)? Answer in MeV.
(b) If the nucleus was initially at rest, calculate the velocities of the alpha particle and the radon-222 nucleus in part (a).
(c) What percentage of the total kinetic energy does the alpha particle carry away?

25. Hafnium-173 has a half-life of 24.0 h. If you begin with 0.25 g, how much will be left after 21 days?

26. How long will it take a 125 mg sample of krypton-89, which has a half-life of 3.16 min, to decrease to 10.0 μg?

27. A scientist at an archeological dig finds a bone that has a carbon-14 activity of 5.70×10^{-2} Bq. If the half-life of carbon-14 is 5.73×10^{3} a, what is the age of the bone? (Assume that the initial activity was 0.23 Bq.)

28. Suppose you began with a sample of 800 radioactive atoms with a half-life of 5 min.
(a) How many atoms of the parent nucleus would be left after 10 min?
(b) How many atoms of the daughter nucleus would be present after 10 min?
(c) How many atoms of the parent nucleus would be left after 25 min?
(d) How many atoms of the daughter nucleus would be left after 25 min?
(e) Write an equation to determine the number of daughter nuclei present at any time.

29. In radioactive dating, ratios of the numbers of parent and daughter nuclei from the same decay chain, such as uranium-238 and lead-206, are determined. Assume that when the sample formed, it contained no daughter nuclei. Consider the analyses of three different rock samples that have been determined to have ratios of uranium-238 to lead-206 of 1.08:1, 1.22:1, and 1.75:1.

(a) Using the results of the previous question, write an equation for the ratio of the number of uranium-238 atoms to lead-206 atoms present at any time. (Hint: the initial number of uranium-238 atoms will divide out.)

(b) Solve the above equation for time, and determine the ages of the three samples. (The half-life of uranium-238 is 4.5×10^9 a.)

(c) Explain whether these samples could have been taken from an area where the rock solidified all at once.

(d) Intuitively, what conclusion can you draw if you measure a ratio of less than one?

30. What is the wavelength of each of the two photons produced in electron-positron annihilation?

31. Heavy water used in the Sudbury Neutrino Observatory is made up of oxygen and deuterium, a radioactive isotope of hydrogen (see Not Your Average Observatory, page 554). One of the reactions that physicists at the observatory are trying to detect is $\nu_e + {}^2_1\text{H} \rightarrow p + p + e^-$, where ν_e is an electron-neutrino. For this reaction to be observed, the neutrino's energy must be greater than the binding energy of a deuterium atom.

(a) Given that the nuclear mass of deuterium is 2.013 553 u, calculate the minimum neutrino energy for this reaction to occur.

(b) If 95.0% of the neutrino's kinetic energy goes into the kinetic energy of the produced electron, calculate the speed of the electron. (Hint: The electron's speed is relativistic.)

(c) Compare the electron's speed with the speed of light in water.

32. Analyze the following reactions in terms of their constituent quarks.

(a) $n \rightarrow p + e^- + \overline{\nu}_e$

(b) $\gamma + n \rightarrow \pi^- + p$

Decades of Triumph and Turmoil

Background

From the late 1800s to the mid-1900s, the world saw change at a rate it had never experienced before. Science progressed rapidly as understanding of the atom deepened. Molecules were mapped, and from that mapping came new products — plastics, pharmaceuticals, stronger alloys. Harnessing the nucleus gave promise of bountiful energy in peacetime and mass destruction in time of war. Through study of the electromagnetic spectrum came an ever-increasing ability to probe inward to understand the workings of our body cells and outward to observe the workings of the universe. Some parts of the spectrum became crowded with use as radio and television stations staked their claims to frequencies.

Along with scientific and technological change came societal change. Two world wars left their legacy of broken lives, shattered countries and economies, and radical changes in social outlooks and value systems. Warriors returned to very different homelands. Changes in production techniques also had a huge impact on society. Augmented by new technologies, the assembly line became the backbone of many huge industries, and the need for unskilled workers plummeted.

A World War I battle scene

It is easy to forget that the scientists whose contributions you have studied during this course lived and worked in the midst of these changes. They, too, were affected, and sometimes even caused or influenced these changes. The goal of this project is to examine the parallel between these scientists' professional lives and what their lives were like when they stepped outside of their offices and laboratories.

Plan and Present

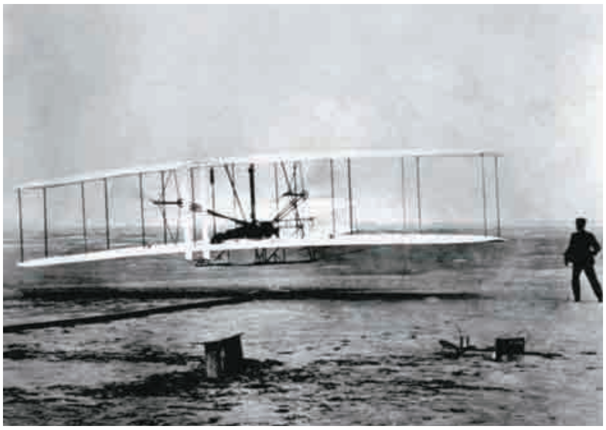
1. As a class, establish clear guidelines for evaluating the finished project. Discuss specifics such as
   - deadlines
   - expectations for the diary or letter: Will there be a minimum length, a minimum number of societal factors to be included, a specified presentation format?
   - expectations for the poster: Will presentation attractiveness and organization be assessed, as well as the content? Will there be a minimum amount of biographical and scientific material that must be included?
   - expectations for the time line: How do you intend to assess a group's contribution to the overall historical time line?
2. As a class, prepare an initial time line for the period of 1881–1950, listing major scientific advances and discoveries alongside major events in society, such as World Wars I and II, the Depression, the birth of jazz, the first automobiles, the introduction of radio and then television shows, and aviation, from the Wright brothers' first experiment on a North Carolina seashore to space exploration.
3. Divide the study up into six time spans: the two-decade period of 1881 to 1900 and the five individual decades between 1901 and 1950. Assign a team to each era. (You could perhaps allocate the number of members per team according to the number of events in each era.)

ASSESSMENT

After you complete this project

- assess your ability to conduct research: Did you find relevant information?
- assess your teamwork skills: How effectively were you able to share your ideas with other members of your group and contribute to the group effort?
- assess your communication skills: How effectively did you communicate your ideas in the poster and written portions of this task?

4. Each team is to research three major scientific or technological events that occurred during its designated time period and prepare a poster on each event. This presentation must include biographical data for the people involved and an outline of the nature and importance of the event. The team is then to research the major societal events and changes that might have affected those scientists.



The famous reconnaissance aircraft SR71 *Blackbird* is a descendent of the Wright brothers' *Wright Flyer*, which made history on December 17, 1903, when Orville Wright piloted the first powered, manned, controlled flight. For its debut, the *Wright Flyer* was in the air for 12 s and covered a distance of 37 m. By contrast, the Blackbird flew 3500 missions and was so fast that a missile had to be fired 48 km ahead of the plane to reach it in time.

5. As a class, construct an overall time line. This could perhaps be a horizontal version of the time line shown on page xiv, and could be posted around the classroom near the top of two or three of the walls. The names of the scientists along with brief outlines of their contributions or applications of these contributions could be placed on one side of the line, with a listing of the corresponding major societal and world events on the other side of the line. The posters could also be displayed.

6. Working individually or in pairs, you will write letters or diaries that represent what one or more of the featured scientists might have written about their everyday lives.

- What type of transportation did the scientist probably use, locally and for long-distance travel?
- What type of lighting was available in that scientist's time?
- What were the major newspaper stories at the time the scientist did his or her most notable work?
- What type of medical treatment was available at that time?

Evaluate

1. Evaluate the extent to which your group met the expectations for the project in relation to the

- posters
- timeline
- written material

2. (a) Which items prepared by your group do you feel were most effective? Explain.

(b) Which items prepared by your group do you feel were least effective? How might they have been improved?

UNIT 5 Review

Knowledge/Understanding

Multiple Choice

In your notebook, choose the most correct answer for each of the following questions. Outline your reasons for your choice.

1. Of the following quantities, which, if any, have the same value to all observers?
   (a) mass of the muon
   (b) average lifetime of the muon
   (c) charge of the muon
   (d) energy of a photon
   (e) speed of light
2. The kinetic energy of a particle travelling near the speed of light is
   (a) always less than the rest energy
   (b) equal to mc^2
   (c) equal to $\frac{1}{2}mv^2$
   (d) equal to $(m - m_0)c^2$
   (e) equal to $(m_0 - m)c^2$
3. When an object with a rest mass of 2.0 kg approaches the speed of light, its mass approaches
   (a) 0 **(b)** 0.5 kg **(c)** 1.0 kg **(d)** c^2 **(e)** ∞
4. If you direct light at a metal surface, the energies of the emitted electrons
   (a) are random
   (b) vary with the speed of light
   (c) vary with the intensity of light
   (d) vary with the frequency of light
   (e) are constant
5. In the Bohr model of the atom, an electron emits energy when it
   (a) accelerates in its orbit
   (b) decelerates in its orbit
   (c) jumps from a higher energy level to a lower energy level
   (d) jumps from a lower energy level to a higher energy level
   (e) is in the ground state
6. The strong nuclear force has a limited range. A consequence of this is the
   (a) magnitude of nuclear binding energies
   (b) instability of large nuclei
   (c) ratio of atomic size to nuclear size
   (d) existence of isotopes
   (e) existence of neutrinos
7. The number of elementary charge units in a nucleus determine the atomic
   (a) size **(b)** weight **(c)** mass **(d)** number **(e)** density
8. The half-life of ^{28}Ni is six days. What fraction of a sample of this nuclide will remain after 30 days?
   (a) $\frac{1}{4}$ **(b)** $\frac{1}{8}$ **(c)** $\frac{1}{16}$ **(d)** $\frac{1}{32}$ **(e)** $\frac{1}{64}$
9. After 4 h, $\frac{1}{16}$ of the initial amount of a certain radioactive isotope remains undecayed. The half-life of the isotope is
   (a) 15 min **(b)** 30 min **(c)** 45 min **(d)** 1 h **(e)** 2 h
10. A particle that will not leave a curved track in a bubble chamber is the
   (a) proton **(b)** positron **(c)** electron **(d)** neutron **(e)** alpha particle

Short Answer

11. If you were travelling in a spaceship at 0.9 c, would you notice any time dilation effects for clocks in the spaceship? Explain your reasoning.
12. A clock on a flying carpet streaks past an Earthling who is looking at her watch. What does the Earthling notice about the passage of time on the moving clock, compared with her watch? What would a wizard on the flying carpet notice about the passage of time on the Earthling's watch, compared to the clock on the carpet? Does it matter which timepiece is considered to be in motion and which is considered to be at rest?
13. Explain the following statement: The speed of light is a constant.
14. Max Planck introduced an hypothesis regarding the energy of vibration of the molecules in order to satisfy the observed spectrum emitted

from a hot body. What was this hypothesis and on whose work did he reportedly base his idea?

15. **(a)** Describe the relationship Phillip Lenard found between the energy of photoelectrons and the frequency of the incident light.

(b) Describe how increasing the light intensity affects the electron flow.

16. **(a)** Use the photon theory of light to explain why a photographer might use a red safety light in a darkroom for black and white photography.

(b) Sunburn is caused by the ultraviolet component of sunlight, not by the infrared component. How does the photon theory account for this?

17. Does it take more or less energy to remove a photoelectron from lead than from aluminum? (See Table 12.1 on page 509.) Explain your reasons.

18. Describe the technique that was used successfully to demonstrate the existence of de Broglie matter waves.

19. **(a)** Some features of the emission spectrum could still not properly be explained by the Bohr model. Name two such features.

(b) Paul Dirac modified Erwin Schrödinger's equation. What was he seeking to include and how successful was he?

20. Differentiate between a transmutation and a radioactive decay.

21. Describe how a knowledge of electromagnetism has been used to develop technologies to probe matter for indirect evidence of its elementary particles.

22. Explain, with the aid of a series of sketches, the relative effects of an electromagnetic force and a strong nuclear force at several stages as a proton is propelled toward a nucleus in a fusion reaction. Build on your explanation to suggest why "cold fusion" is not considered to be scientifically possible.

Inquiry

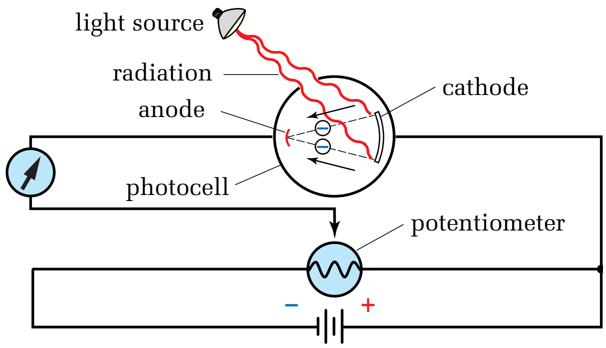
23. Suppose you had a rod of length L aligned parallel to the y-axis of an x-y reference frame labelled S and an identical rod of length L' aligned parallel to the y'-axis of an x'-y' reference frame labelled S' When the two frames are aligned, it is seen that the rods are the same length. Allow the frames to be offset in the x-direction and then set one of them in motion so that the rods move past each other. Argue that the length of either rod will not be seen to change. What would be the physical implications if one of the rods was observed to change?

24. Some people thought that they had disproved Einstein's special theory of relativity by describing the twin paradox. According to this thought experiment, identical twins Al and Bert grow up on Earth. Al rides a rocket, which travels close the speed of light, to Alpha Centauri and then returns. Consider the following points.

- From Bert's point of view of Earth, Al has been travelling at a high rate of speed, so his clock would have slowed down. When Al returns, he should be younger than Bert.
- However, from Al's point of view, it was Bert who was travelling at a high rate of speed. It was Bert's clock that slowed down, so Bert would be younger than Al. Since these two results are contradictory, the special theory of relativity must be wrong.

Explain why the special theory of relativity does not fully describe what is happening in this example. (Hint: Are both frames of reference equivalent?)

25. The phototube shown in the diagram was used to determine the stopping potential (also called "cut-off voltage") for electrons emitted from the cathode (emitter) when different wavelengths of light were incident on its surface. The table that follows the diagram records the values of the wavelengths used and the corresponding stopping potential.



Colour	Wave-length (nm)	Stopping potential (V)	Maximum E_k of photoelectrons (J)	Frequency (Hz)
green	530.0	0.045		
green	500.0	0.244		
blue	460.0	0.402		
violet	410.0	0.731		

(a) Prepare a table similar to the one above and complete the remaining two columns by calculating the maximum kinetic energy of the emitted electrons (using $E = qV$) and the frequency of the light.

(b) Draw a graph with maximum E_k on the vertical axis and frequency on the horizontal axis.

(c) From your graph, determine the work function for the particular emitter material used.

(d) Identify the metal used in the emitter (see Table 12.1 on page 509).

(e) Calculate the slope of the graph and compare it with Planck's constant.

(f) Explain how you feel that the graph would or would not be different if
- the emitter had been made from a different material
- the intensity of the light was doubled in each case

26. When a charged particle passes through a magnetic field that is perpendicular to its motion, its path is deflected into a circular path. If the strength of the field (B) is known and you assume that the particles are singly charged, prove that the radius of the path indicates the momentum of the particle.

27. Assume that a pure sample of a radioisotope contains exactly 1.6×10^4 nuclei with a half-life of 10.0 s.

(a) Determine the expected number of nuclei remaining after time intervals of 10 s, 20 s, 30 s, 40 s, 50 s, and 60 s.

(b) Draw an accurate graph of the data with the time interval (t) on the x-axis and the number of nuclei remaining (N) on the y-axis.

(c) The activity at any given time is given by $A = \Delta N/\Delta t$. What property of the graph does this ratio represent?

(d) Determine the activity of the sample at 10 s, 20 s, 30 s, 40 s, and 50 s.

(e) Draw an accurate graph of the data with the time interval (t) on the x-axis and the activity (A) on the y-axis.

(f) Compare the two graphs.

28. In the initial form of the quark model in the 1960s, three quarks were proposed: up, down, and strange.

(a) Make a table to show the charges on these quark-antiquark combinations..
Include a column of "strangeness," determined as follows: If the particle contains a "strange" quark, assign it a strangeness of –1. If the particle contains an "anti-strange" quark, assign it a strangeness of +1. Sum strangeness the same way in which charge is summed.

(b) Evidence for an underlying simplicity in matter was shown by plotting strangeness versus charge. Why are there apparent holes in the graph?

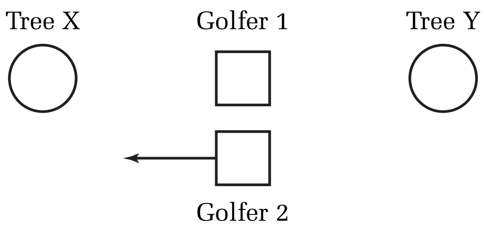
(c) Repeat (a) for all three quark combinations. Include columns for charge and strangeness.

(d) Plot strangeness versus charge for the three quark particles. Comment on the symmetry of the plot.

29. Use conservation laws to determine which of the following reactions are possible. Explain your reasoning in each case.
   (a) $p + p \rightarrow p + n + \pi^+$
   (b) $p + p \rightarrow p + p + n$
   (c) $p + p \rightarrow p + \pi^+$
   (d) $p + p \rightarrow p + p + \pi^0$

Communication

30. In your own words, explain the term "relativity."
31. **(a)** Several golfers are out on a golf course when two trees are struck by lightning. The arrangement is as shown in the diagram. Golfer 1 is at rest relative to the two trees and observes that both trees were struck simultaneously. Golfer 2 is driving a relativistic golf cart. In the golf cart frame of reference, which tree was struck first? Give reasons for your answer.



   (b) During a storm, a passenger in a stretch limousine, travelling close to the speed of light (the ultimate speed limit), noticed that two large hailstones struck the limousine simultaneously, one on the hood of the car and one on the trunk. According to a pedestrian who was standing on the sidewalk as the car sped past, which hailstone struck first? Give reasons for your answer.
32. Explain what it means to say that a certain quantity is quantized.
33. Describe the evidence that matter behaves as a wave.
34. Explain what limits the size of a stable nucleus.
35. Develop a graphic organizer to show how the elementary particles are related to other groups of subatomic particles.

Making Connections

36. Find examples in this textbook where the study of a particular area of physics was advanced by new experimental results that led to a new theory, and vice versa. Express your thoughts in writing about the manner in which science advances, using these examples.
37. Despite the complexity of some observed phenomena and some equations, the following statement is true: The basic ideas underlying all science are simple. Prove this to yourself by examining the chapters in this textbook. For each chapter, write down at least three simple but scientifically correct statements that summarize one or more of the concepts in the chapter. For example, one of the sentences for Chapter 11, Special Theory of Relativity, could be "Energy and matter are equivalent" or "Energy and matter are interchangeable." Make some of your statements general (to apply to an entire unit, for example) and relate some to specific concepts.
38. The Cavendish Laboratory for experimental physics at the University of Cambridge, England, has been responsible for many significant discoveries and inventions in the history of physics. These include the discoveries of the electron and neutron and the inventions of the mass spectrometer, cloud chamber, and the Cockcroft-Walton proton accelerator. Between 1879 and 1937, the chair of the laboratory was occupied by James Clerk Maxwell, Lord Rayleigh, J.J. Thomson, and Ernest Rutherford. Write an essay that examines the research done in this famous laboratory. Identify and discuss some of the factors that have enabled members of the Cavendish Laboratory to be so productive.
39. Draw a circuit diagram for a smoke detector and explain how it works. What determines its sensitivity?
40. Distinguish between fission and fusion. Research and prepare a report on why some elements are most likely to be involved in

nuclear fission reactions while others are most likely to be involved in nuclear fusion reactions.

41. Although physics has come a long way in its understanding of matter and energy, much work remains and it is uncertain that a full understanding is even possible. Write an essay to discuss the status of the standard model. What are its present weaknesses? Express your own views on whether a full understanding of the interactions between matter and energy is possible. Popular books that explore this topic have been written by Stephen Weinberg, Murray Gell-Mann, and Leon Lederman, and will help you to frame your argument.

Problems for Understanding

42. Relativistic speeds are speeds at which relativistic effects become noticeable. Just how fast is this? To answer the question, determine the following.
 (a) At what speed relative to your frame of reference would a particle have to travel so that you would see that its length in the direction of motion had decreased by 1.0%?
 (b) At what speed relative to your frame of reference would a particle have to travel for you to detect that its mass had increased by 0.10%?

43. **(a)** How much energy would be released if a 1.0 kg brick was converted directly into energy?
 (b) For how long could this amount of energy power a 100 W light bulb?

44. The star Alpha Centauri is 4.2 light-years away (a light-year is the distance light travels in one year: 365.25 days).
 (a) If you travelled in a spaceship at a speed of 2.0×10^8 m/s, how long would this distance appear to be?
 (b) How long would a one-way trip take you?
 (c) How much time would pass for someone back on Earth?

45. **(a)** Calculate the energy required to give an electron a speed of $0.90c$, starting from rest.
 (b) Compare this to its rest mass energy.
 (c) In terms of its rest mass, what is the mass of an electron travelling at this speed?

46. Suppose you allowed a 100 W light bulb to burn continuously for one year.
 (a) How much energy would it radiate in this time?
 (b) To what change in mass does this correspond?

47. Radiation of wavelength 362 nm is incident on a potassium surface. What will be the maximum kinetic energy of the electrons emitted from this surface? (Refer to Table 12.1 on page 509.)

48. Calculate the maximum kinetic energy of the electrons emitted from the cathode emitter of a photocell if the stopping potential is 4.7 V.

49. **(a)** A zinc surface is used on the emitter of a photocell. What will be the threshold frequency necessary for a photocurrent to flow? (See Table 12.1 on page 509)
 (b) What is the threshold wavelength for zinc?

50. **(a)** Calculate the de Broglie wavelength of an electron moving with a speed of 5.82×10^5 m/s.
 (b) An electron is accelerated across an electric potential difference of 64.0 V. Calculate the de Broglie wavelength of this electron.

51. An electron drops from the second energy level of the hydrogen atom to the first energy level.
 (a) Calculate the frequency of the photon emitted.
 (b) Calculate the wavelength of the photon.
 (c) In which series does the spectral line belong?

52. Calculate the wavelength of the second line in the Balmer series.

53. A typical classroom helium-neon laser has a power of 0.80 mW and emits a monochromatic beam of red light of wavelength 670 nm.
 (a) Calculate the energy (in J) of each photon in the beam.
 (b) If the laser is left on for 5.0 min, how many photons will be emitted?

54. A photon of light is absorbed by a hydrogen atom in which the electron is already in the second energy level. The electron is lifted to the fifth energy level.

- **(a)** What was the frequency of the absorbed photon?
- **(b)** What was its wavelength?
- **(c)** What is the total energy of the electron in the fifth energy level?
- **(d)** Calculate the radius of the orbit representing the fifth energy orbit.
- **(e)** If the electron subsequently returns to the first energy level in one "jump," calculate the wavelength of the corresponding photon to be emitted.
- **(f)** In which region of the electromagnetic spectrum would the radiation be found?

55. A prediction of the lifetime of the Sun can be calculated by analyzing its observed rate of energy emission, 3.90×10^{26} J/s. (Hint: In making the following calculations, pay close attention to unit analysis.)

- **(a)** Calculate the amount of energy released in the conversion of four protons to one helium nucleus: $4^1_1\text{H} \rightarrow {}^4_2\text{He} + 2^0_1\text{e}$.
- **(b)** If the above is considered as one reaction, how many reactions must occur each second to produce the observed rate of energy emission?
- **(c)** How much helium is produced during each reaction?
- **(d)** How much helium is produced per second?
- **(e)** Let the lifetime of the Sun be defined as the time it takes 10.0% of the Sun's total mass to be converted into helium. (You can make this assumption, since it is accepted that only the reactions in the Sun's core need be considered.) Calculate the Sun's lifetime in years.

56. The Sun's lifetime can also be determined by calculating the total energy available and dividing by the energy radiated per second.

- **(a)** Calculate the mass defect for converting four protons into one helium nucleus.
- **(b)** What fraction of the mass of the initial four protons does this mass defect represent? This is the fraction of the mass of each proton that is converted into energy.
- **(c)** Suppose the Sun's entire mass (1.99×10^{30} kg) was composed of protons. What is the total energy available?
- **(d)** Assume that only 10.0% of the Sun's mass of protons are available to undergo fusion and calculate the lifetime of the Sun in years. (The Sun radiates 3.90×10^{26} J/s.)

57. Consider a sample of rock that solidified with Earth 4.55×10^9 years ago. If it contains N atoms of uranium-235 (half-life: 7.04×10^8 a), how many atoms were in the rock when it solidified?

58. In the very early universe, protons and antiprotons existed with gamma rays. What is the minimum gamma ray energy required to create a proton-antiproton pair? To what wavelength does this amount of energy correspond?

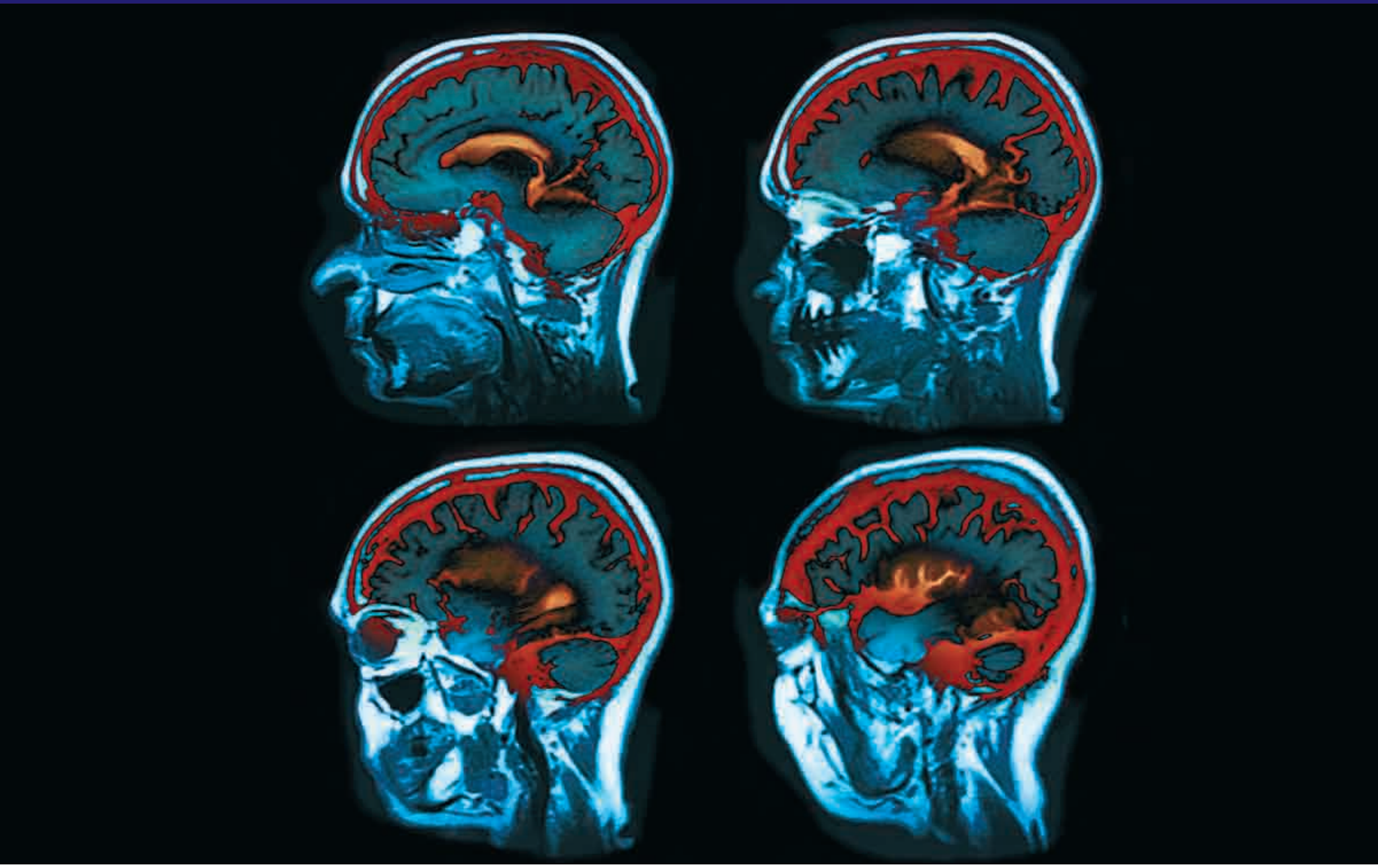
COURSE CHALLENGE

Scanning Technologies: Today and Tomorrow

Consider the following as you complete the final information-gathering stage for your end-of-course project.

- Attempt to combine concepts from this unit with relevant topics from previous units.
- Verify that you have a variety of information items, including concept organizers, useful Internet sites, experimental data, and unanswered questions to help you create an effective final presentation.
- Scan magazines, scientific journals, and the Internet for interesting information to validate previously identified content and to enhance your project.

Physics Course Challenge



These magnetic resonance imaging (MRI) scans reveal four profile views at different depths of a healthy human brain. The folded cerebral cortex — associated with thought processes — is highlighted in red.

Scanning Technologies: Today and Tomorrow

An X-ray image of a tooth or broken bone is commonplace, and ultrasound images of a developing fetus are a regular part of prenatal care. Without the need for a single incision, various forms of non-invasive imaging technology provide clear images of the soft tissues of our bodies. Imaging technology also exposes the contents of locked luggage during airport security checks. Satellites circle Earth, relaying data about geological changes, volcanoes, hurricanes, and crop and vegetation densities.

Understanding the fundamental properties of matter, fields, waves, and energy has opened the door to hundreds of scanning technologies, and continuing research results in yet more scanning methods and continues to push the capabilities of these technologies to new heights. Research costs money, however, and is very time-consuming. Are these new scanning techniques worth the expense and time involved?

ASSESSMENT

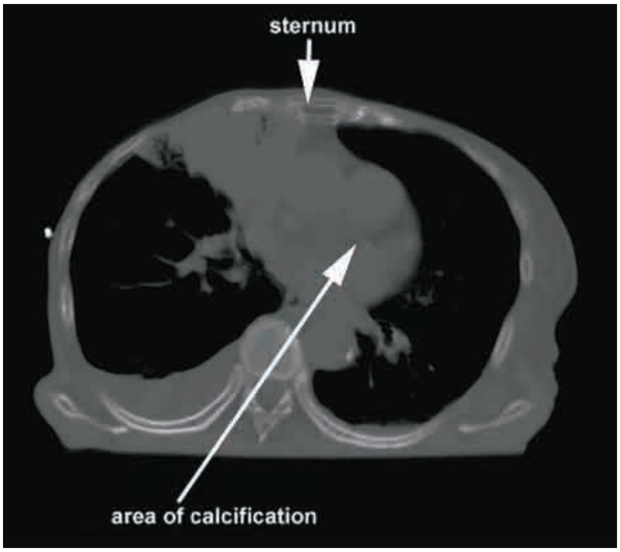
After you complete this Course Challenge, you will be assessed on

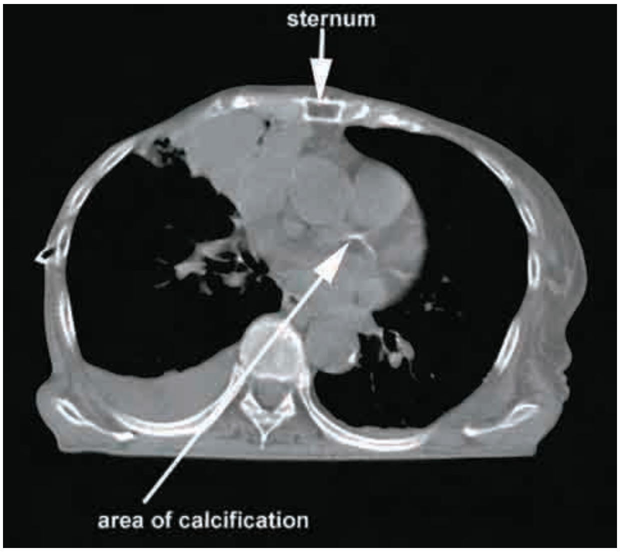
- **the quality of your research**
- **the accuracy and depth of your understanding**
- **your presentation**
- **other criteria you decide on as a class**

This Course Challenge prompts you to examine the costs and benefits of imaging technologies to both the scientific community and society. To help you get started, three fields of scanning technology and some associated issues are presented here.

Medical Issues

Doctors and politicians are often criticized when professional athletes gain access to magnetic resonance imaging (MRI) diagnosis immediately after sustaining an injury, while the general public must often wait months. Questions arise about the real expense of MRI equipment, its availability, and the value of the results as compared to other methods. How does an MRI machine work? What fundamental principles of nature does it exploit? Why is MRI scanning so expensive? Will the costs reduce with time? Will the technology improve with time? Are there better, less expensive options that should be pursued? Will this technology ever be made available to citizens of developing nations? To develop an argument supporting continued use of and research into MRI technology, you need to be able to answer these and other questions.





Motion, such as a beating heart or breathing, causes a blurring of conventional computerized tomography (CT) scan images. New computer technology, involving millions of frames of reference calculations, is able to remove the blur and produce much clearer images.

Photos courtesy of Dr. S. Stergiopoulos, Defence R&D Canada - Human Sciences, Toronto, Canada

PHYSICS FILE

A remote mountaintop in western North America rose 10 cm in four years, from 1996 to 2000. The 10 cm bulge is at the centre of a circle with a 12 km radius, and is only a few kilometres from the South Sister, a volcano that erupted 2000 years ago. The bulge is believed to be the result of growing pressure in an enormous chamber of magma beneath the mountain, and a telltale sign of potential volcanic activity. The 10 cm shift is a very small indicator of the tremendous amount of energy behind it, and would never have been detected without the remote-sensing ability of Earth-orbiting satellites.

Security Issues

Border-crossing and airport security often rely on technology to solve problems associated with screening large numbers of people and baggage in an efficient way. Some debate the effectiveness of the technological solutions compared to the enormous costs required to install and maintain the equipment. Opponents suggest that a human work force could do a more thorough and efficient job. Developing an argument supporting either side of this debate requires an in-depth understanding of the technology and its capabilities, and perhaps even a sense of its future potential.

A security imaging system can be set to detect the presence of explosives, narcotics, currency, or gold. In this case, the computer analyzed the contents of a laptop computer case and identified explosive material, indicated by the bright red area in this scan image.

Space Issues

Earth-orbiting, satellite-scanning technologies are used for environmental data collection, which is required for the development of sustainable agricultural, industrial, and even population-settlement plans. Weather satellites have allowed meteorologists to dramatically improve their forecasts. Surveillance satellites provide governments with information about covert operations.

A wealth of information comes from space, but the launching and maintaining of satellites is extremely expensive. World citizens need to be convinced that the economic costs associated with space-based research and related technologies are worth the rewards.

The Canadian Space Agency (CSA) and the U.S. National Aeronautics and Space Agency (NASA) devote a substantial amount of effort to global education, providing evidence of the benefits of space-related research. These agencies also work diligently to include other nations in large projects, such as the International Space Station. The CSA and NASA also recognize that projects must offer the global business community financial opportunities, as well as knowledge, to be successful in the long term. Does the commercialization of space fit with your vision of the future?

Debating which technologies are worth the investment of monetary and human resources can be accomplished only when all of the facts are known. Think about these questions as you undertake this Course Challenge.

Challenge

Develop and present a case either for or against the use of a particular scanning technology. You will use the knowledge and concepts you have acquired throughout this course, along with additional research, to develop your presentation about the economic, social, or environmental viability of a medical, industrial, or environmental scanning technology. Your class will decide together whether the presentations will be made through

- a formal debate
- research report presentations (either as a written report, an audiovisual presentation, or an information billboard)
- another format of your choice

Materials

All presentations are to be supported by your portfolio of research findings, the results of supporting experiments conducted, and a complete bibliography of references used.

Design Criteria

A. You need to develop a system to collect and organize information that will include data, useful mathematical relationships, and even questions that you use to formulate your final presentation near the end of the course. You can collect your own rough notes in a research portfolio.

B. **Building a Research Portfolio**
Your individual creativity will shape the amount, type, and organization of the material that will eventually fill your portfolio. Do not limit yourself to the items mentioned in the Course Challenge cues scattered throughout textbook; if something seems to fit, include it. The following are suggested items for your research portfolio.

- experiments you have designed yourself, and their findings
- useful equations
- specific facts
- interesting facts
- disputed facts
- conceptual explanations
- diagrams
- graphical organizers
- useful Internet site URLs
- experimental data
- unanswered questions
- pertinent economic or social statistics (Canadian or global)

C. As a class, decide on the type(s) of assessment you will use for your portfolio and for its presentation. Working with your teacher and classmates, select which type of presentation you will use to present your scanning technology arguments.

Action Plan

1. As a class, have a brainstorming session to establish what you already know and to raise questions about various scanning technologies that are currently being used or researched today. For example, what medical value does an MRI offer over other diagnostic methods, and is that difference worth the economic price? How widely available is MRI technology in **(a)** Canada or **(b)** other parts of the developed or underdeveloped world?
2. As a class, design an evaluation scheme, such as a rubric or rubrics for assessing the task. You could decide to assess specific components leading up to the final presentation, as well as the presentation itself.
3. Decide on the grouping, or assessment categories, for this task.
4. Familiarize yourself with what you need to know about the task that you choose. For example, if you choose a debate, it is important to research the proper rules of debating in order to carry out the debate effectively.
5. Develop a plan to find, collect, and organize in your research portfolio the information that is critical to your presentation.
6. Carry out the Course Challenge recommendations that are interspersed throughout the textbook wherever the Course Challenge logo and heading appear, and keep an accurate record of these in your portfolio.
7. When researching concepts, designing experiments or surveys, or following a Course Challenge suggestion in the textbook, you might find that the McGraw-Hill Ryerson Internet site is a good place to begin: www.mcgrawhill.ca/links/physics12
8. Carry out your plan, making necessary modifications throughout the course.
9. Present your arguments to your class. Review each presentation against the assessment criteria that you decided on as a class.

Evaluate Your Challenge

1. Using the assessment criteria you have prepared, evaluate your work and presentation. How effectively did your portfolio and presentation support your arguments? Were others able to follow your line of reasoning, based on the evidence, results, and conclusions you presented? How would you revise your presentation?

2. Evaluate your classmates' Course Challenge presentations.

3. After analyzing the presentations of your classmates, what changes would you make to your own project if you had the opportunity to do it again? Provide reasons for your proposed changes.

4. How did the process required to complete this challenge help you to think about what you have learned in this course?

Background Information

The following sections provide ideas to consider. They are linked to topics covered in the course and relate to the Course Challenge cues in your textbook. Your arguments will be both strengthened and redirected as you gain knowledge from each unit in this course.

Unit 1 Forces and Motion: Dynamics

Frames of Reference

Chapter 1, page 11

Describing motion in two and three dimensions requires the use of vector quantities. Consider the scanning technology that you have selected for investigation. How is an image obtained? Does the scanning machinery move, or does the item that is being scanned move? Does the technology detect motion or the change in orientation of atomic and subatomic particles? Analyze the scanning technology you are investigating from the perspective of frames of reference. Develop a comprehensive description detailing how an image is formed based on the location of particles in a two- or three-dimensional space.

Unit 2 Energy and Momentum

Momentum

Chapter 4, page 150

The conservation of momentum is the principle that allows navigation in space. Conservation of momentum is a fundamental property of our universe. Conservation of momentum applies to planetary, human, and subatomic levels. Investigate possible applications of momentum conservation used in the scanning technology that you are investigating. If the conservation of momentum applies only to atomic and subatomic interactions, you might want to complete your analysis during your study of Unit 5, Matter-Energy Interface, in the textbook.

Energy Transformations

Chapter 5, page 217

Producing scanned images requires very controlled energy transformations. Investigate the energy path used by the technology you have chosen to investigate. Answer questions such as: What energy is directed at the item to be scanned? Is energy absorbed, transmitted, or both? What energy transformations occur within the scanned item? What energy transformations occur at the scanning receiver? Support your presentation with quantitative energy transformation analysis. Is there an economic, social, or safety aspect relating your technology to energy transformation issues?

Unit 3 Electric, Gravitational, and Magnetic Fields

Contact versus Non-Contact

Chapter 7, page 275

You might want to compare contact versus non-contact forces. A century ago, a medical examination conducted to identify an abnormal growth would have involved physical contact, because the doctor used touch to assess the patient. Current medical examinations are able to obtain a much clearer picture of an abnormal growth inside the body without ever coming into direct contact with the patient. Consider the scanning technology you have chosen in these terms.

Field Energy

Chapter 8, page 356

Ultimately, the energy stored in fields will be the basis for the operation of any scanning technology. Satellite-based technologies orbit Earth, held in position by the gravitational field. Medical scans employ powerful magnetic fields to obtain diagnostic imagery. Investigate how fields play a role in the production of images in the technology that you are investigating. You might want to consider your technology in terms of a quantitative application of Coulomb's law.

Unit 4 The Wave Nature of Light

How Far Can It Go?

Chapter 10, page 445

Energy transported in the form of oscillating electric and magnetic fields is the fundamental method used in most scanning technologies. This textbook provides an introduction to some of these applications in Chapter 10, Section 10.2, The Electromagnetic Spectrum. Consider those discussions while you complete your analysis. You might want to direct your arguments in terms of past and future scientific developments. What has been accomplished? What new research is taking place? Are you able to predict how scanning technology might change in the next five years? Monetary and social arguments fit naturally into discussions based on possible changes in the field.

Unit 5 Matter-Energy Interface

Waves and Particles

Chapter 12, page 531

Scientific models evolve when theories are modified and validated by new experimental results. Physicists realize that electromagnetic radiation can be fully described only by using two completely different scientific models. Models are made by humans and therefore change as more knowledge is acquired. You might be able to demonstrate that a complete description of your chosen scanning technology requires both the wave and particle nature of electromagnetic radiation.

Nuclear Energy

Chapter 13, page 574

Nuclear energy provides electrical power not only to our homes, but also to most of the satellites orbiting overhead. Nuclear energy is used to probe living tissue in a variety of medical scanning technologies. Investigate nuclear decay rates of various materials and how they relate to your scanning technology. You might want to introduce safety and societal issues related to the use of nuclear material in the technology that you are investigating.

Wrap-Up

These ideas and questions are provided to help you develop your arguments related to a specific scanning technology. The ultimate shape of your presentation will be determined by the technology you choose to investigate, the issues you choose to address, and your own creativity. In order to prepare a high-quality, in-depth presentation, you will need to limit the amount of information that you attempt to present, focussing on the key points. Attempt to support your ideas with experimental evidence, mathematical verification, and comparisons to accepted scientific models. Give your project added relevance by relating your topic to key societal issues, such as economic or safety considerations.

Use your Course Challenge presentation to assist your learning by drawing together topics from each unit of study. As is often the case with any issue, the quality of discussion improves when knowledgeable links are made between topics.

Precision, Error, and Accuracy

A major component of the scientific inquiry process is the comparison of experimental results with predicted or accepted theoretical values. In conducting experiments, you must realize that all measurements have a maximum degree of certainty, beyond which there is uncertainty. The uncertainty, often referred to as "error," is not a result of a mistake, but rather, it is caused by the limitations of the equipment or the experimenter. The best scientist, using all possible care, could not measure the height of a doorway to a fraction of a millimetre accuracy using a metre stick. The uncertainty introduced through measurement must be communicated using specific vocabulary. Experimental results can be characterized by both their accuracy and their precision.

Precision describes the exactness and repeatabilty of a value or set of values. A set of data could be grouped very tightly, demonstrating good precision, but not necessarily be accurate. The darts in illustration (A) missed the bull's-eye and yet are tightly grouped, demonstrating precision without accuracy.

A B C

Differentiating between accuracy and precision

Accuracy describes the degree to which the result of an experiment or calculation approximates the true value. The darts in illustration (B) missed the bull's-eye in different directions, but are all relatively the same distance away from the centre. The darts demonstrate three throws that share approximately the same accuracy, with limited precision.

The darts in illustration (C) demonstrate accuracy and precision.

Random Error

- Random error results from small variations in measurements due to randomly changing conditions (weather, humidity, quality of equipment, level of care, etc.).
- Repeating trials will reduce but never eliminate random error.
- Random error is unbiased.
- Random error affects precision.

Systematic Error

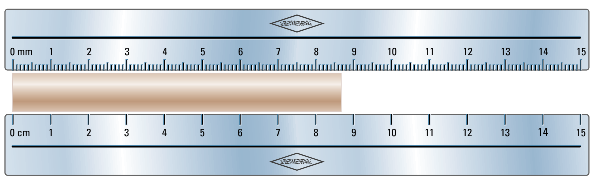
- Systematic error results from consistent bias in observation.
- Repeating trials will not reduce systematic error.
- Three types of systematic error are natural error, instrument-calibration error, and personal error.
- Systematic error affects accuracy.

Error Analysis

Error exists in every measured or experimentally obtained value. The error could deal with extremely tiny values, such as wavelengths of light, or with large values, such as the distances between stars. A practical way to illustrate the error is to compare it to the specific data as a percentage.

Relative Uncertainty

Relative uncertainty calculations are used to determine the error introduced by the natural limitations of the equipment used to collect the data. For instance, measuring the width of your textbook will have a certain degree of error due to the quality of the equipment used. This error, called "estimated uncertainty," has been deemed by the scientific community to be half of the smallest division of the measuring device. A metre stick with only centimetres marked would have an error of ±0.5 cm. A ruler that includes millimetre divisions would have a smaller error of ±0.5 mm. The measure should be recorded showing the estimated uncertainty, such as 21.00 ± 0.5 cm. Use the relative uncertainty equation to convert the estimated uncertainty into a percentage of the actual measured value.



Estimated uncertainty is accepted to be half of the smallest visible division. In this case, the estimated uncertainty is ±0.5 mm for the top ruler and ±0.5 cm for the bottom ruler.

$$\text{relative uncertainty} = \frac{\text{estimated uncertainty}}{\text{actual measurement}} \times 100\%$$

Example:

Converting the error represented by 21.00 ± 0.5 cm to a percentage

$$\text{relative uncertainty} = \frac{0.05\ \cancel{\text{cm}}}{21.00\ \cancel{\text{cm}}} \times 100\%$$

$$\text{relative uncertainty} = 0.2\%$$

Percent Deviation

In conducting experiments, it frequently is unreasonable to expect that accepted theoretical values can be verified, because of the limitations of available equipment. In such cases, percent deviation calculations are made. For instance, the standard value for acceleration due to gravity on Earth is 9.81 m/s^2 toward the centre of Earth in a vacuum. Conducting a crude experiment to verify this value might yield a value of 9.6 m/s^2. This result deviates from the accepted standard value. It is not necessarily due to error. The deviation, as with most high school experiments, might be due to physical differences in the actual lab (for example, the experiment might not have been conducted in a vacuum). Therefore, deviation is not necessarily due to error, but could be the result of experimental conditions that should be explained as part of the error analysis. Use the percent deviation equation to determine how close the experimental results are to the accepted or theoretical value.

$$\text{percent deviation} = \left|\frac{\text{experimental value} - \text{theoretical value}}{\text{theoretical value}}\right| \times 100\%$$

Example:

$$\text{percent deviation} = \frac{\left|9.6\ \cancel{\frac{\text{m}}{\text{s}^2}} - 9.8\ \cancel{\frac{\text{m}}{\text{s}^2}}\right|}{9.8\ \cancel{\frac{\text{m}}{\text{s}^2}}} \times 100\%$$

$$\text{percent deviation} = 2\%$$

Percent Difference

Experimental inquiry does not always involve an attempt at verifying a theoretical value. For instance, measurements made in determining the width of your textbook do not have a theoretical value based on a scientific theory. You still might want to know, however, how precise your measurements were. Suppose you measured the width 100 times and found that the smallest width measurement was 20.6 cm, the largest was 21.4 cm, and the average measurement of all 100 trials was 21.0 cm. The error contained in your ability to measure the width of the textbook can be estimated using the percent difference equation.

$$\text{percent difference} = \frac{\text{maximum difference in measurements}}{\text{average measurement}} \times 100\%$$

Example:

$$\text{percent difference} = \frac{(21.4\ \cancel{\text{cm}} - 20.6\ \cancel{\text{cm}})}{21.0\ \cancel{\text{cm}}} \times 100\%$$

$$\text{percent difference} = 4\%$$

SET 1 Skill Review

1. In Sèvres, France, a platinum–iridium cylinder is kept in a vacuum under lock and key. It is the standard kilogram with mass 1.0000 kg. Imagine you were granted the opportunity to experiment with this special mass, and obtained the following data: 1.32 kg, 1.33 kg, and 1.31 kg. Describe your results in terms of precision and accuracy.
2. You found that an improperly zeroed triple-beam balance affected the results obtained in question 1. If you used this balance for each measure, what type of error did it introduce?
3. Describe a fictitious experiment with obvious random error.
4. Describe a fictitious experiment with obvious systematic error.
5. **(a)** Using common scientific practice, find the estimated uncertainty of a stopwatch that displays up to a hundredth of a second.

   (b) If you were to use the stopwatch in part (a) to time repeated events that lasted less than 2.0 s, could you argue that the estimated uncertainty from part (a) is not sufficient? Explain.

Rounding, Scientific Notation, and Significant Digits

When working with experimental data, follow basic rules to ensure that accuracy and precision are not either overstated or compromised. Consider the 100 m sprint race. Several people using different equipment could have timed the winner of the race. The times might not agree, but would all be accurate within the capability of the equipment used.

Sprinter's Time with Different Devices

Time (s)	Estimated error of device (s)	Device
11.356	±0.000 5	photogate timer
11.36	±0.005	digital stopwatch
11.4	±0.05	digital stopwatch
11	±0.5	second hand of a dial watch

Using the example of the 100 m race, you will solidify ideas you need to know about exact numbers, number precision, number accuracy, and significant digits.

Exact Numbers If there were eight competitors in the race, then the number 8 is considered to be an exact number. Whenever objects are counted, number accuracy and significant digits are not involved.

Number Precision If our race winner wants a very precise value of her time, she would want to see the photogate result. The electronic equipment is able to provide a time value accurate to 1/1000th of a second. The time recorded using the second hand on a dial watch is not able to provide nearly as precise a value.

Number Accuracy and Significant Digits The race winner goes home to share the good news. She decides to share the fastest time with her family. What timing method does she share? She would share the 11 s time recorded using the second hand of a dial watch. All of the other methods provide data that has her taking a longer time to cross the finish line. Is the 11 s value accurate?

The 11 s value is accurate to within ±0.5 s, following common scientific practice of estimating error. The 11.356 s time is accurate to within 0.0005 s. The photogate time is simply more precise. It would be inaccurate to write the photogate time as 11.356 00 s. In that case, you would be adding precision that goes beyond the ability of the equipment used to collect the data, as the photogate method can measure time only to the thousandths of a second. Scientists have devised a system to help ensure that number accuracy and number precision are maintained. It is a system of significant digits, which requires that the precision of a value does not exceed either (a) the precision of the equipment used to obtain it or (b) the least precise number used in a calculation to determine the value. The table on the left provides the number of significant digits for each measurement of the sprinter's times.

There are strict rules used to determine the number of significant digits in a given value.

When Digits Are Significant ✔

1. All non-zero digits are significant (159 — three significant digits).
2. Any zeros between two non-zero digits are significant (109 — three significant digits).
3. Any zeros to the right of *both* the decimal point and a non-zero digit are significant (1.900 — four significant digits).
4. All digits (zero or non-zero) used in scientific notation are significant.

When Digits Are Not Significant ✘

1. Any zeros to the right of the decimal point but preceding a non-zero digit are not significant; they are placeholders. For example, 0.000 19 kg = 0.19 g (two significant digits).
2. Ambiguous case: Any zeros to the right of a non-zero digit are not significant; they are placeholders (2500 — two significant digits). If the zeros are intended to be significant, then scientific notation must be used. For example, 2.5×10^3 (two significant digits) and 2.500×10^3 (four significant digits).

Calculations and Accuracy As a general rule, accuracy is maintained though mathematical calculations by ensuring that the final answer has the same number of significant digits as the least precise number used during the calculations.

Example:

Find the product of these lengths.

12.5 m   16 m   15.88 m

Product = 12.5 m × 16 m × 15.88 m

Product = 3176 m^3

Considering each data point, notice that 16 has only two significant digits; therefore the answer must be shown with only two significant digits.

Total length = 3.2×10^3 m (two significant digits)

Rounding to Maintain Accuracy It would seem that rounding numbers would introduce error, but in fact, proper rounding is required to help maintain accuracy. This point can be illustrated by multiplying two values with differing numbers of significant digits. As you know, the right-most digit in any data point contains some uncertainty. It follows that any calculations using these uncertain digits will yield uncertain results.

> Multiply 3**2** and 13.5**5**. The last digit, being the most uncertain, is highlighted.
>
> 13.5**5**
> × 3**2**
> **2710** Each digit in this line is obtained using an uncertain digit.
> 406**5** In this line only the 5 is obtained using uncertain digits.
> 4**33.60**
>
> The product 4**33.60** should be rounded so that the last digit shown is the only one with uncertainty. Therefore, $4.\mathbf{3} \times 10^2$.

Notice that this value contains two significant digits, which follows the general rule.

Showing results of calculations with every digit obtained actually introduces inaccuracy. The number would be represented as having significantly more precision than it really has. It is necessary to round numbers to the appropriate number of significant digits.

Rounding Rules When extra significant digits exist in a result, rounding is required to maintain accuracy. Rounding is not simply removing the extra digits. There are three distinct rounding rules.

1. **Rounding Down**
   When the digits dropped are less than 5, 50, 500, etc., the remaining digit is left unchanged.

   Example:
   4.123 becomes
   4.12   rounding based on the "3"
   4.1   rounding based on the "23"

2. **Rounding Up**
   When the digits dropped are greater than 5, 50, 500, etc., the remaining digit is increased or rounded up.

   Example:
   4.756 becomes
   4.76   rounding based on the "6"
   4.8   rounding based on the "56"

3. **Rounding with 5, 50, 500, etc.**
   When the digits dropped are exactly equal to 5, 50, 500, etc., the remaining digit is rounded to the *closest even number*.

   Example:
   4.850 becomes
   4.8   rounding based on "50"
   4.750 becomes
   4.8   rounding based on "50"

Always carry extra digits throughout a calculation, rounding only the final answer.

Scientific Notation Numbers in science are sometimes very large or very small. For example, the distance from Earth to the Sun is approximated as 150 000 000 000 m and the wavelength of red light is 0.000 000 65 m. Scientific notation allows a more efficient method of writing these types of numbers.

- Scientific notation requires that a single digit between 1 and 9 be followed by the decimal and all remaining significant digits.
- The number of places the decimal must move determines the exponent.
- Numbers greater than 1 require a positive exponent.
- Numbers less than 1 require a negative exponent.
- Only significant digits are represented in scientific notation.

Example:

1 5 0 0 0 0 0 0 0 0 0 0 .   becomes 1.5×10^{11} m

0.0 0 0 0 0 0 0 6.5   becomes 6.5×10^{-7} m

continued ▶

Skill Set 2 continued from previous page

SET 2 Skill Review

1. There are a dozen apples in a bowl. In this case, what type of number is 12?
2. Put the following numbers in order from most precise to least precise.
   - (a) 3.2, 5.88, 8, 8.965, 1.000 08
   - (b) 6.22, 8.5, 4.005, 1.2000×10^{-8}
3. How many significant digits are represented by each value?
   - (a) 215
   - (b) 31
   - (c) 3.25
   - (d) 0.56
   - (e) 1.06
   - (f) 0.002
   - (g) 0.006 04
   - (h) 1.250 000
   - (i) 1×10^{6}
   - (j) 3.8×10^{4}
   - (k) 6.807×10^{58}
   - (l) 3.000×10^{8}
4. Round the following values to two significant digits.
   - (a) 1.23
   - (b) 2.348
   - (c) 5.86
   - (d) 6.851
   - (e) 6.250
   - (f) 4.500
   - (g) 5.500
   - (h) 9.950
5. Complete the following calculations. Provide the final answer to the correct number of significant digits.
   - (a) 2.358×4.1
   - (b) $102 \div 0.35$
   - (c) $2.1 + 5.88 + 6.0 + 8.526$
   - (d) $12.1 - 4.2 - 3$
6. Write each of the following in scientific notation.
   - (a) 2.5597
   - (b) 1000
   - (c) 0.256
   - (d) 0.000 050 8
7. Write each value from question 6 in scientific notation accurate to three significant digits.

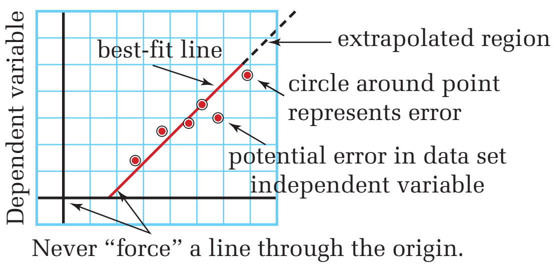
Drawing and Interpreting Graphs

Graphical analysis of scientific data is used to determine trends. Good communication requires that graphs be produced using a standard method. Careful analysis of a graph could reveal more information than the data alone.

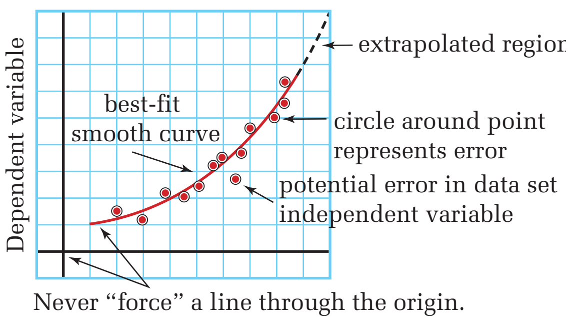
Standards for Drawing a Graph

- Independent variable is plotted along the horizontal axis (include units).
- Dependent variable is plotted along the vertical axis (include units).
- Decide whether the origin (0,0) is a valid data point.
- Select convenient scaling on the graph paper that will spread the data out as much as possible.
- A small circle is drawn around each data point to represent possible error.
- Determine a trend in the data — draw a best-fit line or best-fit smooth curve. Data points should never be connected directly when finding a trend.
- Select a title that clearly identifies what the graph represents.

Constructing a linear graph



Constructing a non-linear graph



Interpolation and Extrapolation

A best-fit line or best-fit smooth curve that is extended beyond the size of the data set should be shown as a dashed line. You are extrapolating values when you read them from the dashed-line region of the graph. You are interpolating values when you read them from the solid-line region of the graph.

Find a Trend

The best-fit line or smooth curve provides insight into the type of relationship between the variables represented in a graph.

A *best-fit line* is drawn so that it matches the general trend of the data. You should try to have as many points above the line as are below it. Do not cause the line to change slope dramatically to include only one data point that does not seem to be in line with all of the others.

A *best-fit smooth curve* should be drawn so that it matches the general trend of the data. You should try to have as many points above the line as are below it, but ensure that the curve changes smoothly. Do not cause the curve to change direction dramatically to include only one data point that does not seem to be in line with all of the others.

Definition of a Linear Relationship

A data set that is most accurately represented with a *straight line* is said to be linear. Data related by a linear relationship can be written in the form

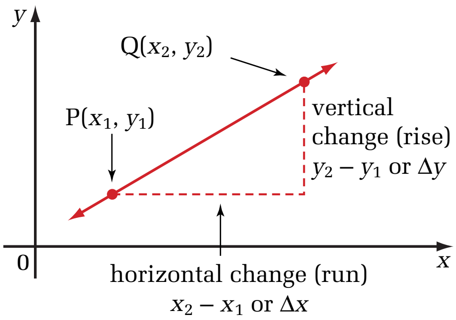
$$y = mx + b$$

Quantity	Symbol	SI unit
y value (dependent variable)	y	obtained from the vertical axis
x value (independent variable)	x	obtained from the horizontal axis
slope of the line	m	rise/run
y-intercept	b	obtained from the vertical axis when *x* is zero

continued ►

Slope (m)

Calculating the slope of a line



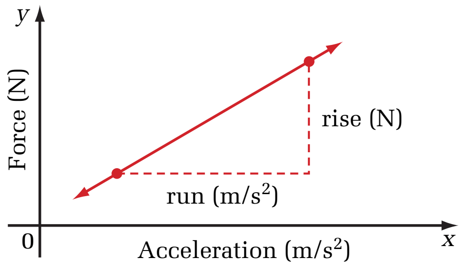
$$\text{slope } (m) = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}}$$

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$$

Mathematically, slope provides a measure of the steepness of a line by dividing the vertical change (rise) by the horizontal change (run). In scientific situations, it is also very important to include units of the slope. The units will provide physical significance to the slope value.

For example:



Including the units throughout the calculation helps verify the physical quantity that the slope represents.

$$m = \frac{\text{rise (N)}}{\text{run (m/s}^2\text{)}} \quad \text{Recall : 1 N = 1 kg} \cdot \text{m/s}^2$$

$$m = \frac{\text{kg} \cdot \cancel{\text{m/s}^2}}{\cancel{\text{m/s}^2}}$$

$$m = \text{kg}$$

In this example, the slope of the line represents the physical quantity of mass.

Definition of a Non-Linear Relationship

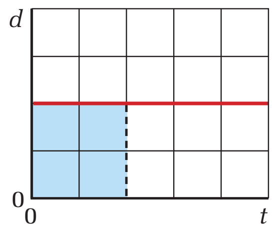
A data set that is most accurately represented with a smooth curve is said to be non-linear. Data related by a non-linear relationship can take several different forms. Two common non-linear relationships are as follows:

(a) parabolic $\quad y = ax^2 + k$

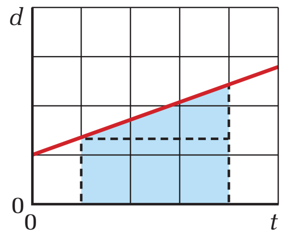
(b) inverse $\quad y = \frac{1}{x}$

Area Under a Curve

Mathematically, the area under a curve can be obtained without the use of calculus by finding the area using geometric shapes.

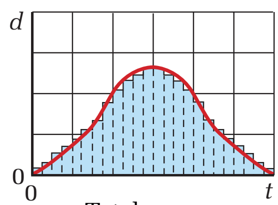


Total area = length × width

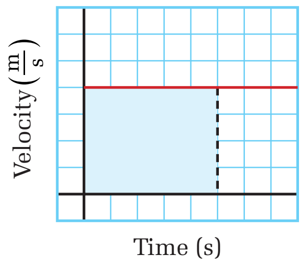


Total area =
area of the rectangle +
area of the triangle

Always include units in area calculations. The units will provide physical significance to the area value. For example, see below.



Total area =
area 1 + area 2 + area 3 …



Including the units throughout the calculation helps verify the physical quantity that the area represents.

Area = (length)(width)
Area = (velocity)(time)
Area = (m/s)(s)
Area = m (base unit for displacement)

The units verify that the area under a speed-versus-time curve represents displacement (m).

SET 3 Skill Review

1. **(a)** Plot the data in Table 1 by hand, ensuring that it fills at least two thirds of the page and has clearly labelled axes that include the units.
   (b) Draw a best-fit line through the plotted data.
   (c) Based on the data trend and the best-fit line, which data point seems to be most in error?
   (d) Interpolate the time it would take to travel 14 m.
   (e) Extrapolate to find how far the object would travel in 20 s.

Table 1

Time (s)	Distance (m)	Time (s)	Distance (m)
0	2	8	17
1	4	9	20
2	7	10	23
3	8	11	24
4	5	12	26
5	12	13	29
6	16	14	28
7	16	15	33

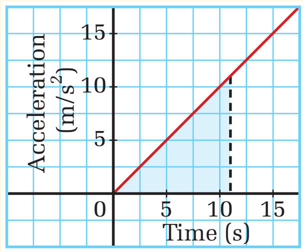
2. **(a)** Plot the data in Table 2 by hand, ensuring that it fills at least two thirds of the page and has clearly labelled axes that include the units.
   (b) Draw a best-fit smooth curve through the plotted data.
   (c) Does this smooth curve represent a linear or non-linear relationship?
   (d) At what force is the position at the greatest value?

Table 2

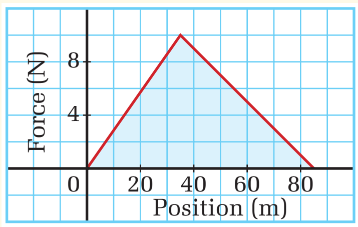
Force (N)	Position (m)	Force (N)	Position (m)
0	0.0	1.1	2.5
0.1	0.5	1.2	2.5
0.2	0.9	1.3	2.4
0.3	1.3	1.4	2.2
0.4	1.6	1.5	2.0
0.5	1.9	1.6	1.7
0.6	2.1	1.7	1.4
0.7	2.3	1.8	1.1
0.8	2.4	1.9	0.7
0.9	2.5	2	0.2
1	2.6		

3. Find the area of the shaded regions under the following graphs. Use the units to determine the physical quantity that the area represents.

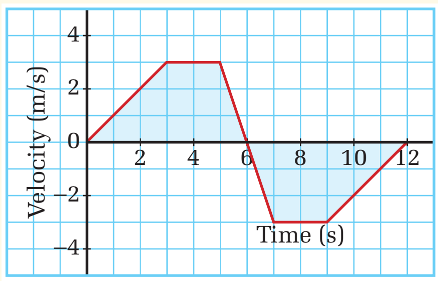
(a)



(b)



(c)



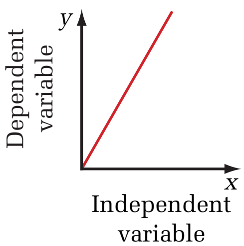
Mathematical Modelling and Curve Straightening

Patterns in quantitative data can be expressed in the form of mathematical equations. These relationships form a type of *mathematical model* of the phenomenon being studied. You can use the model to examine trends and to make testable numerical predictions.

Identifying Types of Relationships

Graphing data is a common way of revealing patterns. Simply by drawing a best-fit curve through the data points, it might be possible to identify a general type of mathematical relationship expressed in the observations. Four common patterns are illustrated below. Each pattern can be expressed algebraically as a proportionality statement ($a \propto b$) or as an equation. In mathematics courses, you might also have studied the graphs and equations of logarithmic, sinusoidal, or other types of relationships.

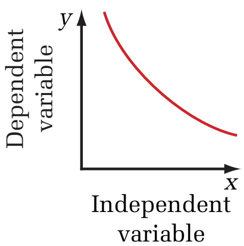
Basic mathematical relationships



A linear

$y \propto x$

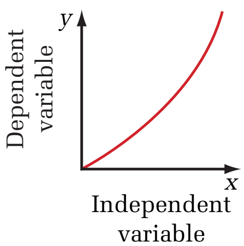
$y = kx$



B inverse

$y \propto \frac{1}{x^{n}}$ or $y \propto x^{-n}$

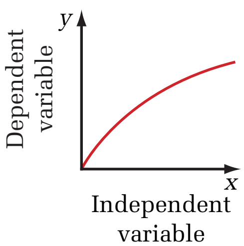
$y = kx^{-n}$



C exponential

$y \propto x^{n}$

$y = kx^{n}$



D root

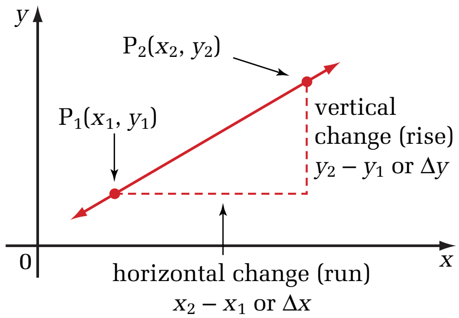
$y \propto \sqrt[n]{x}$ or $y \propto x^{\frac{1}{n}}$

$y = kx^{\frac{1}{n}}$

Linear Relationships

In previous studies, you have used the straight-line graph of a linear relationship to produce a specific mathematical equation that represents the graph. The equation is completely determined by the slope, m, of the graph and its y-intercept, b.

The equation of a straight-line graph



$$\text{slope } (m) = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}} = \frac{\Delta y}{\Delta x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1},\ x_2 \neq x_1$$

Equation of the line: $y = mx + b$

Straightening Non-Linear Graphs

You can often produce a straight-line graph from a non-linear relationship by making an appropriate choice of independent variables for the graph. By analyzing the resulting straight line, you can obtain an equation that fits the data. This procedure, which is called "curve straightening," produces equations of the form

(quantity on the vertical axis =
m (quantity on the horizontal axis) + b

You can straighten a curve by selecting the quantity graphed on the horizontal axis to match the general type of variation shown by the data. If the independent variable is x and you suspect

- inverse variation: plot $\frac{1}{x}$ or $\frac{1}{x^2}$ or $\frac{1}{x^3}$ on the horizontal axis
- exponential variation: plot x^2 or x^3 on the horizontal axis
- root variation: plot $x^{\frac{1}{2}}$ or $x^{\frac{1}{3}}$ on the horizontal axis

There is no mathematical reason why other exponents could not be used. Most phenomena examined in this course, however, are best modelled using integer exponents or roots no greater than three.

Procedure

1. From a table of raw data for two variables, x and y, produce an initial graph of y versus x.
2. Identify the general type of relationship shown by the graph.
3. Modify the independent variable to suit the proposed type of relationship. Add the new quantity to your data table and then draw a new graph of y against this quantity derived from x.
4. If the new graph is a straight line, calculate its slope and y-intercept. Use these values to write and simplify an equation to represent the data.
5. If the new graph is not a straight line, repeat steps 3 and 4, using a different modification of the independent variable until you obtain a straight-line graph.

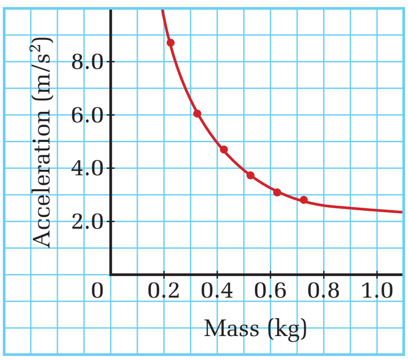
Example

A force of 1.96 N was used to accelerate a lab cart with mass 0.225 kg. The mass of the cart was then systematically increased, producing the accelerations shown below. Find an equation that represents this data.

Mass (kg)	Acceleration $\left(\frac{m}{s^2}\right)$
0.225	8.71
0.325	6.05
0.425	4.70
0.525	3.73
0.625	3.09
0.725	2.81

1. Graph the raw data.

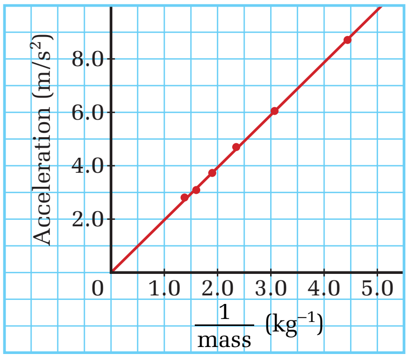
Acceleration against mass



2. Identify the type of variation. This graph shows inverse variation.
3. Modify the independent variable, extend the data table, and regraph the data. Choose the most simple possibility, $\frac{1}{\text{mass}}$, to investigate.

Mass (kg)	Acceleration $\left(\frac{m}{s^2}\right)$	$\frac{1}{\text{mass}}$ $\left(\frac{1}{kg}\right)$
0.225	8.71	4.44
0.325	6.05	3.07
0.425	4.70	2.35
0.525	3.73	1.90
0.625	3.09	1.60
0.725	2.81	1.38

Acceleration against $\frac{1}{\text{mass}}$



4. Since the graph is a straight line, use its slope and y-intercept to obtain its equation.

   slope (m) = 1.96

   y-intercept (b) 0.0500

   Equation of the line

   (quantity on the vertical axis) =
   m (quantity on the horizontal axis) + b

   $\text{acceleration} = 1.96\left(\frac{1}{\text{mass}}\right) + 0.0500$

   $\text{acceleration} = \frac{1.96}{m} + 0.0500$

5. If the graph had not been a straight line, the next most simple variation of the independent variable would have been considered: $\frac{1}{(\text{mass})^2}$.
6. The equation determined from the data is reasonable, because the situation is an example of Newton's second law, $F = ma$. Solved for acceleration, this becomes $a = \frac{F}{m}$,

Skill Set 4 continued from previous page

or $a = F(\frac{1}{m})$, which has the same form as the equation for the graph. The slope of the graph represents the force applied to the cart.
The y-intercept, 0.0500, is probably due to experimental error, as the graph should pass through the origin.

Points to Remember

Make sure that the scales on your graph axes start at zero. Otherwise, you will see a magnified view of only a small portion of the graph. The overall shape of the graph might not be shown, so it will be difficult to identify the type of variation in the data. Part of a gentle curve, for example, can appear to be a straight line.

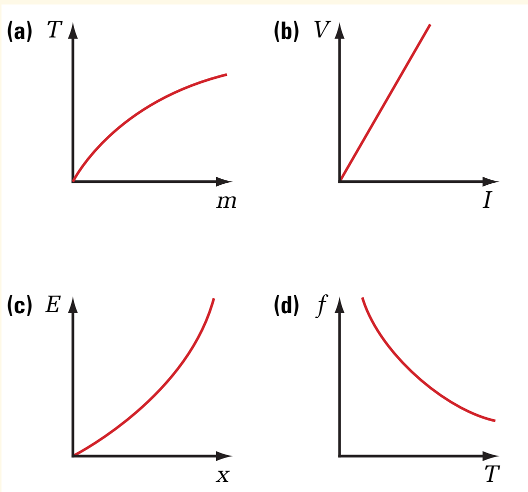
Sometimes it is a good idea to look at only part of a data set. The relationship between force applied to a spring and its extension (Hooke's law), for example, is linear, as long as the force does not exceed a certain value. Rather than trying to find a relationship that fits the entire graph, only the linear portion is usually considered. An initial graph of your data will show parts that are easy to model and will also reveal data points that are far from the best-fit line. Obvious errors are often excluded when developing equations to fit data.

Some computer programs are capable of automatic curve fitting. The resulting equations might fit experimental data well, without being very helpful. If you suspect that a certain phenomenon follows an inverse square law, for example, it would be sensible to choose $\frac{1}{(\text{dependent variable})^2}$ as the quantity to graph. Then you can determine how closely your observations approach an ideal model.

You might want to investigate other aspects of mathematical modelling. If you are familiar with logarithms, for example, consider the advantages and disadvantages of curve straightening by graphing, or the use of log or semi-log graph paper. You might also look into correlation coefficients, statistical measures that can express how well a given equation models a set of data. Finally, you might explore the use of power series to produce approximations of complex relationships.

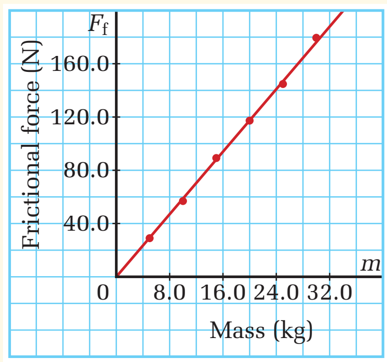
SET 4 Skill Review

1. Name the type of relationship represented by each graph below and write the relationship as a proportion and as a general equation.



2. The graph in the next column shows the force of static friction when you push different masses placed on a horizontal carpet and they start to move.
   (a) Determine the slope and intercept of the graph below.
   (b) Write an equation that represents the data.
   (c) Explain the physical meaning of each numerical coefficient in the equation.

Static friction between leather-soled shoes and a carpet



3. For each of the following relationships, use curve-straightening techniques to determine an equation that represents the data. If possible, validate your solution by giving physical reasons why the relationship must have the form it does.

(a) The gravitational attraction between two lead spheres in a Cavendish apparatus depends on the separation between their centres.

Separation (cm)	Gravitational force ($N \times 10^{-9}$)
55	3.13
70	1.93
85	1.31
100	0.946
115	0.715
130	0.560

(b) The buoyant force on a spherical weather balloon depends on how much the balloon is inflated (the volume of the balloon).

Radius of balloon (m)	Force (N)
2.285	569
2.616	855
2.879	1135
3.102	1422
3.296	1709
3.470	1993
3.628	2281

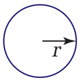
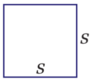
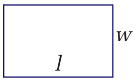
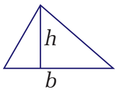
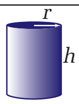
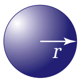
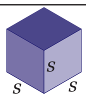
(c) The power dissipated by a light bulb is related to the electric current flowing through the light bulb.

Electrical current (A)	Power (W)
16.0	15
20.6	25
26.3	40
32.0	60
35.7	75
41.2	100

(d) The kinetic energy of a moving car depends on the car's velocity.

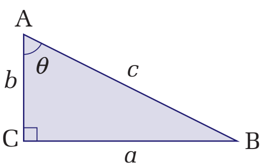
Velocity $\left(\frac{km}{h}\right)$	Kinetic energy (kJ)
15	15.4
25	42.7
35	83.8
45	139.4
55	208.9
65	289.8

A Math Toolbox

	Circumference/ perimeter	Area	Surface area	Volume
	$C = 2\pi r$	$A = \pi r^2$		
	$P = 4s$	$A = s^2$		
	$P = 2l + 2w$	$A = lw$		
		$A = \frac{1}{2}bh$		
			$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
			$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
			$SA = 6s^2$	$V = s^3$

Trigonometric Ratios

The ratios of side lengths from a right-angle triangle can be used to define the basic trigonometric function sine (sin), cosine (cos), and tangent (tan).



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{a}{c}$$

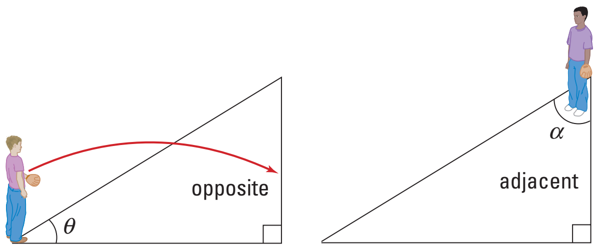
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{a}{b}$$

The angle selected determines which side will be called the opposite side and which the adjacent side. The hypotenuse is always the side across from the 90˚ angle. Picture yourself standing on top of the angle you select. The side that is directly across from your position is called the *opposite* side. The side that you could touch and is not the hypotenuse is the *adjacent* side.



A scientific calculator or trigonometry tables can be used to obtain an angle value from the ratio result. Your calculator performs a complex calculation (Maclaurin series summation) when the $\sin^{-1}$, or $\cos^{-1}$, or $\tan^{-1}$ operation is used to determine the angle value. Sin^{-1} is not simply a $1/\sin$ operation.

Definition of the Pythagorean Theorem

The Pythagorean theorem is used to determine side lengths of a right-angle (90°) triangle. Given a right-angle triangle ABC, the Pythagorean theorem states

$$c^2 = a^2 + b^2$$

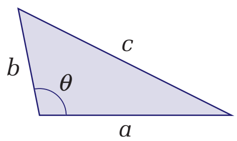
Quantity	Symbol	SI unit
hypotenuse side is opposite the 90° angle	c	m (metres)
side a	a	m (metres)
side b	b	m (metres)

Note: The hypotenuse is always the side across from the right (90°) angle. The Pythagorean theorem is a special case of a more general mathematical law called the "cosine law." The cosine law works for all triangles.

Definition of the Cosine Law

The cosine law is useful when

- determining the length of an unknown side given two side lengths and the contained angle between them
- determining an unknown angle given all side lengths



Angle θ is contained between sides a and b.

The cosine law states $\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos\theta}$.

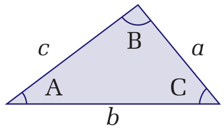
Quantity	Symbol	SI unit
unknown length side c opposite angle θ	c	m (metres)
length side a	a	m (metres)
length side b	b	m (metres)
angle θ opposite unknown side c	θ	(radians)

Note: Applying the cosine law to a right angle triangle, setting $\theta = 90°$, yields the special case of the Pythagorean theorem.

Definition of the Sine Law

The sine law is useful when

- two angles and any one side length are known
- two side lengths and any one angle are known



Given any triangle ABC the sine law states

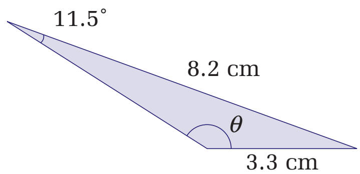
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Quantity	Symbol	SI unit
length side a opposite angle A	a	m (metres)
length side b opposite angle B	b	m (metres)
length side c opposite angle C	c	m (metres)
angle A opposite side a	A	(radians)
angle B opposite side b	B	(radians)
angle C opposite side c	C	(radians)

Note: The sine law generates ambiguous results in some situations because it does not discriminate between obtuse and acute triangles. An example of the ambiguous case is shown below.

Example

Use the sine law to solve for θ.



Sine law: ambiguous case

$$\frac{\sin\theta}{8.2} = \frac{\sin 11.5°}{3.3}$$

$$\sin\theta = 0.5$$

$$\theta = 30°$$

Clearly, angle θ is much greater than 30°. In this case, the supplementary angle is required (180° – 30° = 150°). It is important to recognize when dealing with obtuse angles (> 90°) that the supplementary angle might be required. Application of the cosine law in these situations will help reduce the potential for error.

Algebra

In some situations, it might be preferable to use algebraic manipulation of equations to solve for a specific variable before substituting numbers. Algebraic manipulation of variables follows the same rules that are used to solve equations after substituting values. In both cases, to maintain equality, whatever is done to one side must be done to the other.

Solving for "*x*" before Numerical Substitution		Solving for "*x*" after Numerical Substitution	
(a) $A = kx$	x is multiplied by k, so divide by k to isolate x.	**(a)** $8 = 2x$	x is multiplied by 2, so divide by 2 to isolate x.
$\frac{A}{k} = \frac{\cancel{k}x}{\cancel{k}}$	Divide both sides of the equation by k.	$\frac{8}{2} = \frac{\cancel{2}x}{\cancel{2}}$	Divide both sides of the equation by 2.
$\frac{A}{k} = x$	Simplify.	$4 = x$	Simplify.
$x = \frac{A}{k}$	Rewrite with x on the left side.	$x = 4$	Rewrite with x on the left side.
(b) $B = \frac{x}{g}$	x is divided by g, so multiply by g to isolate x.	**(b)** $8 = \frac{x}{4}$	x is divided by 4, so multiply by 4 to isolate x.
$Bg = \frac{x\cancel{g}}{\cancel{g}}$	Multiply both sides of the equation by g.	$(10)(4) = \frac{4x}{4}$	Multiply both sides of the equation by 4.
$Bg = x$	Simplify.	$40 = x$	Simplify.
$x = Bg$	Rewrite with x on the left side.	$x = 40$	Rewrite with x on the left-hand side.
(c) $W = x + f$	x is added to f, so subtract f to isolate x.	**(c)** $25 = x + 13$	x is added to 13, so subtract 13 to isolate x.
$W - f = x + f - f$	Subtract f on both sides of the equation.	$25 - 13 = x + 13 - 13$	Subtract 13 from both sides of the equation.
$W - f = x$	Simplify.	$12 = x$	Simplify.
$x = W - f$	Rearrange for x.	$x = 12$	Rewrite with x on the left-hand side.
(d) $W = \sqrt{x}$	x is under a square root, so square both sides of the equation.	**(d)** $6 = \sqrt{x}$	x is under a square root, so square both sides of the equation.
$W^2 = (\sqrt{x})^2$		$6^2 = (\sqrt{x})^2$	
$W^2 = x$	Simplify.	$36 = x$	Simplify.
$x = W^2$	Rearrange for x.	$x = 36$	Rewrite with x on the left-hand side.

Definition of the Quadratic Formula

The quadratic equation is used to solve for the roots of a quadratic function. Given a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, the roots of it can be found using

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Statistical Analysis

In science, data are collected until a trend is observed. Three statistical tools that assist in determining if a trend is developing are *mean*, *median*, and *mode*.

Mean: The sum of the numbers divided by the number of values. It is also called the "average."

Median: When a set of numbers is organized in order of size, the median is the middle number. When the data set contains an even number of values, the median is the average of the two middle numbers.

Mode: The number that occurs most often in a set of numbers. Some data sets will have more than one mode.

See examples of these on the following page.

Example 1:
Odd number of data points

Data Set 1: 12, 11, 15, 14, 11, 16, 13

$$\text{mean} = \frac{12 + 11 + 15 + 14 + 11 + 16 + 13}{7}$$
$$\text{mean} = 13$$
$$\text{reorganized data} = 11, 11, 12, 13, 14, 15, 16$$
$$\text{median} = 13$$
$$\text{mode} = 11$$

Example 2:
Even number of data points

Data Set 2: 87, 95, 85, 63, 74, 76, 87, 64, 87, 64, 92, 64

$$\text{mean} = \frac{(87 + 95 + 85 + 63 + 74 + 76 + 87 + 64 + 87 + 64 + 92 + 64)}{12}$$
$$\text{mean} = 78$$
$$\text{reorganized data} = 63, 64, 64, 64, 74, 76, 85, 87, 87, 87, 92, 95$$
$$\text{median} = \frac{(76 + 85)}{2}$$
$$\text{median} = 80$$

An even number of data points requires that the middle two numbers be averaged.

$$\text{mode} = 64, 87$$

In this example, the data set is bimodal (contains two modes).

SET 5 Skill Review

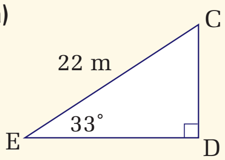
1. Calculate the area of a circle with radius 6.5 m.

2. By how much does the surface area of a sphere increase when the radius is doubled?

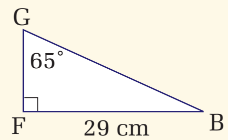
3. By how much does the volume of a sphere increase when the radius is doubled?

4. Find all unknown angles and side lengths.

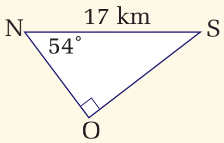
(a)



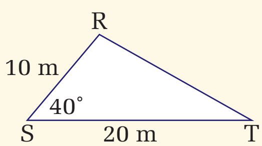
(b)



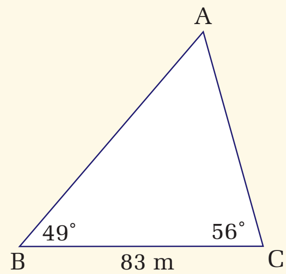
(c)



5. Use the cosine law to solve for the unknown side.



6. Use the sine law to solve for the unknown sides.



7. Solve for x in each of the following.

(a) $42 = 7x$

(b) $30 = x/5$

(c) $12 = x \sin 30°$

(d) $8 = 2x - 12^4$

8. Solve for x in each of the following.

(a) $F = kx$

(b) $G = hk + x$

(c) $a = bx \cos\theta$

(d) $b = d \cos x$

(e) $a = bc + x^2$

(f) $T = 2\pi\sqrt{\frac{1}{x}}$

9. Use the quadratic equation to find the roots of the function.
$4x^2 + 15x + 13 = 0$

10. Find the mean, median, and mode of each data set.

(a) 25, 38, 55, 58, 60, 61, 61, 65, 70, 74, 74, 74, 78, 79, 82, 85, 90

(b) 13, 14, 16, 17, 18, 20, 20, 22, 26, 30, 31, 32, 32, 35

The Metric System: Fundamental and Derived Units

Metric System Prefixes

Prefix	Symbol	Factor
tera	T	1 000 000 000 000 = 10^{12}
giga	G	1 000 000 000 = 10^{9}
mega	M	1 000 000 = 10^{6}
kilo	k	1000 = 10^{3}
hecto	h	100 = 10^{2}
deca	da	10 = 10^{1}
		1 = 10^{0}
deci	d	0.1 = 10^{-1}
centi	c	0.01 = 10^{-2}
milli	m	0.001 = 10^{-3}
micro	μ	0.000 001 = 10^{-6}
nano	n	0.000 000 001 = 10^{-9}
pico	p	0.000 000 000 001 = 10^{-12}
femto	f	0.000 000 000 000 001 = 10^{-15}
atto	a	0.000 000 000 000 000 001 = 10^{-18}

Fundamental Physical Quantities and Their SI Units

Quantity	Symbol	Unit	Symbol
length	l	metre	m
mass	m	kilogram	kg
time	t	second	s
absolute temperature	T	Kelvin	K
electric current	I	ampère (amp)	A
amount of substance	mol	mole	mol

Derived SI Units

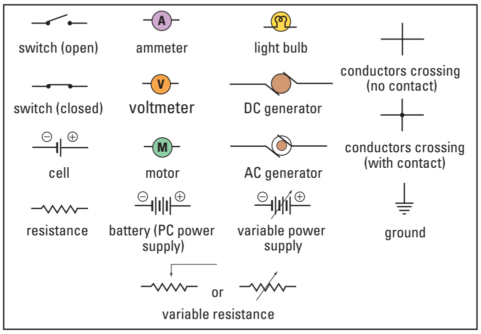
Quantity	Quantity symbol	Unit	Unit symbol	Equivalent unit(s)
area	A	square metre	m^2	
volume	V	cubic metre	m^3	
velocity	v	metre per second	m/s	
acceleration	a	metre per second per second	m/s^2	
force	F	newton	N	$kg \cdot m/s^2$
work	W	joule	J	$N \cdot m$, $kg \cdot m^2/s^2$
energy	E	joule	J	$N \cdot m$, $kg \cdot m^2/s^2$
power	P	watt	W	J/s, $kg \cdot m^2/s^3$
density	ρ	kilogram per cubic metre	kg/m^3	
pressure	p	pascal	Pa	N/m^2, $kg/(m \cdot s^2)$
frequency	f	hertz	Hz	s^{-1}
period	T	second	s	
wavelength	λ	metre	m	
electric charge	Q	coulomb	C	$A \cdot s$
electric potential	V	volt	V	W/A, J/C, $kg \cdot m^2/(C \cdot s^2)$
resistance	R	ohm	Ω	V/A, $kg \cdot m^2/(C^2 \cdot s)$
magnetic field intensity	B	tesla	T	$N \cdot s/(C \cdot m)$, $N/(A \cdot m)$
magnetic flux	Φ	weber	Wb	$V \cdot s$, $T \cdot m^2$, $m^2 \cdot kg/(C \cdot s)$
radioactivity	$\Delta N/\Delta t$	becquerel	Bq	s^{-1}
radiation dose		gray	Gy	$J/kg \cdot m^2/s^2$
temperature (Celsius)	T	degree Celsius	°C	T°C = (T + 273.15) K
		atomic mass unit	u	$1u = 1.660\,566 \times 10^{-27}$ kg
		electron volt	eV	$1\ eV = 1.602 \times 10^{-19}$ J

Physical Constants and Data

Fundamental Physical Constants

Quantity	Symbol	Accepted value
speed of light in a vacuum	c	2.998×10^{8} m/s
gravitational constant	G	6.673×10^{-11} N · m²/kg²
Coulomb's constant	k	8.988×10^{9} N · m²/C²
charge on an electron	e	1.602×10^{-19} C
rest mass of an electron	m_e	9.109×10^{-31} kg
rest mass of a proton	m_p	1.673×10^{-27} kg
rest mass of a neutron	m_n	1.675×10^{-27} kg
Planck's constant	h	6.626×10^{-34} J · s

Electric Circuit Symbols

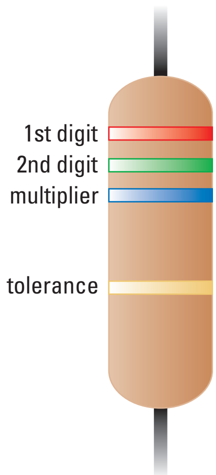


Other Physical Data

Quantity	Symbol	Accepted value
standard atmospheric pressure	P	1.013×10^{5} Pa
speed of sound in air		343 m/s (at 20°C)
water: density (4°C)		1.000×10^{3} kg/m³
latent heat of fusion		3.34×10^{5} J/kg
latent heat of vaporization		2.26×10^{6} J/kg
specific heat capacity (15°C)		4186 J/(kg°C)
kilowatt hour	E	3.6×10^{6} J
acceleration due to Earth's gravity	g	9.81 m/s² (standard value; at sea level)
mass of Earth	m_E	5.98×10^{24} kg
mean radius of Earth	r_E	6.38×10^{6} m
mean radius of Earth's orbit	R_E	1.49×10^{11} m
period of Earth's orbit	T_E	365.25 days or 3.16×10^{7} s
mass of Moon	m_M	7.36×10^{22} kg
mean radius of Moon	r_M	1.74×10^{6} m
mean radius of Moon's orbit	R_M	3.84×10^{8} m
period of Moon's orbit	T_M	27.3 days or 2.36×10^{6} s
mass of Sun	m_s	1.99×10^{30} kg
radius of Sun	r_s	6.96×10^{8} m

Resistor Colour Codes

Colour	Digit represented	Multiplier	Tolerance
black	0	$\times 1$	
brown	1	$\times 1.0 \times 10^{1}$	
red	2	$\times 1.0 \times 10^{2}$	
orange	3	$\times 1.0 \times 10^{3}$	
yellow	4	$\times 1.0 \times 10^{4}$	
green	5	$\times 1.0 \times 10^{5}$	
blue	6	$\times 1.0 \times 10^{6}$	
violet	7	$\times 1.0 \times 10^{7}$	
gray	8	$\times 1.0 \times 10^{8}$	
white	9	$\times 1.0 \times 10^{9}$	
gold		$\times 1.0 \times 10^{-1}$	5%
silver		$\times 1.0 \times 10^{-2}$	10%
no colour			20%



Mathematical Equations

Equations in Unit 1 — Forces and Motion: Dynamics		
Equation	**Variable**	**Name**
$\vec{F} = m\vec{a}$	$\vec{F}$ = net force m = mass $\vec{a}$ = acceleration	Newton's second law
$\lvert\vec{F}_f\rvert = \mu_s\lvert\vec{F}_N\rvert$ $\lvert\vec{F}_f\rvert = \mu_k\lvert\vec{F}_N\rvert$	$\vec{F}_f$ = force of friction μ_s = coefficient of static friction μ_k = coefficient of kinetic friction $\vec{F}_N$ = normal force	friction
$\vec{F}_g = m\vec{g}$	$\vec{F}$ = force of gravity at Earth's surface m = mass $\vec{g}$ = acceleration due to gravity (on Earth)	weight
$\Delta d = v\Delta t$ $v_2 = v_1 + a\Delta t$ $\Delta d = v_1\Delta t + \frac{1}{2}a\Delta t^2$ $v_2^2 = v_1^2 + 2a\Delta d$	Δd = displacement v = velocity v_1 = initial velocity v_2 = final velocity a = acceleration Δt = time interval	motion equations (constant acceleration)
$R = \frac{v_i^2 \sin 2\theta}{g}$ $H_{max} = \frac{v_i \sin^2 \theta}{2g}$	R = range H_{max} = maximum height v_i = initial velocity θ = launch angle g = acceleration due to gravity	projectile range projectile maximum height
$a_c = \frac{v^2}{r}$	a_c = centripetal acceleration v = velocity r = radius	centripetal acceleration
$F_c = \frac{mv^2}{r}$	F_c = centripetal force v = velocity r = radius	centripetal force
$\frac{T^2}{r^3} = k$ $\frac{T_A^2}{r_A^3} = \frac{T_B^2}{r_B^3}$	T = period of planet r = average distance from planet to Sun k = constant T_A = period of planet A T_B = period of planet B r_A = average distance of planet A to Sun r_B = average distance of planet B to Sun	Kepler's third law
$F_g = G\frac{m_1 m_2}{r^2}$	F_g = force of gravity between 2 point masses G = universal gravitational constant m_1 = first mass m_2 = second mass r = distance between the centres of the masses	Newton's law of universal gravitation

Equations in Unit 2 — Energy and Momentum

$\vec{p} = m\vec{v}$	$\vec{p}$ = momentum m = mass $\vec{v}$ = velocity	momentum
$\vec{J} = \vec{F}\Delta t = m\vec{v}_2 - m\vec{v}_1 = \Delta\vec{p}$	$\vec{J}$ = impulse $\vec{F}$ = force Δt = time interval m = mass $\vec{v}_1$ = initial velocity $\vec{v}_2$ = final velocity $\Delta\vec{p}$ = change in momentum	impulse
$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$	m_A = mass of object A m_B = mass of object B $\vec{v}_A$ = velocity of object A *before* collision $\vec{v}_B$ = velocity of object B *before* collision $\vec{v}'_A$ = velocity of object A *after* collision $\vec{v}'_B$ = velocity of object B *after* collision	conservation of momentum
$v'_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1$ $v'_2 = \left(\frac{2m_1}{m_1 + m_2}\right)v_1$	v'_1 = velocity of object 1 *after* collision v'_2 = velocity of object 2 *after* collision m_1 = mass of object 1 m_2 = mass of object 2 v_1 = velocity of object 1 *before* collision	perfectly elastic, head-on collision, using a frame of reference such that $v_2 = 0$
$W = F\Delta d\cos\theta$	W = work F = applied force Δd = displacement θ = angle between force and displacement	work
$E_k = \frac{1}{2}mv^2$	E_k = mechanical kinetic energy m = mass v = velocity	mechanical kinetic energy
$E_g = mg\Delta h$	E_g = gravitational potential energy m = mass Δh = change in height	gravitational potential energy
$E_k + E_g + E_e = E'_k + E'_g + E'_e$	E_k = initial kinetic energy E_g = initial gravitational energy E_e = initial elastic energy E'_k = final kinetic energy E'_g = final gravitational energy E'_e = final elastic energy	conservation of mechanical energy
$F_a = kx$	F_a = applied force k = spring constant x = extension or compression of spring	Hooke's law
$E_e = \frac{1}{2}kx^2$	E_e = elastic potential energy k = spring constant x = extension or compression of spring	elastic potential energy
$v = \sqrt{\frac{2GM_p}{r_p}}$	v = escape speed G = universal gravitational constant M_p = mass of planet r_p = radius of planet	escape speed

Appendix C continued from previous page

$F_c = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$	F_c = centripetal force m = mass of object v = speed of object r = radius of orbit T = period of orbit	centripetal force
$v = \sqrt{\frac{GM}{r}}$	v = speed of orbiting object G = universal gravitational constant M = mass of planet or star r = radius of orbit	speed of satellite
$a_c = \frac{v^2}{r} = \frac{4\pi^2}{T^2} r$	a_c = centripetal acceleration v = speed of orbiting object r = radius of orbit T = period of orbit	centripetal acceleration
$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$	T = period of orbit r = radius of orbit G = universal gravitational constant M = mass of planet or star	orbital period (squared)
$E_g = -\frac{GMm}{r}$ $E_k = \frac{GMm}{2r}$ $E_{total} = -\frac{GMm}{2r}$ $E_{binding} = \frac{GMm}{r}$	E_g = gravitational potential energy E_k = kinetic energy E_{total} = total orbital energy $E_{binding}$ = binding energy	orbital energies
$F_{thrust} = \left(\frac{m_{gas}}{\Delta t}\right)\Delta v_{gas}$	F_{thrust} = thrust force m_1 = mass of expelled gas Δv_{gas} = speed of expelled gas Δt = time interval	rocket thrust
Equations in Unit 3 — Electric, Gravitational, and Magnetic Fields		
$F_Q = k\frac{q_1 q_2}{r^2}$	F_Q = electrostatic force between charges k = Coulomb's constant q_1 = electric charge on object 1 q_2 = electric charge on object 2 r = distance between object centres	Coulomb's law
$\vec{E} = \frac{\vec{F}_Q}{q}$	$\vec{E}$ = electric field intensity $\vec{F}_Q$ = electric force q = electric charge	electric field intensity
$\vec{g} = \frac{\vec{F}_g}{m}$ $g = \frac{Gm}{r^2}$	$\vec{g}$ = gravitational field intensity $\vec{F}_g$ = force of gravity m = mass G = universal gravitational constant r = distance from centre of object	gravitational field intensity
$\lvert\vec{E}\rvert = k\frac{q}{r^2}$	$\vec{E}$ = electric field intensity k = Coulomb's constant q = source charge r = distance from centre of charge	Coulombic electrostatic field

$V = \frac{E_Q}{q}$ $V = k\frac{q}{r}$	V = electric potential difference E_Q = electric potential energy q = electric charge r = distance k = Coulomb's constant	electric potential electric potential due to a point charge
$\Delta V = \frac{W}{q}$	ΔV = electric potential difference W = work done q = electric charge	electric potential difference
$\lvert\vec{E}\rvert = \frac{\Delta V}{\Delta d}$	$\lvert\vec{E}_Q\rvert$ = electric field intensity ΔV = electric potential difference Δd = component of displacement between points, parallel to field	electric field and potential difference
$F_M = qvB\sin\theta$	F_M = magnitude of the magnetic force on moving charged particle q = electric charge on particle v = speed of particle B = magnetic field intensity θ = angle between velocity vector and magnetic field vector	force on a moving charge in a magnetic field
$F_M = IlB\sin\theta$	F_M = magnitude of the magnetic force on moving charged particle B = magnetic field intensity I = electric current in conductor l = length of conductor in magnetic field θ = angle between conductor and magnetic field	force on a current-carrying conductor in a magnetic field
Equations in Unit 4 — The Wave Nature of Light		
$T = \frac{\Delta t}{N}$ $f = \frac{1}{T}$ $f = \frac{N}{\Delta t}$	T = period f = frequency Δt = time interval N = number of cycles	period and frequency
$PD = (n - \frac{1}{2})\lambda = d\sin\theta$ $n = 1, 2, 3, \ldots$ for dark fringes $PD = n\lambda = d\sin\theta$ $n = 0, 1, 2, 3, \ldots$ for light fringes $\lambda \cong \frac{d}{(n - \frac{1}{2})}\left(\frac{y_n}{x}\right)$ $\lambda \cong \left(\frac{d}{n}\right)\left(\frac{y_n}{x}\right)$	PD = path difference n = nodal line number λ = wavelength d = slit separation θ = angle between central bisector and line formed from slit to nodal point y_n = distance from central maximum fringe to fringe n x = distance from slits to screen	path difference dark fringes light fringes
$\lambda \cong \frac{\Delta y d}{x}$	λ = wavelength Δy = space between fringes d = slit separation x = distance from slits to screen	Young's double-slit experiment

Appendix C continued from previous page

$y_m \cong \frac{m\lambda L}{w}$ $(m = \pm1, \pm2, \pm3 \ldots)$ $y_m \cong \frac{(m + \frac{1}{2})\lambda L}{w}$ $(m = \pm1, \pm2, \pm3 \ldots)$	y_m = distance from fringe m to central bisector m = fringe order number L = distance to screen w = width of slit	single-slit interference destructive constructive
$\theta_{min} = \frac{\lambda}{w}$ $\theta_{min} = \frac{1.22\lambda}{D}$	θ_{min} = minimum angle for resolution λ = wavelength of light w = width of rectangular aperture D = diameter of circular aperture	resolution slit aperture circular aperture
$m\lambda = d\sin\theta\,(m = 0, 1, 2, \ldots)$	m = fringe order number λ = wavelength of light d = distance between slit centres θ = angle from central bisector	diffraction grating bright fringes
$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$	c = speed of light μ_0 = permeability of free space ε_0 = permittivity of free space	speed of electromagnetic radiation
$E = hf$	E = energy of photon h = Planck's constant f = frequency of wave	energy of photon
$c = f\lambda$	c = speed of light f = frequency of wave λ = wavelength	wave equation
Equations in Unit 5 — Matter-Energy Interface		
$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$ $L = L_o\sqrt{1 - \frac{v^2}{c^2}}$ $m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$ $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	Δt = dilated time Δt_0 = proper time v = speed of object c = speed of light L = relativistic length L_0 = proper length m = relativistic mass m_0 = proper mass γ = gamma	time dilation length contraction mass increase variable substitution for simplicity
$E_k = mc^2 - m_oc^2$ $E_{k(max)} = hf - W$	E_k = relativistic kinetic energy m = relativistic mass m_0 = proper mass c = speed of light $E_{k(max)}$ = maximum kinetic energy of photoelectron h = Planck's constant f = frequency of electromagnetic radiation W = work function of metal	kinetic energy at relativistic speeds photoelectric effect

$p = \frac{h}{\lambda}$	p = momentum h = Planck's constant λ = wavelength	momentum of a photon
$\lambda = \frac{h}{mv}$	λ = wavelength m = mass v = velocity h = Planck's constant	de Broglie wavelength
A = Z + N	A = atomic mass number Z = atomic number N = number of neutrons	atomic mass number
$N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$	N_0 = original amount of radioactive material N = amount of radioactive material remaining after Δt Δt = time interval $T_{\frac{1}{2}}$ = half life	amount of radioactive material remaining
$v = \frac{2V}{Br}$ $\frac{m}{e} = \frac{2V}{v^2}$	v = speed of electron V = accelerating voltage B = magnetic field strength r = radius of trajectory m = mass of electron e = charge of electron	speed of electrons electron mass to charge ratio

Achieving in Physics

WEB LINK

www.mcgrawhill.ca/links/physics12

This feature directs you to interesting and informative Internet sites. Access is easy when you use the Physics 12 Internet page links.

PROBEWARE

This logo indicates where electronic probes could be used as part of the procedure or as a separate lab.

The following Achievement Chart identifies the four categories of knowledge and skills in science that will be used in all science courses to assess and evaluate your achievement. The chart is provided to help you in assessing your own learning and in planning strategies for improvement with the help of your teacher.

You will find that all written text, problems, investigations, activities, and questions throughout this textbook have been developed to encompass the curriculum expectations of your course. The expectations are encompassed by these general categories: Knowledge/Understanding K/U, Inquiry I, Communication C, and Making Connections MC. You will find, for example, that questions in the textbook have been designated under these categories so that you can determine if you are able to achieve well in each category. Some questions could easily fall under a different category; for each question, the category chosen is the one with which it best complies. (In addition, problems that involve calculation have been designated either as practice problems or, in chapter and unit reviews, as Problems for Understanding.) Keep a copy of this chart in your notebook as a reminder of the expectations of you as you proceed through the course.

Achievement Chart

Knowledge/ Understanding	Inquiry	Communication	Making Connections
■ Understanding of concepts, principles, laws, and theories ■ Knowledge of facts and terms ■ Transfer of concepts to new contexts ■ Understanding of relationships between concepts	■ Application of the skills and strategies of scientific inquiry ■ Application of technical skills and procedures ■ Use of tools, equipment, and materials	■ Communication of information and ideas ■ Use of scientific terminology, symbols, conventions, and standard (SI) units ■ Communication for different audiences and purposes ■ Use of various forms of communication ■ Use of information technology for scientific purposes	■ Understanding of connections among science, technology, society, and the environment ■ Analysis of social and economic issues involving science and technology ■ Assessment of impacts of science and technology on the environment ■ Proposing courses of practical action in relation to science- and technology-based problems

Safety Symbols

The following safety symbols are used in *Physics 12* to alert you to possible dangers. Make sure that you understand each symbol that appears in a lab or investigation before you begin.

Thermal Safety	This symbol appears as a reminder to be careful when handling hot objects.
Sharp Object Safety	This symbol appears when there is danger of cuts or punctures caused by the use of sharp objects.
Fume Safety	This symbol appears when chemicals or chemical reactions could cause dangerous fumes.
Electrical Safety	This symbol appears as a reminder to be careful when using electrical equipment.
Skin Protection Safety	This symbol appears when the use of caustic chemicals might irritate the skin or when contact with micro-organisms might transmit infection.
Clothing Protection Safety	A lab apron should be worn when this symbol appears.
Fire Safety	This symbol appears as a reminder to be careful around open flames.
Eye Safety	This symbol appears when there is danger to the eyes and safety glasses should be worn.
Chemical Safety	This symbol appears when chemicals could cause burns or are poisonous if absorbed through the skin.

Safety symbols used in *Physics 12*

Look carefully at the WHMIS (Workplace Hazardous Materials Information System) safety symbols shown below. These symbols are used throughout Canada to identify dangerous materials in all workplaces, including schools. Make sure that you understand what these symbols mean. When you see these symbols on containers in your classroom, at home, or in a workplace, use safety precautions.

Compressed Gas	Flammable and Combustible Material
Oxidizing Material	Corrosive Material
Poisonous and Infectious Material Causing Immediate and Serious Toxic Effects	Poisonous and Infectious Material Causing Other Toxic Effects
Biohazardous Infectious Material	Dangerously Reactive Material

WHMIS symbols

Practice Problems and Chapter and Unit Review Problems

Chapter 1

Practice Problems

1. 9.6×10^{-13} N
2. 9.3 m/s
3. 0.61 m/s^2
4. (a) 0.249 N **(b)** 0.00127
5. 78 N
6. (a) 58 N **(b)** 16 m/s^{-2}
7. 6.7 m
8. 1.6×10^3 N, 9.1×10^2 N
9. (a) 21 N **(b)** 15 N
10. (a) 74 N **(b)** 34 N
11. negative; 5.9×10^2 N
12. down (negative); 6.9×10^2 N
13. up (positive); 5.9×10^2 N
14. −1.9 m/s^2
15. No, the climber must limit his descent to $a = -2.5$ m/s^2.
16. (a) downward **(b)** −1.1 m/s^2 **(c)** 87 N
17. 1.7×10^2 N
18. 1.8 m/s^2
19. 0.49 m/s^2; 39 N
20. 14 kg; 75 N
21. 62 kg; 1.6 m/s^2
22. 17 N
23. Both of them will rise, with $a = +1.0$ m/s^2.
24. (a) 3.88 N **(b)** 2.04 m/s^2
25. 0.67 s
26. 15 m/s
27. (a) 1.2 m/s^2 **(b)** 0.16 m/s^2 **(c)** 12 s
28. 0.061
29. 0.34 m
30. 0.37

Chapter 1 Review

Problems for Understanding

16. 3.0 m/s[N]
17. 11 kg
18. (a) $v = 0$; $a = -9.8$ m/s^2
(b) 3.5 m/s; −9.8 m/s^2
19. (a) 1.34 m/s^2 **(b)** 334 N
20. 1.1 m/s^2[W]
21. 1.2 N
22. (a) 0.062 m/s^2
(b) 0.40 m/s^2
(c) A friction force of magnitude 3.4 N operates to reduce the ideal acceleration ($a = F/m$)
23. 5.4 m
24. 11 m
25. (a) 5.4 m/s[down]
(b) 3.8×10^4 N[up]
26. 49 N
27. 1.3 m/s^2
28. (a) $a_2 = 2.5a_1$ **(b)** $d_2 = 2.5d_1$
29. (a) 9.00 N **(b)** 132 N **(c)** 293 N **(d)** 0.451
30. 3.3 m/s^2; 13 N
31. (a) 4.6 m/s^2 **(b)** 0.70 N

Chapter 2

Practice Problems

1. −677 m
2. 4.67 m/s
3. 89.6 m, 45.2 m/s [60.3° below the horizontal]
4. 0.156 m
5. 3.05 m/s
6. 0.55 m
7. 74 m
8. (a) 153 m **(b)** 5.00 m/s
9. 85 m
10. 4.0×10^1 m
11. 18 m/s [52° below the horizontal]
12. 2.8 m/s
13. (a) 58.9 m **(b)** 21.0 m **(c)** 4.14 s
14. 33.2°; 2.39 m; 1.40 s
15. 47.0 m/s
16. 8.3×10^{-8} N
17. (a) 48.6 N **(b)** 54.2 N **(c)** 9.62 m/s
18. 5.9×10^3 N
19. 84 m
20. 103 m
21. 13 m/s (47 km/h)
22. 19.1 m/s (68.8 km/h)
23. 20.1°

Chapter 2 Review

Problems for Understanding

20. (a) 3.0×10^1 m **(b)** 3.7 s
21. (a) 0.78 s
(b) at the same position
(c) 4.7 m **(d)** 9.7 m/s
22. (a) 42 m **(b)** 62 m
23. 2.7×10^2 m
24. (a) 2.1 s **(b)** 34 m
(c) 8.5 m
(d) $v_x = 16$ m/s; $v_y = +3.8$ m/s or −3.8 m/s
(e) 38.2°
25. 52 m/s
26. Yes. It travels 330 m.
27. (a) 7.4 s
(b) 67 m
(c) 1.2×10^2 m
(d) x: 34 m, y: 53 m
(e) $v_x = 17$ m/s; $v_y = -23$ m/s
28. (a) 193 m/s
(b) 843 m
(c) $v_x = 162$ m/s, $v_y = 156.3$ m/s
(d) 44.0°
29. (a) 2.0 m/s **(b)** 1.2 m/s^2
30. 7.1×10^2 N
31. (a) 1.33×10^{14} m/s^2
(b) 1.21×10^{-16} N
32. 0.33
33. 8.9 m/s
34. 33°
35. (a) 9.90 m/s
(b) A factor of $\sqrt{2}$
36. 0.62
37. (a) 4.6×10^2 m/s
(b) 2.0 N (for $m = 60.0$ kg)
(c) Toward the centre of Earth; gravity
(d) $mg = 589$ N (for $m = 60.0$ kg)
(e) $N = mg - mv^2/r = 587$ N
(f) $mg - N = ma_c$; because $mg > N$, there is a net acceleration toward the centre of Earth.

Chapter 3

Practice Problems

1. 3.57×10^{22} N
2. 1.99×10^{20} N
3. 5.1×10^{-3} m. This is much smaller than the radii of the bowling balls.
4. 3.61×10^{-47} N
5. 5.0×10^{24} kg
6. 0.25 m
7. $F_{Uranus} = 0.80 \times F_{Earth}$
8. 0.9 × Earth-Moon distance
9. 1.899×10^{27} kg
10. 1.472×10^{22} kg
11. 2.74×10^5 m
12. 1.02×10^3 m/s
13. (a) 6.18×10^4 s (17.2 h)
(b) 7.93×10^2 m/s
14. 4×10^{41} kg $= 2 \times 10^{11} \times M_{Sun}$

15. 7.42×10^3 m/s; 8.59×10^5 m

16. 7.77×10^3 m/s; 5.34×10^3 s (89.0 min)

17. **(a)** 5.21×10^9 s (165 years); 5.43×10^3 m/s
(b) It will complete one orbit, after its discovery, in the year 2011.

Chapter 3 Review

Problems for Understanding

22. 1/8

23. **(c)** F

24. **(b)** $a/3$

25. **(a)** 3.0×10^4 m/s
(b) 6.0×10^{-3} m/s^2

26. 1.8×10^{-8} m/s^{-2}

27. 9.03 m/s^2 = 92% of acceleration due to gravity at Earth's surface

28. 4.1×10^{36} kg = $2.0 \times 10^6 \times m_{\text{Sun}}$

29. 2.67×10^{-10} N

30. **(a)** 5.3×10^5 m
(b) 5.7×10^3 s = 95 min

31. 1.02×10^3 m/s; 2.37×10^6 s = 27.4 days

32. **(a)** Yes.
(b) 5.69×10^{26} kg

33. **(a)** 4×10^{15} kg
(b) 4×10^{27} kg
(c) $m_{\text{Oort}} = 700 m_{\text{Earth}} = 2 m_{\text{Jupiter}}$

Unit 1 Review

Problems for Understanding

29. 1.4 m/s^2

30. **(a)** 2.00 **(b)** 2.00

31. 1.6×10^4 N. The acceleration remains constant.

32. **(a)** 3.1×10^3 N **(b)** 4.5 m

33. 1.1×10^4 N

34. **(a)** 7.00×10^3 N **(b)** 9.16 × true

35. **(a)** 1.5×10^4 N
(b) 3.8×10^3 N
(c) 2.5 m/s^2
(d) 22 m/s = 81 km/h
(e) 9.0 s

36. 17°

37. **(a)** 9.8×10^2 N **(b)** 13 km

38. 3.3 m/s^2; 23 N

39. **(a)** 1.4 s **(c)** 5.0×10^1 m
(b) 1.8 s

40. **(a)** 21.3 m/s **(c)** down
(b) 1.54 m

41. 2.40 m

42. 0.084 m

43. **(a)** 4.4×10^2 N; 1.0 × her weight
(b) 2.0×10^2 N; 0.45 × her weight
(c) same as (a)
(d) 6.8×10^2 N; 1.6 × her weight

44. 2.0×10^2 N

45. **(a)** 6.9×10^3 N **(b)** 64 km/h

46. **(a)** 612 N **(c)** 786 N
(b) 437 N **(d)** 612 N

47. 29 m/s^2

48. **(a)** 5.1×10^2 N **(b)** 5.6×10^2 N

49. **(a)** 1.7×10^2 N **(b)** 29 m/s

50. **(a)** 8.0 m/s^2
(b) 6.9 m/s^2
(c) 6.0×10^1 m/s[down]; 52 m/s[down]

51. **(a)** 0 m/s^2; 2.0×10^1 N
(b) 2.0 m/s^2; 16 N
(c) 0.50

52. 2.4 m/s^2; 0.61 m/s^2

53. Swift-Tuttle: **(a)** 51.69 AU; **(b)** 26.32 AU; **(c)** 135.5 a
Hale-Bopp: **(a)** 369.2 AU; **(b)** 185.1 AU; **(c)** 2511 a
Encke: **(a)** 4.096 AU; **(b)** 2.218 AU; **(c)** 3.303 a
Kopff: **(a)** 5.351 AU; **(b)** 3.467 AU; **(c)** 6.456 a
Hyakutake: **(a)** 1918 AU; **(b)** 959.1 AU; **(c)** 2.970×10^4 a
(d) student sketch
(e) Swift-Tuttle, Hale-Bopp, and Hyakutake

54. **(a)** 4.6×10^2 m/s
(b) 7.9×10^3 m/s

55. **(a)** 0.7445 AU **(c)** 1.732 a
(b) 1.442 AU

56. 2×10^{42} kg; 1×10^{12} m_{Sun}

Chapter 4

Practice Problems

1. **(a)** 11.5 kg m/s[E]
(b) 2.6×10^8 kg m/s[W]
(c) 8.39×10^7 kg m/s[S]
(d) 5.88×10^{-24} kg m/s[N]

2. 43.6 N s[down]

3. 2.58×10^5 N · s[S]

4. 4.52×10^6 N[S]

5. 2.6 kg m/s[horizontal]

6. –38 kg m/s

7. 8.8 kg m/s[up]

8. 2.7 m/s[in the original direction]

9. 0.11 m/s[in the direction that car A was travelling]

10. 2.10 m/s[S]

11. 0.11 m/s[E]

12. -2.43×10^2 m/s

13. 6.4 m/s[40.0° counterclockwise]

14. 1.16 m/s[6.1° clockwise from original direction]

15. v_A = 34.3 km/h[S]; v_B = 67.3 km/h[E]

16. v_2 = 6.32 m/s[41.5° counterclockwise from the original direction of the first ball]; the collision is not elastic: E_k = 12.1 J; E'_k = 10.2 J.

17. 1.24×10^5 kg km/h = 3.44×10^4 kg m/s[N39.5°W]; the collision was not elastic: $E_k = 3.60 \times 10^6$ kg km^2/h^2; $E'_k = 1.80 \times 10^6$ kg km^2/h^2

18. 261 m/s

19. The cart will stop at 0.018 m; therefore, it will not reach the end of the track.

20. 55.5 km/h = 15.4 m/s

21. 18.2 m/s

22. 3.62 m/s; 1.71 m

Chapter 4 Review

Problems for Understanding

28. 18 kg m/s[N]

29. 1.5×10^3 kg

30. 1.20 m/s[S]

31. 6.0×10^3 m/s[forward]

32. **(a)** 0.023 N · s[E]
(b) 0.036 N · s[S]

33. 3.8×10^3 N

34. 3.6×10^{-2} s

35. **(a)** –16 kg m/s
(b) 6.4×10^{-3} s

36. 2.5×10^4 N[E]

37. 2.9×10^4 N

38. 134 m/s[E]

39. 3.1 m/s[E]

40. –2.3 m/s

41. 1.3 m/s[forward]

42. 0.17 m/s[forward]

43. 4.4 m/s[35.2° clockwise]

44. 5.6×10^6 m/s[26.6° with respect to the +x direction]

45. **(b)** **(i)** $v'_1 = -v_1$; $v'_2 \approx 0$;
(ii) $v'_1 \approx 0$; $v'_2 = v_1$;
(iii) $v'_1 = v_1$; $v'_2 = 2v_1$
(c) (i) is the limiting case of a small object hitting a wall: it bounces back with the same speed and opposite direction. In (ii), all of the momentum is transferred to the other particle. In (iii), the massive object continues as if the light object had not been there, while the light object flies off with twice the speed of the massive object.

46. $v_1' = 0.86$ m/s[S]; $v_2' = 1.25$ m/s[N]. In a perfectly elastic head-on collision between identical masses, the two bodies simply exchange velocities.

47. (a) 0.29 m/s[W21°N]
(b) 70%

48. (a) 0.21 m/s (c) 95%
(b) 13 kg m/s

Chapter 5

Practice Problems

1. 1.810×10^4 J
2. 1.22×10^4 m
3. 31.5°
4. 61.6 m
5. 34.6 m/s
6. -2.6×10^2 N
7. 515 kg
8. 15.0 m
9. 4.9×10^{-2} J
10. 13 m/s
11. 7.7 m
12. 4.8 m
13. 0.25 J
14. 250 J
15. $v_A = 2.0$ m/s; $v_B = 2.8$ m/s
16. 5×10^2 N/m
17. (a) 0.414 m (b) −455 N
18. 0.0153 kg
19. 1.0 J
20. 0.30 m
21. 1.4 J
22. (a) 0.28 m (b) 1.3 m/s
(c) 17m/s^2
23. 1.4×10^3 N/m
24. 6.59×10^3 N/m
25. 0.42 m
26. (a) 405 N/m (b) 44.1 m/s^2
27. 11 m/s
28. 14 m/s
29. 7.4×10^2 J

Chapter 5 Review

Problems for Understanding

18. (a) 0.035 N (c) 0.025 J
(b) −0.025 J

19. (a) 16 J (b) 16 J

20. (a) 7.7×10^3 J
(b) 6.7×10^3 J
(c) 9.4 m/s; 8.7 m/s
(d) infinity (no friction); 1.3×10^2 m

21. 3.2×10^2 N·m

22. (a) 9.0 m/s
(b) $E_k = W = 2750$ J (2.8×10^3 J)
(c) 4.1 m

23. 57 N
24. 4.6 m/s
25. 4.5×10^2 N/m
26. (a) 0.38 J (b) 9.6 N
27. 0.19 m
28. $k = m_1 g/x$
29. 3.6 m/s
30. 4.1°C
31. 0.28°C
32. (a) 2.3 m/s (b) 5.3 N
33. 1.3 m/s
34. 0.77 m/s; 0.031 m
35. 5.0 m/s
36. 0.15 m
37. 0.45 m
38. 0.096 m
39. (a) -8.7×10^2 J (b) −1.8 m

Chapter 6

Practice Problems

1. 4.0×10^6 J; 1.16×10^3 m/s
2. 1.9×10^5 J; 5.0×10^3 m/s
3. 1.85×10^4 m/s
4. (a) 1.5×10^9 J (c) -1.5×10^9 J
(b) -3.0×10^9 J (d) 1.5×10^9 J
5. (a) 3.32×10^9 J (c) 7.51×10^6 m
(b) 7.29×10^3 m/s
6. (a) 4.12×10^9 J
(b) thermal energy, acoustic energy
7. 1.57×10^3 m/s; 649 m/s
8. (a) 4.87×10^7 J; 1.27×10^3 m/s
(b) -9.74×10^7 J (d) 4.87×10^7 J
(c) -4.87×10^7 J (e) 528 m/s
9. (a) 1.7×10^8 J; 1.8×10^3 m/s
(b) -3.4×10^8 J (d) 1.7×10^8 J
(c) -1.7×10^8 J (e) 7.7×10^2 m/s
10. 1.4×10^{31} kg, or 7.1 times the mass of the Sun
11. 6.00×10^6 N[forward]
12. 5.01×10^3 kg/s
13. (a) 0.33 m/s; 0.69 m/s; 1.1 m/s; 1.5 m/s; 1.9 m/s; 2.4 m/s
(b) 3.0 m/s, a difference of 0.6 m/s. Throwing all of the boxes at once contributes more to the momentum of the cart, because the cart is lighter without the boxes on it.

Chapter 6 Review

Problems for Understanding

14. 3.13×10^9 J
15. (a) 1.1×10^{11} J (b) 39%
16. -1.78×10^{32} J
17. $0.488 \times v_{Earth}$
18. (a) 6.18×10^5 m/s
(b) 4.22×10^4 m/s
(c) 6.71×10^3 m/s
19. (a) 7.0×10^7 m (b) 650 km
20. (a) 1.6×10^2 m/s. No.
(b) 1.2×10^2 m/s. No.
(c) 12 m/s. Yes.
(d) 2.9×10^8 m/s. No — in fact, the poor pitcher would be crushed by the strong gravity before he could even wind up for the throw!
21. 11.1 km/s; 99.4% of Earth's escape speed
22. 7.9×10^{11} m. This is just past Jupiter's orbit.
23. (a) -4.1×10^{10} J (c) 3.7×10^{10} J
(b) -3.1×10^9 J (d) 3.1×10^9 J
24. (a) $v_{200} = 7.78 \times 10^3$ m/s; $v_{100} = 7.84 \times 10^3$ m/s
(b) $E(r = 200 \text{ km}) = -1.52 \times 10^{10}$ J; $E(r = 100 \text{ km}) = -1.54 \times 10^{10}$ J
25. (a) 2.3×10^7 m/s; $0.077c$
(b) 0.14 s
26. 4.89×10^6 kg
27. (a) 3.4×10^6 N (b) 1.2×10^5 m/s

Unit 2 Review

Problems for Understanding

37. 3.5×10^4 kg m/s[N]
38. (a) 6.6 kg m/s
(b) 4.0×10^1 kg m/s
(c) 3.0×10^3 kg m/s
39. (a) 9.6 kg m/s[N]
(b) −17 kg m/s[N]
(c) 17 kg m/s[S]
(d) 2.6×10^2 N[N]
(e) 2.6×10^2 N[S]
40. (a) 45 N (b) 42 m/s
41. 36 m/s
42. (a) 1.3×10^4 kg m/s
(b) -1.3×10^4 kg m/s
(c) -1.3×10^4 kg m/s
(d) 19 m/s
43. 2.6×10^2 m/s[forward]
44. 1.5 m/s[N27°E]
45. (a) 0.76 m/s[E24°N]
(b) 17%

46. (a) 780 J
(b) It loses 780 J.

47. (a) 3×10^{11} J (b) 5 GW

48. (a) 66 m
(b) 74 m
(c) No change; the result is independent of mass.

49. -7.9×10^3 N

50. (a) 0.24 J (b) 48 J

51. (a) 0.32 m (b) 12 J

52. 15 kg

53. 60.0 m

54. (a) 1.46×10^4 J
(b) 1.46×10^4 J; 12.5 m/s
(c) Needed: coefficient of friction, μ, and slope of hill, θ: $E_k = mgh(1 - \mu/\tan\theta)$; $v = \sqrt{2gh(1 - \mu/\tan\theta)}$. For $\mu = 0.45$ and $\theta = 30.0°$, $E_k = 3.2 \times 10^3$ J, $v = 5.9$ m/s.

55. 3.1 m/s

56. (a) 0.47 m (b) 0.47 m

57. (a) 6.0 N (c) 0.023 J
(b) 0.15 J

58. 1.16×10^3 J. No, work is done by friction forces.

59. (a) 4.4 m/s (b) 3.5 m/s

60. (a) 11.2 km/s
(b) 7.91 km/s
(c) 6×10^{10} m or 10 000 Earth radii

61. 7.3×10^3 m/s

62. At Earth's distance from the Sun, the escape velocity is 42 km/s. Thus, the first comet is bound (it has negative total energy) and the second one is not bound (it has positive total energy).

63. 4.2×10^3 m/s; 1.0×10^4 m/s

64. 6.8 km/s; 15 km/s

65. (a) 1.3×10^{10} J
(b) -1.3×10^{10} J
(c) 6.1×10^3 J
(d) 1.3×10^{10} J
(e) 2.2×10^9 J; 3.1 km/s

66. (a) -7.64×10^{28} J
(b) -5.33×10^{33} J

67. 6.2×10^5 m/s

68. 2.6×10^2 m/s[forward]

Chapter 7

Practice Problems

1. 0.34 N

2. 0.80 m

3. 5.1×10^{-7} C

4. 0.50 N

5. 0.17 N (repulsive)

6. 0.12 m (directly above the first proton)

7. $F_A = 1.2 \times 10^{-2}$ N[W73°S]; $F_B = 1.6 \times 10^{-2}$ N[E63°N]; $F_C = 4.6 \times 10^{-3}$ N[W36°S]

8. 8.74 N[E18.2°N]

9. 2.0×10^{-8} C

10. 7.9×10^{-8} C

11. 1.5×10^5 N/C (to the right)

12. 0.019 N[W]

13. 2.5×10^4 N/C (to the left)

14. -4.0×10^{-4} C

15. 3.8 N/kg

16. 52 N

17. 3.46 kg

18. 2.60 N/kg

19. 2.60 m/s^2

20. -7.8×10^5 N/C (toward the sphere)

21. -1.2×10^{-5} C

22. 0.32 m

23. 5.80×10^9 electrons

24. -1.5×10^6 N/C (toward the sphere)

25. 0.080 m

26. 5.3×10^8 N/C[81.4° above the $+x$-axis]

27. 1.9×10^4 N/C[86.7° above the $+x$-axis]

28. 3.4×10^6 N/C[23.7° above the $-x$-axis]

29. 2.25×10^{14} N/C (toward the negative charge)

30. 2.9×10^7 N/C[73.6° above the $+x$-axis]

31. 5.7×10^{-2} N/kg

32. 3.81×10^7 m

33. 8.09 N/kg

34. 5.82×10^{23} kg

35. 5.0×10^{-11} N/kg

36. 8.09 N/kg

37. 1.03×10^{26} kg

38. -4.7×10^{-2} J

39. 0.18 J

40. 5.1×10^2 m

41. 1.55×10^{-4} C. The signs of the two charges must be the same, either both positive or both negative.

42. 4.8×10^6 N/C

43. 1.5×10^{10} m

44. 2.9×10^{-5} J

45. -4.7×10^{-12} C

46. If the positive charge is placed at 0.0 cm and the negative charge is placed at 10.0 cm, there are two locations where the electric potential will be zero: 6.2 cm and 27 cm.

47. 1.1×10^6 V

48. 8.0 V

49. -2.1×10^6 V

50. 1.6×10^6 V

51. 1.4×10^{-6} C

52. 2.0 V

53. 12 J

54. -2.4×10^4 V

55. (a) 1.9×10^5 V
(b) 1.2×10^{-3} J
(c) A. It takes positive work to move a positive test charge to a higher potential. Since in this case, you invest positive work to move your positive test charge from B to A, A must be at a higher potential.

56. 5.3 cm and 16 cm to the right of the positive charge.

57. any point lying on a line midway between the two charges and perpendicular to the line that connects them

58. The potential is zero 3.4 cm above the origin and 24 cm below the origin.

59. If the distances of the first and second charges, q_1 and q_2, from the point of zero potential are d_1 and d_2, then d_2 must satisfy $d_2 = (-q_2/q_1)d_1$, with $q_2 < 0$. For example, if $q_2 = -8.0\ \mu$C, then $d_2 = 16$ cm and the charge would be located either 24 cm to the right of q_1 or 8.0 cm to the left of q_1. Other solutions can be similarly determined.

60. 4.0 cm to the right of the $-4.0\ \mu$C charge

Chapter 7 Review

Problems for Understanding

18. 9×10^3 N

19. 2.3×10^8 N

20. 5.6 cm

21. $F_A = 4.5 \times 10^{-2}$ N to the left; $F_B = 0.29$ N to the right; $F_C = 0.24$ N to the left

22. $F_A = 3.8$ N[N3.0°E]; $F_B = 4.4$ N[E23°S]; $F_C = 4.7$ N[W26°S]

23. $F_Q = 8.2 \times 10^{-8}$ N; $F_g = 3.6 \times 10^{-47}$ N

24. The charges on Earth (Q_E) and the Moon (Q_{Moon}) must satisfy

$|Q_E| \times |Q_M| = 3.3 \times 10^{27}$ C^2, and they must have opposite signs.

25. 4.2×10^{42}

26. –57 C

27. 5.2×10^{-3} N

28. (a) 8.65×10^{25} kg
(b) 8.81 N/kg
(c) 881 N

29. $2/9\ g_{Earth}$ = 2.18 N/kg

30. (a) 8.24×10^{-8} N
(b) 2.19×10^{6} m/s
(c) 5.14×10^{11} N/C
(d) 27.2 V

31. 1.86×10^{-9} kg = $2.04 \times 10^{21} \times m_{actual}$

32. 9×10^{-5} N[W]

33. 0.51 m

34. 6.0×10^{4} N/C[E37°N]

35. (a) -7.5×10^{-8} J
(b) It loses energy.

36. -2.9×10^{-6} J

37. 2.8×10^{2} C

38. (a) 4.5×10^{3} V
(b) Yes; the spheres have to be at equal potential, because the same point cannot have two different potentials.
(c) big sphere: 52 nC; small sphere: 23 nC

39. (a) $E = 0$; $V = 2.2 \times 10^{5}$ V
(b) $E = 4.3 \times 10^{5}$ N/C; $V = 0$
(c) When the two charges have the same sign, the electric fields at the midpoint have the same magnitude but opposite directions, so they cancel. The potential is the algebraic sum of the potentials due to the individual charges; it is a scalar and, in this case, adds to be greater than zero. When the signs are different, the electric fields point in the same direction and the magnitudes add. However, the potentials have opposite signs and cancel.

40. (a) 2.3 J **(c)** X
(b) 1.2×10^{6} V

41. (a) 4.0×10^{5} V **(b)** R

Chapter 8

Practice Problems

1. (a) 3.0×10^{2} N/C[W]
(b) 3.0×10^{2} N/C[W]
(c) 3.0×10^{2} N/C[W]
(d) double the charge on each plate; halve the area of each plate

2. (a) 5.0×10^{3} N/C[E]
(b) The area of the plates was decreased by a factor of 4.

3. 8.0×10^{2} N[N]

4. 1.4×10^{3} N/C

5. (a) 3.0×10^{3} N/C
(b) 60.0 V

6. (a) 0.222 m
(b) 1.44×10^{-3} N[toward + plate]
(c) 1.44×10^{-3} N[toward + plate]

7. 26 V

8. (a) 4.0×10^{1} V **(b)** 2.0×10^{3} N/C

9. (a) 9.62×10^{-19} C **(b)** 6.00

10. (a) 3.7×10^{-15} kg **(b)** 3.6×10^{3} V

11. 1.13×10^{4} V

12. 1.4×10^{-13} N[toward the bottom of the page]

13. 2.7×10^{-14} N[left 28° up]

14. 30°

15. 9.8×10^{-3} T[N]

16. 4.2×10^{-3} T[up out of page]

17. 3.0×10^{2} N

18. 9.2×10^{-2} T[into the page]

19. 1.8 m

20. (a) 6.4×10^{2} A
(b) If such a large current could be passed through the wire, Earth's magnetic field could be used to levitate the wire. However, this is such a large current that it is probably not practical to do so.

21. (a) 2.884×10^{-17} J
(b) 1.86×10^{5} m/s

22. (a) up (that is, opposite to gravity)
(b) 59 V

23. (a) 1.4×10^{7} m/s
(b) 2.0×10^{6} V

24. 2.8×10^{-2} m

25. 6.8×10^{3} m/s

26. (a) 5.01×10^{-27} kg
(b) tritium

Chapter 8 Review

Problems for Understanding

22. 20 V

23. (a) 1.60×10^{2} V
(b) $V_A = 0.0$ V; $V_C = 8.0 \times 10^{1}$ V; $V_D = 1.2 \times 10^{2}$ V
(c) $V_B - V_A = 4.0 \times 10^{1}$ V; $V_C - V_B = 4.0 \times 10^{1}$ V; $V_D - V_A = 1.2 \times 10^{2}$ V
(d) 2.0×10^{3} N/C
(e) 2.0×10^{-3} N in both cases [toward negative plate]
(f) 4.0×10^{-3} N[toward negative plate]

24. (a) -8.0×10^{-19} C
(b) five electrons

25. 3.1×10^{10} electrons

26. 0.63 N

27. 3.2×10^{5} m/s

28. (a) 1.8×10^{-3} N[up]
(b) 0.18 g

29. 1.0×10^{-26} kg

30. (a) r(slow) = 1.4×10^{-4} m; r(fast) = 2.8×10^{-4} m
(b) T(slow) = T(fast) = 8.9×10^{-11} s
(c) f(slow) = f(fast) = 1.1×10^{10} Hz
(d) The period and frequency is independent of the particle's velocity and the radius of its orbit. The faster electron completes an orbit of larger radius in the same time in which the slower electron completes an orbit of smaller radius.

31. (a) $T_e/T_p = m_e/m_p = 5.4 \times 10^{-4}$
(b) $r_e/r_p = \sqrt{\frac{m_e}{m_p}} = 0.023$

32. (a) 3.0×10^{2} Hz; 6.3×10^{3} m
(b) 3.0×10^{2} Hz; 3.1×10^{3} m

33. (a) 1.1×10^{-17} T
(b) [E]

Unit 3 Review

Problems for Understanding

49. 1.8×10^{8} C

50. 8.23×10^{-8} N

51. 2.3×10^{-9} N

52. ±14 μC

53. 1.5×10^{4} electrons

54. (a) –50.0 N **(c)** 3.33 N
(b) 5.56 N **(d)** –3.70 N

55. ±0.14 μC

56. 1.8×10^{13} C

57. -1.0×10^{4} C

58. 0.12 m

59. 8×10^{27} N

60. 9.2×10^{-26} N

61. 1.1×10^{-5} C

62. (a) 9 N
(b) An additional nuclear force — the strong force — holds the nucleus together.

63. 2000 N/C

64. 6.2×10^{12} electrons

65. (a) 0 J
(b) 8.6×10^{-10} J
(c) equipotential surfaces

66. (a) 4.8×10^{-19} C
(b) three electrons (deficit)
(c) 1.2×10^4 V

67. 0.10 T

68. (a) 3.7 nC
(b) It will be reduced by one half.

69. (a) 2.2×10^{-13} N
(b) 1.3×10^{14} m/s^2[up]

70. 0.9 m

72. (a) 1.8×10^{-10} s (b) 2.8×10^{-4} m

73. 330 N/C

74. (a) $v_{\parallel} = 5.6 \times 10^6$ m/s; $v_{\perp} = 3.2 \times 10^6$ m/s
(b) 0.13 m
(c) 2.2×10^{-7} s
(d) 1.4 m
(e) From the side, a helical path will be seen; two places on consecutive orbits of the proton will be separated by 1.4 m. Face on, the orbit will appear to be circular, with a radius of 0.13 m.

75. 2.5 cm

Chapter 9

Practice Problems

1. 0.011 m
2. 1.3×10^{-5} m
3. red light will have a wider maximum; 7.3×10^{-3} m
4. (a) 0.36 m
(b) The value ignores changes in the speed of light due to the lenses of the imaging system and changes in the density of the atmosphere.
5. 0.00192°
6. Angular resolution improves with shorter wavelengths, so you should use blue lettering.

Chapter 9 Review

Problems for Understanding

31. (a) 0.020 m (b) 0.20 m/s
32. 4.8×10^2 nm
33. 589 nm
34. 2.1×10^{-5} m
35. 3.0 km
36. 72 m
37. 485 nm; 658 nm
38. (a) 15° (b) decrease
39. sin θ will be greater than 1 for $\lambda > 629$ nm.
40. $\lambda_1/\lambda_2 = 3/2$
41. (a) 4th (b) 60°
42. 681 nm
43. 667 nm; red light

Chapter 10

Practice Problems

1. (a) 0.0667 s
(b) The distance that the signal passes is usually larger than the geographic separation between the two points (due to satellite networks); also, the speed of light depends on the medium.
2. 2.25×10^8 m/s
3. (a) 46.8 m
(b) The antenna must be larger than the wavelength of the radiation.
4. (a) $\lambda_{micro} = 0.03$ m; $\lambda_{light} = 3 \times 10^{-6}$ m
(b) The metallic screen is used to stop the microwave radiation by working as an antenna for microwave wavelengths.

Chapter 10 Review

Problems for Understanding

35. 2.48×10^{-13} m
36. 1×10^6 Hz or 1 MHz
37. 2.938 m; 102.1 MHz
38. (a) 0.80 J/s (c) 1/4
(b) 8.0×10^{-5} J/s
39. 8.3 light-minutes
40. 1.1×10^4 m
41. 0.24 s
42. (a) Yes, but with very low frequency.
(b) Greater than 4×10^{14} Hz

Unit 4 Review

Problems for Understanding

50. 0.12 m; 2.5×10^9 Hz; 4.0×10^{-10} s
51. 2.1×10^5 Hz; 1.4×10^3 m
52. 3×10^{-13} m
53. 5.4×10^{16}
54. 4.3×10^{-7} m = violet
55. 1.5×10^2 m
56. 3.4×10^{-2} m
57. 9.4610×10^{15} m
58. Listeners in Vancouver will hear a particular sound after 1.7×10^{-2} s, while listeners in the back of the concert hall will hear the same sound after 0.24 s, so listeners in Vancouver will hear it first.
59. 585 nm
60. (a) 7.1 mm (b) closer
61. 1.2×10^4 lines/cm
62. 8×10^{-7} m
63. (a) 0.42″ = 2.0×10^{-6} rad
(b) 0.0042″ = 2.0×10^{-8} rad
(c) 4.6×10^{12} m apart, or, it could distinguish objects that are 0.77 × Pluto's distance from the Sun apart
(d) better (resolution is proportional to wavelength)
64. 45 m
65. (a) 5.8×10^{-19} J (c) 9.0×10^{-20} J
(b) 1.3×10^{-17} J (d) 4.0×10^{-26} J
66. 2.26×10^8 m/s
67. (a) 5.0×10^{-9} m (b) X ray

Chapter 11

Practice Problems

1. (a) 4.8×10^{-13} s (b) 1.5×10^{-13} s
2. 257 s
3. $0.94c = 2.8 \times 10^8$ m/s
4. 702 km
5. 0.31 m
6. (a) 1.74×10^8 m/s
(b) The sphere's diameter appears contracted only in the direction parallel to the spacecraft's motion. Therefore, the sphere appears to be distorted.
7. 465 μg
8. 1.68×10^{-27} kg
9. $0.9987c = 2.994 \times 10^8$ m/s
10. 4.68×10^{-11} J
11. 1.01×10^{-10} J
12. 2.6×10^8 m/s
13. 7.91×10^{-11} J
14. 1.64×10^{-13} J
15. 1.3×10^9 J
16. 4.3×10^9 kg/s

Chapter 11 Review

Problems for Understanding

18. $0.87c$
19. (a) 3.2 m (c) 6.8×10^{-8} s
(b) 1.9 m
20. (a) 2.5×10^{-27} kg (b) 1.7×10^{-27} kg
21. plot
22. 3.0×10^2 m/s
23. (a) c (c) c
(b) c
24. (a) 3.2 (c) 16 m
(b) 5.8×10^{-8} s

25. 1.2×10^{-30} kg, which is 1.3 times its rest mass

26. **(a)** 4.1×10^{-20} J **(b)** 4.1×10^{-16} J **(c)** 1.3×10^{-14} J **(d)** 5.0×10^{-13} J **(e)** (a) and (b)

27. $0.14c = 4.2 \times 10^7$ m/s

28. 3×10^4 light bulbs

29. 4.8×10^{-30} kg; $m/m_o = 5.3$; $0.98c = 2.9 \times 10^8$ m/s

30. **(a)** 1.4 g **(b)** 29% or 0.40 g

Chapter 12

Practice Problems

1. 4.28×10^{-34} kg m/s
2. 9.44×10^{-22} kg m/s
3. 4.59×10^{-15} m
4. 3.66×10^{25} photons
5. 1.11×10^{10} Hz; radio
6. 1.05×10^{-13} m
7. 7.80×10^{-15} m
8. 1.04×10^{-32} m
9. 2.39×10^{-41} m
10. 5.77×10^{-12} m
11. 2.19×10^6 m/s

Chapter 12 Review

Problems for Understanding

32. **(a)** 1.24×10^{15} Hz **(b)** threshold frequency

33. **(a)** 2.900 eV **(b)** lithium

34. 1.5×10^{15} Hz

35. 2.2 eV

36. 5.8×10^{18} photons/s

37. **(a)** 1.2×10^{-27} kg m/s **(b)** 1.3×10^{-27} kg m/s **(c)** 9.92×10^{-26} kg m/s

38. 1.7×10^{17} Hz

39. 5.5×10^{-33} kg m/s

40. 7.0×10^{-27} kg

41. **(a)** 3.1×10^{-7} m **(b)** 6.14×10^{-10} m **(c)** 4.7×10^{-24} kg m/s

42. **(a)** 4.8×10^{-10} m **(b)** −1.5 eV

43. 486 nm

44. **(a)** 3.08×10^{15} Hz **(b)** 97.3 nm **(c)** -0.850 eV $= -1.36 \times 10^{-19}$ J **(d)** 0.847 nm **(e)** 487 nm

Chapter 13

Practice Problems

1. 0.060 660 00 u $= 1.0073 \times 10^{-28}$ kg
2. 1.237×10^{-12} J
3. 2.858×10^{-10} J
4. 2.6×10^9 a
5. 3.5×10^3 a
6. 8.49×10^{-8} mg
7. 0.141 68 u $= 2.3527 \times 10^{-28}$ kg; 2.114×10^{-11} J
8. 2.818×10^{-12} J
9. **(a)** 0.0265 u $= 4.40 \times 10^{-29}$ kg; 3.96×10^{-12} J **(b)** 5.96×10^{11} J

Chapter 13 Review

Problems for Understanding

20. **(a)** $20p$, $20n$, $18e$ **(b)** $26p$, $30n$, $26e$ **(c)** $16p$, $18n$, $17e$

21. **(a)** 1.477×10^{-11} J **(b)** 1.793×10^{-10} J

22. ${}^{230}_{90}\text{Th} \rightarrow {}^{4}_{2}\text{He} + {}^{226}_{88}\text{Ra}$

23. **(a)** 1/4 **(b)** 1/16 **(c)** 1/4096

24. **(a)** 4.876 MeV **(b)** $v_{He} = 1.520 \times 10^7$ m/s; $v_{Rn} = 2.740 \times 10^5$ m/s **(c)** 98%

25. 1.2×10^{-7} kg

26. 11.5 min

27. 1.2×10^3 a

28. **(a)** 200 **(b)** 600 **(c)** 25 **(d)** 775 **(e)** ${}^{D}N(t) = {}^{P}N(0)\left(1 - \left(\frac{1}{2}\right)^{t/T_{1/2}}\right)$, where ${}^{D}N(t)$ is the number of daughter nuclei at any time, t, ${}^{P}N(0)$ is the number of parent nuclei at time, $t = 0$, and $T_{\frac{1}{2}}$ is the half-life of the parent nucleus.

29. **(a)** $R = 1/[2^{t/T_{1/2}} - 1]$ where R is the ratio of parent to daughter nuclei at any time. **(b)** 4.25×10^9 a; 3.89×10^9 a; 2.93×10^9 a **(c)** Assuming that the samples were not polluted by having daughter nuclei present in the beginning, considering the large differences in ages, it is unlikely that these samples came from the same place. **(d)** More than one half-life has elapsed.

30. 2.4×10^{-12} m (assuming the electron and positron are at rest initially)

31. **(a)** 3.56×10^{-13} J = 2.23 MeV **(b)** $0.981c = 2.94 \times 10^8$ m/s **(c)** The electron travels faster than the speed of light in water, which is 2.25×10^8 m/s, and consequently emits Cherenkov radiation, a phenomenon analogous to a sonic boom.

32. **(a)** $udd \rightarrow uud + e^- + \nu_e$ or $d \rightarrow u + e^- + \nu_e$ **(b)** $\gamma + udd \rightarrow \bar{u}d + uud$

Unit 5 Review

Problems for Understanding

42. **(a)** $0.14c$ **(b)** $0.045c$

43. **(a)** 9×10^{16} J **(b)** 3×10^7 a

44. **(a)** 3.1 light-year **(b)** 4.7 a **(c)** 6.3 a

45. **(a)** 1.1×10^{-13} J **(b)** 1.3 × rest mass energy **(c)** 2.1×10^{-30} kg or 2.3 × rest mass

46. **(a)** 3×10^9 J **(b)** 4×10^{-8} kg

47. 1.12eV $= 1.80 \times 10^{-19}$ J

48. 4.7eV $= 7.5 \times 10^{-19}$ J

49. **(a)** 1.05×10^{15} Hz **(b)** 287 nm

50. **(a)** 1.25 nm **(b)** 0.153 nm

51. **(a)** 2.47×10^{15} Hz **(b)** 1.22×10^{-7} m **(c)** Lyman

52. 486 nm

53. **(a)** 3.0×10^{-19} J **(b)** 8.1×10^{17} photons

54. **(a)** 6.91×10^{14} Hz **(b)** 4.34×10^{-7} m **(c)** -0.544 eV $= -8.70 \times 10^{-20}$ J **(d)** 1.32 nm **(e)** 9.49×10^{-8} m **(f)** UV

55. **(a)** 3.96×10^{-12} J/reaction **(b)** 9.68×10^{37} reactions/s **(c)** 6.64×10^{-27} kg/reaction **(d)** 6.43×10^{11} kg/s **(e)** 9.82×10^9 a

56. **(a)** 4.40×10^{-29} kg **(b)** 0.658% **(c)** 1.18×10^{45} J **(d)** 9.59×10^9 a

57. 88.2 N

58. 1.9 GeV, 6.6×10^{-16} m

Glossary

A

action at a distance the force between two objects not in contact (7.2)

air resistance friction due to the motion of an object through air; proportional to the object's velocity (1.3)

alpha particle one or more helium nuclei ejected from a radioactive nucleus (13.2)

antimatter matter composed of antiparticles, which have the same mass but opposite charge, and/or other properties, compared to particles (13.2)

antineutrino a chargeless, very low-mass particle involved in weak interactions (13.2)

apparent weight the weight measured by a scale; same as true weight, unless the object is accelerating (1.3)

atomic mass number the number (A) that represents the total number of protons and neutrons in an atomic nucleus (13.1)

atomic mass unit the value of mass equal to mass of the most common carbon isotope ($^{12}_{6}C$) divided by 12; $1\ u = 1.6605 \times 10^{-27}$ kg (13.1)

atomic number the number (Z) that represents the number of protons in the nucleus; also represents the charge of the nucleus in units of e (13.1)

B

Balmer series spectral lines of hydrogen that lie in the visible wavelength range (12.3)

baryon a subset of the hadron family, such as the proton and neutron, that are composed of combinations of three quarks (13.3)

beta particle high-speed electrons or positrons ejected from a radioactive nucleus (13.2)

betatron a cyclotron modified to accelerate electrons through magnetic induction, instead of using electric fields (8.3)

binding energy 1. the amount of additional energy an object needs to escape from a planet or star (6.1) 2. the amount of energy that must be supplied to nuclear particles in order to separate them (13.1)

blackbody an object that absorbs and emits all radiation of all possible frequencies (12.1)

Bohr radius the distance from the nucleus of the lowest allowed energy level in the hydrogen atom: $r = 0.0529177$ nm (12.3)

C

centripetal acceleration the centre-directed acceleration of a body moving continuously along a circular path; the quotient of the square of the object's velocity and the radius of the circle (2.2)

centripetal force the centre-directed force required for an object to move in a circular path (2.2)

charge density the charge per unit area (8.1)

chemical symbol a shorthand symbol for an element (13.1)

circular orbit an orbit produced by a centripetal force (6.2)

classical physics the long-established parts of physics, including Newtonian mechanics electricity and magnetism, and thermodynamics, studied before the twentieth century (12.1)

closed system a system that can exchange energy with its surroundings, but not with matter (4.2)

coefficient of kinetic friction for two specific materials in contact, the ratio of the frictional force to the normal force between the surfaces when they are in relative motion (1.2)

coefficient of static friction for two specific materials in contact, the ratio of the frictional force to the normal force between the surfaces when they are not moving relative to each other (1.2)

coherent light that is in phase (the maxima and minima occur at the same time and place) (9.2)

combustion chamber the part of an engine where gases are burned (e.g., a jet engine) (6.3)

Compton effect a phenomenon involving the scattering of an X-ray photon with a "free" electron, in which, through conservation of energy and momentum, some of the photon's energy is transferred to the electron (12.2)

conservation of mechanical energy the change in the total mechanical energy (kinetic plus potential) of an isolated system is zero (5.1)

conservation of momentum the total momentum of two objects before a collision is the same as the total momentum of the same two objects after they collide (4.2)

conservative force a force that does work on an object in such a way that the amount of work done is independent of the path taken (5.3)

constructive interference a situation in which a combined or resultant wave has a larger amplitude than either of its component waves (9.1)

coordinate system consists of perpendicular axes that define an origin or zero position and dimensions (1.1)

Coulomb's constant the proportionality constant in Coulomb's law: $k = 9 \times 10^9\ \text{N} \bullet \text{m}^2/\text{C}^2$ (7.1)

Coulomb's law the force between charges at rest, proportional to the magnitudes of the charges and inversely proportional to the square of the distance between their centres (7.1)

counterweight a heavy, movable mass that balances another mass (1.3)

cyclotron a particle accelerator that subjects particles in a circular path to a large number of small increases in potential in order to accelerate them (8.3)

D

daughter nucleus the nucleus remaining after a transmutation reaction (13.2)

de Broglie wavelength the wavelength associated with a particle; the quotient of Planck's constant and the momentum of the particle (12.2)

destructive interference a situation in which a combined or resultant wave has a smaller amplitude than at least one of its component waves (9.1)

deuterium an isotope of hydrogen, consisting of a proton and neutron in the nucleus (13.1)

diffraction the bending of waves around a barrier (9.1)

diffraction grating a device for producing spectra by diffraction and for the measurement of wavelength (9.3)

dilated time the time measured by an observer who sees a clock that is in a frame of reference that is moving relative to the observer (11.2)

dispersion the separation of light into its range of colours (9.1)

doubly refractive having a different refractive index, depending on the polarization of the light (10.1)

dynamics the study of the motions of bodies while considering their masses and the responsible forces; simply, the study of *why* objects move the way they do (1.2)

E

elastic collision a collision in which both momentum and kinetic energy are conserved (4.3)

elastic potential energy a form of energy that accumulates when an elastic object is bent, stretched, or compressed (5.2)

electric field intensity the quotient of the electric force on a unit charge located at that point (7.2)

electric field a region in space that influences electric charges in that region (7.2)

electric field lines imaginary directed lines that indicate the direction a tiny point charge with zero mass would follow if free to move in the electric field; these lines radiate away from positive charges and toward negative charges (7.2)

electric permittivity a number that characterizes a material's ability to resist the formation of an electric field in it (10.1)

electric potential difference the work done per unit charge between two locations (7.3)

electromagnetic force an infinite range force that operates between all charged particles (13.3)

electromagnetic spectrum the range of frequencies of electromagnetic waves (10.3)

electromagnetic wave a wave consisting of changing electric and magnetic fields (10.1)

electron an elementary particle with negative charge and a mass of 9.11×10^{-31} kg (13.1)

electron volt the energy gained by one electron as it falls through a potential difference of one volt: $1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$ (12.1)

electrostatic force the force between charges at rest; see also: Coulomb's law (7.1)

electroweak force the fundamental force from which the electromagnetic and weak nuclear forces are derived (13.3)

elementary charge the basic unit of charge: $e = \pm 1.602 \times 10^{-19}\ \text{C}$ (13.1)

elementary particle a stable particle that cannot be subdivided into smaller particles (13.3)

empirical equation an equation based on observed data and not on any theory (12.1)

energy the ability to do work (5.1)

equipotential surface a surface in which all points have the same electric or gravitational potential (7.3)

escape energy the amount of energy required for an object to escape from the gravitational force of a planet or star and not return (6.1)

escape speed the minimum speed at the surface of a planet that will allow an object to leave the planet (6.1)

exhaust velocity the backward velocity of the gas ejected from the combustion chamber of a rocket relative to the combustion chamber (6.3)

external force any force exerted by an object that is not part of the system on an object within the system (4.2)

F

Faraday cage a metal screen that is used to shield a region from an external electric field (8.2)

fictitious force a force that must be invoked to explain motion in a non-inertial frame of reference (1.1)

field a region in space that influences a mass, charge, or magnet placed in the region (7.2)

frame of reference a subset of the physical world defined by an observer in which positions or motions can be discussed or compared (1.1)

Fraunhofer diffraction diffraction produced by plane wavefronts of a parallel beam of light (9.2)

free-body diagram a diagram in which all of the forces acting on an object are shown as acting on a point representing the object (1.2)

free fall a situation in which gravity is the only force acting on an object (1.3)

Fresnel diffraction diffraction produced by curved wavefronts, such as that produced by a point source of light (9.2)

frictional forces forces that oppose motion between two surfaces in contact (1.2)

fringe a bright or dark band produced by interference of light (9.2)

G

gamma (γ) an abbreviation of an expression that is used in equations for length contraction and time dilation: $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$ (11.2)

gamma ray high-frequency radiation emitted from a radioactive nucleus (13.2)

geostationary orbit the orbit of a satellite around Earth's equator, which gives the satellite the appearance of hovering over the same spot on Earth's surface at all times (3.2)

gluons exchange particles responsible for holding quarks together (13.3)

gradient a change in a quantity relative to a change in position, or displacement (8.1)

gravitational assist or gravitational slingshot interaction typically between a spacecraft and a planet in which the planet loses a small amount of energy and the spacecraft gains a large amount of energy (6.3)

gravitational field intensity the quotient of the gravitational force and the magnitude of the test mass at a given point in a field; the product of the universal gravitation constant and mass, divided by the square of the distance of a given location from the centre of the object (7.2)

gravitational field lines imaginary directed lines that indicate the direction a tiny test mass would follow if free to move in the gravitational field; these lines radiate inward toward the mass that generates them (7.2)

gravitational force infinite range force that operates between all massive particles (13.3)

gravitational mass the property of matter that determines the strength of the gravitational force; compare to: inertial mass (1.1)

gravitational potential the gravitational potential energy per unit mass (7.3)

graviton exchange particle postulated to be responsible for the gravitational force (13.3)

ground state the lowest possible state that an electron can occupy in an atom (13.3)

H

hadron particles that contain quarks (13.3)

half-life the time in which the amount of a radioactive nuclide decays to half its original amount (13.2)

heat the transfer of thermal energy from one system to another due to their different temperatures (5.3)

heavy water water composed of molecules of oxygen and deuterium instead of oxygen and hydrogen (13.2)

Hooke's law states that the applied force is directly proportional to the amount of extension or compression of a spring (5.2)

Huygens' principle each point on a wavefront can be considered to be a source of a secondary wave, called a "wavelet," that spreads out in front of the wave at the same speed as the wave itself (9.1)

I

impulse the product of the force exerted on an object and the time interval over which the force acts (4.1)

impulse-momentum theorem states that the impulse is equal to the change in momentum of an object involved in an interaction (4.1)

inelastic collision a collision in which momentum is conserved, but kinetic energy is not conserved (4.3)

inertia the natural tendency of an object to stay at rest or in uniform motion in the absence of outside forces; proportional to an object's mass (1.1)

inertial frame of reference a frame of reference in which the law of inertia is valid; it is a non-accelerating frame of reference (1.1)

inertial mass the property of matter that resists a change in motion; compare to: gravitational mass (1.1)

interferometer an instrument for measuring wavelengths of light by allowing light beams to interfere with each other (11.1)

internal force any force exerted on an object in the system due to another object in the system (4.2)

inverse square law the relationship in which the force between two objects is inversely proportional to the square of the distance that separates the centres of the objects; for example, the gravitational and electrostatic forces (7.1)

ion an electrically charged atom or molecule (13.1)

ionizing radiation radiation of sufficient energy to liberate the electrons from the atoms or molecules (13.2)

isolated system a system that does not exchange either matter or energy with its surroundings (4.2)

isotope two or more atoms of an element that have the same number of protons but a different number of neutrons in their nuclei (13.1)

K

Kepler's laws three empirical relationships that describe the motion of planets (3.1)

kinematics the study of the motions of bodies without reference to mass or force; the study of *how* objects move in terms of displacement, velocity, and acceleration (1.2)

L

law of universal gravitation the force of gravity between any two objects is proportional to the product of their masses and inversely proportional to the square of the distance between their centres (3.1)

length contraction a consequence of special relativity, in which an object at rest in one frame of reference will appear to be shorter in the direction parallel to its motion in another frame of reference (11.2)

lepton particles, such as electrons and neutrinos, that do not contain quarks and do not take part in strong nuclear force interactions (13.3)

line spectrum (emission spectrum) a spectrum consisting of bright lines at specific wavelengths, produced by atoms of heated elements (9.3)

linear accelerator a particle accelerator that uses alternating electric fields to accelerate particles in stages (8.3)

Lorentz-Fitzgerald contraction contraction of an object in the direction of its motion (11.1)

M

magnetic field intensity the magnetic force acting on a unit length of a current-carrying wire placed at right angles to the magnetic field, measured in tesla (T) (7.2)

magnetic field lines imaginary directed lines that indicate the direction in which the N-pole of a compass would point when placed at that location; these lines radiate out of the magnet's N-pole and into its S-pole and form closed loops in the magnet (7.2)

magnetic permeability a number that characterizes a material's ability to become magnetized (10.1)

magnetic quantum number determines the orientation of the electron orbitals when the atom is placed in an external magnetic field (13.3)

magnetic resonance imaging a medical imaging technique for obtaining pictures of internal parts of the body in a non-invasive manner (10.2)

mass defect the difference between the mass of a nucleus and the sum of the masses of its constituent particles; the mass equivalent of the *binding energy* of a nucleus (13.1)

mass spectrometer an instrument that can separate streams of particles by mass and measure that mass by application of electric and magnetic deflecting fields (8.3)

mass-to-charge ratio the quotient of a particle's mass to its charge, which is easier to measure than either quantity individually (13.3)

Maxwell's equations a series of four related equations that summarize the behaviour of electric and magnetic fields and their interactions (10.1)

meson a particle composed of a quark and an antiquark (13.3)

microgravity the condition of apparent weightlessness (3.2)

modulation a process of adding data to an electromagnetic wave by changing the amplitude or frequency (10.2)

momentum the product of an object's mass and velocity (4.1)

N

neutrino a chargeless, very low-mass particle involved in weak interactions (13.2)

neutron a particle with zero charge, found in the nucleus of all atoms except the hydrogen atom (13.1)

nodal point a stationary point in a medium produced by destructive interference of two waves travelling in opposite directions (9.1)

non-conservative force a force that does work on an object in such a way that the amount of work done is dependent on the path taken (5.3)

non-elastic or plastic the description of a material that does not return precisely to its original form after the applied force is removed (5.2)

non-inertial frame of reference an accelerating frame of reference (1.1)

nuclear fission the splitting of a large nucleus into two or more lighter nuclei; usually caused by the impact of a neutron and accompanied by the release of energy (13.2)

nuclear fusion the formation of a larger nucleus from two or more lighter nuclei, accompanied by the release of energy (13.2)

nuclear model a model for the atom in which all of the positive charge and most of the mass are concentrated in the centre of the atom, while negatively charged electrons circulate well beyond this "nucleus" (12.3)

nucleon the collective term for a particle (proton and/or neutron) in the atomic nucleus (13.1)

nucleon number the total number of nucleons (protons and neutrons) in the nucleus; also called the "atomic mass number" (13.1)

nuclide the nucleus of a particular atom, as characterized by its atomic number and atomic mass number (13.1)

O

open system a system that can exchange both matter and energy with its surroundings (4.2)

orbital quantum number specifies the shape of an electron's orbital or energy level; has integer values of one less than the principal quantum number (12.3)

P

parabola a geometric figure formed by slicing a cone with a plane that is parallel to the axis of the cone (2.1)

parent nucleus the initial nucleus involved in a transmutation reaction (13.2)

particle accelerator an instrument capable of emitting beams of high-speed, subatomic-sized particles, such as protons and electrons (8.3)

Pauli exclusion principle states that no two electrons in the same atom can occupy the same state; alternatively, no two electrons in the same atom can have the same four quantum numbers (13.3)

periodic motion the motion of an object in a repeated pattern over regular time intervals (5.2)

perturbation deviation of a body in orbit from its regular path, caused by the presence of one or more other bodies (3.2)

photoelastic materials that exhibit doubly refractive properties while under mechanical stress (10.1)

photoelectric effect the emission of electrons from matter by radiation of certain frequencies (12.1)

photon a quantum of light or electromagnetic radiation (12.1)

pion a type of meson (13.3)

plane polarized light or an electromagnetic wave in which the vibrations of the electric field lie in one plane and are perpendicular to the direction of travel (10.1)

polarization the orientation of the oscillations in a transverse wave (10.1)

positron a particle with the same mass as the electron, but with a positive charge; an antielectron (13.2)

potential gradient the quotient of the electric potential difference between two points and the component of the displacement between the points that is parallel to the field (8.1)

principal quantum number describes the orbital or energy level of an electron in an atom (12.3)

projectile an object that is given an initial thrust and allowed to move through space under the force of gravity only (2.1)

proper length the length of an object measured by an observer at rest relative to the object (11.2)

proper time the duration of an event measured by an observer at rest relative to the event (11.2)

proton a positively charged particle found in the nucleus of all atoms (13.1)

Q

quantized a property of a system that occurs only in multiples of a minimum amount (12.1)

quantum a discrete amount of energy, given by the product of Planck's constant (h) and the frequency of the radiation (f): hf (12.1)

quark the family of six types of particles with charges of $\frac{1}{3}$ or $\frac{2}{3}$ of the elementary charge, which comprise all hadrons (13.3)

R

radioactive isotope (radioisotope) an isotope of an element that has an unstable nucleus and therefore disintegrates, emitting alpha, beta, or gamma radiation (13.2)

radioactive material material that contains radioactive nuclei (13.2)

radioactivity the spontaneous disintegration of the nuclei of certain elements, accompanied by the emission of alpha, beta, or gamma radiation (13.2)

range the horizontal distance a projectile travels (2.1)

Rayleigh criterion the criterion for resolution of two point sources, which states that the inner dark ring of one diffraction pattern should coincide with the centre of the second bright fringe (9.3)

reaction mass matter ejected backward from a rocket in order to propel it forward (6.3)

recoil the interaction that occurs when two stationary objects push against each other and then move apart (4.2)

relativistic speeds speeds close to the speed of light (11.2)

resolving power the ability of a telescope or microscope to distinguish objects that are close together (9.3)

rest mass the mass of an object measured by an observer at rest relative to the object (11.3)

restoring force the force exerted by a spring on an object; proportional to the amount of extension or compression of the spring (5.2)

Rydberg constant the constant of proportionality that relates the wavelength of a spectral line in the hydrogen atom and the difference of energy level numbers that produce it: $R = 1.09737315 \times 10^{7}\text{m}^{-1}$ (12.3)

S

Schrödinger wave equation the basic quantum mechanical equation used to determine the properties of a particle (13.3)

simultaneity a concept that describes events that occur at the same time and in the same inertial reference frame (11.2)

spin quantum number specifies the orientation, up or down, of the electron's "spin"; has values $+\frac{1}{2}$ or $-\frac{1}{2}$ when placed in a magnetic field (13.3)

spring constant the amount of force a spring can exert per unit distance of extension or compression (5.2)

standard model a comprehensive model that describes subatomic particles, their properties, and the force particles that govern their interactions (13.3)

Stoke's law states that the drag force on a sphere moving through a liquid is proportional to the radius of the sphere and its velocity (8.1)

stopping potential in the photoelectric effect, the potential difference required to stop the emission of photoelectrons from the surface of a metal (12.1)

strong nuclear force the fundamental force that holds the parts of the nucleus together (13.1)

superposition of waves when two or more waves propagate through the same location in a medium, the resultant displacement of the medium will be the algebraic sum of the displacements caused by each wave (9.1)

synchrocyclotron a modified cyclotron, in which the frequency of the accelerating electric field is adjusted to allow for the relativistic mass increase of the particles (8.3)

synchrotron a cyclic particle accelerator that uses a series of magnets around the circular path and several high-frequency accelerating cavities (8.3)

system of particles an arbitrarily assigned group of objects (4.2)

T

tension the magnitude of the force exerted on and by a cable, rope, or string (1.3)

terminal velocity the velocity of a falling object at which the force of friction is equal in magnitude to the force of gravity (1.3)

test charge a charge of a magnitude that is small enough that it will not affect the field being measured; it is used to determine the strength of an electric field (7.2)

threshold frequency the lowest frequency of light (smallest photon energy) that can eject a photoelectron from a particular metal (12.1)

thrust the force with which gases ejected from a rocket push back on the rocket (6.3)

time dilation a consequence of special relativity in which two observers moving at constant velocity relative to each other will each observe the other's clock to have slowed down (11.2)

torsion balance a sensitive instrument for measuring the twisting forces in metal wires, consisting of an arm suspended from a fibre (7.1)

total energy the sum of the rest mass energy of a particle and its kinetic energy (11.3)

total orbital energy the sum of the mechanical (gravitational potential and kinetic) energies of an orbiting body (6.2)

trajectory the path described by an object moving due to a force or forces (2.1)

transmutation the conversion of one element into another, usually as a result of radioactive decay (13.2)

triangulation a geometrical method for determining distances through the measurement of one side and two angles of a right triangle (10.4)

tritium an isotope of hydrogen, consisting of a proton and two neutrons in the nucleus (13.1)

Tychonic system a planetary model in which the Sun and Moon revolve around Earth, but the other planets revolve around the Sun (3.1)

U

ultraviolet catastrophe the significant discrepancy at ultraviolet and higher frequencies between the predictions based on classical physics and observations of blackbody radiation (12.1)

uniform circular motion motion with constant speed in a circle (2.2)

uniform motion motion at a constant velocity (1.2)

uniformly accelerated motion motion under constant acceleration (1.2)

W

W^+, W^-, Z° bosons exchange particles responsible for the behaviour of the weak nuclear force (13.3)

wave function a mathematical expression that is a solution of the Schrödinger wave equation; describes the behaviour of a particle (13.3)

wave-particle duality both matter and radiation have wave-like properties and particle-like properties (12.2)

weak nuclear force a short-range interaction between elementary particles that is much weaker than the strong nuclear force and governs the process of beta decay; one of the four fundamental forces (13.3)

work the transfer of mechanical energy from one system to another; equivalent to a force acting through a distance (5.1)

work function in the photoelectric effect, the minimum amount of energy necessary to remove an electron from a metal surface (12.1)

work-kinetic energy theorem the relationship between the work done by a force on an object and the resulting change in kinetic energy: $W = \Delta E_k$ (5.1)

work-energy theorem the relationship between the work done on an object by a force and the resulting change in the object's potential and kinetic energy: $W = \Delta E_k + \Delta E_p$ (5.1)

Z

Zeeman effect the splitting of the spectral lines of an atom when it is placed in a magnetic field (12.3)

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The page numbers in **boldface** type indicate the pages where terms are defined. Terms that occur in investigations (*inv*), Sample Problems (*SP*), MultiLabs (*ML*), and QuickLabs (*QL*), are also indicated.

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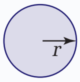
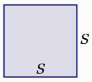
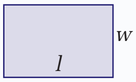
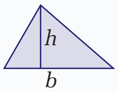
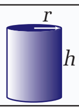
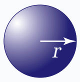
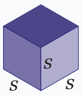
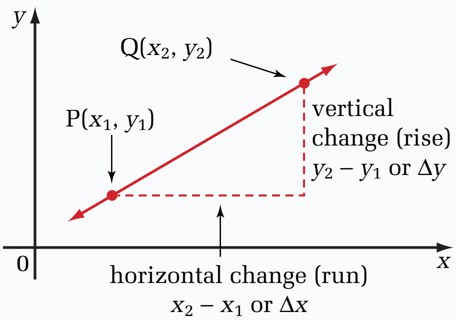
	Circumference/ perimeter	Area	Surface area	Volume
	$C = 2\pi r$	$A = \pi r^2$		
	$P = 4s$	$A = s^2$		
	$P = 2l + 2w$	$A = lw$		
		$A = \frac{1}{2}bh$		
			$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
			$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
			$SA = 6s^2$	$V = s^3$

Slope (m)

Calculating the slope of a line

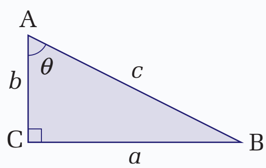


$$\text{slope } (m) = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}}$$

$$m = \Delta y / \Delta x$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$$

Trigonometric Ratios



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

The Greek Alphabet

alpha	Α	α	iota	Ι	ι	rho	Ρ	ρ
beta	Β	β	kappa	Κ	κ	sigma	Σ	σ
gamma	Γ	γ	lambda	Λ	λ	tau	Τ	τ
delta	Δ	δ	mu	Μ	μ	upsilon	Υ	υ
epsilon	Ε	ε	nu	Ν	ν	phi	Φ	ϕ
zeta	Ζ	ζ	xi	Ξ	ξ	chi	Χ	χ
eta	Η	η	omicron	Ο	o	psi	Ψ	ψ
theta	Θ	θ	pi	Π	π	omega	Ω	ω

Fundamental Physical Constants

Quantity	Symbol	Accepted value
speed of light in a vacuum	c	2.998×10^{8} m/s
gravitational constant	G	6.673×10^{-11} N · m²/kg²
Coulomb's constant	k	8.988×10^{9} N · m²/C²
charge on an electron	e	1.602×10^{-19} C
rest mass of an electron	m_e	9.109×10^{-31} kg
rest mass of a proton	m_p	1.673×10^{-27} kg
rest mass of a neutron	m_n	1.675×10^{-27} kg
atomic mass unit	u	1.661×10^{-27} kg
Planck's constant	h	6.626×10^{-34} J · s

Metric System Prefixes

Prefix	Symbol	Factor
tera	T	1 000 000 000 000 = 10^{12}
giga	G	1 000 000 000 = 10^{9}
mega	M	1 000 000 = 10^{6}
kilo	k	1000 = 10^{3}
hecto	h	100 = 10^{2}
deca	da	10 = 10^{1}
		1 = 10^{0}
deci	d	0.1 = 10^{-1}
centi	c	0.01 = 10^{-2}
milli	m	0.001 = 10^{-3}
micro	μ	0.000 001 = 10^{-6}
nano	n	0.000 000 001 = 10^{-9}
pico	p	0.000 000 000 001 = 10^{-12}
femto	f	0.000 000 000 000 001 = 10^{-15}
atto	a	0.000 000 000 000 000 001 = 10^{-18}

Other Physical Data

Quantity	Symbol	Accepted value
standard atmospheric pressure	P	1.013×10^{5} Pa
speed of sound in air		343 m/s (at 20°C)
water: density (4°C)		1.000×10^{3} kg/m³
latent heat of fusion		3.34×10^{5} J/kg
latent heat of vaporization		2.26×10^{6} J/kg
specific heat capacity (15°C)		4186 J/(kg°C)
kilowatt hour	E	3.6×10^{6} J
acceleration due to Earth's gravity	g	9.81 m/s² (standard value; at sea level)
mass of Earth	m_E	5.98×10^{24} kg
mean radius of Earth	r_E	6.38×10^{6} m
mean radius of Earth's orbit	R_E	1.49×10^{11} m
period of Earth's orbit	T_E	365.25 days or 3.16×10^{7} s
mass of Moon	m_M	7.36×10^{22} kg
mean radius of Moon	r_M	1.74×10^{6} m
mean radius of Moon's orbit	R_M	3.84×10^{8} m
period of Moon's orbit	T_M	27.3 days or 2.36×10^{6} s
mass of Sun	m_s	1.99×10^{30} kg
radius of Sun	r_s	6.96×10^{8} m

Derived Units

Quantity	Quantity symbol	Unit	Unit symbol	Equivalent unit(s)
area	A	square metre	m^2	
volume	V	cubic metre	m^3	
velocity	v	metre per second	m/s	
acceleration	a	metre per second per second	m/s^2	
force	F	newton	N	$kg \cdot m/s^2$
work	W	joule	J	$N \cdot m$, $kg \cdot m^2/s^2$
energy	E	joule	J	$N \cdot m$, $kg \cdot m^2/s^2$
power	P	watt	W	J/s, $kg \cdot m^2/s^3$
density	ρ	kilogram per cubic metre	kg/m^3	
pressure	p	pascal	Pa	N/m^2, kg/s^2
frequency	f	hertz	Hz	s^{-1}
period	T	second	s	
wavelength	λ	metre	m	
electric charge	Q	coulomb	C	$A \cdot s$
electric potential difference	V	volt	V	W/A, J/C, $kg \cdot m^2/(C \cdot s^2)$
resistance	R	ohm	Ω	V/A, $kg \cdot m^2/(C^2 \cdot s)$
magnetic field strength	B	tesla	T	$N \cdot s/(C \cdot m)$, $N/A \cdot m$
magnetic flux	Φ	weber	Wb	$V \cdot s$, $T \cdot m^2$, $m^2 \cdot kg/(C \cdot s)$
radioactivity	$\Delta N/\Delta t$	becquerel	Bq	s^{-1}
radiation dose		gray	Gy	$J/kg \cdot m^2/s^2$
radiation dose equivalent		sievert	Sv	$J/kg \cdot m^2/s^2$
temperature (Celsius)	T	degree Celsius	°C	T°C = (T + 273.15) K
		atomic mass unit	u	$1u = 1.660\ 566 \times 10^{-27}$ kg
		electron volt	eV	$1\ eV = 1.602 \times 10^{-19}$ J

Electromagnetic Spectrum